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# APPARENT FINE TUNINGS FROM BROKEN SPACETIME SYMMETRIES

w/ Rothstein, 2022

( also w/ : Kourkoulou & Sun, 2021

Podo & Santoni, 2023 )

## Naturalness problems in particle physics / gravity

1) E.W. hierarchy problem :

$m_h^2 \ll \Lambda^2$ $\quad\quad$ $\Lambda$ = more fundamental scale

2) C.C. problem :

$\langle T_{\mu\nu} \rangle \ll (\mathrm{TeV})^4$

that couples to gravity

<u>NOT</u> enforced by symmetry-based selection rules. $\Longrightarrow$ problem (?)

# This talk

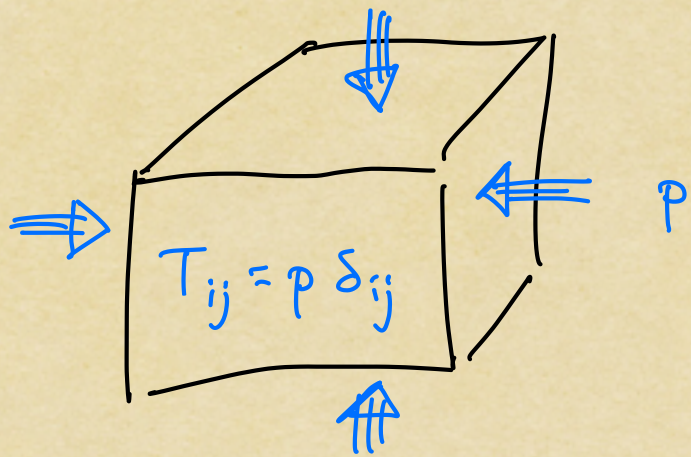
CLAIM: Things like these can happen naturally, at least when some spacetime symmetries are spontaneously broken.

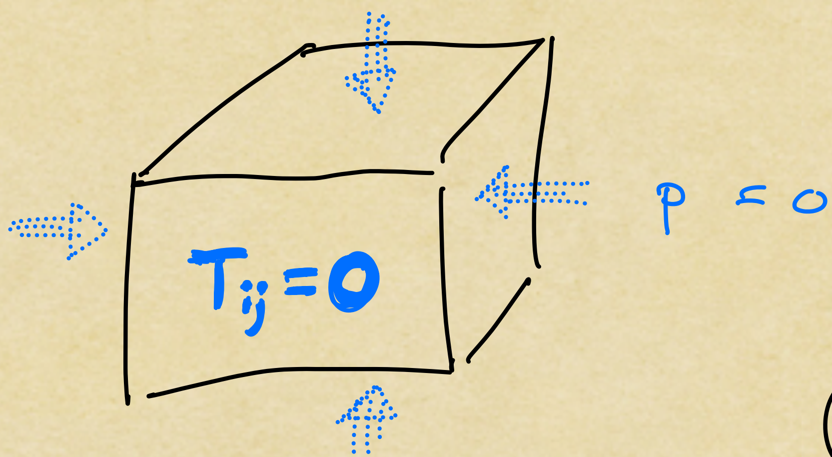
PROOF: By example, of real systems.

in NATURE, unaided (undisturbed?) by humans.

# Experimental Data

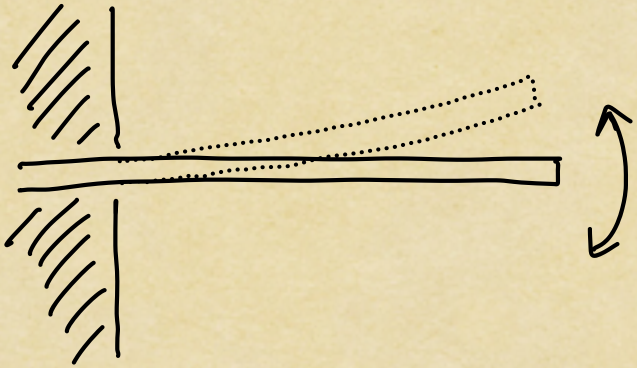
1) <u>Any</u> solid :




(up to surface tension effects)

