Non-dissipative Electric Fluids

Based on arXiv:2211.05791,
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Holotube– 04/04/2023

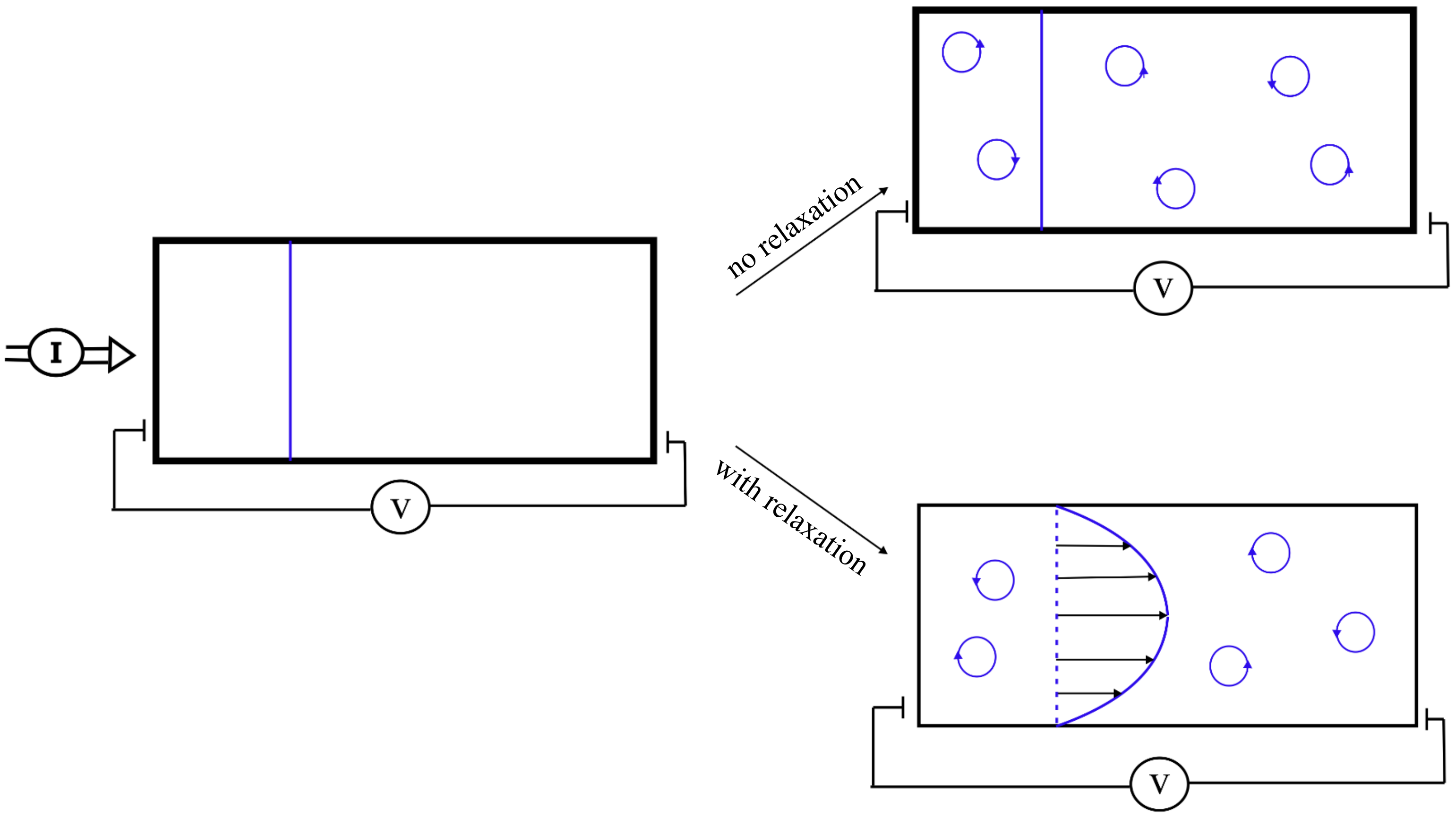
Motivation

Setup:

Charged fluid in a channel

Homogeneous electric field

Wait



Stationary by balancing electric field to chemical potential

Polarization currents flowing

No charge/heat flow

Equilibrium state
[Kovtun JHEP 28 (2016)]

