Bound on equilibration time from hydrodynamic fluctuations

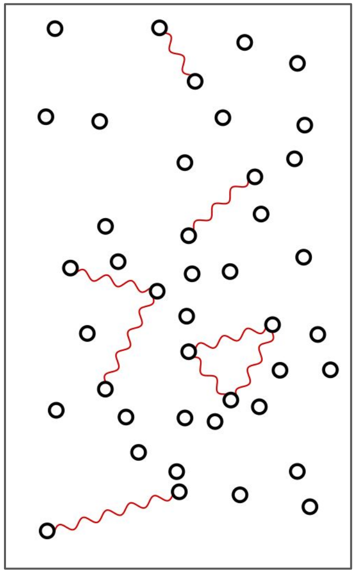
Luca Delacrétaz | U Chicago

HoloTube Seminar
23 January 2024

Interacting systems thermalize

This talk will be about the local equilibration time $\tau_{\rm eq}$

Time-scale for a system to reach local thermal equilibrium, before the (much slower) global equilibration



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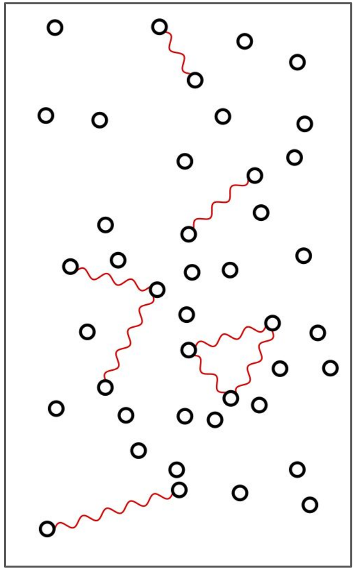
At weak coupling $g \ll 1$ slow thermalization $\tau_{\mathrm{eq}} \propto \dfrac{1}{g^2}$



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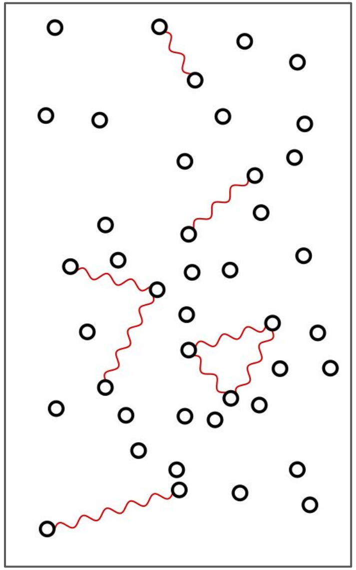
Conjectured "Planckian bound" on thermalization:

$$\tau_{\text{eq}} \gtrsim \frac{\hbar}{T}$$

Sachdev '99, Zaanen '04,
(Review: Hartnoll Mackenzie '21)

Possible relevance for strange metals $\rho_{\text{dc}} \sim T \overset{?}{\sim} \frac{1}{\tau_{\text{eq}}}$



Does there exist a quantum statistical mechanics bound $\tau_{\mathrm{eq}} \gtrsim \frac{\hbar}{T}$ on all many-body systems, where τ_{eq} can be universally defined?

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Yes

Thermalization vs. Chaos

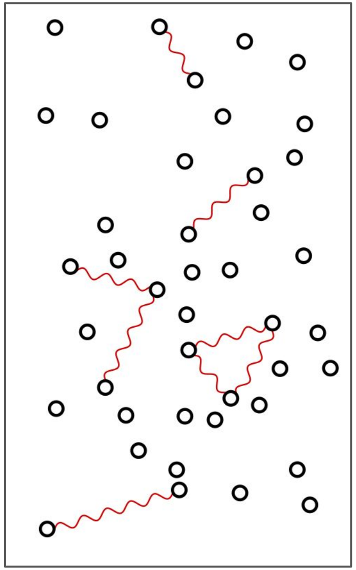
$1/\tau_{\mathrm{eq}}$ has similarities with Lyapunov exponent(s)

$$\langle A(t)B(0)A(t)B(0)\rangle \sim 1 - \frac{1}{N^2}e^{\lambda_L t} + \cdots$$

Both are small at weak coupling, and bounded at strong coupling [Maldacena Shenker Stanford '15]

But τ_{eq} well-defined in general (no need for semiclassical limit) and is more experimentally relevant

It has also proven more difficult to constrain



I. Defining the local equilibration time

"Time scale where hydrodynamics/diffusion emerges"

II. Universal bound from hydrodynamic fluctuations

Hydrodynamics "knows" about its own breakdown

III. Precision tests of thermalization in classical/quantum numerics

Fil rouge:
Effective field theory (EFT) for thermalizing systems (diffusion, hydrodynamics)

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LVD [2310.16948]

III. Precision tests of thermalization in classical/quantum numerics

Ruchira Mishra [2304.03236]
Michailidis, Abanin [2310.10564]
...

Fil rouge:
Effective field theory (EFT) for thermalizing systems (diffusion, hydrodynamics)

Part I:

Defining the local equlibration time

Universal definition of $\tau_{\rm eq}$

For weakly coupled particles, one could define $\tau_{\rm eq} \equiv \tau_{\rm scattering}$

What about in general? Tempting to look for $\langle \mathcal{O}(t)\mathcal{O}\rangle \sim e^{-t/\tau_{\rm eq}}$

But exponential decay does not generically occur, due to hydrodynamics

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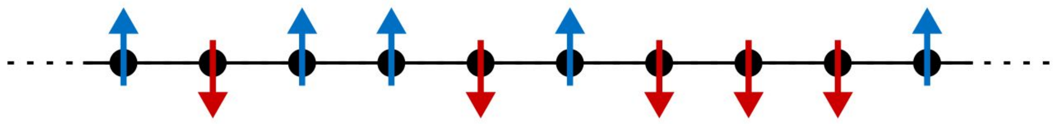

$$H = \sum_i Z_i Z_{i+1} + gX_i + g' Z_i$$

with $g, g' \sim 1$

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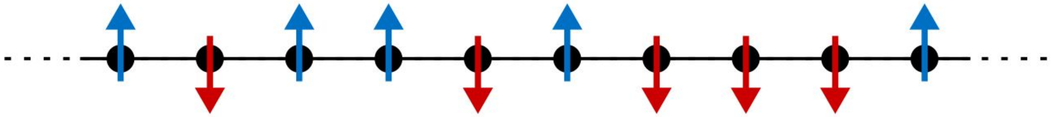

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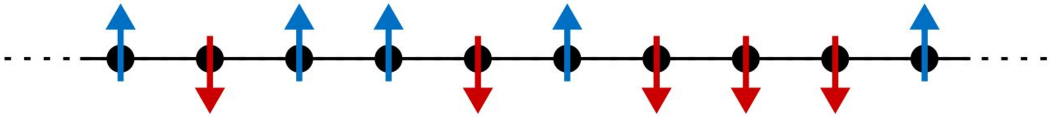

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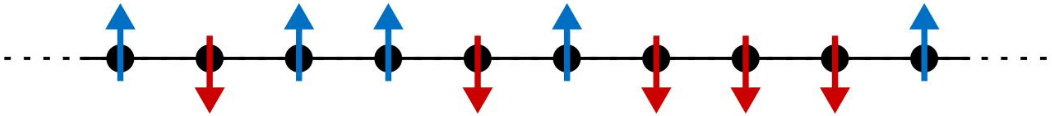

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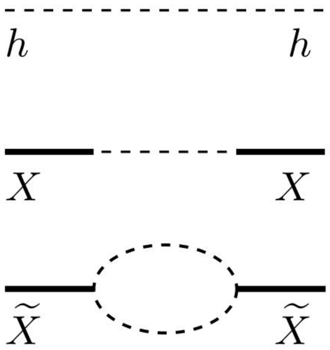
$$\widetilde{X}_i \equiv X_i - \# h_i \quad \rightsquigarrow \quad \langle \widetilde{X}_i(t) \widetilde{X}_i \rangle \sim t^{-1}$$

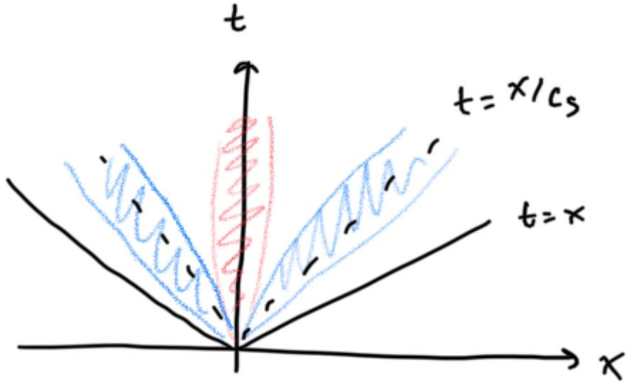


Universal definition of τ_{eq}

Exponential decay does not generically occur, due to hydrodynamics. E.g:

$$S = -\int d^{d+1}x \, (\partial\phi)^2 + m^2\phi^2 + \lambda\phi^3 + \lambda'\phi^4$$

$$\langle T_{\mu\nu}(t,x) T_{\rho\sigma} \rangle \sim \frac{1}{t^{d/2}} e^{-\frac{(x-c_s t)^2}{2\gamma t}} + \frac{1}{t^{d/2}} e^{-\frac{x^2}{4Dt}}$$



