Holographic nodal-antinodal dichotomy from infrared scaling

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with Jewel Kumar Ghosh and Alexander Krikun.

Based on arXiv:2212.09694

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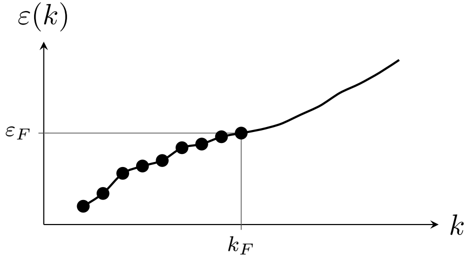
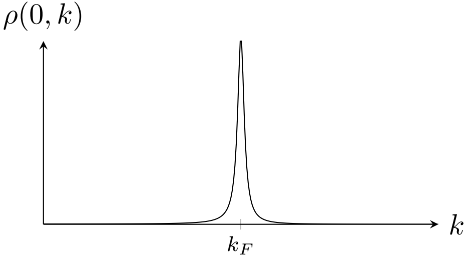
Fermi surfaces, free electrons



$$G(\omega, k) \sim \frac{1}{\omega - \varepsilon(k)} + i\delta(\omega - \varepsilon(k))$$



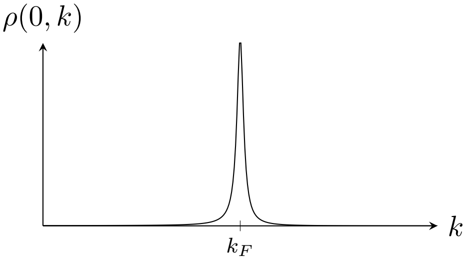
$$\rho(\omega, k) = \mathrm{Tr}\,\mathrm{Im}\,G(\omega, k)$$
$$\sim \delta(\omega - \varepsilon(k))$$

(Non)-Fermi liquids

Fermi liquid: weak electronic interactions

$$G(\omega, k) = \frac{1}{G_0(\omega, k)^{-1} - \Sigma(\omega, k)}$$

- $|\mathrm{Im}\,\Sigma| \ll 1$ near $\omega = 0$, $k = k_F$
- Sharp quasiparticle peak
- Sharp Fermi surface



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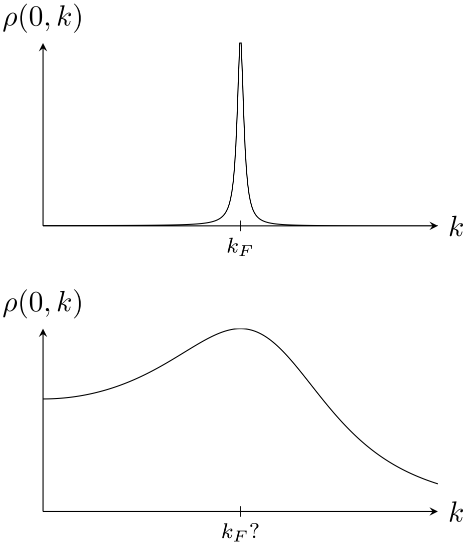
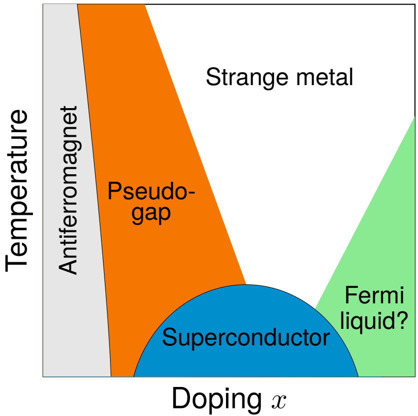
Non-Fermi liquid: No quasiparticles

- Strong interactions
- Experimental example: cuprates
- Holography



Nodal anti-nodal dichotomy in cuprates



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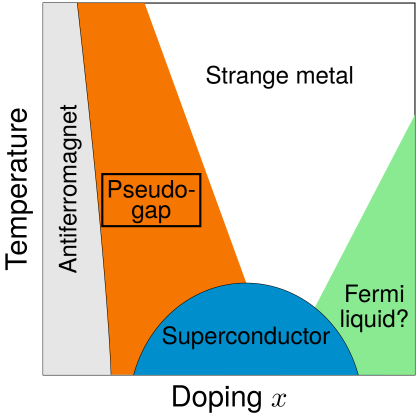
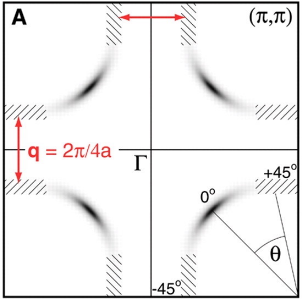
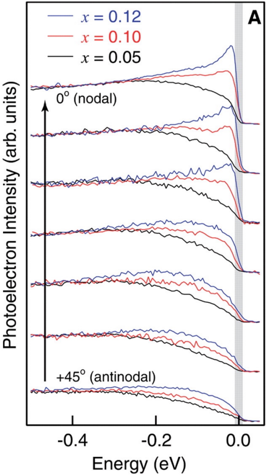


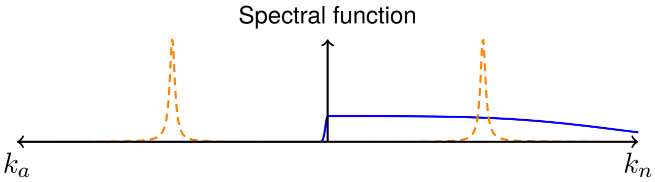
$Ca_{2-x}Na_xCuO_2Cl_2$:

[Shen, Ronning, Lu et al 2005]



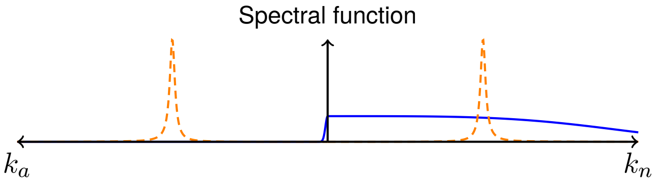


Heuristic mechanism



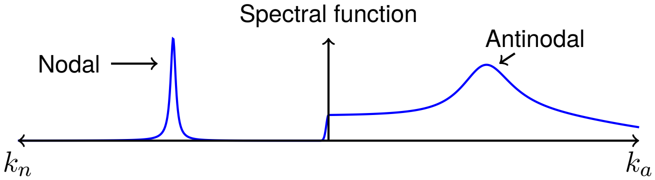
- Direction k_a: resonance peak
- Direction k_n: resonance peak + continuum

