A holographic effective field theory for a metal with a Fermi surface

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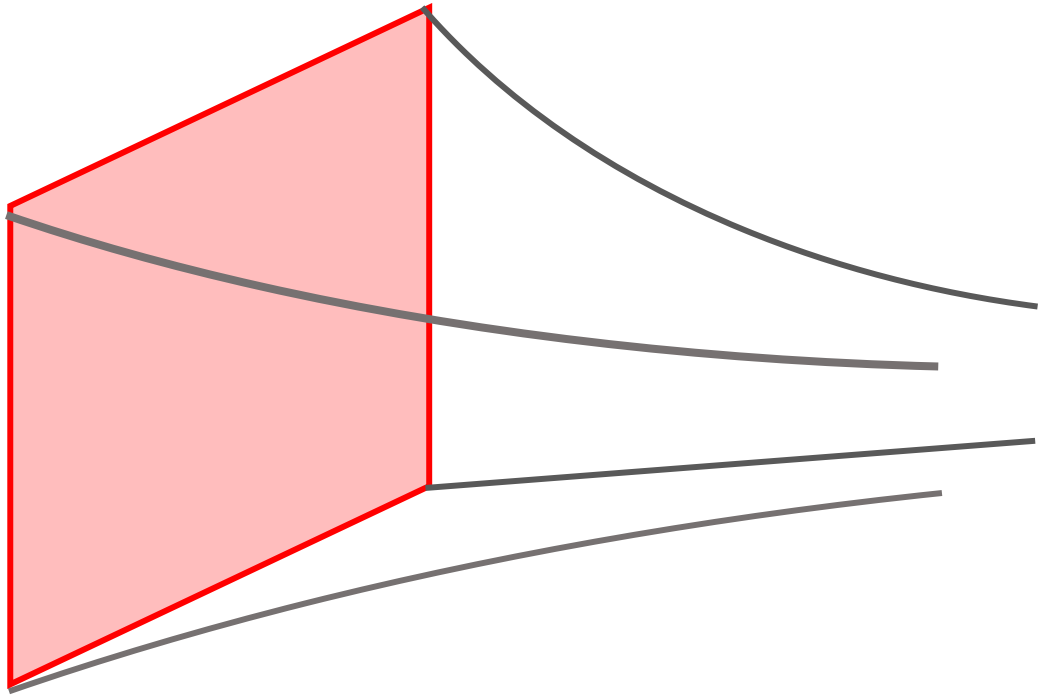
Holography



Quantum field theory
in D space-time dimensions

(strongly coupled)

Duality ↔

Quantum gravity
in D+1 space-time dimensions

(weakly coupled, solves classical equations of motion)

This talk:

Strongly coupled metals (a.k.a *non-Fermi liquids*)

Strange metals: example of non-Fermi liquid

Doped cuprates (e.g. YBCO = Yttrium Barium Copper Oxide)

High temperature superconductors (YBCO has T_c ~ 93 K)

T

Strange metal

Other examples in heavy fermion compounds, twisted bilayer graphene, …

AFM

SC

???

Doping

Outline

- What are the fundamental properties of metals (beyond weak coupling)?
- Review: previous holographic models of metals
- Constructing a new holographic model as an *effective field theory*
- Results from solution of the model

1. Fundamental properties of metals

What is a metal?

UV

Quantum field theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

e.g. non-relativistic electron with chemical potential

$$H = \int d^d\mathbf{x} \left[-\frac{1}{2m} \Psi^\dagger \nabla^2 \Psi - \mu \Psi^\dagger \Psi + (\mathrm{interactions}) \right]$$

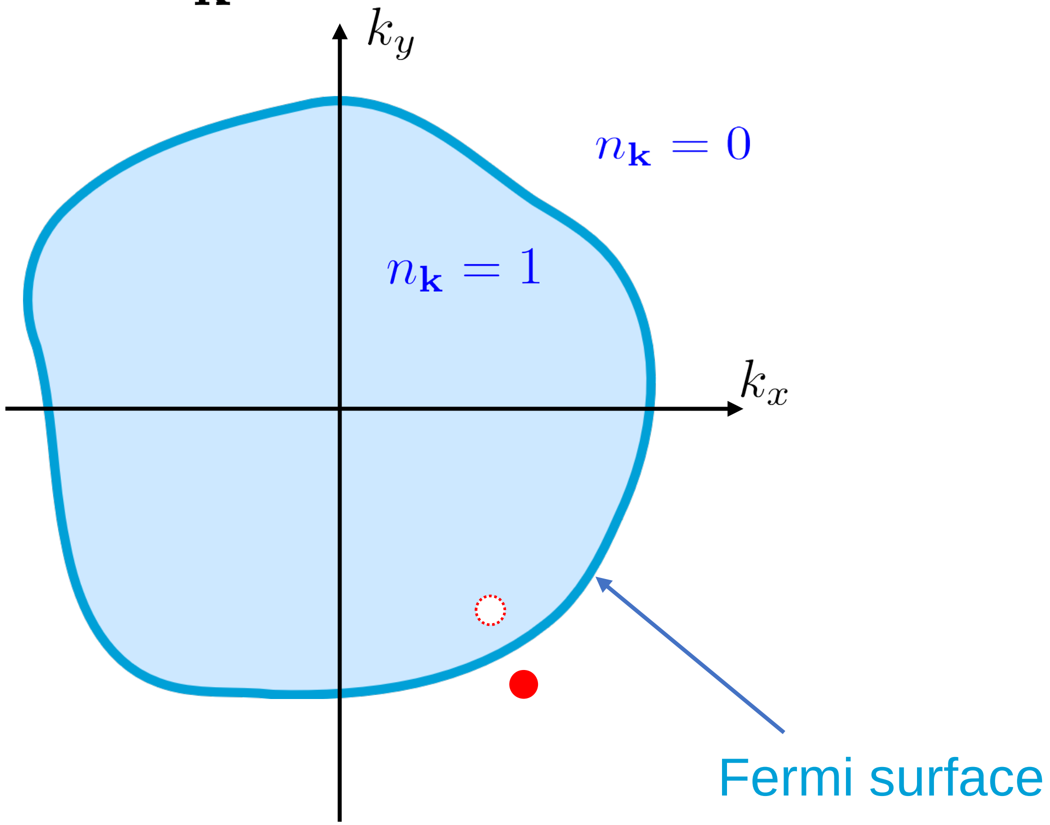
IR Effective field theory

Metal or superfluid

Spontaneously breaks U(1)

IR theory of a metal: non-interacting electrons

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}}$$



UV-IR relation:

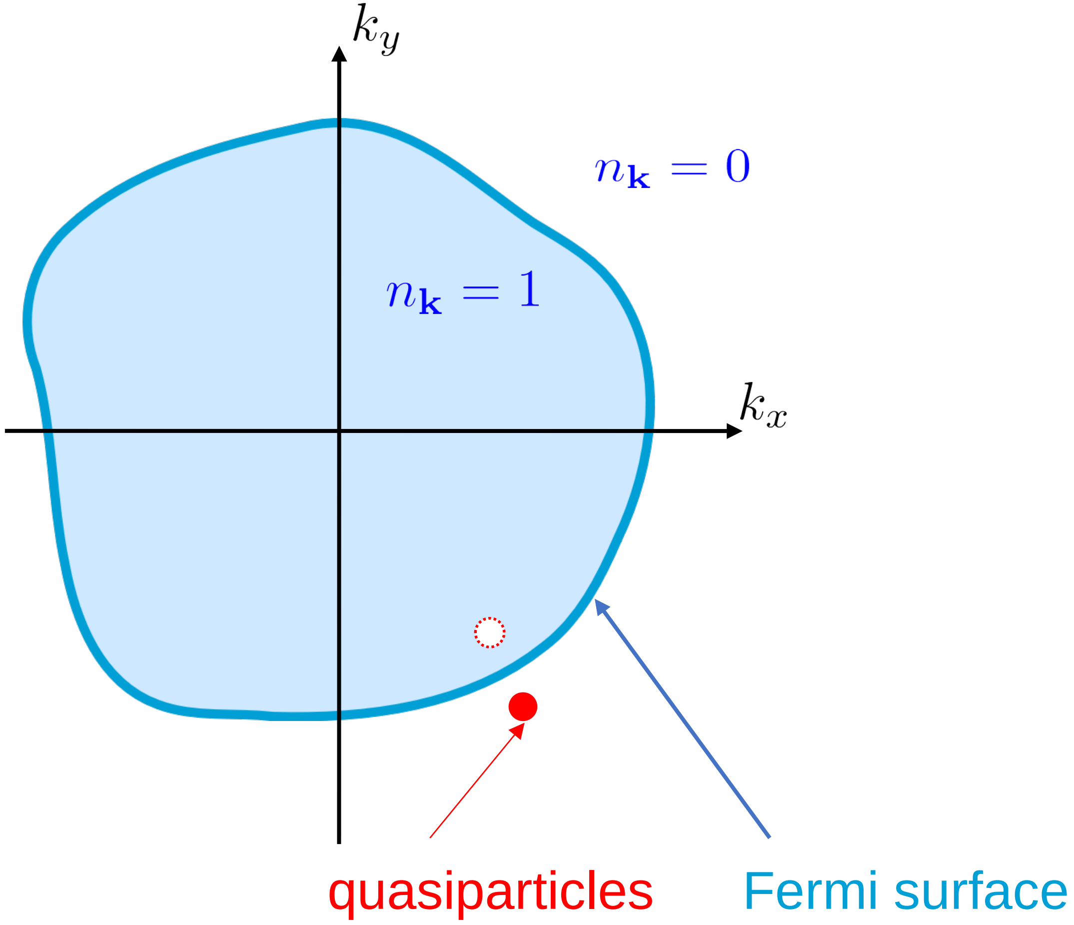
$$\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}$$

Volume enclosed by Fermi surface

UV charge density

IR theory of a metal: Fermi liquid theory

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \hat{n}_{\mathbf{k}} \hat{n}_{\mathbf{k}'}$$



UV-IR relation
(Luttinger's theorem)

$$\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}$$

Volume enclosed by
Fermi surface

UV charge density

UV

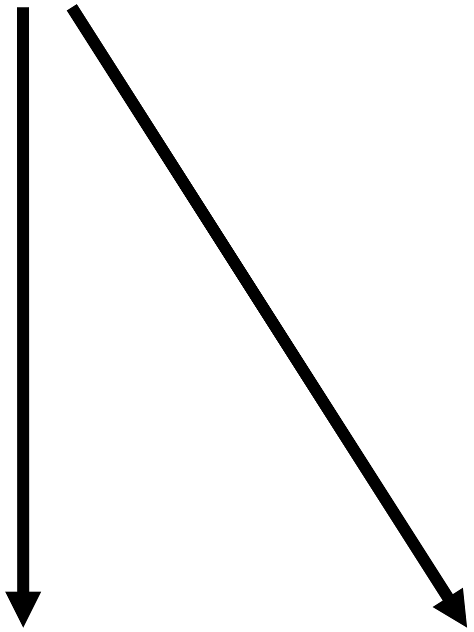
IR

Fermi liquid theory represents
a fixed-point under RG flow

Possibilities for IR fixed point

UV

Continuum theory with global U(1) symmetry and continuous translation symmetry
at nonzero charge density $\rho \neq 0$