Stationary by balancing energy/momentum with environment

Polarization currents flowing

With charge/heat flow

Steady state

OUTLINE

- Hydrodynamics of the equilibrium state
  - Aristotelian geometry
  - Equilibrium conditions
  - Constitutive relations
- Hydrodynamics of the steady state
  - Introducing relaxation
  - Steady state conditions
  - Conductivities and linear stability
- Conclusions and Outlook

Hydrodynamics of the equilibrium state

- Hydrodynamics = EFT for thermalized matter

- Dynamics = conservation equations

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu \qquad\qquad \partial_\mu J^\mu = 0$$

- IR/Mean fields of hydrodynamics = thermodynamic parameters

- EFT = derivative expansion of charges

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \mathcal{O}(\partial^2) \qquad\qquad J^\mu = J^\mu_{(0)} + J^\mu_{(1)} + \mathcal{O}(\partial^2)$$

Thermodynamic parameters

[Jensen et al. PRL 109 (2012) 101601; de Boer et al., SciPostPhys. 9, 018 (2020); Armas, Jain, SciPostPhys. 11, 054 (2021)]

- Split spacetime → space + time*

$$\Sigma_p = \{V \in T\mathcal{M}_p : \tau(V) = 0\} \qquad \nu^\mu \tau_\mu = -1 \qquad h_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu \;\; (= \mathrm{diag}(0,1,1,1))$$

- Thermal vector (energy/momentum conjugate)

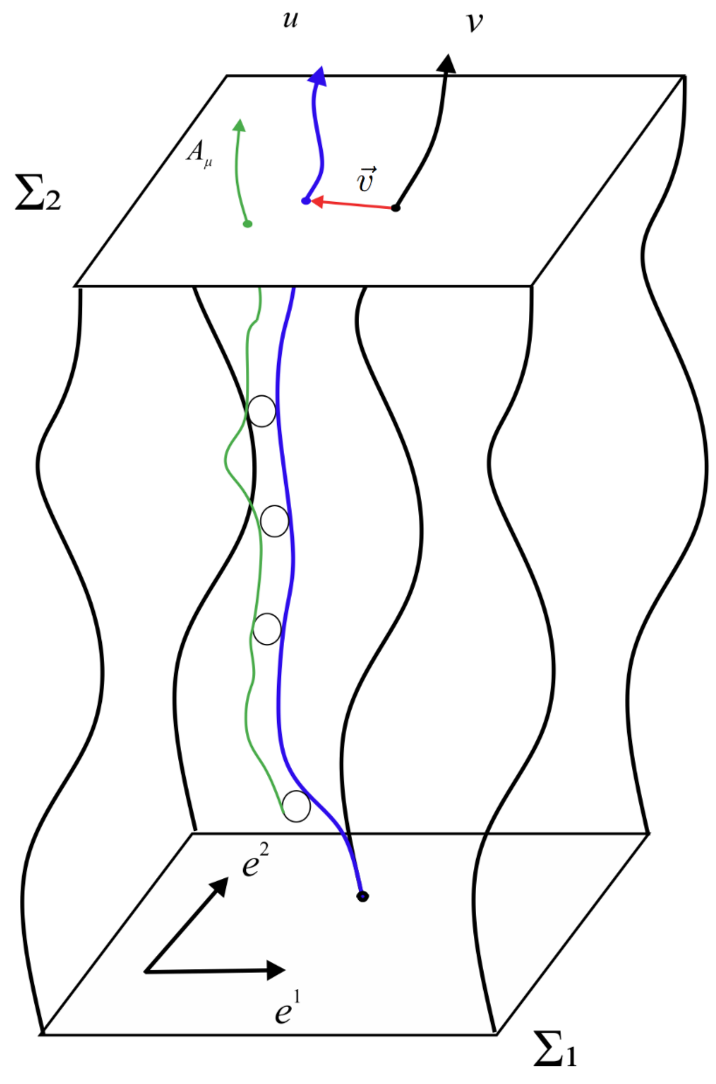
$$T = \frac{1}{\tau_\mu \beta^\mu}\,, \quad u^\mu = T\beta^\mu$$

- Chemical potential (charge conjugate)

$$\mu = T\left(A_\mu \beta^\mu + \Lambda_V\right)$$

- Electric field (polarization conjugate)

$$\mathbb{E}_\mu = -F_{\mu\nu}\nu^\nu\,, \qquad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} = \mathbb{E}_\mu \tau_\nu - \mathbb{E}_\nu \tau_\mu$$



