Fracton gauge theory vs Gravity

Based on arXiv:2310.21610, 2107.13884

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Plan

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- Conservation laws and symmetry algebra
- From Poincaré to Fracton algebra
- Fracton gauge theory from a non-relativistic limit of gravity
- Conclusion

A **fracton** is an emergent topological quasiparticle excitation which is immobile when in isolation.[1][2] Many theoretical systems have been proposed in which fractons exist as elementary excitations. Such systems are known as fracton models. Fractons have been identified in various CSS codes as well as in symmetric tensor gauge theories.

Gapped fracton models often feature a topological ground state degeneracy that grows exponentially and sub-extensively with system size. Among the gapped phases of fracton models, there is a non-rigorous phenomenological classification into "type I" and "type II". Type I fracton models generally have fracton excitations that are completely immobile, as well as other excitations, including bound states, with restricted mobility. Type II fracton models generally have fracton excitations and no mobile particles of any form. Furthermore, isolated fracton particles in type II models are associated with nonlocal operators with intricate fractal structure.[3]

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Applications [edit]

Fractons were originally studied as an analytically tractable realization of quantum glassiness where the immobility of isolated fractons results in a slow relaxation rate. [14] [15] This immobility has also been shown to be capable of producing a partially self-correcting quantum memory, which could be useful for making an analog of a hard drive for a quantum computer. [16] [17] Fractons have also been shown to appear in quantum linearized gravity models [18] and (via a duality) as disclination crystal defects. [19] However, aside from the duality to crystal defects, and although it has been shown to be possible in principle, [20] [21] other experimental realizations of gapped fracton models have not yet been realized. On the other hand, there has been progress in studying the dynamics of dipole-conserving systems, both theoretically[22][23][24] and experimentally,[25][26] which exhibit the characteristic slow dynamics expected of systems with fractonic behavior.

Fracton models [edit]

	U(1) symmetric tensor gauge theory	type-I	type-II
energy spectrum	gapless	gapped	gapped
example models	scalar charge [13]	X-cube [27]	Haah's cubic code [28]
example characteristics	conserved dipole moment	conserved charge on stacks of two-dimensional surfaces	fractal conservation laws, no mobile particles

It has been conjectured [4] that many type-I models are examples of foliated fracton phases; however, it remains unclear whether non-Abelian fracton models[29][30][31] can be understood within the foliated framework.

References [edit]

1. ^ Vijay, Sagar; Haah, Jeongwan; Fu, Liang (2015). "A New Kind of Topological Quantum Order: A Dimensional Hierarchy of Quasiparticles Built from Stationary Excitations". *Physical Review B*. **92** (23): 235136. arXiv:1505.02576. Bibcode:2015PhRvB..92w5136V.

17. ^ Brown, Benjamin J; Loss, Daniel; Pachos, Jiannis K; Self, Chris N; Wootton, James R (2016). "Quantum memories at finite temperature" (PDF). *Reviews of Modern Physics*. **88** (4): 045005. arXiv:1411.6643. Bibcode:2016RvMP...88d5005B, doi:10.1103/RevModPhys.88.045005.

