Large N fractons

Holotube virtual seminar
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based on: { 2205.01132 w/ Amir Raz
see also 2111.03973 w/ Akash Jain

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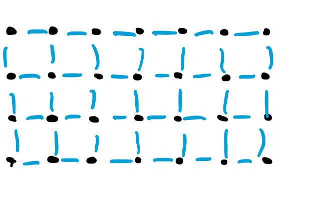
Fractons challenge long-held beliefs concerning the simplicity and universality of low-energy EFT.

→ Interesting, at this time hypothetical phases of quantum matter which seem difficult to describe w/ QFT in the continuum limit.

Usual approach:

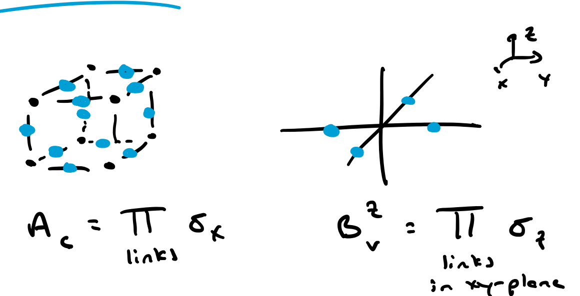
- Coarse grain a complicated system e.g. lattice QCD, string compactification, etc.



- get EFT description of relevant dofs
- Microscopic details → { low-energy constants, irrelevant operators

Now, a spin system that seems to defy us.

"X-cube" [Vijay, Haah, Fu] (hypercubic lattice in 3d "3d toric code")



$A_c = \prod_{\text{links}} \sigma_x$ $\quad B_v^z = \prod_{\text{links in xy-plane at } v} \sigma_z$

$H = \alpha \sum_c A_c + \beta \sum_{v,i} B_v^i$ → "integrable system"

$[A_c, A_{c'}] = [A_c, B_v^i] = [B_v^i, B_v^j] = 0$

Simultaneously diagonalize

ground states satisfy
$A = B = -1 \;\; \forall c, v, i$ → strong constraint.

Exotic features:

① Ground state degeneracy is UV-sensitive, does not depend on topology (a la $d \sim 2^{2g-2}$)
$d \sim 4^{\#\text{planes}}$
$= 2^{2L_x + 2L_y + 2L_z - 3}$ $\quad S(T \to 0) \sim L$

② Excitations of restricted mobility. (finite energy)
- fracton: stuck at a link
- lineon: can move along a line of the lattice
- planeon: can move in a plane

③ Subsystem Symmetry. Ordinary spin models at zero field invariant under $\vec{\sigma}_v \to -\vec{\sigma}_v$.
X-cube invariant under spin flips that act independently on planes. $\vec{\sigma}_v \to f(\text{plane})\, \vec{\sigma}_v$. ($\pm 1$)

① & ② a consequence of ③.

- Subsystem symmetry is spontaneously broken
$d \sim \text{vol}(\text{subsystem})$
- Restricted mobility a consequence of conservation laws

To recap, the physical features that challenge us are { non-topological, UV-sensitive ground state degeneracy; restricted mobility excitations

Exotic Spacetime Symmetry.

> Can QFT describe this?!

Today I am going to showcase a class of models where the answer is YES.

Start w/ symmetry, which we know how to implement.

Models For this talk, let's fix lots of symmetry.

- translations $(H, \vec{P})$
- rotations (M_{ab})
- $u(1)$ (Q) ⟶ $Q = \int d^dx\, \rho$
- dipole moment (D^i) ⟶ $D^i = \int d^dx\, \rho x^i$

Isolated charges are the fractons!

What does such an EFT look like?

Suppose we have a complex scalar ϕ. [Pretko]

$Q: \; \phi \to e^{i\alpha}\phi$ $\qquad [D^i, P^j] = i\delta^j_i Q$

$D^i: \; \phi \to e^{i\vec{d}\cdot\vec{x}}\phi$

Invariant action:

$$S = \int dt\, d^dx \left\{ i\bar{\phi}\partial_t\phi - V(|\phi|^2) - \cancel{\frac{|\vec{\nabla}\phi|^2}{2m}} - 2\mathrm{Re}\left(\lambda\bar{\phi}^2 D_{ii}(\phi,\phi)\right) - \lambda_1 D_{ij}(\bar{\phi},\bar{\phi}) D^{ij}(\phi,\phi) + \dots \right\}$$

$D_{ij}(\phi,\phi) = \phi\,\partial_i\partial_j\phi - \partial_i\phi\,\partial_j\phi \to e^{2i\vec{d}\cdot\vec{x}} D_{ij}(\phi,\phi)$

- terms w/ spatial derivatives are at least quartic in ϕ.
(dipole $\sim \bar{\phi}(x_1)\phi(x_2)$ w/ $x_1 \to x_2$
these terms are $\sim$ dipole dipole, like $|\phi|^2$ = charge charge)

? ! ?