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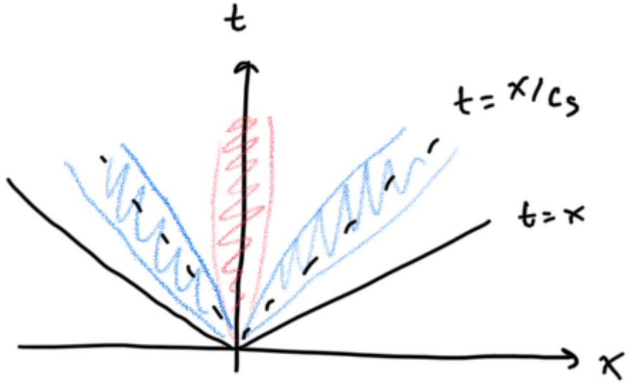

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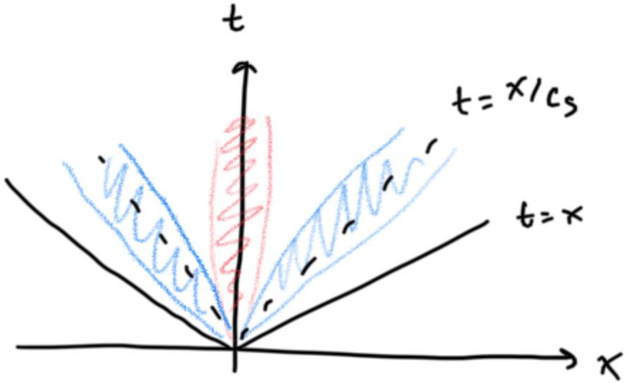

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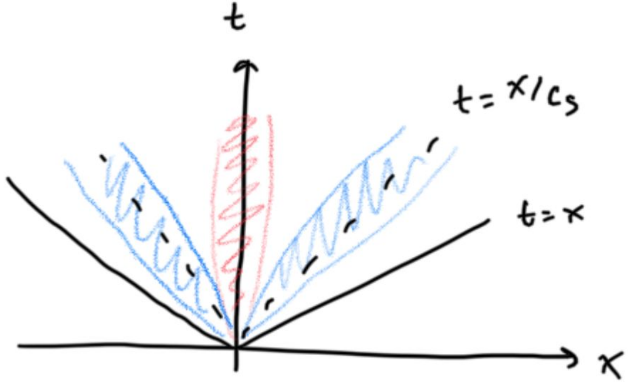

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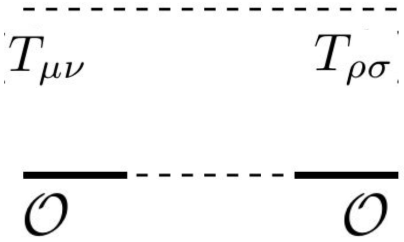

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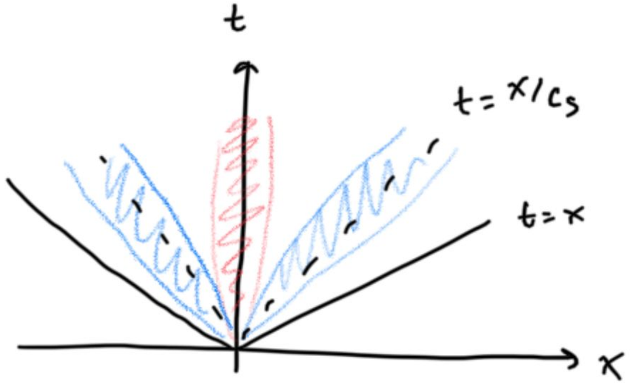

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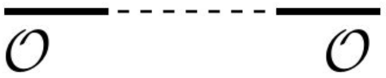

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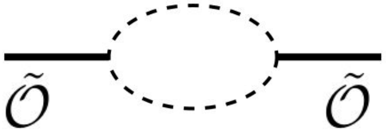


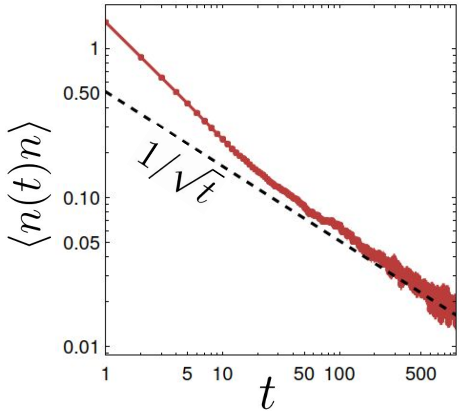
Universal definition of τ_{eq}

We know what systems thermalize to: hydrodynamics (which then controls global equilibration)

Define τ_{eq} (loosely) as the timescale at which hydrodynamics emerges

In the simplest example with a single conservation law, diffusion

$$\langle n(x=0,t)n(0,0)\rangle = \frac{\chi T}{(4\pi D t)^{d/2}} + \cdots$$



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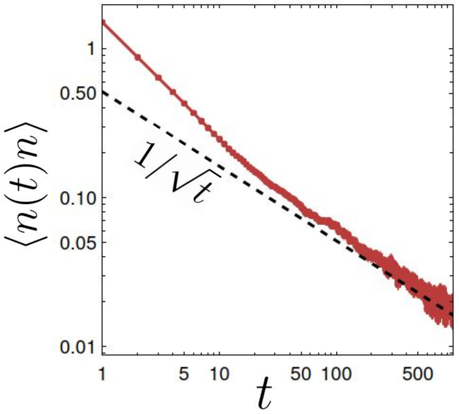

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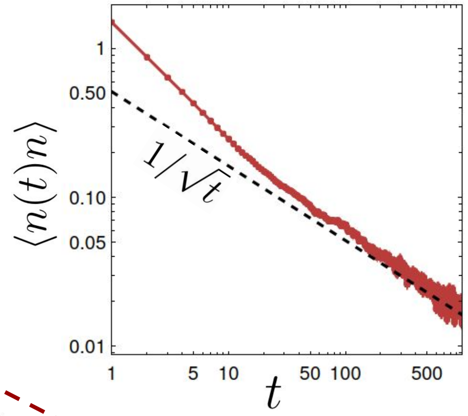

Higher derivative terms, e.g.

$$j_i = -D\nabla_i n + a_{0,1}\nabla^2\nabla_i n + \cdots$$

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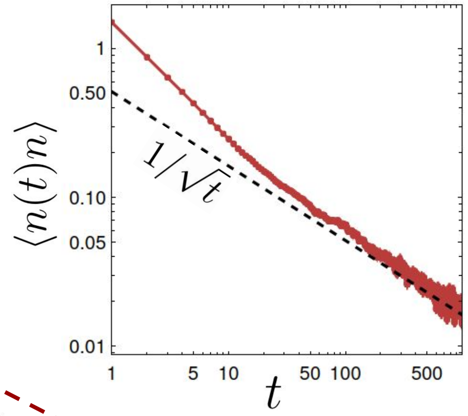
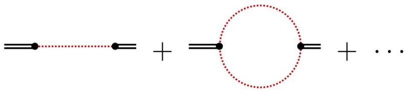


Loops $j \supset D' n \nabla n$



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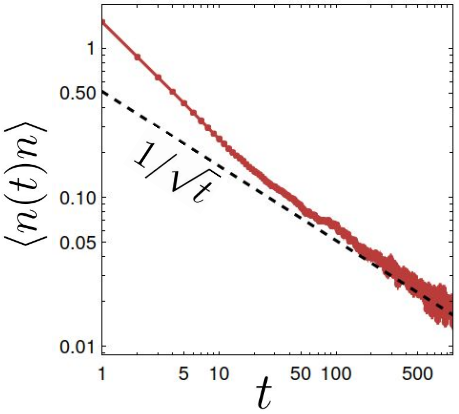
Universal definition of τ_{eq}

$a_{\ell,m}$ is an ℓ-loop, $O(\nabla^{2m})$ higher-derivative correction to diffusion

Each such coefficient defines a time-scale, providing sharp universal definitions for τ_{eq}

$$\tau_{\text{eq}} \equiv \max\{(a_{\ell,n}/a_{0,0})^{1/(n+\frac{1}{2}d\ell)}\}$$

$$\begin{aligned}\langle n(x=0,t)n\rangle = \frac{1}{t^{d/2}}\Big\{ & \quad a_{0,0} + \frac{a_{0,1}}{t} + \frac{a_{0,2}}{t^2} + \cdots \\ & + \frac{1}{t^{d/2}}\Big(a_{1,0} + \frac{a_{1,1}}{t} + \frac{a_{1,2}}{t^2} + \cdots \Big) \\ & + \frac{\log t}{t^d}\Big(a_{2,0} + \frac{a_{2,1}}{t} + \frac{a_{2,2}}{t^2} + \cdots \Big) \\ & + \cdots \Big\}\end{aligned}$$



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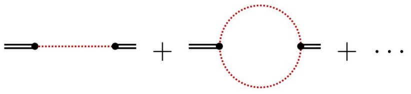

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Corrections to hydrodynamics

What do we know about these corrections?

