Holographic Meissner effect

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arXiv: 2207.07182 [hep-th]

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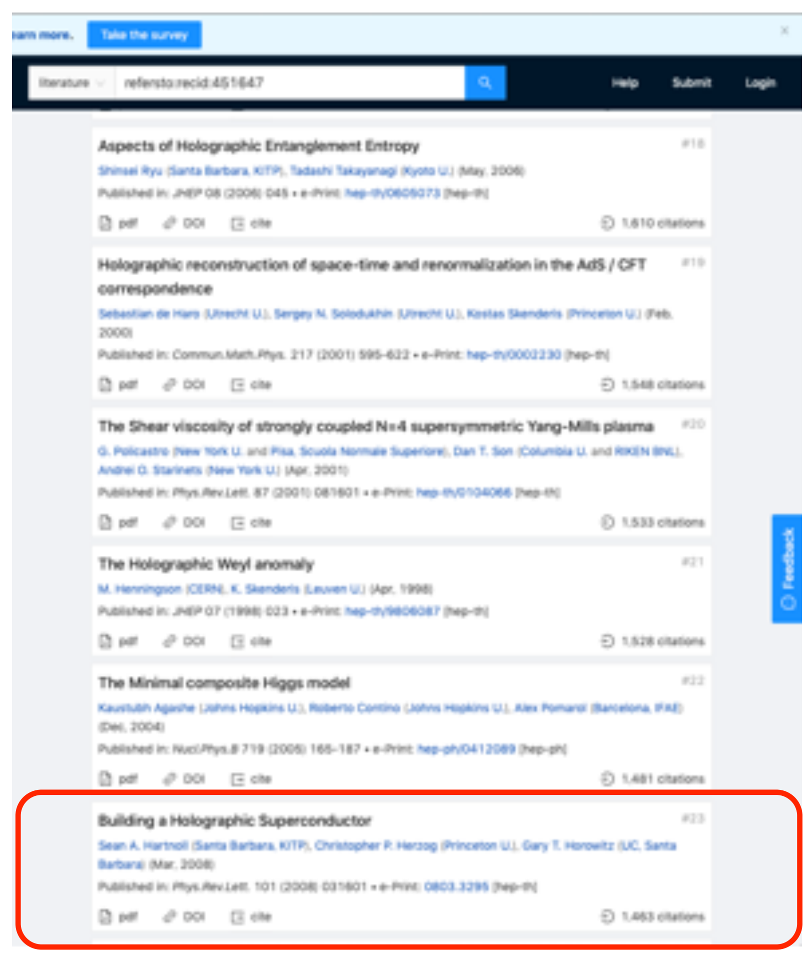
2022/11 Holotube

Introduction

In AdS/CFT, there is a system called holographic superconductor (HSC) which is dual to a superconductor.

This system has been studied extensively. For example, here is the list of citations of Maldacena's original paper (presumably, it covers most of AdS/CFT papers).

HSC is #23



Introduction

- One characteristic feature of superconductivity is the Meissner effect.
- However, for HSCs,
  - Meissner effect was rarely discussed and
  - It has been shown only numerically.
- The reason is simple: In most applications, the bdy. Maxwell field is added as an external source and is not dynamical (more later).
- I explain how to implement the Meissner effect in HSCs and show the effect analytically.
- The magnetic penetration length & GL parameter κ has a nontrivial dependence on the U(1) coupling e.

Plan

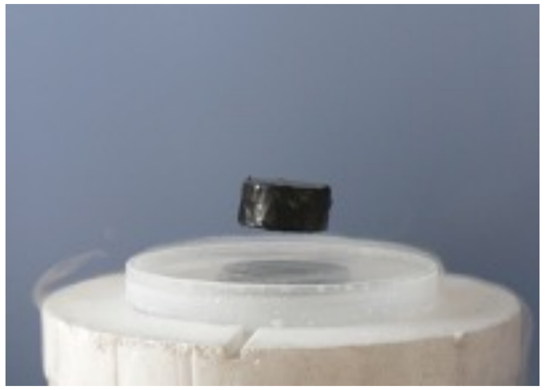
- Review: Superconductivity & Ginzburg-Landau theory
- Review: Holographic superconductors
- Holographic Meissner effect (bulk 4-dim)
  - "Holographic semiclassical eq."
  - Dirichlet BC case
  - Holographic semiclassical eq. case
- Bulk 5-dim. case

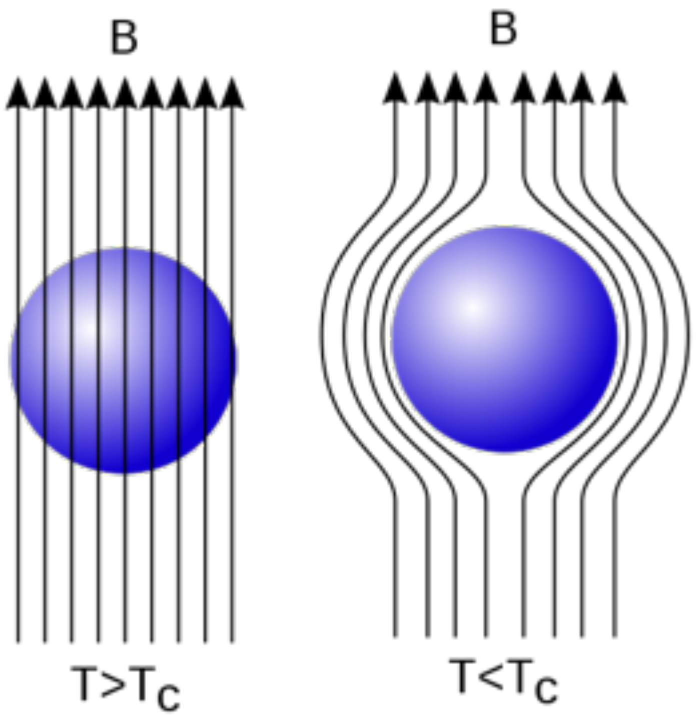
Superconductivity

2 characteristic features of SC:

- Zero resistivity/diverging conductivity
- Meissner effect:
  Magnetic field cannot enter the material

Phenomenologically,
Ginzburg-Landau theory describes SC well.





Wikipedia

GL theory of superconductivity

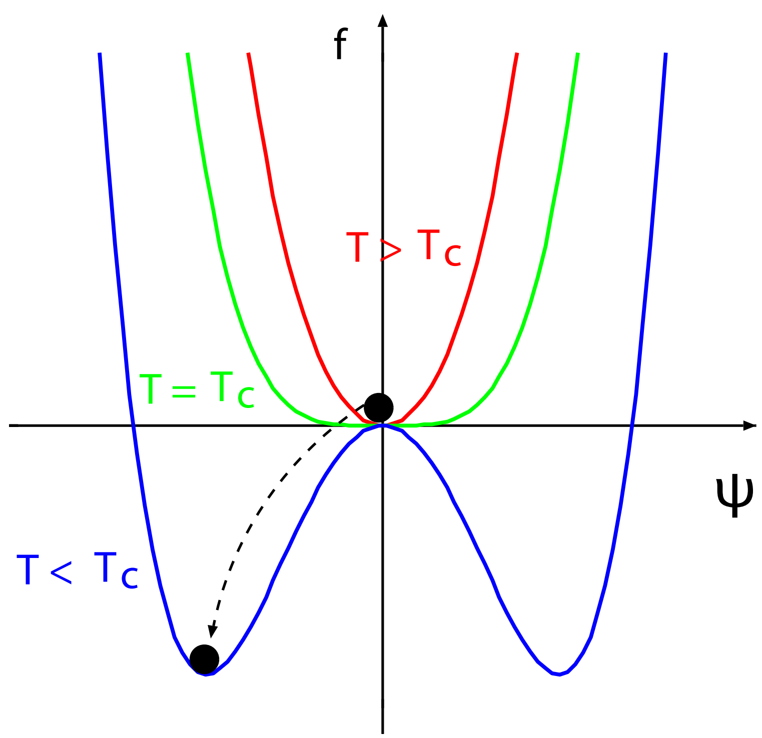
$$f = \left|(\partial_i - iA_i)\psi\right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 \cdots + \frac{1}{4}F_{ij}^2$$

$$a = a_0(T - T_c) + \cdots$$

The "macroscopic wave fn." ψ has the familiar Higgs-like potential.
"Mass term" is proportional to temperature