<u>Option 1:</u> treat quartic terms as a perturbation

$$S = \int dt\, d^dx \left\{ i\bar{\phi}\partial_t\phi + \mu|\phi|^2 - 2\mathrm{Re}\left(\lambda\phi^2 D_{ii}(\bar{\phi},\bar{\phi})\right) + \dots \right\}$$

(expand around; small)

<u>But:</u> any loop vanishes under dim reg.

$\sim \frac{1}{\omega - \mu}$ $\qquad \int d^dk\, k^2 = 0$

besides, λ is dimensionful, cannot be neglected.

<u>Option 2:</u> $V(|\phi|^2) = \lambda_4|\phi|^4 - \mu|\phi|^2$ study $\mu \gg 1$

try to condense $\phi = v e^{i\chi}$ $\qquad \chi \to \chi + \alpha + \vec{d}\cdot\vec{x}$

$L_{\text{eff}} = a\dot{\chi}^2 - (\nabla^2\chi)^2 + \dots$ quadratic theory.

<u>Option 3:</u> (TODAY) <u>Large N!</u>

Study a vector model w/ $U(N) \sim SU(N) \times U(1)$ (ordinary) (dipole)

$$S_I = \int dt\, d^dx \left\{ \bar{\phi}^a\partial_t\phi^a + 2\mathrm{Re}\left[\frac{\lambda}{N}\phi^a\phi^b D_{ii}(\bar{\phi}^a,\bar{\phi}^b)\right] - \mu|\phi|^2 + \frac{\lambda_4}{N}|\phi|^4 \right\}$$

take $N \gg 1$. 1970s: $L \sim |\partial\vec{\phi}|^2 + \frac{\lambda}{N}|\vec{\phi}|^4 + m^2|\vec{\phi}|^2$

solve via Hubbard-Stratonovich.

$L \sim |\partial\vec{\phi}|^2 - \frac{N}{\lambda}\sigma^2 + \sigma|\vec{\phi}|^2$ $\qquad \int$ in $\sigma \sim |\phi|^2$

$\frac{S_{\text{eff}}}{N} \sim \ln\det(-\partial^2 + \sigma) + \int\left(-\frac{\sigma^2}{\lambda} + \sigma\right)$

Fails here because the quartic term has derivatives.

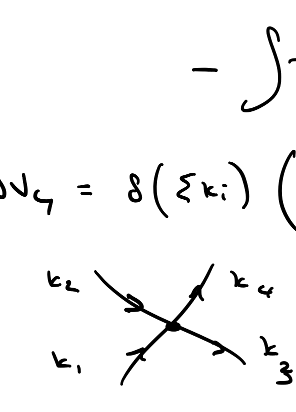
GENERALIZED Hubbard-Stratonovich (a la Chern-Simons-matter, SYK)
aka G/Σ.

$G(x_1,x_2) = \bar{\phi}^a(x_1)\phi^a(x_2)$ (@ Lagrange multiplier $\Sigma(x_1,x_2)$)

$G_{ij} = G(k_i,k_j)$

$$S = N\ln\det(i\omega\delta + \Sigma) + \int Dk_1 \dots Dk_4\, G_{12}G_{34}\, V_4(k_i) - \int Dk_1 Dk_2\, \Sigma_{12}G_{12} - \mu\int Dk_1 Dk_2\, G_{12}\delta_{12}$$

$NV_4 = \delta(\textstyle\sum k_i)\left(\frac{\lambda}{2}(\vec{k}_1 - \vec{k}_2)^2 + \frac{\bar{\lambda}}{2}(\vec{k}_3 - \vec{k}_4)^2 + \lambda_4\right)$



Many-body QM w/ $2 \to 2$.

