Quantum Chaos and Unitary Black Hole Evaporation

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Based on work with Larus Thorlacius, arxiv:2203.06434

Introduction

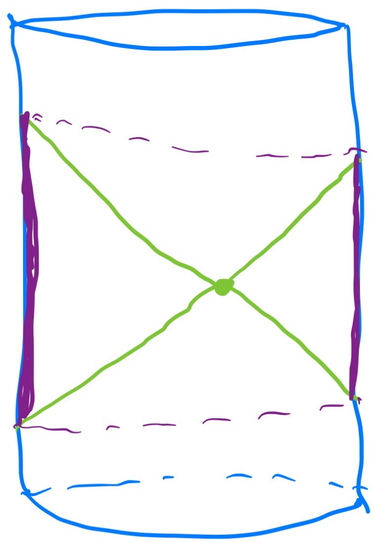
- Use AdS/CFT – though arguments generalize to any holographic model (i.e. quantum theory on a fixed background with a continuous time) at large but finite N
- Use HKLL map to build small black hole states
- Apply (Energy) Eigenstate Thermalization Hypothesis and the theory of quantum chaos to small black hole Hilbert subspace
- Provides estimates of corrections to semiclassical physics
- Provides definitions of special states that behave like firewalls
- Provides definitions of typical states that behave quasi-classically
  - Infalling observables computable

AdS/CFT and HKLL

- HKLL = Hamilton, Kabat, Lifschytz, Lowe

Gravity $\qquad\qquad$ CFT_3 on

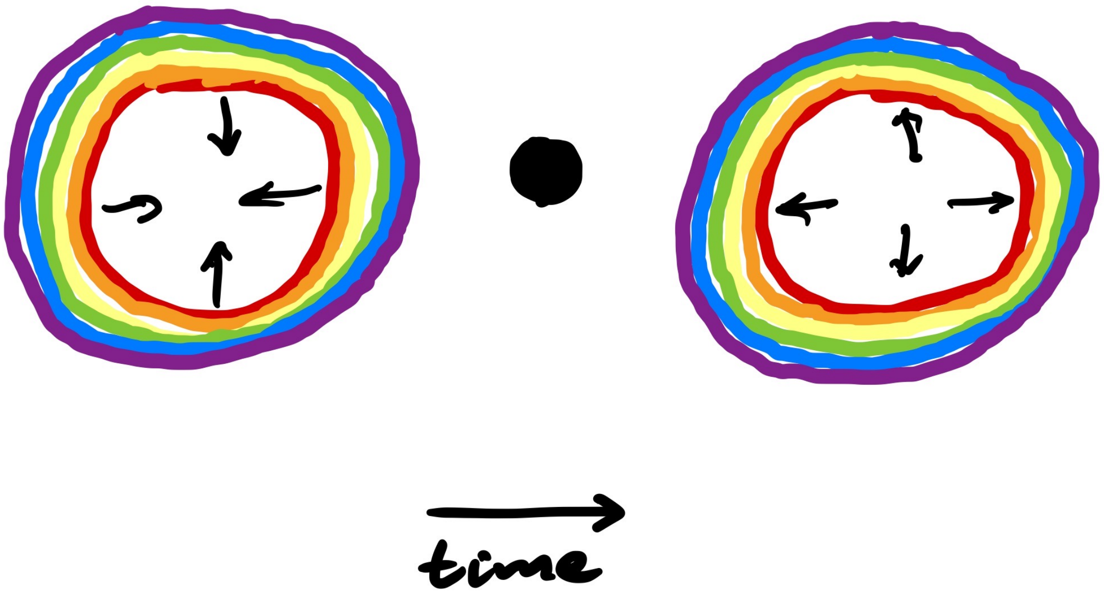
$AdS_4 = \mathbb{R} \times S_2$



Small Black Holes

- Mukhanov (2003), Zurek (1982)

$T \sim \frac{1}{M}$



- Let shell of radiation expand out to size

$$D = M^3$$

- Arrange that

$$R_{AdS} \gg D$$

- Use HKLL to build the initial and final states out of wavepackets of light excitations