Heuristic mechanism



- Direction k_a: resonance peak
- Direction k_n: resonance peak + continuum

- Continuum broadens k_a peak



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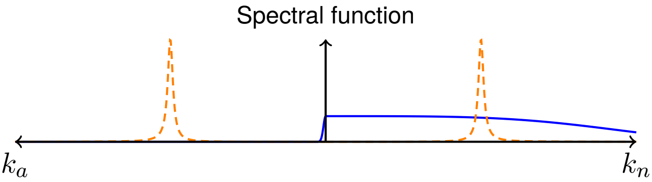

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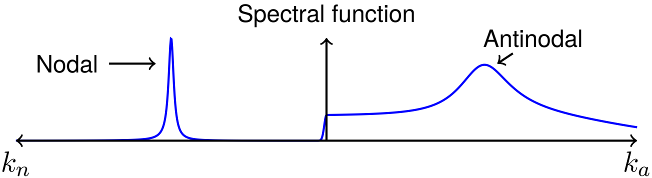
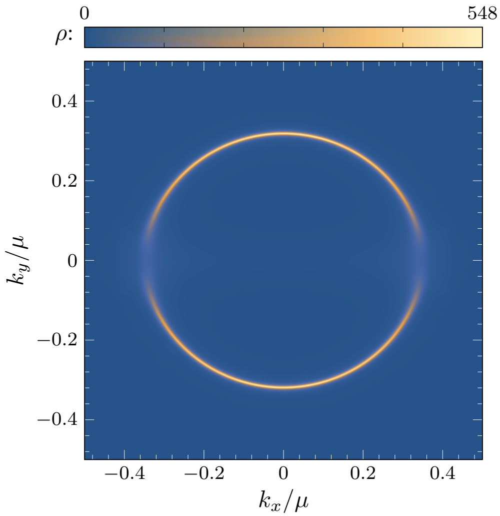


Antinodal ⇔ continuum

Disclaimer

- Will consider two-fold rotational symmetry only
- Four-fold symmetry tricky (More on this later)



Review: Fermion spectral functions in holography

Holographic fermions

Asymptotically-AdS$_4$

$$\mathrm{d}s^2 \overset{r\to\infty}{=} \frac{\mathrm{d}r^2}{r^2} + r^2\left(-\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2\right)$$

Black brane horizon in bulk (at $r = 0$)

Probe fermion

$$S = i \int \mathrm{d}^4x\, \sqrt{-G}\, \bar{\Xi}\left(\not{D} - m\right)\Xi + \text{(boundary terms)}$$

Dual to boundary fermion operator (“electron”)

Holographic fermions

[Iqbal, Liu, 0809.3808]

Define projectors $P_\pm = \frac{1}{2}(1 \pm \Gamma^{\underline{r}})$

Projection	Field theory interpretation
$\psi \propto P_+ \Xi$	$\lim_{r\to\infty} r^{\frac{3}{2}-m}\psi$ $=$ source for fermionic operator
$\chi \propto -iP_- \Xi$	$\lim_{r\to\infty} r^{\frac{3}{2}+m}\chi$ $\propto$ VEV of fermionic operator

Green's function

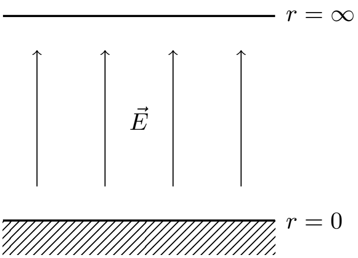
$$G(\omega, k) = \lim_{r\to\infty} r^{2m}\Gamma^{\underline{0}}\frac{\chi}{\psi}$$

Retarded Green's function: ingoing boundary conditions

[Son, Starinets, hep-th/0205051; Herzog, Son, hep-th/0212072]

Extremal AdS–Reissner–Nordström

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{\mathrm{d}r^2}{U(r)} - U(r)\,\mathrm{d}t^2 + V(r)\left[\mathrm{d}x^2 + \mathrm{d}y^2\right] \\
A &= \frac{\mu r}{r+1}\mathrm{d}t \\
U(r) &= \frac{r^2(r^2+4r+6)}{(r+1)^2} \\
V(r) &= (r+1)^2
\end{aligned}
$$



Chemical potental μ

Extremal horizon at $r = 0$

Extremal AdS–Reissner–Nordström

In appropriate basis: $\psi = (\psi_+, \psi_-)$, $\chi = (\chi_+, \chi_-)$

[Faulkner, Liu, McGreevy, Vegh, 0907.2694]

$$\left[\sqrt{U}\,\partial_r + m\sigma^3 - \frac{\omega + qA_t}{\sqrt{U}}(i\sigma^2) \pm \frac{k}{\sqrt{V}}\sigma^1\right]\begin{pmatrix}\chi_\pm(r)\\ \psi_\pm(r)\end{pmatrix} = 0$$

From now, will consider ψ_+, χ_+ only and drop plusses

Extremal AdS–Reissner–Nordström

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From now, will consider ψ_+, χ_+ only and drop plusses

Near Fermi surface: small ω — solve Dirac equation perturbatively?

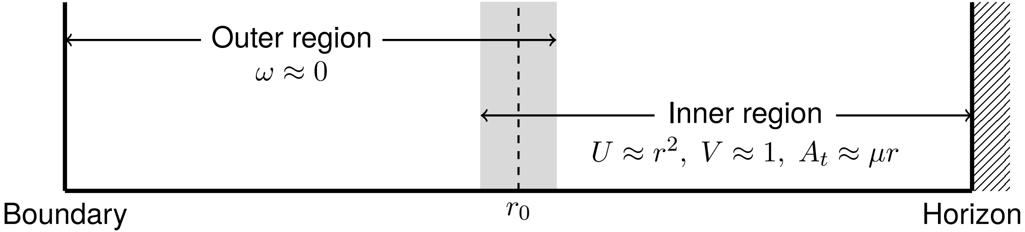
Near horizon:

$$U(r) \approx r^2, \qquad V(r) \approx 1, \qquad A_t(r) \approx \mu r$$

$\dfrac{\omega}{\sqrt{U}}$ always important at small enough r

Matched expansions

[Faulkner, Liu, McGreevy, Vegh, 0907.2694]
[Faulkner, Iqbal, Liu, McGreevy, Vegh, 1101.0597]



Outer region: $\left[\sqrt{U}\,\partial_r + m\sigma^3 - \frac{qA_t}{\sqrt{U}}(i\sigma^2) + \frac{k}{\sqrt{V}}\sigma^1\right]\begin{pmatrix}\chi(r)\\ \psi(r)\end{pmatrix} = 0$

