Holography for QFTs in de Sitter

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Outline

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Introduction

- ➢ The physics of quantum fields in de Sitter is important:
- ➢ Observations suggest that the cosmological constant in our Universe is positive.
- ➠ Our Universe is asymptotically de Sitter.

- ➢ We believe that the very Early Universe underwent a period of exponential expansion, the inflationary period, where the description was also quasi-de Sitter.
- ➠ In slow-roll inflation, many of the cosmological observables are well-approximated by QFT in a fixed dS background.

QFT in de Sitter

- Weakly coupled QFT in a fixed de Sitter background has been studied through the years.
- It is well-known that light fields, $m \ll H$ exhibit infrared divergences at loop order. [Starobinski (1984) ...]
- The meaning and implications of these IR divergences are still debated [Starobinski, Yokohama, Ford, Antoniadies, Iliopoulos, Tomaras, Tsamis, Woodard, Weinberg, Burgess, Marolf, Morisson, Zaldariaga, Senatore, Sundrum, Polyakov ....].
- In this work we aim to use holography to discuss strongly coupled QFTs in a fixed de Sitter background.

References

- This is talk in based on work with José Manuel Penín and Ben Withers

  Massive holographic QFTs in de Sitter, SciPost Phys. 12, 182 (2022) and on-going work

- Earlier relevant work includes

  A. Buchel, Ringing in de Sitter spacetime, Nucl. Phys. B 928, 307 (2018)

Holographic cosmology

- This work is conceptually distinct from dS/CFT and holographic cosmology. [Strominger (2001)], [Maldacena (2002) ... [McFadden, KS (2009)] ....
- In dS/CFT one seeks to describe a dS$_{d+1}$ Universe with dynamical gravity via d-dimensional CFT with no gravity.
- Here we want to describe a d dimensional strongly couple QFT on fixed de Sitter background using AdS gravity in $d+1$ dimensions.

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Conformal boundary

- There are very common misconceptions about the conformal boundary of AdS.
- Many assume that if you write the metric as

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\left(g_{(0)ij}(x) + O(r)\right) dx^i dx^j$$

then the boundary is at $z = 0$ and the boundary metric is $g_{(0)ij}(x)$.
- In general, this is not correct.

- If the r=constant slices are non-compact then part of the conformal boundary is located at each value of r.

Boundary conformal boundary

- What is correct is that if the metric takes the form

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\left(g_{(0)ij}(x) + O(r)\right) dx^i dx^j$$

AND the the r=constant slices are compact then

the boundary is at $r = 0$ and $g_{(0)ij}(x)$ is a representative of the boundary conformal structure.

- The conformal boundary does not depend on which coordinates we are using.

AdS and its conformal structure

The metric in global coordinates is given by

$$ds^2 = \frac{1}{\sin^2 \bar{r}} \left(-dT^2 + d\bar{r}^2 + \cos^2 \bar{r} d\Omega_{d-1}^2 \right)$$

where $0 < \bar{r} \leq \pi/2$.

- The $\bar{r}$=constant slices are compact.

  (What we usually call AdS is the universal cover of AdS. The time variable in AdS is compact $-\pi < T < \pi$.)
- The conformal boundary of AdS_{d+1} is at $\bar{r} = 0$ and the boundary is the Einstein Universe $R \times S^{d-1}$.
- The bulk metric divergences there: there is a second order pole. So there is no well-defined boundary metric.
- There is however a well-defined conformal structure, i.e. a metric up to a Weyl transformation.

The boundary conformal structure

- To obtain a boundary metric we use a *defining function*, *i.e.* a function $\omega(x)$ which is positive in the interior but has a single zero at the boundary. We then define

$$g_{(0)} = \lim_{\bar{r} \to 0} \omega^2 g$$

  This limit exits because the second order pole in g is canceled by the zero of ω^2.

