HoloTube, 21/6/2022

A simple semiholographic model of non-Fermi liquids

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2012.15679

2206.01215

Outline

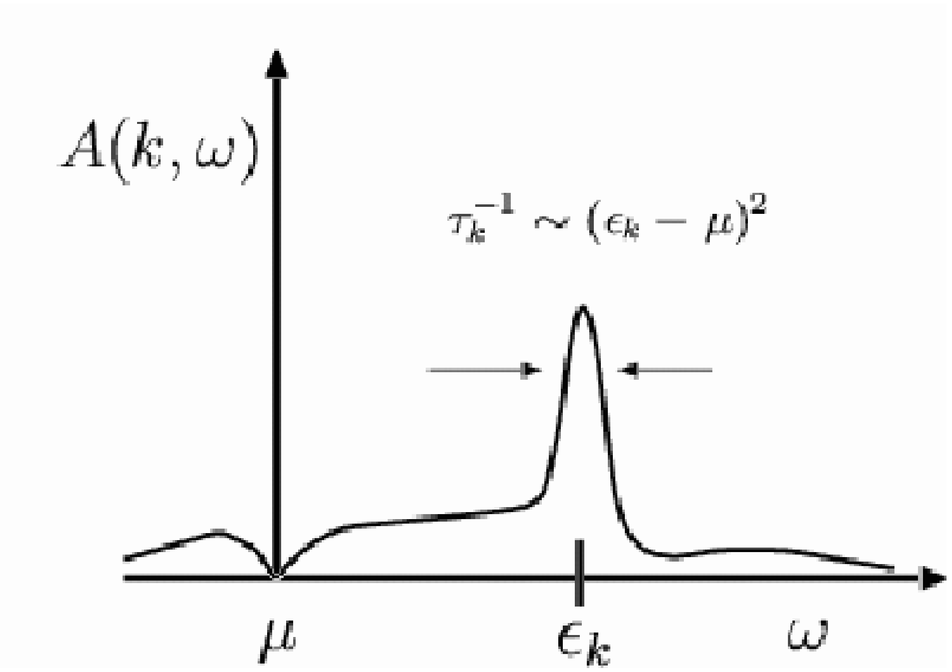
- Non-Fermi Liquid phenomenology
- Holographic models of strange metals
- Semi-holographic approach

Fermi liquid

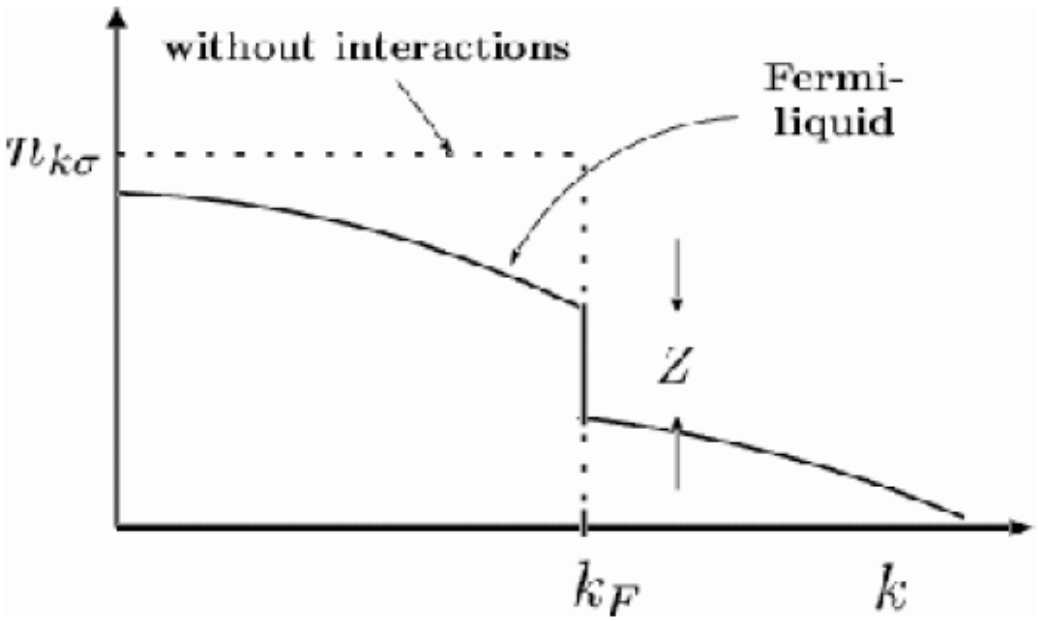
A Fermi liquid is a state adiabatically connected to the ground state of free fermions. The electrons "renormalize" into coherent quasiparticles

$$|\psi^{N+1}\rangle = Z^{1/2} c^\dagger |\psi^N\rangle + \sum c^\dagger c c^\dagger |\psi^N\rangle + \dots$$

$$G(k,\omega) = \frac{Z_k}{\omega - \tilde{\epsilon}_k + i/\tau_k} + G_{inc} \qquad \frac{1}{\tau_k} << \tilde{\epsilon}_k$$



spectral function



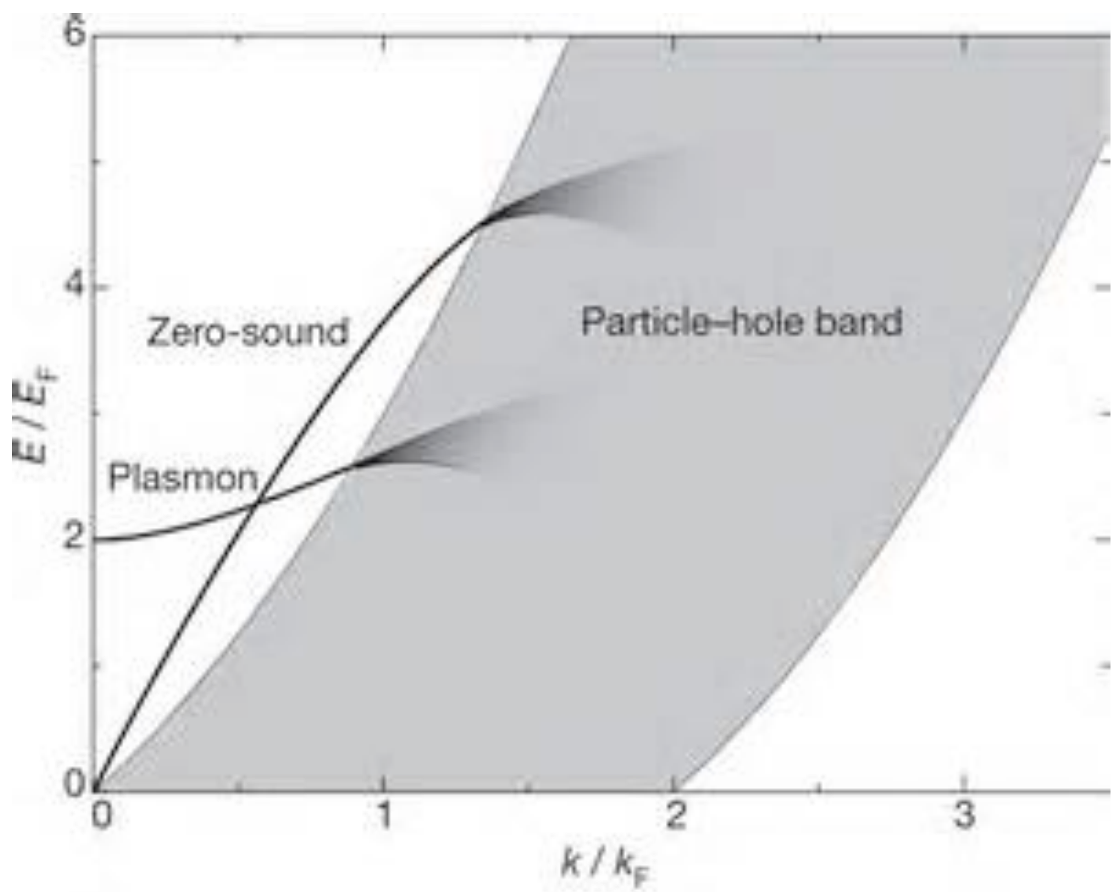
occupation number

[Varma, Nussinov, van Zaarlos, 01]

specific heat $C_v \sim \gamma T$

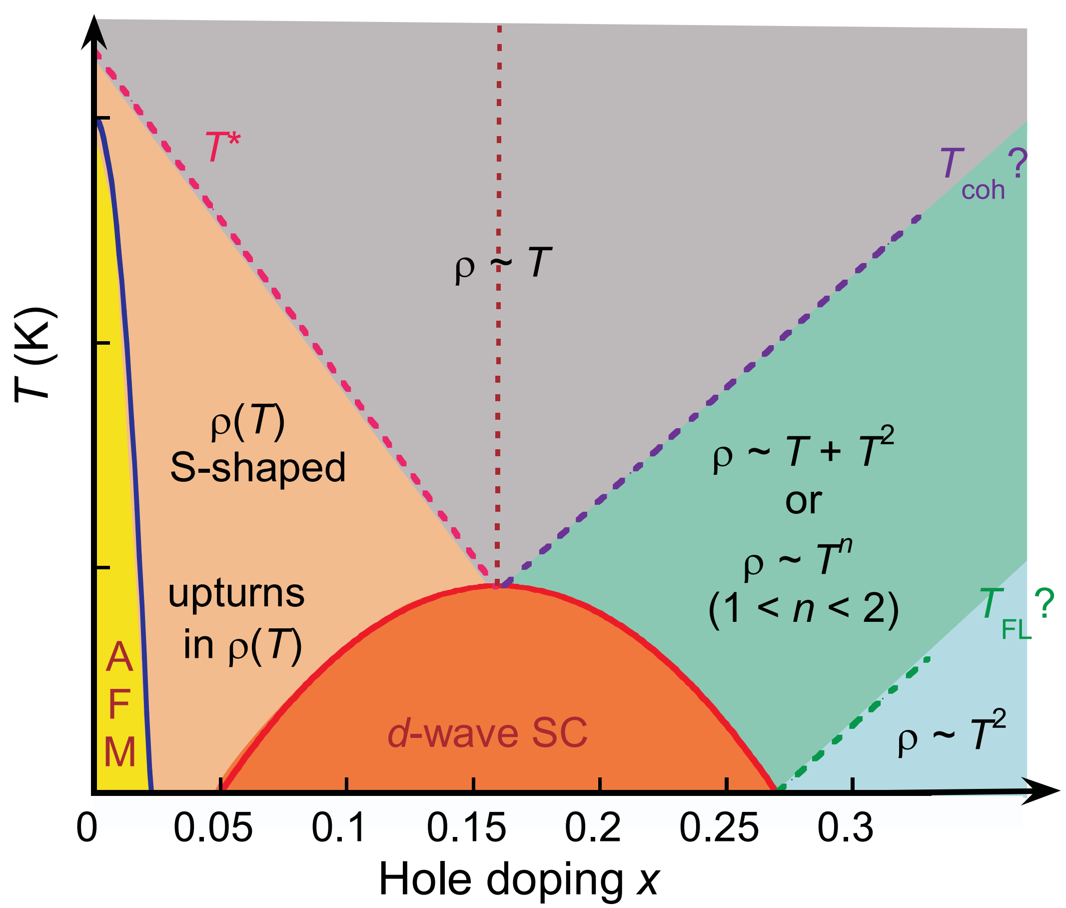
resistivity $\rho \sim \frac{1}{\tau} = \mathrm{Im}\Sigma \sim \omega^2 + T^2$

