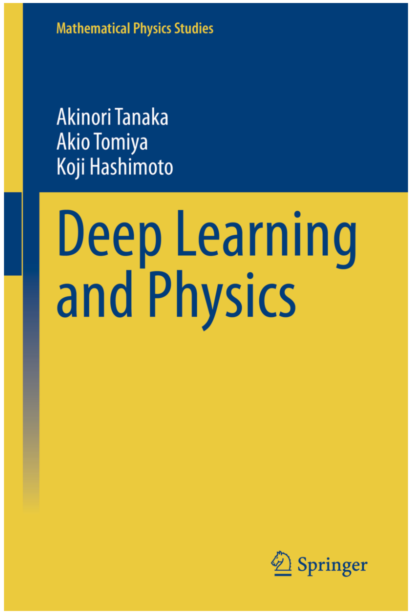
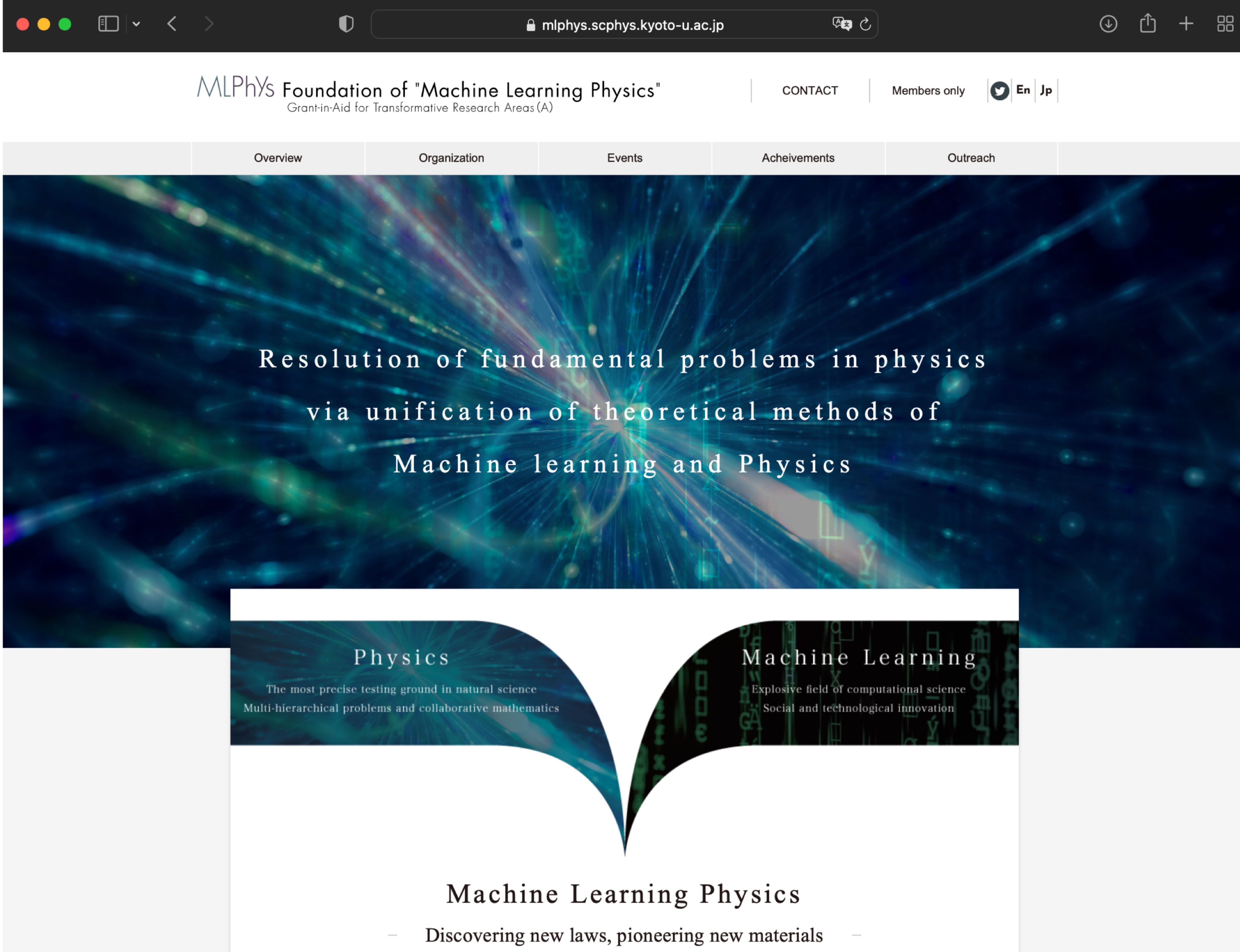
Holotube, 14 Feb 2023

Machine Learning the Bulk in AdS/CFT

Koji Hashimoto (Kyoto U.)

"Deriving dilaton potential in improved holographic QCD from chiral condensate" 2209.04638
"Deriving dilaton potential in improved holographic QCD from meson spectrum" 2108.08091
w/ K.Ohashi (Keio), T.Sumimoto (Osaka u)
"Neural ODE and Holographic QCD" 2006.00712
w/ H.Y.Hu, Y.Z.You (UCSD)
"Deep Learning and AdS/QCD" 2005.02636
w/ T. Akutagawa, T. Sumimoto (Osaka u)
"Deep Boltzmann Machine and AdS/CFT" 1903.04951
"Deep Learning and Holographic QCD" 1809.10536
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)
"Deep Learning and AdS/CFT" 1802.08313
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)