- However, any other $\omega'(x) = \omega(x) e^{\sigma(x)}$ is as good, so what is well-defined here is the conformal class

$$g_{(0)} \sim e^{2\sigma(x)} g_{(0)}$$

- For AdS we may pick $\omega = \sin \bar{r}$, and this leads to the representative:

$$ds_0^2 = -dt^2 + d\Omega_{d-1}^2$$

  This metric is conformally flat and any other conformally flat metric is as good.

Different representatives of conformal structure

- Modulo issues that are associated with the holographic conformal anomaly, any representative is as good.
- One can change representative by doing a bulk diffeomorphism.
- A conformally flat conformal structure can represented by
  - Minkowski metric: Poincaré coordinates
  - AdS metric: AdS slicing of AdS
  - dS metric: dS slicing of AdS
  - FRW metric: FRW slicing of AdS [Giatagianas, Tetradis]
- This does not change the boundary of AdS, which is always the Einstein Universe $R \times S^{d-1}$.
- ➠ Different representatives describe the same boundary in different ways.

- A CFT is invariant under Weyl transformations (module conformal anomalies), so in AdS/CFT it does not matter which representative one is using.

dS_3 slicing of AdS_4

- The dS-slicing of AdS is given by

$$ds^2 = dz^2 + e^{-2z}\left(1 - \frac{H^2}{4}e^{2z}\right)^2 ds^2_{dS_3}$$

where

$$ds^2_{dS_3} = -dt^2 + e^{2Ht}d\vec{y}^2 = \frac{-d\eta^2 + d\vec{y}^2}{H^2\eta^2}$$

where $-\infty < \eta < 0$.

Map to Poincaré and global coordinates

- The coordinate transformation

$$z = \log\left(-\frac{r}{H\tau}\frac{2\tau^2 - 2\sqrt{\tau^4 - r^2\tau^2}}{r^2}\right), \qquad \eta = \tau\frac{\tau^2 - r^2 - \sqrt{\tau^4 - r^2\tau^2}}{\tau^2 - \sqrt{\tau^4 - r^2\tau^2}}$$

maps the metric to Poincaré coordinates

$$ds^2 = \frac{1}{r^2}(dr^2 - d\tau^2 + d\vec{y}^2)$$

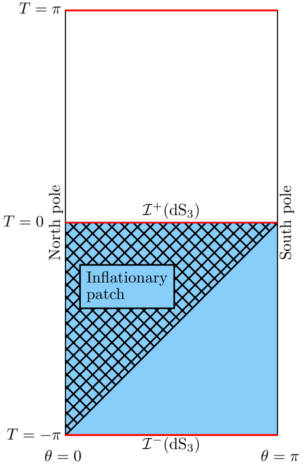
- and the further transformation

$$r = \frac{\sin\bar{r}}{\cos T + \cos\theta\cos\bar{r}}, \; \tau = \frac{\sin T}{\cos T + \cos\theta\cos\bar{r}}, \; R = \frac{\sin\theta\cos\bar{r}}{\cos T + \cos\theta\cos\bar{r}}$$

where $d\vec{y}^2 = dR^2 + R^2 d\Phi^2$, maps to global coordinates

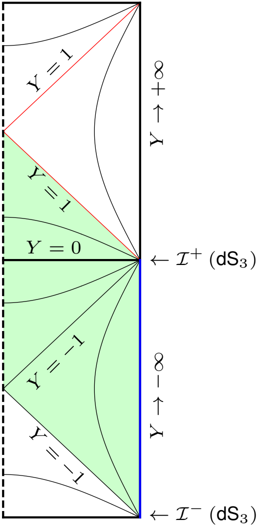
$$ds^2 = \frac{1}{\sin^2\bar{r}}\left(-dT^2 + d\bar{r}^2 + \cos^2\bar{r}\, d\Omega_2^2\right)$$

Boundary and global issues



- Boundary $R \times S^2$ is at $\bar{r} = 0$. Azimuthal angle is suppressed.
- dS_3 is conformal to a portion of $R \times S^2$
- The spacelike conformal boundaries of dS are shown in red.
- As $Y \equiv \frac{\sin T}{\sin \bar{r}} \to -\infty$ we get the blue square region.
- As $Y \to \infty$ we get the white square region.