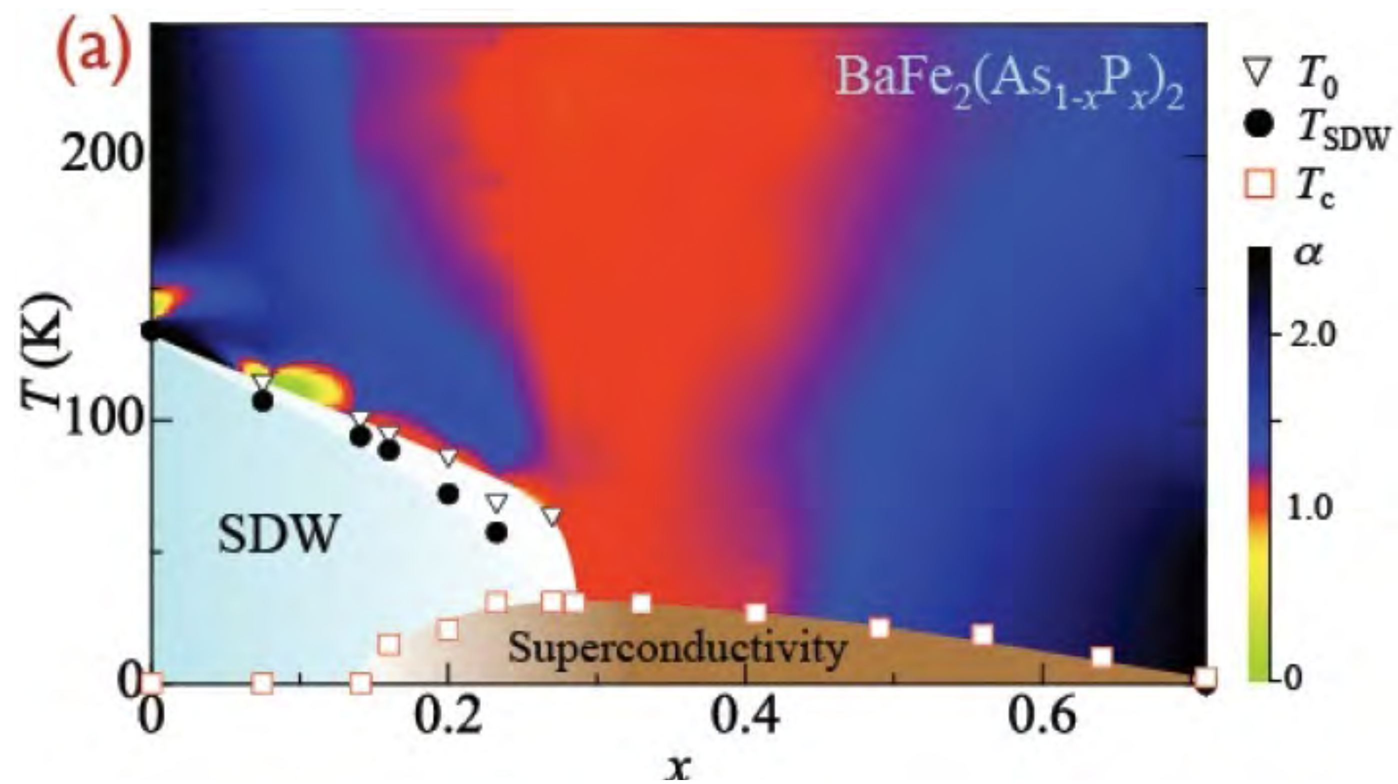
Zero sound = oscillations in the volume and shape of FS



Many systems of correlated electrons (e.g. high-Tc superconductors, heavy fermions) exhibit a “strange metal” behavior that challenges the Fermi liquid paradigm

This behaviour is often associated with the vicinity to a quantum critical point, and points to a short lifetime of excitations





Planckian dissipation $\tau_P = \frac{\hbar}{k_B T}$ Sachdev, Zaanen

At criticality observables have a scaling behavior $f(\omega\tau_P)$

One expects a general lower bound based on uncertainty principle

$$\tau \gtrsim \tau_P$$

Saturation of the bound should be associated with strong coupling

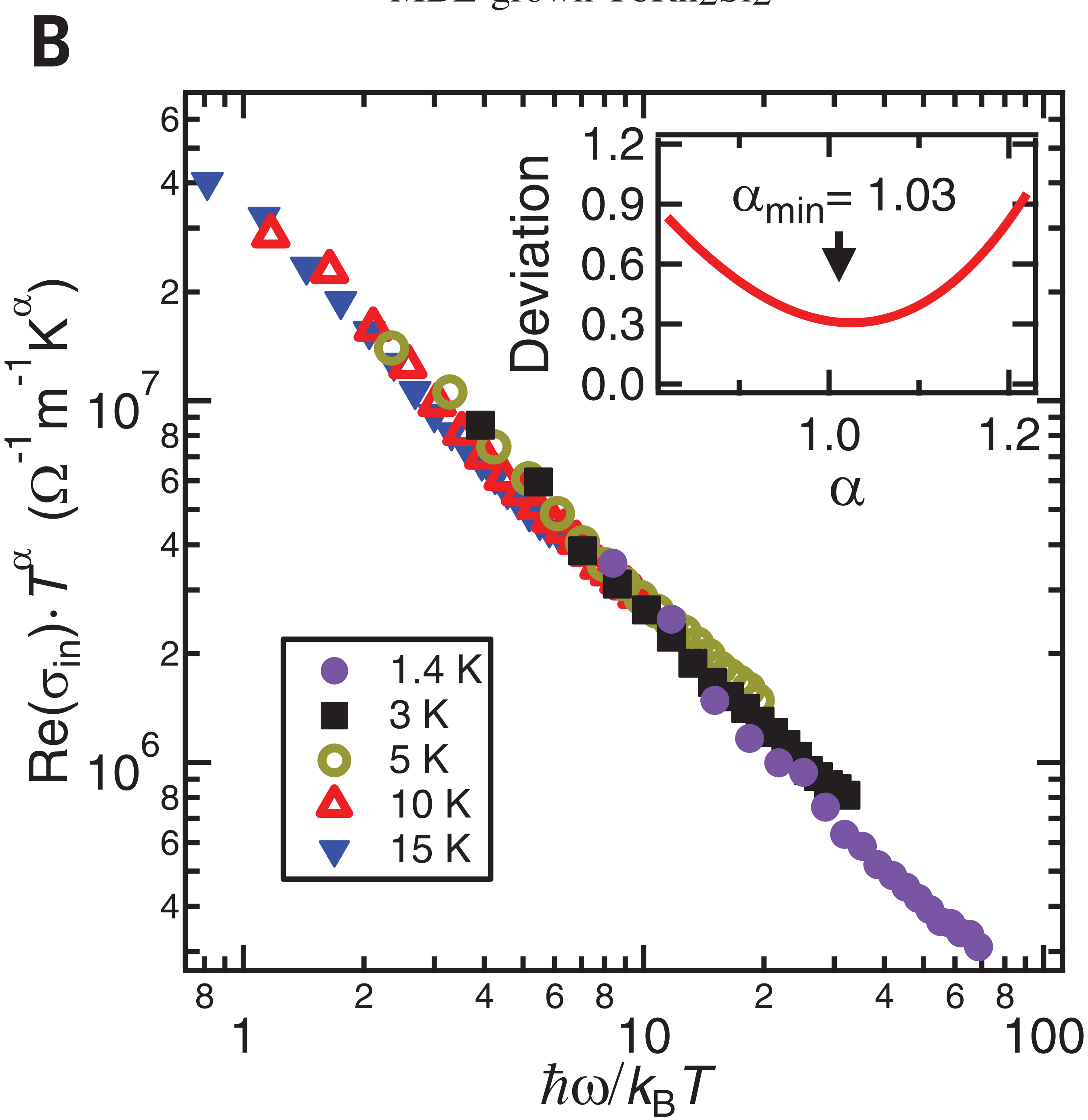
The intuition/evidence for this comes from the related viscosity bound, saturated by holographic systems at infinite coupling