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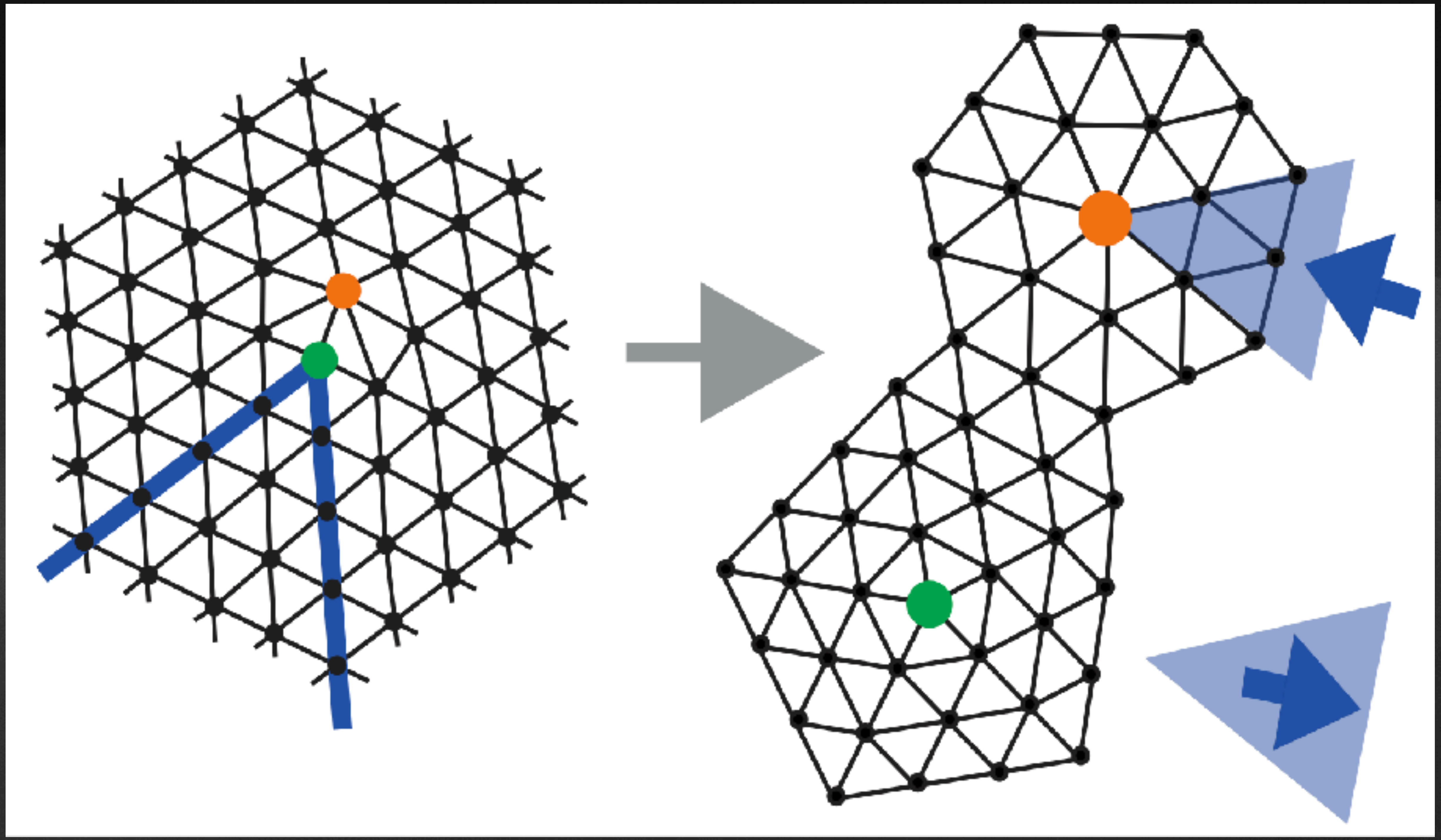

