Disordered quantum critical fixed points from holography

Andrew Lucas

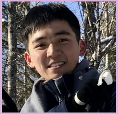
University of Colorado Boulder

HoloTube Seminar

November 28, 2023

✆

Acknowledgements



Xiaoyang Huang
Boulder

Koushik Ganesan
Boulder

Subir Sachdev
Harvard

Leo Radzihovsky
Boulder

ALFRED P. SLOAN FOUNDATION

GORDON AND BETTY MOORE FOUNDATION

The problem

A longstanding challenge in quantum field theory: analytically tractable IR fixed points with

- strong coupling
  (no quasiparticles/not close to Gaussian fixed point)
- finite U(1) charge density, Fermi surfaces
- finite disorder

The problem

A longstanding challenge in quantum field theory: analytically tractable IR fixed points with

- strong coupling
  (no quasiparticles/not close to Gaussian fixed point)
  use holography!
- finite U(1) charge density, Fermi surfaces
  study charged black holes!
- finite disorder

The problem

A longstanding challenge in quantum field theory: analytically tractable IR fixed points with

- strong coupling
  (no quasiparticles/not close to Gaussian fixed point)
  use holography!
- finite U(1) charge density, Fermi surfaces
  study charged black holes!
- finite disorder

We will find new, stable, IR fixed points with finite disorder and clarify some subtleties in earlier papers.

Scaling exponents

Start with some UV QFT (which might be emergent in a lattice description in condensed matter...):

$$S = \int \mathrm{d}^{d+1}x \, \mathcal{L}$$

in d *spatial* dimensions. (We don't need a Lagrangian description, but such "notation" will be useful for the talk.)

Scaling exponents

Start with some UV QFT (which might be emergent in a lattice description in condensed matter...):

$$S = \int \mathrm{d}^{d+1}x \, \mathcal{L}$$

in d *spatial* dimensions. (We don't need a Lagrangian description, but such "notation" will be useful for the talk.)

The theory may have non-trivial **dynamical scaling exponent** z:

$$[t] = z \cdot [x],$$

Scaling exponents

Start with some UV QFT (which might be emergent in a lattice description in condensed matter...):

$$S = \int \mathrm{d}^{d+1}x \, \mathcal{L}$$

in d *spatial* dimensions. (We don't need a Lagrangian description, but such "notation" will be useful for the talk.)

The theory may have non-trivial **dynamical scaling exponent** z:

$$[t] = z \cdot [x],$$

and non-trivial **hyperscaling-violating exponent** θ:

$$s \sim T^{(d-\theta)/z}.$$

Adding disorder

Suppose this QFT has scalar operator $\mathcal{O}$, with scaling dimension Δ:

$$\langle \mathcal{O}(x,0)\mathcal{O}(y,0)\rangle \sim |x-y|^{-2\Delta}.$$

Adding disorder

Suppose this QFT has scalar operator $\mathcal{O}$, with scaling dimension Δ:

$$\langle \mathcal{O}(x,0)\mathcal{O}(y,0)\rangle \sim |x-y|^{-2\Delta}.$$

We add **random-field disorder** coupled to $\mathcal{O}$:

$$S = S_0 - \int \mathrm{d}^d x \mathrm{d}t \; h(x)\mathcal{O}(x,t)$$

where the disorder profile is random:

$$\overline{h(x)} = 0, \quad \overline{h(x)h(y)} = D \cdot \delta^{(d)}(x-y),$$

Note that we want the *same* disorder profile at all t.

Adding disorder

Let $[x] = -1$. Then if $[\mathcal{O}] = \Delta$,

$$[D] = d - \theta + z - 2\Delta.$$

Adding disorder

Let $[x] = -1$. Then if $[\mathcal{O}] = \Delta$,

$$[D] = d - \theta + z - 2\Delta.$$

We see that $[D] = 2\nu$ if we write

$$\Delta = \frac{d - \theta}{2} + z - \nu.$$

Adding disorder

Let $[x] = -1$. Then if $[\mathcal{O}] = \Delta$,

$$[D] = d - \theta + z - 2\Delta.$$

We see that $[D] = 2\nu$ if we write

$$\Delta = \frac{d-\theta}{2} + z - \nu.$$

Harris relevant	$\nu > 0$
Harris marginal	$\nu = 0$
Harris irrelevant	$\nu < 0$

[Harris; *J. Phys.* **C7** 1671 (1974)]