Fluctuating hydro:
Alder Wainwright ‘70
Martin Siggia Rose ‘75
Forster Nelson Stephen ‘77
Andreev ‘78, Spohn ‘91, …

Quantum dissipation:
Caldeira Leggett ‘83
Leggett Chakravarty Dorsey Fisher Garg
Zwerger ‘87
…

Macroscopic fluct. theory:
Bertini De Sole Gabrielli
Jona-Lasinio Landim ‘01
Doyon Perfetto Sasamoto
Yoshimura ‘22

Symmetry-based EFTs:
Haehl Loganayagam Rangamani ‘15
Crossley Glorioso Liu ‘15, Chen-Lin LVD
Hartnoll ‘18, Jensen Marjieh Pinzani-Fokeeva
Yarom ‘18, Akyuz Goon Penco ‘23

Precision tests of thermalization:
Glorioso LVD Chen Nandkishore Lucas ‘20
LVD Mishra ‘23
Michalidis Abanin LVD ‘23

$$\langle n(x=0,t)n\rangle = \frac{1}{t^{d/2}}\left(1+\frac{a_{0,1}}{t}+\frac{a_{0,2}}{t^2}+\cdots\right) + \frac{1}{t^d}\left(a_{1,0}+\frac{a_{1,1}}{t}+\frac{a_{1,2}}{t^2}+\cdots\right) + \cdots$$

Corrections to hydrodynamics

What do we know about these corrections?

Higher-derivative corrections $a_{0,m}$ are independent non-universal parameters (“Wilsonian coefficients”) → hard to constrain universally

However, most loop corrections $a_{\ell>0,m}$ are universally fixed by the EFT

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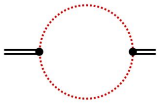
Consider 1-loop correction $a_{1,0}$

- ➢ Leading correction in $d = 1, 2$
- ➢ $a_{1,0} = \dfrac{\chi D'^2}{4\sqrt{\pi D} D^2}$

$$D' \equiv \frac{dD(n)}{dn}$$



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Bound on thermalization

At times when the 1-loop correction is large, one does not observe standard hydro! Hydrodynamics therefore can only emerge at times

$$\tau_{\rm eq} \;\geq\; \frac{(T\chi)^{2/d}}{4\pi D}\left(\frac{D'}{D}\right)^{4/d}, \qquad D' \equiv \frac{dD(n)}{dn}$$

This is a classical bound (no $\hbar$).

But this bound has quantum implications, e.g. if one knows about $D(n)$.

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Example: diffusion in a CFT $d > 1$

Scale invariance → $s = s_o T^{d+1}, \quad D = D_o/T$

(e.g.: 3d Ising, $s_o \approx 0.455$, High-T QGP: $\frac{\pi^2}{45}\left(4(N_c^2-1)+7N_cN_f\right)$)

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Can be read as lower bound on diffusivity or viscosity: $\frac{\eta}{s} \geq \frac{1}{(s_o)^{d/2} T\tau_{\rm eq}}$

Long sought after! [Kovtun Son Starinets '06, Hartnoll '14]

Similar in spirit to previous attempts [Kovtun Moore Romatschke '11, Chafin Schäfer '12, Kovtun '14], which relied on positivity of analytic corrections (these turn out not to be positive definite [Chen-Lin LVD Hartnoll '18]).

Bound on thermalization

$$\tau_{\rm eq} \geq \frac{(T\chi)^{2/d}}{4\pi D}\left(\frac{D'}{D}\right)^{4/d}, \qquad D' \equiv \frac{dD(n)}{dn}$$

Example: diffusion in a CFT $d > 1$

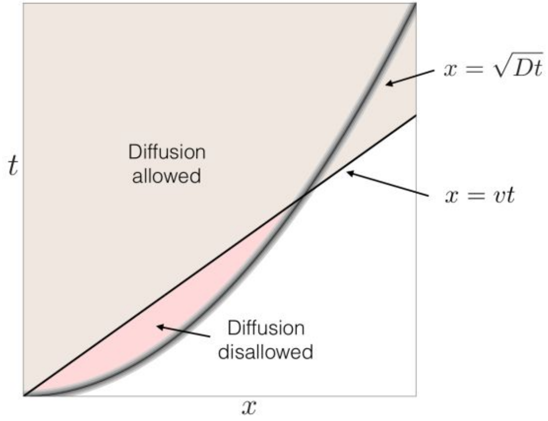
Fluctuation bound: $$T\tau_{\rm eq} \geq \frac{1}{D_o}\frac{1}{(s_o)^{2/d}}$$

CFTs also have a sharp lightcone:
diffusion cannot emerge too soon or it would be outside

[Hartman Hartnoll Mahajan '17] $$T\tau_{\rm eq} \gtrsim D_o$$

Combining the two: $$T\tau_{\rm eq} \gtrsim \frac{1}{(s_o)^{1/d}}$$

→ Planckian bound!



Bound on thermalization

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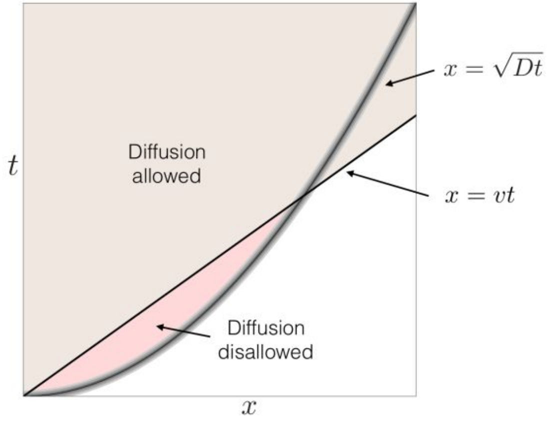


Sharp causality bounds possible at large N

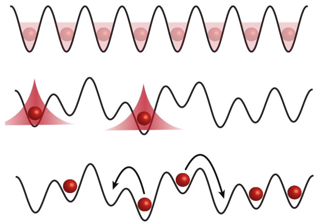
[Heller Serantes Spalinski Withers '22]

Part III:

Precision tests of thermalization

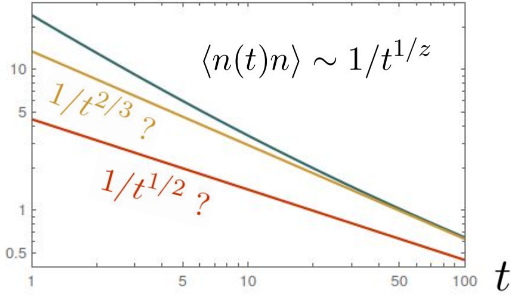
Thermalization in quantum many-body systems

[Abanin Altman Bloch Serbyn '18]



- Alternatives to thermalization in quantum many-body systems? (MBL, glassiness, scars…)

- Even if system thermalizes, identifying dissipative universality class with limited ressourses can be challenging



- "Thermalization" comes with much more predictive power beyond decay of autocorrelation function $\langle n(t)n \rangle \sim 1/t^{1/z}$
  - Decay of arbitrary microscopic operators [LVD '20]
  - Nonlinear response [with **Ruchira Mishra** '23]
  - Universal corrections [with **Alex Michailidis** and **Dima Abanin** '23]

EFT for fluctuating hydrodynamics

The generating functional of a system with a conservation law

$$Z[A^1_\mu, A^2_\mu] \equiv \mathrm{Tr}\left(U[A^1]\rho_\beta U^\dagger[A^2]\right)$$

with time evolution operator

$$U[A] = \mathcal{T}\exp\left\{-i\int_{-\infty}^{\infty} dt\left(H - \int d^dx j^\mu A_\mu(t,x)\right)\right\}$$

can be used to generate arbitrary correlation functions of densities or currents.