Eliminate χ algebraically $\Rightarrow$ second order ODE for ψ

$$\psi(r) = Cf(r) + Dg(r)$$

Non-normalisable (f) — Normalisable (g)

$$G(\omega, k) \propto \frac{D}{C}$$

Matched expansions

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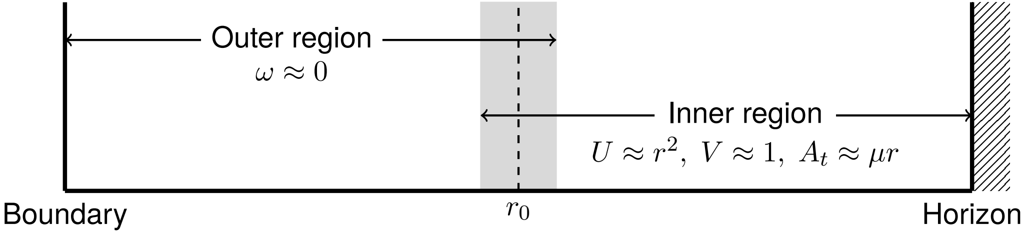

$$\psi(r) = Cf(r) + Dg(r) \qquad \Rightarrow \qquad G \propto \frac{D}{C} = \frac{\psi(r_0)f'(r_0) - \psi'(r_0)f(r_0)}{\psi(r_0)g'(r_0) - \psi'(r_0)g(r_0)}$$

Define (everything evaluated at r_0)

$$a = mf - f'\sqrt{U} \qquad b = f\left(qA_tU^{-1/2} - kV^{-1/2}\right)$$

$$c = mg - g'\sqrt{U} \qquad d = g\left(qA_tU^{-1/2} - kV^{-1/2}\right)$$

Matched expansions

[Faulkner, Liu, McGreevy, Vegh, 0907.2694]
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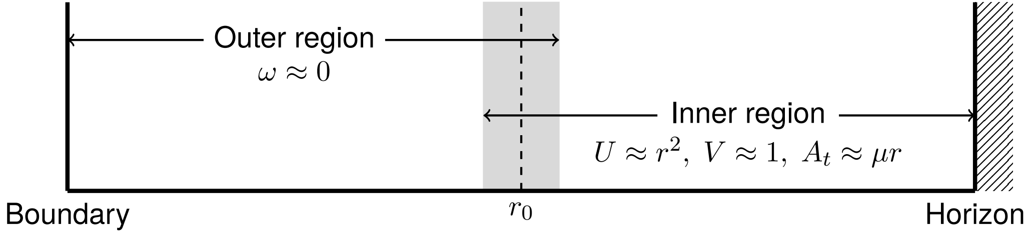

Replace ψ' with χ using EOM:

$$G(\omega, k) = \frac{a + b\mathcal{G}}{c + d\mathcal{G}}, \qquad \mathcal{G} = \frac{\chi(r_0)}{\psi(r_0)}$$

$\mathcal{G}$ = IR Green's function

Compute $\mathcal{G}$ using solution in inner region

Matched expansions

[Faulkner, Liu, McGreevy, Vegh, 0907.2694]
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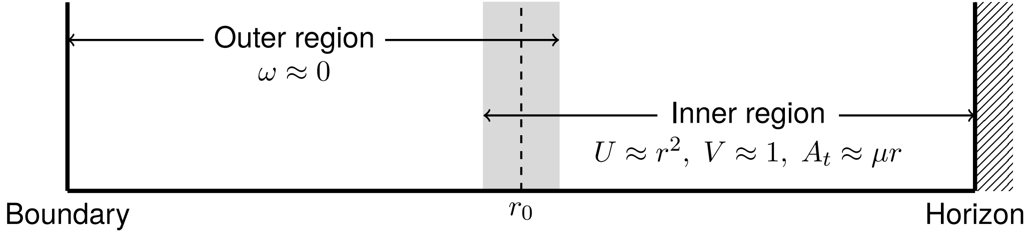

$a, b, c, d \in \mathbb{R}$

$$\rho(\omega, k) = \operatorname{Im} G(\omega, k) = \frac{bc - ad}{|c + d\mathcal{G}|^2} \operatorname{Im} \mathcal{G}$$

Low frequency scaling of $\operatorname{Im} \mathcal{G}$ determines low frequency scaling of ρ

Determine presence of continuum from $\operatorname{Im} \mathcal{G}$

Matched expansions

[Faulkner, Liu, McGreevy, Vegh, 0907.2694]
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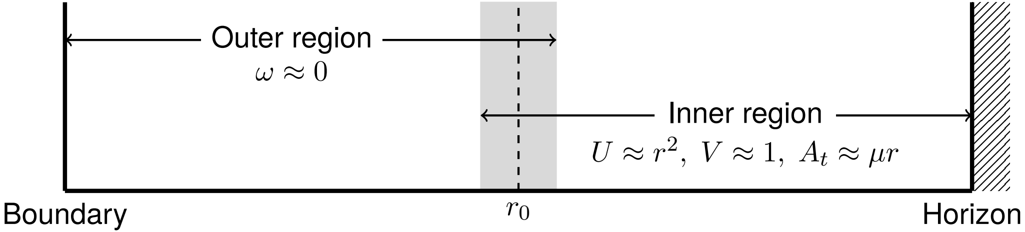

Inner region:

$$\left[r\,\partial_r + m\sigma^3 - \left(\frac{\omega}{r} + q\mu\right)(i\sigma^2) + k\sigma^1 \right] \begin{pmatrix} \chi(r) \\ \psi(r) \end{pmatrix} = 0$$

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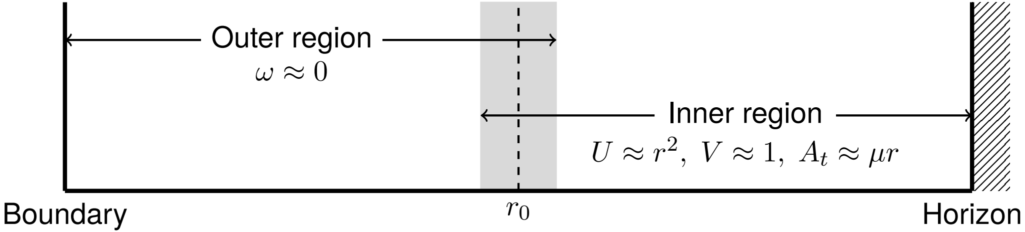