- ψ = 0 for $T > T_c$
  ψ ≠ 0 for $T < T_c$ and SSB $|\psi|^2 = -\frac{a}{b}$
- Maxwell field then becomes massive (just like Higgs mechanism)

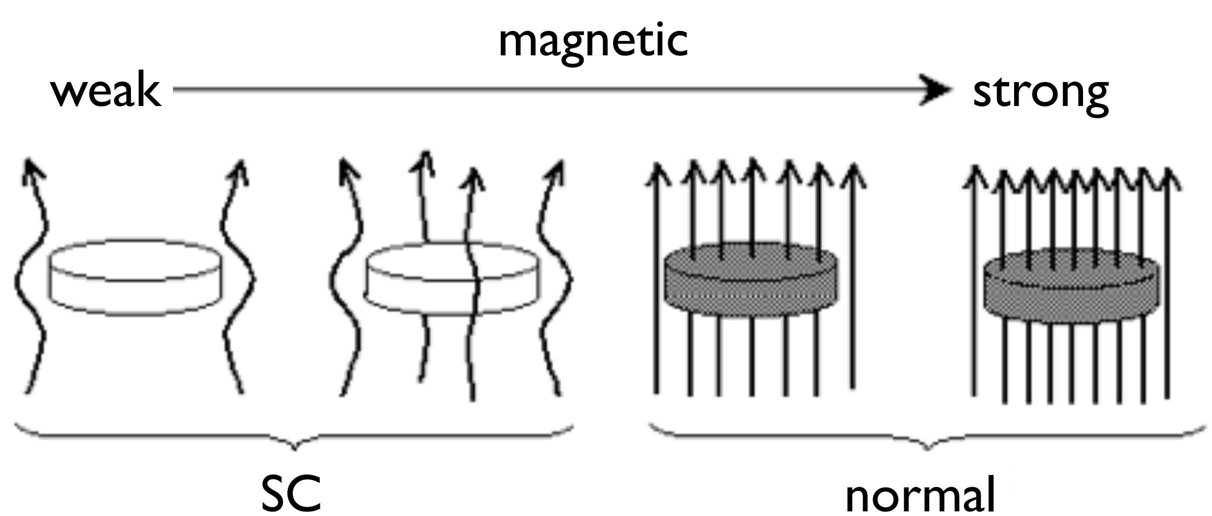
→ Meissner effect
Magnetic field cannot enter the SC.



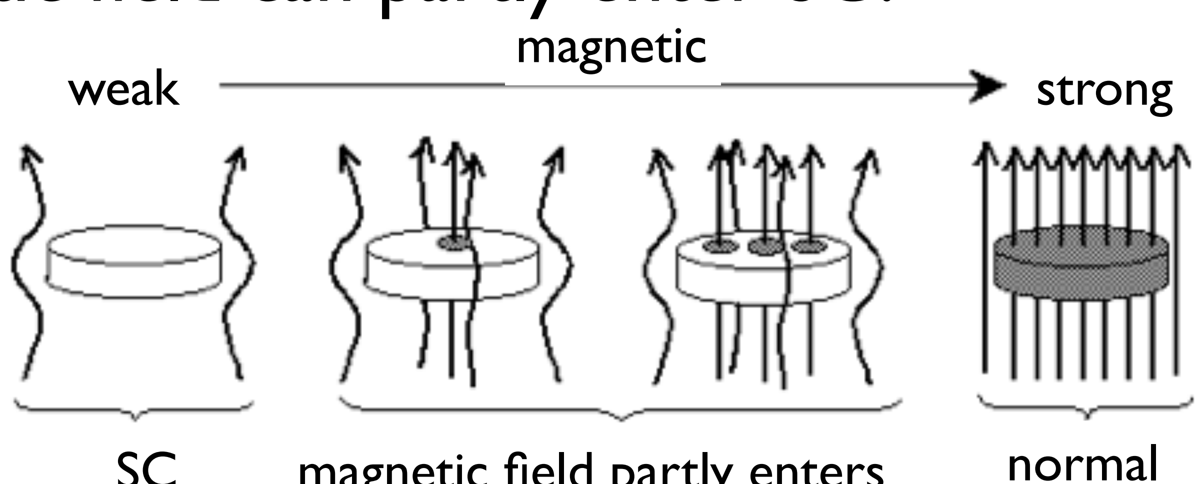
Type I & II

There are 2 kinds of SCs:

- Type I: magnetic field is completely expelled. The SC state is broken at high enough magnetic field.



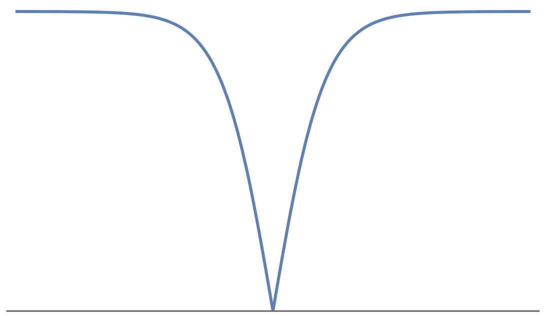
- Type II: magnetic field can partly enter SC.



Wikipedia

In Type II SCs, magnetic field can enter SC keeping SC state.

The magnetic field enters by forming vortices.
ψ =0 at vortex core and magnetic field enters there.



According to GL theory,

$$\nabla_j F^{ij} = -2e^2 |\psi|^2 A^i$$

→ supercurrent (diamagnetic)

$$A_\phi \propto \sqrt{r} e^{-r/\lambda}$$

$$\boxed{\lambda^2 = \frac{1}{2e^2 |\psi|^2}}$$

This is one way to see the Meissner effect. Note Maxwell field is dynamical.

We see analogous expression in HSC, but this is impossible in the standard HSC.

- The vortices create supercurrent from London eq: $J = -2|\psi|^2 A$
- The Ampere law $\nabla \times B = J$ then tells the magnetic field is induced, and compensates the external magnetic field.

As we see below, in standard HSCs, the London eq holds, but there is no Ampere law because there is no dynamical Maxwell field on bdy.
The magnetic field can always enter and no Meissner effect. In a sense, HSC is an "extreme" type II.

Plan

- Superconductivity & Ginzburg-Landau theory (review)
- Holographic superconductors (review)
- Holographic Meissner effect
  - "Holographic semiclassical eq."
  - Dirichlet BC case
  - Holographic semiclassical eq. case
- Bulk 5-dim. case

Holographic superconductors

Typically, Einstein-Maxwell-complex scalar system:

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - \frac{1}{g^2}\left(F_{MN}^2 + \left|\nabla_M \psi - iA_M \psi\right|^2 + m^2 |\psi|^2\right)\right]$$

Hartnoll - Herzog - Horowitz, 0803.3295; 0810.1563
Gubser, 0801.2977

M,N...: bulk indices
μ,ν...: bdy indices

- Phase structure
  - T>Tc: AdS BH w/ ψ = 0
  - T<Tc: AdS BH w/ ψ ≠ 0 → ψ: order parameter
    ~ dual to "macroscopic wave fn"
- Dual to some kind of superconductors → diverging conductivity

Setup

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - \frac{1}{g^2}\left(F_{MN}^2 + \left|\nabla_M\psi - iA_M\psi\right|^2 + m^2\left|\psi\right|^2\right)\right]$$

Matter fields are coupled w/ gravity, and the system is hard to solve. So, we employ the "probe approx" $g \gg 1$, where matter fields are decoupled from gravity.

