Critical and near-critical relaxation of holographic superfluids

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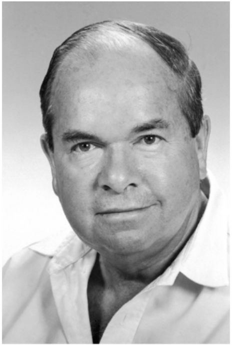
The Nobel Prize in Physics 2003



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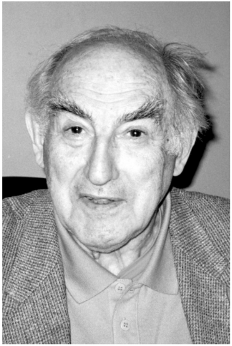

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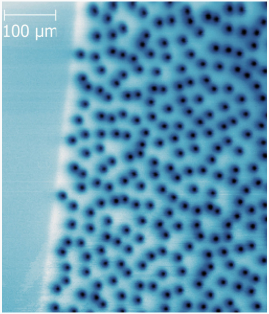
Ginzburg-Landau and Gross-Pitaevskii equations

- Complex (time-dependent) Ginzburg-Landau equation is one of the most-studied nonlinear equations in physics
- Wide range of applications from nonlinear waves to second-order phase transitions, superconductivity, (superfluidity) and Bose-Einstein condensation
- Gross-Pitaevskii equation is the superfluid counterpart
- Both simple, phenomenological models are widely used



[Wells, Pan, Wang, Fedoseev, Hilgenkamp; '15]

Relaxation to critical points

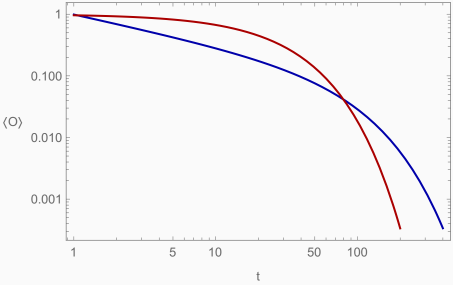
Homogeneous **Superfluids:** $\partial_t \phi \sim \frac{\delta F}{\delta \phi}$. F is free energy; ϕ order parameter.

How is $\partial_t \phi$ governed if $\delta F / \delta \phi = 0$ (as it happens at the critical point)?

Holographic analogue: at zero wave vector, Quasi-normal mode describing condensate/amplitude fluctuations becomes massless $\Rightarrow$ No exponential decay to equilibrium. Relaxation?

For final states **near the critical point** half-life time of exponential falloff diverges as the end-state is taken towards critical point $\Rightarrow$ **critical slowing down**.

Critical slowing down



Relaxation towards critical point hard to engineer experimentally; try to build phenomenological model to describe "near" critical relaxation (power law $\rightarrow$ exponential decay).

Very timely due to recent interest in fluctuations of order parameter which is usually neglected in hydrodynamics (recent experimental results in context of strange metals) *and* push to employ holography for phenomenological model building

Plan

- Study phenomenology of critical quenches numerically in holographic bulk model
- Build phenomenological model in the boundary theory describing the *homogeneous* dynamics
- Determine input parameters in holographic model
- Make independent predictions and check them

$\Rightarrow$ use powerful toolkit of holography to gain insight into the universal behavior of strongly coupled dynamics through the classical theory of gravity with one additional dimension.

$\Rightarrow$ use holography to develop and test phenomenological models which may be used in other areas of physics

Drawback: Fluctuations at critical point are suppressed in large N limit; mean field limit

Ginzburg-Landau

Time-dependent Ginzburg-Landau eqs [Schmid; '66],[Aranson, Kramer; '01]

$$\frac{1}{\gamma}\partial_t \psi = \Delta\psi + \alpha\psi + \beta\left|\psi\right|^2\psi,$$

where ψ is complex function of time and space and β characterizes nonlinear dispersion.

In presence of external vector potential $\boldsymbol{A}$ with effective potential Φ

$$\frac{1}{\gamma}(\partial_t - iq\Phi)\psi = (\nabla - iq\boldsymbol{A})^2\psi + \alpha\psi + \beta\left|\psi\right|^2\psi$$

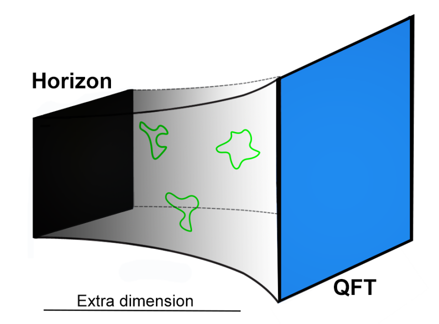
Gross-Pitaevskii

At finite T: add dissipative terms [Yan, Lan, Tian, Yang, Yao, Zhang; '22]

$$\begin{aligned}
\partial_t \psi \quad &= -\frac{(i+\gamma)}{\tau}\left(-\frac{\nabla^2}{2}\psi + \mu(|\psi|^2 - 1)\psi\right), \\
\partial_t \psi \quad &+ i\eta\psi\partial_t|\psi|^2 = -\frac{i}{\tau}\left[\left(-\frac{\nabla^2}{2}\psi + \mu(|\psi|^2 - 1)\psi\right)\right].
\end{aligned}$$

