Superconductivity without quasiparticles: Quantum critical Eliashberg theory and its holographic dual

HoloTube: The Applied Holography Webinars Network, June 28, 2022

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collaborators, references, funding …

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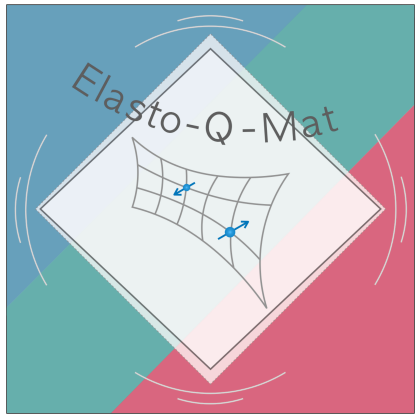
- I. Esterlis and J.S., Phys. Rev. B **100**, 115132 (2019)
- D. Hauck, M. J. Klug, I. Esterlis, J. S., Ann. of Phys. **417**, 168120 (2020)
- G. A. Inkof, K. Schalm, J. S., npj-Quantum Materials **7**, 56 (2022).
- V. Stangier, I. Esterlis, and J.S. preprint

GORDON AND BETTY MOORE FOUNDATION

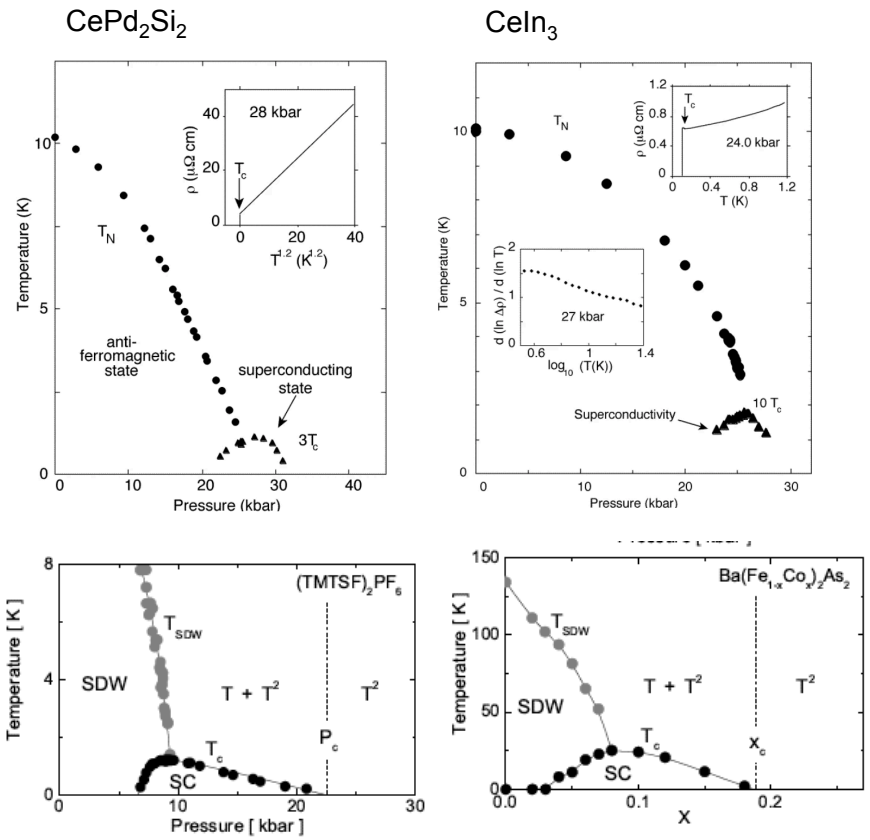
DFG Deutsche Forschungsgemeinschaft German Research Foundation

CRC– TRR 288



superconductivity and quantum criticality

T_c largest right at the QCP



N. D. Mathur, N. D. et al Nature **394**, 39 (1998),
N. Doiron-Leyraud, et al. Phys. Rev. B **80,** 214531 (2009),
S. Karahara et a. Phys. Rev B **81**, 184519 (2010).

no Cooper instability
(for instantaneous pairing interaction)

critical non Fermi liquid

$$\Sigma(\omega) \sim -i\,\mathrm{sign}(\omega)\,|\omega|^{1-\gamma}$$

$$\chi_{\mathrm{pair}} = \frac{\chi^0_{\mathrm{pair}}}{1 - \lambda_{\mathrm{pair}}\chi^0_{\mathrm{pair}}}$$

superconductivity only if

$$\lambda_{\mathrm{pair}} > \lambda^*_{\mathrm{pair}}$$

A. Balatsky, Philos. Mag. Lett. 68, 251 (1993);
B. A. Sudbo, Phys. Rev. Lett. 74, 2575 (1995);
C. L. Yin and S. Chakravarty, Int. J. Mod. P. B 10, 805 (1996).

two quite different answers…

A: superconductivity by critical bosons

$$S_{\rm int} \sim g_{ijk} \int \phi_k \psi_i^\dagger \psi_j$$

bosonic dynamics gets dominated by fermions

critical fermions

$$\Sigma\left(\omega\right) \sim -i{\rm sign}\left(\omega\right)\left|\omega\right|^{1-\gamma}$$

singular pairing interaction

$$\lambda_{\rm pair} \to \lambda_{\rm pair}(\omega) \propto \left|\omega\right|^{-\gamma}$$

generalized Cooper instability

D. T. Son, Phys. Rev. D **59**, 094019 (1999)
Ar. Abanov, A. Chubukov, and A. Finkel'stein, EPL **54**, 488 (2001)
Ar. Abanov, A. Chubukov, and J. S. EPL **55**, 369 (2001)
A. V. Chubukov and J. S., PRB **72**, 174520 (2005)
J.-H. She and J. Zaanen, PRB **80**, 184518 (2009)
M. A. Metlitski, D. F. Mross, S. Sachdev, and T. Senthil, PRB **91**, 115111 (2015)