* + compatible connection

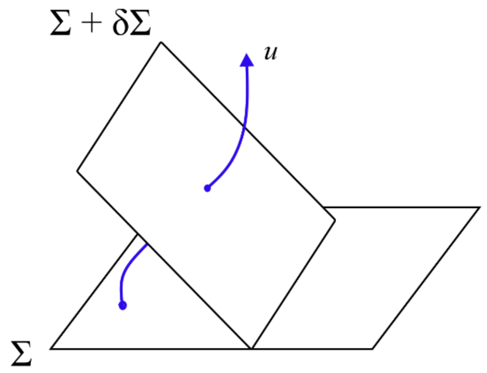
Equilibrium conditions

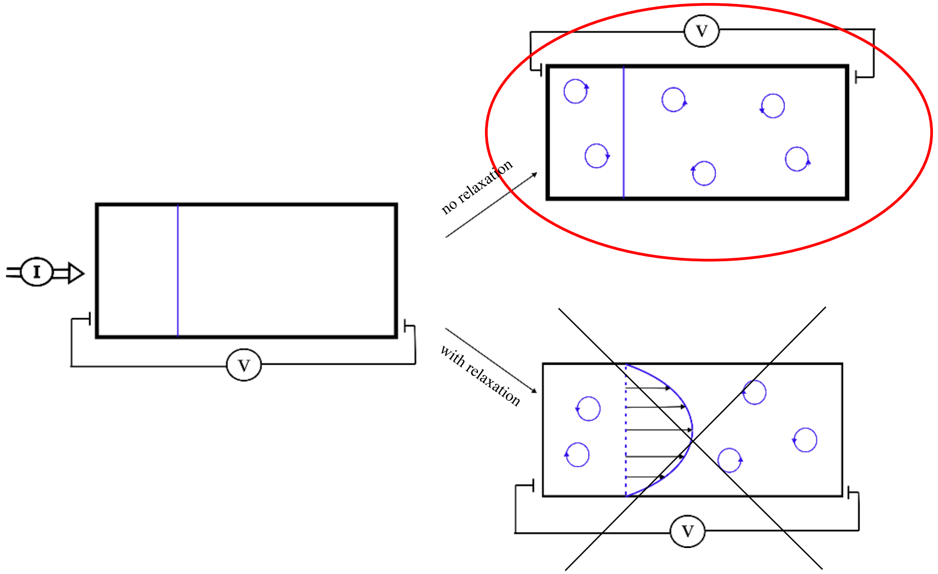
- Thermal equilibrium w.r.t. co-moving frame

$$\mathcal{L}_\beta \tau_\mu = 0 \longrightarrow \frac{\partial_\mu T}{T} - u^\nu \left(\partial_\nu \tau_\mu - \partial_\mu \tau_\nu\right) = 0$$

$$\mathcal{L}_\beta A_\mu = 0 \longrightarrow \mathbb{E}_\mu - \partial_\mu \mu = u^\nu \left(\mathbb{E}_\nu \tau_\mu - \mu \partial_{[\nu} \tau_{\mu]}\right)$$

$$\mathcal{L}_\beta h_{\mu\nu} = 0 \qquad \partial_{[\mu} F_{\nu\rho]} = 0 \qquad \mathcal{L}_\beta \mathbb{E}_\mu = 0$$