- Key point: build wavepackets that are moving in a restricted element of solid angle

$$\delta\Omega \sim \frac{M^2}{D^2}$$

- Number of states will match the Bekenstein-Hawking entropy of the black hole = ¼ horizon area

$$S \sim M^2$$

- Zurek: process slight irreversibility (extra 4/3 if treated like black body)

Quantum Chaos

- Any Hamiltonian can be diagonalized
- Energy eigenstates remain energy eigenstates
- There is no chaos
- However, energy eigenstates may be highly delocalized
- Chaos can emerge in some semiclassical limit
  - Interactions of localized wavepackets should appear chaotic
  - Requires dense spectrum of states

Black Holes as Fast Scramblers

- Susskind-Sekino: Argued Hilbert subspace of black holes are chaotic, and scramble rapidly in (asymptotic) time

$$t \sim M \log S$$

- Number of states e^{M^2}

Eigenstate Thermalization Hypothesis

- Berry, Srednicki, …
- For some finite set of observables with good classical limits

$$A_{\alpha\beta} = \mathcal{A}(E_\alpha)\delta_{\alpha\beta} + e^{-S\left(\left(E_\alpha+E_\beta\right)/2\right)/2} R_{\alpha\beta},$$

- Matrix elements between energy eigenstates
- A(E) smooth in the energy
- R random matrix (e.g. random gaussian entries)

Black Hole Energy Eigenstates

- Key point: small black holes in AdS these are highly nonclassical
- Superpositions of black holes + clouds of ingoing/outgoing radiation
- No well defined spacetime geometry
- Very different from large black holes in AdS, much better understood (don’t evaporate, can use finite temperature/Euclidean time methods)

Typical Black Hole States

- Superpositions with finite energy width

$$|\psi\rangle = \sum_\alpha C_\alpha |\alpha\rangle,$$

$$\Delta_\psi E = \left(\sum_\alpha |C_\alpha|^2 \left(E_\alpha - \langle E \rangle_\psi \right)^2 \right)^{1/2},$$

$\Delta E \sim 1/M^3$ typical from previous construction.

- Opens door to self-averaging over many energy eigenstates, level spacing $\sim e^{-M^2}$

Fluctuations in Observables

$$\delta A = \langle \psi | A | \psi \rangle - \langle \psi' | A | \psi' \rangle$$

$$\langle E \rangle_\psi = \langle E \rangle_{\psi'}$$

$$\Delta_{\psi'} E = c \Delta_\psi E$$

- Apply ETH $\delta A = \delta A_{diag} + \delta A_{off-diag}$

$$\delta A_{\text{diag}} = \frac{1}{2}(1 - c^2)\left(\Delta_\psi E\right)^2 A''\left(\langle E \rangle_\psi\right),$$

A'' is a smooth function, so can't depend on the black hole microstate

- Contribution from off-diagonal random terms
- Can make this term large if you pick an eigenstate of R

$$|\delta A_{\mathrm{off-diag}}|_{\max} \sim \mathcal{O}(1)$$

- Likelihood of picking such a state $\sim \epsilon^{e^{M^2}}$
- Typical state, instead get self-averaging