$\Longleftrightarrow$

B: holographic superconductivity

$$S = S_{\rm gravity} + S_{\rm gauge} + \int d^{d+2}x\sqrt{g}\left(D_a\psi^* D^a\psi + m^2\psi^*\psi\right)$$

pairing near AdS-black horizon

- superconductivity of a critical state of matter
- transition at Breitenlohner-Freedman bound

$$m^2_{\rm eff} < m^2_{\rm BF}$$

Cooper pairs form in holographic metals

S. S. Gubser, *Phys. Rev.* D **78**, 065034 (2008)
S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, PRL **101**, 031601 (2008)

superconductivity:
from microscopics to continuum's theory

Karlsruhe Institute of Technology

<u>conventional s.c.</u>

BCS theory

L. P. Gor'kov (1959)

$$\psi(\boldsymbol{r},\tau) = V_{\mathrm{ep}} F(\boldsymbol{r},\tau;\boldsymbol{r},\tau)$$

$$e^* = 2e \quad m = m(\rho_F, V_{\mathrm{ep}})$$

Ginzburg-Landau theory

Gor'kov ~1970

anomalous propagator $F(\boldsymbol{r},\tau;\boldsymbol{r}',\tau') = -\langle T\psi_\uparrow(\boldsymbol{r},\tau)\,\psi_\downarrow(\boldsymbol{r}',\tau')\rangle$

$$F\left(\frac{\boldsymbol{r}+\boldsymbol{r}'}{2}, \frac{\tau+\tau'}{2}; \boldsymbol{k}, \epsilon\right)$$

inhomogeneous

non-eq. dynamics

internal space-time structure of the pair

superconductivity:
from microscopics to continuum's theory

Karlsruhe Institute of Technology

<u>conventional s.c.</u>

BCS theory

L. P. Gor'kov (1959)

$$\psi(\boldsymbol{r},\tau) = V_{\mathrm{ep}} F(\boldsymbol{r},\tau;\boldsymbol{r},\tau)$$

$$e^* = 2e \quad m = m(\rho_F, V_{\mathrm{ep}})$$

Ginzburg-Landau theory

<u>quantum critical s.c.</u>

microscopic Hamiltonian

$$F(\boldsymbol{r},\tau;\boldsymbol{k},\epsilon)$$

$$\updownarrow$$

$$\psi(\boldsymbol{r},\tau,\zeta)$$

holographic s.c. in AdS_{D+1}

- mass and charge of the scalar field
- physical interpretation of the holographic dimension
- origin of gravity
- …

Sachdev-Ye-Kitaev (SYK) model

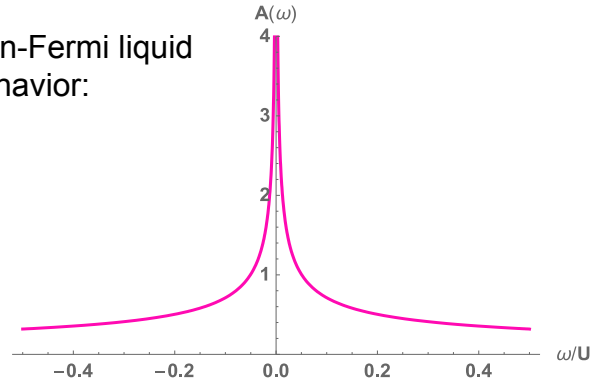
model of non-dispersive fermions with random interactions

$$H = -\mu \sum_{i=1}^{N} c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{ijkl}^{N} U_{ij,kl} c_i^\dagger c_j^\dagger c_k c_l \qquad \overline{|U_{ij,kl}|^2} = U^2$$

bi-local fields (propagators, self energies)… $$Z = \int DG(\tau,\tau')\, D\Sigma(\tau,\tau')\, e^{-S}$$

saddle point (large N):

Non-Fermi liquid behavior:



$$G(\omega) = -A_0 \frac{\mathrm{sign}(\omega)\, e^{\mathrm{sign}(\omega) i\pi\Delta}}{|\omega|^{1-2\Delta}}$$

IR-solution is re-parametrization invariant $\tau \to f(\tau)$

$$Z \sim \int Df e^{\alpha \frac{N}{U\beta} \int d\tau Sch[f,\tau]} \qquad Sch[\phi,\tau] = \frac{\phi'''(\tau)}{\phi'(\tau)} - \frac{3}{2}\left(\frac{\phi''(\tau)}{\phi'(\tau)}\right)^2$$

closely related to gravity theories in AdS_2 (+ dilaton fields)

D. Stanford, J. Maldacena, Phys. Rev D 94 (2016)

S. Sachdev and J. Ye, PRL, **70** 3339, (1993), A. Georges, O. Parcollet, and S. Sachdev PRL **85**, 840 (2000),
A. Kitaev, talk at KITP http://online.kitp.ucsb.edu/online/joint98/kitaev/, February, 2015.

Yukawa-SYK-model of electron-boson coupling

$$H = -\mu \sum_{i=1}^{N} \sum_{\sigma=\pm} c_{i\sigma}^{\dagger} c_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \left(\pi_k^2 + \omega_0^2 \phi_k^2 \right) + \frac{\sqrt{2}}{N} \sum_{ij,\sigma}^{N} \sum_{k}^{M} g_{ij,k} c_{i\sigma}^{\dagger} c_{j\sigma} \phi_k ,$$

I. Esterlis and J.S., Phys. Rev. B **100**, 115132 (2019)

related models:

Y. Wang, Phys. Rev. Lett. **124**, 017002 (2020).
D. Chowdhury and E. Berg, Phys. Rev. Research **2**, 013301 (2020).
L. Classen and A. Chubukov, Phys. Rev. B **104**, 125120 (2021).

extensions to finite dimensions:

J. Kim, E. Altman, and X. Cao, Phys. Rev. B **103**, L081113 (2021) → Dirac systems
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021) → compressible Fermi systems

Yukawa-SYK-model of electron-boson coupling

$$H = -\mu \sum_{i=1}^{N} \sum_{\sigma=\pm} c_{i\sigma}^{\dagger} c_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \left(\pi_k^2 + \omega_0^2 \phi_k^2 \right) + \frac{\sqrt{2}}{N} \sum_{ij,\sigma}^{N} \sum_{k}^{M} g_{ij,k} c_{i\sigma}^{\dagger} c_{j\sigma} \phi_k ,$$

random electron-phonon coupling $g_{ij,k} = g'_{ij,k} + i g''_{ij,k}$

$$\overline{\left(g'_{ij,k} \right)^2} = (1 - \tfrac{\alpha}{2}) \overline{g}^2 \qquad \overline{\left(g''_{ij,k} \right)^2} = \tfrac{\alpha}{2} \overline{g}^2$$