Inner region, ingoing solution:

$$\psi(r) = \sqrt{r}\left[\left(2\kappa + m + ik + 2i\frac{\omega}{r}\right) W_{\kappa,\nu}\left(-2i\frac{\omega}{r}\right) - W_{\kappa+1,\nu}\left(-2i\frac{\omega}{r}\right)\right]$$

$$\chi(r) = -i\sqrt{r}\left[\left(2\kappa - m - ik + 2i\frac{\omega}{r}\right) W_{\kappa,\nu}\left(-2i\frac{\omega}{r}\right) + W_{\kappa+1,\nu}\left(-2i\frac{\omega}{r}\right)\right]$$

$$\kappa = \frac{1}{2} + iq\mu, \qquad \nu = \sqrt{m^2 + k^2 - q^2\mu^2}.$$

Matched expansions

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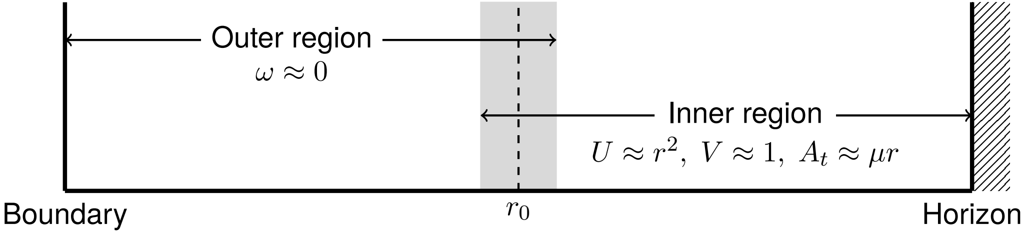

$$\operatorname{Im}\mathcal{G} = \operatorname{Im}\frac{\chi(r_0)}{\psi(r_0)} \sim \omega^{2\,\mathrm{Re}\,\nu}, \qquad \nu = \sqrt{m^2 + k^2 - q^2\mu^2}.$$

Presence of continuum depends on k

IR geometries

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{U(r)} - U(r)\mathrm{d}t^2 + V(r)\left[\mathrm{d}x^2 + \mathrm{d}y^2\right]$$

$r \to 0$: suppose $U \approx r^{\alpha_U}$, $V \approx r^{\alpha_V}$,

$$\mathrm{d}s^2 \approx \zeta^{\theta/z}\left(\frac{-\mathrm{d}t^2 + \mathrm{d}\zeta^2}{\zeta^2} + \frac{\mathrm{d}x^2 + \mathrm{d}y^2}{\zeta^{2/z}}\right), \qquad \zeta = r^{1-\alpha_U}$$

$$z = \frac{2(\alpha_U - 1)}{\alpha_U + \alpha_V - 2}, \qquad \theta = \frac{2(\alpha_U - 2)}{\alpha_U + \alpha_V - 2}$$

AdS-RN: $\alpha_U = 2$, $\alpha_V = 0$ $\qquad \Rightarrow \qquad z \to \infty$

IR geometries

$$\mathrm{d}s^2 \approx \zeta^{\theta/z}\left(\frac{-\mathrm{d}t^2+\mathrm{d}\zeta^2}{\zeta^2}+\frac{\mathrm{d}x^2+\mathrm{d}y^2}{\zeta^{2/z}}\right)$$

Lifshitz scaling

$$(t,\zeta)\to\lambda(t,\zeta),\qquad (x,y)\to\lambda^{1/z}(x,y),\qquad \mathrm{d}s^2\to\lambda^{\theta/z}\mathrm{d}s^2$$

Hyperscaling violation [Huijse, Sachdev, Swingle, 1112.0573]

$$S\propto T^{(2-\theta)/z}$$

Spectral functions in Lifshitz geometries

$$\mathrm{d}s^2 \approx \zeta^{\theta/z}\left(\frac{-\mathrm{d}t^2+\mathrm{d}\zeta^2}{\zeta^2}+\frac{\mathrm{d}x^2+\mathrm{d}y^2}{\zeta^{2/z}}\right)$$

$$\left[\zeta\,\partial_\zeta+\bar{m}\zeta^{\theta/2z}\sigma^3-\left(\bar{\omega}\zeta+\bar{q}\zeta^{\theta/z-\nu_A}\right)(i\sigma^2)+\bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix}=0$$

$\nu_A \geq 0$ related to scaling of A_t at small r

Null energy condition + non-divergent entropy as $T \to 0$:

- Positive z: $z \geq 1$, $\theta \leq 1$

Spectral functions in Lifshitz geometries

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Spectral functions in Lifshitz geometries

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$\nu_A \geq 0$ related to scaling of A_t at small r

Null energy condition + non-divergent entropy as $T \to 0$:

- Positive z: $z \geq 1$, $\theta \leq 1$
- Negative z: $z \leq 0$, $\theta \geq 1$

When $z > 1$: $\operatorname{Im}\mathcal{G}(\omega,k) \sim \exp\left[-c\left(k^z/\omega\right)^{1/(z-1)}\right]$ [Faulkner, Polchinski, 1001.5049]

No continuum $\Rightarrow$ sharp quasiparticle peaks

What about negative z?

Negative z

IR EOM:

$$\left[\zeta\,\partial_\zeta + \bar{m}\zeta^{\theta/2z}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\theta/z-\nu_A}\right)(i\sigma^2) + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Deep IR $\zeta \to \infty$

- $\bar{\omega}$ term dominates
- Suppose $\theta > 2$ and $z < 0 \Rightarrow \bar{k}$ term is first subleading

Negative z

IR EOM:

$$\left[\zeta\,\partial_\zeta + \bar{m}\zeta^{\theta/2z}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\theta/z-\nu_A}\right)(i\sigma^2) + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

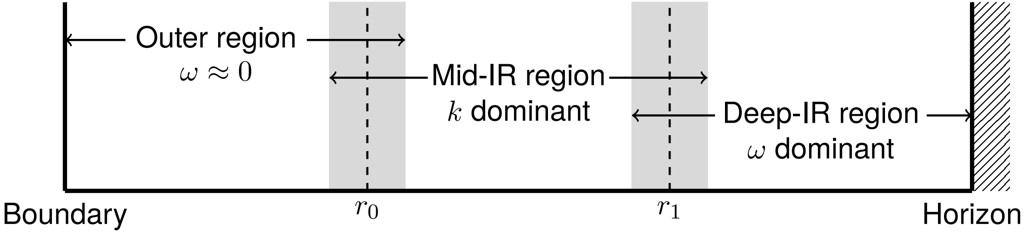
Deep IR $\zeta \to \infty$

- $\bar{\omega}$ term dominates
- Suppose $\theta > 2$ and $z < 0 \Rightarrow \bar{k}$ term is first subleading

$$\left[\zeta\,\partial_\zeta - i\bar{\omega}\zeta\sigma^2 + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

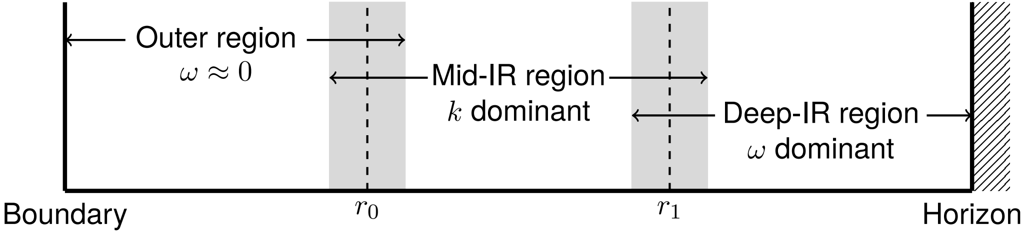
Can't solve IR equation exactly: two-step matching