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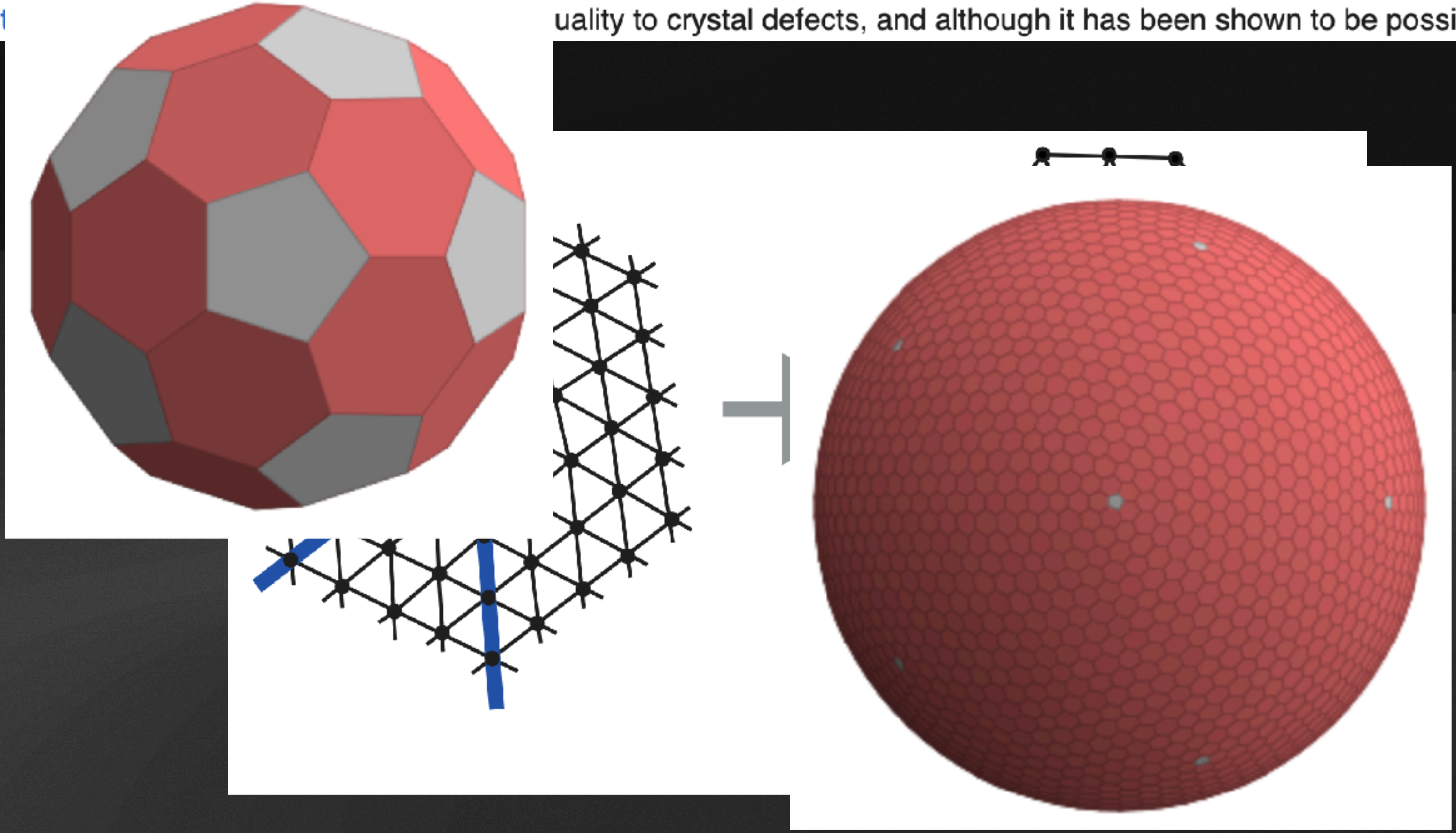

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Kinematics of dipole conserving systems

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The conservation of momentum and dipole moment imply