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

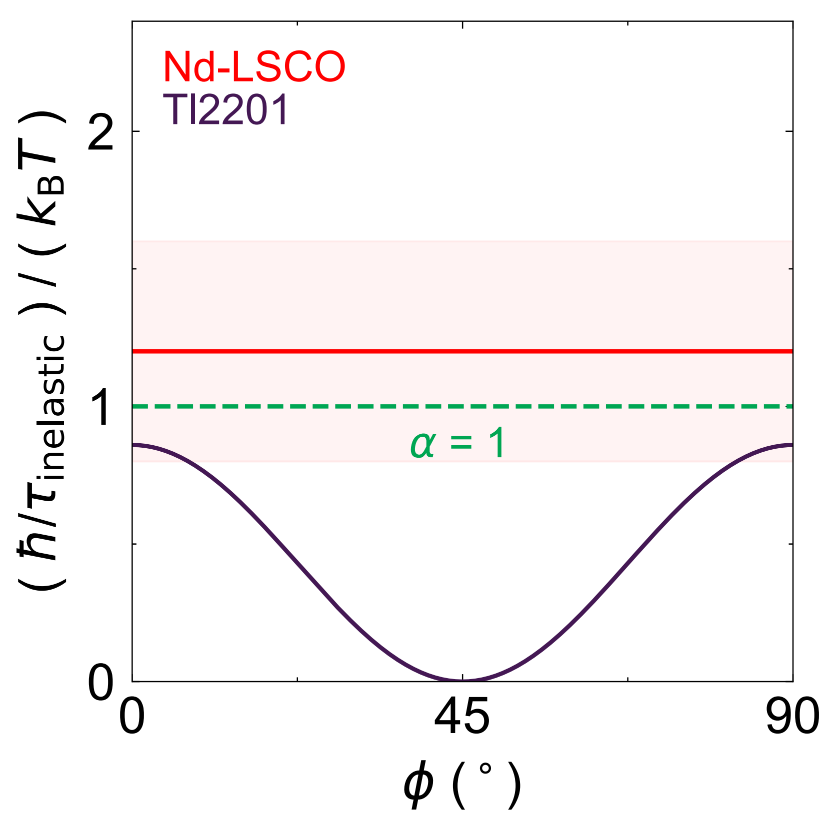
Kovtun, Son, Starinets

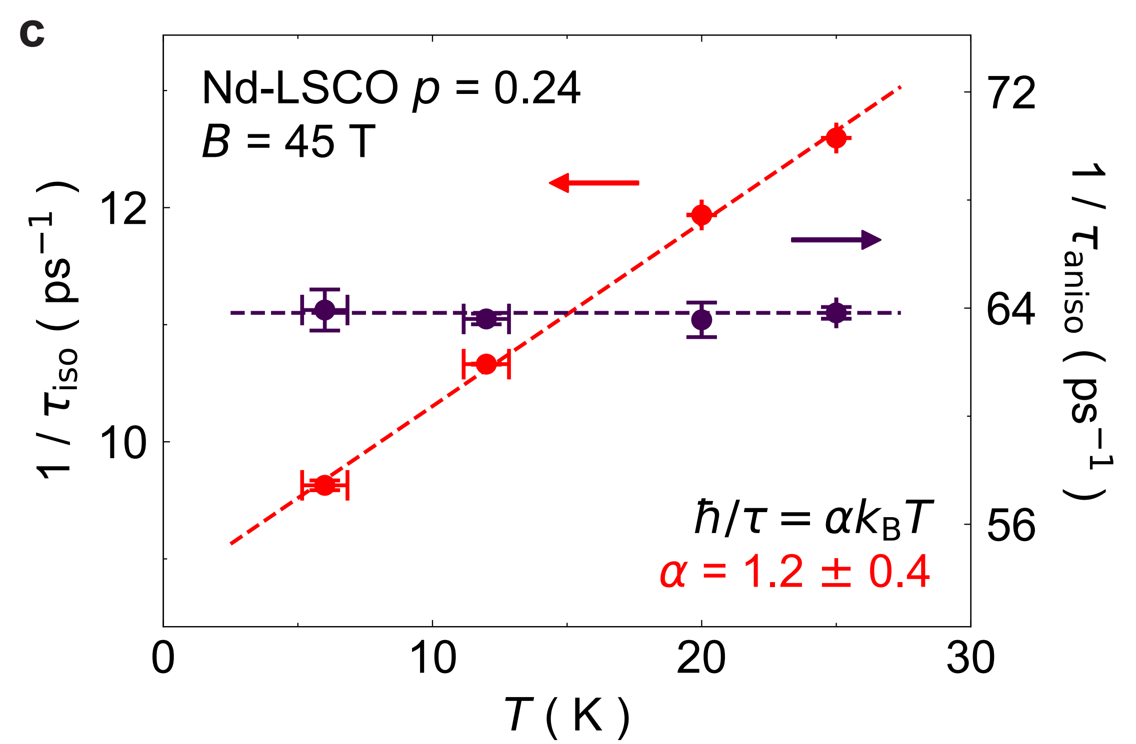
Optical conductivity

MBE-grown YbRh2Si2



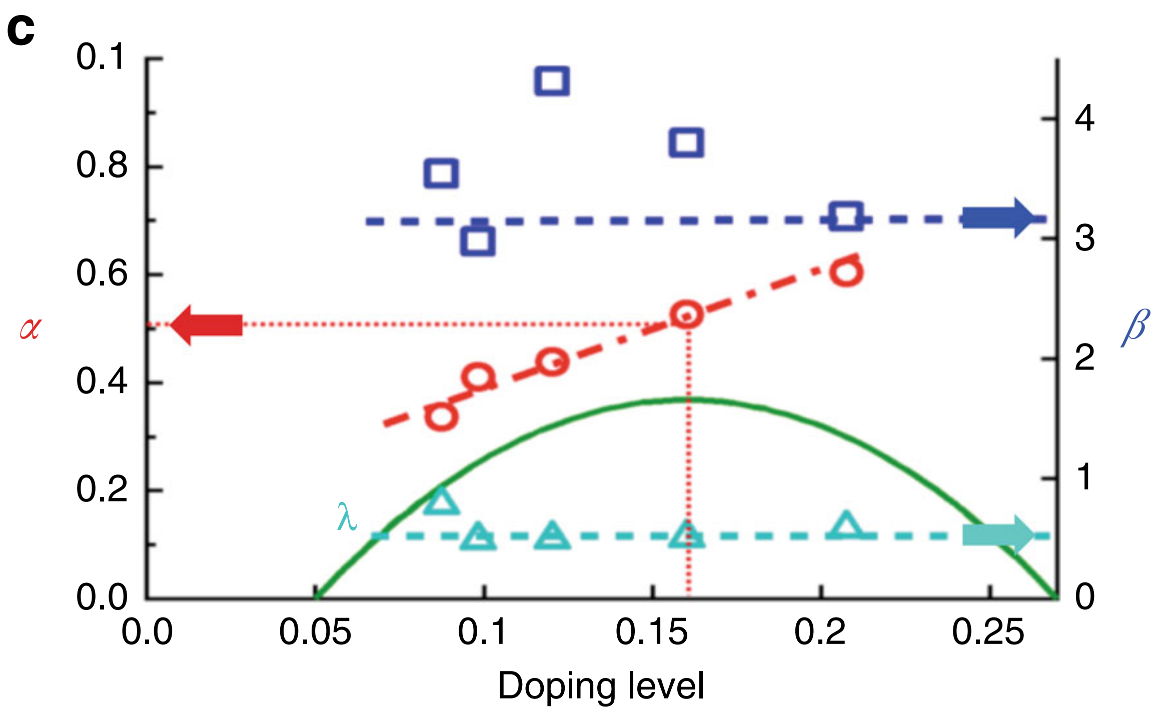
Prochaska et al., Science 2020





Grissonnanche et al., Nature 2021

Single-particle lifetime



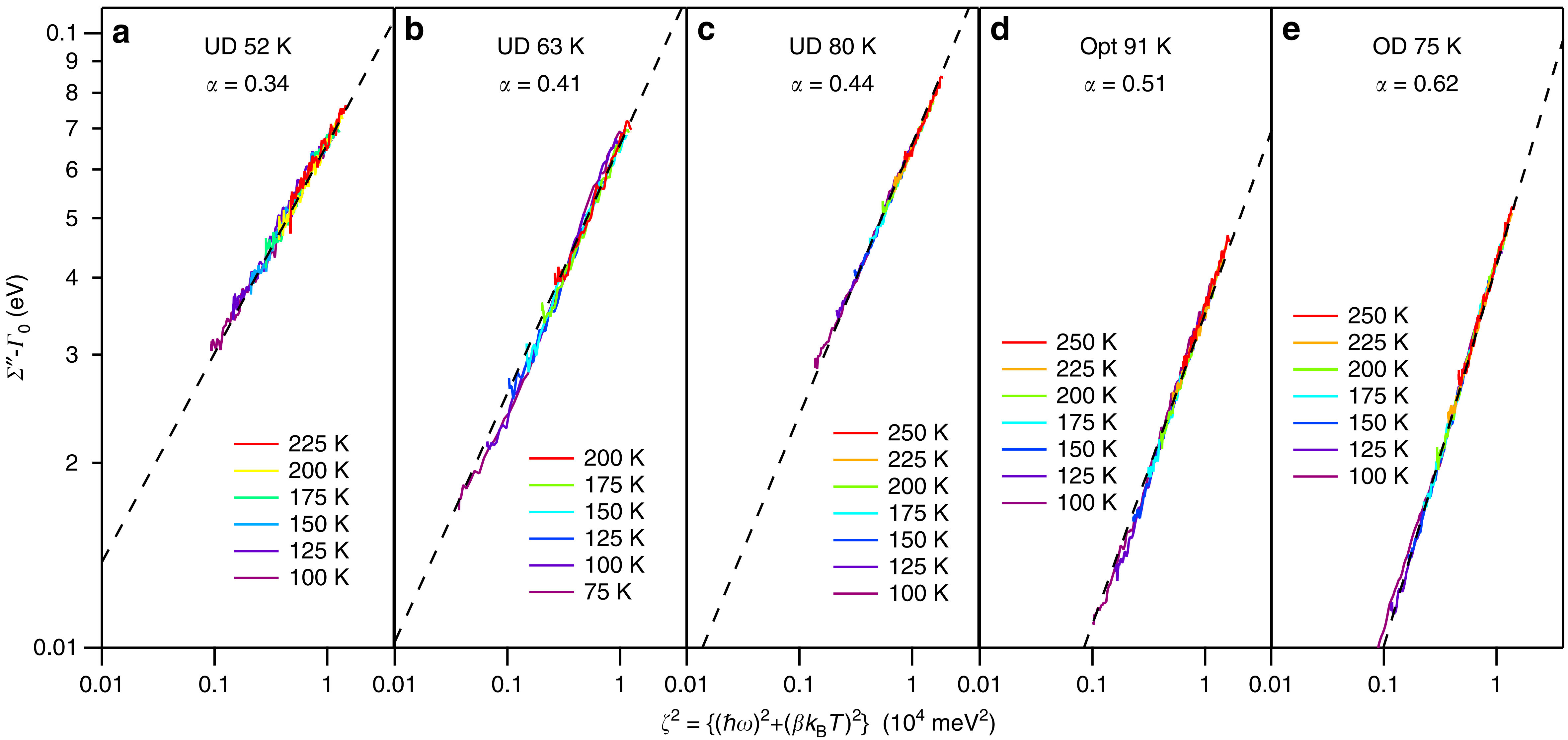
Reber et al., Nature Comm. 2019

Fit from ARPES measurements of $Bi_2Sr_2CaCu_2O_{8+\delta}$ using a phenomenological form of the self-energy

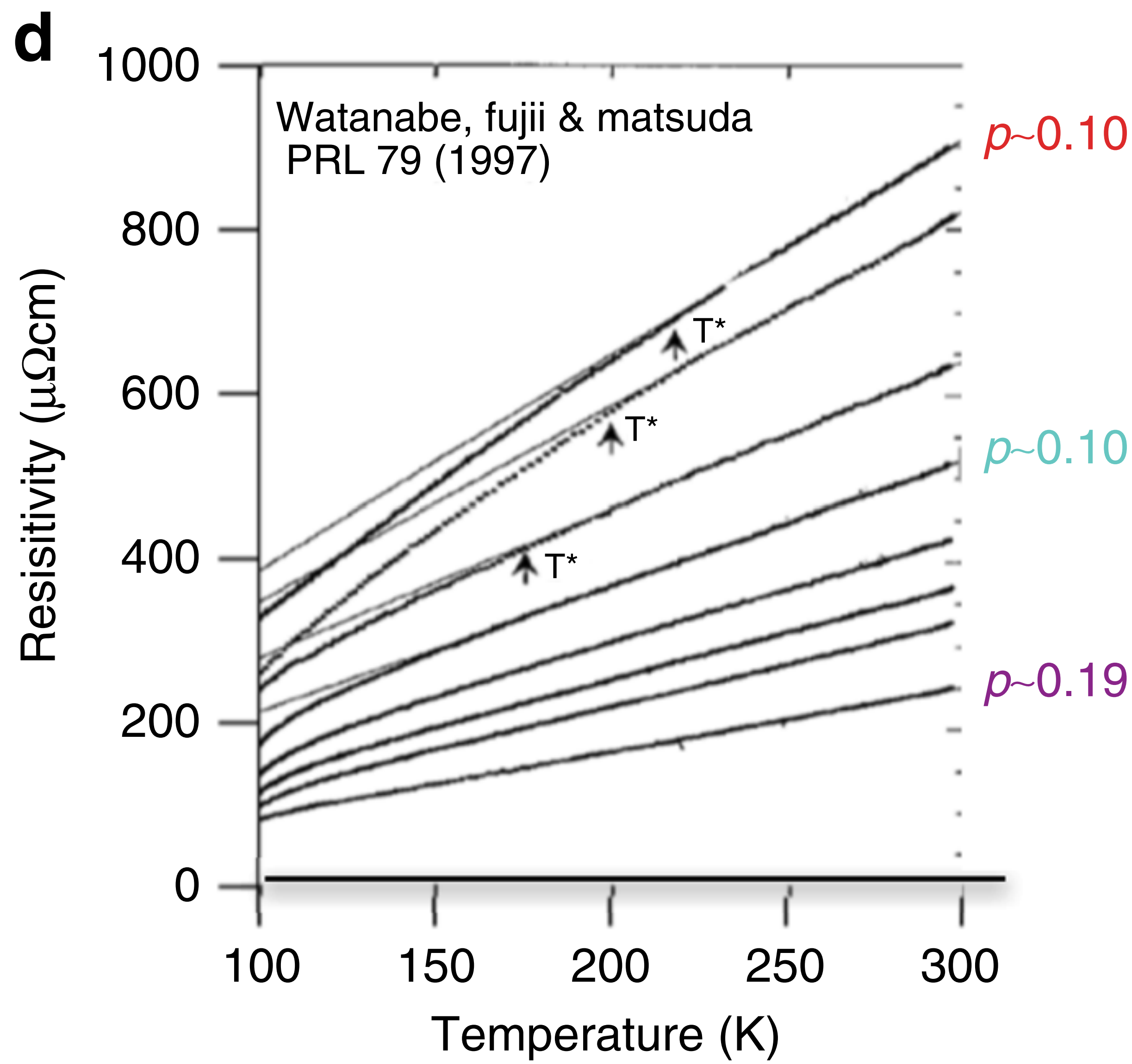
$$\mathrm{Im}\Sigma(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_B T)^2\right]^{\alpha}}{(\hbar\omega_0)^{2\alpha-1}} \quad \sim T^{2\alpha} f(\omega/T)$$

$$\beta \sim \pi$$

for $\alpha = \frac{1}{2}$ $\qquad \frac{1}{\tau} = \mathrm{Im}\Sigma(\omega = 0) = \frac{O(1)}{\tau_P}$ Marginal Fermi Liquid


a
UD 52 K
α = 0.34
225 K
200 K
175 K
150 K
125 K
100 K
b
UD 63 K
α = 0.41
200 K
175 K
150 K
125 K
100 K
75 K
c
UD 80 K
α = 0.44
250 K
225 K
200 K
175 K
150 K
125 K
100 K
d
Opt 91 K
α = 0.51
250 K
225 K
200 K
175 K
150 K
125 K
100 K
e
OD 75 K
α = 0.62
250 K
225 K
200 K
175 K
150 K
125 K
100 K
Σ″-Γ0 (eV)
0.1
9
8
7
6
5
4
3
2
0.01
0.01
0.1
1
ζ2 = {(ħω)2+(βkBT)2} (104 meV2)

