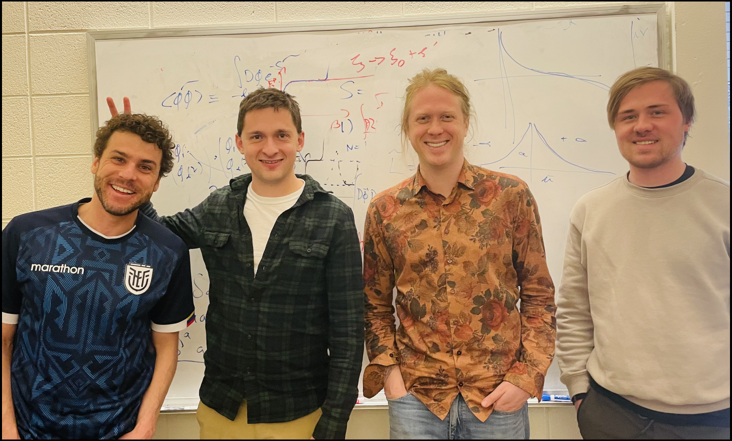
QFT in (Winter-)Wonderland

Paul Romatschke, CU Boulder

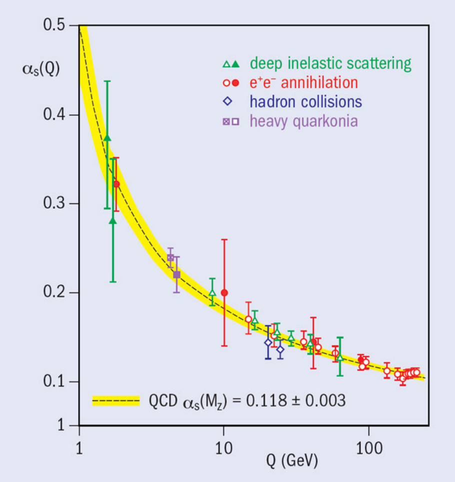
Wonderland Physics

QFT in wonderland would not be possible without my wonderful group:

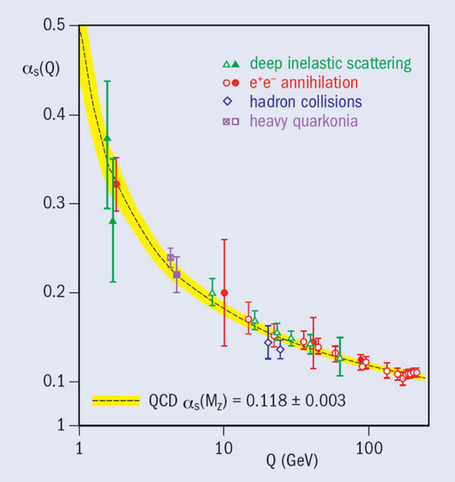


Max Weiner, Scott Lawrence, Seth Grable & Ryan Weller

Motivation



0.5
$\alpha_s(Q)$
0.4
0.3
0.2
0.1
1
1
10
100
Q (GeV)
deep inelastic scattering
e^+e^- annihilation
hadron collisions
heavy quarkonia
QCD $\alpha_s(M_Z)$ = 0.118 ± 0.003



QCD: asymptotic freedom; confinement; low energy bound states

Properties/Tools

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- Asymptotic Freedom: Perturbation Theory

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- Confinement: N/A

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- Low energy bound states: Numerical (Monte Carlo)

Problems with Tools

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- confinement and bound states in regime where coupling is **LARGE**. Cannot use perturbation theory
- Using $N \gg 1$ for $SU(N)$ could work, but we can't solve large N SU(N) either
- Holographic models capture some properties, but hard to know what results are model-independent

Plan for this Talk

- Properties of PT-symmetric field theories
- Solving large N scalar theories
- A wonderful solvable theory with asymptotic freedom
- QFT in Wonderland: what's next?