$$\dot{P}_i = 0$$

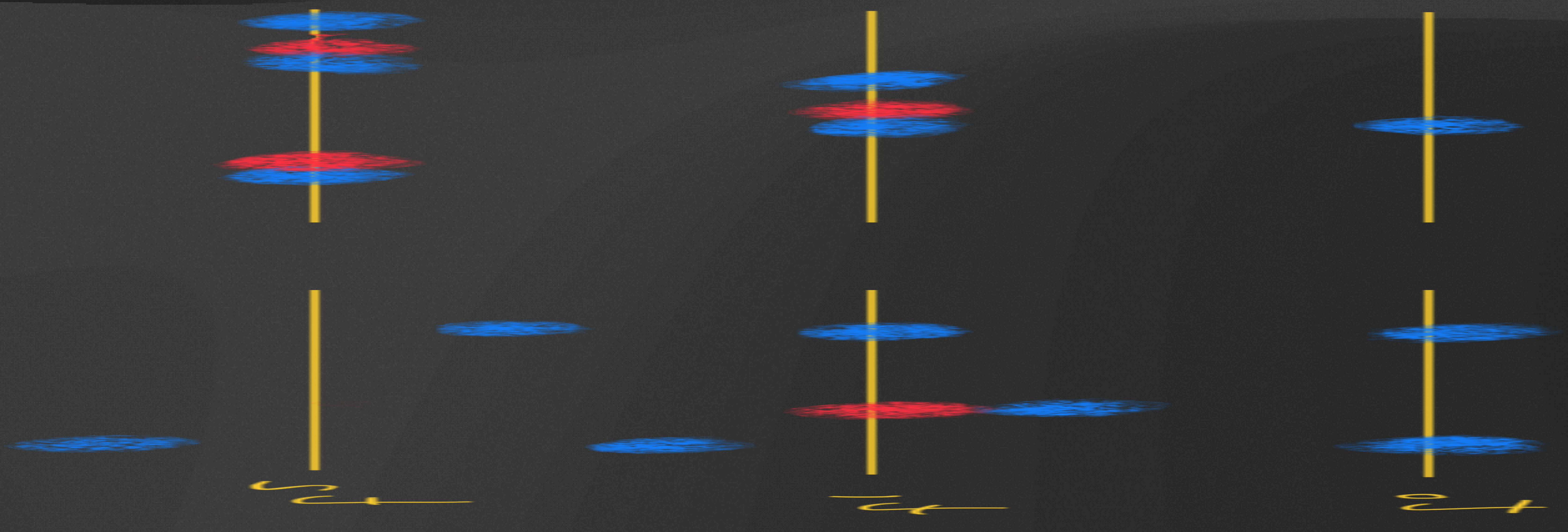
$$\dot{Q} = 0$$

$$\dot{Q}_i = 0$$

$$\dot{x}_i = 0$$

Kinematics of dipole conserving systems

However, motion is not strictly prohibited if dipoles can be absorbed or emitted



$$\dot{P}_i = 0$$

$$\dot{Q} = 0$$

$$\dot{Q}_i = 0$$

Conservation laws and symmetry algebra

Conservation laws and symmetry algebra

The charge conservation is captured by the continuity equations

$$Q = \int d^d x\, \varrho, \qquad Q^i = \int d^d x\, x^i \varrho$$

$$\partial_0 \varrho + \partial_i j_i = 0 \quad , \quad \partial_0 p_j + \partial_i \tau_{ij} = 0$$

$$P_i = \int d^d x\, p_i, \qquad J_{ij} = \int d^d x \left(x^i p_j - x^j p_i \right)$$

$$j_i = \partial_j K_{ji} \quad , \quad \tau_{[ij]} = \partial_k L_{kij}$$

The conservation of energy, momentum, angular momentum, charge and dipole moment form a group with algebra

$$[\mathbf{P}_i, \mathbf{Q}^j] = \delta_i^j \mathbf{Q}$$

$$[\mathbf{J}_{ij}, \mathbf{Q}^k] = \delta^k_{[j} \mathbf{Q}_{i]}, \qquad [\mathbf{J}_{ij}, \mathbf{P}_k] = \delta_{k[j} \mathbf{P}_{i]}, \qquad [\mathbf{J}_{ij}, \mathbf{J}_{kl}] = \delta_{[i[k} \mathbf{J}_{l]j]},$$

Gromov 1812.05104

Grosvenor, Hoyos, P-B, Surówka 2105.01084

An “electrodynamics” version this systems can be obtained with the gauge principle

$$\delta A_{ij} = \partial_i \partial_j \varepsilon, \quad \delta\phi = -\dot{\varepsilon}$$

The “electromagnetic” fields are defined as

$$F_{ijk} = \partial_i A_{jk} - \partial_j A_{ik}, \quad F_{0ij} = \dot{A}_{ij} + \partial_i \partial_j \phi$$

And the action for the minimally couple system would contain the terms

$$S = \frac{1}{2}\int d^{d+1}x \left(F_{0ij}F_{0ij} - \frac{1}{2}F_{ijk}F_{ijk} \right) \qquad S_M \sim \int d^{d+1}x \left(\varrho\phi + K_{ij}A_{ij} \right)$$

Pretko 1606.08857

Notice that the gauge fields action can be express as

$$S = \frac{1}{4}\int d^{d+1}x F_{\mu\nu\rho}F^{\mu\nu\rho}$$

Provided the constraint is imposed

$$\tau^{\nu}A_{\mu\nu} = -\partial_{\mu}\phi$$

The field strength and gauge transformations read

$$F_{\mu\nu\rho} = \partial_{\mu}A_{\nu\rho} - \partial_{\nu}A_{\mu\rho} \qquad \delta A_{\mu\nu} = \partial_{\mu}\partial_{\nu}\varepsilon$$