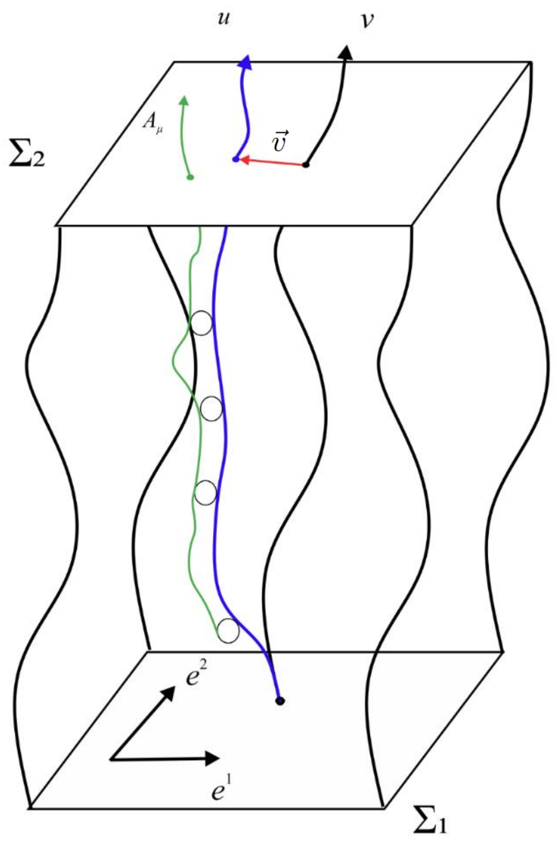




Constitutive relations

- From thermodynamics: $$W_{(0)} = \int d^{d+1}x \; |e| P(\mathrm{S}) \;, \quad e = \det(\tau_\mu, e^a_\mu)$$

	Elementary	Composite
Scalars:	$T,\ \mu$	$h_{\mu\nu}u^\mu u^\nu,\ h^{\mu\nu}\mathbb{E}_\mu\mathbb{E}_\mu,\ \mathbb{E}_\mu u^\mu$
One-forms:	$\tau_\mu,\ \mathbb{E}_\mu,\ \partial_\mu\mu$	$h_{\mu\nu}u^\mu$
Vectors:	$\nu^\mu,\ u^\mu$	$h^{\mu\nu}\mathbb{E}_\nu\ , h^{\mu\nu}\partial_\mu\mu$
Covariant 2-tensors:	$h_{\mu\nu}$	$\tau_\mu\tau_\nu,\ \tau_\mu\mathbb{E}_\nu,\ \tau_\mu\partial_\nu\mu\ldots$
Contravariant 2-tensors:	$h^{\mu\nu}$	$\nu^\mu\nu^\nu,\ \nu^\mu u^\nu,\ u^\mu u^\nu$



- Free energy as a generating functional:
  [Jensen et al. PRL 109 (2012) 101601]

$$\delta W_{(0)}[\tau, h, A, F] = \int d^{d+1}x\; e \left(\boxed{-T^\mu\delta\tau_\mu + \frac{1}{2}T^{\mu\nu}}\delta h_{\mu\nu} + \boxed{J^\mu}\delta A_\mu + \frac{1}{2}\boxed{M^{\mu\nu}}\delta F_{\mu\nu}\right)$$

Energy-stress-momentum tensor

U(1) current

Magnetization/Polarization tensor

$$T^\mu{}_\nu = -T^\mu\tau_\nu + T^{\mu\rho}h_{\rho\nu}$$

Constitutive relations

Current takes expected form

$$J^\mu = nu^\mu + \frac{1}{e}\partial_\nu \left(2e\nu^{[\mu}\mathbb{P}^{\nu]}\right)$$

$$J^0 = \partial_i \mathbb{P}^i$$

$$J^i = \partial_t \mathbb{P}^i$$

Velocity-dependent polarization

$$\vec{\mathbb{P}} = \left(\frac{\partial P}{\partial \vec{\mathbb{E}}}\right) = \kappa_{\mathbb{E}}\vec{\mathbb{E}} + \beta_{\mathbb{P}}\vec{v}$$

$$\beta_{\mathbb{P}} = \left(\frac{\partial P}{\partial(\vec{\mathbb{E}}\cdot\vec{v})}\right) \ , \quad \kappa_{\mathbb{E}} = 2\left(\frac{\partial P}{\partial \vec{\mathbb{E}}^2}\right)$$

Constitutive relations

New heat and stress terms

$$T^{\mu}{}_{\nu} = -\varepsilon u^{\mu}\tau_{\nu} - \underline{\left(P - \mathbb{P}^{\sigma}\mathbb{E}_{\sigma}\right) h^{\mu\rho}h_{\rho\sigma}u^{\sigma}\tau_{\nu}} + P h^{\mu\rho}h_{\rho\nu} + \rho_{\mathrm{m}}u^{\mu}u^{\rho}h_{\rho\nu} - \kappa_{\mathbb{E}}\mathbb{E}_{\alpha}\mathbb{E}_{\beta}h^{\alpha\mu}h^{\beta\rho}h_{\rho\nu} - \underline{\beta_{\mathbb{P}}\mathbb{E}_{\alpha}h^{\alpha\rho}\nu^{\mu}h_{\rho\nu}}$$

