

Double holography and Page curves in Type IIB

Christoph Uhlemann

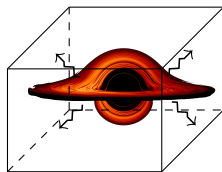


HoloTube
The Applied Holography Webinars Network
Jan 10, 2023

Introduction

Information paradox \geq 2019:

For some black holes in AdS coupled to a bath the radiation entropy is consistent w/ unitarity.



Fine-grained radiation entropy with replica wormhole contributions:

$$S_{\text{rad}} = \min_I \left\{ \text{ext}_I \left[\frac{\text{Area}(\partial I)}{4G_N} + S_{\text{semi-cl}}[\Sigma_{\text{rad}} \cup I] \right] \right\}$$

→ Page curves for radiation entropy in models of (massive) gravity:

2d JT gravity, braneworld models & double holography in $d > 2$,
bath can be QFT or gravitating, black holes evaporating/eternal

Motivation: 4d black holes in UV-complete quantum gravity, i.e. top-down models, microscopic QFT duals, no inherent averaging

Outline:

- braneworlds & double holography
- top-down string theory braneworlds
- islands and Page curves in Type IIB
- double holography in string theory

2105.00008: Islands and Page curves in 4d from Type IIB

2011.10050, 2112.14648: Localization calculations w/ Lorenzo Coccia

2206.11292: Double Holography in IIB w/ Andreas Karch, Hao-Yu Sun

braneworlds & double holography

Braneworlds & double holography

Bottom-up holographic dual for BCFT_4 from Karch-Randall branes:

$$S = \int_{\mathcal{M}} d^5x \sqrt{g} (R - 2\Lambda) + \lambda \int_{\Sigma} d^4x \sqrt{g_{\text{ind}}}$$

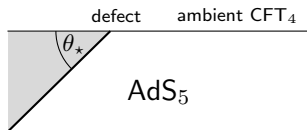
brane with tension in AdS_5 , $\lambda < \lambda_c$:

Braneworlds & double holography

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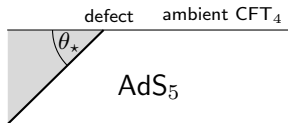


$$ds^2 = \frac{d\theta^2 + ds_{\text{AdS}_4}^2}{\sin^2\theta}$$

AdS_5 cut off by 'end-of-the-world' brane along AdS_4 , leaving a half space as conformal boundary, brane angle $\leftrightarrow \lambda$

Braneworlds & double holography

'Intermediate' holographic description & double holography:



$$ds^2 = \frac{d\theta^2 + ds_{\text{AdS}_4}^2}{\sin^2\theta}$$

- (a) CFT₄ on half space coupled to CFT₃ on boundary
- (b) AdS₄ gravity + cut-off CFT₄ on brane, coupled to 'ambient' CFT₄ on half space (RS holography)
- (c) AdS₅ gravity + ETW brane

Braneworld model for 4d gravity coupled to non-gravitational bath

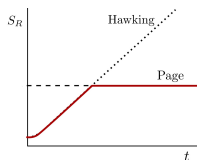
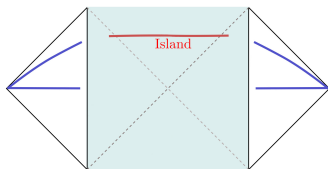
Braneworlds & double holography

Page curves

Braneworlds & double holography

Page curve for AdS black hole in equilibrium with QFT bath:

[Penington, Almheiri, Engelhardt, Marolf, Maxfield, Mahajan, Maldacena, Zhao, . . .]



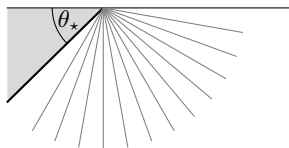
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Prepare in pure state, collect radiation in bath. Initial radiation entropy growth à la Hawking, island limits growth eventually.