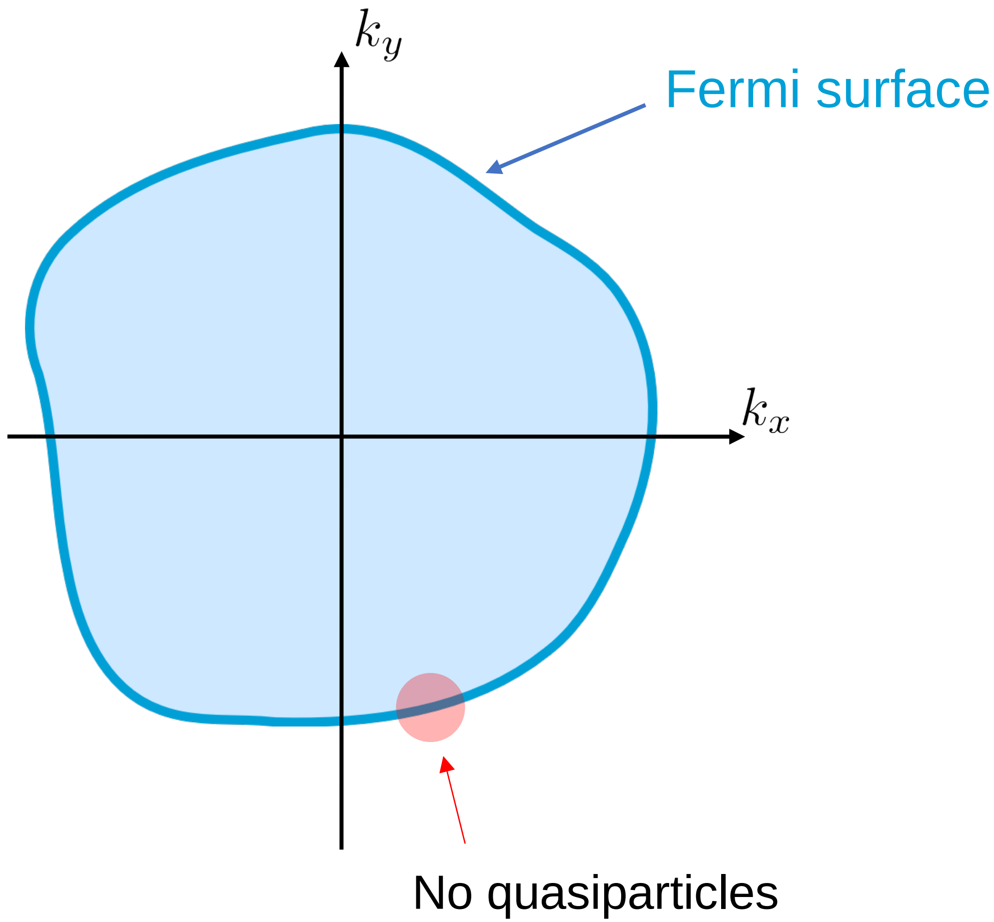
Fermi liquid

Strongly coupled fixed-point(s)
“Non-Fermi liquid metal”

???

General features of metals (beyond Fermi liquid)

They still have a Fermi surface!



The Fermi surface still obeys Luttinger's theorem!

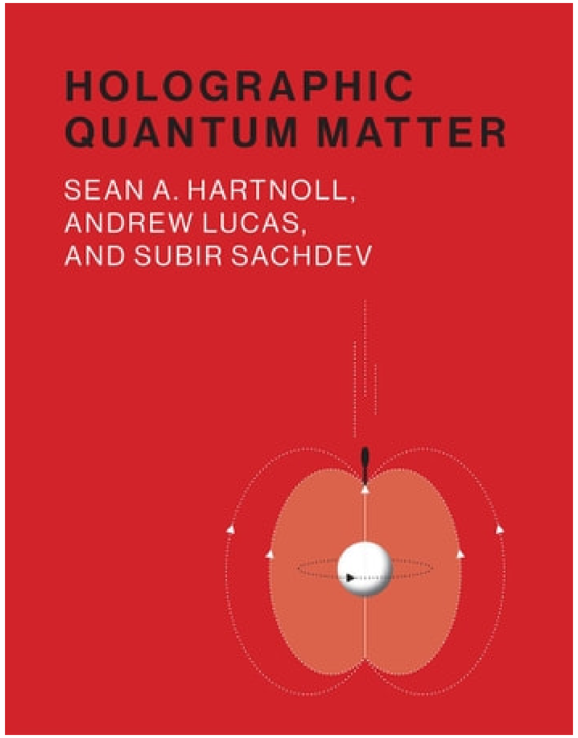
UV-IR relation
(Luttinger's theorem)

Volume enclosed by
Fermi surface

$$\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}$$

UV charge density

2. Review: previous holographic models



Holographic construction of metal

UV

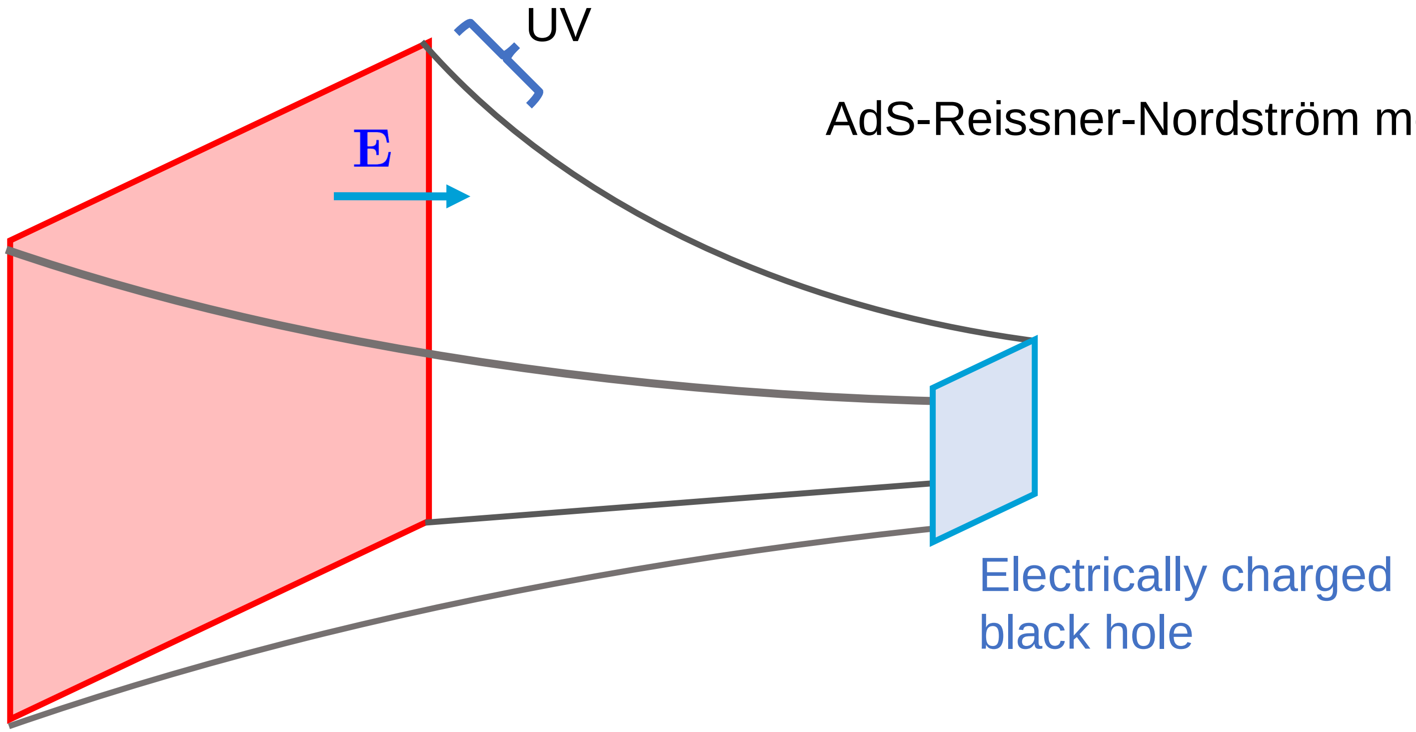
Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

For holography: take this to be a strongly coupled CFT perturbed by a chemical potential

IR Effective field theory

Metal

AdS-Reissner-Nordström metric



The dual field theory has nonzero entropy density
even in the limit T=0 (!!)