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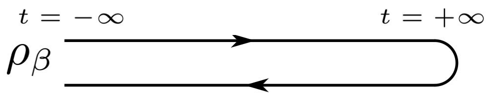

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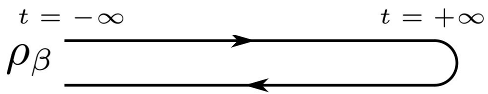

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can be used to generate arbitrary correlation functions of densities or currents.

The effective field theory (EFT) of diffusion consists in representing it with a local effective action of the hydrodynamic and noise fields [Crossley Glorioso Liu '15]

$$Z[A^1_\mu, A^2_\mu] \simeq \int Dn D\phi_a\, e^{i\int dt d^dx\, \mathcal{L}}$$

EFT for fluctuating hydrodynamics

Using symmetries and properties of the generating functional, the EFT takes the form

$$\mathcal{L} = i\sigma(n)(\nabla\phi_a)^2 - \phi_a\left(\dot{n} - \nabla\left(D(n)\nabla n\right)\right) + \cdots$$

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Degrees of freedom: density and associated noise field

EFT for fluctuating hydrodynamics

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'Wilsonian coefficients' of the EFT

Higher derivative terms, e.g.

$$j_i = -D\nabla_i n + a_{0,1}\nabla^2\nabla_i n + \cdots$$

and higher order in ϕ_a terms

EFT for fluctuating hydrodynamics

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$$\mathcal{L} = i\sigma(n)(\nabla\phi_a)^2 - \phi_a\Big(\dot{n} - \nabla\left(D(n)\nabla n\right)\Big) + \cdots$$

Leading nonlinearities come expanding $\sigma(n) = \sigma + \sigma'\delta n + \ldots$, $D(n) = D + D'\delta n + \ldots$

⇒ Nonlinear response tied to linear response! (~ NLsM)

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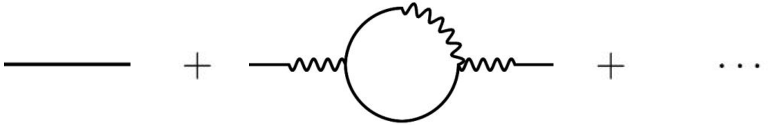
More generally, the EFT parametrizes all corrections to diffusion, extending previous approaches (e.g: MSR [Martin Siggia Rose '73], MFT [Bertini De Sole Gabrielli Jona-Lasinio Landim '01])

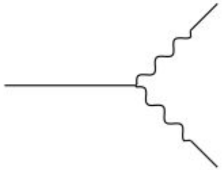
EFT for precision tests of thermalization

1. Universal corrections



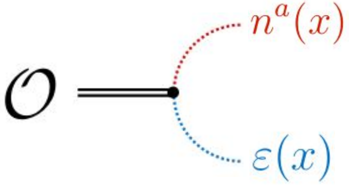
$$\frac{1}{\sqrt{t}} \quad + \quad \frac{1}{t} \quad + \quad \cdots$$

2. Nonlinear response



$$\langle n(x_1,t_1)n(x_2,t_2)\cdots n(x_N,t_N)\rangle$$

3. Decay of arbitrary microscopic operators from operator matching equations



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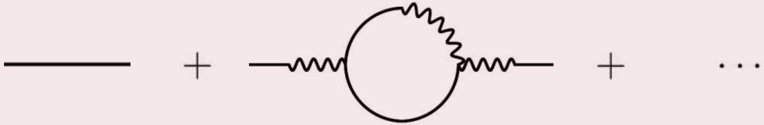

$$\frac{1}{\sqrt{t}} + \frac{1}{t} + \cdots$$

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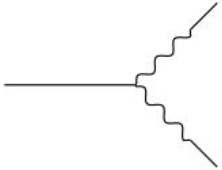

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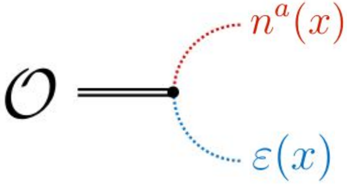
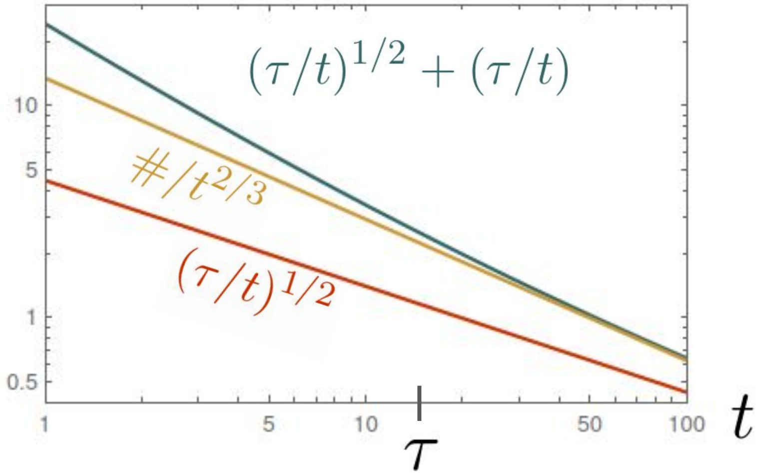
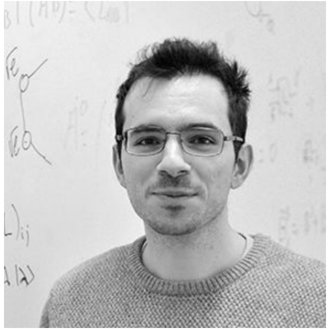


Universal corrections to diffusion





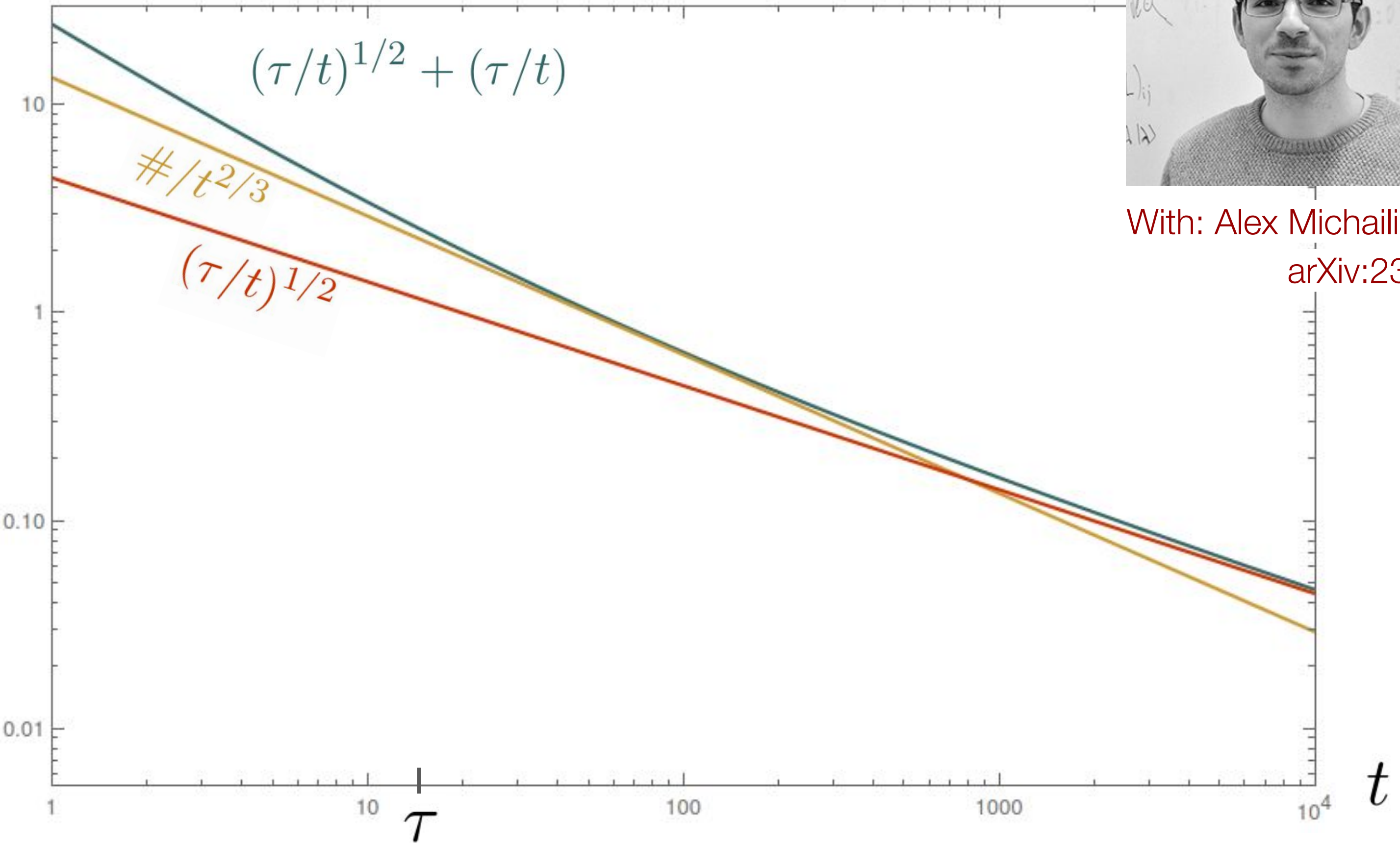
With: Alex Michailidis & Dima Abanin
arXiv:2310.10564