$\alpha = 0$	Gaussian orthogonal ensemble → time reversal symmetry for each configuration	**superconductivity**
$\alpha = 1$	Gaussian unitary ensemble → max. TRS-breaking	**no s.c.**
$0 < \alpha < 1$	“partial” TRS-breaking	**pair breaking**

I. Esterlis and J.S., Phys. Rev. B **100**, 115132 (2019)
D. Hauck, M. J.Klug, I. Esterlis, J.S., Ann. of Phys. **417**, 168120 (2020)

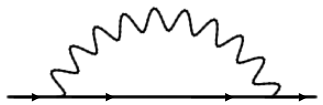
bilocal fields,… $$Z = \int DGD\Sigma DFD\Phi DDD\Pi e^{-NS}$$

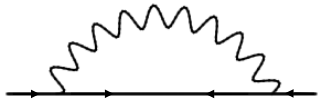
additional bilocal pairing fields: $F(\tau,\tau') \sim c_\uparrow(\tau)c_\downarrow(\tau')$ and $\Phi(\tau,\tau')$

solution at $N \to \infty$

Nambu-Gor'kov propagator, self energy

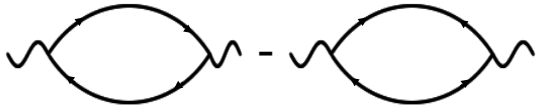
$$\hat{G}\left(i\omega\right)^{-1} = \begin{pmatrix} i\omega - \Sigma\left(i\omega\right) & \Phi\left(i\omega\right) \\ \Phi\left(i\omega\right) & i\omega + \Sigma\left(-i\omega\right) \end{pmatrix}$$

$\Sigma\left(i\omega\right) =$ 

$\Phi\left(i\omega\right) =$ 

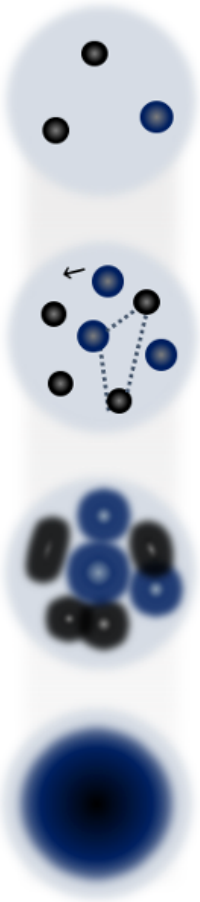
boson propagator, self energy

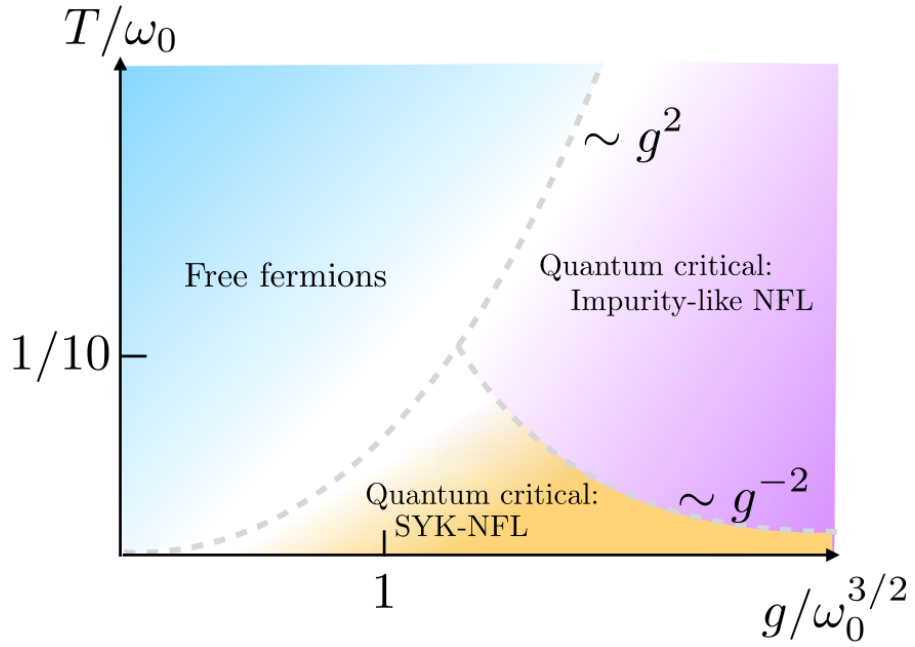
$$D^{-1}\left(i\omega\right) = \omega^2 + \omega_0^2 - \Pi\left(i\omega\right)$$

$\Pi\left(i\omega\right) =$ 

Eliashberg equations of superconductivity become exact!

normal state: critical ground state





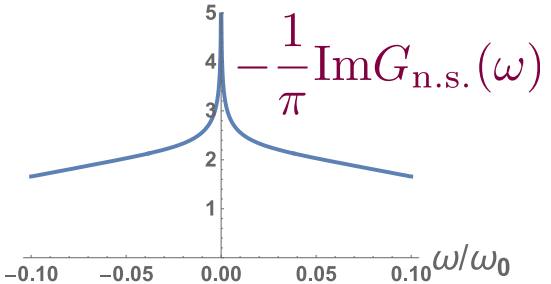
$$G_{\mathrm{n.s.}}(\tau_1-\tau_2) \propto \frac{\tanh(\pi q\mathcal{E}) + \mathrm{sign}(\tau_1-\tau_2)}{|\tau_1-\tau_2|^{\frac{1+\gamma}{2}}}$$

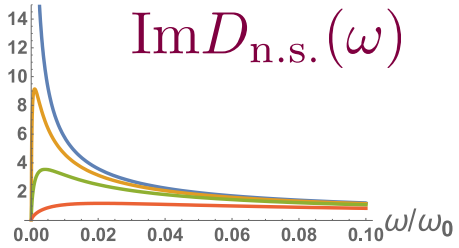
$$D_{\mathrm{n.s.}}(\tau_1-\tau_2) \propto \frac{1}{|\tau_1-\tau_2|^{1-\gamma}}$$