Then, one can simply use pure gravity solution (Schwarzschild-AdS BH) and solve matter fields in the background.

SAdS4 BH

We first consider the 4-dim bulk (for simplicity):

$$ds_4^2 = r^2(-fdt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2 f}$$

$$= \frac{1}{u^2}(-fdt^2 + dx^2 + dy^2) + \frac{du^2}{u^2 f}$$

AdS radius: $L = 1$

$f = 1 - r^{-3} = 1 - u^3$

$u = 1/r$

$r = u = 1$: horizon

$u = 0, r = \infty$: asymptotic infinity, "boundary"

BH: only T as scale → no characteristic T/no phase transition
→ chemical potential μ

At high temp. phase, $A_t = \mu(1-u)$

System parametrized by μ/T. We fix T and vary μ.

- μ_c/T < (const) → Normal phase
- μ_c/T > (const) → SC phase

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Bdy. Maxwell

We have the bulk Maxwell field, but on the bdy, the Maxwell field is added as an external source and is not dynamical.

This is because we usually impose the Dirichlet BC on the bdy.

Recall the standard dictionary. After solving the bulk EOM, extracting u→0 behavior, one gets

$$A_\mu \sim \mathcal{A}_\mu + \left\langle J^\mu \right\rangle u + \cdots \quad (u \to 0)$$

$\mathcal{A}_\mu$: bdy Maxwell
J^μ : U(1) current

fixed: Dirichlet ↓

or
$$A_\mu\Big|_{bdy} = \mathcal{A}_\mu$$

In other words, the bdy Maxwell has the action $\delta S = \int d^3x\, \mathcal{A}_\mu \left\langle J^\mu \right\rangle$ but no kinetic term.

It is possible to make it dynamical. Compere - Marolf, 0805.1902

Simply consider the following bdy EOM:

$$\frac{1}{e^2}\partial_\nu \mathcal{F}^{\mu\nu} = \left\langle J^\mu \right\rangle$$

All quantities are bdy ones including e. Here $\left\langle J^\mu \right\rangle$ is expectation value of bdy op computed by standard AdS/CFT recipes.

We call it "holographic semiclassical eq."

In other words, add the following action to the bdy CFT:

$$S_{bdy} = -\frac{1}{4e^2}\int d^3x \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$$

Then, what one should do is

- Solve the bulk EOM and obtain e.g. $\left\langle J^{\mu} \right\rangle$ by standard AdS/CFT recipes.
- Impose the semiclassical eq as BC instead of the Dirichlet BC.

We show Meissner effect analytically by imposing the holographic semiclassical eq. on holographic superconductors.

Previous studies

Actually, holographic Meissner effect was discussed in the past and I show you a few examples. They show the effect by constructing a single vortex numerically.

cf. Holographic vortex

Albash-Johnson, 0906.2396
Montull-Pomarol-Silva, 0906.2396
Maeda-Natsuume-Okamura, 0910.4475
Keranen-Keski-Vakkuri-Nowling-Yogendran, 0912.4280
…

Previous studies 1

- Probe limit, bulk 4-dim (Neumann BC) & 5-dim (holographic semiclassical eq.)

Domenech-Montull-Pomarol-Salvio-Silva, 1005.1776

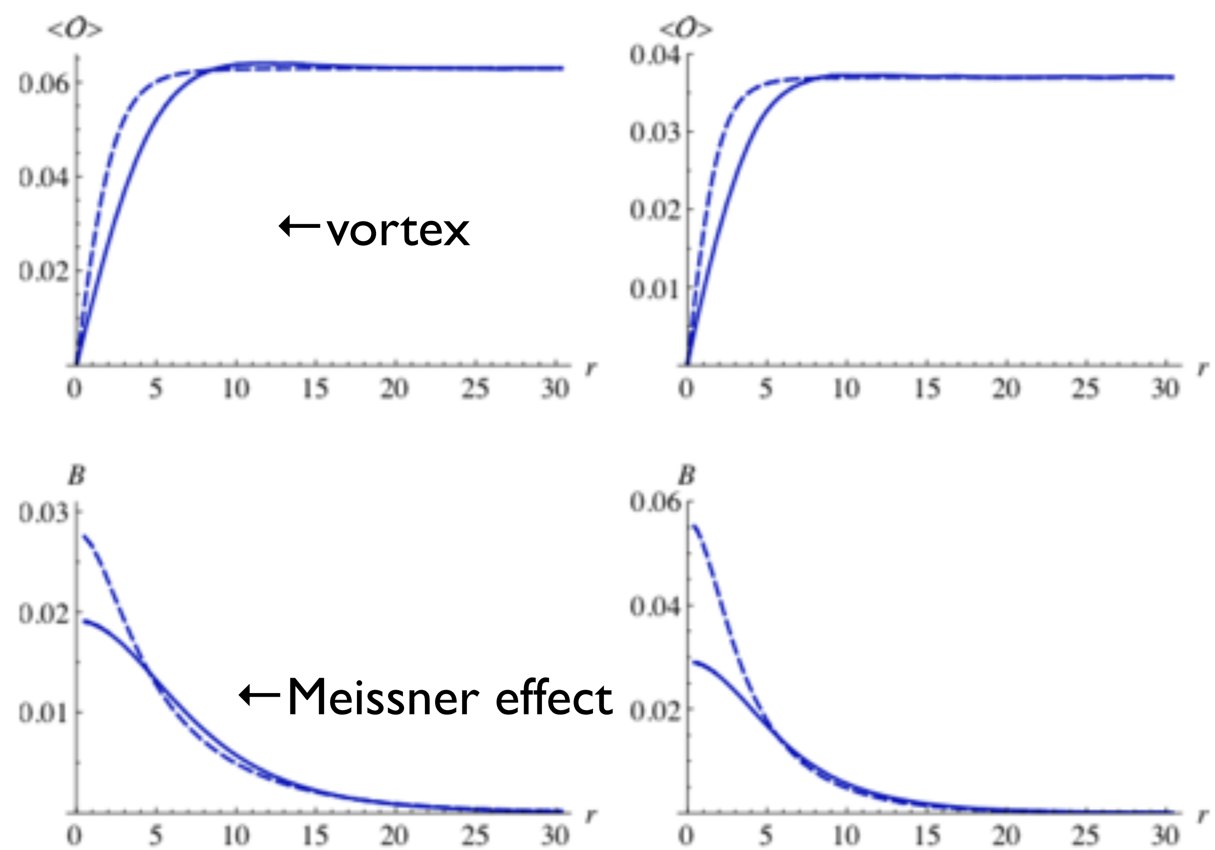

Figure 2: The modulus of $\langle \mathcal{O} \rangle$ (up to a factor L^{d-3}/g^2) and B as functions of r from our holographic model in the $n = 1$ superconductor vortex solution for $d = 2 + 1$ (solid lines on the left) and $d = 3 + 1$ (solid lines on the right). The dashed lines are the corresponding profiles in the GL theory. Presented in units of $\mu = 1$.

We have more to say about the Neumann BC.