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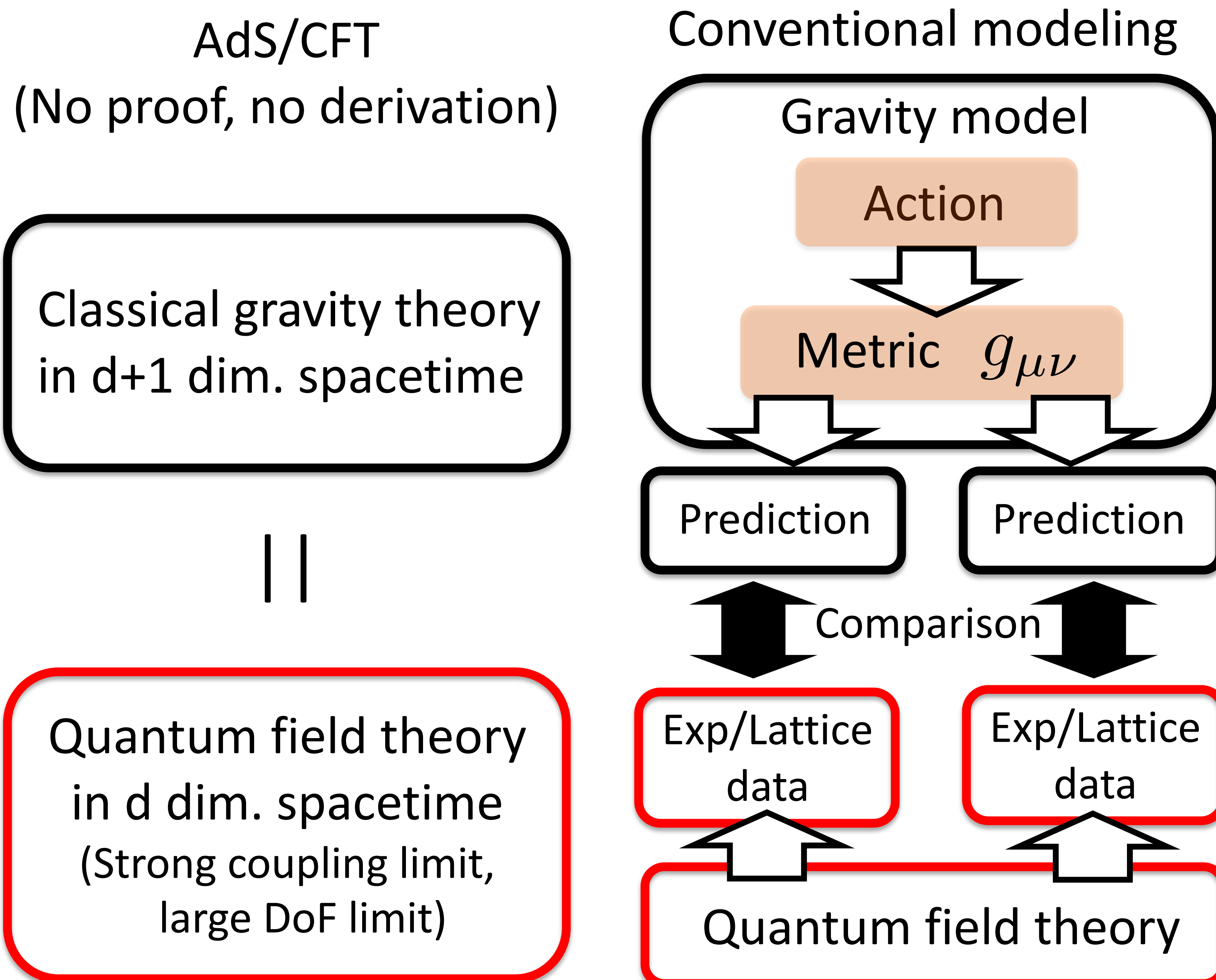
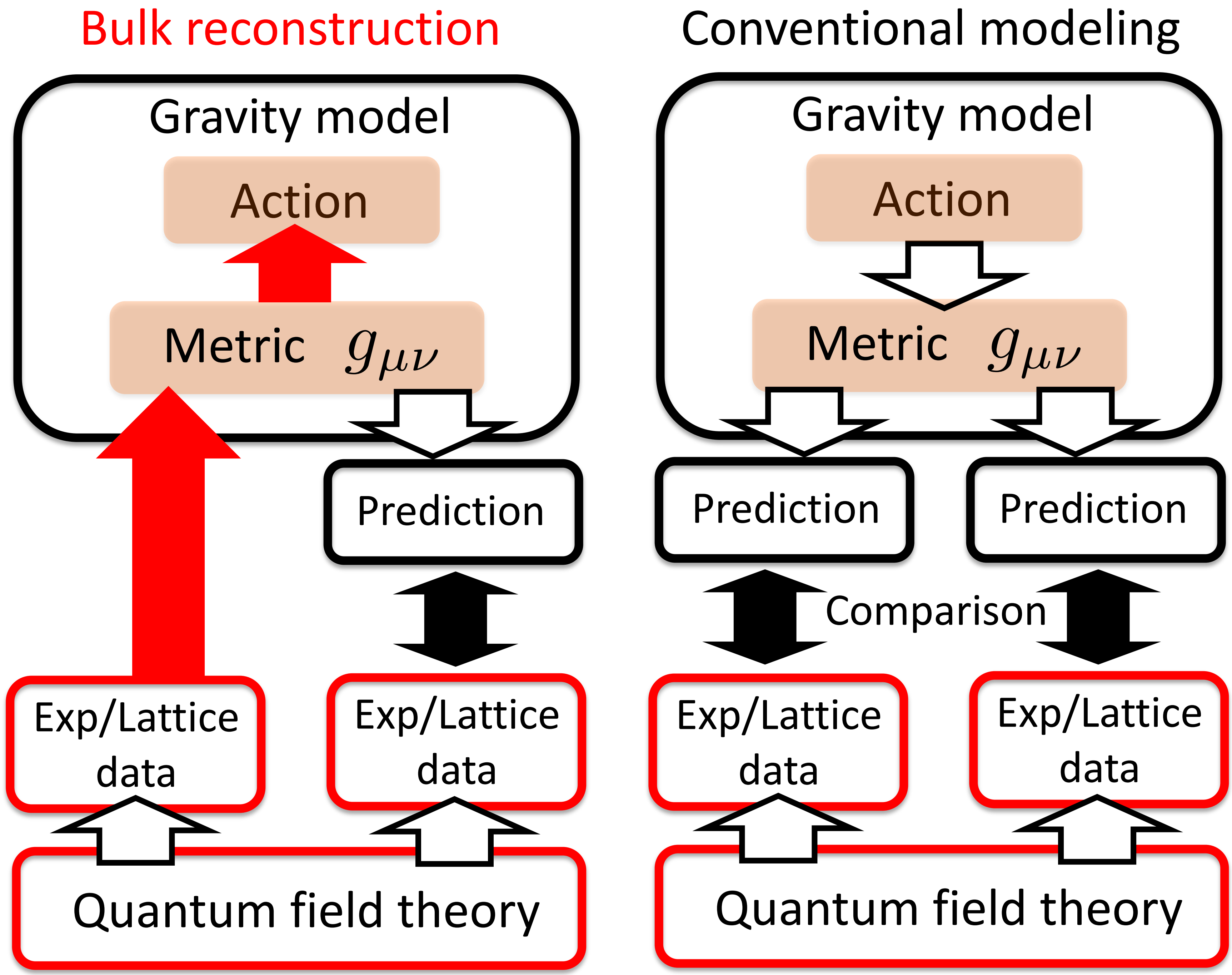
Overview
Organization
Events
Acheivements
Outreach
Resolution of fundamental problems in physics
via unification of theoretical methods of
Machine learning and Physics
Physics
The most precise testing ground in natural science
Multi-hierarchical problems and collaborative mathematics
Machine Learning
Explosive field of computational science
Social and technological innovation
Machine Learning Physics
Discovering new laws, pioneering new materials

Bulk reconstruction by deep learning



AdS/CFT
(No proof, no derivation)
Classical gravity theory in d+1 dim. spacetime
=
Quantum field theory in d dim. spacetime
(Strong coupling limit, large DoF limit)
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Prediction
Comparison
Exp/Lattice data
Exp/Lattice data
Quantum field theory