[Lucas, Sachdev, Schalm; *Phys. Rev.* **D89** 066018 (2014)]

Adding disorder

Let $[x] = -1$. Then if $[\mathcal{O}] = \Delta$,

$$[D] = d - \theta + z - 2\Delta.$$

We see that $[D] = 2\nu$ if we write

$$\Delta = \frac{d-\theta}{2} + z - \nu.$$

Harris relevant	$\nu > 0$
Harris marginal	$\nu = 0$
Harris irrelevant	$\nu < 0$

[Harris; *J. Phys.* **C7** 1671 (1974)]

[Lucas, Sachdev, Schalm; *Phys. Rev.* **D89** 066018 (2014)]

Harris marginal (or relevant!) disorder could change IR fixed point.

Holographic setup

Einstein-Maxwell-dilaton theories give holographic models with tunable z and θ:

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - 2(\partial\Phi)^2 - V(\Phi) - \frac{Z(\Phi)}{4}F_{ab}F^{ab}\right].$$

where

$$V(\Phi) = V_0 \mathrm{e}^{-\beta\Phi}, \quad Z(\Phi) = Z_0 \mathrm{e}^{\alpha\Phi}$$

with α, β depending on z, θ.

Einstein-Maxwell-dilaton theories give holographic models with tunable z and θ:

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - 2(\partial\Phi)^2 - V(\Phi) - \frac{Z(\Phi)}{4}F_{ab}F^{ab}\right].$$

where

$$V(\Phi) = V_0\mathrm{e}^{-\beta\Phi}, \quad Z(\Phi) = Z_0\mathrm{e}^{\alpha\Phi}$$

with α, β depending on z, θ.

F_{ab} is the Maxwell flux, so the dual field theory has a conserved U(1) charge. (Added bonus feature for us!)

Einstein-Maxwell-dilaton theories give holographic models with tunable z and θ:

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - 2(\partial\Phi)^2 - V(\Phi) - \frac{Z(\Phi)}{4}F_{ab}F^{ab}\right].$$

where

$$V(\Phi) = V_0\mathrm{e}^{-\beta\Phi}, \quad Z(\Phi) = Z_0\mathrm{e}^{\alpha\Phi}$$

with α, β depending on z, θ.

F_{ab} is the Maxwell flux, so the dual field theory has a conserved U(1) charge. (Added bonus feature for us!)

Expand around background:

$$\mathrm{d}s^2 \sim \frac{1}{r^2}\left[r^{2\theta/(d-\theta)}\mathrm{d}r^2 - \frac{\mathrm{d}t^2}{r^{2d(z-1)/(d-\theta)}} + \mathrm{d}\mathbf{x}^2\right]$$

Holographic setup

In usual QFT, disorder is handled through replica method, which is subtle to physically interpret.

Holographic setup

In usual QFT, disorder is handled through replica method, which is subtle to physically interpret.

But in holography, random-field disorder is simple to add! Consider the scalar field ψ dual to $\mathcal{O}$:

$$S = S_{\mathrm{EMD}} + \int \mathrm{d}^{d+2}x\sqrt{-g}\left[\frac{1}{2}(\partial\psi)^2 - B(\Phi)\psi^2\right].$$

For suitable $B(\Phi)$, this can encode operator with generic scaling dimension Δ (i.e. generic ν).

In usual QFT, disorder is handled through replica method, which is subtle to physically interpret.

But in holography, random-field disorder is simple to add! Consider the scalar field ψ dual to $\mathcal{O}$:

$$S = S_{\mathrm{EMD}} + \int \mathrm{d}^{d+2}x\sqrt{-g}\left[\frac{1}{2}(\partial\psi)^2 - B(\Phi)\psi^2\right].$$

For suitable $B(\Phi)$, this can encode operator with generic scaling dimension Δ (i.e. generic ν).

The *UV boundary conditions encode disorder realization:*

$$\psi(r,x) = h(x)r^{\#} + \cdots .$$

The inhomogeneous geometry encodes the influence of disorder. Solve Einstein equations to deduce the IR fixed point ($r \to \infty$).