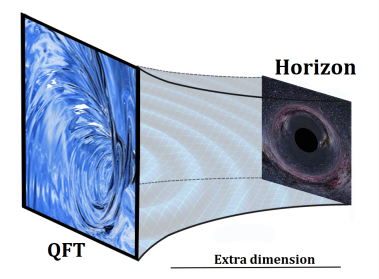
τ: characteristic time scale of dynamics, μ is the chemical potential, dissipative parameter γ: Keldysh self-energy through the fluctuation-dissipation theorem [Stoof; '97], dissipative parameter η: second law of thermodynamics holds [Carlson; '96].

Methodology



Holography as a blackbox

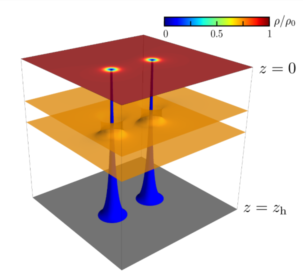
Quantum field theory	$\Leftrightarrow$	Quantum Gravity in AdS



large N limit	$\Leftrightarrow$	classical gravity
equilibrium state at finite T & ρ	$\Leftrightarrow$	black hole with T & ρ
linear response G^{ret}	$\Leftrightarrow$	QNMs of black hole
real-time non-equilibrium dynamics	$\Leftrightarrow$	time-dependent gravity

[Maldacena; '97], [Witten; '98], [Kovtun, Starinets; '05], [Chesler, Yaffe; '08]

Holography and Gross-Pitaevskii



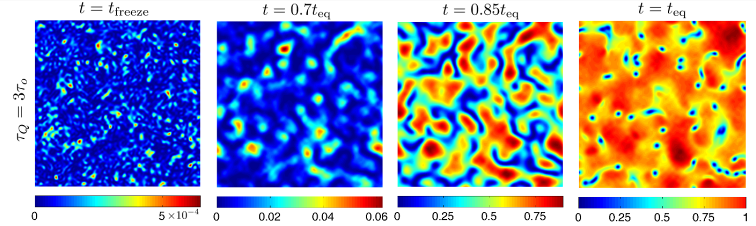
State of the art:

[Wittmer, Schmied, Gasenzer, Ewerz; '20], [Yan, Lan, Tian, Yang, Yao, Zhang; '22]

Simulate the motion of a vortex dipole and matching the data with the phenomenological dissipative Gross-Pitaeviskii models

Insights into holographic dissipation mechanism; Selection of phenomenological models. Predictions? Fixed μ out-of-equilibrium?

Holography and Kibble-Zurek

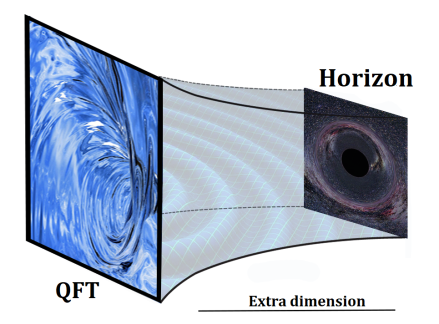


State of the art:

[Chesler, Garcia-Garcia Liu; '14], [Sonner, del Campo, Zurek; '14], [Bhaseen, Gauntlett, Simons, Sonner, Wiseman; '12], [del Campo, Gómez-Ruiz, Li, Xia, Zeng, Zhang; '21], [Li, Shi, Zhang; '21]

- Dynamic after smooth quench across continuous transition from disordered phase to ordered phase
- Formalism to predict rate of defect formation (smaller than the Kibble-Zurek prediction)
- Breakdown of Kibble-Zurek scaling for sufficiently fast quenches

Holographic Model



Building the holographic superconductor

Consider (probe approximation/large charge q expansion)

- Radial electric field on top of fixed AdS_4 Schwarzschild background
- Massive scalar charged under gauge field

Action

$$S = S_{\text{grav}} + \frac{1}{2\kappa^2}\int_{\mathcal{M}} \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\psi|^2 - m^2|\psi|^2\right],$$

where $F = \mathrm{d}A,\; D_\mu\cdot = (\nabla_\mu - iqA_\mu)\cdot,\; L = u_h = 2\kappa^2 = q = 1$ and

$$A = A_t\mathrm{d}t, \quad \psi = \psi_1 + i\psi_2, \quad \mathrm{d}s^2 = \frac{1}{u^2}\left[-f(u)\,\mathrm{d}t^2 - 2\,\mathrm{d}t\mathrm{d}u + \mathrm{d}x^2 + \mathrm{d}y^2\right],$$

with $f(u) = 1 - u^3 \;\Rightarrow T = |f'(1)|/(4\pi) = 3/(4\pi)$.