Penrose diagram



- Each point is an S^2 which shrinks to zero size at the origin of coordinates indicated by the dashed line.
- Lines are level sets of $Y (= \sin T / \sin \bar{r})$
- Blue line corresponds to the blue square area of the boundary.
- The green shaded region shows the development of data prescribed in the blue dS_3 region at the boundary.

Outline

From CFT to QFT

- A CFT is Weyl invariant, so it is the same as in all conformally related spacetimes
- We would like to deform the CFT by a mass term:

$$S = S_{\text{CFT}} + \int d^d x \sqrt{-\det g}\, m\, O(x).$$

- Since m breaks conformal symmetry there is no longer a relation to vacuum QFT on Minkowski spacetime under a Weyl transformation.
- Instead a massive theory in dS is equivalent to QFT on Minkowski spacetime in the presence of a spacelike defect: The Weyl tranformation to Minkowski spacetime yields

$$S = S_{\text{CFT}} + \int d^d x \frac{m}{-H\eta} O(x).$$

⇒ The future conformal boundary of dS_3 is described by a singular spacelike source function in $\mathbb{R}^{1,2}$.

Holographic implementation

- It is well-known how to deform a CFT holographically from the studies of holographic RG flows in the early days of AdS/CFT

  [Boonstra, KS, Townsend (1998)] [Girardello etal (1998)][Freedman etal (1999)] [KS, Townsend (1999)]....
- We need to turn on the scalar ϕ that is dual to O
- Look for dS-sliced asymptotically AdS domain-wall solutions

$$ds^2 = dz^2 - P(z) ds^2_{dS_3}, \qquad \phi = \phi(z)$$

- As $z \to \infty$
  - the metric should approach that of AdS is AdS-sliced coordinates
  - the scalar should behave as a sources, $\phi \to e^{(d-\Delta)z} m$

The model

- Following [Buchel (2017)], we consider a free massive field in AdS:

$$S = \frac{1}{2\kappa^2}\int_{\mathcal{M}_4}\sqrt{-G}d^4x\left(R+6-\frac{1}{2}(\partial\phi)^2+\phi^2\right),$$

- The field ϕ is dual to a dimension $\Delta = 2$ operator.
- One can solve the field equations perturbatively in m.

$$P = -e^{-2z}\left(1-\frac{H^2}{4}e^{2z}\right)^2 - \frac{(-144+112He^z-32H^2e^{2z}+4H^3e^{3z}+H^4e^{4z})}{1152\left(1+\frac{H}{2}e^z\right)^2}m^2 + O(m^4)$$

$$\bar{\phi} = \frac{e^z}{\left(1+\frac{H}{2}e^z\right)^2}m - \frac{e^{2z}(40+12He^z+14H^2e^{2z}+H^3e^{3z})}{576H\left(1+\frac{H}{2}e^z\right)^6}m^3 + O(m^5).$$

This solution was first obtained (in different coordinates) in [Buchel (2017)].

Global solution

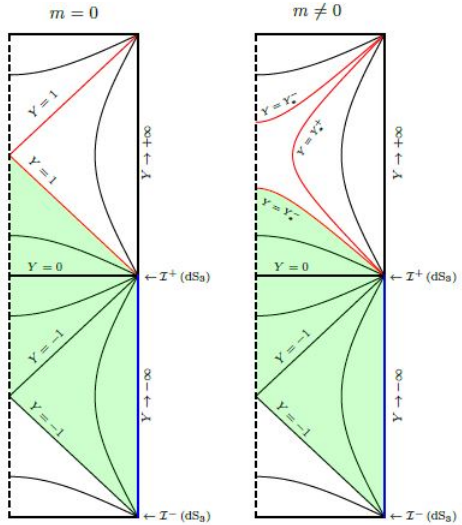
- One may transform to global coordinates

$$
\begin{aligned}
ds^2 &= \Omega(Y)^2 \frac{1}{\sin^2 \bar{r}} \left(-dT^2 + d\bar{r}^2 + \cos^2 \bar{r} d\Omega_2^2\right), \\
\bar{\phi} &= F(Y)
\end{aligned}
$$