Previous studies 2

- No probe limit, bulk 4-dim, Neumann BC

Dias-Horowitz-Iqbal-Santos, 1311.3674

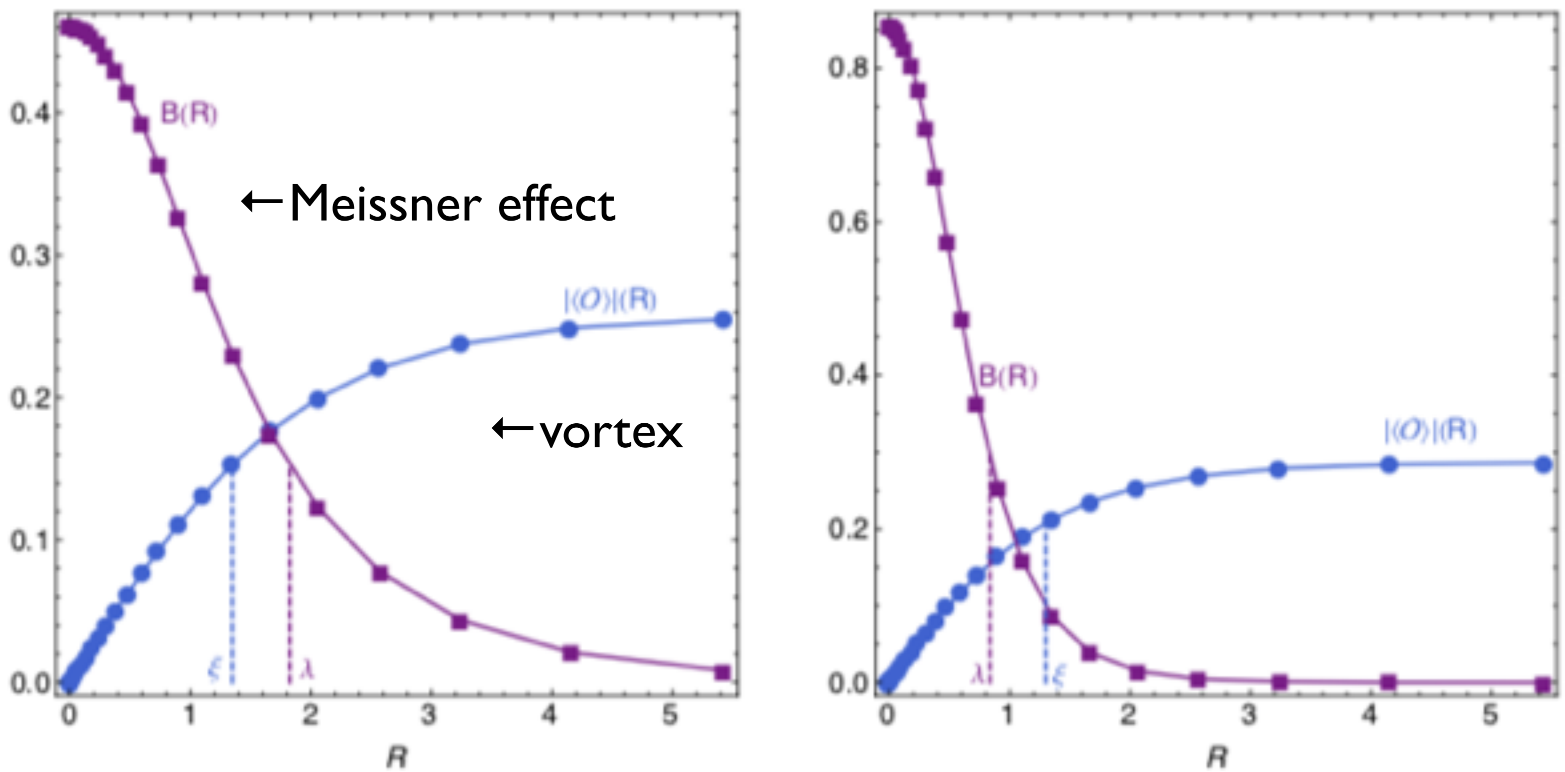

FIG. 14: Profile of magnetic field $B(R)$ and order parameter $\langle\mathcal{O}(R)\rangle$ for a single vortex with $qL = 1$ *(left panel)* and $qL = 3$ *(right panel)*. λ and ξ are found from exponential fits and measure the rate of fall-off of magnetic field and order parameter, respectively.

So, the Meissner effect has been shown, but it is desirable to show the effect more clearly.

Dirichlet BC case

First, let us check that standard HSC has no Meissner effect.

For T<Tc, a uniform condensate is a solution, so add a small magnetic field there.

$$A_y = Y e^{iqx}$$

Bulk EOM: $$0 = \left\{ -\partial_u (f \partial_u) + q^2 + 2 |\varphi_0|^2 \right\} Y, \quad \psi = u\varphi$$

One can integrate the eq formally:

$$\begin{aligned} Y &= \mathcal{Y} - \int_0^u \frac{du'}{f(u')} \int_{u'}^1 du''(q^2 + 2|\varphi_0|^2) Y \\ &= \mathcal{Y} \left\{ 1 - \int_0^u \frac{du'}{f(u')} \int_{u'}^1 du''(q^2 + 2|\varphi_0|^2) + \cdots \right\} \end{aligned}$$

$$Y = \mathcal{Y}\left\{1 - \int_0^u \frac{du'}{f(u')} \int_{u'}^1 du''(q^2 + 2|\varphi_0|^2) + \cdots\right\}$$

- The 1st term reps. the magnetic field.
  The magnetic field can enter SC (no matter how small) and no Meissner effect.

$$B = \partial_x Y\big|_{u=0} = iq\mathcal{Y}$$

- The 2nd term reps. current.
  According to the standard AdS/CFT recipe,

$$\begin{aligned}\langle J_y \rangle &= \partial_u Y\big|_{u=0} \\ &= \mathcal{Y}\left(-q^2 - \int_0^1 du\, 2|\varphi_0|^2 + \cdots\right)\end{aligned}$$

$$\left\langle J_y \right\rangle = \mathcal{Y}\left(-q^2 - \int_0^1 du\, 2\,|\,\varphi_0\,|^2 + \cdots \right)$$

normal current↓ (diamagnetic)

↓supercurrent (diamagnetic)

This is London eq. w/ added normal component.

Supercurrent itself exists, but there is no Ampere law on the bdy $\nabla \times B = J$, so the magnetic field does not decrease and no Meissner effect.

The 1st term exists even for pure bulk Maxwell theory. This is a diamagnetic current but should not be confused w/ the supercurrent (which is also diamagnetic). It can be interpreted as the magnetization current due to magnetization.

Holographic semiclassical eq case

We now impose holographic semiclassical eq as the BC:

$$\partial_j \mathcal{F}^{ij} = e^2 \left\langle J^i \right\rangle$$

What is the role of the normal current?
Consider only normal current & external current. The semiclassical eq gives

$$\left\langle J_y \right\rangle = \mathcal{Y}\left(\underline{-q^2} - \int_0^1 du\, 2\,|\varphi_0|^2 \right)$$

$A_y = Ye^{iqx}$

$$q^2 \mathcal{Y} = -e^2 q^2 \mathcal{Y} + e^2 J_{\text{ext}}$$

$$q^2 \mathcal{Y} = \frac{e^2}{1+e^2} J_{\text{ext}}$$

cf. $\nabla \times B = \mu_m J$

It shifts magnetic const. from $\mu_0 = e^2$ to $\mu_m = e^2 / (1+e^2)$

Magnetization current

Recall elementary EM.
Magnetic moment produces magnetization M and steady magnetiz. current Jm.

$$\nabla \times B = \mu_0 (J_{\mathrm{ext}} + J_m) \\ = \mu_0 (J_{\mathrm{ext}} + \nabla \times M)$$

At linear order

$$M = \frac{1}{\mu_0} \frac{\chi}{1+\chi} B$$

So

$$\nabla \times B = \mu_0 (1+\chi) J_{\mathrm{ext}} =: \mu_m J_{\mathrm{ext}}$$

χ : magnetic susceptibility
μ_m : magnetic const.