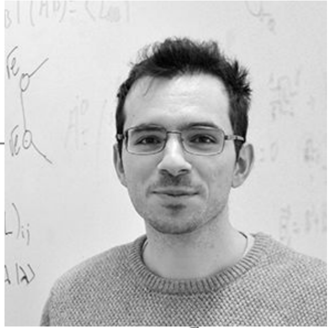
Difficult to reliably extract even the dynamic exponent with limited resources, because scaling corrections to diffusion are still large.

But thanks to the EFT, we know these corrections!

⇒ use EFT for more reliable extraction of transport parameters

Universal corrections to diffusion





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Universal corrections to diffusion

$$
\begin{aligned}
\langle n(x=0,t)n\rangle \;=\; \frac{1}{t^{d/2}}\Big\{ \quad & a_{0,0} + \frac{a_{0,1}}{t} + \frac{a_{0,2}}{t^2} + \cdots \\
+ \frac{1}{t^{d/2}}\Big(& a_{1,0} + \frac{a_{1,1}}{t} + \frac{a_{1,2}}{t^2} + \cdots \Big) \\
+ \frac{1}{t^{d}}\Big(& a_{2,0} + \frac{a_{2,1}}{t} + \frac{a_{2,2}}{t^2} + \cdots \Big) \\
+ & \qquad \cdots \qquad \Big\}
\end{aligned}
$$

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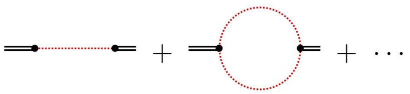
& \cdots \Big\}
\end{aligned}
$$

Loops



Higher derivative terms, e.g.

$$j_i = -D\nabla_i n + a_{0,1}\nabla^2\nabla_i n + \cdots$$

Universal corrections to diffusion

$$y \equiv x/\sqrt{Dt}$$

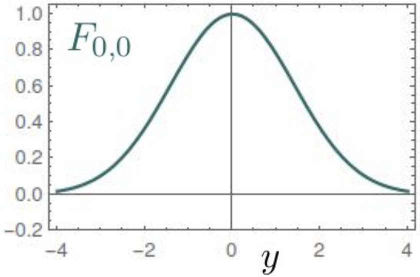
$$\begin{aligned}\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\Big\{ & \quad F_{0,0}(y) + \frac{F_{0,1}(y)}{t} + \frac{F_{0,2}(y)}{t^2} + \cdots \\ & + \frac{1}{\sqrt{t}}\Big(F_{1,0}(y) + \frac{F_{1,1}(y)}{t} + \frac{F_{1,2}(y)}{t^2} + \cdots \Big) \\ & + \frac{1}{t}\Big(F_{2,0}(y) + \frac{F_{2,1}(y)}{t} + \frac{F_{2,2}(y)}{t^2} + \cdots \Big) \\ & + \cdots \Big\}\end{aligned}$$

EFT produces entire universal scaling *functions* that capture scaling corrections to diffusion at intermediate times

Universal corrections to diffusion

$$y \equiv x/\sqrt{Dt}$$

$$\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\Big\{ \boxed{F_{0,0}(y)} + \frac{F_{0,1}(y)}{t} + \frac{F_{0,2}(y)}{t^2} + \cdots$$
$$+ \frac{1}{\sqrt{t}}\Big(F_{1,0}(y) + \frac{F_{1,1}(y)}{t} + \frac{F_{1,2}(y)}{t^2} + \cdots \Big)$$
$$+ \frac{1}{t}\Big(F_{2,0}(y) + \frac{F_{2,1}(y)}{t} + \frac{F_{2,2}(y)}{t^2} + \cdots \Big)$$
$$+ \cdots \Big\}$$



EFT produces entire universal scaling *functions* that capture scaling corrections to diffusion at intermediate times

$$F_{0,0}(y) = \frac{\chi}{\sqrt{4\pi D}} e^{-y^2/4}$$

Universal corrections to diffusion

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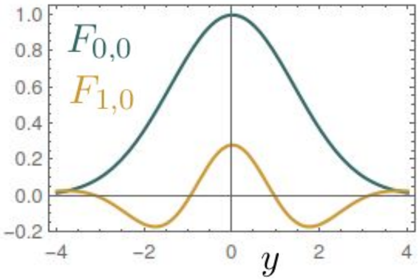
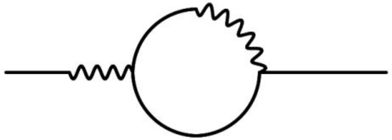
$$\begin{aligned}\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\Bigg\{ & \quad F_{0,0}(y) + \frac{F_{0,1}(y)}{t} + \frac{F_{0,2}(y)}{t^2} + \cdots \\ & + \frac{1}{\sqrt{t}}\left(\boxed{F_{1,0}(y)} + \frac{F_{1,1}(y)}{t} + \frac{F_{1,2}(y)}{t^2} + \cdots \right) \\ & + \frac{1}{t}\left(F_{2,0}(y) + \frac{F_{2,1}(y)}{t} + \frac{F_{2,2}(y)}{t^2} + \cdots \right) \\ & + \qquad \cdots \qquad \Bigg\}\end{aligned}$$



EFT produces entire universal scaling *functions* that capture scaling corrections to diffusion at intermediate times

$$F_{1,0}(y) = \frac{\chi D'^2}{D^{5/2}}\left[\frac{4+y^2}{8\sqrt{\pi}}e^{-y^2/2} + \frac{y(y^2-10)}{16}e^{-y^2/4}\mathrm{Erf}(y/2)\right]$$



Precision test of thermalization:

Universal 1-loop correction to diffusion

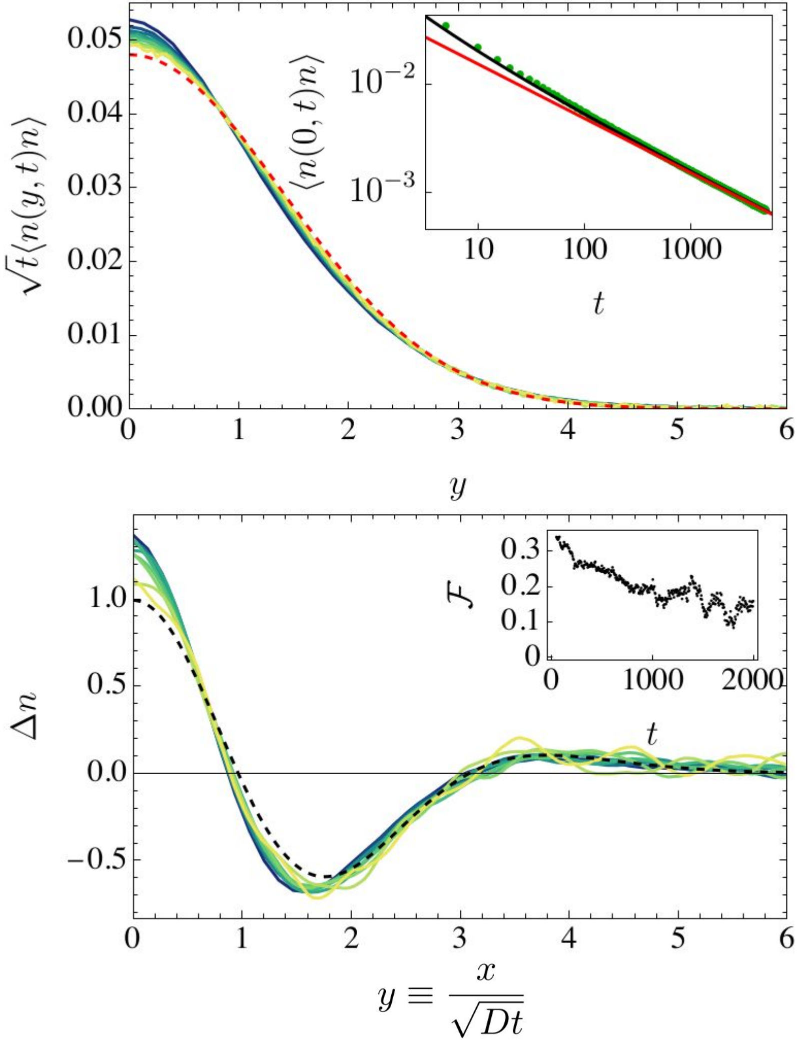
$$\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\left[e^{-x^2/4Dt} + \frac{a_{1,0}}{\sqrt{t}}F_{1,0}\left(\frac{x^2}{Dt}\right) + \cdots\right]$$