Braneworlds & double holography

4d brane black hole coupled to CFT_4 bath on a fixed background:

[Karch,Raju,Randall et al]

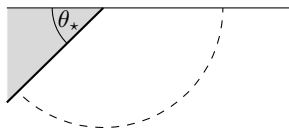


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Braneworlds & double holography

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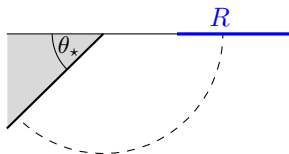
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eternal AdS_4 black hole slices

Braneworlds & double holography

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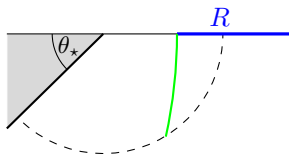
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Collect Hawking radiation in CFT region R , compute entropy:

Braneworlds & double holography

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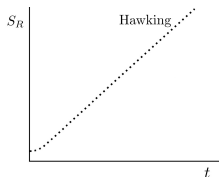


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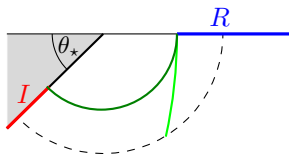
- HM surface: area grows in time



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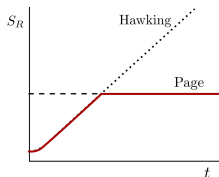


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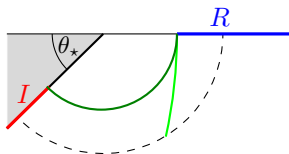
- HM surface: area grows in time
- island surface: constant area



Braneworlds & double holography

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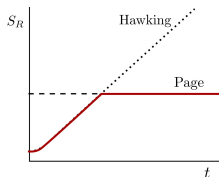


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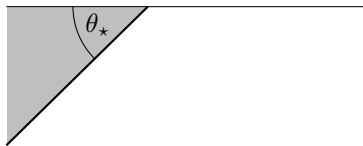
Island surface in 4d intermediate description from R/T in 5d
Competition between island and HM surfaces \rightarrow Page curves

Braneworlds & double holography

A puzzle

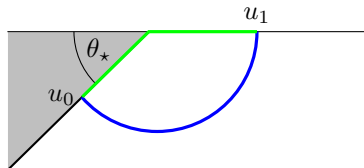
Braneworlds & double holography

RS holography intermediate description for BCFT = gravity on ETW brane coupled to ambient CFT:



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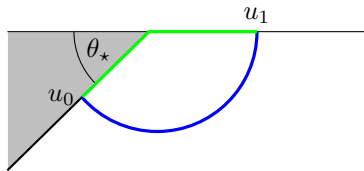
[Omiya,Wei '21]: Null geodesics connecting a point on the brane to a point in the bath are faster through the bulk than along the brane

$$\Delta t^2 = 2u_0u_1(1 - \cos \theta_*)$$

Time difference grows w/ separation, vanishes in near-critical limit

Braneworlds & double holography

RS holography intermediate description for BCFT = gravity on ETW brane coupled to ambient CFT:



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IR non-localities in intermediate picture? How to address?

Main part: uplift discussion to string theory

RS branes not part of string theory, bottom-up models with no clear UV completion. Does the story hold up?

Intermediate picture poorly understood, seemingly with dramatic non-localities. Rain on the Page curve parade?

[Wishlist: Page curves for 4d black holes in UV-complete string theory, top-down double holography w/ local intermediate picture]

Uplifting braneworlds to string theory

D3/D5/NS5 BCFTs

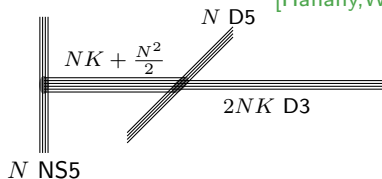
BPS boundary conditions for $\mathcal{N} = 4$ SYM: D3 ending on D5/NS5

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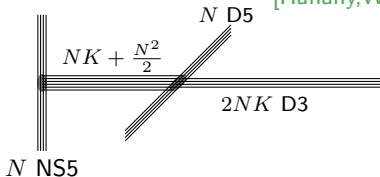


$2NK$ semi-infinite D3 end on combination of N D5 and N NS5,
 $N/2 + K$ D3 end on each NS5, $N/2 - K$ net D3 on each D5.