$\gamma \approx 0.68$

q fermion charge

$\mathcal{E}(n)$ spectral asymmetry

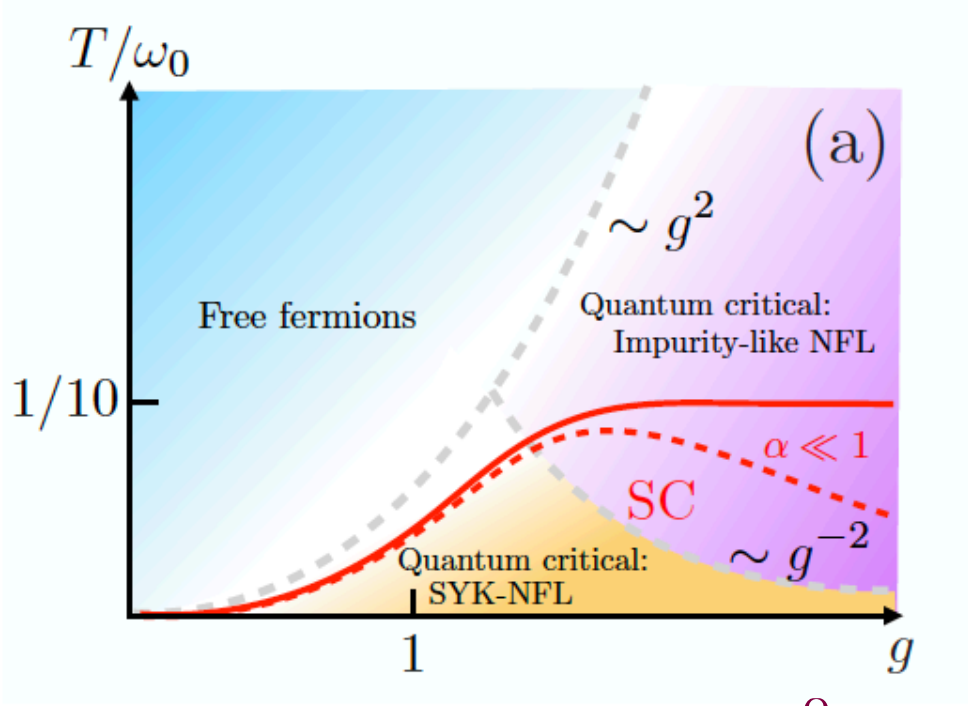


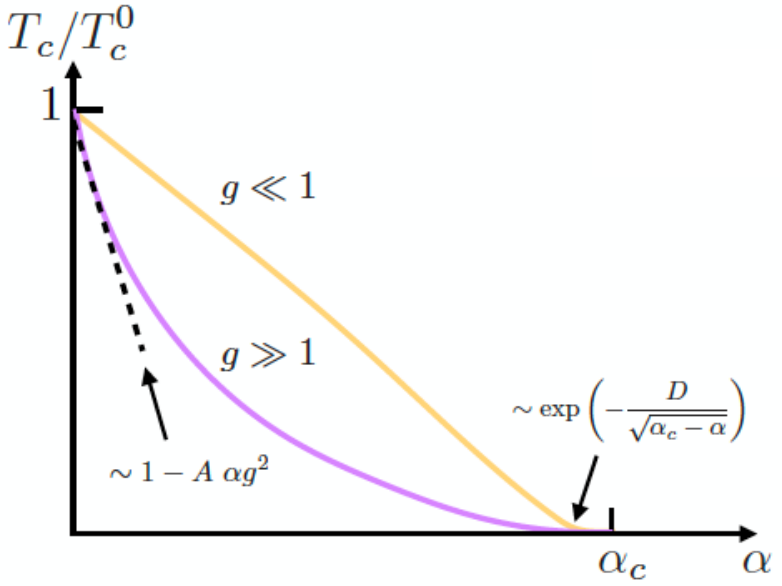


boson mass

$$\omega_r^2 = \omega_0^2 - \Pi(0) \quad \propto T^{\gamma}$$

pair breaking (partial time reversal symmetry breaking)





$$T_c\,(\alpha \sim Z_B) \ll T_c^0$$

small coherent weight
makes s.c. fragile against
pair breaking

$$T_c\,(\alpha \approx \alpha_c) = T^* \exp\left(-\frac{D}{\sqrt{\alpha_c - \alpha}}\right)$$

spontaneously broken
conformal invariance

like in holographic models in AdS_2

D. Hauck, M. J. Klug, I. Esterlis, J. S., Ann. of Phys. **417**, 168120 (2020)

superconducting transition temperature

linearized gap equation of the Yukawa-SYK model

generalized Cooper instability

$$\Phi(\epsilon) = g^2 \int_T^\Lambda \frac{d\epsilon'}{2\pi} \frac{\Phi(\epsilon')}{|\epsilon - \epsilon'|^\gamma |\epsilon'|^{1-\gamma}}$$

γ-model

A. Abanov, and A. V. Chubukov, Phys. Rev. B **102**, 024524 (2020),
Y.-M. Wu, A. Abanov, and A. V. Chubukov, Phys. Rev. B 102, 094516 (2020)

same gap equation occurs in theories with pairing due to critical:

- antiferromagnetic + ferromagnetic spin fluctuations
- gauge-field induced composite fermion pairing
- nematic fluctuations
- massless gluons in high-density quark matter
- U(1) and Z_2 gauge fluctuations in spin liquids

N. E. Bonesteel, I. A. McDonald, and C. Nayak, PRL 77, 3009 (1996), D. T. Son, Phys. Rev. D **59**, 094019 (1999),. Ar. Abanov, A. Chubukov, and A. Finkel'stein, EPL **54**, 488 (2001) , Ar. Abanov, A. Chubukov, and J. S. EPL **55**, 369 (2001), R. Roussev and A. J. Millis, Phys. Rev. B 63, 140504R (2001), A. V. Chubukov and J. S., PRB **72**, 174520 (2005), J.-H. She and J. Zaanen, PRB **80**, 184518 (2009), M. A. Metlitski, D. F. Mross, S. Sachdev, and T. Senthil, PRB **91**, 115111 (2015)