3D CFT ⟷ Asymptotically AdS_4 geometry in the UV region

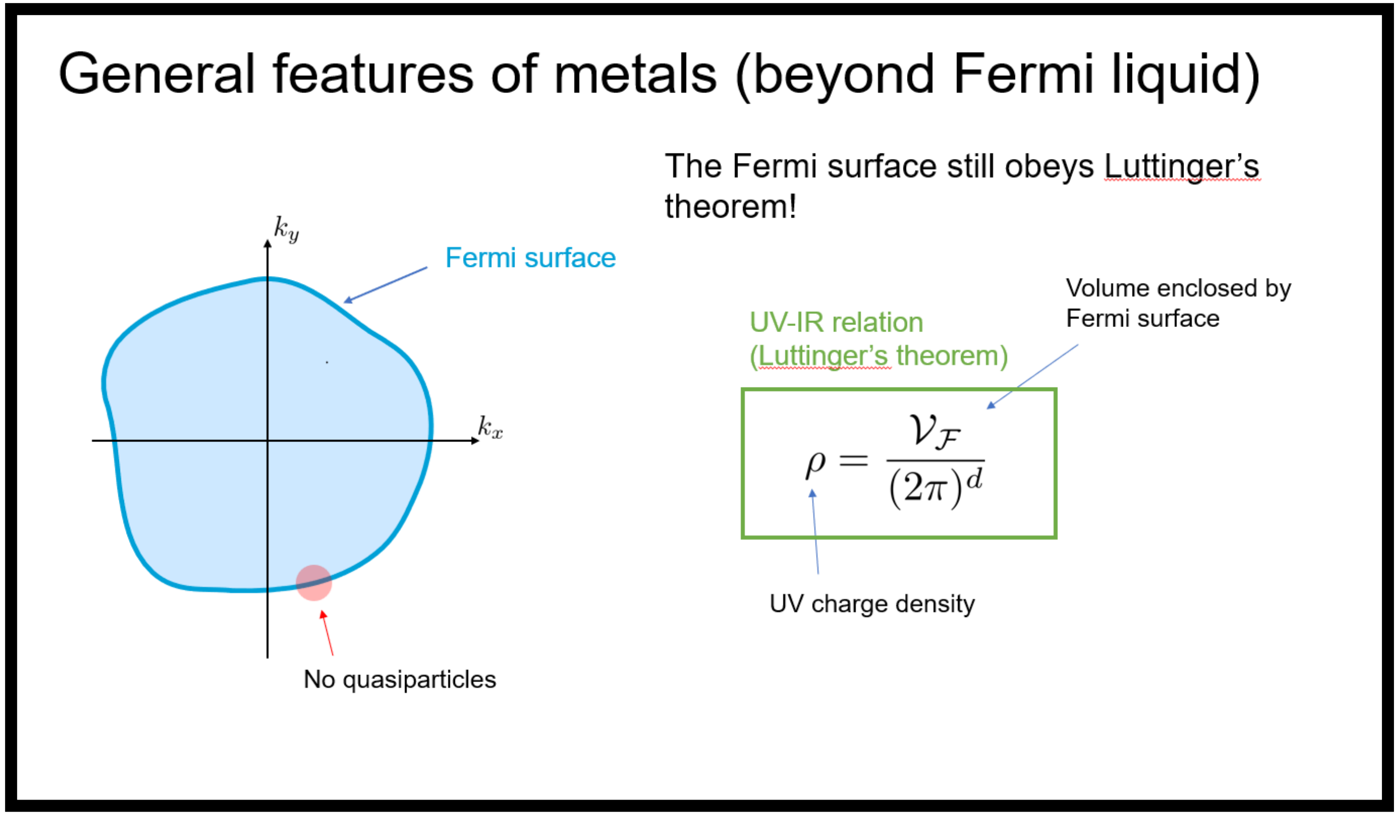
Nonzero charge density ⟷ Electric field in the UV region in the direction normal to the boundary

The missing Fermi surface

- These models have *no trace of a Fermi surface**

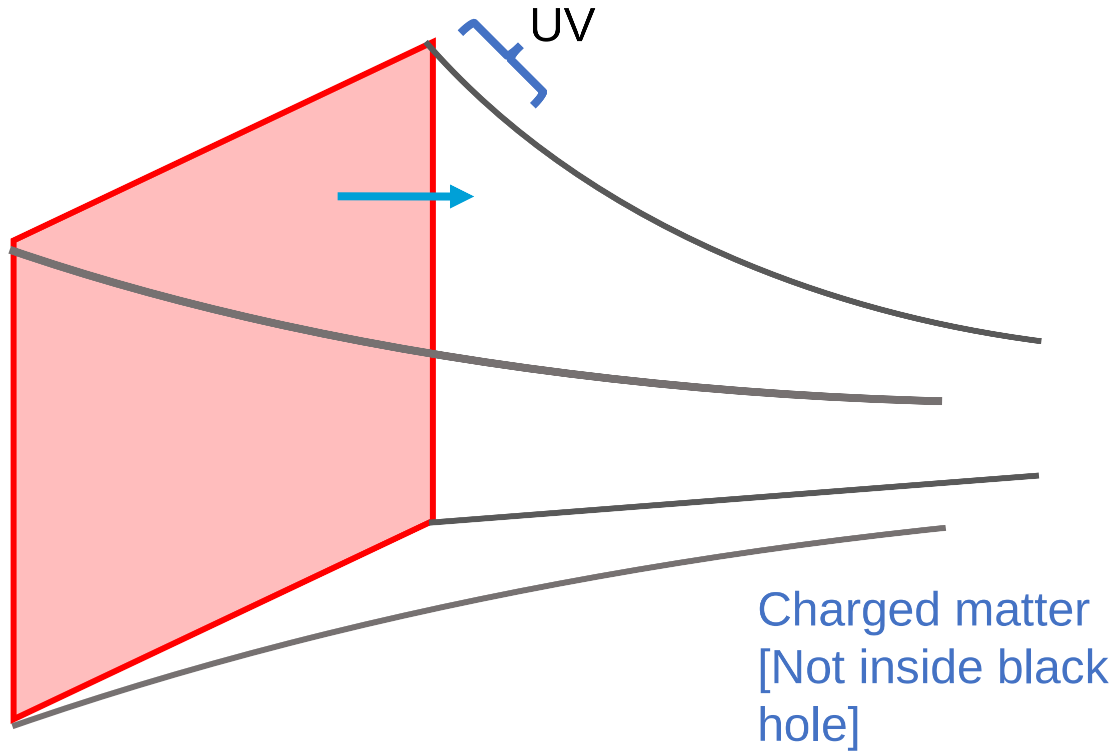
* at least not a Fermi surface satisfying Luttinger's theorem

Recall from before:



There are some hints that the Fermi surface may come back if we include quantum corrections in the bulk

Electron star models



[Hartnoll and Tavanfar, PRD
2011]

3. A new model

What are we actually trying to do?