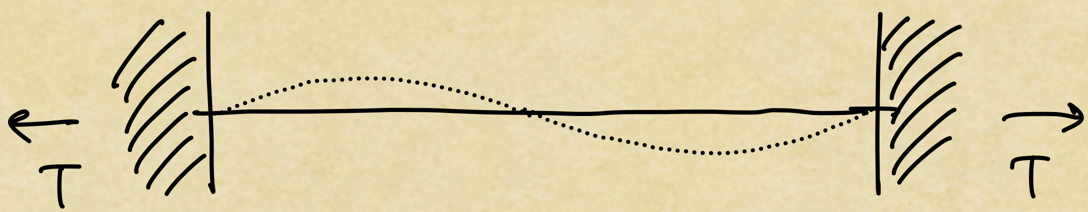
2) Any "bar, beam, rod" w/ free bdy. conditions :



$$\omega \propto k^2$$

$$\Big\uparrow \; T \to 0$$

as opposed to "string" w/ tension :

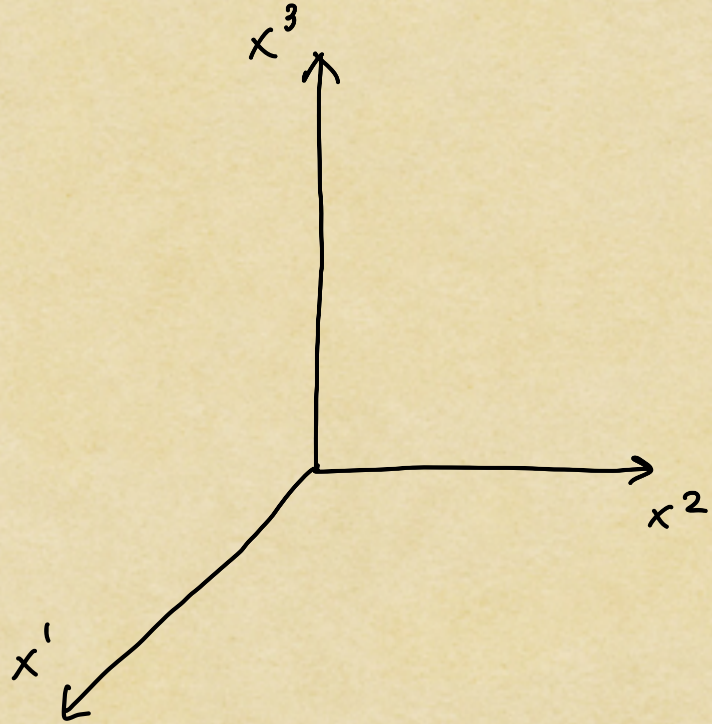


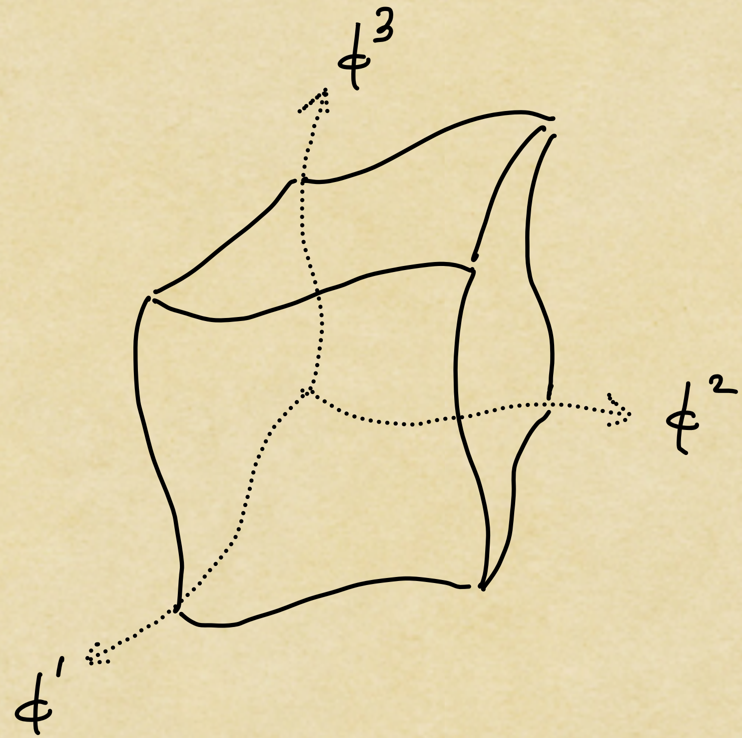

$$\omega \propto \sqrt{T}\, k$$

well known to Euler, Bernoulli, ... Freivogel (thanks!)

QFT / EFT understanding ?