Eliashberg equation

$$\Phi(\epsilon) = g^2 \int_T^\Lambda \frac{d\epsilon'}{2\pi} \frac{\Phi(\epsilon')}{|\epsilon - \epsilon'|^\gamma |\epsilon'|^{1-\gamma}} \quad \gamma \ll 1 \quad \Longrightarrow \quad \frac{d}{d\epsilon}\epsilon^{1-\gamma}\frac{d}{d\epsilon}\epsilon^\gamma \Phi(\epsilon) = -\frac{\gamma g^2}{\pi}\frac{\Phi(\epsilon)}{\epsilon}$$

scalar field $\psi(\zeta) = \zeta^{\frac{1-\gamma}{2}}\Phi(1/\zeta)$ with coordinate $\zeta = 1/\epsilon$

$$-\partial_\zeta^2\psi + \frac{m^2}{\zeta^2}\psi = 0 \qquad m^2 = -\frac{1}{4} + \frac{\gamma^2}{4} - \frac{g^2\gamma}{\pi}$$

static Klein-Gordon equation of pairs with mass m in AdS_2

D. T. Son, Phys. Rev. D **59**, 094019 (1999), A. V. Chubukov and J. S., PRB **72**, 174520 (2005), D. Hauck, M. J. Klug, I. Esterlis, J. S., Ann. of Phys. 417, 168120 (2020), A. Abanov, and A. V. Chubukov, Phys. Rev. B **102**, 024524 (2020), Y.-M. Wu, A. Abanov, and A. V. Chubukov, Phys. Rev. B 102, 094516 (2020)

On the level of the action

fluctuating field: anomalous Gor'kov Green's function

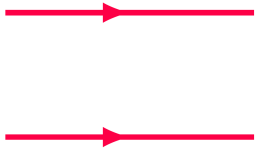
$$F(\tau,\tau') \to F(\varepsilon,\omega)$$

FT of the relative time

FT of the absolute time

Gaussian fluctuations near the quantum critical normal state

$$S^{(\mathrm{sc})}/N = \int_{\omega,\epsilon} \frac{F^{\dagger}(\omega,\epsilon)\,F(\omega,\epsilon)}{\Pi_{\mathrm{n.s.}}(\omega,\epsilon)} - \frac{\bar{g}^2\lambda_p}{2}\int_{\omega,\epsilon,\epsilon'} F^{\dagger}(\omega,\epsilon)\,D_{\mathrm{n.s.}}(\epsilon-\epsilon')\,F(\omega,\epsilon')$$

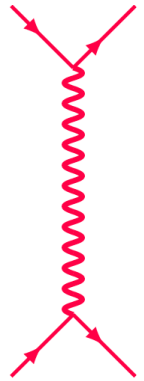


particle-particle propagator

$$\Pi_{\mathrm{n.s.}}(\omega,\epsilon) = G_{\mathrm{n.s.}}\left(\frac{\omega}{2}-\epsilon\right)G_{\mathrm{n.s.}}\left(\epsilon+\frac{\omega}{2}\right)$$

boson propagator
= singular paring interaction

$$D_{\mathrm{n.s.}}(\epsilon) \propto |\epsilon|^{-\gamma}$$



a geometric interpretation

scalar field $\psi(z,t) = z^{\frac{1-\gamma}{2}} F\left(\epsilon \sim z^{-1}, t\right)$

$$S = \int d\tau dz \frac{1}{z^2}\left(-z^2 |\partial_t \psi|^2 + z^2 |\partial_z \psi|^2 + m^2 \psi^* \psi\right)$$

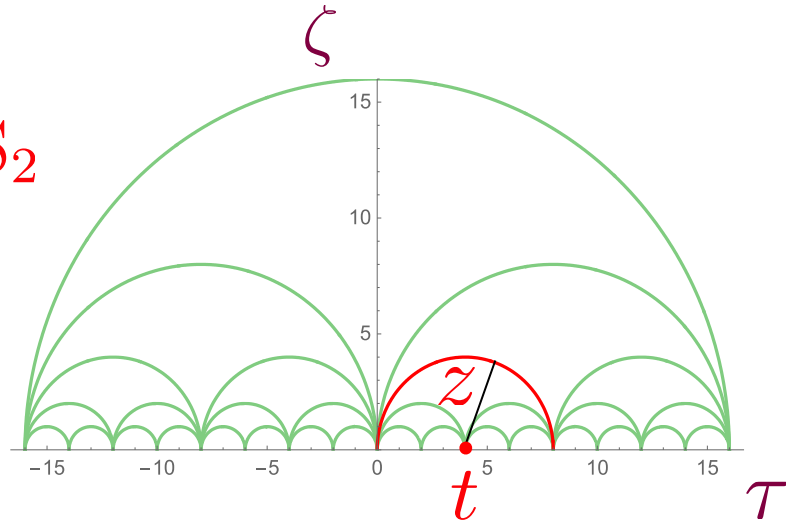
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2}\left(d\tau^2 - dz^2\right)$$

$$S = \int d^2x \sqrt{-g}\left(-g_{\mu\nu}\partial^\mu \psi^* \partial^\nu \psi + m^2 \psi^* \psi\right)$$

- this seems unstable (stabilized by boundary conditions)
- this is not AdS
- somewhat similar to Lorentzian de Sitter space

kinematic space = the space of geodesics

$$(\zeta, \tau) \in \mathrm{AdS}_2 \leftrightarrow (t, z) \in \mathrm{dS}_2$$



Radon transformation

$$\int_\Gamma \psi(\tau, \zeta) dl \rightarrow \psi(t, z)$$

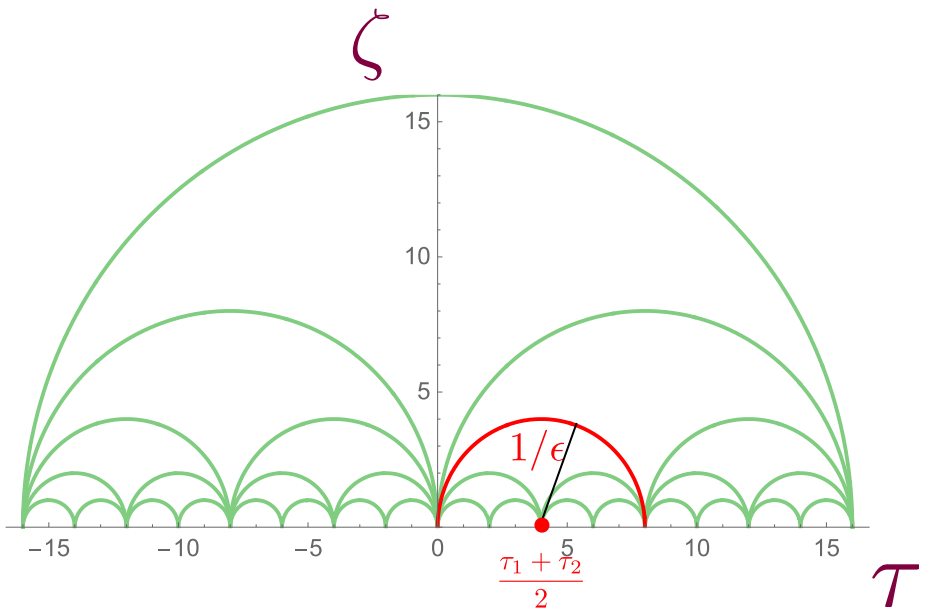
field theory describes geodesics of the AdS