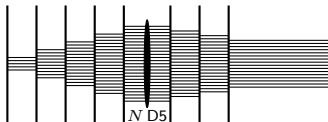
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4d $\mathcal{N} = 4$ SYM with gauge group $U(2NK)$ on half space, coupled to 3d quiver SCFT with $N - 1$ nodes, N flavors at maximal node

Holographic duals for D3/D5/NS5 BCFTs

AdS_4 , S_1^2 , S_2^2 warped over Riemann surface Σ [D'Hoker,Estes,Gutperle]

$$ds^2 = f_4^2 ds_{\text{AdS}_4}^2 + f_1^2 ds_{S_1^2}^2 + f_2^2 ds_{S_2^2}^2 + 4\rho^2 ds_{\Sigma}^2$$

Specified by Σ + harmonic h_1, h_2 : Janus, 3d SCFTs [Assel,Bachas],
4d BCFTs [Aharony,Berdichevsky,Berkooz].

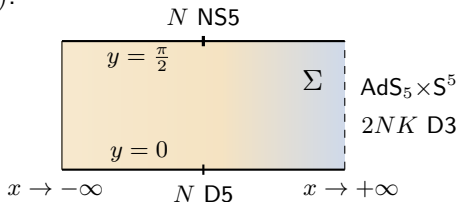
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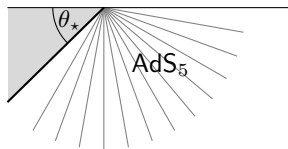
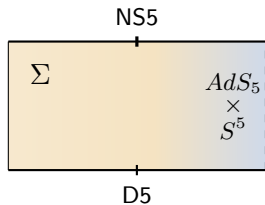
BCFT(N, K):



Σ =strip with D5, NS5 sources on boundary, $\text{AdS}_5 \times S^5$ at $x \rightarrow \infty$,
geometry closes off smoothly on other boundaries.

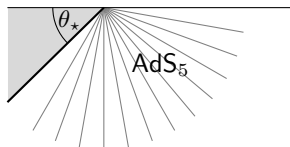
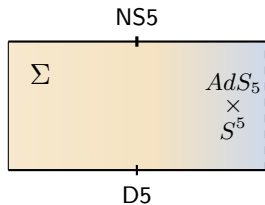
Connection to braneworld models

ETW brane 'resolved' into geometry + fluxes around 5-branes,
 $AdS_5 \times S^5$ region ends smoothly



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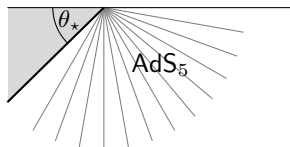
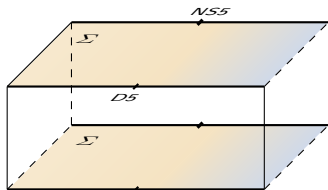
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AdS_4 at each point on Σ vs. AdS_4 warped over angular coordinate,
fibers joined at 3d boundary.

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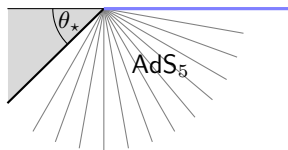
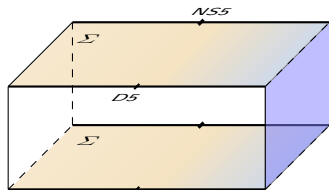
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4d ambient CFT at $x \rightarrow \infty$. Intermediate holographic description
later, assume it exists and use BCFT dual for EE computations.