$\mathcal{P}T$-symmetric Field Theories

$\mathcal{PT}$-symmetric Field Theories

FALL 2020

Show 50 entries Search:

Date	Time	Title	Speaker	Affiliation
01/09/2020	16:00 CET	Dynamics of Fluids without Boost Symmetries	Jelle Hartong	University of Edinburgh
08/09/2020	16:00 CET	Diffusion in a magnetic field	Danny Brattan	University of Genova
15/09/2020	16:00 CET	Hydrodynamics Off Equilibrium	Paul Romatschke	University of Colorado Boulder
22/09/2020	16:00 CET	Holographic QCD and Gravitational Waves	Aldo Cotrone	University of Firenze
29/09/2020	18:00 CET	How right was Landau?	John McGreevy	University of California at San Diego
06/10/2020	16:00 CET	The complex life of hydrodynamic modes	Andrei Starinets	University of Oxford
13/10/2020	16:00 CET	Unmasking PT symmetry	Carl Bender	Washington University
27/10/2020	16:00 CET	Gravitational turbulence in large D	Christiana Pantelidou	Trinity College Dublin

$\mathcal{P}\mathcal{T}$-symmetric Quantum Mechanics

- "Normal" Hamiltonian

$$\mathcal{H} = \frac{p^2}{2m} + \lambda x^4 \,.$$

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$\mathcal{P}T$-symmetric Field Theory (1/2)

- "Normal" action

$$S = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \lambda\phi^4 \right] .$$

Bounded action, renormalizable, positive β-function (trivial for $\lambda > 0$)

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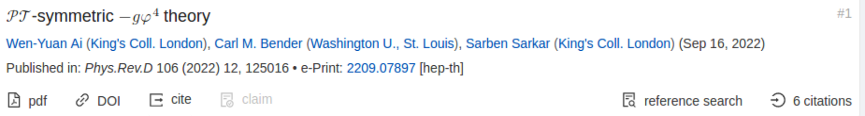
- $\mathcal{P}T$-symmetric action

$$S = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - g\phi^4\right] .$$

  Unbounded action, renormalizable, negative β-function

$\mathcal{PT}$-symmetric Field Theory (2/2): ABS conjecture


$\mathcal{PT}$-symmetric $-g\varphi^4$ theory #1

Wen-Yuan Ai (King's Coll. London), Carl M. Bender (Washington U., St. Louis), Sarben Sarkar (King's Coll. London) (Sep 16, 2022)

Published in: *Phys.Rev.D* 106 (2022) 12, 125016 • e-Print: 2209.07897 [hep-th]

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- Fantastically simple way to get results for $\lambda < 0$...
- ...but probably wrong for ϕ^4 theory!
- However: can be proven for large N scalar field theory!

Lesson# 1: Hermitian and bounded action is **sufficient, but not necessary** for consistent quantum field theory;

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Consequence: Do not dismiss theories just because the potential seems unbounded!

Solving large N scalar theories

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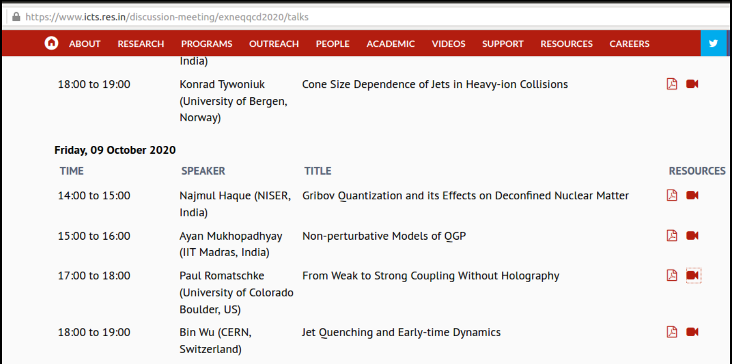

https://www.icts.res.in/discussion-meeting/exneqqcd2020/talks

ABOUT RESEARCH PROGRAMS OUTREACH PEOPLE ACADEMIC VIDEOS SUPPORT RESOURCES CAREERS

TIME	SPEAKER	TITLE	RESOURCES
	India)		
18:00 to 19:00	Konrad Tywoniuk (University of Bergen, Norway)	Cone Size Dependence of Jets in Heavy-ion Collisions	