A minimal generalisation to generic background geometries could be

$$F_{\mu\nu\lambda} = \nabla_\mu A_{\nu\lambda} - \nabla_\nu A_{\mu\lambda} \qquad \delta A_{\mu\nu} = \nabla_\mu \nabla_\nu \epsilon$$

However, $\delta F_{\mu\nu\rho} = [\nabla_\mu, \nabla_\nu]\partial_\rho \epsilon = -R^\alpha{}_{\rho\mu\nu}\,\partial_\alpha \epsilon$

Therefore, a possible way out is to require the background to satisfy $\nabla_\mu R^{\mu\nu\rho\sigma} = 0$

And propose the action

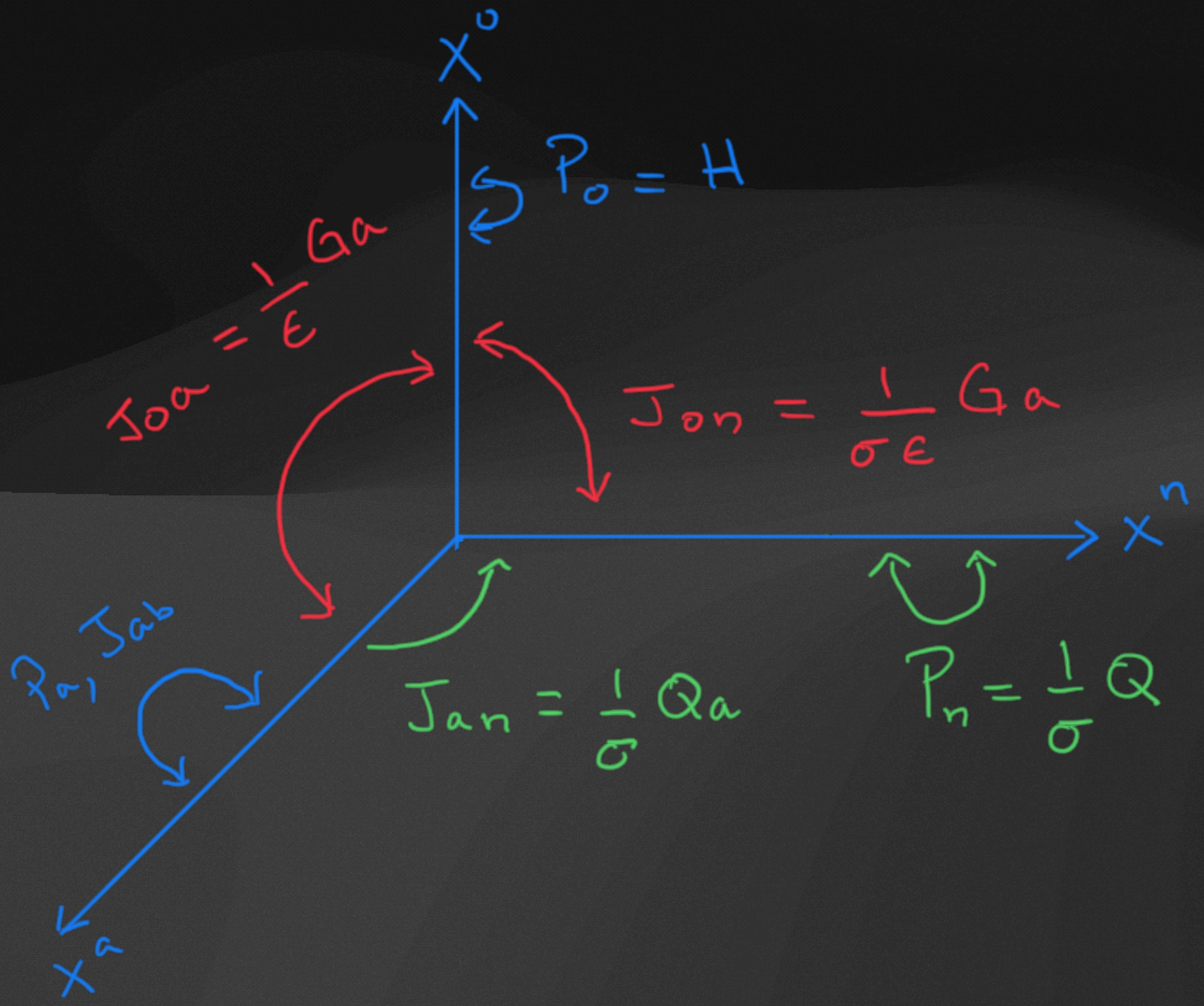
$$S = -\int d^{d+1}x\,\sqrt{|g|}\,\left(\frac{1}{2}F_{\mu\nu\lambda}F^{\mu\nu\lambda} - R^{\mu\nu\rho\sigma}A_{\mu\rho}A_{\nu\sigma}\right)$$