The previous holographic models are attempting to find an exact description of this *entire* RG flow

Instead:

Just find a holographic description of the IR theory

UV Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

IR Effective field theory

Metal

What we usually do in condensed matter physics:

UV Some material, e.g. YBCO

???

IR We narrow down what the IR theory could be through *experimental* probes of the material

We want a holographic *effective* field theory

UV Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

Need to impose an "emergeability" condition

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

Holographic

IR Effective field theory

Metal

Emergeability condition

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

UV

Global U(1) symmetry
and continuous translation symmetry

Nonzero charge density
$\rho \neq 0$

IR

UV global symmetries have to act on the IR theory

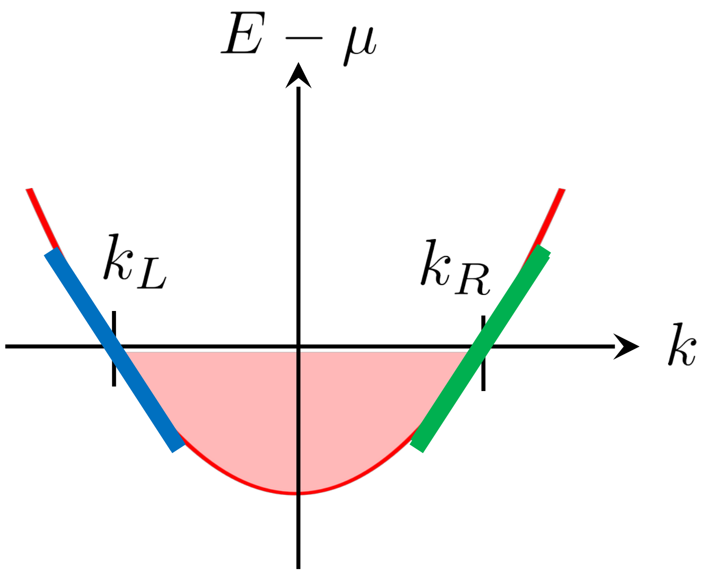
~~$\rho_{\mathrm{IR}} = \rho_{\mathrm{UV}}$~~

Charge density is *not* an RG invariant

Example: electron gas in 1 spatial dimension

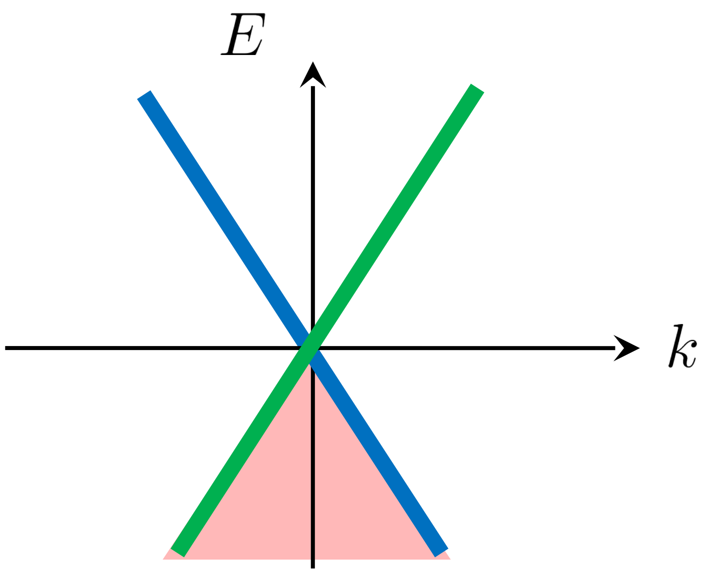
$$H = \int d^d\mathbf{x} \left[-\frac{1}{2m}\Psi^\dagger \nabla^2 \Psi - \mu \Psi^\dagger \Psi \right]$$

UV theory $(\rho \neq 0)$



IR theory: (1+1)-D Dirac fermion

$(\rho = 0)$



Emergent

$\mathrm{U}(1)_L \times \mathrm{U}(1)_R$

symmetry

Chiral anomaly

$$\partial_\mu (j^{(R)})^\mu = \frac{1}{2\pi} E$$

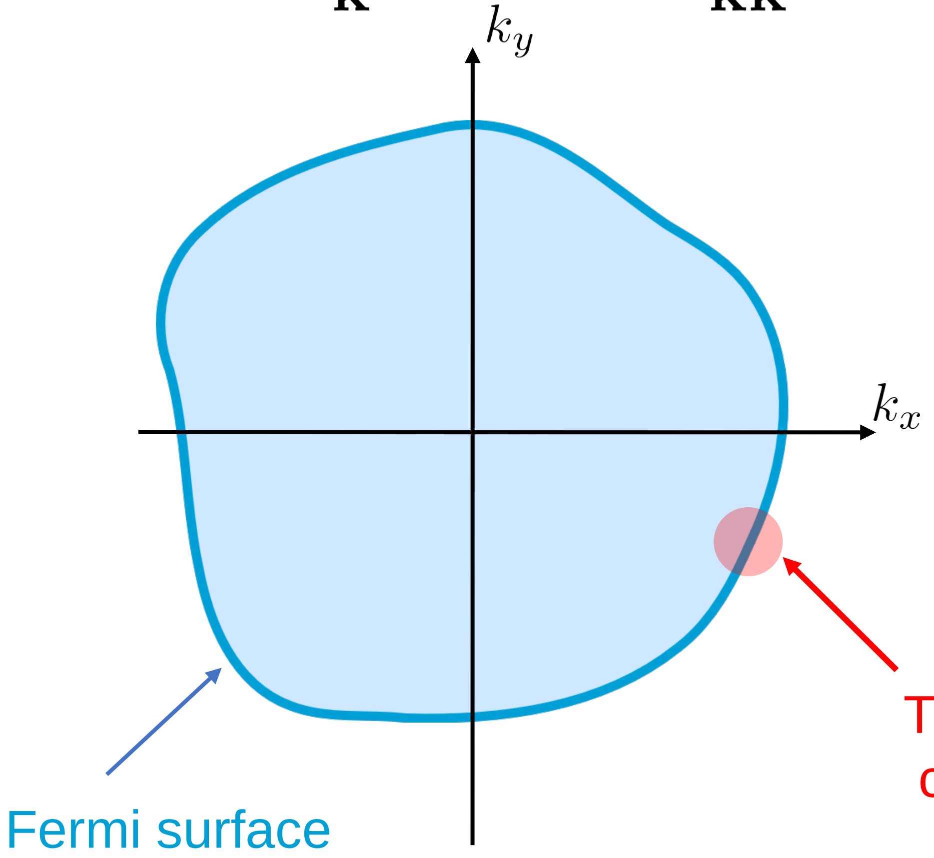
[Example of a ‘t Hooft anomaly]