Friday, 09 October 2020

TIME	SPEAKER	TITLE	RESOURCES
14:00 to 15:00	Najmul Haque (NISER, India)	Gribov Quantization and its Effects on Deconfined Nuclear Matter	
15:00 to 16:00	Ayan Mukhopadhyay (IIT Madras, India)	Non-perturbative Models of QGP	
17:00 to 18:00	Paul Romatschke (University of Colorado Boulder, US)	From Weak to Strong Coupling Without Holography	
18:00 to 19:00	Bin Wu (CERN, Switzerland)	Jet Quenching and Early-time Dynamics	

Solving large N scalar theories (1/2)

- Euclidean field theory action

$$S_E = \int d^4x \left[\frac{1}{2} \partial_\mu \vec{\phi} \partial_\mu \vec{\phi} + \frac{\lambda}{N} \left(\vec{\phi}^2 \right)^2 \right]$$

where $\vec{\phi} = (\phi_1, \phi_2, \ldots, \phi_N)$

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where $\vec{\phi} = (\phi_1, \phi_2, \ldots, \phi_N)$

- Exact transform (Hubbart-Stratonovic)

$$e^{-\int_x \frac{\lambda}{N}\left(\vec{\phi}^2\right)^2} = \int \mathcal{D}\zeta\, e^{-\int_x\left[i\zeta\vec{\phi}^2 + \frac{N\zeta^2}{4\lambda}\right]}$$

leads to

$$Z = \int \mathcal{D}\zeta\, e^{-\frac{N}{2}\mathrm{tr}\ln[-\Box+2i\zeta]-\frac{N}{4\lambda}\int_x \zeta^2}\,.$$

Solving large N scalar theories (2/2)

- At large N, can solve this path integral using method of steepest descent; saddle is Fourier-zero mode of ζ; get

$$\ln Z = N\beta V p(m) + \mathcal{O}(N^0)\,, \quad p(m) = p_{\rm free}(m) + \frac{m^4}{16\lambda}\,,$$

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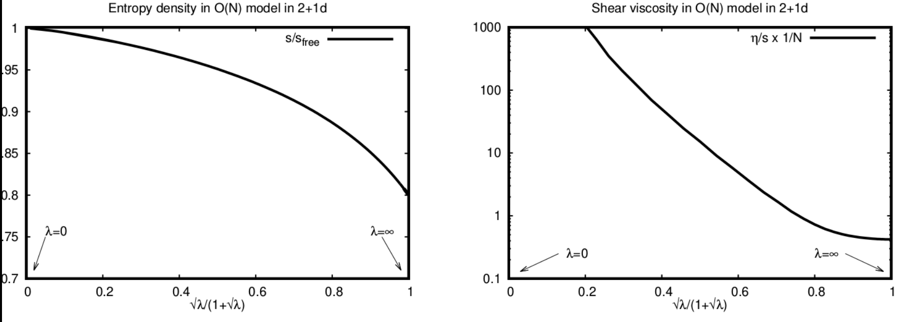
- Very fruitful result for massless fields in $d = 3$ (no renormalization)
- For years, I was stuck on $d = 4$: positive β function, Landau pole; the resolution of this puzzle is what's new in this talk

Results for large N scalar theories in 3d



3d: massless interacting theory exists (can be put on the lattice), lot's of results about IR interacting CFT; exact non-perturbative thermodynamics and transport, see (click on): [1904.09995],[2104.06435]

Problems for large N scalar theories in 4d

- In 4d, large N pressure is (in dim-reg)

$$p(m) = \frac{m^4}{16\lambda} + \frac{m^4}{64\pi^2}\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2 e^{\frac{3}{2}}}{m^2}\right) + \frac{m^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_2(n\beta m)}{n^2}\,.$$

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- Problem: β function is positive, coupling runs as

$$\lambda_R(\bar{\mu}) = \frac{4\pi^2}{\ln\frac{\Lambda_{LP}^2}{\bar{\mu}^2}}\,.$$