Let's study $d = 1$, $z = 1$, $\theta = 0$, with Harris marginal disorder $\nu = 0$.

$$S = \int \mathrm{d}^3x \sqrt{-g} \left(R + 2 - \frac{1}{2}(\partial\psi)^2 + \frac{3}{4}\psi^2 \right).$$

A line of fixed points?

Let's study $d = 1$, $z = 1$, $\theta = 0$, with Harris marginal disorder $\nu = 0$.

$$S = \int \mathrm{d}^3x\sqrt{-g}\left(R + 2 - \frac{1}{2}(\partial\psi)^2 + \frac{3}{4}\psi^2\right).$$

Claim: there is a line of Lifshitz fixed points in the IR with

$$z_* = 1 + \frac{D}{8}, \quad \theta_* = 0.$$

Both analytical and numerical arguments provided.

[Hartnoll, Santos; *Phys. Rev. Lett.* **112** 231601 (2014)]

Step 1: the geometry can be taken to be homogeneous.

$$R_{ab} - \frac{R}{2} g_{ab} = T_{ab} \sim D\bar{T}_{ab}(k=0) + DT'_{ab}(k \neq 0)$$

Since on average $\overline{T'_{ab}} = 0$, it will only correct the homogeneous metric at order D^2. The homogeneous part can be self-consistent at $\mathrm{O}(D)$.

Step 1: the geometry can be taken to be homogeneous.

$$R_{ab} - \frac{R}{2} g_{ab} = T_{ab} \sim D\bar{T}_{ab}(k=0) + DT'_{ab}(k \neq 0)$$

Since on average $\overline{T'_{ab}} = 0$, it will only correct the homogeneous metric at order D^2. The homogeneous part can be self-consistent at $\mathrm{O}(D)$.

Step 2: Attempt perturbation theory. The bulk disordered scalar has solutions:

$$\psi(k,r) = h(k)\sqrt{r} f(kr)$$

with $f(0) = \text{const.}$. We deduce that

$$\bar{T}_{ab}(k=0) \sim \int \mathrm{d}k\; k^2 r f(kr) \sim \frac{1}{r^2},$$

which has the same scaling as

$$R_{ab} \sim R g_{ab} \sim r^{-2}.$$

Step 3: Naive perturbation theory will get log-divergences:

$$g_{tt} \sim r^{-2}\left(1 + \frac{D}{4}\log r + \cdots\right).$$

Step 3: Naive perturbation theory will get log-divergences:

$$g_{tt} \sim r^{-2}\left(1 + \frac{D}{4}\log r + \cdots\right).$$

Like in ordinary QFT, these logs suggest a *critical exponent*:

$$g_{tt} \sim r^{-2z}, \qquad z = 1 + \frac{D}{8},$$

and indeed, a slightly more sophisticated ansatz for g_{ab} finds this solves Einstein's equations with the $\bar{T}_{ab}$ from **Step 2**. This leads to a **Lifshitz IR geometry**

$$\mathrm{d}s^2(r \to \infty) \sim \frac{\mathrm{d}r^2}{r^2} - \frac{\mathrm{d}r^2}{r^{2z}} + \frac{\mathrm{d}x^2}{r^2}.$$

Step 3: Naive perturbation theory will get log-divergences:

$$g_{tt} \sim r^{-2}\left(1 + \frac{D}{4}\log r + \cdots\right).$$

Like in ordinary QFT, these logs suggest a *critical exponent*:

$$g_{tt} \sim r^{-2z}, \quad z = 1 + \frac{D}{8},$$

and indeed, a slightly more sophisticated ansatz for g_{ab} finds this solves Einstein's equations with the $\bar{T}_{ab}$ from **Step 2**. This leads to a **Lifshitz IR geometry**

$$\mathrm{d}s^2(r \to \infty) \sim \frac{\mathrm{d}r^2}{r^2} - \frac{\mathrm{d}r^2}{r^{2z}} + \frac{\mathrm{d}x^2}{r^2}.$$

Note that in holography, this argument requires a non-perturbative resummation of solutions to Einstein's equations.