B. I. Czech, L. Lamprou, S. McCandlish and J. Sully, JHEP10, 175 (2015).
S R. Das, A. Ghosh, A. Jevicki and K. Suzuki, JHEP07 184 (2018)

holographic map

$$F(\omega,\epsilon)\to F\left(\frac{\tau_1+\tau_2}{2},\epsilon\right)$$

$$F\left(\frac{\tau_1+\tau_2}{2},\epsilon\right)=|\epsilon|^{\frac{\gamma-1}{2}}\int_\Gamma \psi(\tau,\zeta)\,dl$$



Radon transformation

$$\Gamma:\quad |\epsilon|^{-1}=\sqrt{\left(\tau-\frac{\tau_1+\tau_2}{2}\right)^2+\zeta^2}$$

geodesics of Eucledian AdS_2

$$ds^2=g_{ab}dx^a dx^b=\frac{1}{\zeta^2}\left(d\tau^2+d\zeta^2\right)$$

SYK superconductor = holographic superconductor

$$F\left(\frac{\tau_1+\tau_2}{2},\epsilon\right)=|\epsilon|^{\frac{\gamma-1}{2}}\int_\Gamma \psi\left(\tau,\zeta\right)dl$$

holographic superconductor in AdS_2 with Euclidean signature

$$S^{(\mathrm{sc})}=N\int d\tau d\zeta\left(\frac{m^2}{\zeta^2}\left|\psi\right|^2+\left|\partial_\tau\psi\right|^2+\left|\partial_\zeta\psi\right|^2\right)$$

dynamics

RG scale

$$S^{(\mathrm{sc})}/N=\int_{\omega,\epsilon}\frac{F^\dagger\left(\omega,\epsilon\right)F\left(\omega,\epsilon\right)}{\Pi_{\mathrm{n.s.}}\left(\omega,\epsilon\right)}-\frac{\bar{g}^2\lambda_p}{2}\int_{\omega,\epsilon,\epsilon'}F^\dagger\left(\omega,\epsilon\right)D_{\mathrm{n.s.}}\left(\epsilon-\epsilon'\right)F\left(\omega,\epsilon'\right)$$

SYK superconductor = holographic superconductor

$$F\left(\frac{\tau_1+\tau_2}{2},\epsilon\right)=|\epsilon|^{\frac{\gamma-1}{2}}\int_\Gamma \psi\left(\tau,\zeta\right)dl$$

holographic superconductor in AdS_2 with Euclidean signature at T=0

$$S^{(\mathrm{sc})}=N\int d\tau d\zeta\left(\frac{m^2}{\zeta^2}|\psi|^2+|\partial_\tau\psi|^2+|\partial_\zeta\psi|^2\right)$$

positive contribution to the mass (no Cooper instability in NFL with instantaneous pairing)

negative contribution to the mass (generalized Cooper instab.)

$$S^{(\mathrm{sc})}/N=\int_{\omega,\epsilon}\frac{F^\dagger\left(\omega,\epsilon\right)F\left(\omega,\epsilon\right)}{\Pi_{\mathrm{n.s.}}\left(\omega,\epsilon\right)}-\frac{\bar{g}^2\lambda_p}{2}\int_{\omega,\epsilon,\epsilon'}F^\dagger\left(\omega,\epsilon\right)D_{\mathrm{n.s.}}\left(\epsilon-\epsilon'\right)F\left(\omega,\epsilon'\right)$$

holographic instability $m^2=m^2_{\mathrm{BF}}=-1/4$ (Breitenlohner Freedman condition)

=

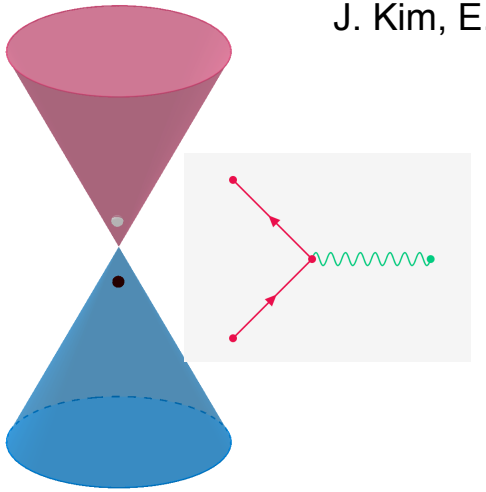
Eliashberg instability

P. Breitenlohner and D. Z. Freedman, Ann. Phys. **144**, 249 (1982)

finite-dimensions: Dirac systems

$$S = \int d^d x d\tau \left(\sum_{l=1}^{N} \overline{\psi_l} i \partial_\mu \gamma^\mu \psi_l + \frac{1}{2} \sum_{r=1}^{M} \phi_r \left(m_0^2 - \partial_\mu \partial^\mu \right) \phi_r + \frac{1}{N} \sum_{lm,r}^{N,M} g_{lm,r} \overline{\psi_l} \psi_m \phi_r \right)$$

J. Kim, E. Altman, and X. Cao, Phys. Rev. B **103**, L081113 (2021)



critical solutions for d=0,1,2

fermions $G(k) \propto k_\mu \gamma^\mu |k|^{2\Delta - d - 2}$

phonons $D(q) \propto |q|^{d+1-4\Delta}$

superconductivity for zero density (real coupling constants)

anomalous self energy condenses in l=1 longitudinal channel

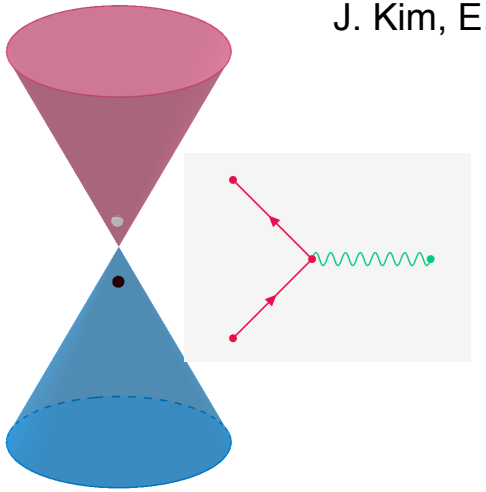
$$\Phi(k) = \sum_{m=-1}^{1} \phi_m(|k|) Y_{1,m}\left(\hat{k}\right) \frac{k_\mu \gamma^\mu}{|k|} + \cdots$$