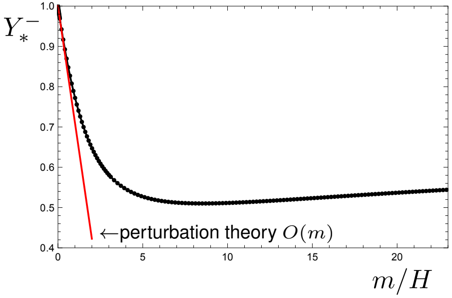
with

$$
\begin{aligned}
\Omega^2(Y) &= 1 - \frac{1}{12(Y-1)^2} \frac{m^2}{H^2} - \frac{5}{432(Y-1)^3} \frac{m^4}{H^4} + O(m)^6, \\
F(Y) &= \frac{1}{1-Y} \frac{m}{H} + \frac{3-5Y}{72(Y-1)^3} \frac{m^3}{H^3} + \frac{-175 + 619Y - 645Y^2 + 129Y^3}{51840(Y-1)^5} \frac{m^5}{H^5} + O(m)^7,
\end{aligned}
$$

Penrose diagram



Location of singularity at finite m



- The null $Y = 1$ singularity splits into a spacelike and timelike singularity for finite m.
- Perturbatively in m:

$$Y_*^{\pm} = 1 \pm \frac{1}{2\sqrt{3}} \frac{m}{H} + O(m)^2$$

- At finite m, we obtained the solution using the shooting method (source m at $Y = -\infty$, regular at $Y = -1$)

One-point functions

- Correlators can be extracted as usual using holographic renormalization.
- One-point functions take the form dictated by dS-invariance and Ward identites:

$$\langle O \rangle_0 = \frac{H^2}{2\kappa^2} \mathcal{F}\left(\frac{m}{H}\right),$$
$$\langle T_{\mu\nu} \rangle_0 = -\frac{H^3}{2\kappa^2} \frac{m}{3H} \mathcal{F}\left(\frac{m}{H}\right) g^{dS}_{\mu\nu},$$

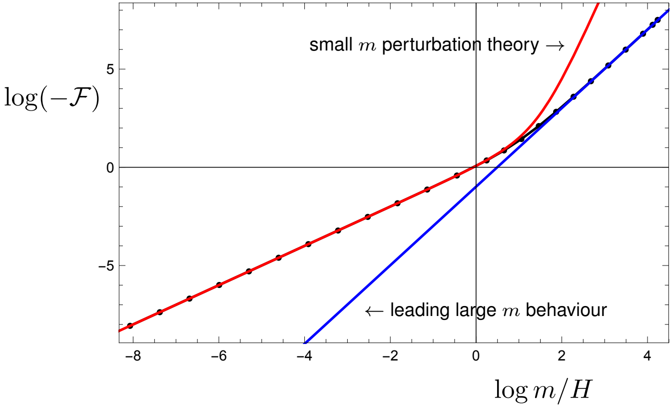
- For small m/H:

$$\mathcal{F} = -\frac{m}{H} - \frac{5}{72}\frac{m^3}{H^3} + \frac{43}{17280}\frac{m^5}{H^5} + O(m)^7$$

- As $m/H \to \infty$:

$$\mathcal{F} = \mathcal{F}_{\text{asy}} \frac{m^2}{H^2} \qquad \mathcal{F}_{\text{asy}} \simeq -0.37$$