From Poincaré to Fracton algebra

Contraction of Poincare algebra

$\epsilon \to 0$

$\sigma \sim 0$



$$[\mathbf{J}_{\mu\nu}, \mathbf{P}_\rho] = \eta_{\rho[\nu}\mathbf{P}_{\mu]}$$

$$[\mathbf{J}_{\mu\nu}, \mathbf{J}_{\rho\sigma}] = \eta_{[\mu[\rho}\mathbf{J}_{\sigma]\nu]}$$

Contraction of Poincare algebra

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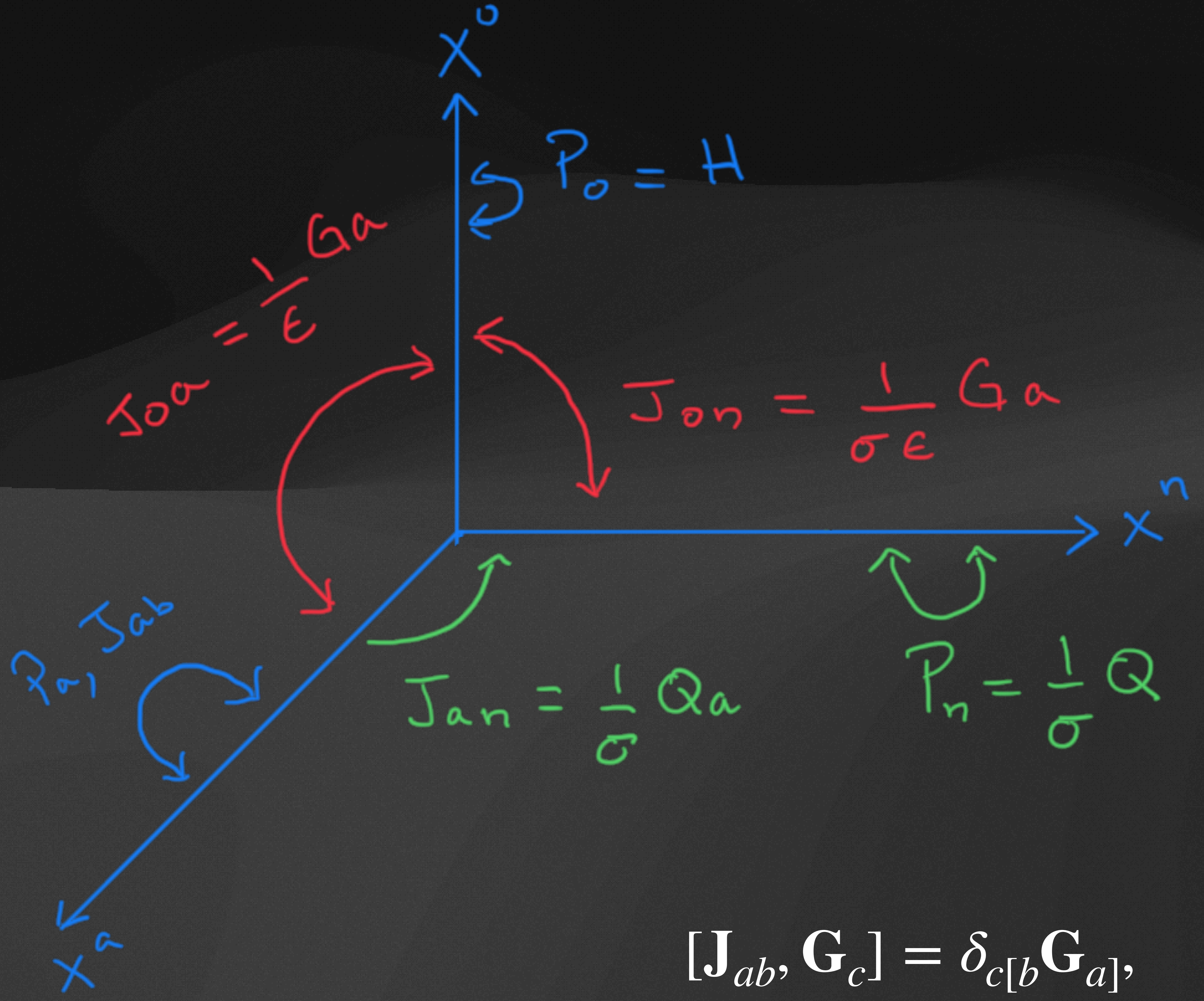