- Radial coordinate $u \in [0,1]$
- At boundary ($u = 0$): field theory coordinates (t, x, y)

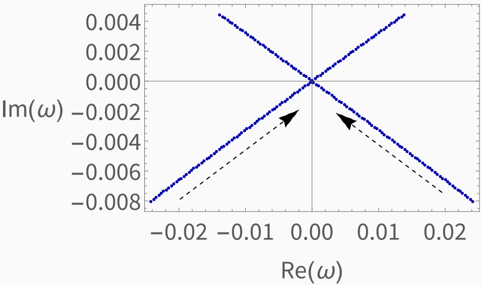
Static holographic superconductor

Equations of motion:

$$\nabla_m F^{mn} - iq(\psi^\star D^n\psi - \psi D^n\psi^\star) = 0\,,$$
$$D_m\left(D^m\psi\right) - m^2\psi = 0, \qquad \text{(and c.c.)}\,.$$

Simple solution: $A_t = \mu\,(1-u), \qquad \psi \equiv 0; \qquad$ (normal fluid)

Consider time dependent scalar fluctuation about this background:

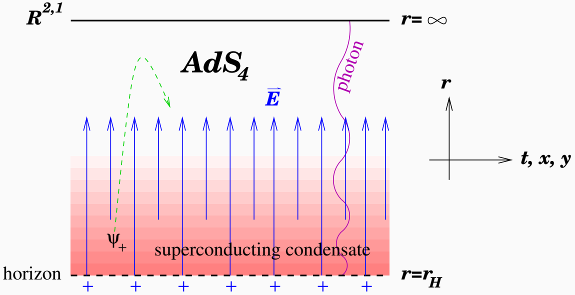


Shown: Quasi-normal modes of scalar fluctuations for $\mu \in [4, 4.1]$

$\Rightarrow$ positive imaginary part above some μ indicating instability.

Bulk picture

[Gubser; '08]: Electrically charged black hole: effective mass of scalar depends on radial direction: $m_{\text{eff}}^2 = m^2 + q^2 g^{tt} A_t^2$; may become sufficiently negative near horizon ⇒ unstable to forming scalar hair



[Gubser, Pufu; '08]: Superconducting condensate floats above horizon balanced by gravitational & electrostatic forces. Condensate carries finite fraction of total charge density → more electric flux above condensate than right at horizon.

Building the holographic superconductor

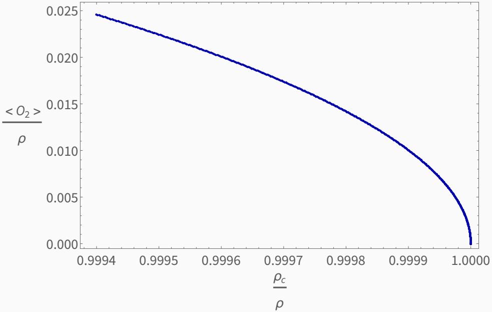
Choose $m^2 = -2 \geq m^2_{\text{BF}} = -\frac{d^2}{4}$. Asymptotic expansions (near $u = 0$)

$$\psi_i(t, \boldsymbol{x}, u) = \psi_i^{(l)}(t, \boldsymbol{x})u + \psi_i^{(s)}(t, \boldsymbol{x})u^2 + \dots, \qquad (i = 1, 2)$$

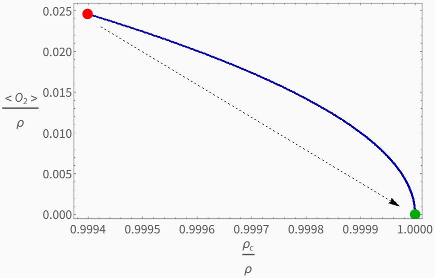
$$A_t(t, \boldsymbol{x}, u) = \mu(t, \boldsymbol{x}) + \rho(t, \boldsymbol{x})u + \dots$$

At equilibrium & $A_t(u = 1) = 0$: μ chemical potential; ρ charge density

Superfluid: set $\psi_i^{(l)}$ to zero $\Rightarrow$ spontaneous symmetry breaking;
Condensate encoded in $\psi_i^{(s)}$



Time dependent quenches



The holographic model

$$S = S_{\text{grav}} + \frac{1}{2\kappa^2}\int_{\mathcal{M}} \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\psi|^2 - m^2|\psi|^2\right].$$

The radial Maxwell equation reads (we don't consider spatial dependence)

$$\nabla_m F^{mu} - iq(\psi^\star D^u\psi - \psi D^u\psi^\star) = 0 \quad \Rightarrow \dot{\rho} = 0 \ \text{ for } u = 0, \psi_i^{(l)} = 0$$

Want: $\dot{\rho} \neq 0$.