Non-perturbative evaluation of $\mathcal{F}$



2-point functions

- These are computed using the methodology developed for holographic RG flows [Bianchi, Freedman, KS (2001)] ....
- We need to solve linearised equations around the background:

$$G_{ab} = G^{DW}_{ab}(z) + H_{ab}(z,x), \quad \phi = \bar{\phi}(z) + H_\phi(z,x)$$

- Decomposition

$$\begin{aligned} H_{zz} &= X \\ H_{z\mu} &= P(z)(\partial_\mu V + V_\mu) \\ H_{\mu\nu} &= P(z)(-2\psi g^{dS}_{\mu\nu} + 2\nabla^{dS}_{(\mu}\partial_{\nu)}\chi + 2\nabla^{dS}_{(\mu}\omega_{\nu)} + \gamma_{\mu\nu}) \\ H_\phi &= S \end{aligned}$$

$\gamma_{\mu\nu}$ is TT and ω_μ, V_μ are divergence-less w.r.t. $g^{dS}_{\mu\nu}$

Using dS isometries

- Gauge redundancy

$$H_{ab} \to H_{ab} + 2\nabla_{(a}\xi_{b)}, \quad H_\phi \to H_\phi + \xi^a \partial_a \bar{\phi}$$

- Take $X = V = V_\mu = 0$ we go to the FG gauge. Leftover redundancy solved with gauge invariant variables.
- We further use the dS isometries to decompose as:

$$\partial_j \Phi = i k_j \Phi, \quad \Box_{dS_3} \Phi = \lambda \Phi$$

$$\Rightarrow \Phi = \Phi_{k,\lambda}(z) \eta J_\nu(k\eta) e^{ik_i y^i}, \quad \lambda = H^2(1-\nu^2)$$

where we work with conformal time:

$$ds^2_{dS_3} = \frac{-d\eta^2 + d\vec{y}^2}{H^2 \eta^2}$$

- So the dynamical equation to be solved is the radial equation involving $\Phi_{k,\lambda}(z)$.

Tensors

- Decomposition:

$$\gamma_{0i} = -h_i J_\nu(k\eta) e^{iky} \gamma(z)$$
$$\gamma_{ij} = \frac{1}{k^2\eta^2}(\eta\partial_\eta - 2)(\eta J_{k\nu}(k\eta))\partial_{(i} e^{iky} h_{j)} \gamma(z)$$

where h_i is a constant polarization vector satisfying: $h_i k^i = 0$.
- Equation: $\gamma'' + \frac{3}{2}\frac{P'}{P}\gamma' - \frac{\lambda}{P}\gamma = 0$, is solved order by order in m.
- 2-point function:

$$\langle T_{\mu\nu}(\nu_1, k_1) T_{\rho\sigma}(\nu_2, k_2)\rangle = \Pi_{\mu\nu\rho\sigma} \mathcal{A}(\nu_1, k_1)$$

where $\Pi_{\mu\nu\rho\sigma}$ is TT projector and

$$\mathcal{A}(\nu, k) = \frac{H^3}{2\kappa^2}\left[\nu(\nu^2 - 1) + \frac{3\nu^2 + 8\nu - 19}{24(\nu - 2)}\frac{m^2}{H^2} + \left(\frac{35}{864} - \frac{23}{1536(\nu - 2)} + \frac{3}{256(\nu - 2)^2}\right)\frac{m^4}{H^4} + O(m)^6\right]$$

- $\mathcal{A}(\nu_1, k_1)$ contains a polynomial in ν and poles in $(\nu - 2)$.