No Landau frame

$$T^{\mu}{}_{\nu}u^{\nu} = -\left(\varepsilon - \rho_{\mathrm{m}}u^{2} - \mathbb{P}\cdot\mathbb{E}\right)u^{\mu} + \left(\mathbb{P}\cdot\mathbb{E} - \beta_{\mathbb{P}}(\mathbb{E}\cdot u)\right)\nu^{\mu} - \kappa_{\mathbb{E}}(\mathbb{E}\cdot u)h^{\mu\nu}\mathbb{E}_{\nu}$$

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Prelude

- Equilibrium conditions = Ideal hydro equations
- Fix clock-form and metric ("Minkowski" spacetime)

$$\tau_\mu = \delta^0_\mu \qquad \nu^\mu = -\delta^\mu_0 \qquad h_{\mu\nu} = \delta^i_\mu \delta^j_\nu \delta_{ij} \qquad h^{\mu\nu} = \delta^\mu_i \delta^\nu_j \delta^{ij}$$

- E.g. momentum conservation: $\mathbb{E}^i - \partial^i \mu = 0 \qquad \left(E_\mu - \partial_\mu \mu = u^\nu \left(E_\nu \tau_\mu - \mu \partial_{[\nu} \tau_{\mu]}\right)\right)$

- Equilibrium state → steady state = modifying ideal hydro equations

Introducing relaxation (0th Order)

Relax energy/momentum ("Minkowski" spacetime):

$$0 = \partial_t n + \partial_i J^i$$
$$0 = \partial_t \varepsilon + \partial_i J^i_\varepsilon - \mathbb{E}_i J^i + \hat{\Gamma}_\varepsilon$$
$$0 = \partial_t P_i + \partial_j T^{ij} - n\,\mathbb{E}_i + \hat{\Gamma}^i_{\vec{P}}$$

Most general 0th order relaxations: $\hat{\Gamma}^i_{\vec{P}} = \Gamma_{\vec{\mathbb{P}}} \mathbb{P}^i + \Gamma_{\vec{P}} P^i + \mathcal{O}(\partial)$, $\hat{\Gamma}_\varepsilon = \Gamma_\varepsilon + \mathcal{O}(\partial)$

$$\vec{\mathbb{P}} = \left(\frac{\partial P}{\partial \vec{\mathbb{E}}}\right) = \kappa_{\mathbb{E}} \vec{\mathbb{E}} + \beta_{\mathbb{P}} \vec{v} \qquad \vec{P} = \left(\frac{\partial P}{\partial \vec{v}}\right) = \rho_{\mathrm{m}} \vec{v} + \beta_{\mathbb{P}} \vec{\mathbb{E}}$$

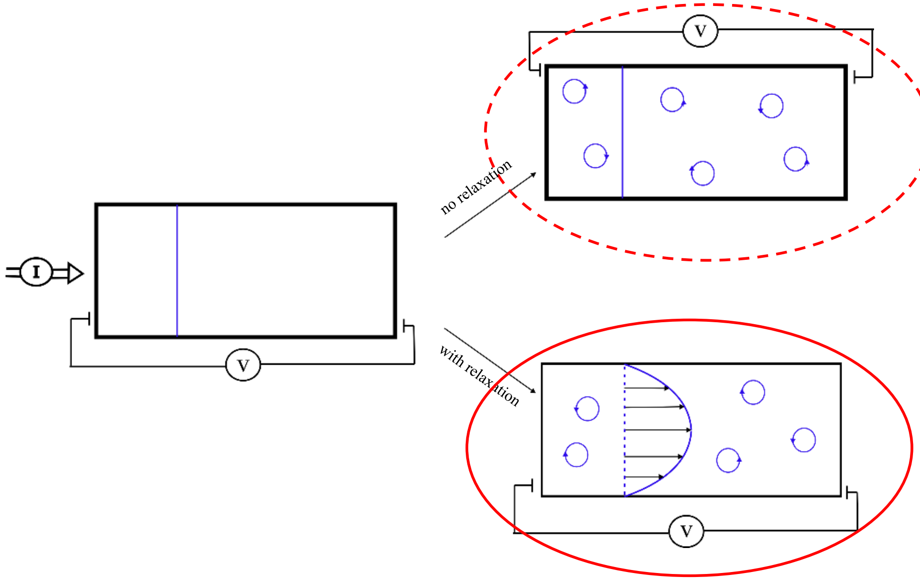
Keep constant temperature, chemical potential E-field $\mathcal{L}_\beta \tau_\mu = 0$ $\mathcal{L}_\beta h_{\mu\nu} = 0$ $\partial_{[\mu} F_{\nu\rho]} = 0$ $\mathcal{L}_\beta \mathbb{E}_\mu = 0$