Matching procedure



$$\mathcal{G}(\omega, k) = \frac{\chi(r_0)}{\psi(r_0)}$$

Matching procedure



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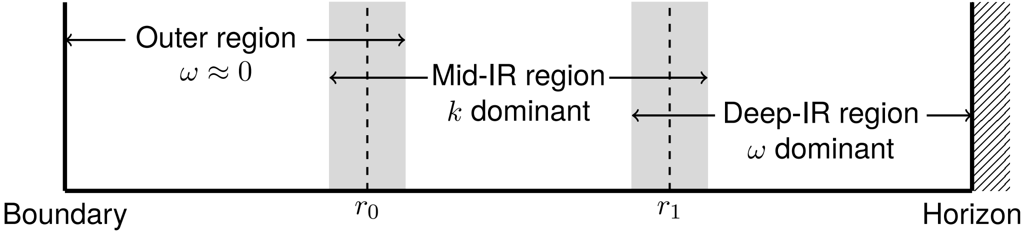
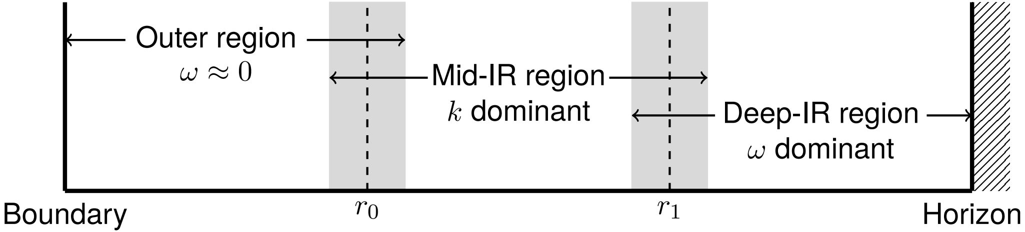


$$\mathcal{G}(\omega, k) = \frac{\chi(r_0)}{\psi(r_0)}$$

Outer region:

$$\left[\sqrt{U}\,\partial_r + m\sigma^3 - \frac{\cancel{\omega} + qA_t}{\sqrt{U}}(i\sigma^2) + \frac{k}{\sqrt{V}}\sigma^1\right]\begin{pmatrix}\chi(r)\\ \psi(r)\end{pmatrix} = 0$$

Matching procedure

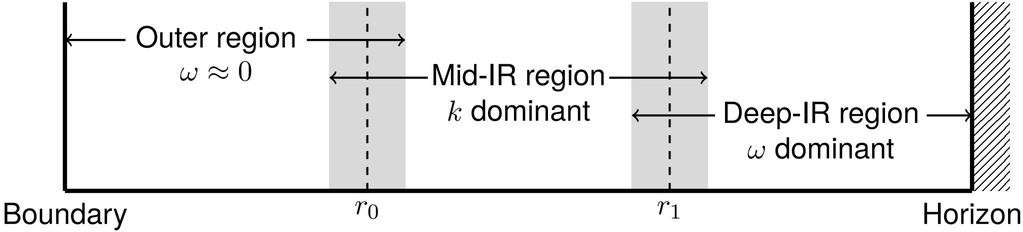


$$\mathcal{G}(\omega, k) = \frac{\chi(r_0)}{\psi(r_0)}$$

Mid-IR region: $\zeta = r^{1-\alpha_U}$

$$\left[\zeta\,\partial_\zeta + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Matching procedure



$$\mathcal{G}(\omega, k) = \frac{\chi(r_0)}{\psi(r_0)}$$

Deep-IR region: $\zeta = r^{1-\alpha_U}$

$$\left[\partial_\zeta - i\bar{\omega}\sigma^2\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Solutions in the IR regions

Deep IR: $$\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Ingoing solution: $$\psi_R(\zeta) = e^{i\bar{\omega}\zeta}, \qquad \chi_R(\zeta) = -ie^{i\bar{\omega}\zeta}.$$

Solutions in the IR regions

Deep IR: $\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$

Ingoing solution: $\psi_R(\zeta) = e^{i\bar{\omega}\zeta}, \qquad \chi_R(\zeta) = -ie^{i\bar{\omega}\zeta}.$

Mid-IR: $\left[\zeta\,\partial_\zeta + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$

General solution:

$$\psi_L(\zeta) = a\exp\left(z\bar{k}\zeta^{1/z}\right) + b\exp\left(-z\bar{k}\zeta^{1/z}\right)$$
$$\chi_L(\zeta) = a\exp\left(z\bar{k}\zeta^{1/z}\right) - b\exp\left(-z\bar{k}\zeta^{1/z}\right)$$

Fix a, b by matching $\psi_L(\zeta_1) = \psi_R(\zeta_1)$ and $\chi_L(\zeta_1) = \chi_R(\zeta_1)$, $\quad \zeta_1 = (\bar{\omega}/\bar{k})^{z/(z-1)}$

Matching solutions

Leading order, low frequency:

$$\psi(\zeta) \approx \begin{cases} e^{i\bar{\omega}\zeta}, & \zeta \geq \zeta_1, \\ \sqrt{2}\cosh\left(z\bar{k}\zeta^{1/z} - i\pi/4\right), & \zeta \leq \zeta_1 \end{cases}$$

$$\chi(\zeta) \approx \begin{cases} -ie^{i\bar{\omega}\zeta}, & \zeta \geq \zeta_1, \\ -\sqrt{2}\sinh\left(z\bar{k}\zeta^{1/z} - i\pi/4\right), & \zeta \leq \zeta_1 \end{cases}$$

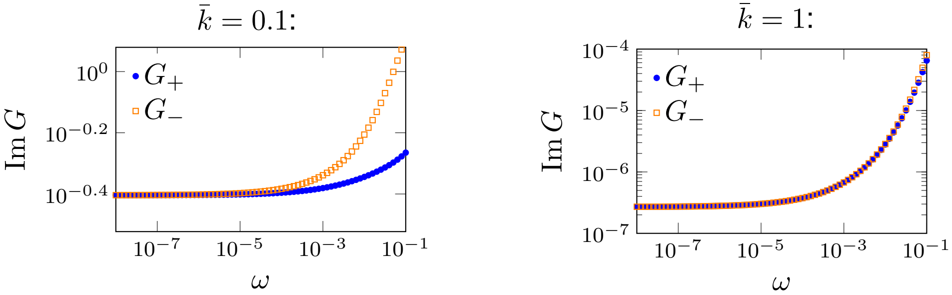
$$\mathcal{G}(\omega, k) \equiv \frac{\chi(\zeta_0)}{\psi(\zeta_0)} \approx i.$$

Continuum $\Rightarrow$ no sharp peaks

Example: $z = -2$, $\theta = 4$

$$U(r) = r^2 + \frac{r^{4/3}}{(1+r)^4}, \qquad V(r) = r^2 + \frac{r^{1/3}}{(1+r)^4}.$$

Set $\bar{m} = \bar{q} = 0$



Negative z

IR EOM:

$$\left[\zeta\,\partial_\zeta + \bar{m}\zeta^{\theta/2z}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\theta/z-\nu_A}\right)(i\sigma^2) + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Similar conclusion for $\bar{m}$ or $\bar{q}$ dominant, as long as $z < 0$ and $\theta > 0$.