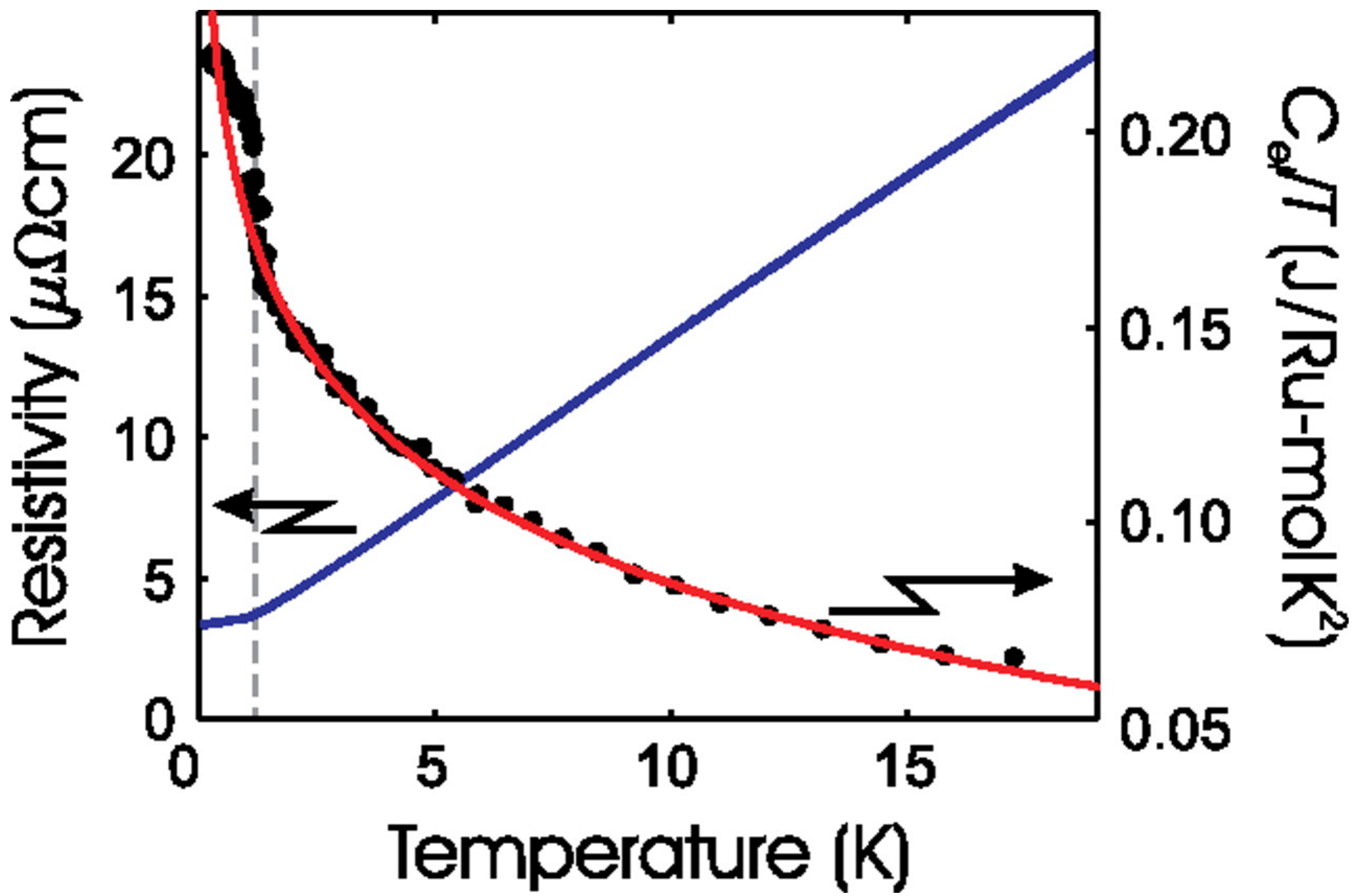
Linear-in-T resistivity



T^* “pseudogap” temperature

Anomalous specific heat

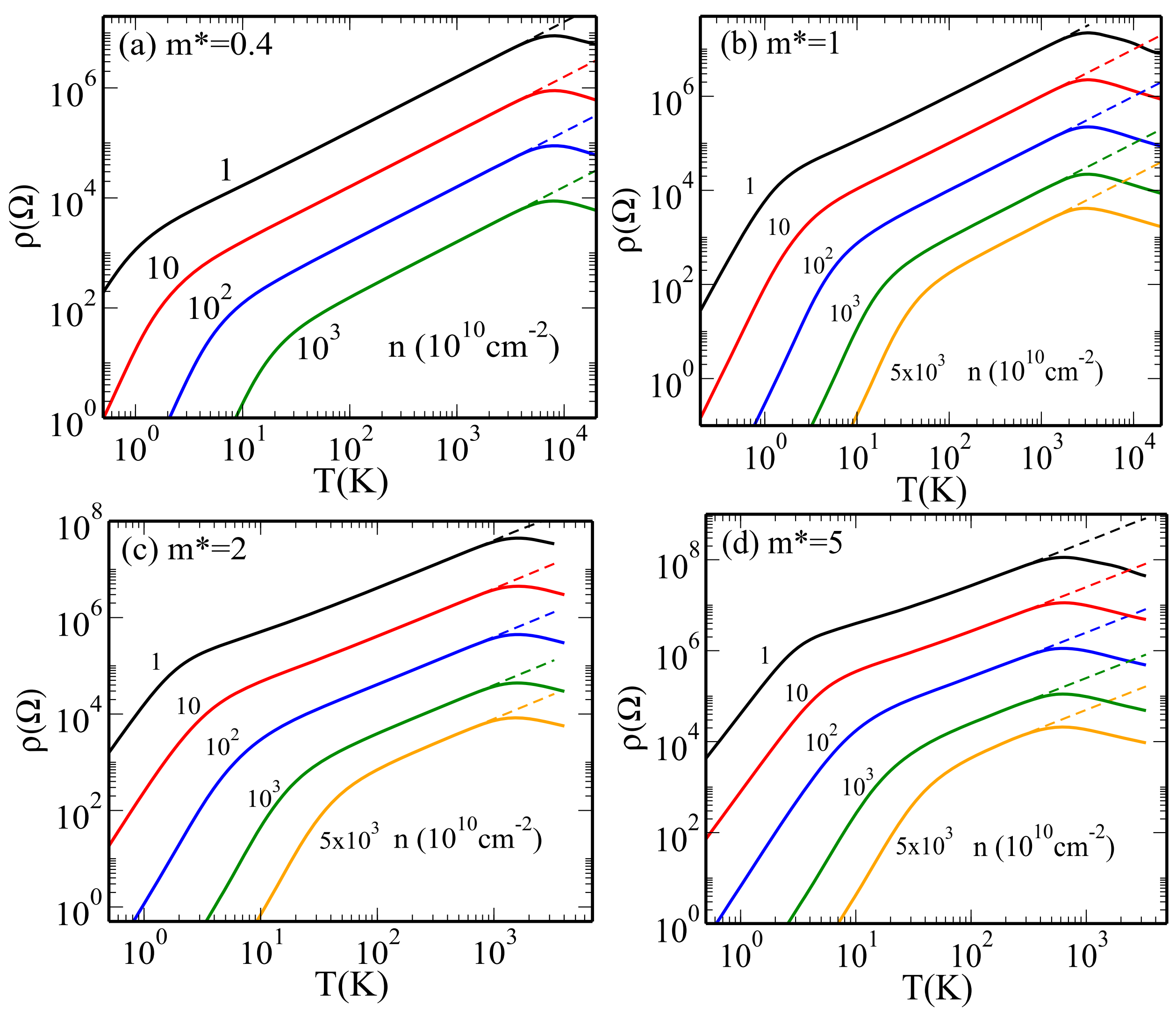
$Sr_3Ru_2O_7$



Kivelson et al., PNAS 2011

Counterpoint:

Phonon-electron scattering in normal diluted metals can give linear resistivity at intermediate temperatures, unrelatedly to quantum criticality



Hwang, Das Sarma PRB 2019

Field-theoretic models for breakdown of FL

- 1D case (Luttinger liquid)
- Orthogonality catastrophe (hidden Fermi liquid)
- Pomeranchuk instability
- Coupling to gauge theory (fundamental or emergent)
- Coupling to critical sector

Anderson's hidden Fermi liquid [Casey, Anderson]

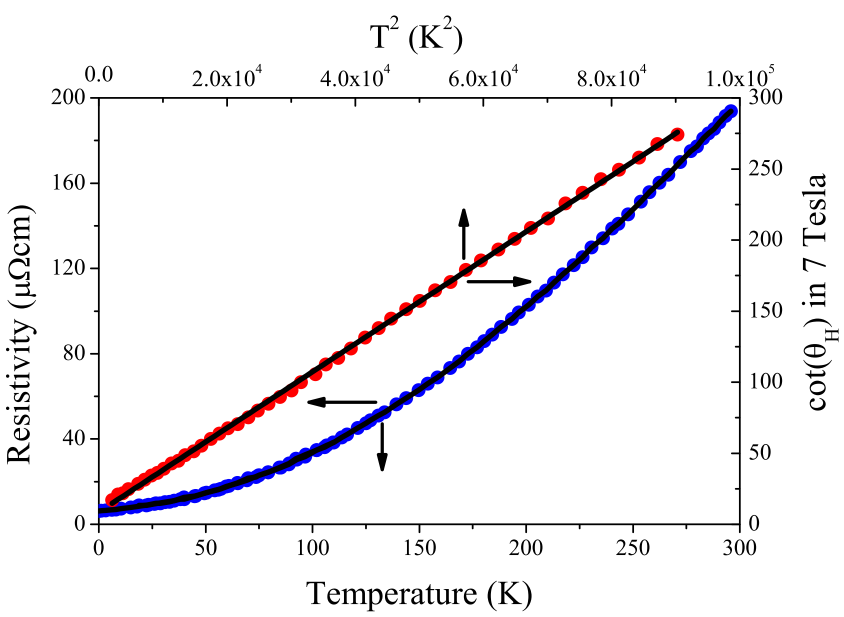
Hubbard model: electrons on a lattice, with hopping, exchange and on-site repulsion

$$H_{eff} = P \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} P = \sum t_{ij} \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} \qquad P = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$$

$\hat{c}$ is the physical electron, whereas the Fermi liquid hides in the unprojected, unphysical Hilbert space

The pseudoparticles scatter off the lattice, the physical particles must decay into the HFL with rate $\Gamma \sim p\pi T$

Linear resistivity, inverse Matthiessen law



Pomeranchuk instability

The collective modes of the FL become unstable for

$$F_l \leq -(2l+1)$$

$$F = F_0 + \sum_k (\tilde{\epsilon}_k - \mu)\delta n_k + \sum_{k,k'} f(k,k')\delta n_k \delta n_{k'} \qquad f(\theta) = \sum F_\ell \cos(\ell\theta)$$

[Davison, Goykhman, Parnachev '13] proposed a connection between the 2-charge 5d STU black hole and a singular FL with

$$F_0 \sim \mathcal{O}(1)\,, \quad F_1 \sim N^{4/3}\,, \quad F_2 = -5 + N^{-2/3}\,, \quad k_F \sim v_F^{-1} \sim N^{2/3}$$

Note that a regular FL cannot have a purely gravitational description since $\frac{\eta}{s} \sim \frac{\mu^3}{T^3}$

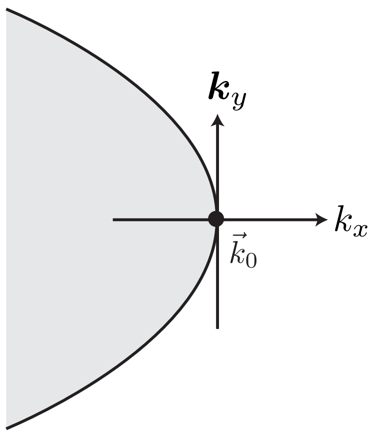
Coupling FL to Maxwell field / massless modes

Transverse photon propagator in the medium

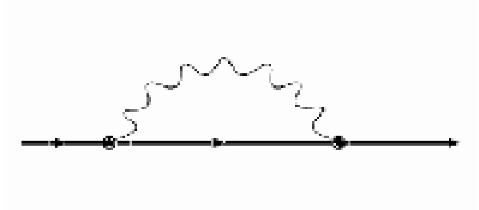
$$D_{ij} = (\delta_{ij} - \frac{q_i q_j}{q^2})\frac{e^2}{\omega^2 - q^2 - 4\pi e^2 M(q,\omega)}$$

$$M = -i\frac{\pi \nu_F v_F}{4}\frac{\omega}{q}$$

Patch theory



Electron self-energy $\quad \Sigma \sim \omega^{2/3}$



The model is always strongly coupled, difficult to solve consistently
The large-N expansion breaks down [Lee, 09]

For transport, it has been argued that the effect of impurities is crucial

Adding disorder [Patel et al, 22]

Same model with N flavors and random Yukawa couplings (inspired by SYK models)

$$\int dt d^d x \sum_{a,b,c} (g_{abc} + g'_{a,b,c}(x)) \phi_a(x) \psi_b(x) \psi_c(x)$$

$$\overline{g_{abc} g_{a'b'c'}} = g^2 \delta_{aa'} \delta_{bb'} \delta_{cc'} \qquad \overline{g'_{abc}(x) g_{a'b'c'}(x')} = g'^2 \delta^d (x - x') \delta_{aa'} \delta_{bb'} \delta_{cc'}$$

$$M(\omega, q) \sim \omega \qquad \Sigma(\omega, k = k_F) \sim \omega \log \omega$$

marginal FL [Varma, 09]

$\rho \sim \Gamma + g'^2 T$ at large N

Emergent U(1): slave boson model

Electrons on a lattice, without double occupancy $\sum_\sigma c_{i\sigma}^\dagger c_{j\sigma} \leq 1$

$$c_{i\sigma} = f_{i\sigma} b_i$$

fractionalization of charge and spin

spinon holon

$$\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1 \quad \Rightarrow \quad \text{Lagrange multiplier} \quad \lambda_i$$

Local U(1) introduced:

$$f_i \to e^{i\phi} f_i$$

$$b_i \to e^{-i\phi} b_i$$

Quantum fluctuations make the U(1) field dynamical

This can also realize a marginal Fermi liquid, but typically suffers from the same large-N problem