One way to achieve this is by quenching the scalar source $\psi^{(l)}$ (see e.g. [Bhaseen, Gauntlett, Simons, Sonner, Wiseman; '12])

In the case of $\psi^{(l)} = 0$, we can quench the charge density with an external source in terms of the Null fluid action

$$S_{\text{nf}} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}A_\mu J^\mu_{\text{ext}} \Rightarrow 2\kappa^2 J^u_{(\text{nf})} = \dot{\rho}_{\text{ext}}\, u^2$$

S_{nf} is null-fluid action added by hand to obtain Vaidya-like solution.

The holographic model

$$S = S_{\text{grav}} + \frac{1}{2\kappa^2} \int_{\mathcal{M}} \mathrm{d}^4 x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D\psi|^2 - m^2 |\psi|^2 \right] + S_{\text{nf}} \,.$$

The radial Maxwell equation reads

$$\nabla_m F^{mu} - iq(\psi^\star D^u \psi - \psi D^u \psi^\star) = 2\kappa^2 \, J^u_{(\text{nf})} \Rightarrow \dot{\rho} = \dot{\rho}_{\text{ext}} \quad \text{for } u = 0, \psi_i^{(l)} = 0$$

Quench Profile

$$\rho(t) = \rho_{\text{initial}} + \frac{1}{2}(\rho_{\text{final}} - \rho_{\text{initial}})(1 + \tanh[\Omega(t - t_s)]);$$

where $\Omega = 10$ (rapidity), $t_s = 1.5$ (center).

Comments:

- Shortly after quench ($t_s \sim 2$) $\rho(t) =$const and thus $\dot{\rho} = 0$
- Intermediate and late time behavior independent of quench protocol
- Solve system of PDEs numerically
- Monitor that all equations and constraint are satisfied at all times

Quench Profile

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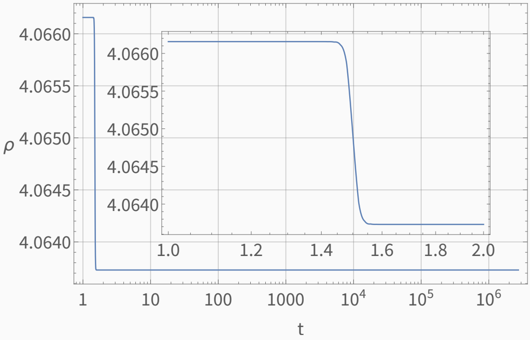


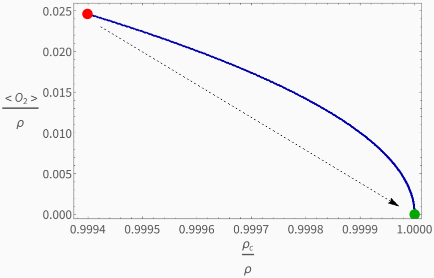
Numerical methods

Usually: Combine pseudo-spectral methods in spatial directions with 4th order Runge-Kutta or Adams-Bashforth (both explicit time-marching algorithms)
Problem: Our problem requires stable simulation over large time intervals $\Rightarrow$ implicit algorithm needed (for example Crank-Nicolson, not subject to CFL condition like explicit schemes)

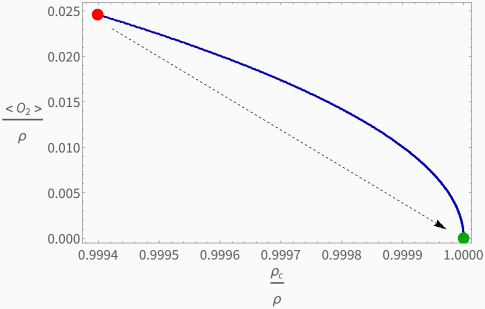
Here: (Pseudo)-spectral methods in space in time (highly implicit)

- Highly accurate (in space and time) and stable
- Divide time interval in multiple domains (size dynamically adjustable)
- Use Chebychev-Lobatto grid in radial direction, right-sided Radau grid in time direction (which does not include initial time slice).
- For more details see [Hennig, Ansorg; '08], [Flory, **SG**, Tejera-Morales; '22],[Ammon, **SG**, Jimenez-Alba, Macedo, Melgar; '16],[**SG**; '17]