The emergent symmetry and anomaly is precisely what allows a (1+1)-D Dirac fermion to be emergeable from a theory with nonzero charge density

Emergent symmetry of Fermi liquid theory in 2 spatial dimensions

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \hat{n}_{\mathbf{k}} \hat{n}_{\mathbf{k}'}$$



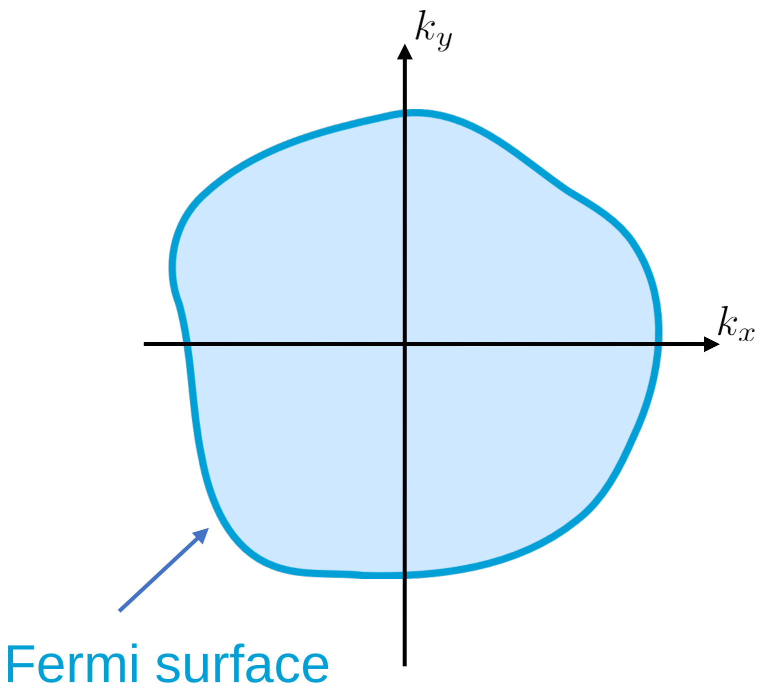
G_{IR}
$= \{$ Smooth functions from $S^1 \to \mathrm{U}(1)$ $\}$
$= \mathrm{LU}(1)$

"Loop group"

The charge at each point of the Fermi surface is *individually* conserved

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

- The emergent LU(1) symmetry (and its anomaly) turns out to be precisely the information needed to derive Luttinger's theorem



UV-IR relation
(Luttinger's theorem)

Volume enclosed by Fermi surface

$$\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}$$

UV charge density

We want a holographic *effective* field theory

UV Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

Need to impose an “emergeability” condition

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

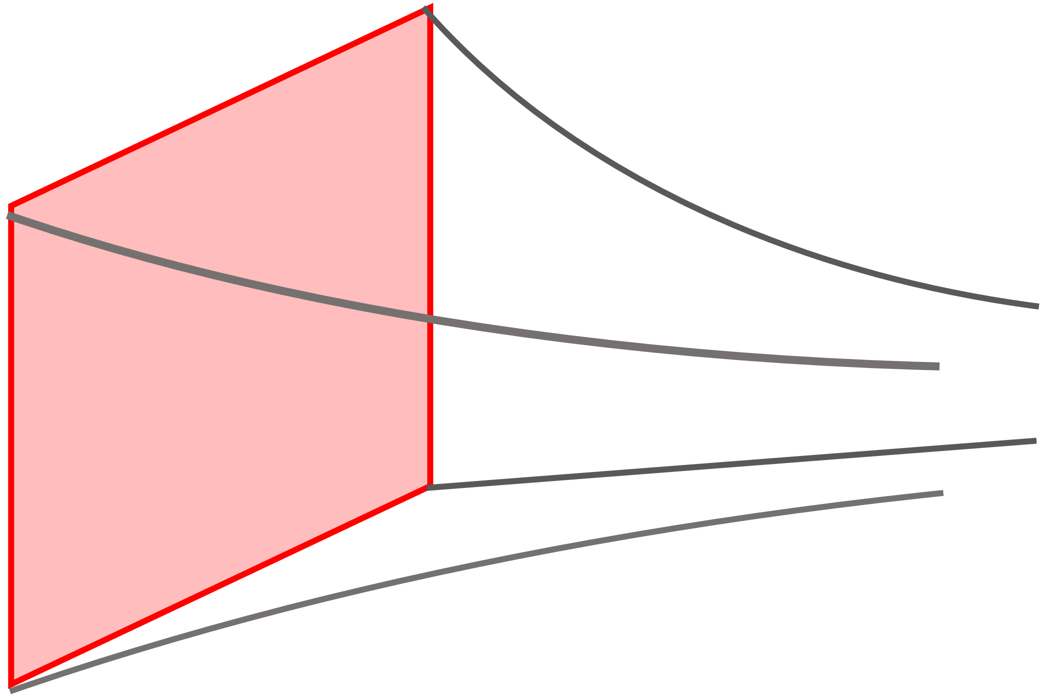
Just try to find a holographic effective field theory of a metal

IR Effective field theory

Metal

Impose that the field theory has a global LU(1) symmetry (with anomaly)

Holography with a global LU(1) symmetry



Duality ←→

Quantum field theory
in 3 space-time dimensions

[with global LU(1) symmetry]

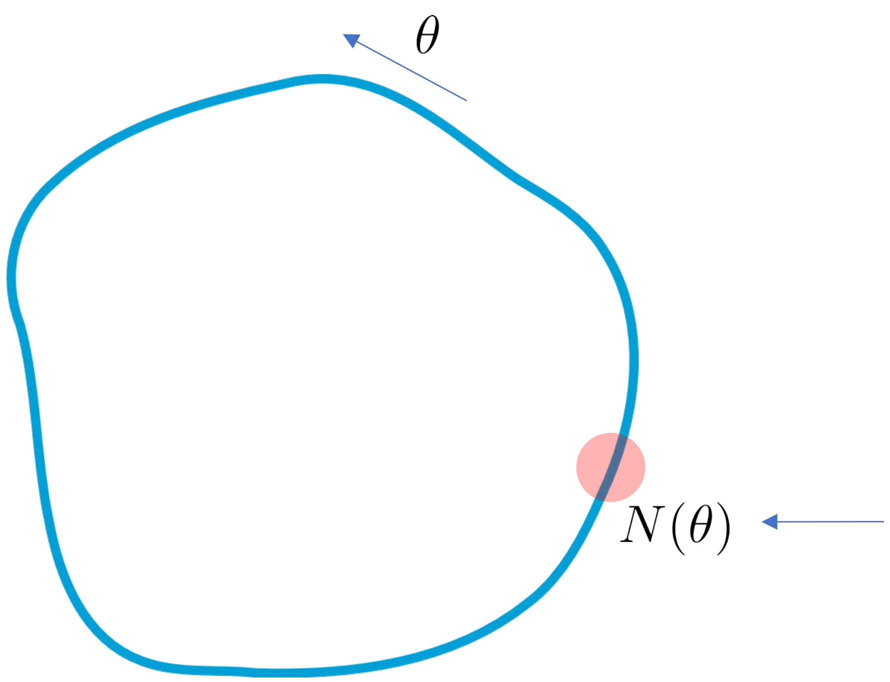
[LU(1) has ‘t Hooft anomaly]