Holography

Several features of strange metals point to a connection with holography:

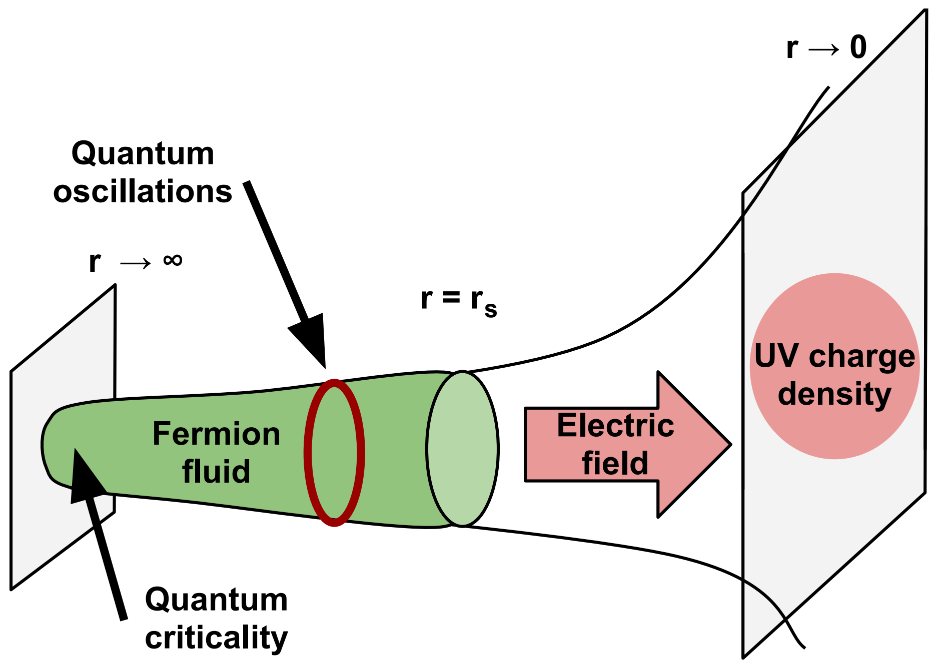
Strong coupling, absence of well-defined quasiparticles

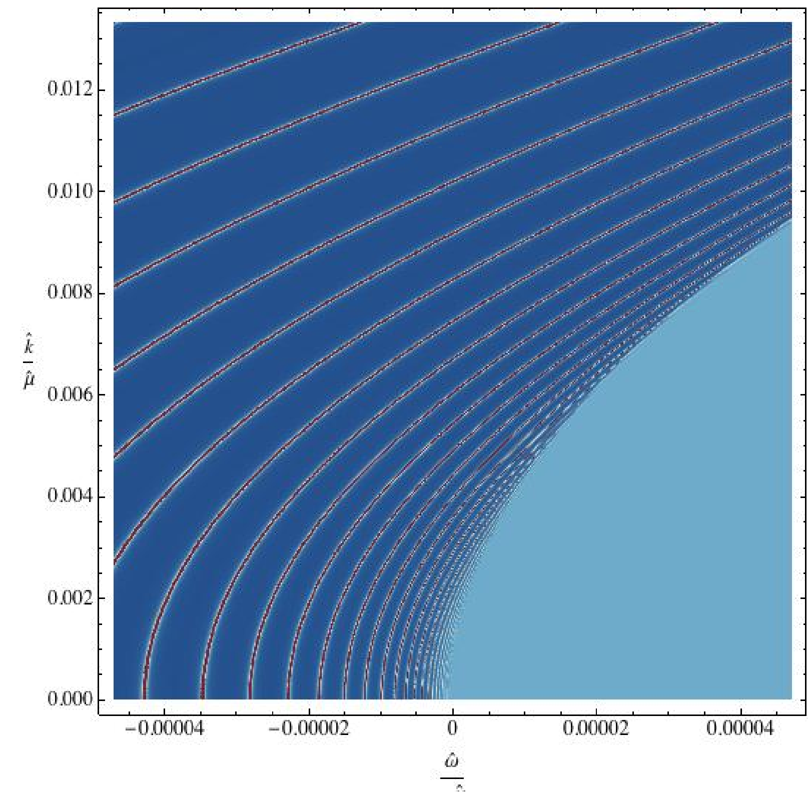
Vicinity of a quantum critical point with non trivial exponents

Saturation of transport bounds

Semi-locality (weak dependence on momentum)

Electron star in AdS [Hartnoll, Tavanfar, Hofman, Vegh '10-'11]





Dual to a boundary system with a large number of FS

Massless excitations interacting with a critical sector described by IR Lifshitz geometry

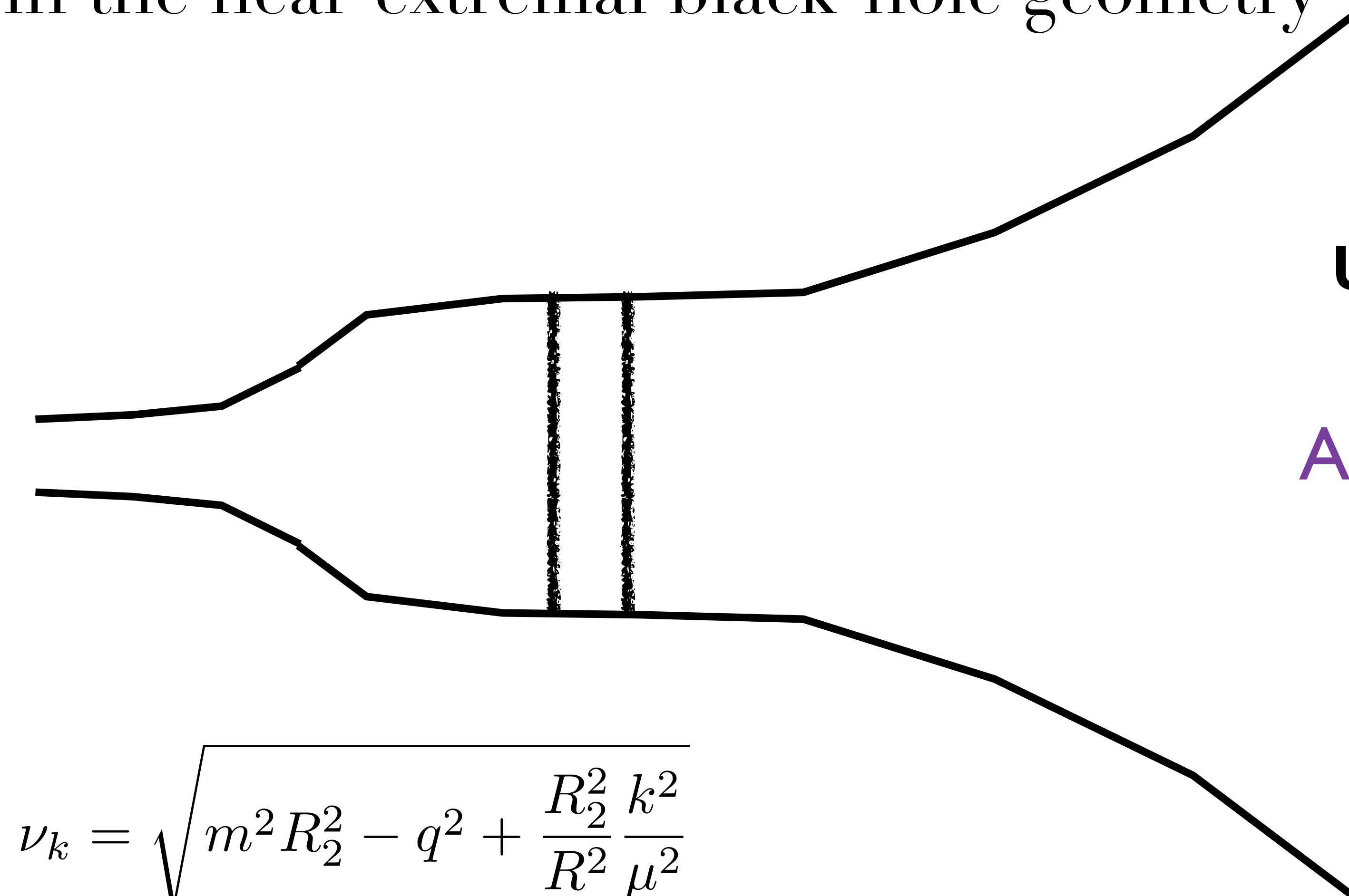
Crossover from FL to NFL at $\omega \sim k^z$

Volume of FS satisfies Luttinger, but the area measured by quantum oscillation is smaller

Holographic non-Fermi liquid

[Liu, McGreevy, Vegh '09]

Probe fermions in the near-extremal black-hole geometry



IR

$AdS_2 \times R^2$

$\psi_k(z,t)\ e^{ikx}$

$$\delta_k = \frac{1}{2} + \nu_k \qquad \nu_k = \sqrt{m^2 R_2^2 - q^2 + \frac{R_2^2}{R^2}\frac{k^2}{\mu^2}}$$

$$\mathcal{G} \propto \omega^{\nu(k)}$$

UV

AdS_4

$\psi(z,t,x)$

m, q

$$\Delta = \frac{3}{2} + mR$$

matching UV and IR

$$G_R(\omega,k) = \frac{A(\omega,k) + B(\omega,k)\mathcal{G}}{C(\omega,k) + D(\omega,k)\mathcal{G}}$$

Fermi surface for

$$C(\omega = 0, k \neq 0) = 0$$

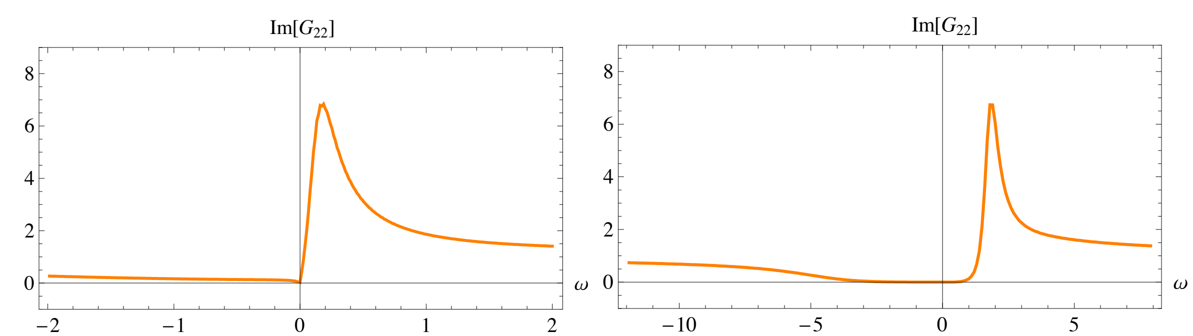
Around the Fermi surface $$G_R(k,\omega) = \frac{A}{k - k_F - \frac{1}{v_F}\omega - \Sigma(\omega,k)}$$

$$\Sigma(\omega,k) \sim c(k)\omega^{\nu} \qquad \nu = \nu_{k_F}$$

$\nu > 1$ Stable quasiparticles but non-FL scattering rate

$\nu < 1$ $\mathrm{Re}\Sigma \approx \mathrm{Im}\Sigma$ no quasiparticles

$\sigma_{dc} \sim T^{-\nu}$ $+O(N^2)$ contribution from other dof



ν free parameter

Solution unstable to the formation of a condensate

The Faulkner-Polchinski model '10

The role of the AdS_2 in the IR is to provide modes that are critical for all momenta, so they scatter efficiently at all angles

UV holographic modes can be replaced by free fermions

$$
\begin{aligned}
S &= \int dt \Big[\sum_k \Big(\chi_{\mathbf{k}}^\dagger (i\partial_t - \epsilon_{\mathbf{k}} + \mu) \chi_{\mathbf{k}} + \sum_k (g_{\mathbf{k}} \chi_{\mathbf{k}}^\dagger \psi_{\mathbf{k}} + c.c. \Big) \Big] \\
&+ S_{\mathrm{CFT}}
\end{aligned}
$$

Hybridization of the free fermions with CFT operators of dimension $\Delta_\psi = \frac{\nu + 1}{2}$