A puzzle

Using the bulk action

$$S = \int \mathrm{d}^3x\sqrt{-g}\left(R + 2 - \frac{1}{2}(\partial\psi)^2 + \frac{3}{4}\psi^2\right)$$

we find that in the Lifshitz geometry:

$$m^2 = -\frac{3}{4} = \Delta(\Delta - d - z)$$

so in the IR theory:

$$\Delta_* \approx \frac{3}{2} + \frac{3D}{16} > \Delta_{\text{marginal}} = \frac{3}{2} + \frac{D}{8}.$$

[Ganesan, Lucas; *JHEP* **06** 023 (**2020**)]

A puzzle

Using the bulk action

$$S = \int \mathrm{d}^3x\sqrt{-g}\left(R + 2 - \frac{1}{2}(\partial\psi)^2 + \frac{3}{4}\psi^2\right)$$

we find that in the Lifshitz geometry:

$$m^2 = -\frac{3}{4} = \Delta(\Delta - d - z)$$

so in the IR theory:

$$\Delta_* \approx \frac{3}{2} + \frac{3D}{16} > \Delta_{\mathrm{marginal}} = \frac{3}{2} + \frac{D}{8}.$$

[Ganesan, Lucas; *JHEP* **06** 023 (**2020**)]

How can irrelevant disorder support a Lifshitz fixed point?

Resolving the puzzle

Revisit our arguments before?

Resolving the puzzle

Revisit our arguments before?

Step 1 (homogeneity) is fine. 👍

Resolving the puzzle

Revisit our arguments before?

Step 1 (homogeneity) is fine. 👍

Step 2 assumed that the perturbation to the geometry was mild (no change to asymptotics). But in a Lifshitz geometry:

$$\psi(k,r) \sim h(k) r^{1-z/2} f(kr),$$

which leads to (since $z > 1$):

$$T_{ab} \sim r^{-1-z} \ll R_{ab} \sim r^{-2}$$

This is where our earlier argument breaks down!

Resolving the puzzle

Improve **Step 3** by the ansatz:

$$\mathrm{d}s^2 \sim \frac{\mathrm{d}r^2}{r^2} - \frac{\mathrm{d}t^2}{r^{2z(r)}} + \frac{\mathrm{d}x^2}{r^2} B(r).$$

Resolving the puzzle

Improve **Step 3** by the ansatz:

$$\mathrm{d}s^2 \sim \frac{\mathrm{d}r^2}{r^2} - \frac{\mathrm{d}t^2}{r^{2z(r)}} + \frac{\mathrm{d}x^2}{r^2} B(r).$$

If $B \approx 1$ and z slowly varies, then we can set:

$$\bar{T}_{ab} \to \bar{T}_{ab}(z(r)) \sim r^{-1-z(r)}.$$

Improve **Step 3** by the ansatz:

$$\mathrm{d}s^2 \sim \frac{\mathrm{d}r^2}{r^2} - \frac{\mathrm{d}t^2}{r^{2z(r)}} + \frac{\mathrm{d}x^2}{r^2} B(r).$$

If $B \approx 1$ and z slowly varies, then we can set:

$$\bar{T}_{ab} \to \bar{T}_{ab}(z(r)) \sim r^{-1-z(r)}.$$

Use asymptotic methods to show that Einstein-matter system approximately solved by (here Λ is a UV cutoff on disorder):

$$z(r) \sim 1 + \frac{1}{\log(r\Lambda)} \log\left(1 + \frac{D}{8}\log(r\Lambda)\right).$$

[Ganesan, Lucas; *JHEP* **06** 023 (**2020**)]

Resolving the puzzle

$$z(r) \sim 1 + \frac{1}{\log(r\Lambda)} \log\left(1 + \frac{D}{8}\log(r\Lambda)\right).$$

As $r \to \infty$, $z \to 1$. There is no Lifshitz fixed point in the IR.

Resolving the puzzle

$$z(r) \sim 1 + \frac{1}{\log(r\Lambda)} \log\left(1 + \frac{D}{8}\log(r\Lambda)\right).$$

As $r \to \infty$, $z \to 1$. There is no Lifshitz fixed point in the IR.