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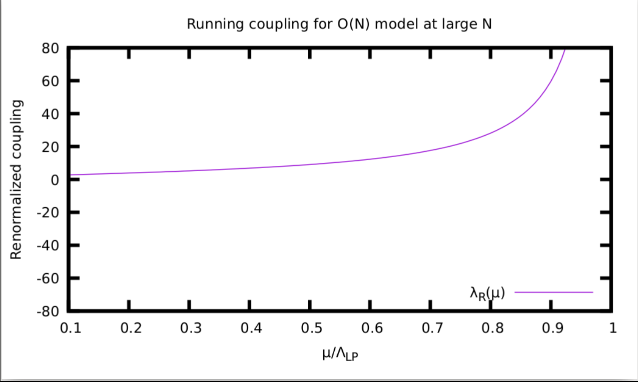

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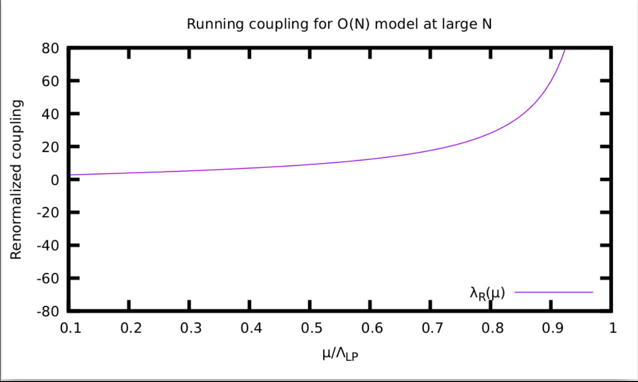
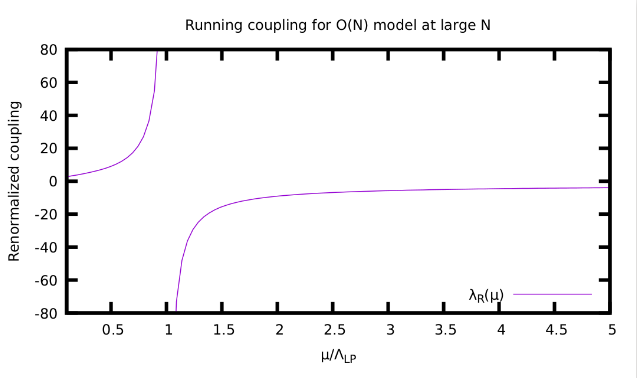


Large N exact; positive β-function; Landau pole

Problems for large N scalar theories in 4d



Above the Landau pole: negative $\lambda_R(\bar{\mu})$; potential unbounded

A wonderful solvable theory with asymptotic freedom

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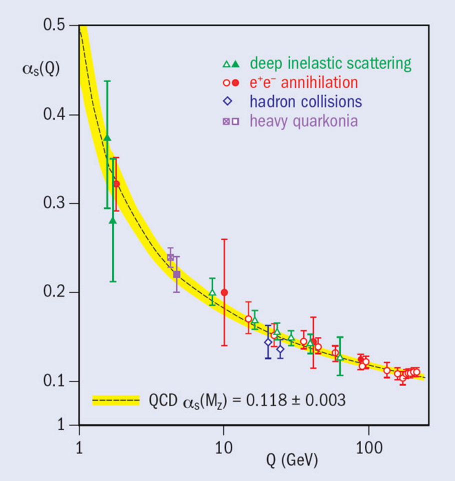
- So let's check what happens to **observables**



Reminder: $\alpha_s(Q)$ is **not** an observable; it is **inferred** from matching experiment to theory (here: pQCD)

A wonderful solvable theory with asymptotic freedom

- Renormalized pressure of O(N) model in 3+1d:

$$p(m) = \frac{m^4}{16\lambda_R(\bar{\mu})} + \frac{m^4}{64\pi^2}\ln\frac{\bar{\mu}^2 e^{\frac{3}{2}}}{m^2} + \frac{m^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_2(n\beta m)}{n^2}\,.$$