Bulk dynamics of 3D solid :



- d.o.f. : $\phi^I(\vec{x},t)$ , 3 scalar fields

- equilibrium configuration (ground state):

$$\langle \phi^I(x) \rangle = \alpha \, x^I \qquad (*)$$

$\uparrow$ compression/dilation level

(can also rotate, shear, boost)

- Symmetries: Poincaré + ?

$\uparrow$ broken by $(*)$ $\Longrightarrow$ need $\begin{cases} \phi^I \to \phi^I + a^I \\ \phi^I \to H \cdot \phi^I \end{cases}$

• Action: $$S = \int G(B^{IJ})\, d^4x + \text{higher } \partial\text{'s}$$

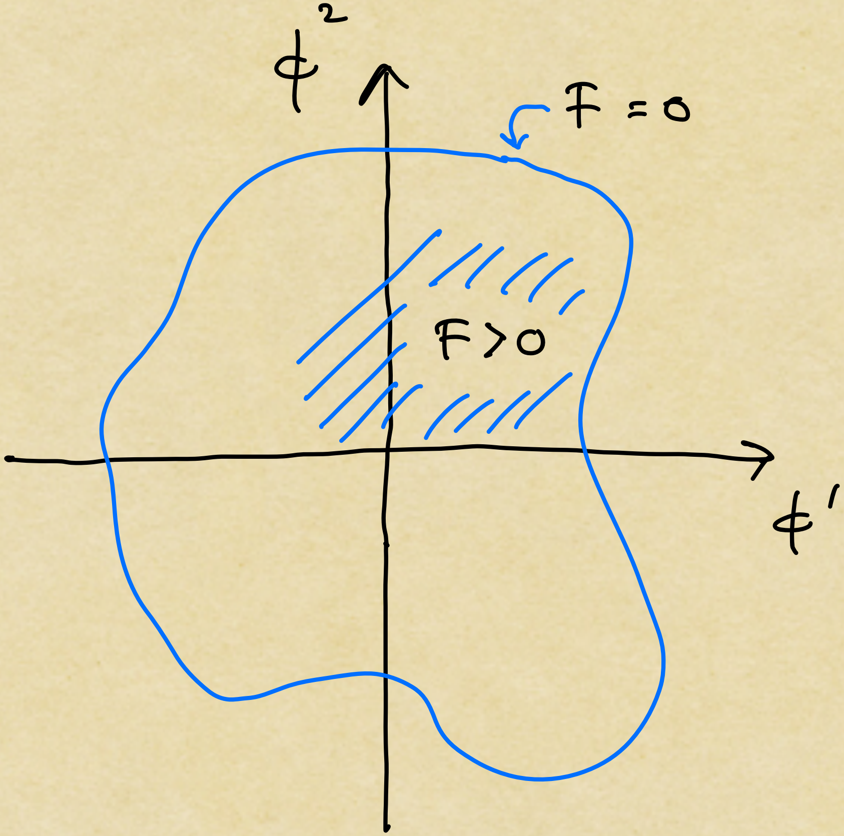
$$B^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J$$

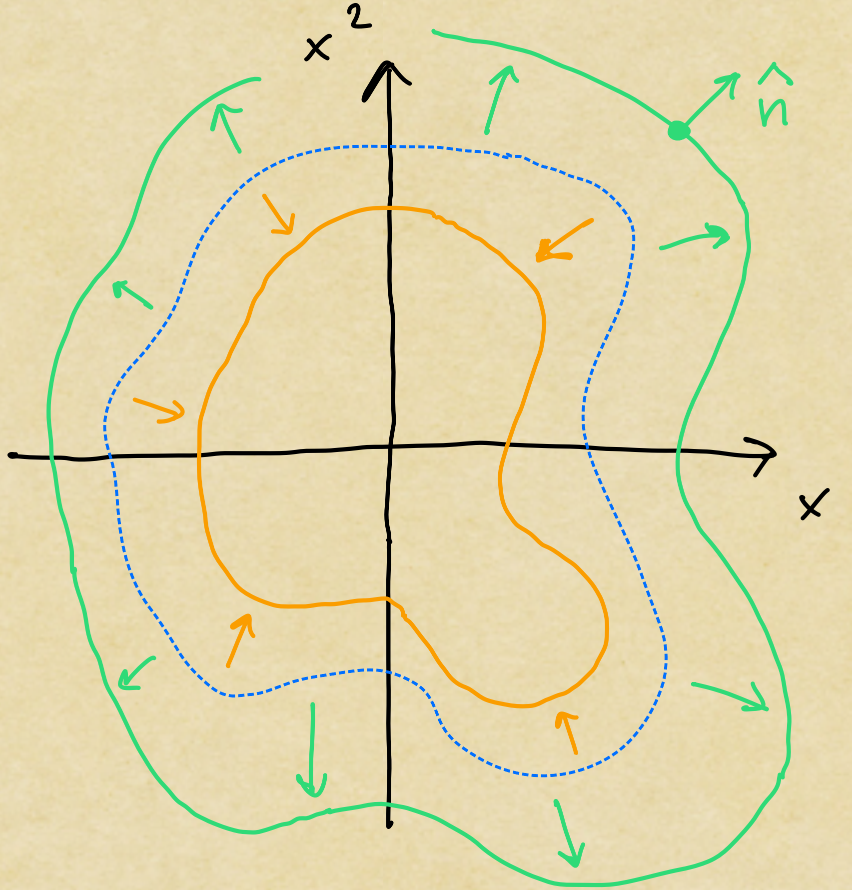
$G$ : invariant under $H$, related to eq. of state.