The breakdown of the "Lifshitz fixed point" is only visible if

$$r > r_{\mathrm{IR}} \sim \frac{1}{\Lambda} \mathrm{e}^{8/D},$$

which is non-perturbatively deep in the IR.

$$z(r) \sim 1 + \frac{1}{\log(r\Lambda)} \log\left(1 + \frac{D}{8}\log(r\Lambda)\right).$$

As $r \to \infty$, $z \to 1$. There is no Lifshitz fixed point in the IR.

The breakdown of the “Lifshitz fixed point” is only visible if

$$r > r_{\mathrm{IR}} \sim \frac{1}{\Lambda}\mathrm{e}^{8/D},$$

which is non-perturbatively deep in the IR.

Claim: This $z(r)$ is actually associated with **marginally irrelevant disorder**. In the right language, it is a **one-loop effect**.

[Ganesan, Lucas, Radzihovsky; *Phys. Rev.* **D105** 066016 (2022)]

$$z(r) \sim 1 + \frac{1}{\log(r\Lambda)} \log\left(1 + \frac{D}{8}\log(r\Lambda)\right).$$

As $r \to \infty$, $z \to 1$. There is no Lifshitz fixed point in the IR.

The breakdown of the "Lifshitz fixed point" is only visible if

$$r > r_{\mathrm{IR}} \sim \frac{1}{\Lambda}\mathrm{e}^{8/D},$$

which is non-perturbatively deep in the IR.

Claim: This $z(r)$ is actually associated with **marginally irrelevant disorder**. In the right language, it is a **one-loop effect**.

[Ganesan, Lucas, Radzihovsky; *Phys. Rev.* **D105** 066016 (2022)]

This effect can only be seen by a *careful* non-perturbative analysis of Einstein's equations.

Insight comes from **conformal perturbation theory**. Calculate

$$\overline{\langle F \rangle} = \overline{\frac{1}{Z(h)} \int \mathrm{D}\mathcal{O} \cdots F \mathrm{e}^{-S_0 + \int h\mathcal{O}}},$$

where the overline denotes averaging over h.

Insight comes from **conformal perturbation theory**. Calculate

$$\overline{\langle F \rangle} = \overline{\frac{1}{Z(h)} \int \mathrm{D}\mathcal{O} \cdots F \mathrm{e}^{-S_0 + \int h\mathcal{O}}},$$

where the overline denotes averaging over h.

Due to $1/Z(h)$, this average is very hard. The strategy is to use the **replica method**. Consider n copies of the theory with fields $\mathcal{O}_a$ ($a = 1, \ldots, n$), and calculate

$$\overline{\langle F \rangle}_D = \int \mathrm{D}\mathcal{O}_a \cdots F_a \exp\left[-\sum_{a=1}^{n} S_{0,a} - S_{\mathrm{dis}}\right].$$

$$S_{\mathrm{dis}} = -D \sum_{a,b=1}^{n} \int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}x \mathcal{O}_a(t_1, x) \mathcal{O}_b(t_2, x)$$

Take the $n \to 0$ limit at the end of the calculation. 🤨

Notice that if t_1 is close to t_2, diagonal term has

$$\begin{aligned}\int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}x \mathcal{O}_a(t_1, x)\mathcal{O}_a(t_2, x) &= \int \mathrm{d}t \mathrm{d}x \mathrm{d}t' \frac{C_{\mathcal{O}\mathcal{O}T}}{|t|'} T_{tt,a}(t, x) + \cdots \\ &= \log b \int \mathrm{d}t \mathrm{d}x T_{tt}\end{aligned}$$

if we integrate out short time scales $(b\Lambda)^{-1} < |t'| < \Lambda^{-1}$.

Notice that if t_1 is close to t_2, diagonal term has

$$\int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}x \mathcal{O}_a(t_1, x)\mathcal{O}_a(t_2, x) = \int \mathrm{d}t \mathrm{d}x \mathrm{d}t' \frac{C_{\mathcal{O}\mathcal{O}T}}{|t|'} T_{tt,a}(t, x) + \cdots$$
$$= \log b \int \mathrm{d}t \mathrm{d}x T_{tt}$$

if we integrate out short time scales $(b\Lambda)^{-1} < |t'| < \Lambda^{-1}$.