Resummed propagator reproduces the holographic form

The interacting F-P model

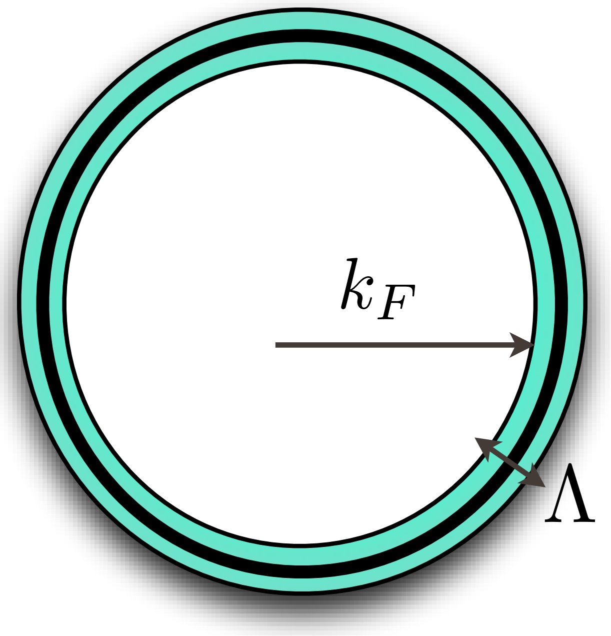
[Mukhopadhyay, GP '13]

$$
\begin{aligned}
S &= N^2 S_{\rm CFT} \\
&+ \int dt \Big[\sum_k \Big(\chi^\dagger_{\mathbf{k}} (i\partial_t - \epsilon_{\mathbf{k}} + \mu) \chi_{\mathbf{k}} + N \sum_k (g_{\mathbf{k}} \chi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + c.c.\Big) \\
&+ N\eta \sum \chi^\dagger \chi \phi + N\tilde{g} \sum \chi^\dagger \chi \chi^\dagger \psi \\
&+ \sum (\lambda + V(\mathbf{q}))\, \chi^\dagger \chi\, \chi^\dagger \chi
\end{aligned}
$$

The factors of N are chosen to reproduce the FP propagator at O(1) and suppress corrections to the vertex functions

RG analysis similar to the FL case shows these are the only potentially relevant interactions

Renormalization group with a FS [Polchinski, Sarkar]



$\Lambda \to s\Lambda \qquad \mathbf{k} = \mathbf{k}_* + \mathbf{k}_\perp$

$[\mathbf{k}_*] = 0 \qquad [\mathbf{k}_\perp] = 1 \qquad [\chi] = -\frac{1}{2}$

4-fermion interaction $\quad \int dt\, d\mathbf{k}_*^i\, d\mathbf{l}^i\, \chi_1^\dagger \chi_2 \chi_3^\dagger \chi_4\, \delta^d(\sum \mathbf{k}_*^i + \sum \mathbf{l}_*^i)$

Generically of dimension 1 *irrelevant*

When $\sum \mathbf{k}_*^i = 0$ *marginal*

Only instability, leading to BCS superconductivity

Lindhard function

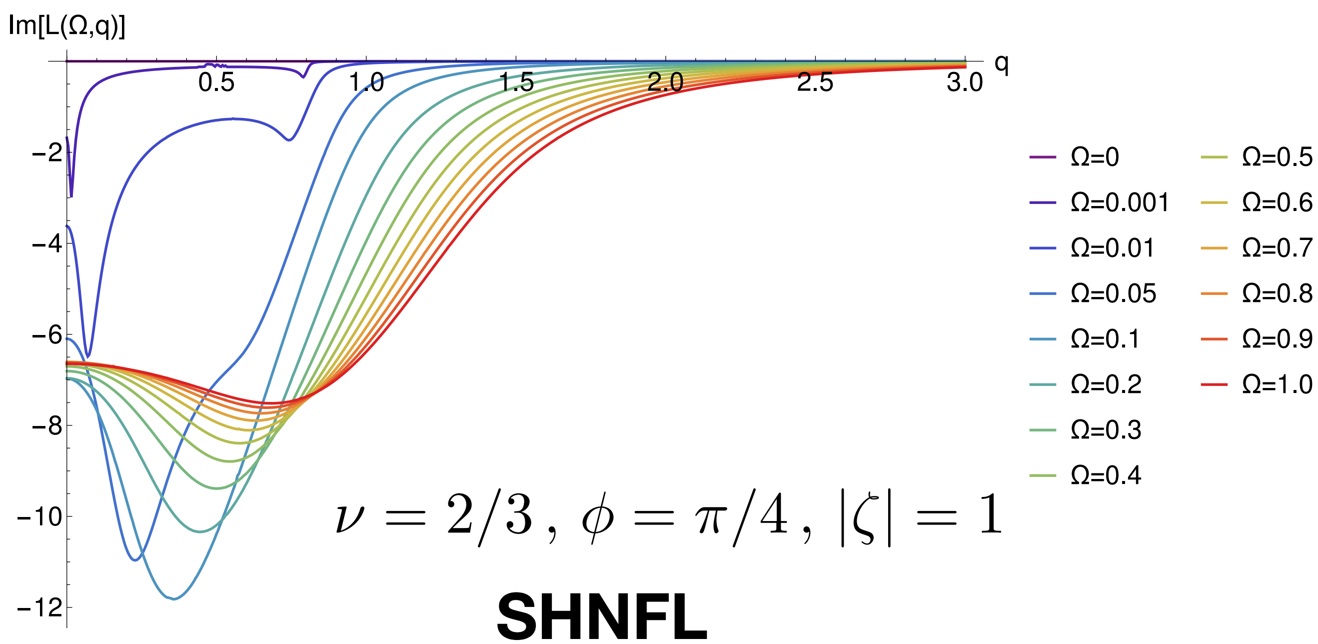
[Doucot, Ecker, Mukhopadhyay, GP ’17]

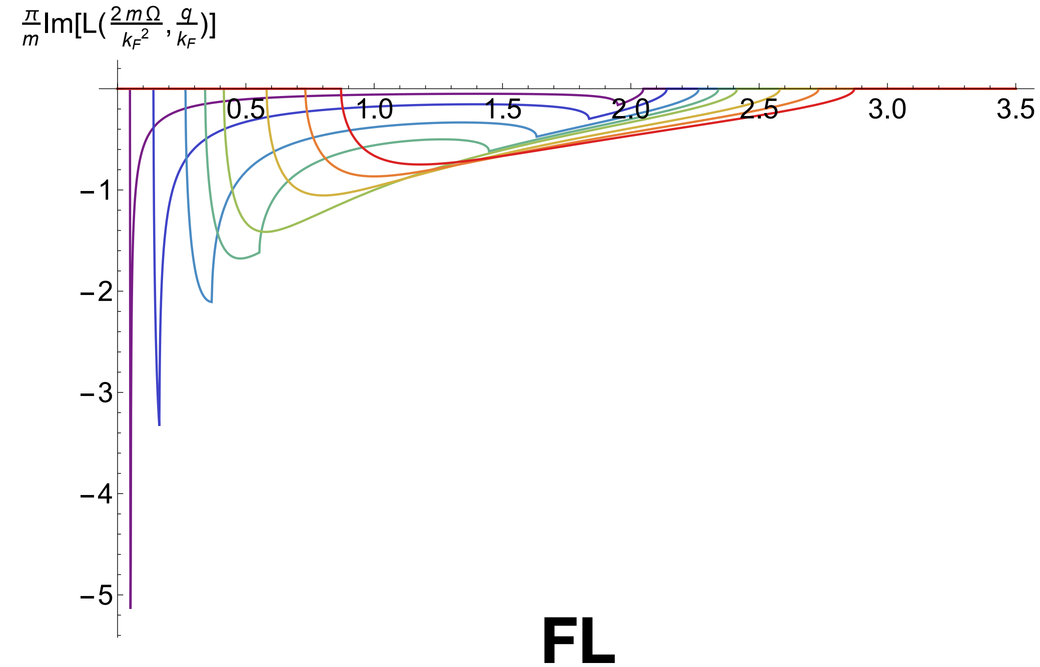
$$G_R(\omega, \mathbf{k}) = \frac{1}{\zeta\omega^\nu - \epsilon_\mathbf{k}}, \quad \epsilon_\mathbf{k} = \frac{\mathbf{k}^2}{2m} - \frac{k_F^2}{2m} \qquad \zeta = |\zeta| e^{i\phi} \qquad 0 < \phi < \pi(1-\nu)$$

$$\frac{1}{2} < \nu < 1$$

$$\mathcal{L}(\Omega, \mathbf{q}) = -2i \int_\mathbf{k} \int_\omega G_F(\omega_+, \mathbf{k}_+)\, G_F(\omega_-, \mathbf{k}_-) \qquad \omega_\pm = \omega \pm \frac{\Omega}{2}, \quad \mathbf{k}_\pm = \mathbf{k} \pm \frac{\mathbf{q}}{2}.$$

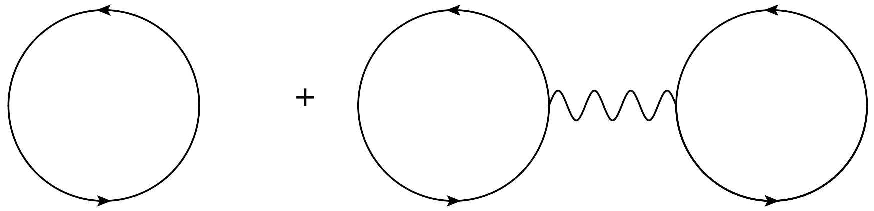
gives the 1-loop correction to the photon propagator

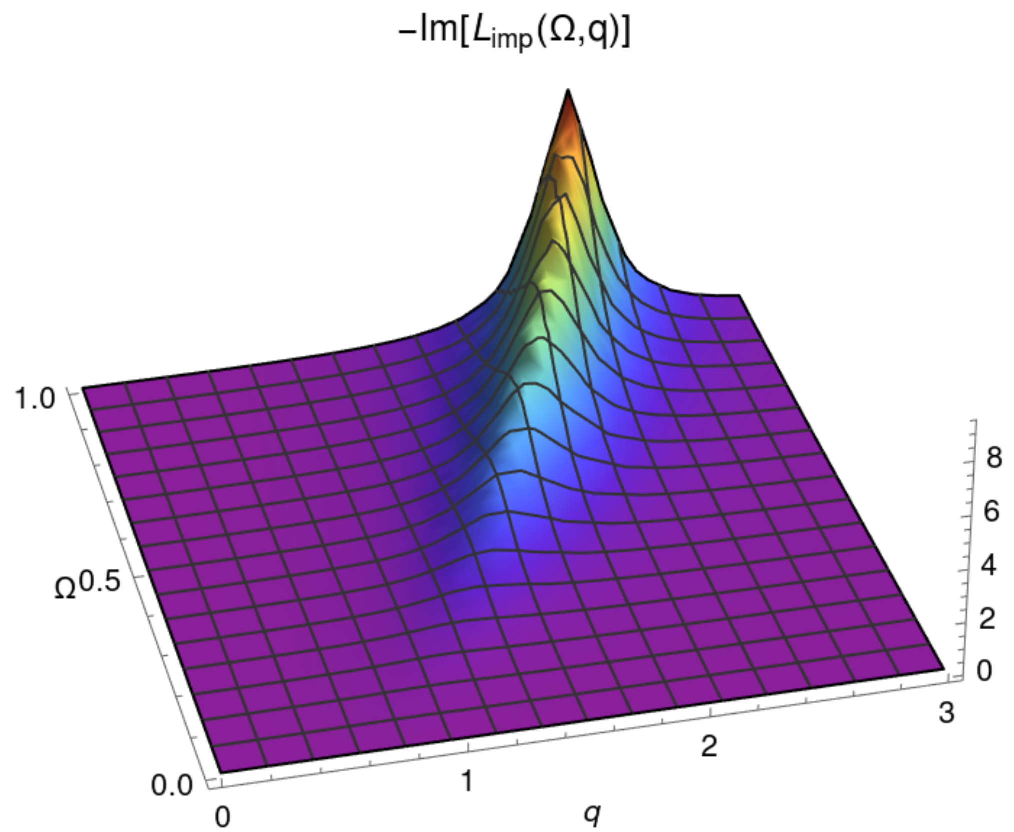


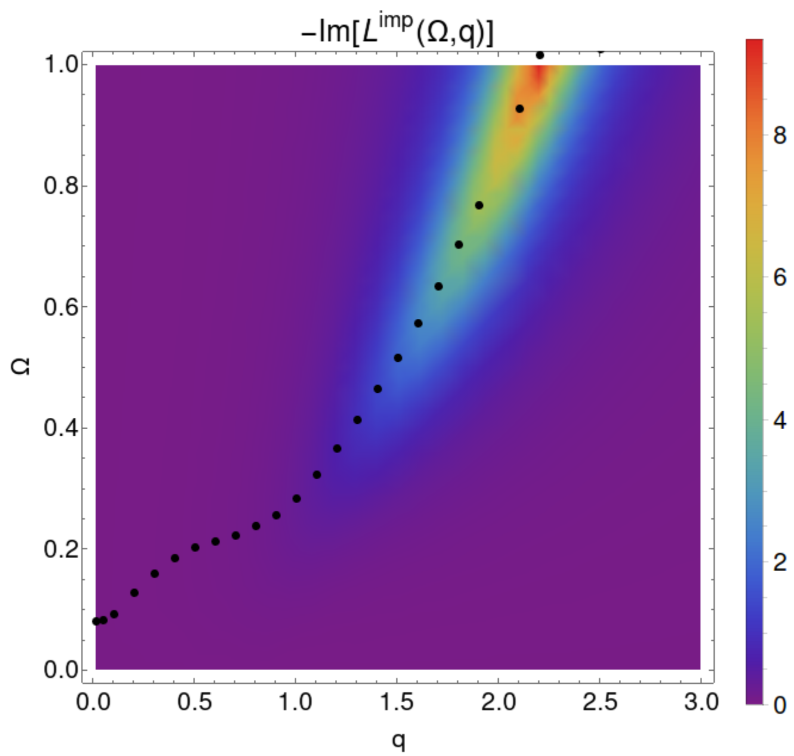


Improved Lindhard function

$$\mathcal{L}^{\mathrm{imp}}(q,\Omega) = \frac{\mathcal{L}(q,\Omega)}{1 - V(q)\mathcal{L}(q,\Omega)}$$