• Phonons: $$\phi^I(x) = \alpha\left( x^I + \pi^I(x)\right)$$

$$S \to \int d^4x \left[ \frac{1}{2} \dot{\vec{\pi}}^2 - \frac{1}{2} c_T^2 (\nabla \times \pi)^2 - \frac{1}{2} c_L^2 (\nabla \cdot \pi)^2 + \dots \right]$$

- Boundary? Boundary is fixed in comoving space :




$$\Longrightarrow \qquad S = \int d^4x \;\; \theta\big(F(\phi^I)\big)\; G\big(B^{IJ}\big)$$

- EOM:

$$\partial_\mu \left( \theta(F(\phi)) \frac{\partial G}{\partial B^{IJ}} \partial^\mu \phi^J \right) = 0$$

$$\Longrightarrow \begin{cases} \text{Bulk:} \quad T_{ij} = \text{const} \quad \Longrightarrow \quad \phi^I = \alpha x^I \ \checkmark \\ \text{Bdy:} \quad T_{ij} \hat{n}^j = 0 \end{cases}$$

$\Longrightarrow$ $T_{ij} = 0$ everywhere (fixes $\alpha$)

**NOT** enforced by symmetry

Equivalently :

$$S = \int d^4x \; \vartheta\big(F(\phi)\big)\, G\big(B^{IJ}\big)$$

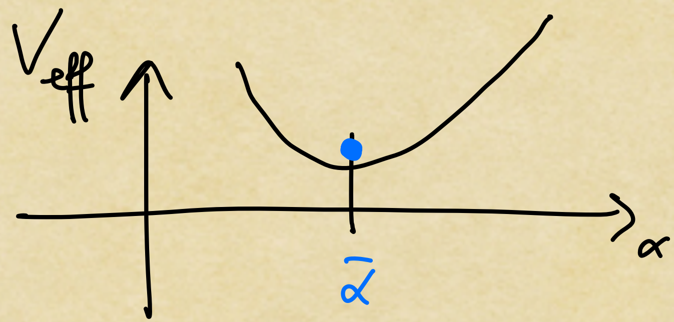
$$\longrightarrow \quad V_{eff}(\alpha) = -\,\mathcal{V} \cdot G(\alpha)/\alpha^3$$

$\uparrow$ $\phi^I = \alpha\, x^I$

$\uparrow$ comoving volume ( = const )

$$T_{ij} = 0$$

$$V'_{eff}(\alpha) = 0 \iff T_{ij} = 0 \iff 3G - G'\cdot\alpha = 0$$



• Phonons (again) : $\phi^I(x) = \alpha\,(x^I + \pi^I(x))$

$$B^{IJ} \simeq \alpha^2 \left( \delta^{IJ} + \partial\pi + \partial\pi\,\partial\pi \right)$$

$$S \to \int d^4x \; \Theta\left( \bar{F} + \frac{\overline{\partial F}}{\partial \phi^I} \alpha \pi^I + \dots \right) \cdot$$

$$\left[ \bar{G} + \frac{\overline{\partial G}}{\partial B^{IJ}} \alpha^2 (\partial\pi + \partial\pi\,\partial\pi) + \dots \right]$$

Does the relaxation condition set some coefficients to zero? Only at the boundary.