$$\cancel{\mathcal{L}_\beta A_\mu = 0 \longrightarrow \mathbb{E}_\mu - \partial_\mu \mu = u^\nu \left(\mathbb{E}_\nu \tau_\mu - \mu \partial_{[\nu} \tau_{\mu]}\right)}$$

$$n\left(\mathbb{E}^i - \partial^i \mu\right) = \Gamma_{\vec{\mathbb{P}}} \mathbb{P}^i + \Gamma_{\vec{P}} P^i + \mathcal{O}(\partial) \qquad n v^i \left(\mathbb{E}_i - \partial_i \mu\right) = \Gamma_\varepsilon + \mathcal{O}(\partial)$$

$$\Gamma_\varepsilon = \vec{v} \cdot \hat{\Gamma}^i_{\vec{P}}$$

$$\vec{v} = \left(\frac{n - \kappa_{\mathbb{E}} \Gamma_{\vec{\mathbb{P}}} - \beta_{\mathbb{P}} \Gamma_{\vec{P}}}{\beta_{\mathbb{P}} \Gamma_{\vec{\mathbb{P}}} + \rho_{\mathrm{m}} \Gamma_{\vec{P}}}\right) \vec{\mathbb{E}} - \frac{n}{\beta_{\mathbb{P}} \Gamma_{\vec{\mathbb{P}}} + \rho_{\mathrm{m}} \Gamma_{\vec{P}}} \vec{\partial} \mu + \mathcal{O}(\partial)$$



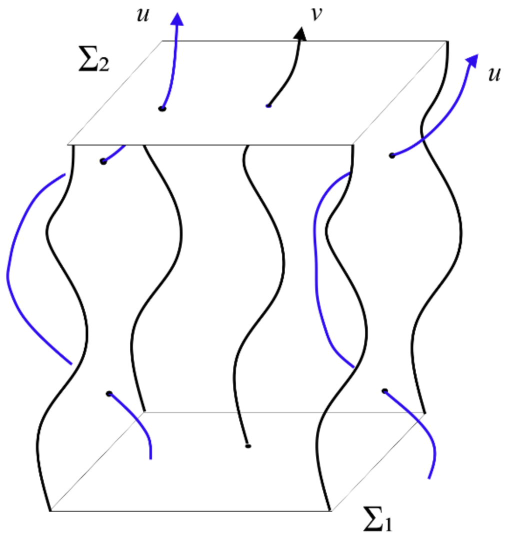
"Drude" constraint

Global charge/entropy conservation

Kinematic constraint $$\Gamma_\varepsilon = \vec{v}\cdot\hat{\Gamma}^i_{\vec{P}}$$

2nd law $$\partial_\mu s^\mu = \Gamma_\varepsilon - \vec{v}\cdot\hat{\Gamma}^i_{\vec{P}}$$

$$\partial_\mu (u_\nu T^{\mu\nu}) = \Gamma_\varepsilon - \vec{v}\cdot\hat{\Gamma}^i_{\vec{P}}$$ Flow energy con.



Stability and conductivities

Perturb around steady state: constant T, μ, $\vec{\mathbb{E}} = (\mathbb{E}_x, 0)$, $v_x = \left(\frac{n - \kappa_{\mathbb{E}}\Gamma_{\vec{\mathbb{P}}} - \beta_{\mathbb{P}}\Gamma_{\vec{P}}}{\beta_{\mathbb{P}}\Gamma_{\vec{\mathbb{P}}} + \rho_{\mathrm{m}}\Gamma_{\vec{P}}}\right)\mathbb{E}_x$

Zero-k modes in 2+1d: $\omega = -i\Gamma_{\text{eff.}}$, $\Gamma_{\text{eff.}} = \frac{1}{\rho_{\mathrm{m}}}\left(\beta_{\mathbb{P}}\Gamma_{\vec{\mathbb{P}}} + \rho_{\mathrm{m}}\Gamma_{\vec{P}}\right)$