Now include the supercurrent as well. w/ a delta-fn source,

$$q^2 \mathcal{Y} = -e^2 (q^2 + 2I)\mathcal{Y} + e^2 \qquad I = \int_0^1 du\, |\varphi_0|^2$$

$$\mathcal{Y} \propto \frac{1}{(1+e^2)q^2 + 2e^2 I} \to e^{-x/\lambda}$$

w/ the magnetic penetration length:

$$\boxed{\lambda^2 = \frac{1}{2\mu_m I} = \frac{1+e^2}{2e^2 I}}$$

cf. GL:

$$\lambda_{\mathrm{GL}}^2 = \frac{1}{2e^2 |\psi|^2}$$

- At weak coupling e<<1, holographic result reduces to the GL result.
- In the limit e→∞, GL implies λ=0. Strong Meissner. "extreme type I"
- For HSC, λ remains finite. "extreme type I" cannot be reached.

Neumann BC

In previous studies, one typically imposes Neumann BC or J=0.

$$A_i \sim \mathcal{A}_i + \left\langle J^i \right\rangle u + \cdots \quad (u \to 0)$$

fixed: Dirichlet ↓ ↓fixed: Neumann

In our language, Neumann BC corresponds to the e→∞ limit since the kinetic term is gone, but the nontrivial result even in the limit, so the BC is possible.

$$\cancel{\partial_j \mathcal{F}^{ij}} = e^2 \left\langle J^i \right\rangle$$

What happens is

- J=0 does not mean no supercurrent because J consists of normal current as well as supercurrent J=Jn+Js.

$$\cancel{q^2 \mathcal{Y}} = -e^2 (q^2 + 2l)\mathcal{Y}$$

- The normal current has “induced” kinetic term.

GL parameter

SC has 2 characteristic length scales:

- Magnetic penetration length λ (gauge field mass) → W-boson mass $\lambda_{\mathrm{GL}}^2 = \frac{1}{2e^2|\psi|^2} = \frac{1}{2e^2}\frac{b}{|T-T_c|}$
- Correlation length ξ (order parameter mass) → Higgs mass $\xi_{\mathrm{GL}}^2 = \frac{1}{|T-T_c|}$

Then, SC is characterized by a dimensionless parameter, GL parameter:

$$\kappa^2 = \frac{\lambda^2}{\xi^2}$$

$$\kappa_{\mathrm{GL}}^2 = \frac{b}{2e^2}$$

Whether SC is type I or II depends on κ:

$\kappa^2 < 1/2$: Type I

$\kappa^2 > 1/2$: Type II→longer λ so magnetic penetration becomes possible

In the bulk 4-dim, the analytic expression is not possible for κ, but in the bulk 5-dim, an analytic expression is possible.

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SAdS5 case

2 Differences from the SAdS4 case:

- Holographic renormalization is necessary for Maxwell field.
- There exits a simple analytic solution when m^2=-4

$$\varphi_0 = \sqrt{\frac{24(\mu-\mu_c)}{\pi T}}\frac{u}{1+u^2}$$

BF bound

mean-field

w/ critical temperature $\mu_c = 2\pi T$

Herzog, 1003.3278
Natsuume - Okamura, 1801.03154

The solution is a special case of a 1-parameter family of analytic solutions for "holographic Lifshitz superconductors."

Then, you can compute everything explicitly.

SAdS5 case

In this case, one can evaluate I:

$$I=\int_0^1 du\frac{|\varphi_0|^2}{u}=\frac{6(\mu-\mu_c)}{\pi T}\to\lambda^2=\frac{1}{2\mu_m(\pi T)^2 I}=\frac{1}{12\mu_m}\frac{1}{(\mu-\mu_c)\pi T}$$

In bulk 5-dim., magnetic const:

$$\mu_m=\frac{e^2}{1-e^2\ln(\pi T)}$$

The correlation length:

$$\xi^2=\frac{1}{2(\mu-\mu_c)\pi T}$$

from scalar QNM computation

Then GL parameter:

$$\kappa^2=\frac{\lambda^2}{\xi^2}=\frac{1}{6\mu_m}=\frac{1-e^2\ln(\pi T)}{6e^2}$$

SAdS5: details

In (4+1)-dimensions,

$$\left\langle J_y \right\rangle = \frac{1}{u}\partial_u Y - q^2 Y \ln\epsilon \Big|_{u=\epsilon}$$

log divergent↓ ↓counterterm

from the CT action:

$$S_{CT} = -\int d^4x \frac{1}{4g^2}\sqrt{-\gamma}\gamma^{\mu\nu}\gamma^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \times \ln u$$

$\gamma_{\mu\nu}$: bdy metric

ϵ : UV cutoff

In (4+1)-dimensions,

$$Y = \mathcal{Y}\left\{1-\int_0^u \frac{u'du'}{f(u')}\int_{u'}^{u_0} du'' \frac{1}{u''}(q^2+2|\varphi_0|^2)+\cdots\right\} \qquad u_0 = 1/r_0$$

$$\left\langle J_y \right\rangle = \frac{1}{u}\partial_u Y - q^2 Y \ln\epsilon\Big|_{u=\epsilon}$$

$$= \mathcal{Y}\left\{-\int_{\epsilon}^{u_0} du \frac{1}{u}(q^2+2|\varphi_0|^2)+\cdots\right\} - q^2\mathcal{Y}\ln\epsilon$$

$$= \mathcal{Y}\left\{-q^2(\ln u_0 - \cancel{\ln\epsilon}) - \int_0^{u_0} du\frac{2}{u}|\varphi_0|^2 - \cancel{q^2\ln\epsilon}\right\}$$

$$= \mathcal{Y}\left\{-q^2\ln u_0 - \int_0^{u_0} du\frac{2}{u}|\varphi_0|^2\right\}$$

Holographic semiclassical eq then gives (when no supercurrent)

$$q^2\mathcal{Y} = -e^2 q^2 \ln u_0 \times \mathcal{Y} + e^2 J_{\text{ext}}$$

$$= \frac{e^2}{\underbrace{1+e^2\ln u_0}_{\mu_m}} J_{\text{ext}}$$

$$\kappa^2 = \frac{\lambda^2}{\xi^2} = \frac{1}{6\mu_m} = \frac{1 - e^2 \ln(\pi T)}{6e^2}$$

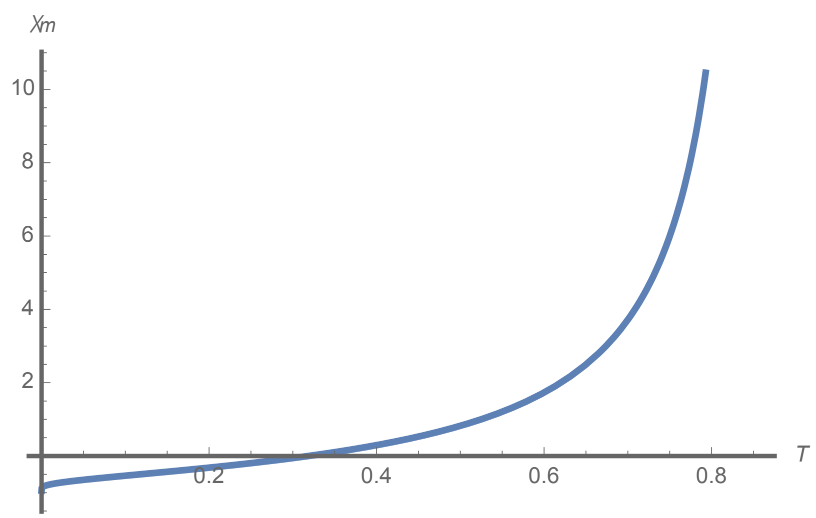
cf. GL: $$\kappa_{\mathrm{GL}}^2 = \frac{b}{2e^2}$$ $$\frac{b}{2}|\psi|^4$$