$$S = S_{bulk} + S_{bdy}$$

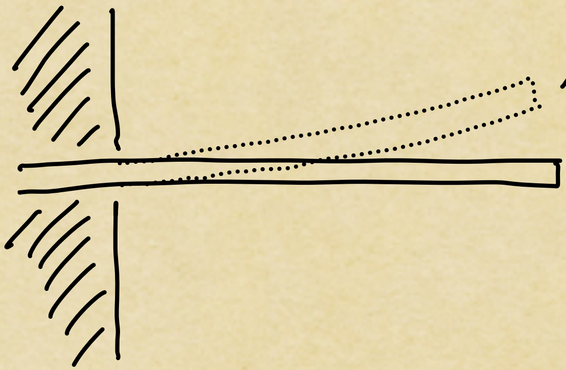
$$S_{bulk} = \int d^4x \left[ \frac{1}{2}\dot{\vec{\pi}}^2 - \frac{1}{2} c_T^2 (\nabla \times \pi)^2 - \frac{1}{2} c_L^2 (\nabla \cdot \pi)^2 + \dots \right]$$

$$S_{bdy} = \frac{1}{2} c_T^2 \int dt \, d\vec{\Sigma} \cdot \left( \vec{\pi} \times (\vec{\nabla} \times \vec{\pi}) \right)$$

Not immediately obvious what the constraint is.

## 3D solid $\longrightarrow$ 1D bar

Bar = (1+1) solid embedded in (3+1)



$X^\mu(\sigma, \tau)$ , $\phi(\sigma, \tau)$

embedding coordinates

solid comoving coordinate

$$S = \int d\tau\, d\tau\; \vartheta(\phi - \phi^*) \sqrt{g_{ind}}\; G(B)$$

$$g^{ind}_{\alpha\beta} = \eta_{\mu\nu}\, \partial_\alpha X^\mu\, \partial_\beta X^\nu \qquad B = g^{\alpha\beta}_{ind}\, \partial_\alpha \phi\, \partial_\beta \phi$$

Ground state:

$$\left.\begin{aligned} X^0(\sigma,\tau) &= \tau \\ X^1(\sigma,\tau) &= \sigma \end{aligned}\right\} \text{ U.G.}$$

$$X^2 = X^3 = 0$$

$$\phi(\sigma,\tau) = \alpha \cdot \sigma$$

$$V_{eff}(\alpha) = -L\,\frac{G(\alpha)}{\alpha} \implies \text{relaxes to } \bar{\alpha}$$

$$\Updownarrow$$

$$T_{\sigma\sigma} = 0$$

Perturbations:

$$\phi(\sigma,\tau) = \bar{\alpha}\left(\sigma + \varphi(\sigma,\tau)\right)$$

$$X^{2,3}(\sigma,\tau) = 0 + \pi^{2,3}(\sigma,\tau)$$

$$X^\mu \longrightarrow g^{\text{ind}}_{\alpha\beta} = \eta_{\alpha\beta} + \partial_\alpha \vec{\pi}\cdot\partial_\beta\vec{\pi}$$

enforced by symmetries

$$S \supset \int \frac{\delta S}{\delta g^{\text{ind}}_{\alpha\beta}}\,\delta g^{\text{ind}}_{\alpha\beta} = \int T^{\alpha\beta}\,\partial_\alpha\vec{\pi}\cdot\partial_\beta\vec{\pi}$$

$$\supset \int T^{\sigma\sigma}\,\partial_\sigma\vec{\pi}\cdot\partial_\sigma\vec{\pi} = 0 \qquad \text{at } \alpha = \bar{\alpha}\,.$$

NOT enforced by symmetries

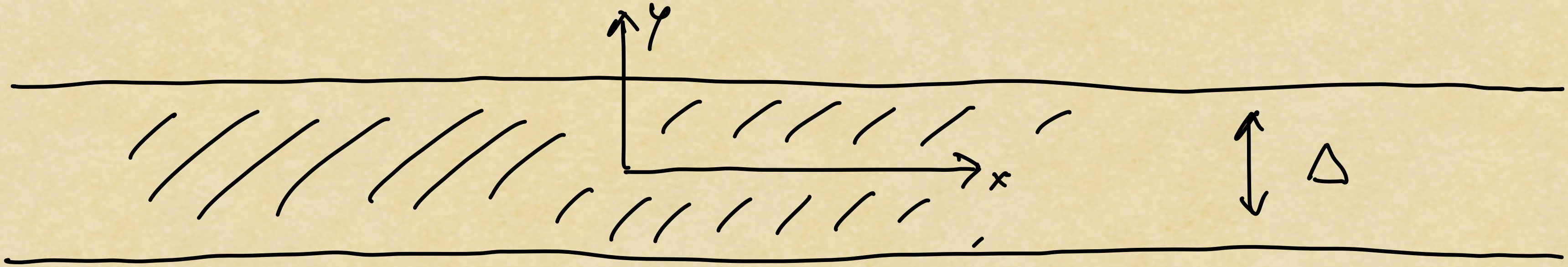
$$S_\pi = \int d\sigma \, d\tau \left[ \frac{1}{2} \dot{\vec{\pi}}^{\,2} - \cancelto{0}{\frac{1}{2} T \left(\partial_\sigma \vec{\pi}\right)^2} \right.$$