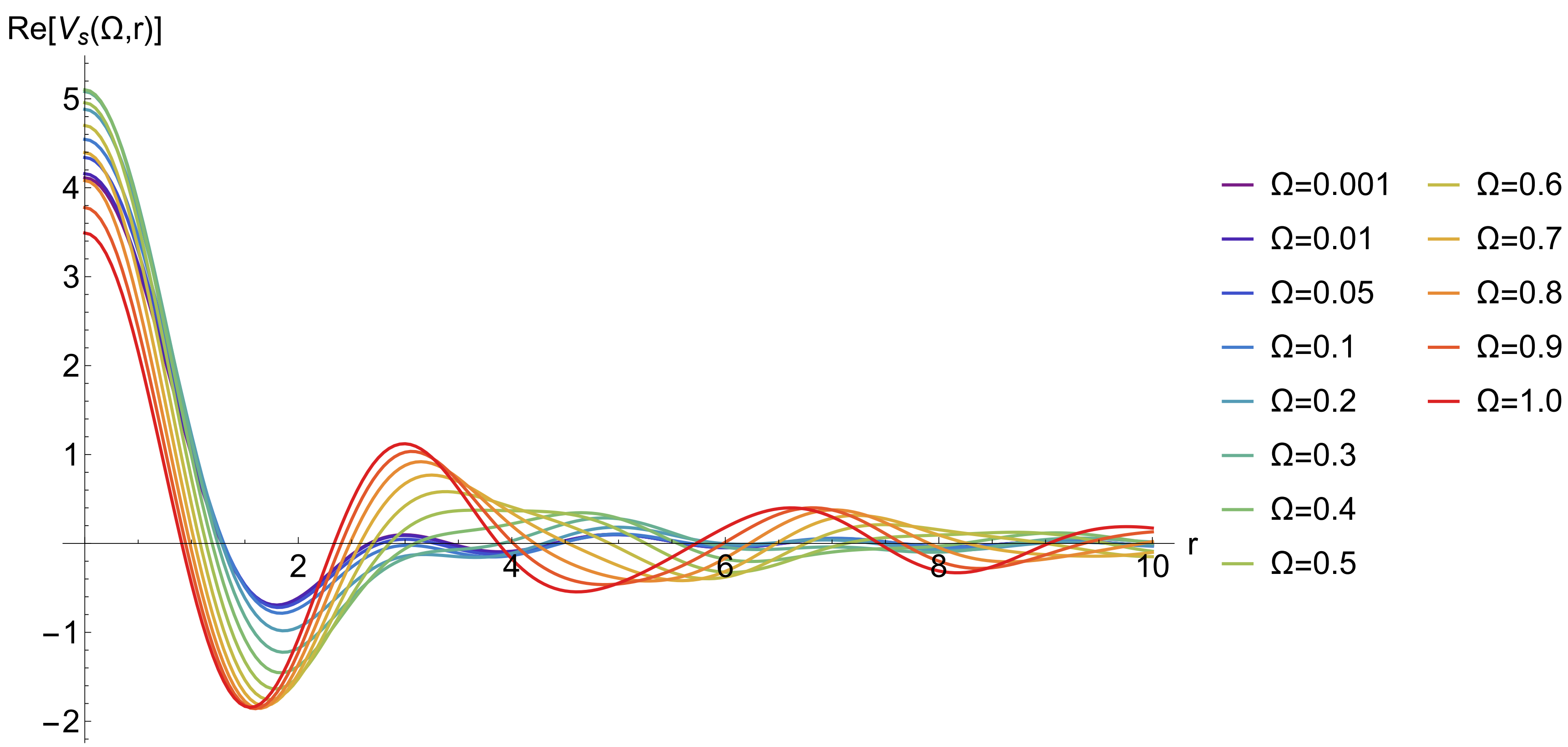




Collective mode inside the particle-hole continuum!

In FL this does not exist because of Landau damping

Screened Coulomb potential



Dynamical attraction at finite distance

Possible Kohn-Luttinger superconductivity

The model is not UV complete, some observables show dependence on the cutoff

Improved model

[Doucot, Mukhopadhyay, GP, Samanta '20]

Hybridization with another species of free fermions (interactions with other bands)

$$S = S[\chi, \psi] + S[f] + \lambda \int \chi^\dagger f f^\dagger f + c.c.$$

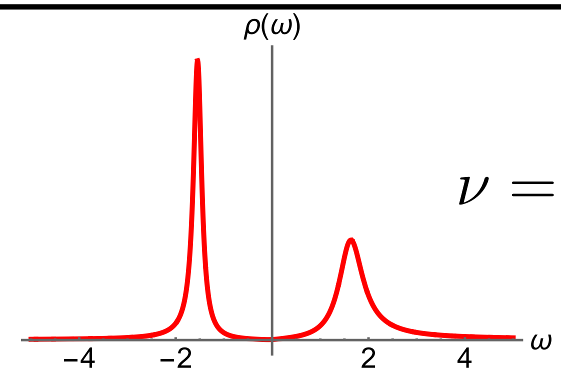
At leading order in λ

$$G^{-1}(\omega, \mathbf{k}) = \omega + i\gamma(\omega^2 + \pi^2 T^2) + \alpha \mathcal{G}(\omega, T) - (\mathbf{k}^2 - 1)$$

$$\mathcal{G}(\omega, T) = e^{i(\phi + \pi\nu/2)} (2\pi T)^\nu \frac{\Gamma(\frac{1}{2} + \frac{\nu}{2} - i\frac{\omega}{2\pi T})}{\Gamma(\frac{1}{2} - \frac{\nu}{2} - i\frac{\omega}{2\pi T})} \sim \begin{cases} \omega^\nu\,, & T \to 0 \\ T^\nu\,, & \omega \to 0 \end{cases}$$

$E_F = 1$

Parameters $\quad \alpha, \gamma, \nu, \phi$

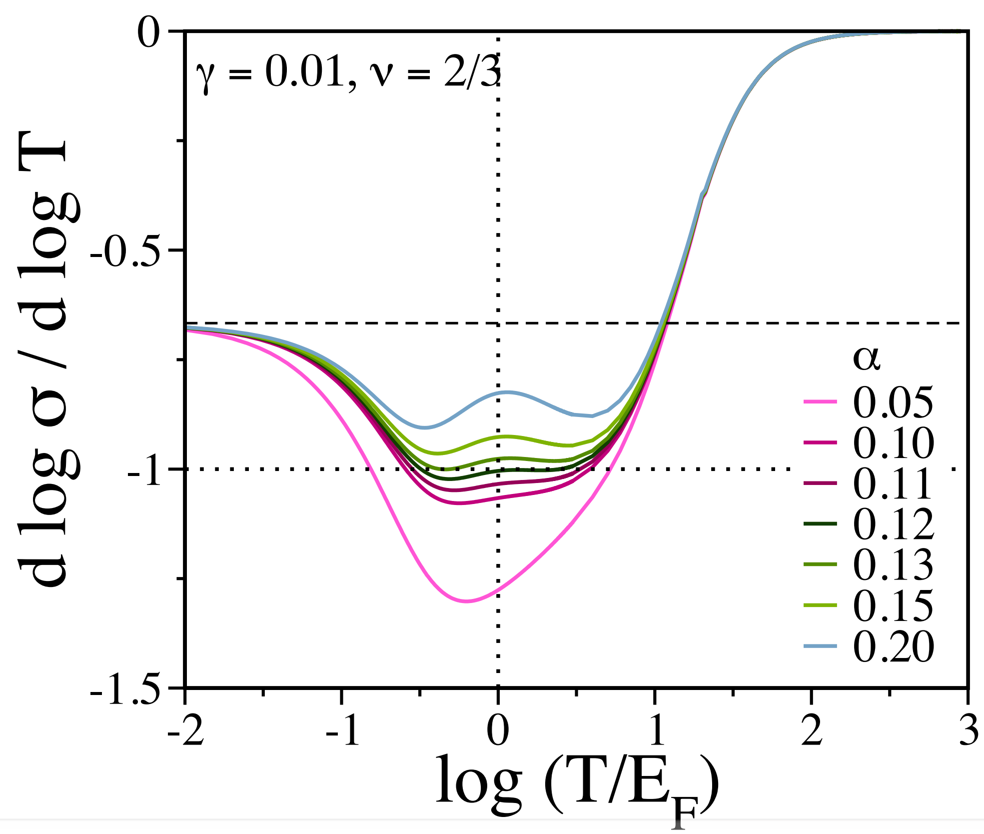


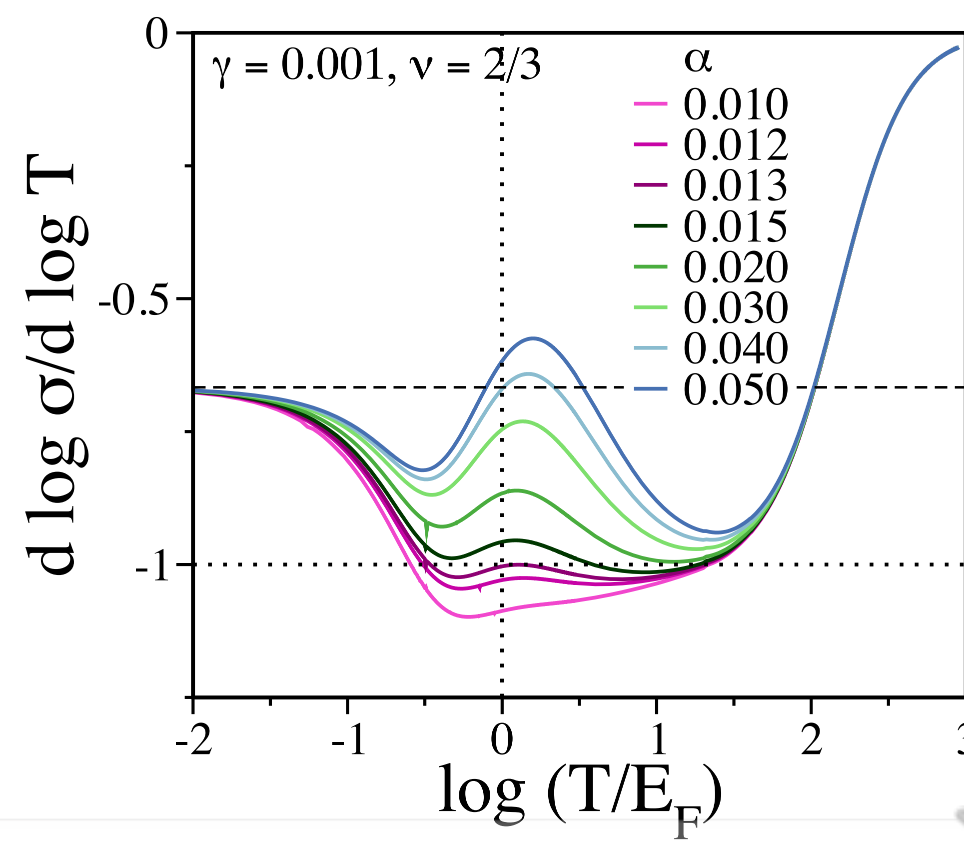
$\nu = 2/3 \quad \gamma = 0.001\,, \alpha = 0.35$