Bulk reconstruction
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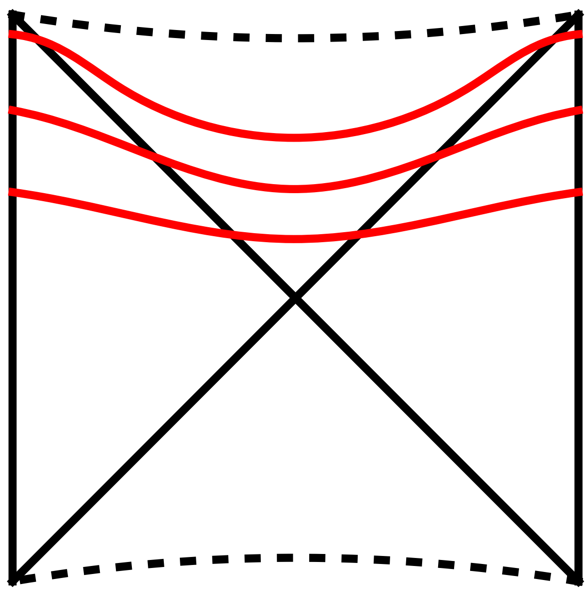
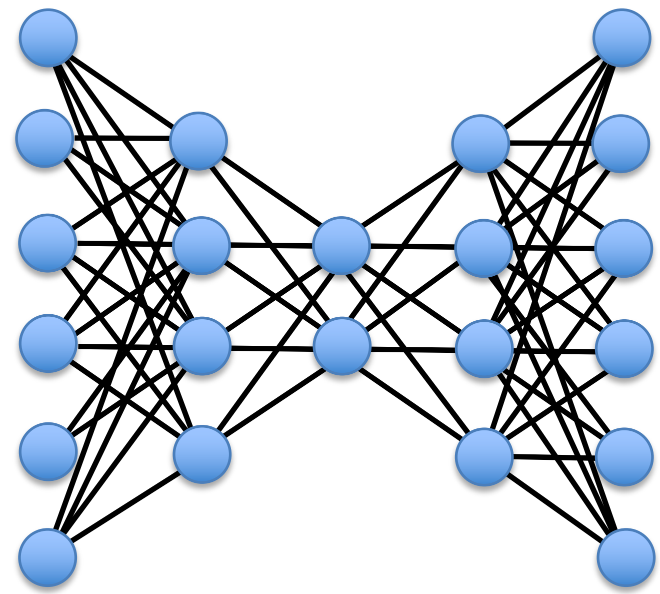
Comparison of solvers

Reconstruction method	No use of Einstein eq	Lattice input
Holographic renormalization [deHaro Solodukhin Skenderis 00]		✓
Entanglement, Complexity [Hammersley 07] [Bilson 08]… [KH Watanabe 21]	✓	
Correlators [Hammersley 06] [Hubeny Liu Rangamani 06]	✓	
AdS/DL [KH Tanaka Tomiya Sugishita 18]	✓	✓
Wilson loop [KH 20]	✓	✓

Similarity!?



Wormholes in Penrose diagram of maximally extended eternal AdS Schwarzschild black hole [Iizuka, Sugishita, KH `17]



Deep Autoencoder

Emergent spacetime as a neural network

Quantum gravity in (d+1)-dim.

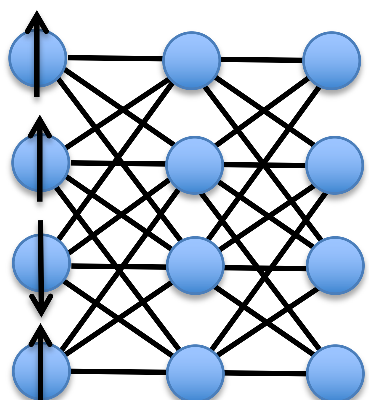
'tHooft \`93
Susskind \`94
Maldacena \`97

||

Quantum mechanics in d-dim.

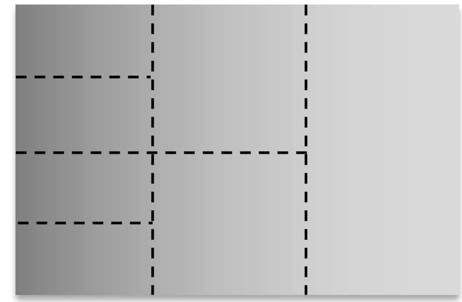
General spacetime

|| ?



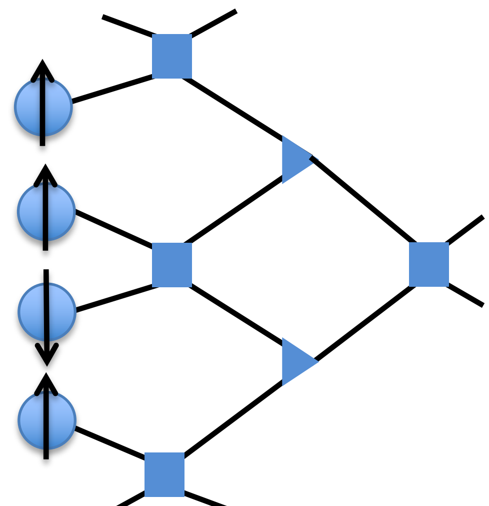
Neural network

Anti de Sitter spacetime



|| Swingle \`10

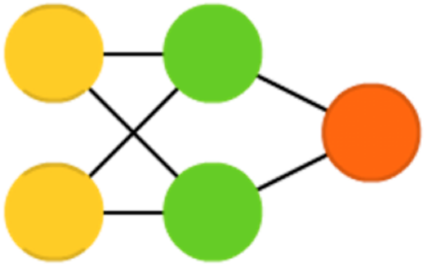
Carleo, Troyer \`17



Tensor network

Basics of machine learning

Neural network



$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

"Unit" (circle) : Vector component

"Weight" (line) : Linear transformation to be optimized

"Activation function" (hidden line-end) : Nonlinear component-wise transf.

$$\varphi(x) \equiv \frac{1}{1+e^{-x}}$$

Training protocol :

1) Prepare many sets $\{(x_j, f)\}$: (input, output)

2) Train the network (adjust W) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right) \right|$

Bulk reconstruction by deep learning

② Space emergent from data

Simplest holographic model

Classical scalar field theory in unknown 5-dim. spacetime

1802.08313
1809.10536

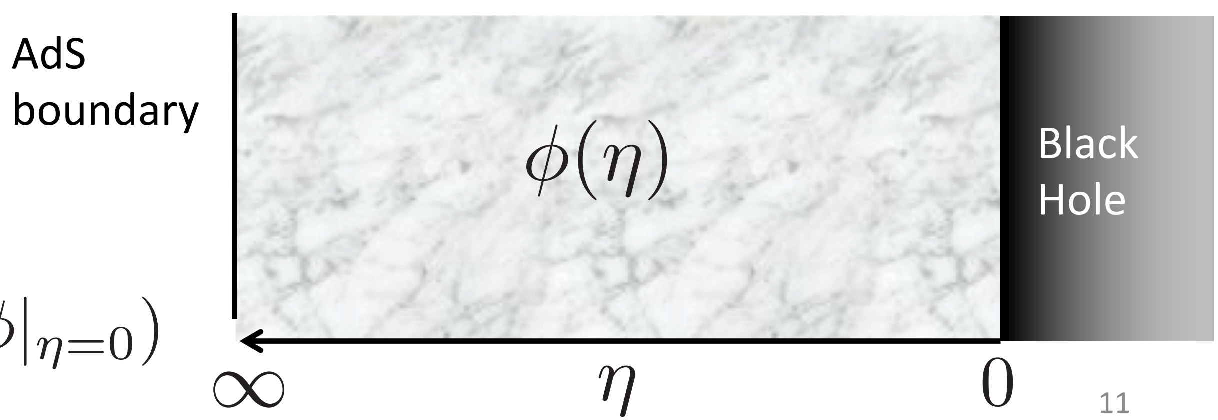
$$S = \int d\eta d^4x \sqrt{\det g}\,\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$\begin{cases} ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2) \\ V[\phi] = -\dfrac{3}{L^2}\phi^2 + \dfrac{\lambda}{4}\phi^4 \end{cases}$$