V. Stangier, I. Esterlis, J. S., unpublished

finite-dimensions: Dirac systems

$$S = \int d^d x d\tau \left(\sum_{l=1}^{N} \overline{\psi_l} i \partial_\mu \gamma^\mu \psi_l + \frac{1}{2} \sum_{r=1}^{M} \phi_r \left(m_0^2 - \partial_\mu \partial^\mu \right) \phi_r + \frac{1}{N} \sum_{lm,r}^{N,M} g_{lm,r} \overline{\psi_l} \psi_m \phi_r \right)$$

J. Kim, E. Altman, and X. Cao, Phys. Rev. B **103**, L081113 (2021)



holographic superconductor in AdS_{d+2}

$$F_m \left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2}, \frac{\tau_1 + \tau_2}{2}, |k| \right) = |k|^{2\Delta - \frac{d}{2} - 1} \int_\Gamma \psi_m \left(\boldsymbol{x}, \tau, \zeta \right) dl$$

holographic variable $\zeta \sim \left(\boldsymbol{k}^2 + \epsilon^2 \right)^{-1/2}$

superconductivity for zero density (real coupling constants)

anomalous self energy condenses in l=1 longitudinal channel

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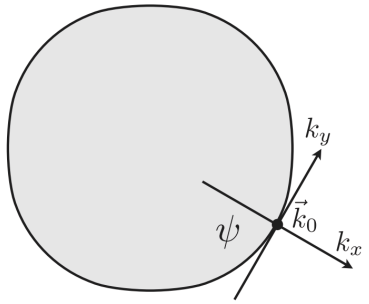
V. Stangier, I. Esterlis, J. S., unpublished

finite-dimensions: compressible fermions

$$S = \int d^d k d\tau \sum_{l=1}^{N} \psi^{\dagger}_{\boldsymbol{k}l} \left(\partial_\tau - \varepsilon\left(\boldsymbol{k}\right)\right) \psi_{\boldsymbol{k}l} + \frac{1}{2} \int d^d q d\tau \sum_{r=1}^{M} \phi_{\boldsymbol{q}r} \left(m_0^2 - \partial_\tau^2 + q^2\right) \phi_{\boldsymbol{q}r}$$
$$+ \frac{1}{N} \int d^d k d^d q d\tau \sum_{lm,r}^{N,M} g_{lm,r}\left(\boldsymbol{k}\right) \psi^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}l} \psi_{\boldsymbol{k}m} \phi_{\boldsymbol{q}r}$$

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021)



critical point

$$\Sigma\left(\boldsymbol{k}, \omega\right) \sim i \mathrm{sign}\left(\omega\right) \left|\omega\right|^{2/3}$$

$$\Pi\left(\boldsymbol{q}, \omega\right) \sim \frac{\left|\omega\right|}{\left|\boldsymbol{q}\right|}$$

holographic superconductor in $AdS_2 \otimes R_2$

holographic variable: $\zeta \sim \left|\epsilon\right|^{-1}$

charged black holes at low energies → AdS_2

J. Maldacena, J. Michelson, and A. Strominger, JHEP **9902**, 011 (1999).

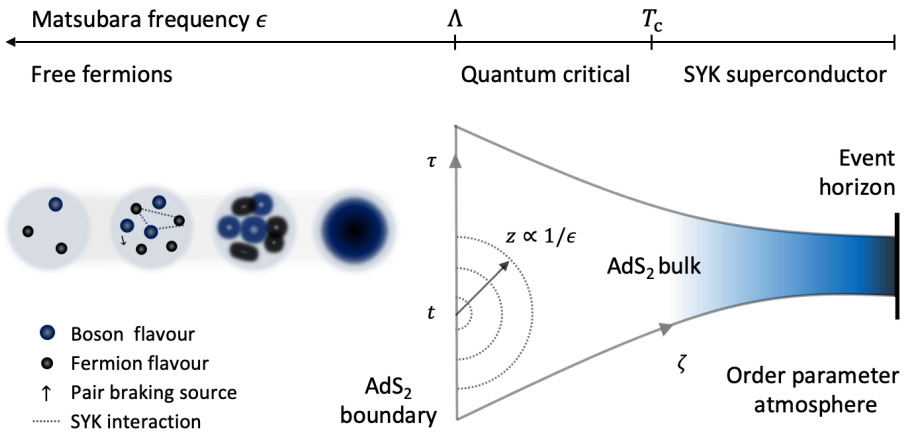
back to SYK, T>0, finite particle number

the holographic map can be extended

$$S^{(\mathrm{sc})} = S_{\mathrm{AdS}_2} = N \int d^2x \sqrt{g} \left(D_a \psi^* D^a \psi + m^2 \left| \psi \right|^2 \right)$$

black hole horizon at finite T $\quad \zeta_T^{-1} = 2\pi T$

$$ds^2 = g_{ab} dx^a dx^b = \frac{1}{\zeta^2} \left((1 - \zeta^2/\zeta_T^2) d\tau^2 + \frac{1}{(1 - \zeta^2/\zeta_T^2)} d\zeta^2 \right)$$



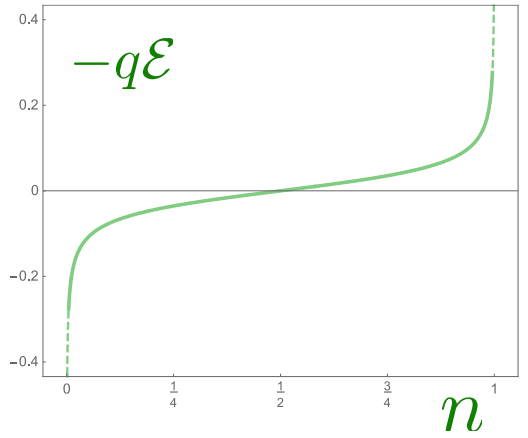
away from half filling

$$D_a = \partial_a - i q^\star A_a$$

Cooper pair charge: $\quad q^\star = 2q$

boundary electric field:

$$A_a = \left(\frac{i\mathcal{E}}{\zeta} \left(1 - \zeta/\zeta_T \right), 0 \right)$$