Critical Quenches

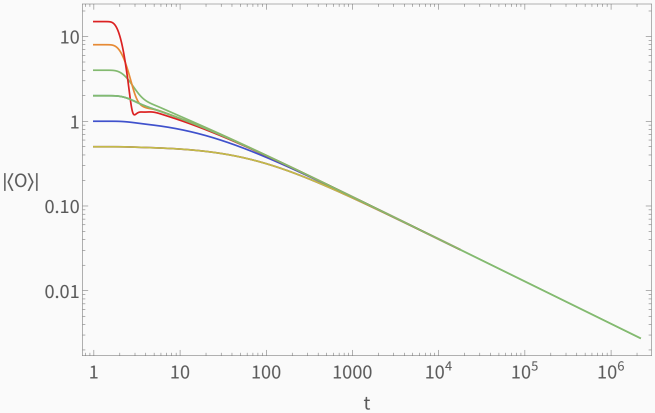


Critical Quenches



- For exactly critical quenches starting from the superfluid phase (red) onto the critical point (green): Quasi-normal mode which usually drives system to equilibrium (Higgs mode/Amplitude mode) becomes massless.
- No exponential decay!
- How does the system relax?
- Relaxation purely encoded in nonlinearity of the equations

Power law decay amplitude

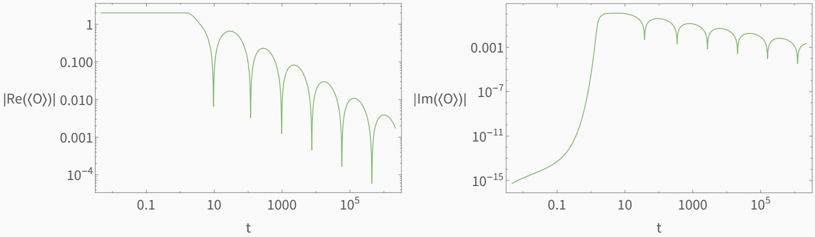


Observation

At late times, all quenches: power law decay with exponent $-\frac{1}{2}$ (instead of exp.)

$$|\langle \mathcal{O} \rangle| \sim \frac{4.07}{\sqrt{t}}$$

Log-periodic oscillations

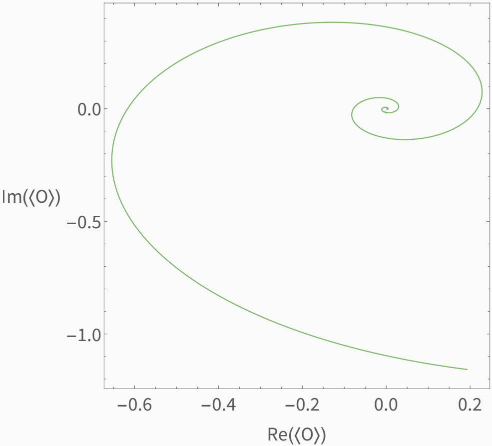


Observation

Phase falloff $\dot{\varphi} \sim 1/t$ means $\varphi(t) \sim \log(t)$, i.e. log-periodic oscillations $\Rightarrow$ discrete scale invariance?

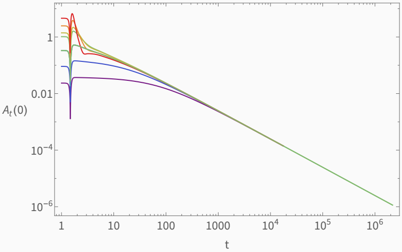
Power law and log periodic oscillations also observed in crit. quenches in Kondo model [Erdmenger, Flory, Newrzella, Strydom, Wu; '16]

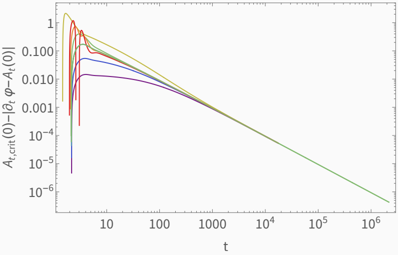
Dicrete scale invariance



Order parameter: power law decay with oscillations in logarithmic time $\Rightarrow$ characteristic for discrete scale invariance and **complex critical exponents** $\langle \mathcal{O} \rangle \sim t^{\alpha + i\beta}$ $\Rightarrow$ Self similar systems and fractal structure

Decay gauge field





Observation

At late times, all quenches: power law decay with exponent -1 (instead of exp.)

$$\dot{\varphi} - (A_t(0) - \rho_c) \sim \frac{0.93}{t}$$

Found both solutions as scaling solutions; analytical solution possible?

Boundary model

Postulate the phenomenological equation (for homogeneous case)

$$\left[\partial_t - iC_1\left(\mathcal{A}_t(t) - \rho + C_5|\Psi(t)|^2\right)\right]\Psi(t)$$
$$\equiv -(C_2 + iC_3)\left[|\Psi(t)|^2 - C_4(\rho - \rho_c)\right]\Psi(t),$$

where $\Psi = \phi e^{i\varphi}$, $\rho, \rho_c, \varphi, \mathcal{A}_t, C_i \in \mathbb{R}$, $\phi > 0$.