Key feature: zero-derivative terms other than $\bar{\omega}$ decay as $\zeta \to \infty$

Nodal-antinodal dichotomy

Anisotropy

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{U(r)} - U(r)\,\mathrm{d}t^2 + V(r)\,\mathrm{d}x^2 + W(r)\,\mathrm{d}y^2$$

$r \to 0$: suppose $U \approx r^{\alpha_U}$, $V \approx r^{\alpha_V}$, $W \approx r^{\alpha_W}$

$$\mathrm{d}s^2 \approx \zeta^{\theta/\bar{z}} \left(\frac{-\mathrm{d}t^2 + \mathrm{d}\zeta^2}{\zeta^2} + \frac{\mathrm{d}x^2}{\zeta^{2/z_x}} + \frac{\mathrm{d}y^2}{\zeta^{2/z_y}} \right), \qquad \zeta = r^{1-\alpha_U}$$

$$z_x = \frac{2(\alpha_U - 1)}{\alpha_U + \alpha_V - 2} \qquad z_y = \frac{2(\alpha_U - 1)}{\alpha_U + \alpha_W - 2}$$

$$\bar{z} = \left(\frac{1}{z_x} + \frac{1}{z_y} \right)^{-1} \qquad \theta = \frac{4(\alpha_U - 2)}{2\alpha_U + \alpha_V + \alpha_W - 4}$$

Anisotropy

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{U(r)} - U(r)\,\mathrm{d}t^2 + V(r)\,\mathrm{d}x^2 + W(r)\,\mathrm{d}y^2$$

$r \to 0$: suppose $U \approx r^{\alpha_U}$, $V \approx r^{\alpha_V}$, $W \approx r^{\alpha_W}$

$$\mathrm{d}s^2 \approx \zeta^{\theta/\bar{z}} \left(\frac{-\mathrm{d}t^2 + \mathrm{d}\zeta^2}{\zeta^2} + \frac{\mathrm{d}x^2}{\zeta^{2/z_x}} + \frac{\mathrm{d}y^2}{\zeta^{2/z_y}} \right), \qquad \zeta = r^{1-\alpha_U}$$

Anisotropic Lifshitz scaling:

$$(t,\zeta) \to \lambda(t,\zeta), \qquad x \to \lambda^{1/z_x} x, \qquad y \to \lambda^{1/z_y} y$$

Anisotropy

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{U(r)} - U(r)\,\mathrm{d}t^2 + V(r)\,\mathrm{d}x^2 + W(r)\,\mathrm{d}y^2$$

$r \to 0$: suppose $U \approx r^{\alpha_U}$, $V \approx r^{\alpha_V}$, $W \approx r^{\alpha_W}$

$$\mathrm{d}s^2 \approx \zeta^{\theta/\bar{z}} \left(\frac{-\mathrm{d}t^2 + \mathrm{d}\zeta^2}{\zeta^2} + \frac{\mathrm{d}x^2}{\zeta^{2/z_x}} + \frac{\mathrm{d}y^2}{\zeta^{2/z_y}} \right), \qquad \zeta = r^{1-\alpha_U}$$

Hyperscaling violation:

$$S \propto T^{(2-\theta)/\bar{z}}, \qquad \bar{z} = \left(\frac{1}{z_x} + \frac{1}{z_y} \right)^{-1}$$

Fermions in anisotropic backgrounds

Momentum in x direction

$$\left[\zeta\partial_\zeta + \bar{m}\zeta^{\theta/2\bar{z}}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\frac{\theta}{\bar{z}}-\nu_A}\right)(i\sigma^2) + \bar{k}_x\zeta^{1/z_x}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Momentum in y direction

$$\left[\zeta\partial_\zeta + \bar{m}\zeta^{\theta/2\bar{z}}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\frac{\theta}{\bar{z}}-\nu_A}\right)(i\sigma^2) + \bar{k}_y\zeta^{1/z_y}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Nodal-antinodal dichotomy:

$$\theta/\bar{z} < 0, \quad z_x < 0, \quad z_y > 0$$

Anisotropic Q-lattices

Q-lattice Model

[Donos, Gauntlett, 1401.5077]

$$S = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^4x \sqrt{-g} \left[R - \frac{1}{4} \cosh^{1/3}(3\phi)\, F^2 + 6\cosh\phi - \frac{3}{2}(\partial\phi)^2 - 6\sinh^2\phi\,(\partial\eta)^2 \right]$$

$$F = \mathrm{d}A$$

Have fixed parameters in model to some nice values

Admits anisotropic, extremal black brane solutions

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{U(r)} - U(r)\,\mathrm{d}t^2 + V(r)\,\mathrm{d}x^2 + W(r)\,\mathrm{d}y^2$$

$$A = A_t(r)\,\mathrm{d}t$$

$$\phi = \phi(r)$$

$$\eta = px$$

Black brane backgrounds

UV $r \to \infty$

$$U(r) \approx V(r) \approx W(r) \approx r^2 \qquad A_t \approx \mu \qquad \phi(r) \approx \lambda r^{-1}$$

Will set $p = 0.1\mu$ and $\lambda = \mu$

Black brane backgrounds

UV $r \to \infty$

$$U(r) \approx V(r) \approx W(r) \approx r^2 \qquad A_t \approx \mu \qquad \phi(r) \approx \lambda r^{-1}$$

Will set $p = 0.1\mu$ and $\lambda = \mu$

IR $r \to 0$

$$U(r) \approx U_0 r^{7/4}, \qquad V(r) \approx V_0 r^{-1/4}, \qquad W(r) \approx W_0 r^{3/4},$$

$$A_t(r) \approx A_0 r, \qquad e^{\phi(r)} \approx e^{\phi_0} r^{-1/4}$$

Black brane backgrounds

UV $r \to \infty$

$$U(r) \approx V(r) \approx W(r) \approx r^2 \qquad A_t \approx \mu \qquad \phi(r) \approx \lambda r^{-1}$$