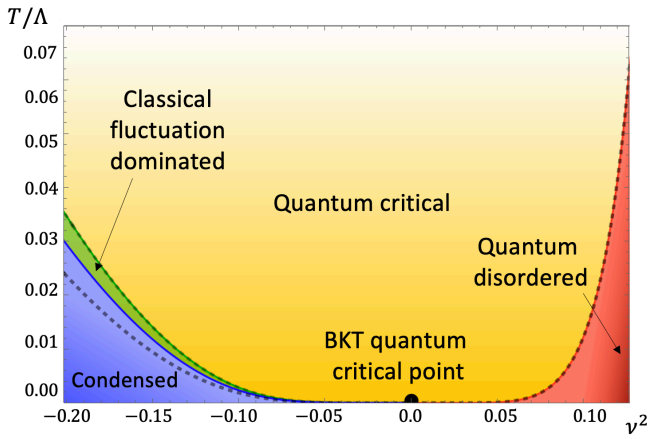
dynamic pairing susceptibility

not easy to calculate within Eliashberg approach, but easy in holography

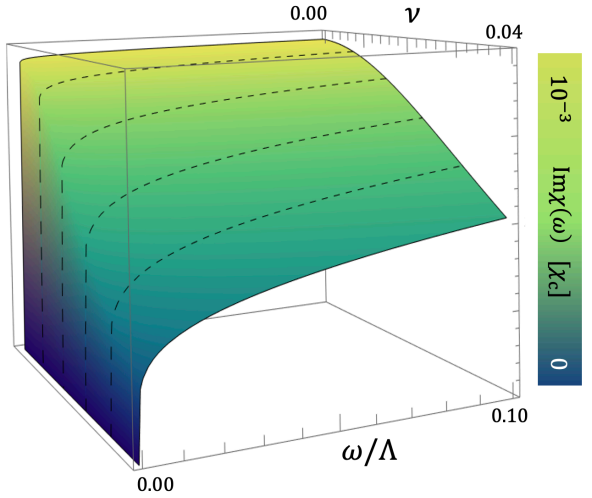
N. Iqbal, H. Liu, M. Mezei, Phys. Rev. D91, 025024 (2015)

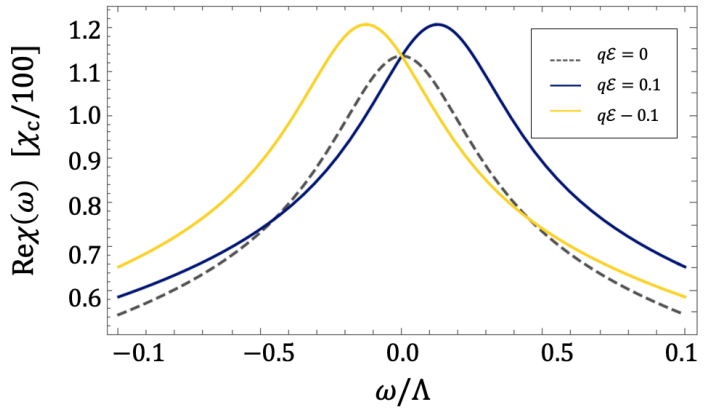
$$\chi_{\mathrm{AdS}_2} = \frac{1 - g\mathcal{G}}{1 - f\mathcal{G}}. \qquad \mathcal{G}(T,\omega) = \frac{2\nu - \gamma}{2\nu + \gamma} T^{2\nu} \frac{\Gamma(u-\nu)\,\Gamma(v+\nu)}{\Gamma(u+\nu)\,\Gamma(v-\nu)} \qquad \begin{aligned} u &= \tfrac{1}{2} + i2q\mathcal{E} \\ v &= \tfrac{1}{2} - i\tfrac{\omega - 4\pi T q\mathcal{E}}{2\pi T} \end{aligned}$$

phase diagram



dynamic susceptibility





dynamic response, non-equilibrium behavior … fluctuations beyond Eliashberg, …

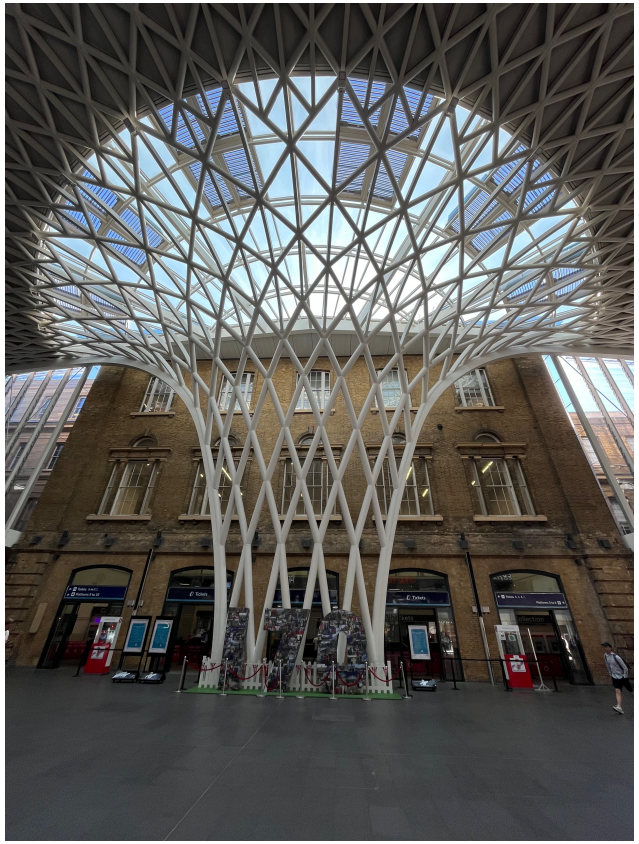
Conclusions:

explicit map → concrete calculations for specific many-body problems using the holographic principle

- phase diagrams
- dynamic response
- pairing mechanisms

curved space is everywhere

outside China Museum of Art,
Shanghai, August 21, 2019



King's Cross Station,
London, June 16, 2022

Stuttgart Hauptbahnhof
soon