- Complex pre-factor $(C_2 + iC_3)$ dissipation, phase rotation
- Neglect spatial derivatives or higher orders of Ψ and $(\rho - \rho_c)$
- Eq for ϕ decouples $\Rightarrow$ analytical solution
- Assume that $\rho(t) \equiv \rho$ (constant in time)
- Set charge of complex scalar under gauge field to unity: $C_1 = 1$.

Validity: Since we neglect higher order of Ψ and $(\rho - \rho_c)$, the equation should describe the dynamics near the critical points, i.e. small enough condensates and small enough deviations from the critical charge density.

Determining parameters

Claim:

- Phenomenological model describes full non-linear dynamics after quench (where $\rho =$const) if initial and final state are sufficiently close to critical point
- All parameters are fixed within linear response theory and from the properties of the equilibrium states

Advantage: Equilibrium solutions and linear response properties are (computationally) much easier to obtain than full real time dynamics.

Goal: Determine the parameters and cross check with real time evolution for sub-, super- and critical quenches.

Determining parameters C_4, C_5

$$\left[\partial_t - iC_1\left(\mathcal{A}_t(t) - \rho + C_5|\Psi(t)|^2\right)\right]\Psi(t)$$
$$\equiv -(C_2 + iC_3)\left[|\Psi(t)|^2 - C_4(\rho - \rho_c)\right]\Psi(t),$$

(Non-trivial) static solutions: (Recall $\Psi = \phi e^{i\varphi}$)

$$\phi = \sqrt{C_4(\rho - \rho_c)}, \qquad \mathcal{A}_t = \rho - C_5\phi^2 = \rho - C_4 C_5(\rho - \rho_c)$$

Within Holography this corresponds to constructing the phase diagram (static case!) near the critical point choosing the condensate to be real and fitting

$$\langle \mathcal{O} \rangle = \sqrt{C_4(\rho - \rho_c)},\ \mu = \rho - C_5\langle \mathcal{O} \rangle^2$$
$$\Rightarrow C_4 \approx 4.09192 \quad C_5 \approx 0.14967$$

Note: As you might have noticed the conformal dimensions do not match in those expressions. We assume all physical quantities to be normalized to appropriate powers of $\bar{T} = 4\pi T/3$ (Recall $T = 3/(4\pi)$).

Determining parameter C_2

$$\left[\partial_t - iC_1\left(\mathcal{A}_t(t) - \rho + C_5|\Psi(t)|^2\right)\right]\Psi(t)$$
$$\equiv -(C_2 + iC_3)\left[|\Psi(t)|^2 - C_4(\rho - \rho_c)\right]\Psi(t),$$

Solve equation analytically (for $\rho > \rho_c$ or $\rho < \rho_c$). Late time expansion for quenches ending near critical point for final states in the superfluid phase yields (exponential decay to eq. $\rightarrow$ linear response)

$$\phi(t) - \sqrt{C_4(\rho - \rho_c)} \propto e^{-2C_2C_4(\rho-\rho_c)t} + \ldots$$

or in the normal phase

$$\phi(t) \propto e^{C_2C_4(\rho-\rho_c)t} + \ldots .$$

Within Holography: lowest QNM(s) at zero wavevector $\sim \delta f\, e^{-i\omega t}$

- Superfluid phase: $\omega_{\text{Ampl}} = -0.2469\, i\,(\rho - \rho_c)$
- Normal phase: $\omega_{\pm} = -(\pm 0.38087 - 0.12348 i)\,(\rho - \rho_c)$, ω_+ corresponds to Ψ and ω_- to $\bar{\Psi}$

$$\boxed{\Rightarrow C_2 \approx 0.03018}$$

Determining parameter C_3 in normal phase

Late time behavior for final states in the normal phase

$$\dot{\varphi}(t) = C_3 C_4 (\rho - \rho_c) + \ldots$$

Within Holography, we find from the same QNM as on previous slide:

$$\omega_\pm = -(\pm 0.38087 - 0.12348i)\,(\rho - \rho_c),\ (\omega_+ \text{ from } \Psi,\ \omega_- \text{ from } \bar{\Psi})$$

$$\boxed{\Rightarrow C_3 \approx 0.09308}$$

Determining parameter C_3 in superfluid phase

Late time behavior for final states in the superfluid phase

$$\phi(t) = \sqrt{C_4}\sqrt{\rho - \rho_c} + \frac{\sqrt{C_4}}{2}\sqrt{\rho - \rho_c}\left(1 - \frac{C_4(\rho - \rho_c)}{\phi_0^2}\right) e^{-2C_2 C_4 t(\rho - \rho_c)} + \dots$$

$$\begin{aligned}\dot{\varphi}(t) - C_1\mathcal{A}_t(t) =& - C_1\rho + C_1 C_4 C_5(\rho - \rho_c) \\ & - \frac{C_4(C_1 C_5 - C_3)}{\phi_0^2}\left(C_4(\rho - \rho_c) - \phi_0^2\right)(\rho - \rho_c) e^{-2C_2 C_4 t(\rho - \rho_c)} + \dots\end{aligned}$$