Test in a classical lattice gas [Spohn '12]

$$0100 \xrightarrow{r(1+\delta)} 0010$$
$$1101 \xrightarrow{r(1-\delta)} 1011$$
$$1100 \xrightarrow{r(1+\epsilon)} 1010$$
$$0101 \xrightarrow{r(1-\epsilon)} 0011$$

($D(n)$ known analytically → no fitting parameter)

arXiv:2310.10564 with **Alex Michailidis** and **Dima Abanin**



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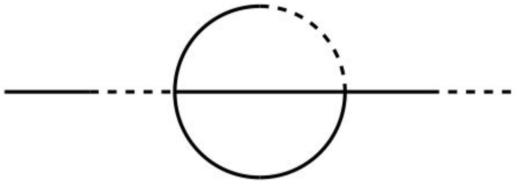
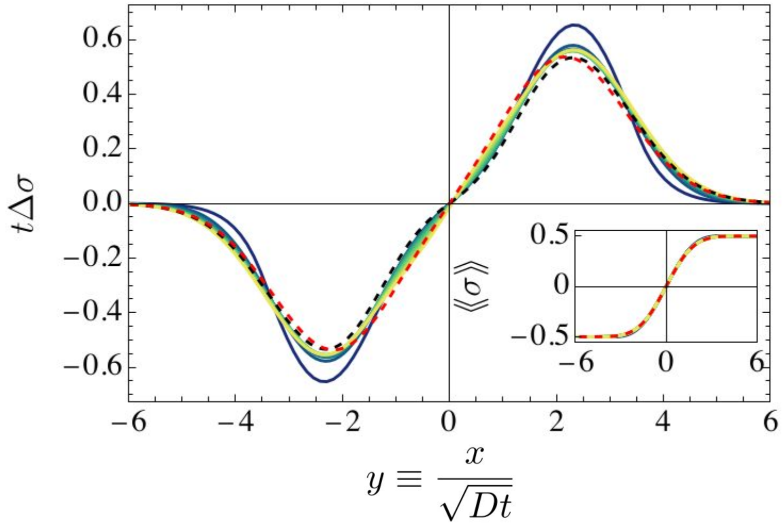


$$\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\left[e^{-x^2/4Dt} + \frac{a_{1,0}}{\sqrt{t}}F_{1,0}(\tfrac{x^2}{Dt}) + \cdots\right]$$

2-loop correction + higher derivative correction

$$\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\left[e^{-x^2/4Dt} + \frac{a_{2,0}}{t}\log(t)F_{2,0}(\tfrac{x^2}{Dt}) + \frac{a_{0,1}}{t}F_{0,1}(\tfrac{x^2}{Dt}) + \cdots\right]$$

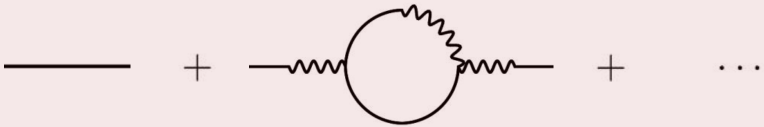


(Floquet-XXZ chain with dephasing/staggered field)

arXiv:2310.10564 with **Alex Michailidis** and **Dima Abanin**

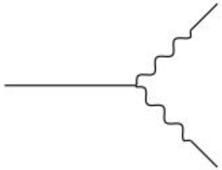
EFT for precision tests of thermalization

1. Universal corrections



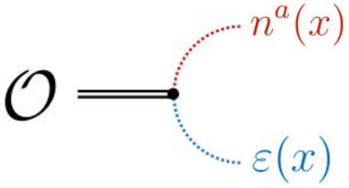
$$\frac{1}{\sqrt{t}} \quad + \quad \frac{1}{t} \quad + \quad \cdots$$

2. Nonlinear response



$$\langle n(x_1, t_1) n(x_2, t_2) \cdots n(x_N, t_N) \rangle$$

3. Decay of arbitrary microscopic operators from operator matching equations



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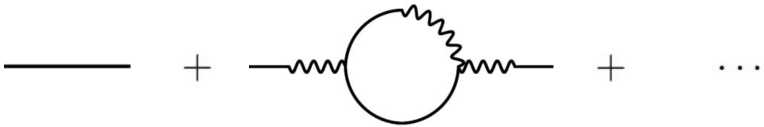

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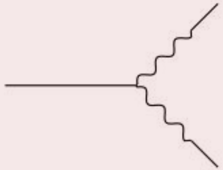

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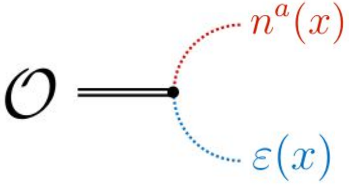

Non-linear response in diffusive systems

Ruchira Mishra
[arXiv:2304.03236]

Simple scaling argument for higher-point functions:

$$\langle n(t)n\rangle \sim \frac{1}{t^{1/2}} \quad \Rightarrow \quad n \sim \frac{1}{t^{1/4}} \qquad \langle n(Nt)\cdots n(2t)n(t)\rangle \sim \frac{1}{t^{N/4}} \quad ?$$

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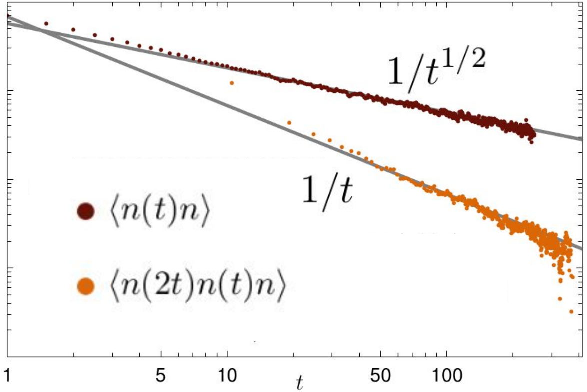
Gaussian diffusive theory does not have nonlinear response: these require non-gaussianities.

The leading non-gaussianity is suppressed by $\delta n \sim \frac{1}{t^{1/4}}$

An N point function needs N-2 such nonlinearities :

$$\langle n(Nt)\cdots n(2t)n(t)\rangle \sim \frac{1}{t^{N/4}}\frac{1}{t^{(N-2)/4}} \sim \frac{1}{t^{(N-1)/2}}$$



Non-linear response in diffusive systems

with **Ruchira Mishra**
[arXiv:2304.03236]

Getting the precise form of higher point functions requires the EFT :

$$\mathcal{L} = i\sigma(n)(\nabla\phi_a)^2 - \phi_a\left(\dot{n} - \nabla\left(D(n)\nabla n\right)\right) + \cdots$$

Leading nonlinearities come from cubic terms

$$\mathcal{L}^{(3)} = i\sigma' n(\nabla\phi_a)^2 + \frac{1}{2}D'\nabla^2\phi_a n^2$$

Can compute all higher point functions using these and higher vertices

$$\langle n(x_N, t_N)\cdots n(x_2, t_2)n(x_1, t_1)\rangle$$

Non-linear response in diffusive systems

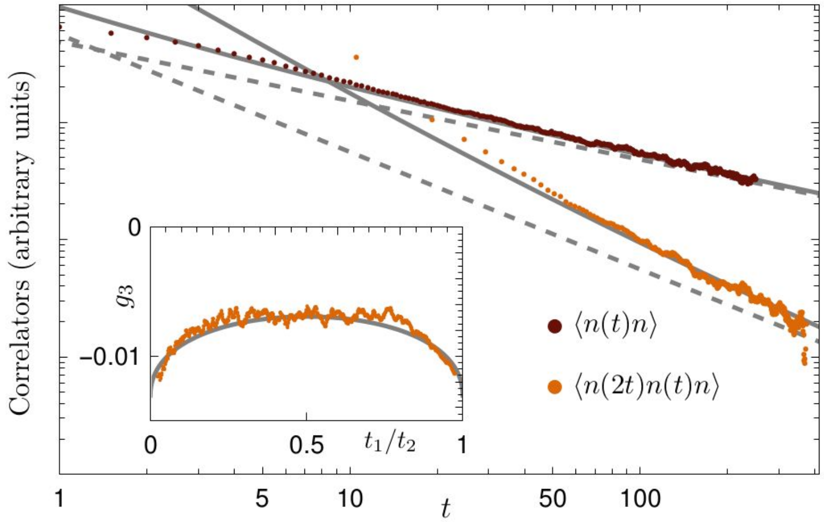
with **Ruchira Mishra**
[arXiv:2304.03236]