Gravitational theory
in 4 space-time dimensions

[with dynamical LU(1) gauge field]

[Gauge field has a Chern-Simons term]

LU(1) gauge field



G_{IR}
$= \{$ Smooth functions from $S^1 \to \mathrm{U}(1)$ $\}$
$= \mathrm{LU}(1)$

"Loop group"

Generators are parameterized by θ

Total charge of the U(1) subgroup: $Q = \int N(\theta) d\theta$

An $\mathrm{LU}(1)$ gauge field on a space-time M is a family of vector fields $a_\mu(\theta)$ parameterized by θ

Gauge transformation: $a_\mu(\theta) \to a_\mu(\theta) + \partial_\mu \lambda(\theta)$

This is basically equivalent to a $\mathrm{U}(1)$ gauge field on $M \times S^1$

The Chern-Simons term

The action for the LU(1) gauge field will include a Chern-Simons term

$$\frac{m}{24\pi^2}\int_{M_4\times S^1} A\wedge dA\wedge dA$$

$m\in\mathbb{Z}$

4D space-time

An important remark:

- The metric that satisfies the Einstein equations lives on M_4, *not* $M_4 \times S^1$
- The Fermi surface coordinate is an *internal* label for the symmetry group, not a space-time dimension
- e.g. If the symmetry group were U(1) x U(1), we would not define a metric that lives on a space-time with two disconnected components.

Another important remark

- LU(1) conservation law in 3-dimensional space-time M_3 is *not* the same as a U(1) conservation law in $M_3 \times S^1$

$\mathrm{LU}(1)$ conservation law in M_3 :

$$\partial_\mu j^\mu(\theta) = (\text{anomaly term})$$

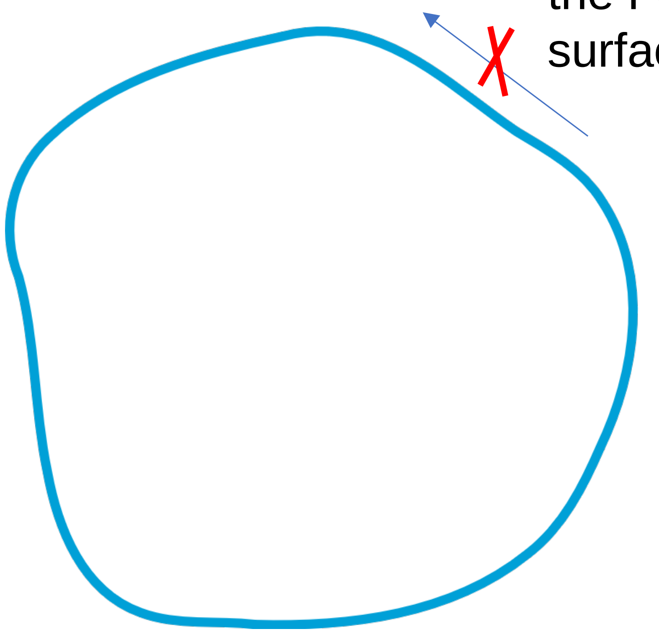
This index ranges over the 3 coordinates of M_3

$\mathrm{U}(1)$ conservation law in $M_3 \times S^1$

$$\partial_\mu j^\mu(\theta) + \partial_\theta j^\theta = (\text{anomaly term})$$

LU(1) conservation enforces that $j^\theta = 0$

No flow of charge *along* the Fermi surface



The Maxwell term in the bulk

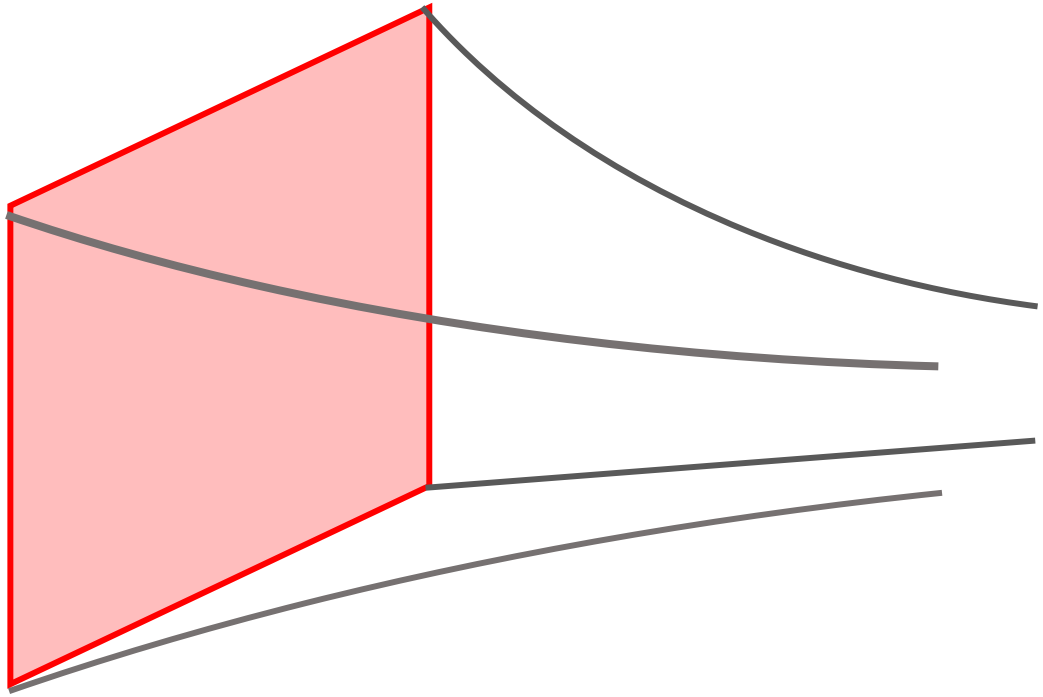
Indices range over the 4 space-time coordinates, *not* including the Fermi surface coordinate θ

$$\int_{M_4} d^4x\sqrt{-g}\int d\theta\, f_{\mu\nu}(\theta)\, f^{\mu\nu}(\theta)$$

$$f_{\mu\nu}(\theta) = \partial_\mu a_\nu(\theta) - \partial_\nu a_\mu(\theta)$$

Metric on the 4-D space-time

The holographic model



Quantum field theory
in 3 space-time dimensions

Duality

$$S[g,a] = S_{\text{gravitational}} + S_{\text{Maxwell}} + S_{\text{Chern-Simons}}$$