Comparing the prefactors of the exponential decay yields

$$\Rightarrow \frac{\text{Amplitude}_\phi}{\text{Amplitude}_{\dot{\varphi}(t) - C_1\mathcal{A}_t(t)}} = \frac{1}{2\sqrt{C_4(\rho - \rho_c)}}\frac{1}{C_1 C_5 - C_3}$$

Determining parameter C_3 in superfluid phase

In Holography, QNMs ω are eigenvalues of generalized eigenvalue problem $(\boldsymbol{A}\omega - \boldsymbol{B})\,\boldsymbol{x} = 0$, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are differential operators of a non-hermitian Sturm-Liouville problem. Eigenvector $\boldsymbol{x}$ corresponding to the eigenvalue ω carries information about how much each field theory operator contributes to QNM excitation. Framework recently developed in [Arean, Baggioli, **SG**, Landsteiner; '21]

In this framework, compute contributions of scalar and gauge field fluctuations to amplitude mode and take ratio of the expectation values:

$$\frac{\text{Amplitude}_{|\langle\delta\mathcal{O}\rangle|}}{\text{Amplitude}_{\langle\delta\dot{\varphi}\rangle-\langle\delta a_t\rangle}} = \frac{17.67}{2\sqrt{C_4(\rho-\rho_c)}} \overset{!}{=} \frac{1}{2\sqrt{C_4(\rho-\rho_c)}}\frac{1}{C_1 C_5 - C_3}$$

$$\boxed{\Rightarrow C_3 \approx 0.09308}$$

Checks

$$\left[\partial_t - iC_1\left(\mathcal{A}_t(t) - \rho + C_5|\Psi(t)|^2\right)\right]\Psi(t)$$
$$\equiv -(C_2 + iC_3)\left[|\Psi(t)|^2 - C_4(\rho - \rho_c)\right]\Psi(t),$$

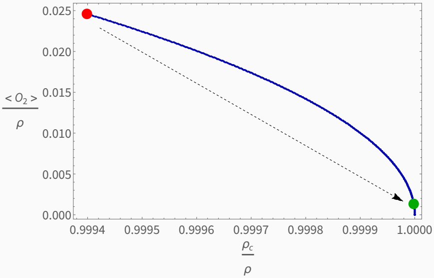
Set $\rho = \rho_c$ for exactly critical quenches and we find

$$\phi(t) = \frac{1}{\sqrt{2C_2 t + \frac{1}{\phi_0^2}}} \approx \frac{4.07}{t^{1/2}} + \dots,$$
$$\dot{\varphi} - C_1(\mathcal{A}_t - \rho_c) = \frac{C_1 C_5 + C_3}{2C_2 t + \frac{1}{\phi_0^2}} \approx \frac{0.94}{t} + \dots\ .$$

Recall for exactly critical quenches, we observed (by fitting the late time behavior, small deviation since initial condensate ~ 1)

$$|\langle \mathcal{O} \rangle| \sim \frac{4.07}{\sqrt{t}}, \qquad \dot{\varphi} - (A_t(0) - \rho_c) \sim \frac{0.93}{t}$$

Sub-critical Quenches



Sub-critical Quenches

Analytical solution to phenomenological equation analytically: Real part yields

$$\phi(t) = \sqrt{\frac{C_4(\rho-\rho_c)}{1-\left(1-\frac{C_4(\rho-\rho_c)}{\phi_0^2}\right)e^{-2C_2C_4t(\rho-\rho_c)}}}$$

with $\phi(0)=\phi_0$. Imaginary part then yields expression for $\dot{\varphi}-C_1\mathcal{A}_t$.

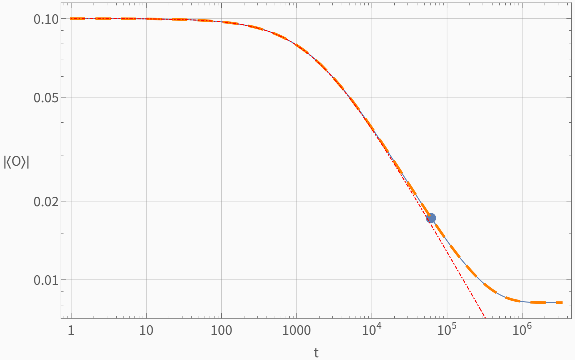
$$\dot{\varphi}-C_1\mathcal{A}_t = -C_1\rho - C_3C_4(\rho-\rho_c) + \frac{C_4(C_3+C_1C_5)(\rho-\rho_c)}{1-e^{-2C_2C_4t(\rho-\rho_c)}\left(1-\frac{C_4(\rho-\rho_c)}{\phi_0^2}\right)}.$$

System will initially react like critical quench (power law decay), only after "handover-timescale" which we define as

$$t_{\mathrm{ho}} \sim \frac{1}{|\rho-\rho_c|}$$

the system relaxes exponentially.