As an example, focus on the 3-point function in a simple kinematic configuration:

$$\langle n(t_2,0)n(t_1,0)n(0,0)\rangle = \frac{(\chi T)^2}{8\pi D\sqrt{t_1(t_2-t_1)}}\left[\frac{\sigma'}{\sigma}+\frac{D'}{D}\left(1+2\sqrt{\frac{t_2}{t_1}}+2\sqrt{1-\frac{t_2}{t_1}}\right)\right]$$

Numerics:

Katz-Lebowitz-Spohn model, a classical lattice gas where $D(n)$ and $\sigma(n)$ are known analytically



Non-linear response in diffusive systems

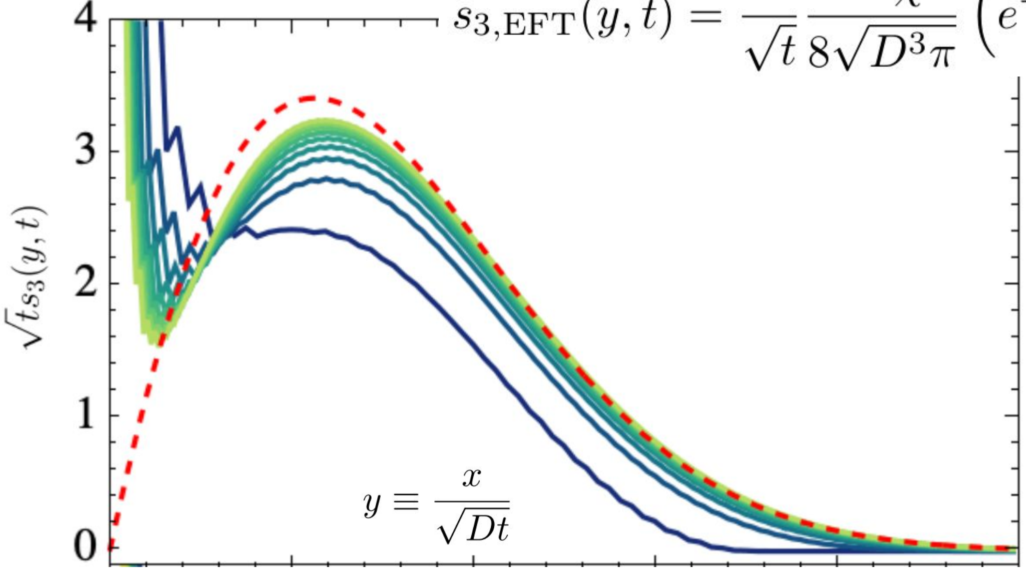
with **Ruchira Mishra**
[arXiv:2304.03236]

w/ **Michailidis** & **Abanin**
[arXiv:2310.10564]

Can use this to help costly quantum numerics!

In some configurations, 3pt function is only sensitive to D'

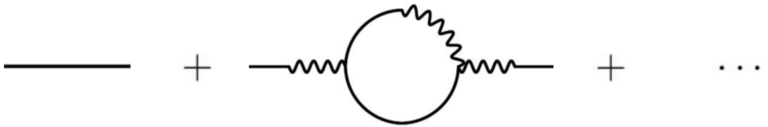
$$s_{3,\mathrm{EFT}}(y,t) = \frac{1}{\sqrt{t}} \frac{D'\chi}{8\sqrt{D^3\pi}} \left(e^{-y^2/4} + \mathrm{Erf}(y/2) - 1 \right)$$



Numerics:
Floquet quantum spin chain with dephasing

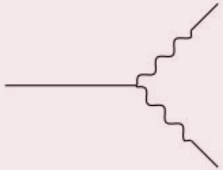
EFT for precision tests of thermalization

1. Universal corrections



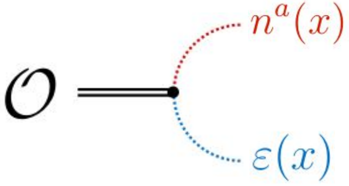
$$\frac{1}{\sqrt{t}} + \frac{1}{t} + \cdots$$

2. Nonlinear response



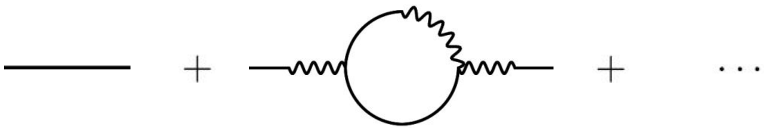
$$\langle n(x_1,t_1)n(x_2,t_2)\cdots n(x_N,t_N)\rangle$$

3. Decay of arbitrary microscopic operators from operator matching equations



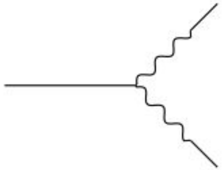
EFT for precision tests of thermalization

1. Universal corrections



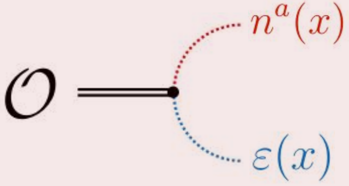
$$\frac{1}{\sqrt{t}} \quad + \quad \frac{1}{t} \quad + \quad \cdots$$

2. Nonlinear response



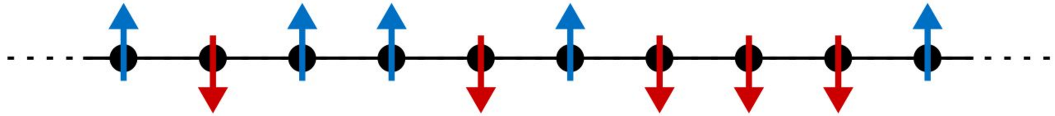
$$\langle n(x_1, t_1) n(x_2, t_2) \cdots n(x_N, t_N) \rangle$$

3. Decay of arbitrary microscopic operators
   from operator matching equations



Operator matching equations

Exponential decay $\langle \mathcal{O}(t)\mathcal{O}\rangle \sim e^{-t/\tau_{\mathrm{eq}}}$ does not generically occur, due to hydro:



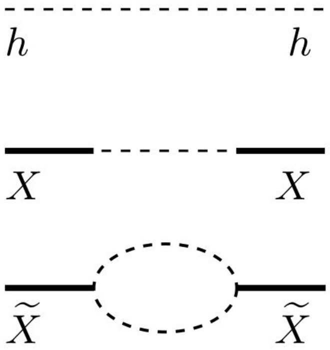
$$H = \sum_i Z_i Z_{i+1} + g X_i + g' Z_i$$

with $g, g' \sim 1$

$$\langle h_i(t) h_i\rangle \sim t^{-1/2}$$

$$X_i \quad \rightsquigarrow \quad \langle X_i(t) X_i\rangle \sim t^{-1/2} \cdot \chi^2_{hX}$$

$$\widetilde{X}_i \equiv X_i - \# h_i \quad \rightsquigarrow \quad \langle \widetilde{X}_i(t) \widetilde{X}_i\rangle \sim t^{-1}$$



These hydrodynamic “tails” come from operator matching equations [LVD ‘20]

Example: SO(3) spin chain

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

Thermalization and hydrodynamics of this model long debated

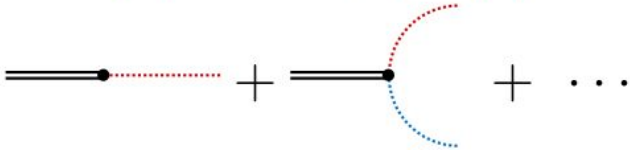
[Srivastava Liu Viswanath Müller ’94, ..., Bagchi ’13, ..., Das Chakrabarty Dhar Kundu Huse Moessner Ray Bhattacharjee ’18, De Nardis Medenjak Karrasch Ilievski ’20]

[Glorioso LVD Chen Nandkishore Lucas ‘20]