Will set $p = 0.1\mu$ and $\lambda = \mu$

IR $r \to 0$

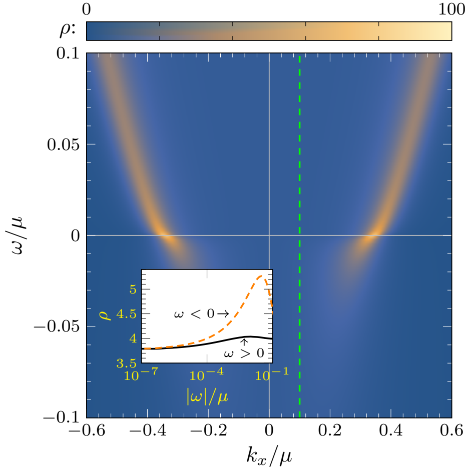
$$U(r) \approx U_0 r^{7/4}, \qquad V(r) \approx V_0 r^{-1/4}, \qquad W(r) \approx W_0 r^{3/4},$$

$$A_t(r) \approx A_0 r, \qquad e^{\phi(r)} \approx e^{\phi_0} r^{-1/4}$$

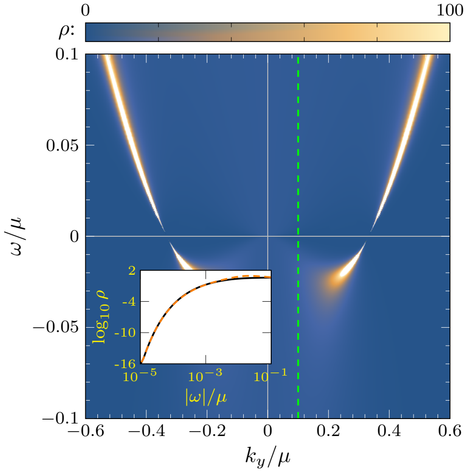
IR scaling: $\theta/\bar{z} = -1/3, \qquad z_x = -3, \qquad z_y = 3$

$$\theta/\bar{z} < 0, \quad z_x < 0, \quad z_y > 0$$

Fermion spectral functions, $m = 0$, $q = 1$

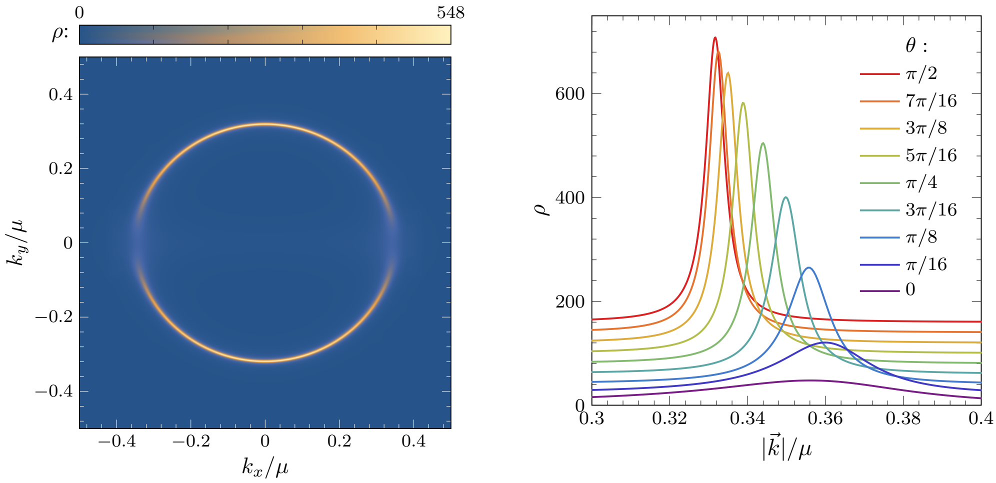


$k_y = 0$

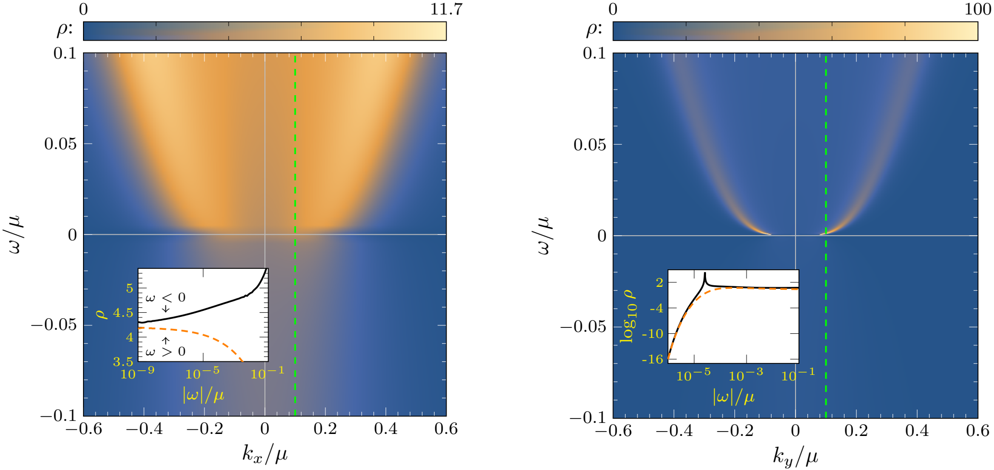


$k_x = 0$

Fermion spectral functions, $m = 0$, $q = 1$

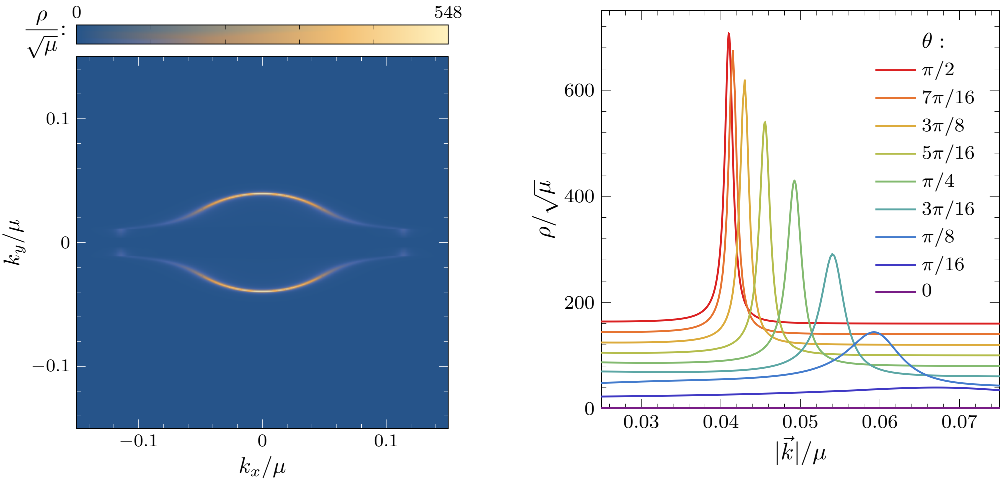


Fermion spectral functions, $m = 1/4$, $q = 1$



$k_y = 0$ $k_x = 0$

Fermion spectral functions, $m = 1/4$, $q = 1$



Discussion

Summary

Minimally coupled fermion, IR scaling with $\theta/\bar{z} < 0, \quad z_x < 0, \quad z_y > 0$:

$\Rightarrow$ fermion continuum in only one direction

$\Rightarrow$ nodal-antinodal dichotomy

Independent of precise method of generating anisotropic scaling

Robustness

$$\left[\zeta\partial_\zeta + \bar{m}\zeta^{\theta/2\bar{z}}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\frac{\theta}{\bar{z}}-\nu_A}\right)(i\sigma^2) + \bar{k}_i\zeta^{1/z_i}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Relied on $\bar{m}$, $\bar{q}$, $\bar{k}_x$ terms decaying as $\zeta \to \infty$

What about non-minimal interactions?

Dipole interaction [Edalati, Leigh, Phillips, 1010.3238]

$$S_{\text{bulk}} = i\int \mathrm{d}^4x\,\sqrt{-G}\,\bar{\Xi}\left(\not{D} - m - ig\not{F}\right)\Xi + \text{(boundary terms)}$$

- Adds term to EOM $\sim \zeta^{-\nu_A}$
- Decays as $\zeta \to \infty$ since $\nu_A \geq 0$ ✓

Robustness

$$\left[\zeta\partial_\zeta + \bar{m}\zeta^{\theta/2\bar{z}}\sigma^3 - \left(\bar{\omega}\zeta + \bar{q}\zeta^{\frac{\theta}{\bar{z}}-\nu_A}\right)(i\sigma^2) + \bar{k}_i\zeta^{1/z_i}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

Relied on $\bar{m}$, $\bar{q}$, $\bar{k}_x$ terms decaying as $\zeta \to \infty$

What about non-minimal interactions?

Yukawa interaction with scalar Φ

$$S_{\text{bulk}} = i\int \mathrm{d}^4x\,\sqrt{-G}\,\bar{\Xi}\left(\not{D} - m - g\Phi\right)\Xi + \text{(boundary terms)}$$

- Depends on IR scaling of Φ
- Q-lattice: $\Phi \propto \phi^n$ ✓
- Q-lattice: $\Phi \propto e^{n\phi}$ × for $n \geq 1/2$

Outlook

Classify effects of non-minimal interactions?

Four-fold rotational symmetry?

- Lattice?
- Spin-two field?

Bosonic spectral functions?

- Minimally coupled scalar: $\rho_{\text{scalar}} \propto \omega$ when $\rho_{\text{fermion}} \propto \omega^0$
- Dichotomy in density–density two-point function?

Non-holographic model?

Outlook

Classify effects of non-minimal interactions?

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- Minimally coupled scalar: $\rho_{\text{scalar}} \propto \omega$ when $\rho_{\text{fermion}} \propto \omega^0$
- Dichotomy in density–density two-point function?

Non-holographic model?

Thank you!

Backup slides

Overlap of mid-IR and deep-IR

$$\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2 + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

- Mid-IR: neglect $\bar{\omega}$
- Deep-IR: neglect $\bar{k}$

Both terms same size at $\zeta = \zeta_1 = (\bar{\omega}/\bar{k})^{z/(z-1)}$?

Overlap of mid-IR and deep-IR

$$\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2 + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

- Mid-IR: neglect $\bar{\omega}$
- Deep-IR: neglect $\bar{k}$

Both terms same size at $\zeta = \zeta_1 = (\bar{\omega}/\bar{k})^{z/(z-1)}$?

Deep-IR: Let $s = \zeta/\zeta_1, \quad \kappa = \bar{k}^{z/(z-1)}\bar{\omega}^{1/(1-z)} \ll 1$

$$\psi(s) = e^{i\kappa s} + \delta\psi(s), \qquad \chi(\zeta) = -ie^{i\kappa s} + \delta\chi(s)$$

Sub into EOM:

$$\left[s\partial_s - i\kappa s\sigma^2 + \kappa s^{1/z}\sigma^1\right]\begin{pmatrix}\delta\chi(s)\\ \delta\psi(s)\end{pmatrix} = \begin{pmatrix}-1\\ i\end{pmatrix}\kappa s^{1/z}\exp(i\kappa s)$$

Overlap of mid-IR and deep-IR

$$\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2 + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

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Both terms same size at $\zeta = \zeta_1 = (\bar{\omega}/\bar{k})^{z/(z-1)}$?

Deep-IR: Let $s = \zeta/\zeta_1$, $\quad \kappa = \bar{k}^{z/(z-1)}\bar{\omega}^{1/(1-z)} \ll 1$

$$s\partial_s\begin{pmatrix}\delta\chi(s)\\ \delta\psi(s)\end{pmatrix} \approx \begin{pmatrix}-1\\ i\end{pmatrix}\kappa s^{1/z}\exp(i\kappa s)$$

Solution:

$$\delta\chi(s) \approx -i\delta\psi(s) \approx \kappa(-i\kappa)^{1/z}\,\Gamma\left(\frac{1}{z}, -i\kappa s\right)$$

Overlap of mid-IR and deep-IR

$$\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2 + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

- Mid-IR: neglect $\bar{\omega}$
- Deep-IR: neglect $\bar{k}$

Both terms same size at $\zeta = \zeta_1 = (\bar{\omega}/\bar{k})^{z/(z-1)}$?

Deep-IR: Let $s = \zeta/\zeta_1$, $\quad \kappa = \bar{k}^{z/(z-1)}\bar{\omega}^{1/(1-z)} \ll 1$

$$s\partial_s \begin{pmatrix}\delta\chi(s)\\ \delta\psi(s)\end{pmatrix} \approx \begin{pmatrix}-1\\ i\end{pmatrix}\kappa s^{1/z}\exp(i\kappa s)$$

Solution:

$$\delta\chi(s) \approx -i\delta\psi(s) \approx z\kappa s^{1/z} + \mathcal{O}(\kappa^{1+1/|z|})$$

Vanishes when $\kappa \to 0$

Overlap of mid-IR and deep-IR

$$\left[\zeta\partial_\zeta - i\bar{\omega}\zeta\sigma^2 + \bar{k}\zeta^{1/z}\sigma^1\right]\begin{pmatrix}\chi(\zeta)\\ \psi(\zeta)\end{pmatrix} = 0$$

- Mid-IR: neglect $\bar{\omega}$
- Deep-IR: neglect $\bar{k}$

Both terms same size at $\zeta = \zeta_1 = (\bar{\omega}/\bar{k})^{z/(z-1)}$?

Similar in mid-IR