Data: $(m_q, \langle \bar{q}q \rangle)$

⇕

$(\phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=0})$



② Space emergent from data

Relation to QCD data

Boundary condition for the metric components

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2)$$

- AdS boundary ($\eta \sim \infty$) : $f \sim g \sim \exp[2\eta/L]$
- Black hole horizon ($\eta \sim 0$) : $f \sim \eta^2, \; g \sim \text{const.}$

Solve eq. of motion to get response $\langle \bar{\psi}\psi \rangle_{m_q}$. [Klebanov, Witten `98]

- AdS boundary ($\eta \sim \infty$) : $\phi = m_q e^{-\eta} + \langle \bar{\psi}\psi \rangle e^{-3\eta}$
- Black hole horizon ($\eta \sim 0$) : $\partial_\eta \phi \big|_{\eta=0} = 0$

② Space emergent from data

Equation of motion as a feedforward NN

Eq. of motion $$\partial_\eta^2\phi + h(\eta)\partial_\eta\phi - \frac{\delta V[\phi]}{\delta\phi} = 0$$

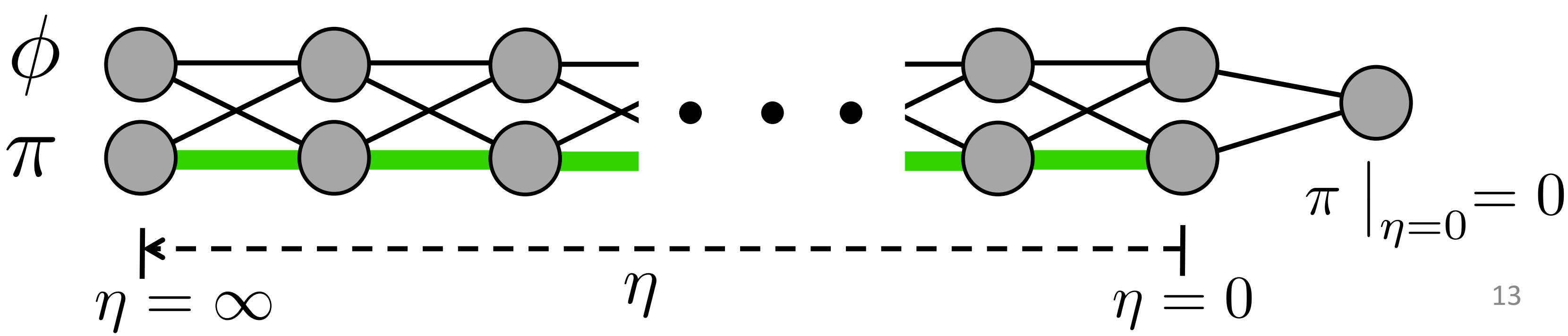
metric $$h(\eta) \equiv \partial_\eta \left[\log\sqrt{f(\eta)g(\eta)^{d-1}}\right]$$

⇩

Discretization
Hamilton form
$$\begin{cases} \phi(\eta+\Delta\eta) = \phi(\eta) + \Delta\eta\,\pi(\eta) \\ \pi(\eta+\Delta\eta) = \pi(\eta) + \Delta\eta\left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta\phi(\eta)}\right) \end{cases}$$

⇩

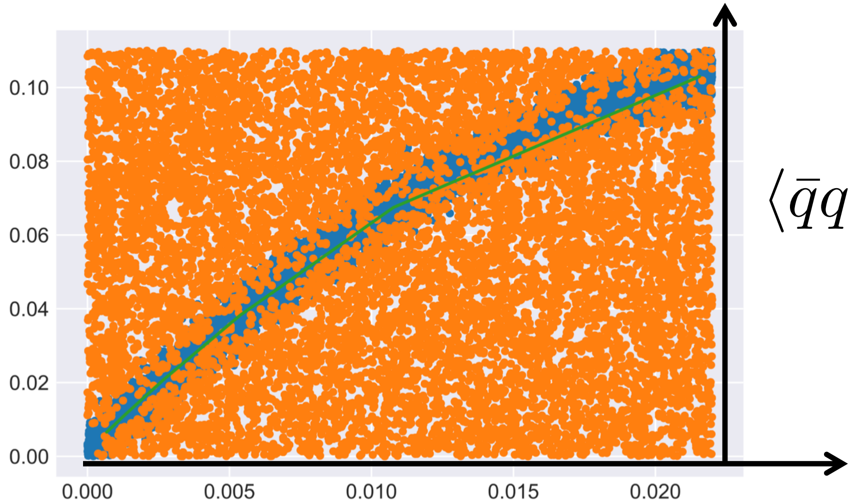
Feedfoward neural network for classification



② Space emergent from data

Training with QCD data : quark condensate

Lattice QCD data at T=207[MeV]

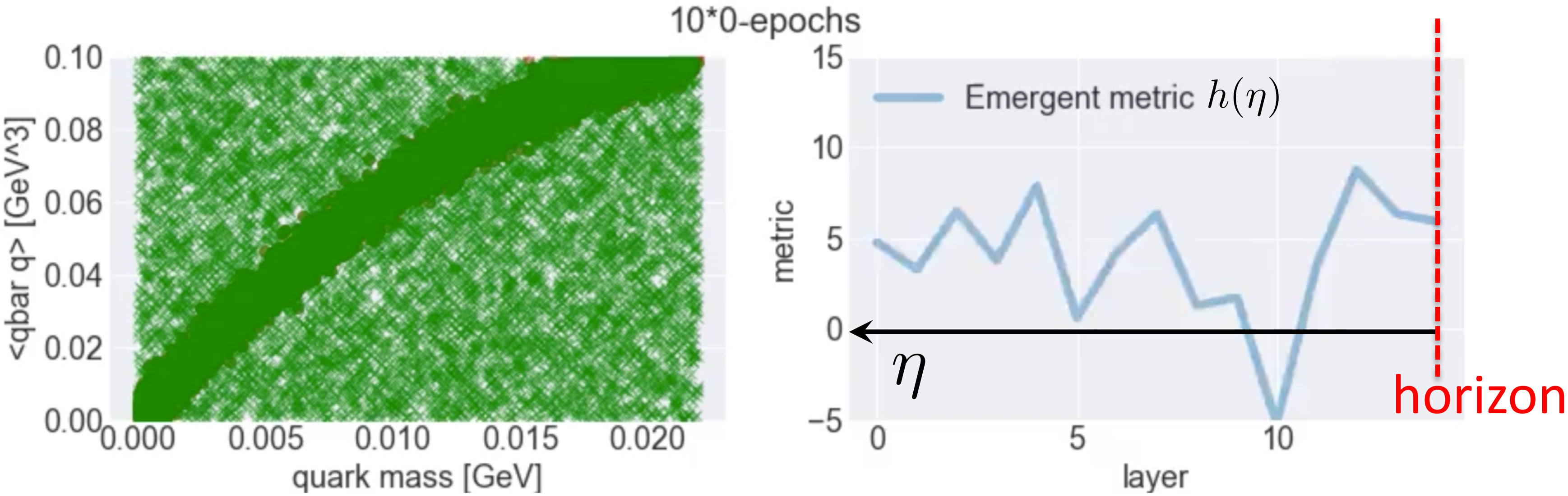


$\langle \bar{q}q \rangle$: Quark condensate

m_q : Quark mass

② Space emergent from data

Training with QCD data : quark condensate



Trained values of potential :

$1/L = 237(3)[\text{MeV}]$, $\lambda/L = 0.0127(6)$

AdS QCD model for meson spectra

[Karch, Kaz, Son, Stephanov `06]

Classical gauge theory in 5-d dilaton gravity background

$$S = \int d^4x dz\, e^{-\Phi} \sqrt{-g}\, \left(F_{MN}\right)^2$$

Dilaton $\Phi(z)$, metric $ds^2 = e^{2A(z)} \left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right)$

AdS boundary ($z \sim 0$) : $B(z) \equiv \Phi(z) - A(z) \sim \log z$

Solve EoM for gauge field $A_\mu(z, x^\mu) = v_n(z) \rho_\mu(x^\mu)$

$$\frac{\partial}{\partial z} \left(e^{-B} \frac{\partial}{\partial z} v_n \right) + \omega^2 e^{-B} v_n = 0$$

When frequency takes a proper discrete value $\omega^2 \sim m_n^2$,
gauge field is normalizable : vector meson spectra.