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- Note: no dependence on fictitious scale $\bar{\mu}$ (good observable)

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- Actual pressure is $p(m)$, with m the solution to saddle point condition

$$0 = \frac{dp(m)}{dm^2} = \frac{m^2}{32\pi^2} \ln \frac{\Lambda_{LP}^2 e^1}{m^2} - \frac{mT}{4\pi^2} \sum_{n=1}^{\infty} \frac{K_1(n\beta m)}{n} .$$

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- Traditionally, people select $m_1 = 0$ on the basis that m_2 is **too close to the cutoff**
- I beg to differ: you can't pick and choose! Physics has a preferred solution:

$$p(m_1) = 0 \,, \quad p(m_2) = \frac{\Lambda_{LP}^4 e^2}{128\pi^2} \,.$$

[2211.15683]

Lesson #2: The perturbative vacuum is unstable; the true vacuum is non-perturbative and has smaller free energy than the perturbative vacuum.

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Consequence: much of the literature on 4d O(N) model is wrong or at least incomplete (including some of my own papers!)

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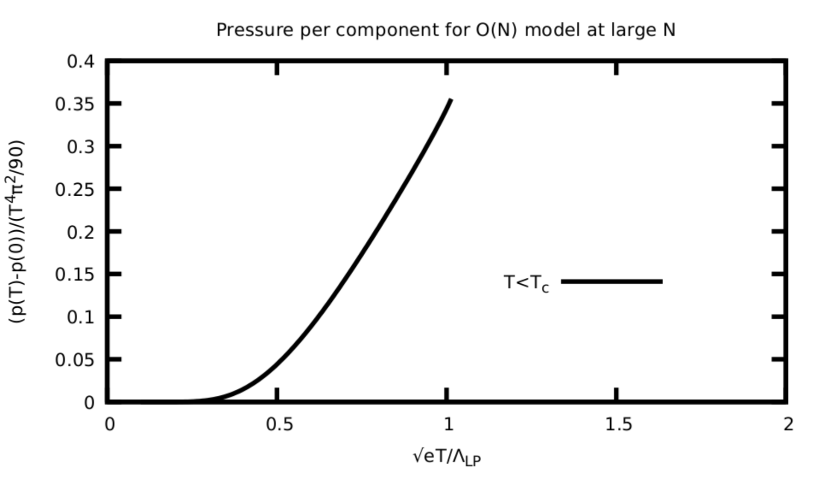
- Track solutions numerically away from deep infrared $T > 0$
- Solution with lower free energy is physically preferred
- Leads to result for physical observable pressure $p = p(m_2(T))$



Pressure per component for O(N) model at large N
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
(p(T)-p(0))/(T^4π^2/90)
T<T_c
0
0.5
1
1.5
2
√eT/Λ_LP

[2211.15683]

A wonderful solvable theory with asymptotic freedom

- For high energy $T \gg \sqrt{e}\Lambda_{LP}$, all solutions m as well as $p(m)$ are complex

A wonderful solvable theory with asymptotic freedom

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- Let's see what we get

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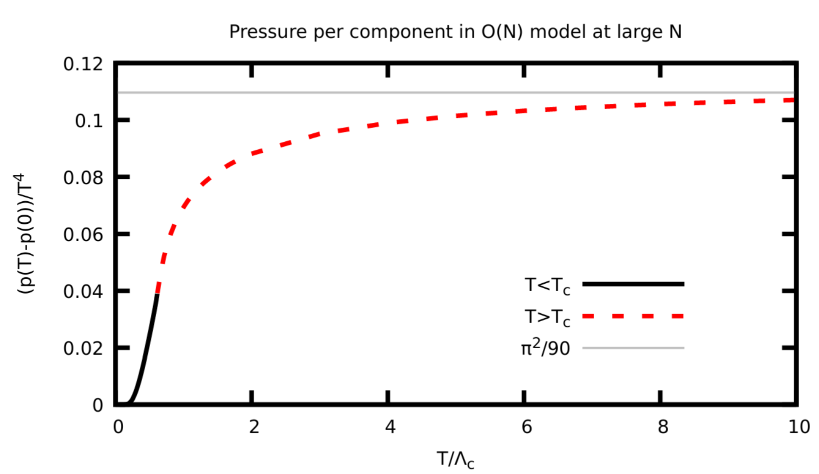