Braneworld parameters in 10d

In brane world models, brane angle $\theta_\star \sim c_{3d}/c_{4d}$. In 10d, F_{S^3}/C_T for 3d *long quivers* from susy localization [L. Coccia, CU '20]

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In BCFT(N, K) with general N, K : c_{3d}/c_{4d} controlled by N/K .

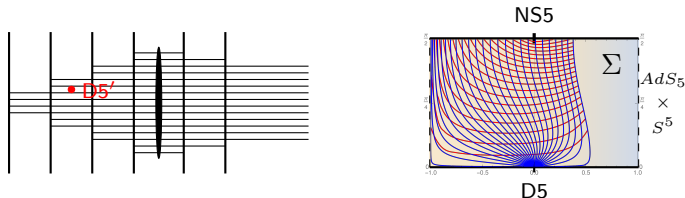
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Link quiver diagram and brane setup to sugra solutions via Wilson loops/D5' and susy localization: quiver coordinates [Coccia, CU '21]



Black holes and Page curves in IIB

Black holes and Page curves

Black holes in stringy braneworlds:

$$ds^2 = f_4^2 ds_{\text{AdS}_4}^2 + f_1^2 ds_{S_1^2}^2 + f_2^2 ds_{S_2^2}^2 + 4\rho^2 ds_{\Sigma}^2$$

Black holes and Page curves

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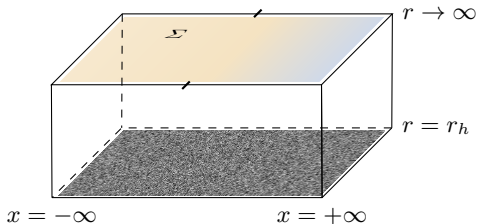
$\text{AdS}_4 \rightarrow \text{AdS}_4$ black hole throughout Σ , solves Type IIB EOM

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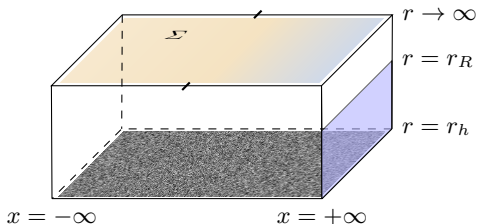


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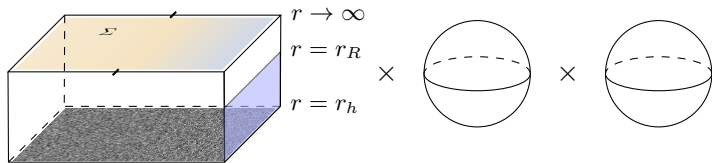
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AdS_4 black hole coupled to 4d CFT in intermediate description.
Radiation region in ambient 4d CFT geometry at $x = \infty$.

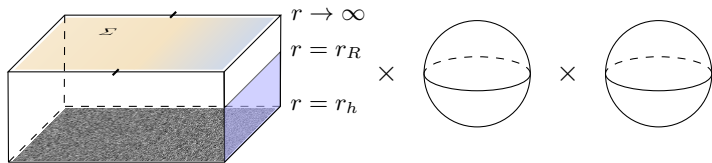
Black holes and Page curves

Radiation entropy: 8d Ryu-Takayanagi surfaces in 10d geometry, wrap $S_{1/2}^2$, split AdS_4 at $x = \infty$, fixed t , extend along Σ



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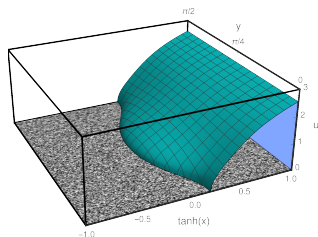
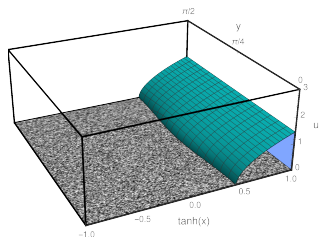
RT surfaces from PDE on Σ , boundary conditions on $\partial\Sigma$ from regularity \Rightarrow 10d analog of 'Neumann at ETW brane'

PDE with 5-brane singularities, no help from susy \rightarrow numerics ...