$$|\delta A_{\mathrm{off-diag}}| \sim e^{-S(\langle E \rangle)/2}.$$

- Quantifies microstate dependent contributions to an observable

Transition Amplitudes

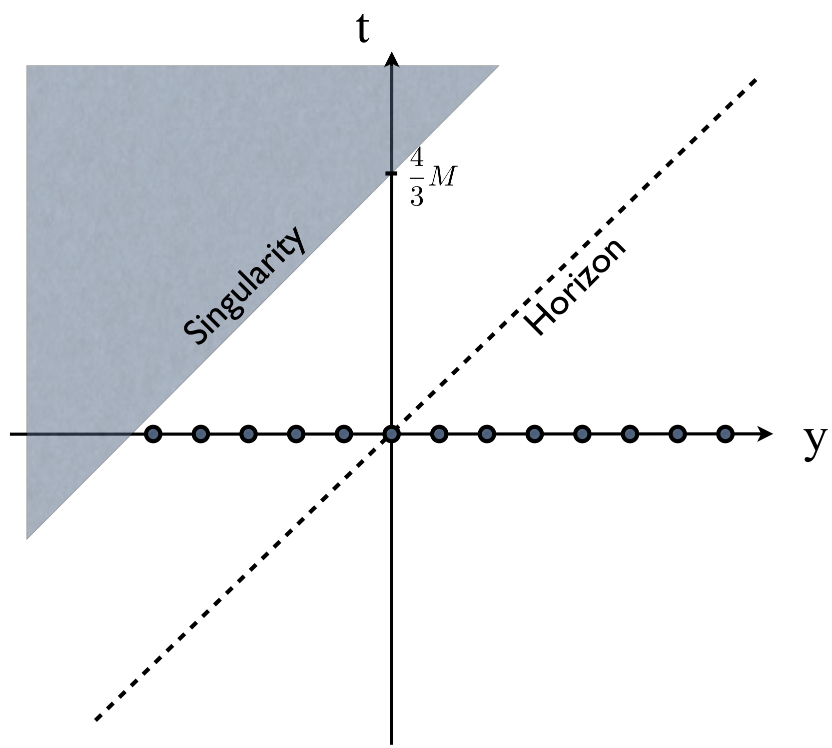
- Example: semiclassical ingoing state has non-vanishing amplitude to go to any other semiclassical outgoing state
- Implies quantum corrections are as big as classical effects near the horizon
- Effect only shows up when you “project” onto the outgoing state – requires non-local operation
- Otherwise, local observables will experience self-averaging over e^S outgoing states

Infalling Observables

- Apply above construction, and combine with results of Lowe, Thorlacius (2015) arxiv: 1508.06572. Also insist states look semiclassical prior/after to black hole formation
- Earlier work explains why this is insensitive to perturbations that act earlier than $\delta t \sim M \log S$

Infalling effective field theory

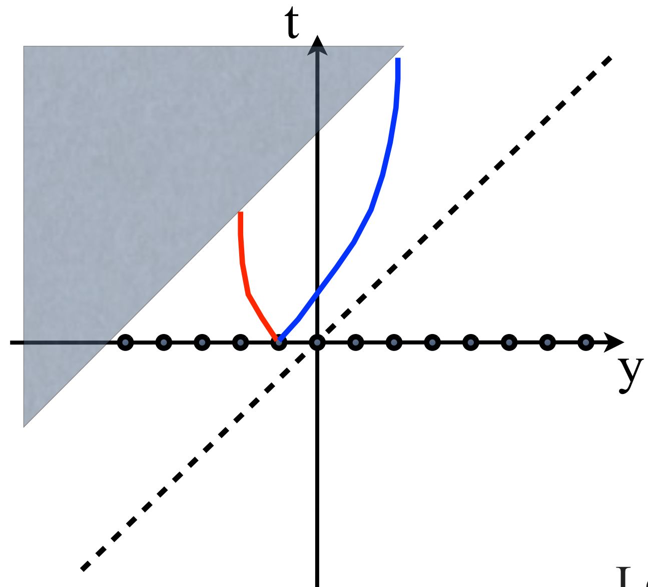
- Encode infalling observables as effective field theory with physical cutoff



$$ds^2 = -dt^2 + v^2(r)dy^2 + r^2 d\Omega^2$$

$$v(r) = -\sqrt{\frac{2M}{r}}$$

$$r(y,t) = 2M\left(1 + \frac{3}{4M}(y-t)\right)^{2/3}$$



Left: $t < 4M/3$

Right: $t < t_{scr} \equiv 4M \log(4M)$

- Continuum of soft near-horizon modes is regulated (no firewall shocks)
- Only need to go back a scrambling time to predict infalling observables
- Can proceed to build local observables outside the horizon, together with infalling Hamiltonian, and propagate across the horizon
- Number of observables constrained holographically
- Apply ETH arguments to compute using just diagonal piece with unobservable exponentially small corrections

Conclusions

- Energy eigenstates are delocalized quantum states
  - Look nothing like classical black holes
- Quantum states with good classical limits must involve a superposition of a band of energy eigenstates
- Because black holes have such a dense spectrum of states, any observable will experience self-averaging for such states
- Question becomes not why is evolution unitary but how does semiclassical limit emerge (typical of quantum chaotic systems)
- Applying ETH arguments explains this