② Space emergent from data

Bring the bulk EoM to neural network

2005.02636

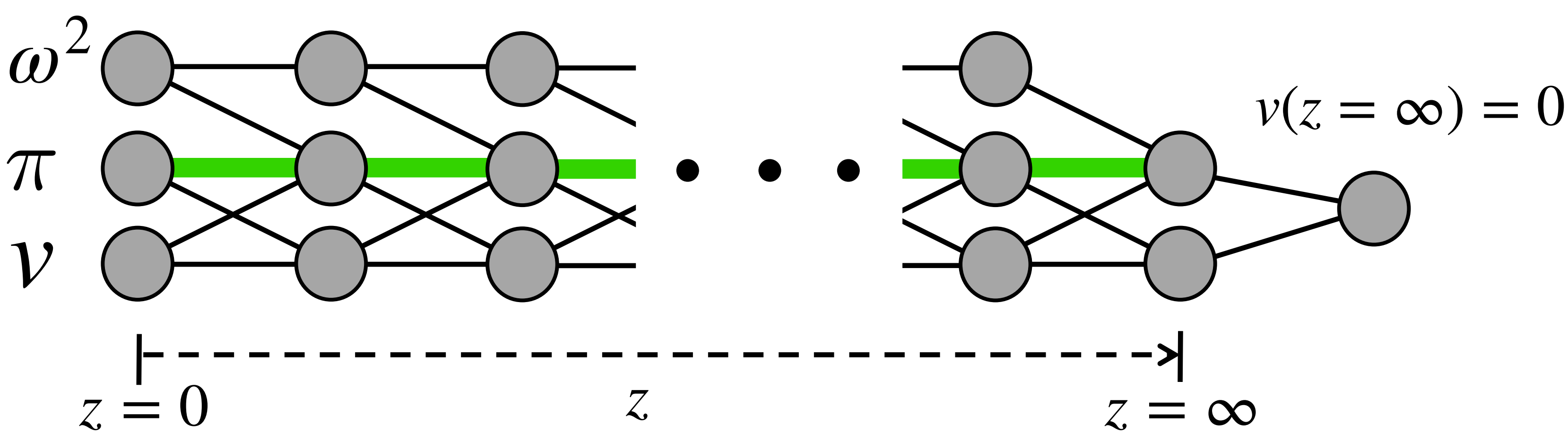
Bulk EoM $$\frac{\partial}{\partial z}\left(e^{-B}\frac{\partial}{\partial z}v_n\right)+\omega^2 e^{-B}v_n=0$$

⇩

Discretization
Hamilton form
$$\begin{cases} v_n(z+\Delta z)=v_n(z)+\Delta z\,\pi_n(z) \\ \pi_n(z+\Delta z)=\pi_n(z)+\Delta z\left(B'(z)\pi_n(z)-\omega^2 v_n(z)\right) \end{cases}$$