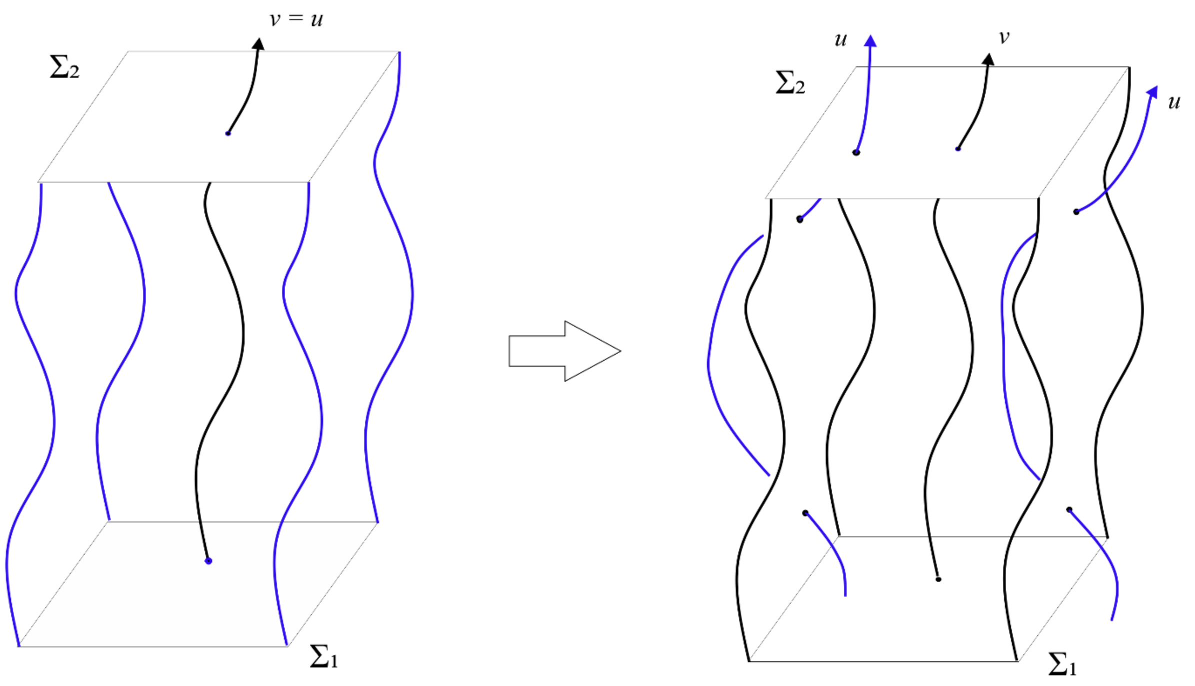
Relaxation times < 0 possible: $\Gamma_{\vec{\mathbb{P}}} = -\left(n + \frac{\beta_{\mathbb{P}}\Gamma_{\vec{P}}}{\kappa_{\mathbb{E}}}\right)$ $\vec{v} = \frac{n}{\Gamma_{\vec{P}}\left(\rho_{\mathrm{m}} - \frac{\beta_{\mathbb{P}}^2}{\kappa_{\mathbb{E}}}\right)}\left(\vec{\mathbb{E}} + \vec{\mathbb{P}} - \vec{\partial}\mu\right)$

Non-trivial heat/charge transport (Drude-like): $\vec{Q} = sT\vec{v} = \frac{sTn\tau}{\rho_{\mathrm{m}}}\vec{\mathbb{E}}$ $\sigma_{\mathrm{DC}} = \vec{J}/\vec{E} = \frac{n^2\tau}{\rho_{\mathrm{m}}}$

$$\tau = \Gamma_{\vec{P}}^{-1}\left(\frac{1 - \frac{\kappa_{\mathbb{E}}\Gamma_{\vec{\mathbb{P}}}}{n} - \frac{\beta_{\mathbb{P}}\Gamma_{\vec{P}}}{n}}{1 + \frac{\beta_{\mathbb{P}}\Gamma_{\vec{\mathbb{P}}}}{\rho_{\mathrm{m}}\Gamma_{\vec{P}}}}\right)$$

Conclusions

- Extended polarized hydro to include finite equilibrium velocity
- New susceptibility at equilibrium, no Landau frame
- Defined Drude-like steady state by relaxing hydro equations
- Stable with finite thermoelectric conductivities
- Satisfies 2nd law and energy conservation



Outlook

- Work out diffusive corrections (work in progress)

- Extend to fully covariant theory

- Introduce steady state in holographic models
  [B. Withers, JHEP 8 (2016)]

Thank you!