So this amounts to a *rescaling of time*:

$$\frac{\mathrm{d}t}{\mathrm{d}\log b} = D\frac{C_{\mathcal{O}\mathcal{O}T}}{C_{TT}}.$$

[Aharony, Narovlansky; *Phys. Rev. Lett.* **121** 071601 (2018)]

Notice that if t_1 is close to t_2, diagonal term has

$$\int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}x \mathcal{O}_a(t_1, x)\mathcal{O}_a(t_2, x) = \int \mathrm{d}t \mathrm{d}x \mathrm{d}t' \frac{C_{\mathcal{O}\mathcal{O}T}}{|t|'} T_{tt,a}(t,x) + \cdots$$
$$= \log b \int \mathrm{d}t \mathrm{d}x T_{tt}$$

if we integrate out short time scales $(b\Lambda)^{-1} < |t'| < \Lambda^{-1}$.

So this amounts to a *rescaling of time*:

$$\frac{\mathrm{d}t}{\mathrm{d}\log b} = D \frac{C_{\mathcal{O}\mathcal{O}T}}{C_{TT}}.$$

[Aharony, Narovlansky; *Phys. Rev. Lett.* **121** 071601 (2018)]

In the holographic theory we studied,

$$\frac{C_{\mathcal{O}\mathcal{O}T}}{C_{TT}} = \frac{1}{8}$$

But$D$ *also flows under RG*. A natural RG scheme is to fix that

$$\frac{\mathrm{d}}{\mathrm{d}\log b}\langle 1\rangle_D = 0$$

which requires sending

$$(D+\delta D)\langle\mathcal{O}\mathcal{O}\rangle = D\langle\mathcal{O}\mathcal{O}\rangle - \frac{D^2}{2}\log b\frac{C_{\mathcal{O}\mathcal{O}T}}{C_{TT}}\langle T\mathcal{O}\mathcal{O}\rangle + t\text{-rescaling},$$

But D *also flows under RG.* A natural RG scheme is to fix that

$$\frac{\mathrm{d}}{\mathrm{d}\log b}\langle 1\rangle_D = 0$$

which requires sending

$$(D+\delta D)\langle\mathcal{O}\mathcal{O}\rangle = D\langle\mathcal{O}\mathcal{O}\rangle - \frac{D^2}{2}\log b\,\frac{C_{\mathcal{O}\mathcal{O}T}}{C_{TT}}\langle T\mathcal{O}\mathcal{O}\rangle + t\text{-rescaling},$$

Ultimately, we fix

$$\frac{\mathrm{d}\delta D}{\mathrm{d}\log b} = -\beta_D = -D^2\frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}}.$$

[Ganesan, Lucas, Radzihovsky; *Phys. Rev.* **D105** 066016 (2022)]

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]

Putting this together we conclude that as a function of energy,

$$D(E) = \frac{D}{1 + D\frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}} \log \frac{\Lambda}{E}}$$

which reproduces the holographic prediction that

$$D \sim \overline{\psi^2} \sim \frac{D}{1 + \frac{D}{8} \log(\Lambda r)}.$$

Integrate $\mathrm{d}t / \mathrm{d} \log E \sim D$ to obtain the effective z found before!

Putting this together we conclude that as a function of energy,

$$D(E) = \frac{D}{1 + D\frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}}\log\frac{\Lambda}{E}}$$

which reproduces the holographic prediction that

$$D \sim \overline{\psi^2} \sim \frac{D}{1 + \frac{D}{8}\log(\Lambda r)}.$$

Integrate $\mathrm{d}t/\mathrm{d}\log E \sim D$ to obtain the effective z found before!

What is special about the holographic models is that the OPE is simple, due to the large N limit?