Resummation

- Resummation yields single poles corresponding to normalisable modes:

$$\nu_n^t = n + O(m^2), \qquad n = 2, 3, 4 \ldots$$

where we computed the corrections though m^6. For example,

$$\nu_2^t = 2 + \frac{1}{32}\frac{m^2}{H^2} - \frac{103}{36684}\frac{m^4}{H^4} + \frac{50929}{212336640}\frac{m^6}{H^6} + O(m)^8,$$

- The resummation reads:

$$\mathcal{A}(\nu, k) = \frac{3H^3}{2\kappa^2}\left(\frac{\nu}{3}(\nu^2 - 1) + \frac{\nu}{24}\frac{m^2}{H^2} + \sum_{j=2}^{\infty}\frac{r_j^t}{\nu - \nu_j^t}\right) - \frac{7}{12}m\langle O\rangle_0$$

with residues

$$r_2^t = -\frac{m^2}{8H^2}\left(1 - \frac{23}{576}\frac{m^2}{H^2} - \frac{14477}{6635520}\frac{m^4}{H^4} + \frac{66506857}{1337720832000}\frac{m^6}{H^6} + O(m)^8\right),$$

$$r_3^t = \ldots$$

$$\ldots$$

Scalars

- Gauge invariant variables:

$$\zeta = -\psi + \frac{P'}{2P}\frac{S}{\bar{\phi}'}, \quad \hat{\phi} = -\left(\frac{S}{\bar{\phi}'}\right)', \qquad \hat{\nu} = \chi' + \frac{S}{P\bar{\phi}'}$$

- The Hamiltonian and momentum constraint equations give:

$$\hat{\phi} = \frac{2H^2 P}{P'}\hat{\nu} - \frac{2P}{P'}\zeta', \qquad \hat{\nu} = -\frac{2(3H^2+\lambda)P'}{Q_\lambda}\zeta + \frac{Q_{-3H^2}}{H^2 Q_\lambda}\zeta'$$

- Dynamical equation

$$\hat{\phi}'' + \Big(-\frac{4\bar{\phi}}{\bar{\phi}'} + \frac{2H^2}{P'} - \frac{2P}{P'} - \frac{\bar{\phi}^2 P}{3P'} - \frac{P\bar{\phi}'^2}{6P'}\Big)\hat{\phi}' +$$
$$\Big(-10 - \bar{\phi}^2 - \frac{8\bar{\phi}^2}{\bar{\phi}'^2} + \frac{40H^2\bar{\phi}}{\bar{\phi}'P'} - \frac{40\bar{\phi}P}{\bar{\phi}'P'} - \frac{20\bar{\phi}^3 P}{3\bar{\phi}'P'} - \frac{10\bar{\phi}P\bar{\phi}'}{3P'} - \frac{\lambda}{P}\Big)\hat{\phi} = 0$$

which is solved pertrubatively in m.

Scalar 2-point function

- 2-point function:

$$\langle O_{\nu_1}(k_1) O_{\nu_2}(k_2)\rangle = a(\nu_1, k_1)\delta_{\nu_1,\nu_2}\delta^{(2)}(k_1 + k_2)$$

- After resummation only single poles at the location of normalizable modes:

$$\langle O_\nu(k) O_\nu(-k)\rangle = H\left(\nu + \frac{r_1^s}{\nu - \nu_1^s} + \sum_{\pm} \frac{r_{2,\pm}^s}{\nu - \nu_{2,\pm}^s} + \sum_{j=3}^{\infty} \frac{r_j^s}{\nu - \nu_j^s}\right)$$

where the normalisables modes are [Buchel (2017)]

$$\nu_n^s = n + O(m^2), \qquad n = 1, 2, 3, 4 \dots$$

again computed through order m^6. E.g.

$$\nu_1^s = 1 + \frac{1}{12}\frac{m^2}{H^2} - \frac{1}{54}\frac{m^4}{H^4} + \frac{1591}{622080}\frac{m^6}{H^6} + O(m)^8,$$

and residues:

$$r_1^s = \frac{m^2}{6H^2}\left(1 - \frac{1}{4}\frac{m^2}{H^2} + \frac{109}{4536}\frac{m^4}{H^4} + \frac{109672267}{100590033600}\frac{m^6}{H^6} + O(m)^8\right), \; r_2^s = \dots$$