$$\left. + \text{ higher } \partial\text{'s } \left( k^2, R, \ldots \right) \right]$$

$$\Longrightarrow \quad \frac{1}{2} \dot{\vec{\pi}}^{\,2} - \frac{1}{2} \left( \partial_\sigma^2 \vec{\pi} \right)^2$$

$$\Longrightarrow \quad \omega^2 = k^4$$

# Dimensional reduction 2D → 1D




$$\phi^I(\vec{x},t) = \tilde{\alpha}\left(x^I + \pi^I(\vec{x},t)\right) \qquad I = 1,2$$

Static Green's function:

$$G^{ij}(x) = \int dt\, dy\, dy' \langle \pi^i(x,y,t)\, \pi^j(0,y',0) \rangle$$

Classical RG flow:

$$G^{11}(k) \sim \frac{1}{c_L^2 k^2} \xrightarrow[k \cdot \Delta \to 0]{} \frac{1}{\bar{c}_L^2 k^2}$$

$$\left( \bar{c}_L^2 = f(c_L^2, c_T^2) \right)$$

$$G^{22}(k) \sim \frac{1}{c_T^2 k^2} \longrightarrow \frac{1}{\beta k^4}$$

$$\left( \beta = \frac{\bar{c}_L^2}{c_L^2} \cdot \Delta^2 \right)$$

# Surface Tension

Back to 3D solid:

$$S = S_{bulk} + S_{bdy}$$

$$S_{bulk} = \int \Theta(F(\phi)) \left[ G(B^{IJ}) + \partial\text{'s} \right]$$

$$S_{bdy} = \ell \int \delta(F(\phi)) \, |\partial F| \, \underbrace{\mathcal{L}_{bdy}(\partial\phi, \phi)}_{\sim \rho_m c_s^2}$$

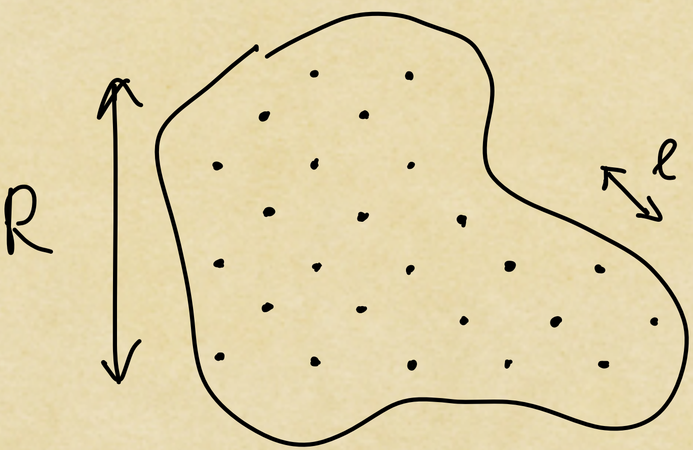
comoving UV length scale

$$+ \ell^2 \delta' + \ell^3 \delta'' + \ldots$$

Bdy. EOM modified :

$$T_{ij} = \mathcal{O}\left(\ell/R\right) \rho_m c_s^2 \ll \rho_m c_s^2$$

comoving curvature
radius of solid




Why are solids large?

## Potential application to CC problem

$$\phi^I = \alpha x^I \longrightarrow T_{ij} = 0$$

$$\phi = \beta t \longrightarrow T_{00} = 0 \quad ?$$

$$S = \int d^4x \, \vartheta\left(F(\phi, \phi^I)\right) \, G(\partial\phi, \partial\phi^I)$$

"supersolid", bounded in space <u>and time</u>.

EOM $\Longrightarrow$ $T_{\mu\nu} = 0$ everywhere, but
NO relaxation : initial conditions have to be right.

# Conclusions

- Explicit examples of apparent fine-tunings <u>in Nature</u>
- More examples, apparently unrelated:

    1) scalars at finite density

    2) Vacuum energy in "framids".

- General lesson?
- Applications for hierarchy and CC problems?