Finite disorder fixed point

Now consider operator with scaling dimension

$$\Delta = \Delta_{\text{marginal}} - \nu,$$

such that $[D] = 2\nu$. Then we get

$$\beta_D = -2\nu D + D^2 \frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}}.$$

There is now a flow to a fixed point at

$$D_* = \frac{2\nu}{d} \frac{C_{TT}}{C_{\mathcal{O}\mathcal{O}T}}.$$

Finite disorder fixed point

Now consider operator with scaling dimension

$$\Delta = \Delta_{\mathrm{marginal}} - \nu,$$

such that $[D] = 2\nu$. Then we get

$$\beta_D = -2\nu D + D^2 \frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}}.$$

There is now a flow to a fixed point at

$$D_* = \frac{2\nu}{d} \frac{C_{TT}}{C_{\mathcal{O}\mathcal{O}T}}.$$

The resulting theory has Lifshitz scaling:

$$z_* = 1 + \frac{2\nu}{d},$$

and is a stable, strongly coupled, disordered fixed point!

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]

Finite disorder fixed point

The holographic confirmation proceeds similarly to before; for $d = z = 1$ and $\theta = 0$, we find

$$g_{tt}(r) \approx -\frac{1}{r^2}\left[1 + \frac{D}{16\nu} r^{2\nu}\right]^{-2},$$

which crosses over to the IR geometry at

$$r_{\mathrm{IR}} \sim \left(\frac{D}{\nu}\right)^{-1/2\nu},$$

which is non-perturbatively large in ν and D!

The holographic confirmation proceeds similarly to before; for $d = z = 1$ and $\theta = 0$, we find

$$g_{tt}(r) \approx -\frac{1}{r^2}\left[1 + \frac{D}{16\nu}r^{2\nu}\right]^{-2},$$

which crosses over to the IR geometry at

$$r_{\mathrm{IR}} \sim \left(\frac{D}{\nu}\right)^{-1/2\nu},$$

which is non-perturbatively large in ν and D!

We find that at the IR fixed point,

$$[D]_* = 0,$$

so the disorder supporting it is *exactly marginal.*

Comparison to weakly coupled theories

The IR Lifshitz exponent for charge-neutral perturbed CFTs was:

$$z_* = 1 + \frac{2\nu}{d}.$$

This same scaling has also been found in a large-N vector model in $d = 2$, $z = 1$, $\theta = 0$. Disorder is weakly relevant with

$$\nu = \frac{16}{3\pi^2 N}$$

and the disordered IR fixed point has

$$z_* = 1 + \nu = 1 + \frac{2\nu}{d}.$$

[Goldman, Thomson, Nie, Bi; *Phys. Rev.* **B101** 144506 (2020)]

More disordered fixed points

The holographic models are not limited to perturbations of charge-neutral CFTs! We can use the EMD backgrounds to study perturbations of finite density theories with non-trivial z and θ. Similar analysis reveals a flow to a new fixed point where

$$z_* \approx z + \frac{2\nu}{d}(z - \theta) > z, \qquad \theta_* = \theta.$$

More disordered fixed points

The holographic models are not limited to perturbations of charge-neutral CFTs! We can use the EMD backgrounds to study perturbations of finite density theories with non-trivial z and θ. Similar analysis reveals a flow to a new fixed point where

$$z_* \approx z + \frac{2\nu}{d}(z-\theta) > z, \qquad \theta_* = \theta.$$

Holography is crucial for these theories, where is no known analogue of conformal perturbation theory!

Transport and scaling exponents

At very low finite temperature, the disordered fixed point described approximately by usual Lifshitz black brane.

At very low finite temperature, the disordered fixed point described approximately by usual Lifshitz black brane.

We can calculate the thermoelectric transport coefficients. For small ν, the IR has perturbatively weak disorder, and thus there is a large conductivity:

$$\sigma_{\mathrm{dc}} = \frac{\rho^2}{D_*} \cdot \frac{K_0}{T^{(d-\theta_*+2)/z_*}},$$

where $K_0 \sim T^0$ and ρ is the U(1) charge density; D_* is the *universal* disorder strength at the fixed point.

At very low finite temperature, the disordered fixed point described approximately by usual Lifshitz black brane.

We can calculate the thermoelectric transport coefficients. For small ν, the IR has perturbatively weak disorder, and thus there is a large conductivity:

$$\sigma_{\mathrm{dc}} = \frac{\rho^2}{D_*} \cdot \frac{K_0}{T^{(d-\theta_*+2)/z_*}},$$

where $K_0 \sim T^0$ and ρ is the U(1) charge density; D_* is the *universal* disorder strength at the fixed point.