Example: SO(3) spin chain

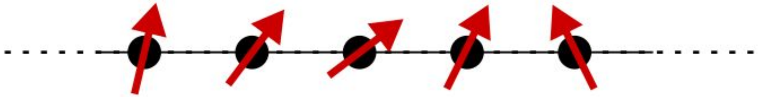
$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} \quad \rightsquigarrow \quad \mathcal{L}_{\text{eff}}[n^a, \varepsilon]$$

$$
\begin{aligned}
S_i^a &= n^a(x) \\
\vec{S}_i \cdot \vec{S}_{i+1} &= \varepsilon(x) \\
S_{i-1}^a \, \vec{S}_i \cdot \vec{S}_{i+1} &= \# \, n^a(x) + \#' \varepsilon(x) n^a(x) + \#'' \nabla^2 n^a(x)
\end{aligned}
$$

$=$ 

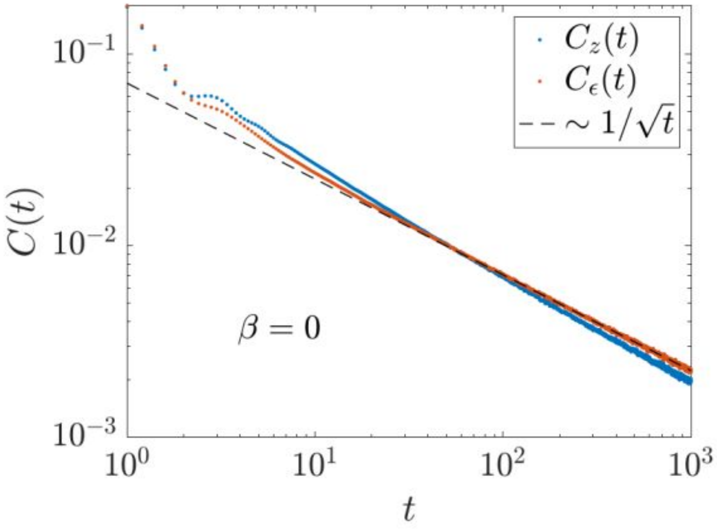
$$
\begin{aligned}
\vec{S}_i \cdot (\vec{S}_{i+1} \times \vec{S}_{i+2}) &= \cancel{\# \, \varepsilon(x)} + \#' \, \nabla \varepsilon + \cdots \\
S_{i-2}^a S_{i-1}^b \vec{S}_i \cdot (\vec{S}_{i+1} \times \vec{S}_{i+2}) &= \# \, n^a(x) \nabla n^b(x) + \cdots
\end{aligned}
$$

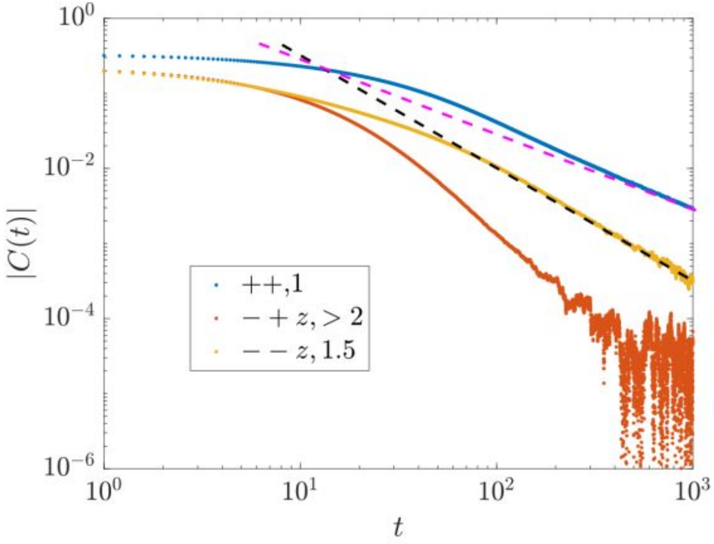
Example: SO(3) spin chain



Hydrodynamic densities diffuse: $\mathcal{O} = \# n + \cdots \quad \Rightarrow \quad \langle \mathcal{O}(t)\mathcal{O}\rangle \sim \frac{1}{\sqrt{t}}$

Composite operators also power law decay: $\mathcal{O} = \# n^a \nabla n^b + \cdots \quad \Rightarrow \quad \langle \mathcal{O}(t)\mathcal{O}\rangle \sim \frac{1}{t^2}$





[LVD '20]
[Glorioso LVD Chen Nandkishore Lucas '20]

I. Defining the local equilibration time

"Time scale where hydrodynamics/diffusion emerges"

II. Universal bound from hydrodynamic fluctuations

Hydrodynamics "knows" about its own breakdown

III. Precision tests of thermalization in classical/quantum numerics

Thanks!

Extra slides

Power law corrections to hydrodynamics

Fluctuation corrections to hydrodynamic correlators have been extensively studied since their discovery in numerics
[Alder Wainwright ‘70, Martin Siggia Rose ‘75, Forster Nelson Stephen ‘77, Andreev ‘78, …]

They arise from nonlinearities in the Navier-Stokes equations and equation of state

However, previous work exclusively focused on q=0 observables.

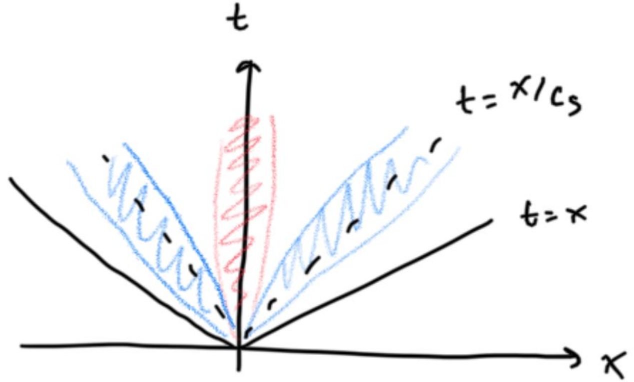
$$\eta(\omega) = \lim_{q\to 0} \frac{1}{\omega} \operatorname{Im} G^R_{T_{xy}T_{xy}}(\omega, q)$$
$$= \eta + \#\omega^{(d-2)/2} + \cdots$$

Scaling is entirely different at finite q, in the regime $\omega - c_s q \sim q^2$!

Power law corrections to hydrodynamics

Along the sound front, diffusive modes kinematically decouple and we can focus on sound modes:

$$\langle T(t_o, x_o) T \rangle \sim \frac{1}{t_o^{d/2}} e^{-\frac{(|x_o| - c_s t_o)^2}{2\gamma t_o}}$$



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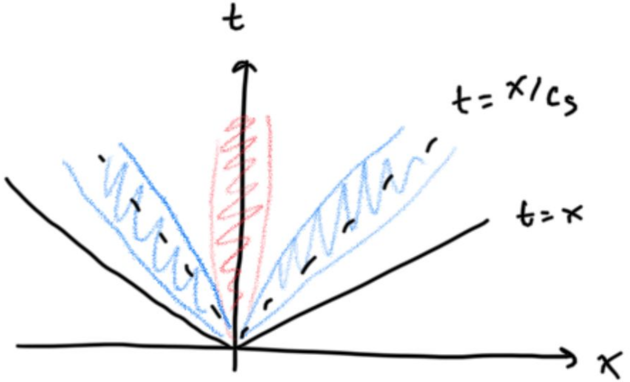

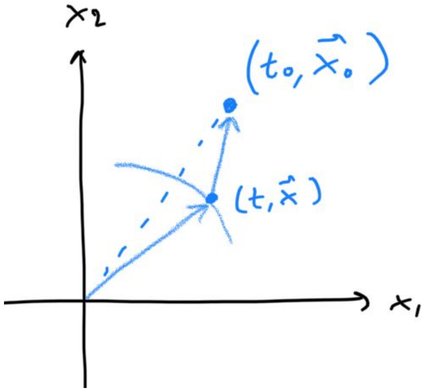

Naively expect $\vec{x} = c_s t\, \hat{x}_o + \delta\vec{x}$ with $\delta\vec{x} \sim \sqrt{\gamma t}$

Correct scaling: $\delta x_{||} \sim \sqrt{\gamma t}$, $\delta x_{\perp} \sim (\gamma t)^{1/4} (c_s t)^{1/2}$

Loop corrections are more suppressed than expected!

Power law corrections to hydrodynamics

Loop corrections along the sound front therefore take the form:

$$\langle T_{\mu\nu}(t,x)T_{\rho\sigma}\rangle \sim \frac{1}{t^{d/2}}e^{-\frac{(x-c_S t)^2}{2\gamma t}}\left[\left(\frac{\gamma}{c_s\delta x}\right)^{\frac{d-1}{2}} + \frac{1}{t^{(d-2)/2}}\left(\frac{\gamma}{c_s\delta x}\right)^{\frac{d-1}{2}} + \cdots\right]$$

In particular, shear viscosities are actually well-defined in 2+1d.