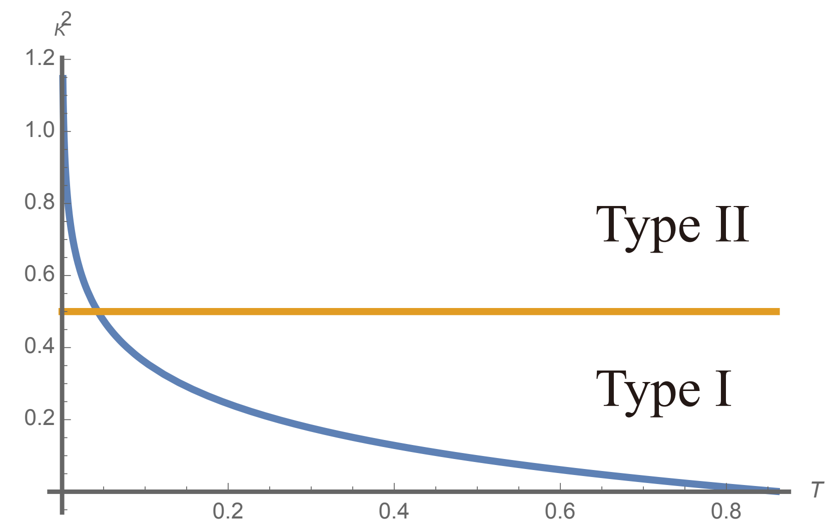
GL parameter of HSC is determined analytically for the first time.

- Focus on πT<<1. At weak coupling e<<1, holographic result reduces to the GL result.
- In the limit e→∞, GL implies κ_{GL}=0. Strong Meissner. "extreme type I." For HSC, κ remains finite even in the limit. "extreme type I" cannot be reached. The Neumann-like BC should be possible even for 5-dim. bulk (in probe limit).
- In general, type I or type II depends on temperature.
  - As T→0, κ→∞ (extreme type II).
  - As one increases T, κ decreases.

$$\kappa^2 = \frac{\lambda^2}{\xi^2} = \frac{1}{6\mu_m} = \frac{1 - e^2 \ln(\pi T)}{6e^2}$$

$\boxed{e/g = 1}$





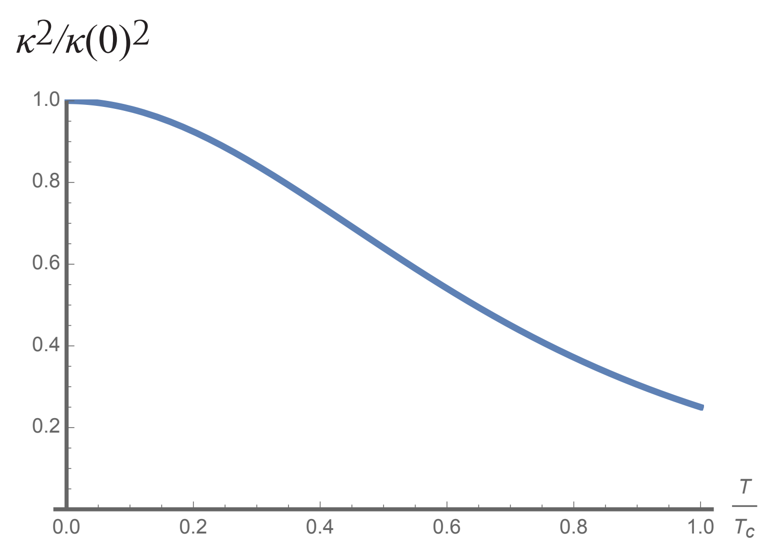
diamagnetic paramagnetic

Temperature dependence of GL parameter

Interestingly, many superconducting materials (including high-Tc) show similar behavior: As one increases T, κ decreases.

e.g. Tinkham, "*Introduction to superconductivity*" gives an empirical rough estimate (Sect. 4.2)
"Of course, this is only a rough approximation…"

$$\kappa^2 \propto \left(\frac{1}{1+t^2}\right)^2$$



This does not imply the same physics. For this formula, it comes from T-dependence on "b."

Dual GL theory

w/ all information we have, one can write down the dual GL theory:

Natsuume - Okamura, 1801.03154

$$F = \int d^3x \frac{1}{4} | D_i \phi |^2 - \frac{1}{2}(\mu - \mu_c) | \phi |^2 + \frac{1}{96} | \phi |^4 + \cdots - (\phi J^\dagger + \phi^\dagger J) + \frac{1}{4\mu_m} \mathcal{F}_{ij}^2$$

$$D_i = \partial_i - i \mathcal{A}_i$$

Just like GL, this should be regarded as leading terms near the critical pt. e.g. $-\underbrace{\frac{253 - 336 \ln 2}{82944}}_{\sim 0.00024} |\phi|^6$

Dynamic case: time-dependent Ginzburg-Landau eq at linear level (from QNM computation of scalar at high temp)

$$-\Gamma^{-1} \partial_t \phi = \frac{1}{4} \partial_i^2 \phi - a\phi + \cdots$$

$$\Gamma = \frac{2}{5}(1 + 3i)$$

Recently, several attempts to determine the dual theory numerically

Magnetic susceptibility: QCD

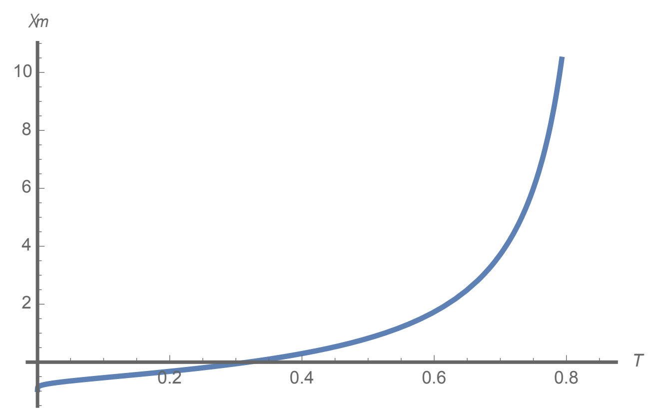
Putting aside HSC, consider Einstein-Maxwell theory.
The dual theory is $\mathcal{N}$=4 SYM + U(1).
Our computation of μ and χ themselves is valid there (in probe limit).

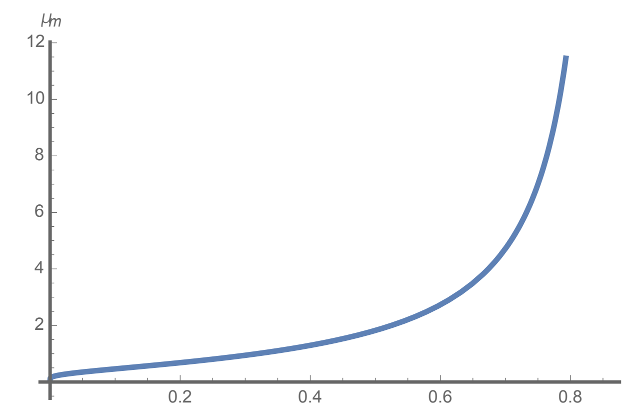
Just for fun, let us compare w/ QCD.