Scaling consistent with older predictions of

[Lucas, Sachdev, Schalm; *Phys. Rev.* **D89** 066018 (2014)]

We checked whether there is a sharp **Drude peak** in ac conductivity, associated with **coherent** charge transport (dominated by slow momentum relaxation):

$$\sigma(\omega) = \frac{\sigma_{\mathrm{dc}}}{1 - \mathrm{i}\omega\tau}.$$

We checked whether there is a sharp **Drude peak** in ac conductivity, associated with **coherent** charge transport (dominated by slow momentum relaxation):

$$\sigma(\omega) = \frac{\sigma_{\mathrm{dc}}}{1 - \mathrm{i}\omega\tau}.$$

We find that

$$\tau \sim T^{-(2+d-\theta_*)/z_*}.$$

We checked whether there is a sharp **Drude peak** in ac conductivity, associated with **coherent** charge transport (dominated by slow momentum relaxation):

$$\sigma(\omega) = \frac{\sigma_{\mathrm{dc}}}{1 - \mathrm{i}\omega\tau}.$$

We find that

$$\tau \sim T^{-(2+d-\theta_*)/z_*}.$$

This calculation only valid if $\tau T \gg 1$. Hence, coherent transport if

$$z_* < 2 + d - \theta_*.$$

Incoherent transport likely in other regimes?

Incoherent transport (momentum-insensitive) does dominate if $\omega \gg T$ (still below crossover to IR fixed point):

$$\sigma(\omega) \sim \omega^{2+(d-\theta_*-2)/z_*}.$$

Incoherent transport (momentum-insensitive) does dominate if $\omega \gg T$ (still below crossover to IR fixed point):

$$\sigma(\omega) \sim \omega^{2+(d-\theta_*-2)/z_*}.$$

Demanding that the IR fixed point has

$$[\rho] = d - \theta_* - \Phi_* = 0$$

requires an anomalous dimension for the density. Simultaneously,

$$[\sigma_{\mathrm{inc}}] = 3(d - \theta_*) + 2(z_* - 1 + \Phi_*).$$

[Davison, Goutéraux, Hartnoll; *JHEP* **10** 112 (**2015**)]
[Davison, Gentle, Goutéraux; *Phys. Rev.* **D100** 086020 (2019)]

Incoherent transport (momentum-insensitive) does dominate if $\omega \gg T$ (still below crossover to IR fixed point):

$$\sigma(\omega) \sim \omega^{2+(d-\theta_*-2)/z_*}.$$

Demanding that the IR fixed point has

$$[\rho] = d - \theta_* - \Phi_* = 0$$

requires an anomalous dimension for the density. Simultaneously,

$$[\sigma_{\rm inc}] = 3(d - \theta_*) + 2(z_* - 1 + \Phi_*).$$

[Davison, Goutéraux, Hartnoll; *JHEP* **10** 112 (**2015**)]
[Davison, Gentle, Goutéraux; *Phys. Rev.* **D100** 086020 (2019)]

Our IR fixed point consistent with both of these requirements.

Summary

We have found novel disordered IR fixed points using holographic models.

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]

Summary

We have found novel disordered IR fixed points using holographic models.

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]

Starting with a charge-neutral theory, our predictions are consistent with conformal perturbation theory, and remarkably even with some weakly-coupled models.

We have found novel disordered IR fixed points using holographic models.

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]

Starting with a charge-neutral theory, our predictions are consistent with conformal perturbation theory, and remarkably even with some weakly-coupled models.

Starting with generic z and θ, we didn't find that IR exponents are universal functions of ν in all models (but were in EMD models). The IR fixed points pass all sanity checks (marginal disorder, consistent thermodynamic/transport scalings, etc.).

Summary

We have found novel disordered IR fixed points using holographic models.

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]

Starting with a charge-neutral theory, our predictions are consistent with conformal perturbation theory, and remarkably even with some weakly-coupled models.

Starting with generic z and θ, we didn't find that IR exponents are universal functions of ν in all models (but were in EMD models). The IR fixed points pass all sanity checks (marginal disorder, consistent thermodynamic/transport scalings, etc.).

Our technical methods generalize to many other holographic models!