MDMA +

$$[\mathbf{Q}_a, \mathbf{Q}] = \sigma^2 \mathbf{P}_a$$

$$[\mathbf{Q}_a, \mathbf{Q}_b] = -\sigma^2 \mathbf{J}_{ab}$$

$$[\mathbf{J}_{ab}, \mathbf{G}_c] = \delta_{c[b} \mathbf{G}_{a]}, \qquad [\mathbf{G}_a, \mathbf{Q}_b] = \delta_{ab} \mathbf{K}, \qquad [\mathbf{Q}_a, \mathbf{K}] = \sigma^2 \mathbf{G}_a$$

Fracton gauge theory from a non-relativistic limit of gravity

Fracton gauge theory from a non-relativistic limit of gravity

We assume the Poincare group in d+2 is broken down to Lorentz in d+1

$$\mathbf{A} = e^{\Psi^A \mathbf{J}_{An}} e^{\Phi^A \mathbf{P}_A} \left(\tilde{\mathbf{A}} + d \right) e^{-\Phi^B \mathbf{P}_B} e^{-\Psi^B \mathbf{J}_{Bn}}$$

$$\mathbf{A} = E^A \mathbf{P}_A + \frac{1}{2} \Omega^{AB} \mathbf{J}_{AB}$$

$$\mathbf{F} = d\mathbf{A} + \mathbf{A} \wedge \mathbf{A} = T^A \mathbf{P}_A + \frac{1}{2} R^{AB} \mathbf{J}_{AB}$$

The action is

$$S = -\frac{1}{2}\int \langle {}^*\mathbf{F} \wedge \mathbf{F} \rangle$$

Since the gauge group is broken $ISO(d,2) \to SO(d,1)$

$\langle \ldots \rangle$ Is the most general bilinear form that is invariant under $SO(d,1)$

$$\left\langle \mathbf{J}_{An}\mathbf{J}_{Bn} \right\rangle = \hat{\alpha}_2 \eta_{AB}, \qquad \left\langle \mathbf{P}_A \mathbf{P}_B \right\rangle = \hat{\alpha}_3 \eta_{AB}$$

$$\left\langle \mathbf{J}_{AB}\mathbf{J}_{CD} \right\rangle = \hat{\alpha}_0 \left(\eta_{AC}\eta_{BD} - \eta_{AD}\eta_{BC} \right), \qquad \left\langle \mathbf{P}_n \mathbf{P}_n \right\rangle = \hat{\alpha}_1$$

After doing the contraction, introducing some gauge fixing, and requiring the existence of a space-like Killing vector we obtain

$$\delta a_\mu = \mathfrak{L}_\xi a_\mu - \partial_\mu \epsilon + b_a e^a{}_\mu$$

$$\tilde{F}^a = D\tilde{v}^a$$

$$\delta \tilde{v}^a{}_\mu = \mathfrak{L}_\xi \tilde{v}^a{}_\mu + D_\mu b^a - \theta^a{}_b v^b{}_\mu$$

$$\tilde{f} = da + e^a \tilde{v}_a$$

$$\delta \psi^a = \mathfrak{L}_\xi \psi^a + b^a - \theta^a{}_b \psi^b$$

After doing the contraction, introducing some gauge fixing, and requiring the existence of a space-like Killing vector we obtain