At high temp., paramagnetic like our case

But more careful comparison is necessary.

Magnetic susceptibility: comparison





diamagnetic paramagnetic

Bali et al., 1406.0269 [hep-lat]

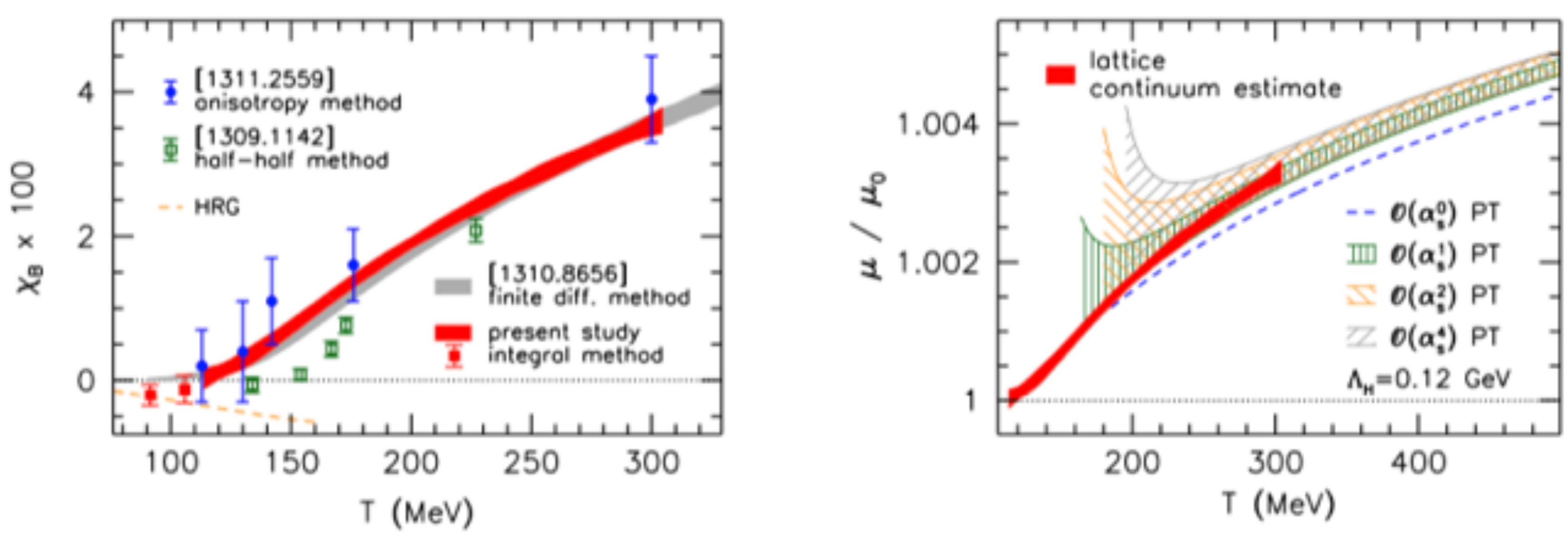


Figure 9. Left panel: magnetic susceptibility of QCD as a function of the temperature. Results with different lattice approaches are collected. Right panel: QCD magnetic permeability in units of the vacuum permeability μ_0, and a comparison to perturbation theory, truncated at various orders of the strong coupling.

Other applications of holographic semiclassical eq.

Similar story for bdy gravity.

We have gravity in the bulk, but on the bdy., gravity is an external source and is not dynamical just like the Maxwell field:

$$\delta S = \int d^4x \, h_{\mu\nu} \left\langle T^{\mu\nu} \right\rangle$$

$h_{\mu\nu}$: bdy graviton
$T^{\mu\nu}$: gauge theory EM tensor

Again, one can make it dynamical. Simply consider the following bdy EOM:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_4 \left\langle T_{\mu\nu} \right\rangle$$

All quantities are bdy ones including *G*.

In general, it is not easy: one should solve the bulk eq and the semiclassical eq simultaneously.

But in principle, one can apply it to the probs of gauge theories w/ gravity, e.g. neutron star mergers and cosmology.

One recent application to cosmology:

Ecker - van deer Schee - Mateos - Casalderrey-Solana, 2109.10355

Summary

- In most applications of AdS/CFT, the Maxwell field is added as an external source.
- One can make it dynamical by changing the BC on the AdS bdy ("holographic semiclassical eq").
- As an application, we study the holographic Meissner effect.
    - In standard HSCs, there is no Meissner effect because the Maxwell field is nondynamical.
    - We show the Meissner effect analytically.
    - The magnetic penetration length & GL parameter take a nontrivial form due to the change of the magnetic const.
- It is interesting to explore the other backreaction probs.