⇩

Neural-Network representation



② Space emergent from data

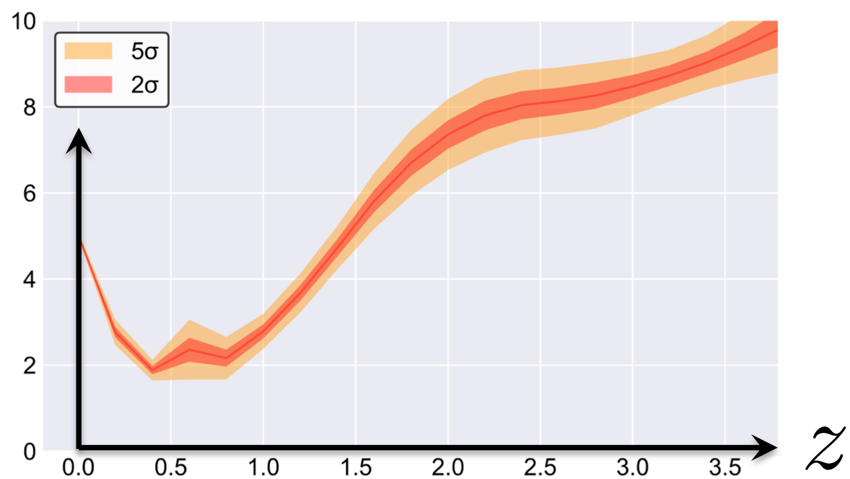
Training with QCD data: hadron spectra

2005.02636

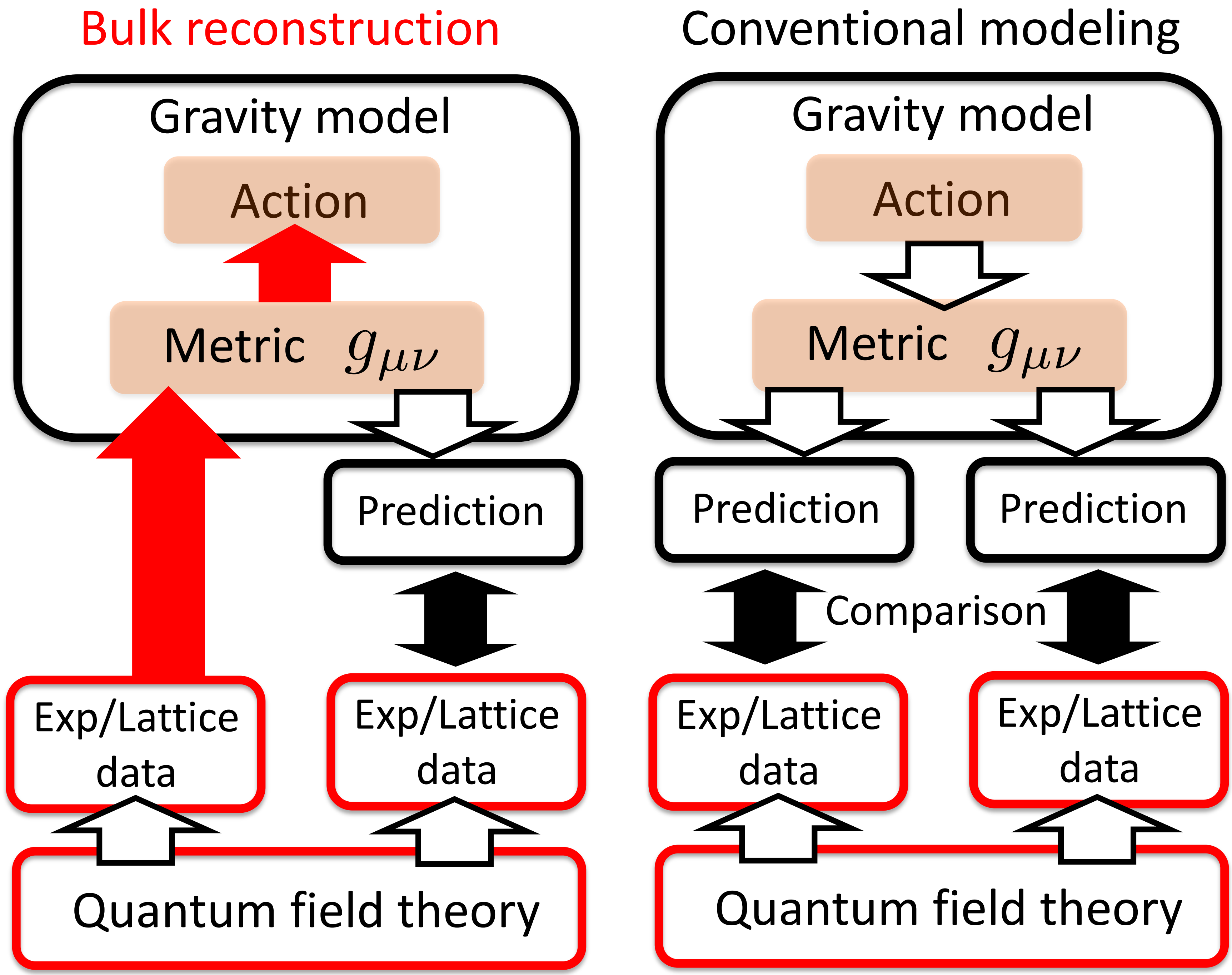
Input : PDG data for rho meson mass

$$m_\rho^{(1)} = 0.77\ \mathrm{GeV},\ m_\rho^{(2)} = 1.45\ \mathrm{GeV}$$

Result: Emergent metric $B'(z) = \Phi'(z) - A'(z)$



Bulk reconstruction by deep learning


Bulk reconstruction
Conventional modeling
Gravity model
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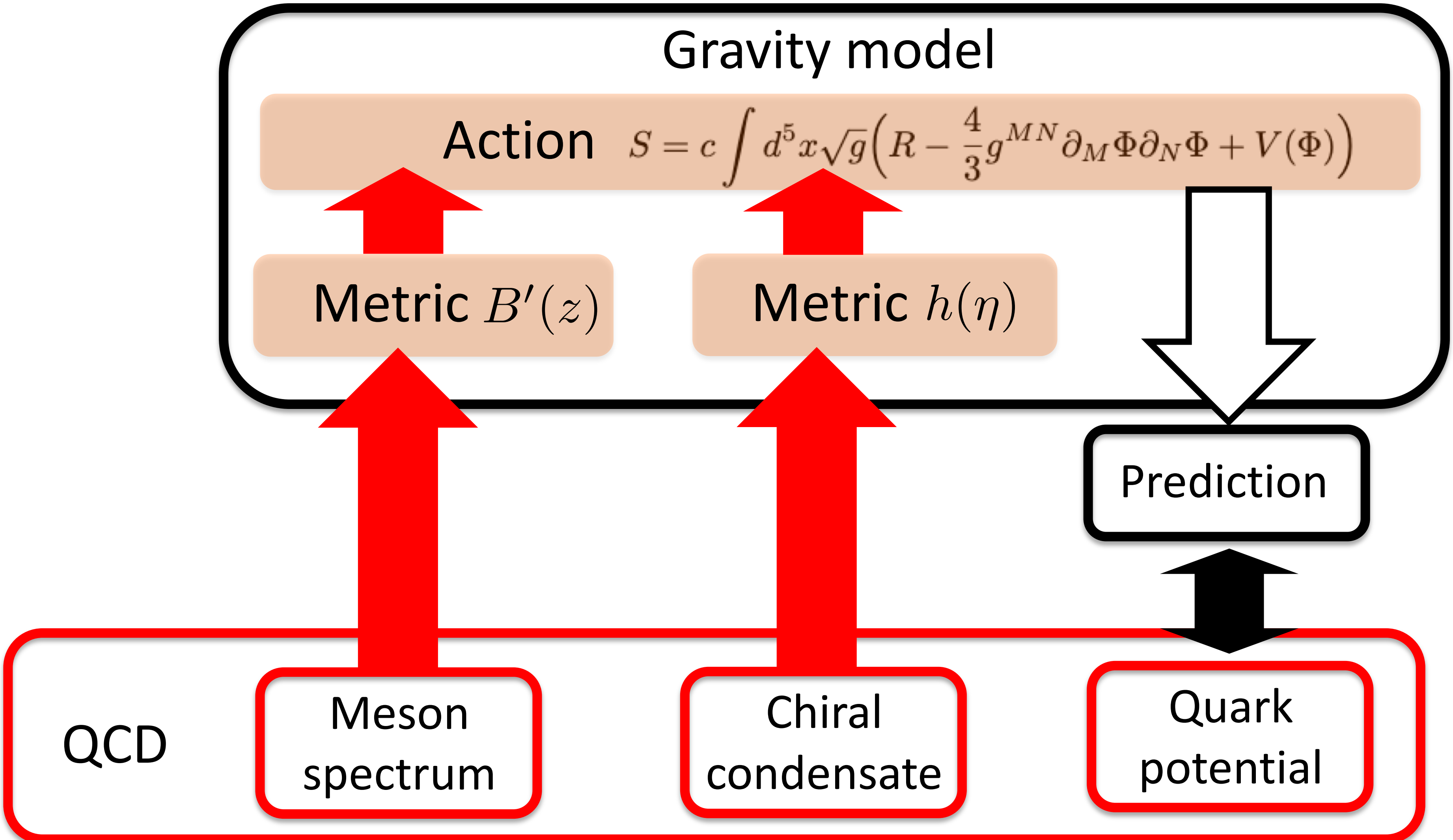
Quantum field theory

③ Gravity reconstructed

Two independent information of metric



③ Gravity reconstructed

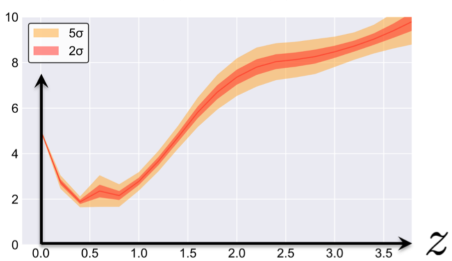
Two independent information of metric

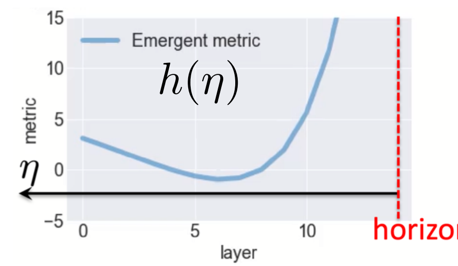
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

Metric $B'(z)$ Metric $h(\eta)$

$B'(z) = \Phi'(z) - A'(z)$





Meson spectrum

Chiral condensate

3. Gravity reconstructed

Deriving the dilaton potential (T=0)