[with global LU(1) symmetry]

“Bottom-up holography”

[LU(1) has anomaly]

Boundary conditions

- Asymptotic metric is AdS_4

$$ds^2 = \frac{L^2}{r^2}(-dt^2 + dx^2 + dy^2 + dz^2)$$

- Asymptotic solutions to the equations of motion for the LU(1) gauge field take the form

$$a_\alpha = a_\alpha^{(0)} + r a_\alpha^{(1)}$$

- The holographic dictionary tells us to identify

$$a_\alpha^{(0)} = A_\alpha$$

[background LU(1) gauge field in the dual QFT]

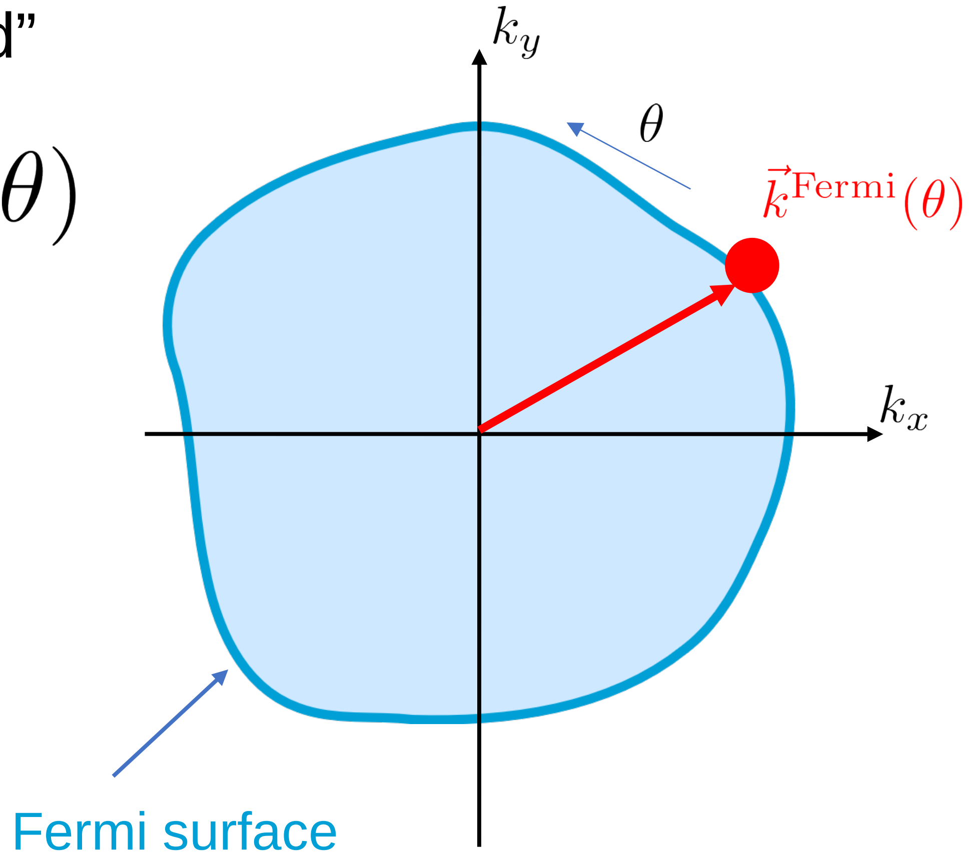
$$a_\alpha^{(1)} = \langle j_\alpha \rangle$$

[LU(1) current in the dual QFT]

Properties to impose on the dual QFT

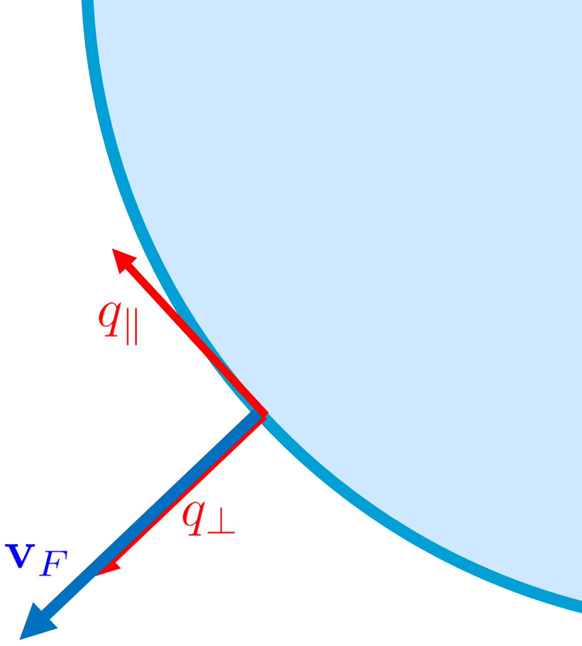
- The charge density of the LU(1) symmetry is *zero*
- We need to apply a "phase space magnetic field"

$$f_{\theta i} := \partial_\theta a_i - \partial_i a_\theta = \partial_\theta k_i^{\mathrm{Fermi}}(\theta)$$



4. Results from solution of the model

Results from solution of the model



Chern-Simons level

Green's function of the LU(1) charge density [for $m > 0$]

$$G_{j^t(\theta)j^t(\theta')} = \left[-\frac{m|\partial_\theta \mathbf{k}_F(\theta)|}{(2\pi)^2} \frac{q_\perp}{\omega - q_\perp} + \frac{\alpha^{-1} q_{\|}^2}{\sqrt{q_\perp^2 - \omega^2}} + O\left(\frac{1}{k_F}\right) \right] \delta(\theta - \theta')$$

Fermi-liquid-like behavior (undamped ballistic mode)

Non-Fermi liquid corrections

Results from solution of the model

Optical conductivity: $j^i = \sigma^{ij}(\omega) E_j$ (at $\mathbf{q} = 0$)

$$\sigma^{ij}(\omega) = \frac{i}{\omega}\mathcal{D}^{ij} + \sigma^{ij}_{\text{inc}} + O\left(\frac{1}{k_F}\right)$$

"Drude" or "coherent" conductivity

"Incoherent" conductivity (absent in Fermi liquid theory)

Conclusions

- I have presented a new holographic model which incorporates the essential physics of strongly coupled metals, including the Fermi surface
- A jumping off point to build models of strongly coupled metals
- One future direction: weakly break the LU(1) symmetry to model scattering and get nonzero DC resistivity