e.g. Ahn et al, 2211.01760

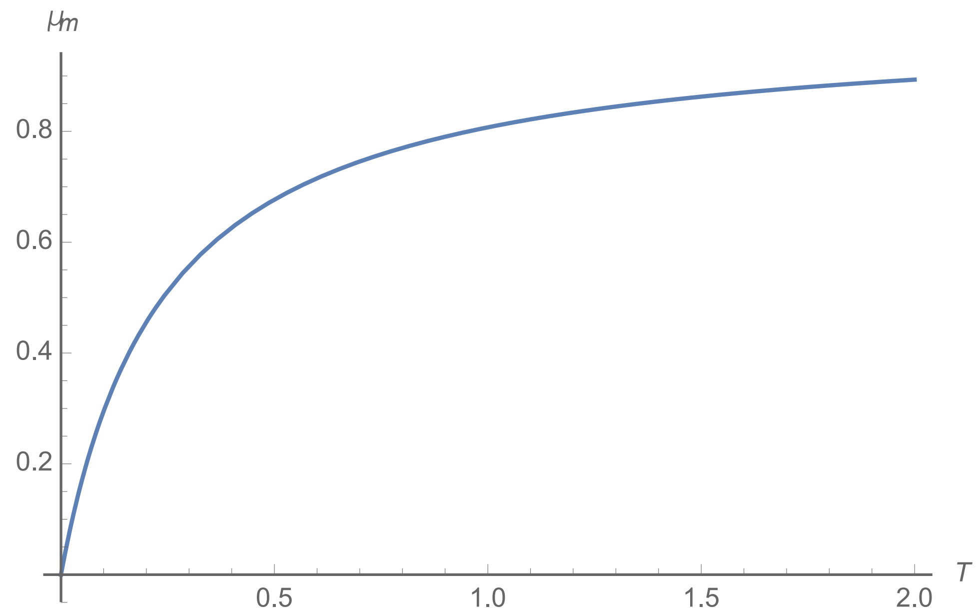
Backup

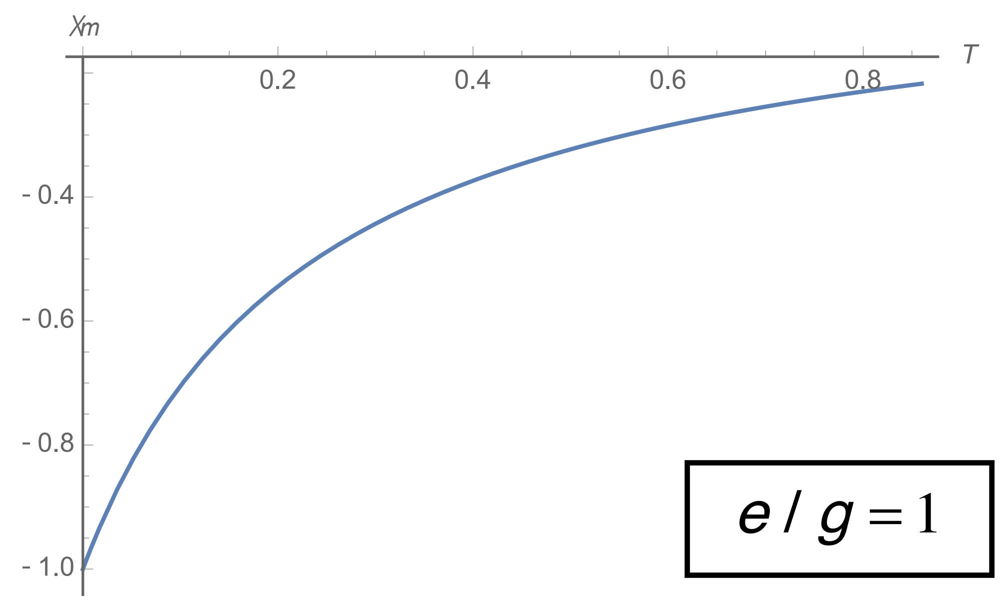
SAdS4 case

Restoring T, one gets

$$\mu_m = \frac{e^2}{1 + e^2 / r_0}$$

$$\chi_m = -\frac{e^2 / r_0}{1 + e^2 / r_0}$$





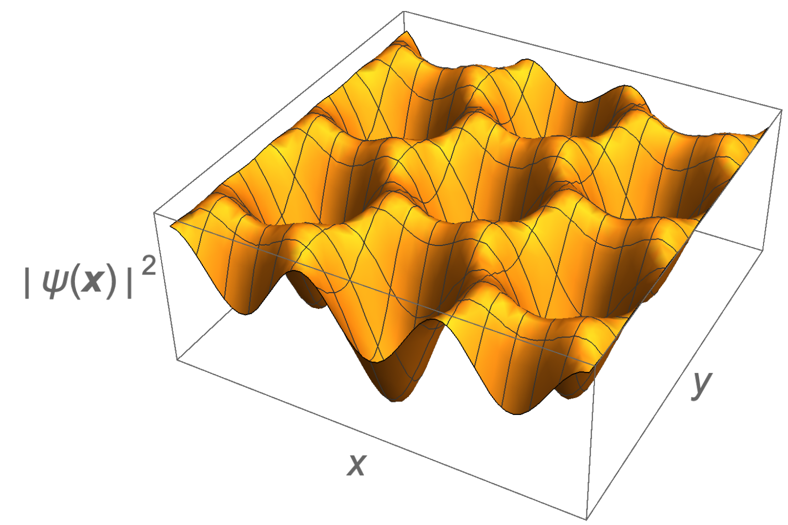
$$\boxed{e / g = 1}$$

Near upper critical magnetic field

We consider a small magnetic field.

As one increases the magnetic field, more and more vortices are created, and they form a vortex lattice.

Eventually, SC state is completely broken at the upper critical magnetic field H_{c2}.



According to the GL theory,

$$B = H - e^2 |\psi|^2$$

B reduces by the amount $|\psi|^2$ which implies Meissner effect.

Holographic semiclassical eq case

Impose the holographic semiclassical eq. I only quote the final result.

For the bulk 4-dim (in the hydro limit q→0)

$$B = H - \frac{e^2}{1+e^2}(\text{numerical factors})\left|\langle \mathcal{O} \rangle\right|^2$$

cf. GL: $$B = H - e^2 |\psi|^2$$

Once again,

- At weak coupling e<<1, holographic result reduces to the GL result.
- There is a nontrivial e→∞ limit unlike the GL theory.
- This comes from the nontrivial magnetic const. The magnetic const. obtained here agrees w/ the small magnetic field case.

Critical magnetic field

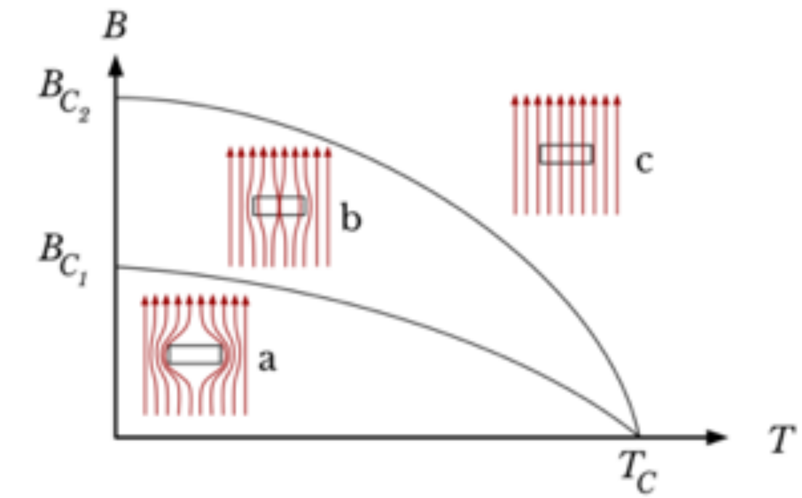
- Upper critical magnetic field: $H_{c2} = a = \sqrt{2}\kappa H_c$
  SC state is completely destroyed.

$$\kappa = \sqrt{\frac{b}{2e^2}}$$

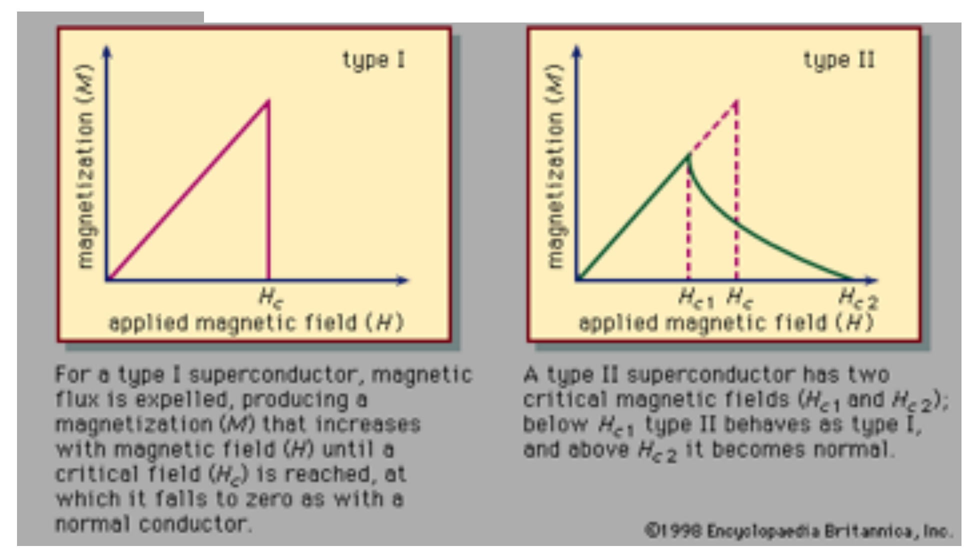
- Critical magnetic field: $H_c = e\dfrac{a}{\sqrt{b}}$

  Uniform |ψ| thermodynamically favored.

- Lower critical magnetic field: H_{c1}
  Vortex begins to form.



For Type II or $\kappa > 1/\sqrt{2}$, $H_c < H_{c2}$



For Type I, $H_c > H_{c2}$
As one lowers H, ψ=0 remains as the supercooled state, and vortex is formed for $H < H_{c2}$.