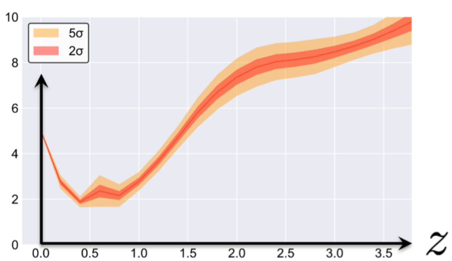
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

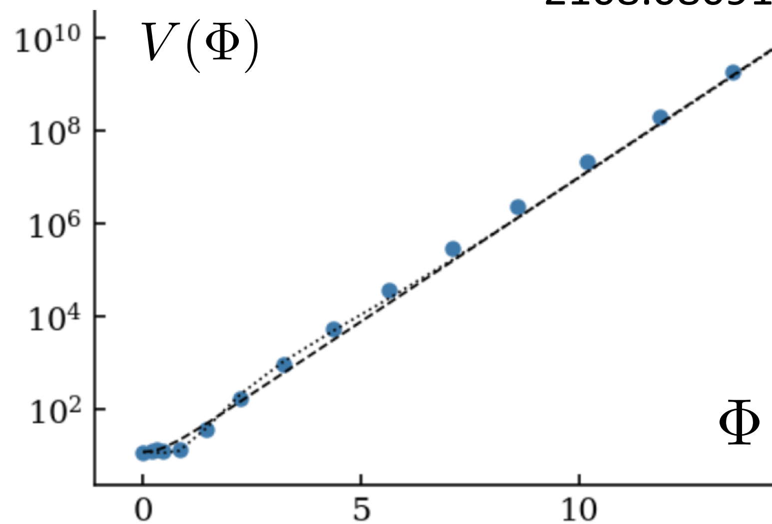
2108.08091

Metric $B'(z)$

$B'(z) = \Phi'(z) - A'(z)$



Meson spectrum



- - - - - - $V = 12\cosh(1.433\Phi)$

·········· $V = 12\cosh(1.430\Phi) - 16.778\Phi^2 + 5.943\Phi^4$

Cf. [Gubser, Nellore, 0804.0434]

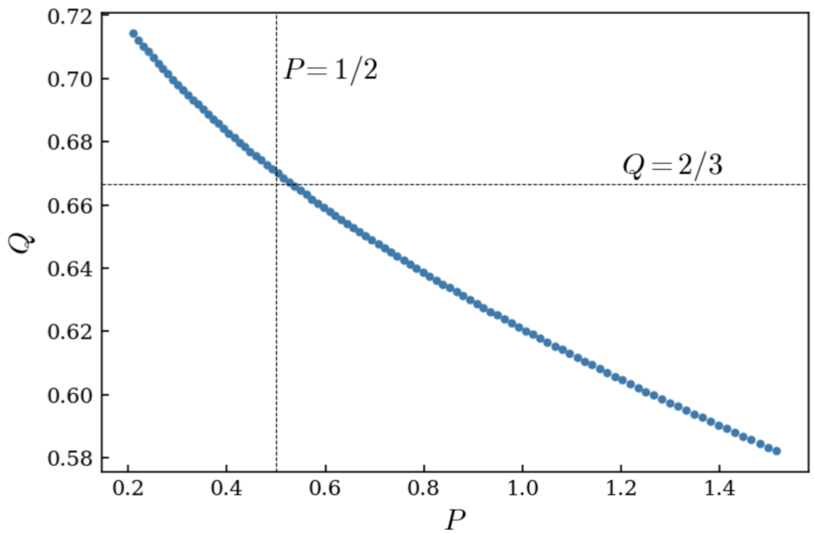
③ Gravity reconstructed

It's a nice dilaton potential !

Gravity model

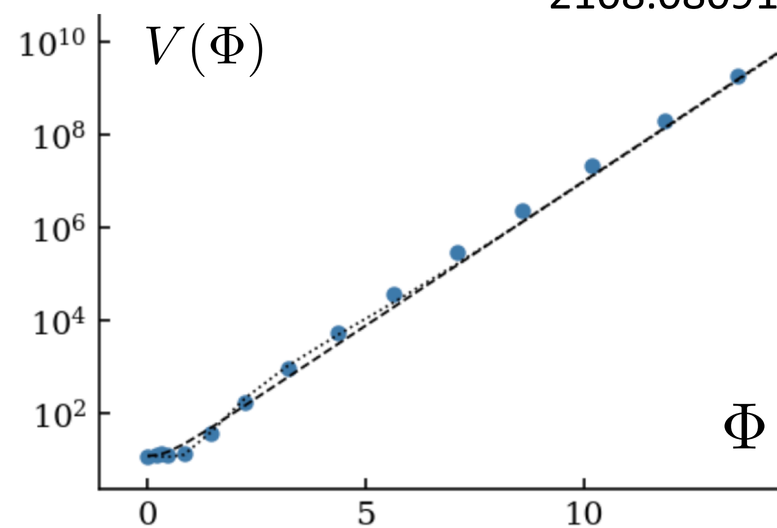
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

2108.08091



Fit the asymptotic part by $V(\Phi) \sim e^{2Q\Phi}\Phi^P$ for different values of dilaton initial cond.

Cf. [Gursoy, Kiritsis, 0707.1324]
[Gursoy, Kiritsis, Nitti, 0707.1349]



$V = 12\cosh(1.433\Phi)$

$V = 12\cosh(1.430\Phi) - 16.778\Phi^2 + 5.943\Phi^4$

Cf. [Gubser, Nellore, 0804.0434]