(G, Σ) are the weakly coupled large N dofs.

Translationally invariant ansatz:
$\langle G_{12}\rangle = N G(k_2)\,\delta(k_1 + k_2)$
$\langle \Sigma_{12}\rangle = \Sigma(k_2)\,\delta(k_1 + k_2)$ $\qquad S \sim N S_0(G,\Sigma)$ weakly coupled.

eoms for (G,Σ) are:

$G(k) = \frac{1}{i\omega + \Sigma(k)}$, $\quad \Sigma =$

$\Sigma(k) = -\mu + 2\int Dk'\, G(k')\, N\tilde{V}_4(-k,-k',k,k')$

$= \mathrm{Re}\,\lambda\,(\vec{k} - \vec{k}')^2 + \lambda_4$

These eq'ns can be solved! Σ = polynomial in $\vec{k}$.

rotationally invariant ansatz: $\Sigma(k) = a_1\vec{k}^2 + a_0$

get coupled eq'ns for (a_0, a_1) w/ nontrivial sol'ns.
(depend non-analytically on $\mathrm{Re}\,\lambda$)

Quartic interactions generate self-energy, attenuate high momentum behaviour!

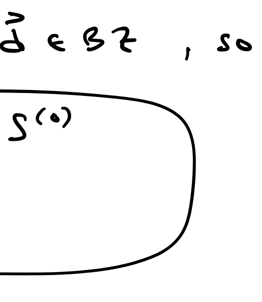
* In fact, there are <u>many</u> sol'ns.

Dipole symmetry acts as $G(\vec{k}) \to G(\vec{k} + \vec{d})$
$\Sigma(\vec{k}) \to \Sigma(\vec{k} + \vec{d})$

($G(x_1,x_2) = \bar{\phi}(x_1)\phi(x_2) \to e^{i\vec{d}\cdot(\vec{x}_2 - \vec{x}_1)}\bar{\phi}(x_2)\phi(x_1)$)

G depends on $\vec{k}$ ⟹ <u>dipole SSB</u>!

<u>This makes sense</u>, $G \sim$



so G is a good order parameter for dipole SSB.

we then find $Z \sim \text{vol}(\text{zm})\, e^{-S^{(0)}}$ ← $O(N)$ on-shell action
$\text{vol}(\vec{d}) = ?$

In a lattice regularization, $\vec{d} \in BZ$, so $\text{vol}(\text{zm}) \sim \text{vol}(BZ)$.

> $Z = Z_{1\text{-loop}}\, N_{\text{sites}}\, e^{-S^{(0)}}$

$\text{vol}(BZ) \sim \left(\frac{2\pi}{a}\right)^d V \sim N_{\text{sites}}$

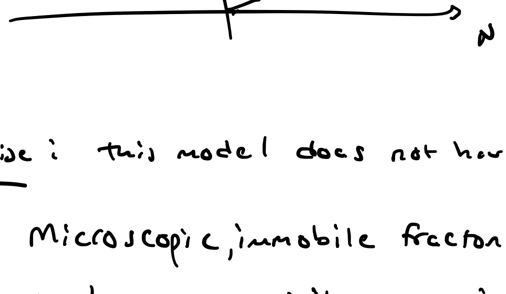
- UV/IR mixing. sensitive to details of lattice.

<u>But:</u> innocuous, only appears through an overall symmetry factor, zero mode field range.

- loops softened at large $\vec{k}$!

We've done a lot more. e.g. { condensed phase @ large μ; dipole → subsystem.

$d > 2$:



dipole broken everywhere, gapless phases.
$\vec{\varphi} \to \vec{\varphi} + \vec{d}$
$\chi \to \chi + \alpha + \vec{d}\cdot\vec{x}$

<u>Surprise:</u> this model does not have <u>fractons</u> at low energy.

{ Microscopic, immobile fractons are dressed by interactions to become mobile quasiparticles.

However, we can make models w/ fractons by coupling to "E/M."

<u>LOTS of work in progress:</u>

- Field theory models w/ two time derivatives.
- Renormalization.
- Models w/ subsystem symmetry.
- Coupling to tensor gauge theory to make <u>fractons</u>.
- Generalizations to other systems w/ exotic symmetry, like LLL.

THANK YOU!