Black holes and Page curves

Radiation entropy: 8d Ryu-Takayanagi surfaces in 10d geometry, wrap $S_{1/2}^2$, split AdS_4 at $x = \infty$, fixed t , extend along Σ

HM surfaces @ $t = 0$:

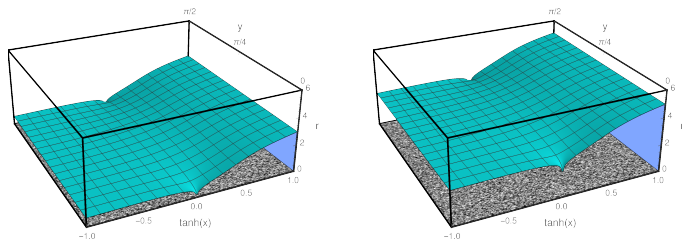


- cross horizon before reaching ‘resolved ETW brane region’, end in second exterior region \Rightarrow area grows in time

Black holes and Page curves

Radiation entropy: 8d Ryu-Takayanagi surfaces in 10d geometry, wrap $S_{1/2}^2$, split AdS_4 at $x = \infty$, fixed t , extend along Σ

Island surfaces:

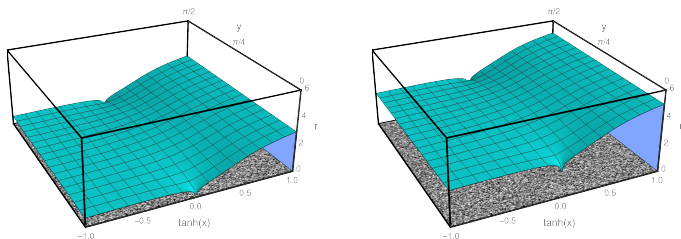


- stretch all through Σ to $x = -\infty$, detect D5/NS5
- do not cross horizon \Rightarrow constant area, limit entropy growth

Black holes and Page curves

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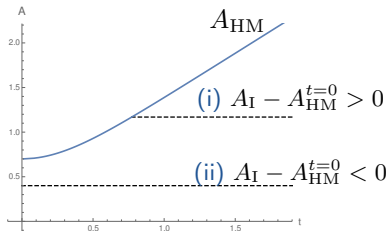
Island surfaces limiting entropy growth for 4d black holes in 10d Type IIB setups engineered to uplift braneworld models



Black holes and Page curves

Entropy curve from competition between island and HM surfaces:

- (i) HM dominates initially, island later \rightarrow Page curve
- (ii) island dominates right away \rightarrow constant entropy

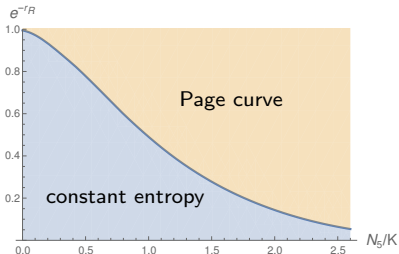
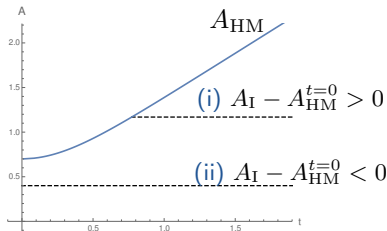


Both compatible with unitarity, order parameter $\Delta A_{t=0}$.

Black holes and Page curves

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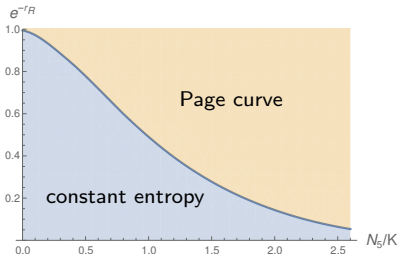
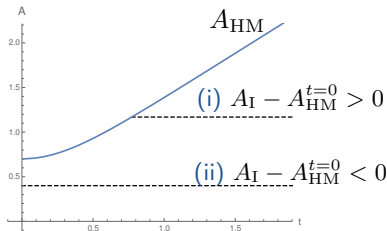


Both compatible with unitarity, order parameter $\Delta A_{t=0}$.