A simple representation of conformal correlators

- When $m^2 = 0$ the 2-point should reduce to a CFT correlator:

$$\langle O_{\nu_1}(k_1) O_{\nu_2}(k_2) \rangle \sim \nu_1 \delta_{\nu_1,\nu_2} \delta^{(2)}(k_1 + k_2)$$

  where ν_1 is the index of the Bessel function.
- This is a surprising simple representation of the CFT correlator
  - No explicit momentum dependence, apart from the momentum conserving delta function
- Recall that in momentum space (for $\Delta = 2, d = 3$):

$$\langle O(\omega_1, k_1) O(\omega_2, k_2) \rangle \sim \sqrt{|k_1^2 - \omega_1^2|} \delta(\omega_1 + \omega_2) \delta^{(2)}(k_1 + k_2)$$

- The fact that the two agree follows from expanding $e^{i\omega t}$ in terms of Bessel functions.
- Similar results hold for any Δ and d.

Outline

Model for dual QFT

- We deformed the CFT with an operator O of dimension 2.
- In $d = 3$ a free massless fermion ψ is a CFT and has an operator of dimension 2, namely a mass term $O = \bar{\psi}\psi$
- Thus a free massive fermion in dS has some of the features of the dual QFT.

Conformal perturbation theory

- We can use conformal perturbation theory in Minkowski with a singular source for O and then Weyl transform to de Sitter.
- In the free-fermion CFT:

$$\begin{aligned}\langle O(x_1)\rangle_0 &= 0 \\ \langle O(x_1)O(x_2)\rangle_0 &= \frac{1}{8\pi^2}\frac{1}{|x_{12}|^4} \\ \langle O(x_1)O(x_2)O(x_3)\rangle_0 &= 0 \\ \langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_0 &= \mathrm{non-zero}\end{aligned}$$

where the subscript 0 indicates that the computation was done in the massless theory.

1-point function

- Computing in MInkowski

$$\langle O(x_1)\rangle = 0 - \int d^3x_2 m(x_2)\langle O(x_1)O(x_2)\rangle_0 + \mathcal{O}(m^2) = \frac{m}{4H\tau_1^2} + \mathcal{O}(m^2)$$

and transforming to de Sitter

$$\langle O\rangle_{dS_3} = -H^2\frac{1}{4}\frac{m}{H} + \mathcal{O}(m^2)$$

which matches the holographic result, up to a constant.

- Note that in $\lambda\phi^4$ theory in dS$_4$ [Bunch, Davies (1978)]:

$$\langle\phi^2\rangle_{dS_4} \sim \frac{H^4}{m^2}$$

2-point function

- Two-point functions up to $\mathcal{O}(m^2)$

$$\langle O(x_1)O(x_2)\rangle = \langle O(x_1)O(x_2)\rangle_0 - \int d^dx_3 m(x_3)\langle O(x_1)O(x_2)O(x_3)\rangle_0 + \frac{1}{2}\int d^dx_3 d^dx_4 m(x_3)m(x_4)\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_0$$

which yields

$$\langle O(x_1)O(x_2)\rangle = \langle O(x_1)O(x_2)\rangle_0 + \mathcal{O}(m^2)$$

which is also in agreement with the holographic result: no order m contribution.

Outline

Conlusions

- We studied strong coupled QFTs in dS_3 via holography.
- We found no signs of IR instabilities. Perhaps this is unsurprising given that the QFT was a deformation of a CFT.
- 2-point functions are expressed in a spectral representation as a sum over simple poles.
- The poles correspond to normalizable modes.

Outlook

- Extend the work to dS_4 and FRW, and general potential.
- Make connection with cosmological observables.
- Explore the novel Bessel basis for CFT correlators.