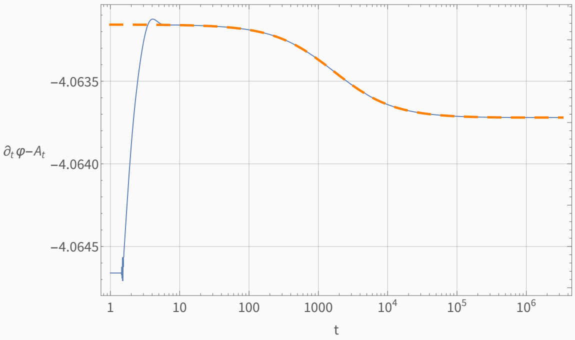
Relaxation after sub-critical quench



Legend

Dashed orange line: solution to phenomenological equation; blue line: numerical non-linear time evolution; blue dot: hand-over time; red dashed line: solution for critical quench

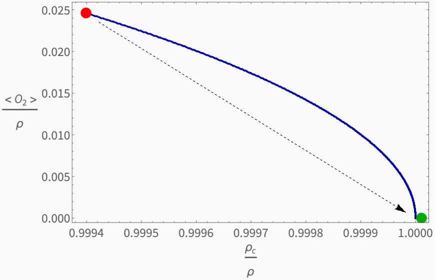
Relaxation after sub-critical quench



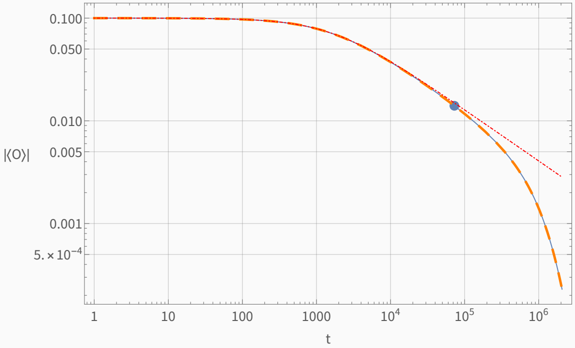
Legend

Dashed orange line: solution to phenomenological equation; blue line: numerical non-linear time evolution;

Super-critical Quenches



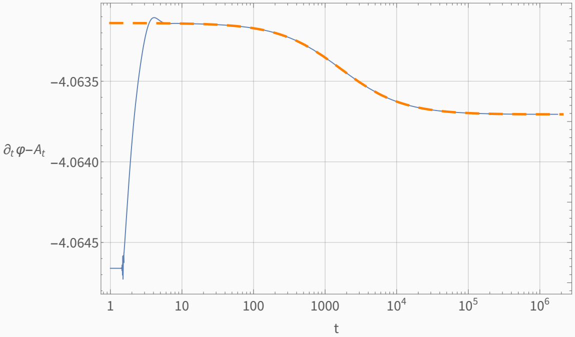
Relaxation after super-critical quench



Legend

Dashed orange line: solution to phenomenological equation; blue line: numerical non-linear time evolution; blue dot: hand-over time; red dashed line: solution for critical quench

Relaxation after super-critical quench



Legend

Dashed orange line: solution to phenomenological equation; blue line: numerical non-linear time evolution;

Conclusions and Outlook

Conclusions

- Studied relaxation of critical and near-critical quenches exhibiting power-law decay and discrete scale invariance
- Established and successfully tested phenomenological model capturing the non-linear out-of-equilibrium dynamics
  - All parameters of the model may be determined within linear response and static equilibrium
  - Non-trivial prediction of the full non-linear dynamics
  - Nice test and application of the framework to compute the amplitudes of Quasi-normal modes, recently developed in [Arean, Baggioli, **SG**, Landsteiner; '21]

Conclusions and Outlook

Outlook

- Extension to include spatial dependence
- Similar study in the case of broken spacetime dimensions or other spontaneously broken symmetries in general?
- Origin of discrete scale invariance in horizon dynamics?
- Full solution from scaling solutions?
- Amplitude/Higgs mode near phase transition recently investigated in linear regime from holography in [Donos, Pantelidou; '22]. Connections?
- Different types of phase transitions?
- Beyond mean-field?

Thank you for your attention!!