[2211.15683]

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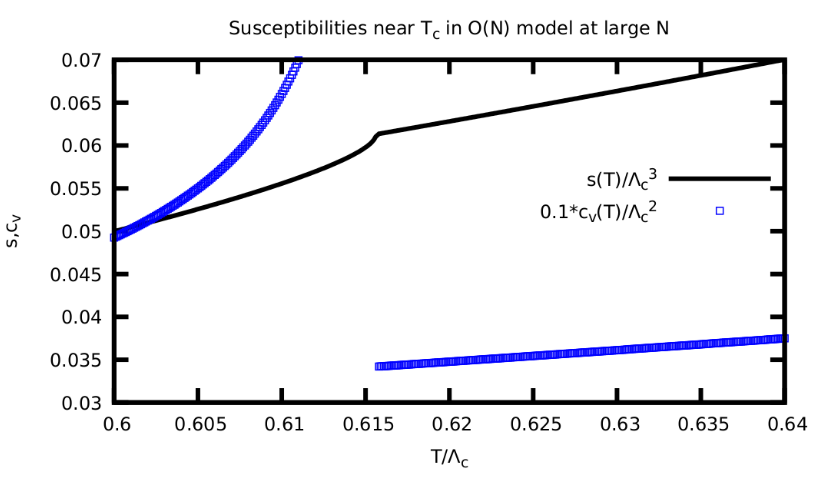

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- Observables in 4d O(N) model are well-defined, positive-definite and show no sign of unphysical behavior
- 4d O(N) model does exhibit a second order phase transition at $T \simeq \sqrt{e}\Lambda_{LP}$, separating low- and high-temperature phases

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- The theory is asymptotically free in the UV!

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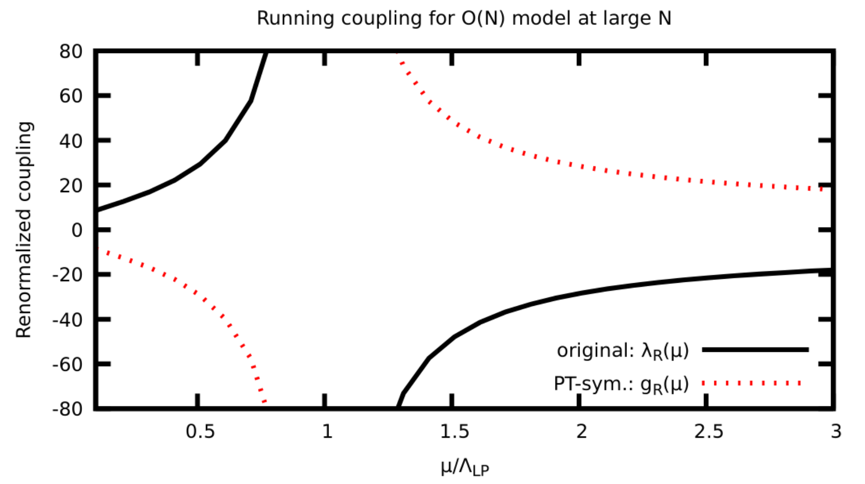

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We knew this for a long time already: $\mathcal{N}$=4 SYM has well-behaved observables in the limit $\lambda \to \infty$; why should it be any different for ϕ^4 theory?

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We knew this for a long time already: $\mathcal{N}=4$ SYM has well-behaved observables in the limit $\lambda \to \infty$; why should it be any different for ϕ^4 theory?

The main difference in scalar theory is that we can "see around" the Landau pole in the regime $\lambda_R < 0$ using the $\mathcal{PT}$-symmetric ABS conjecture. This is how we find two phases in scalar theory.