③ Gravity reconstructed

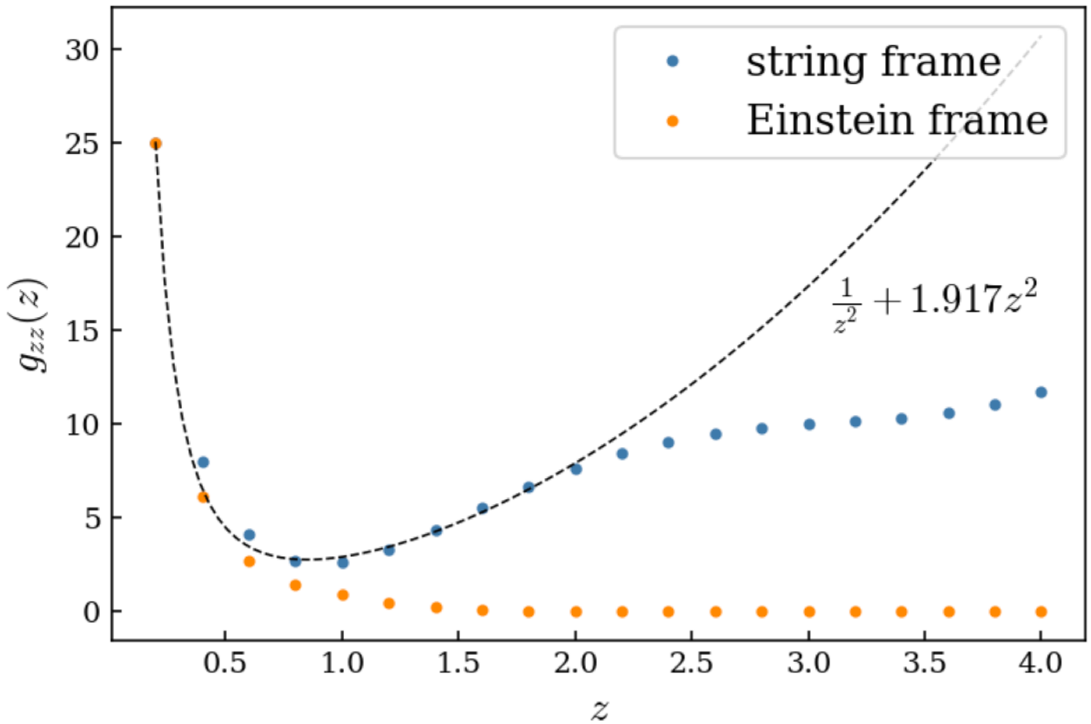
String frame metric has a bottom

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

Prediction

Quark potential

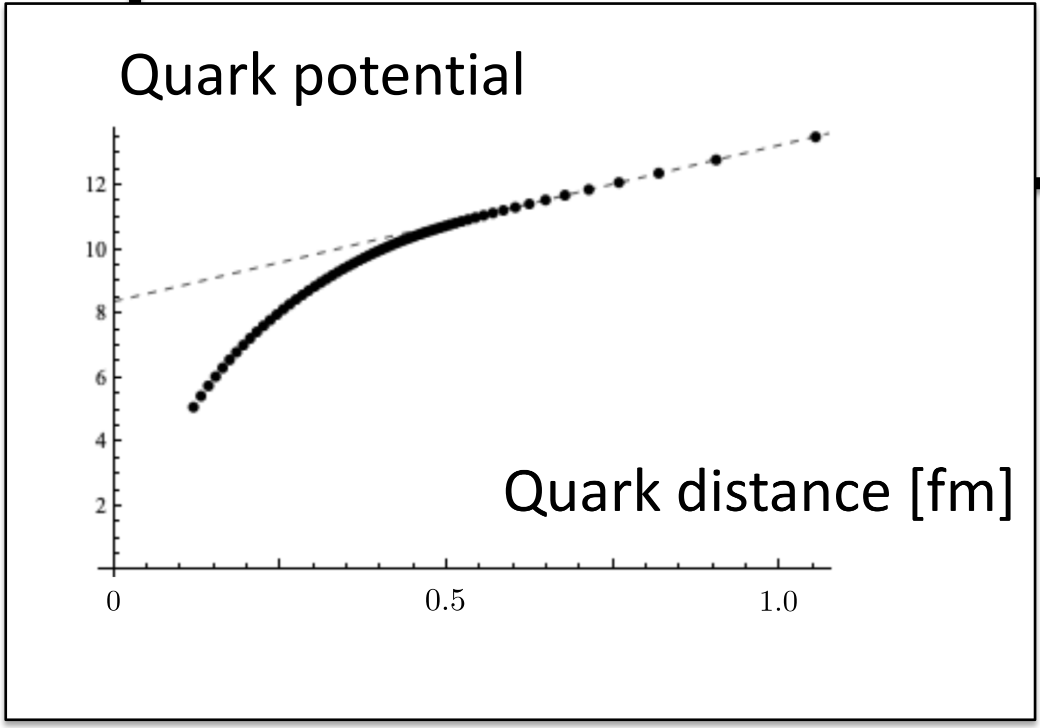


③ Gravity reconstructed

Prediction of string breaking (T=0)

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$



Prediction

Quark potential

③ Gravity reconstructed

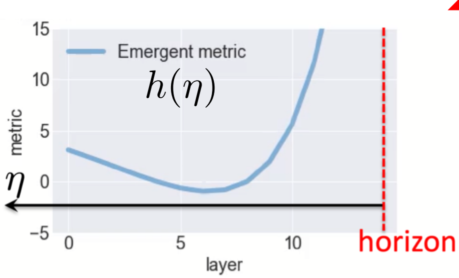
Deriving the dilaton potential (finite T)

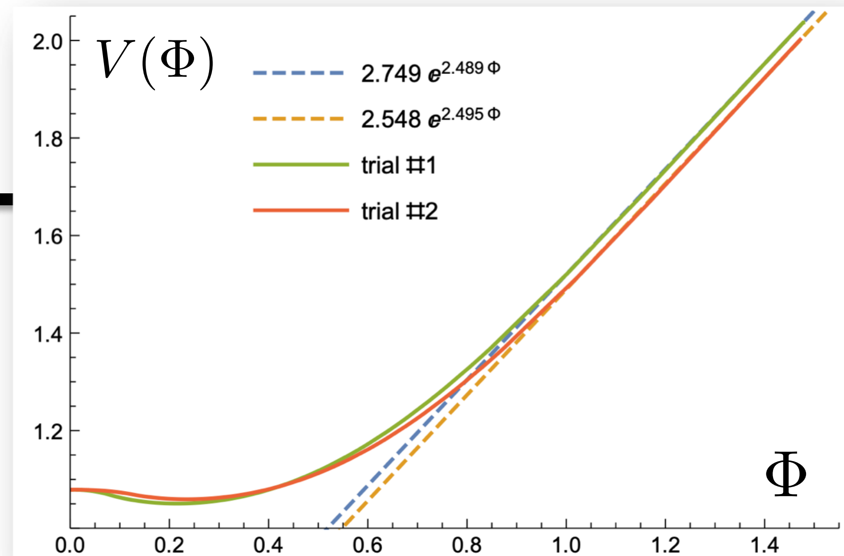
Gravity model

Action $S = c\int d^5x\sqrt{g}\Big(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\Big)$

2209.04638

Metric $h(\eta)$





Chiral condensate

Prediction of string breaking (finite T)

Gravity model

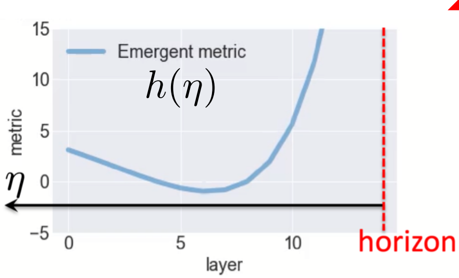
Action $S = c \int d^5x \sqrt{g} \left(R - \frac{4}{3} g^{MN} \partial_M \Phi \partial_N \Phi + V(\Phi) \right)$

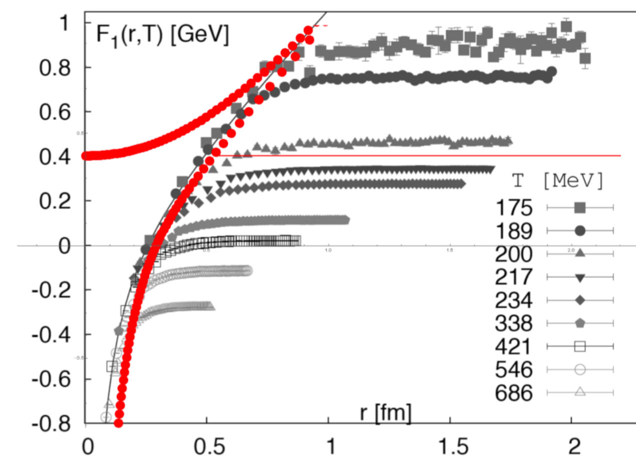
Metric $h(\eta)$

Prediction

Chiral condensate

Quark potential





Lattice data (grey) : Petreczky, J.Phys.G37(2010)094009

Bulk reconstruction by deep learning