Black holes and Page curves

Entropy curve from competition between island and HM surfaces:

- (i) HM dominates initially, island later \rightarrow Page curve
- (ii) island dominates right away \rightarrow constant entropy



Both compatible with unitarity, order parameter $\Delta A_{t=0}$.

Radiation collected far enough in bath \rightarrow non-trivial entropy curve.

Consistent braneworld results: [Geng,Karch,Perez-Pardavila,Raju,Randall]

Black holes and Page curves

Page curves for 4d black holes coupled to bath in full 10d string theory from competition between island and HM surfaces.

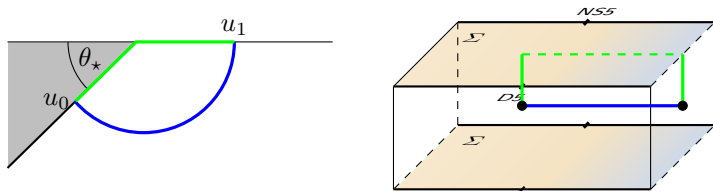
Results in 'minimal 10d model' validate braneworld discussions. Vast space of 10d solutions to explore.

10d setups more complicated, but microscopically well defined. Give access to new questions, allow to address puzzles . . .

Double holography in string theory

double holography in string theory

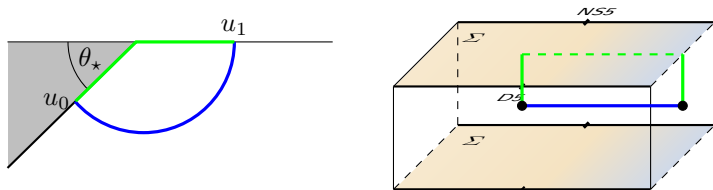
Does uplift to 10d eliminate shortcuts right away? Send signal from point in 'resolved ETW brane region' to the CFT bath:



Similar features as braneworlds: faster through bulk than “along the brane”. Times agree only for $N/K \rightarrow \infty \sim$ near-critical limit.

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Naive 10d intermediate description = gravity in ‘resolved ETW brane’ region coupled by hand to ambient CFT no good either ✗

double holography in string theory

Instead, make idea behind double holography precise – isolate 3d defect d.o.f., geometrize them and couple dual to ambient CFT:

$$\text{BCFT} = \text{3d defect d.o.f.} \oplus \text{4d ambient d.o.f.}$$

double holography in string theory

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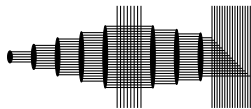
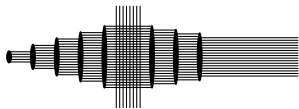
geometrize only 3d d.o.f.:

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Use brane construction + 1st principles AdS/CFT to derive *proper* intermediate description, coupling of intermediate gravity to bath

double holography in string theory

Isolate 3d d.o.f. in BCFT: terminate semi-infinite D3-branes on D5:



double holography in string theory

Isolate 3d d.o.f. in BCFT: terminate semi-infinite D3-branes on D5:



Instead of 4d $U(2NK)$ node, 3d quiver ends with $2NK$ flavors

$$U(R) - U(2R) - \dots - U(R^2) - \dots - U(2NK + S) - \widehat{U(2NK)}$$

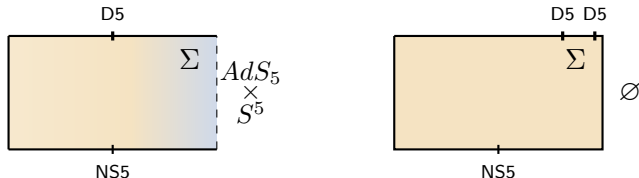
|
[N]

double holography in string theory

Isolate 3d d.o.f. in BCFT: terminate semi-infinite D3-branes on D5:



Holographic dual for 3d CFT $AdS_4 \times S^2 \times S^2 \times \Sigma$, as before:

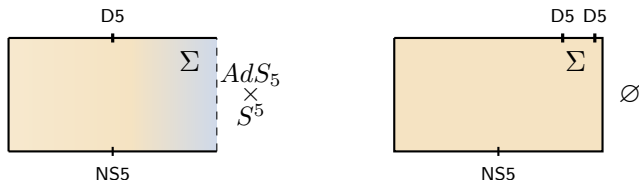


double holography in string theory

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Proper intermediate description from first principles AdS/CFT:

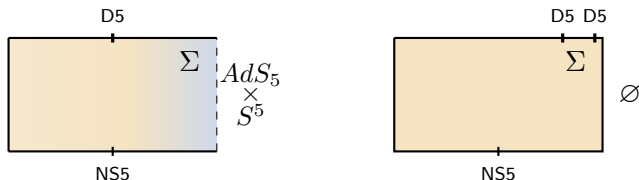
$$Z_{\text{BCFT}} = \int \mathcal{D}A e^{S_{4d \mathcal{N}=4}[A]} Z_{\text{3d CFT}}[\hat{A}]$$

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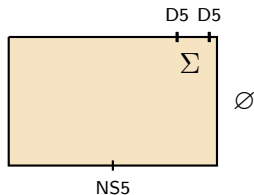
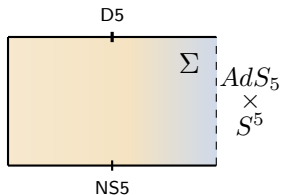
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double holography in string theory

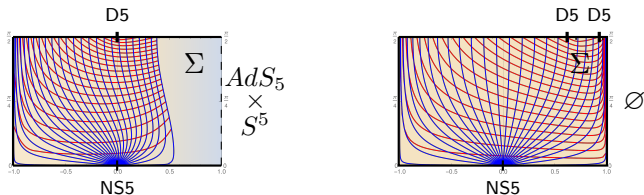
What's wrong with the naive/braneworld intermediate description?

Compare full BCFT dual to intermediate 3d-only dual:



double holography in string theory

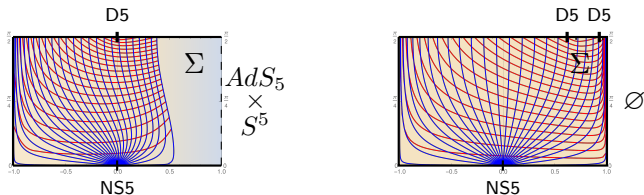
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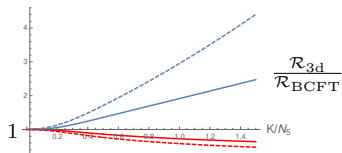
'brane coordinates' relate points on Σ , left end = 3d end of quiver

double holography in string theory

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Even deep in 3d region geometries only agree in near-critical limit $K/N \rightarrow 0$, when $\Delta t^2 \rightarrow 0$. 3d dual *not* a subset of full BCFT dual.

double holography in string theory


10d intermediate description from 1st principles: defect dual *not* a subset of the full BCFT dual; two genuinely different solutions.

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intermediate picture
black hole question \longleftrightarrow BCFT question

BCFT problem can be solved in full dual, Page curves stand 

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Bottom-up intermediate picture as gravity on ETW brane too simplistic. Islands outside the horizon?

Summary

Summary

Top-down string theory models for 4d black holes coupled to bath. Islands, Page curves from 10d R/T surfaces + double holography.

Proper intermediate description from brane construction and standard AdS/CFT. Refines bottom-up models, resolves puzzles.

String theory versions of wedge holography, information transfer with gravitating bath, non-geometric entropies: [2105.00008, ...]

Thank you!