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- The resulting calculations are technical, but doable: [2212.03254]
- One finds that the traditional view is incorrect; neither relevant nor irrelevant operators change the results qualitatively at large N

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- This is a singlet bound state of two vectors: $\vec{\phi} \cdot \vec{\phi}$
- It's the QFT wonderland: this 'colorless' state emerges from the theory Lagrangian, and it's the only such state at large N

QFT in Wonderland: What's next?

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- On linking 4d O(N) model and QCD
- Beyond scalars: fermionic theories in 4d
- Beyond wonderland theory: wonderland experimental consequences

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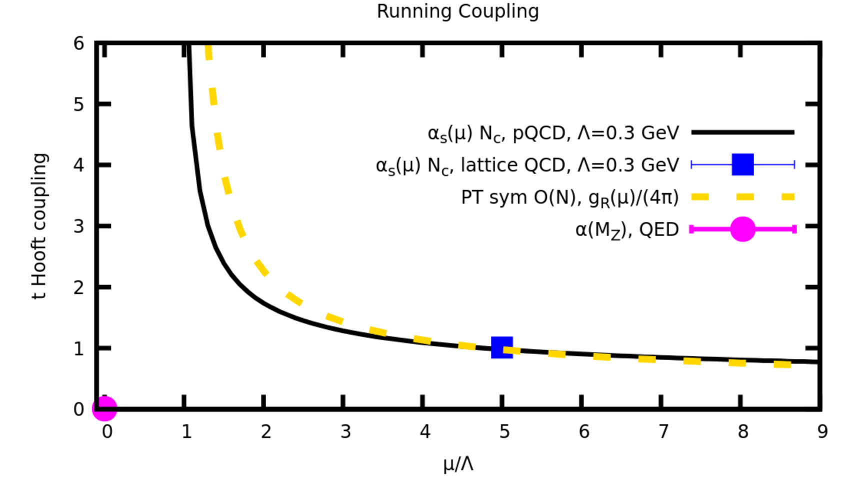
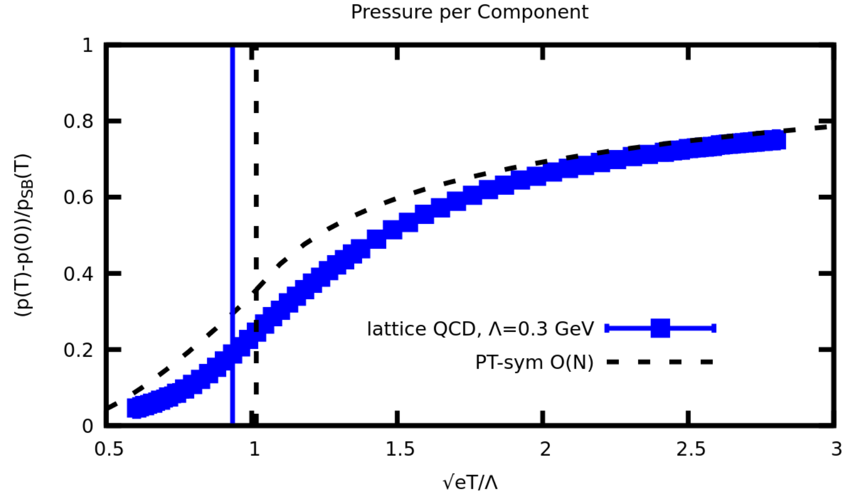
- Test: O(N) model has phase transition at $T_c \simeq \sqrt{e}\Lambda_{LP}$. Plot pressure vs. QCD pressure in temperature units of $\sqrt{e}/\Lambda$

QFT in Wonderland: Running coupling



[2212.03254]

QFT in Wonderland: O(N) vs. QCD



[2212.03254]

QFT in Wonderland: Fermionic Theories

- Solution techniques also work for N-component fermions in 4d
- Look out for arXiv preprint by Seth Grable and Max Weiner very soon!

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- At one-loop, the Higgs mass value is off from experimental value. Stay tuned.


Thanks for listening! I'm off now
to enjoy more of QFT in Wonderland!