$$S = S_1 + \sigma^2 S_2 + O(\sigma^4)$$

$$S_1 = -\frac{1}{2}\int d^{d+1}x\sqrt{|g|}\ \left[\frac{\alpha_0}{2}R^{ab}{}_{\mu\nu}R_{ab}{}^{\mu\nu} + \alpha_3 T^a{}_{\mu\nu}T_a{}^{\mu\nu} - \alpha_3\partial_{[\mu}\tau_{\nu]}\partial^{[\mu}\tau^{\nu]}\right]$$

$$S_2 = -\frac{1}{2}\int d^{d+1}x\sqrt{|g|}\left[\alpha_2\left(\tilde{F}^a{}_{\mu\nu} - R^{ab}{}_{\mu\nu}\psi_b\right)\left(\tilde{F}_a{}^{\mu\nu} - R_{ac}{}^{\mu\nu}\psi^c\right)\right.$$

$$-\alpha_0 R^{ab\mu\nu}\left(\tilde{v}_{a\mu} - D_\mu\psi_a\right)\left(\tilde{v}_{b\nu} - D_\nu\psi_b\right) + \alpha_1\left(\tilde{f}_{\mu\nu} - T^a{}_{\mu\nu}\psi_a\right)\left(\tilde{f}^{\mu\nu} - T^{b\mu\nu}\psi_b\right)$$

$$\left.+2\alpha_3\left(\tilde{v}^a{}_\mu - D_\mu\psi^a\right)\left(\tilde{v}_a{}^\mu - D^\mu\psi_a\right)\right]$$

In flat space with $\alpha_3 = 0$

$$S = -\frac{1}{2}\int d^{d+1}x \left[\tilde{F}^a{}_{\mu\nu}\tilde{F}_a{}^{\mu\nu} + \alpha_1 \tilde{f}_{\mu\nu}\tilde{f}^{\mu\nu}\right]$$

This is Preto's action after gauge fixing $a_\mu = (\phi, 0, \ldots, 0)$

And setting the second term to zero

With $\alpha_3 \neq 0$ We obtain the Proca extension

$$S = -\int d^{d+1}x \left[\frac{1}{2}D_{[\mu}A^a{}_{\nu]}D^{[\mu}A_a{}^{\nu]} + m^2\left(A^a{}_\mu - \partial_\mu \psi^a\right)\left(A_a{}^\mu - \partial^\mu \psi_a\right)\right] + \ldots$$

In curved space

$$S = -\frac{1}{2}\int d^{d+1}x\sqrt{|g|}\,\Bigg[\frac{\alpha_0}{2}R^{ab}{}_{\mu\nu}R_{ab}{}^{\mu\nu} + \alpha_3 T^a{}_{\mu\nu}T_a{}^{\mu\nu} - \alpha_3\partial_{[\mu}\tau_{\nu]}\partial^{[\mu}\tau^{\nu]}$$

$$+\alpha_2\left(\tilde{F}^a{}_{\mu\nu} - R^{ab}{}_{\mu\nu}\psi_b\right)\left(\tilde{F}_a{}^{\mu\nu} - R_{ac}{}^{\mu\nu}\psi^c\right) + \alpha_1\left(\tilde{f}_{\mu\nu} - T^a{}_{\mu\nu}\psi_a\right)\left(\tilde{f}^{\mu\nu} - T^{b\mu\nu}\psi_b\right)\Bigg]$$

Without Stueckelberg field and mass term with the gravitational sector on shell

$$S_2 = - \int d^{d+1}x \sqrt{|g|} \left[\frac{1}{2} \tilde{F}^{a}{}_{\mu\nu} \tilde{F}_{a}{}^{\mu\nu} - R^{ab\mu\nu} A_{a\mu} A_{b\nu} \right]$$

If

$$D^{\mu} R_{ab\mu\nu} = 0$$

Conclusion

- Dipole conserving gauge theories are intimately related to Carrol theories and can be obtained from Poincare gauge theories in a similar manner.
- The pseudo-Carrol parameter can not be send exactly to zero, otherwise the generated theory would be absent of fracton gauge fields.
- The singular nature of the limit requires the existence of Higgs fields that allow us preserve the invariance of the system.
- That explains why the fracton gauge invariance is broken in curved space.
- The dynamics of dipole conserving systems is far from trivial, even though their “matter” excitations naively are not allowed to freely move. Probably Carrol systems will show similar behaviour.