$T = 0.05 E_F$

DC conductivity

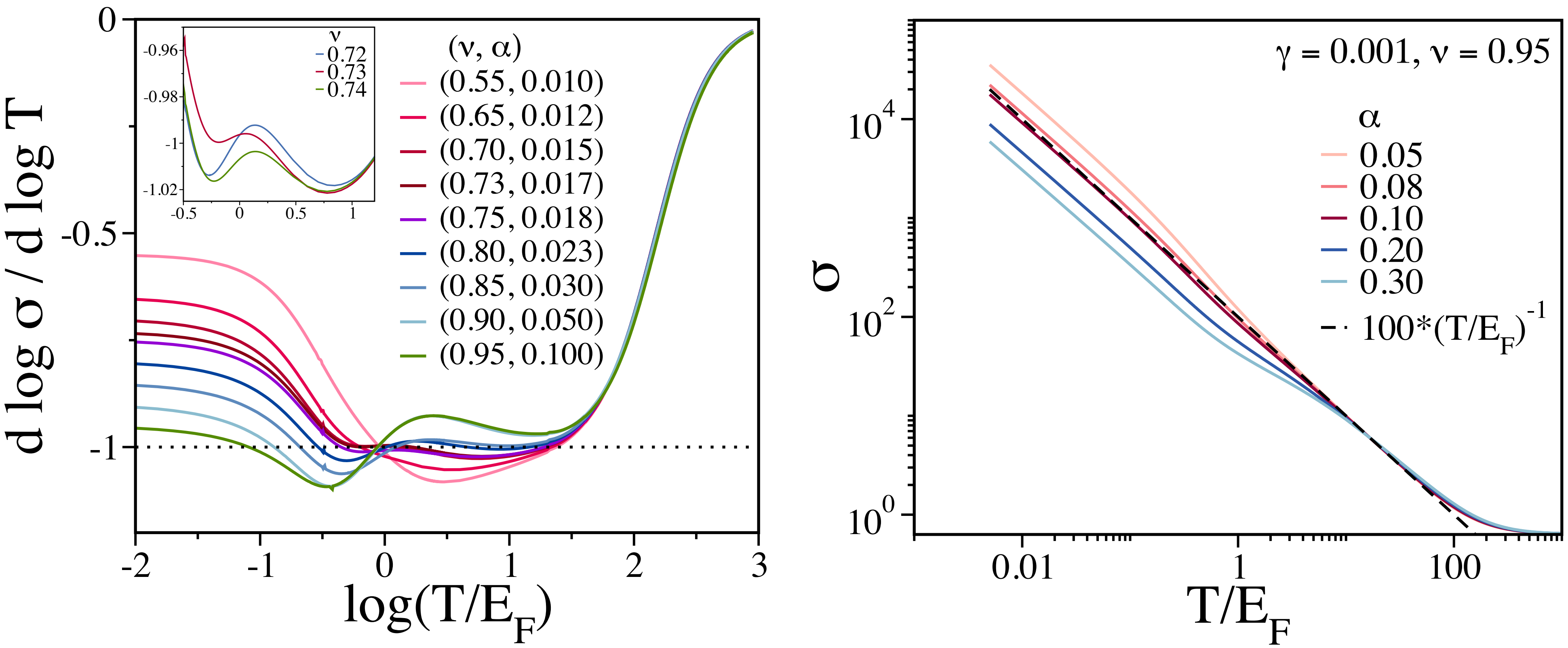
$$\sigma_{DC} = -\frac{e^2}{2\hbar}\int \frac{d\omega}{2\pi}\int \frac{d^2k}{4\pi^2} k^2 \rho(\omega,k,T)^2 \frac{\partial n_F(\omega,T)}{\partial \omega}$$



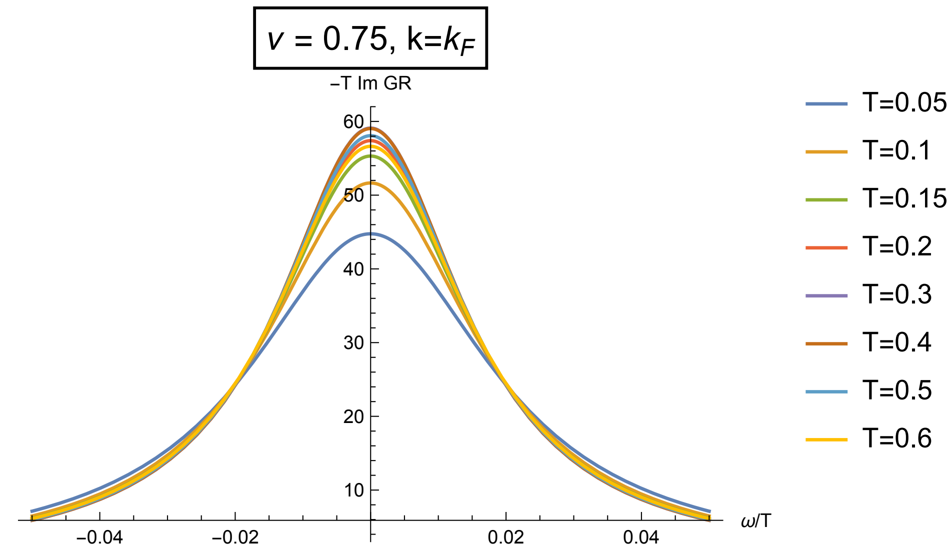


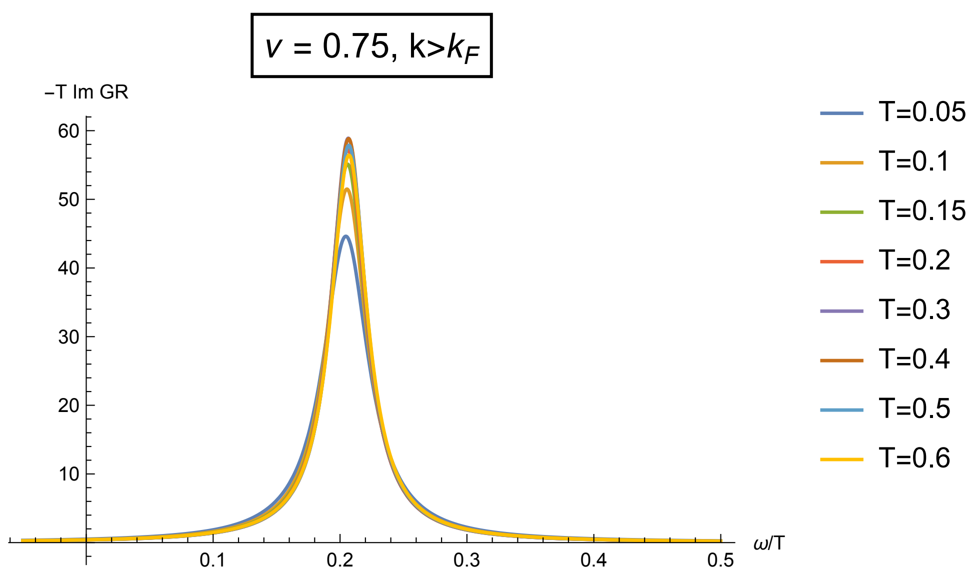
Emergent linear-in-T resistivity at intermediate temperatures, for a range of values of the critical exponent $\nu \gtrsim 2/3$

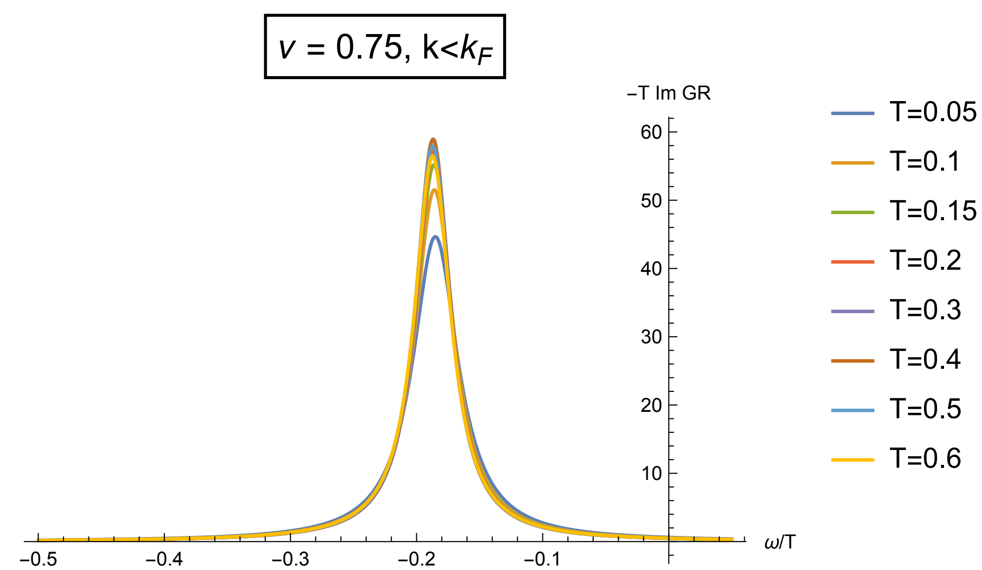
However fine tuning of $\alpha/\gamma \approx 13$



Approximate scale invariance of the spectral function

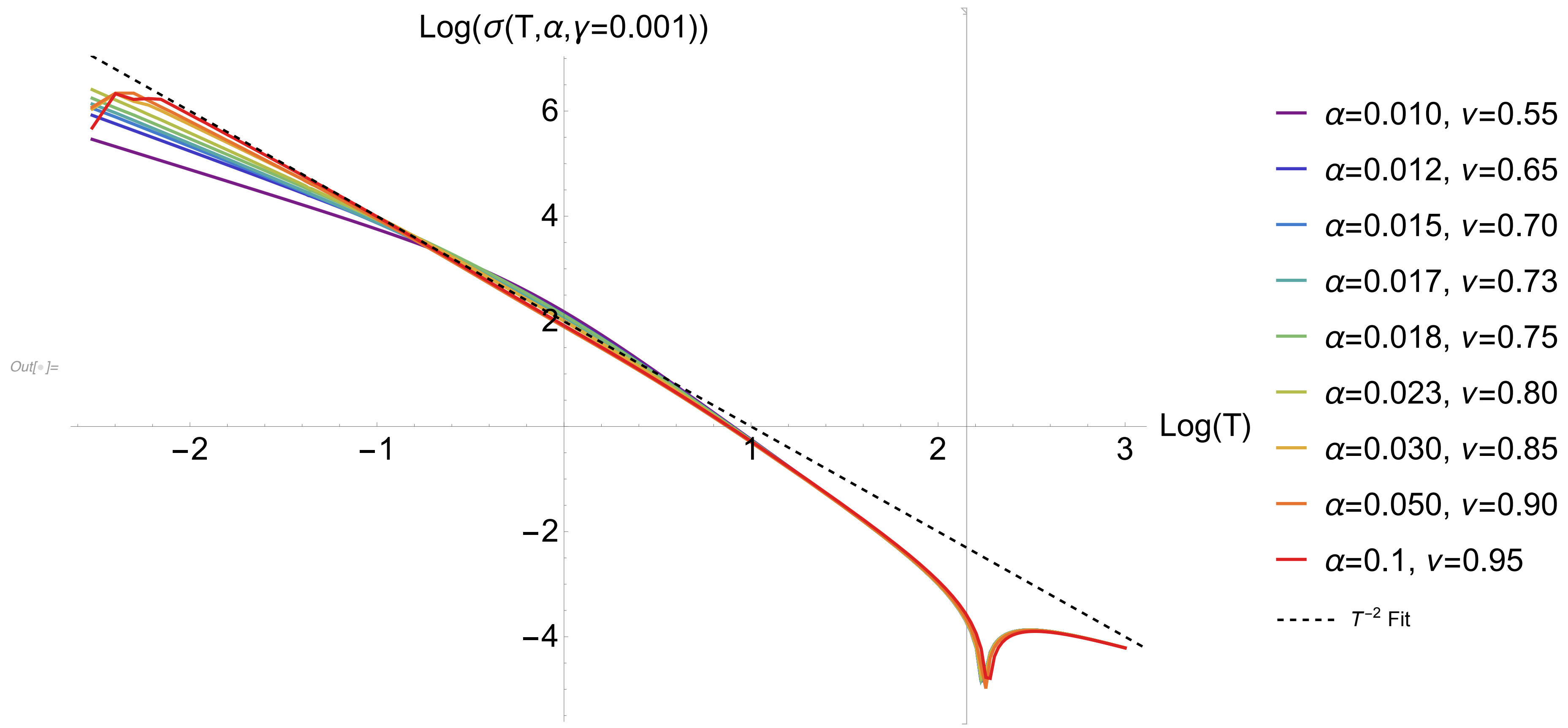




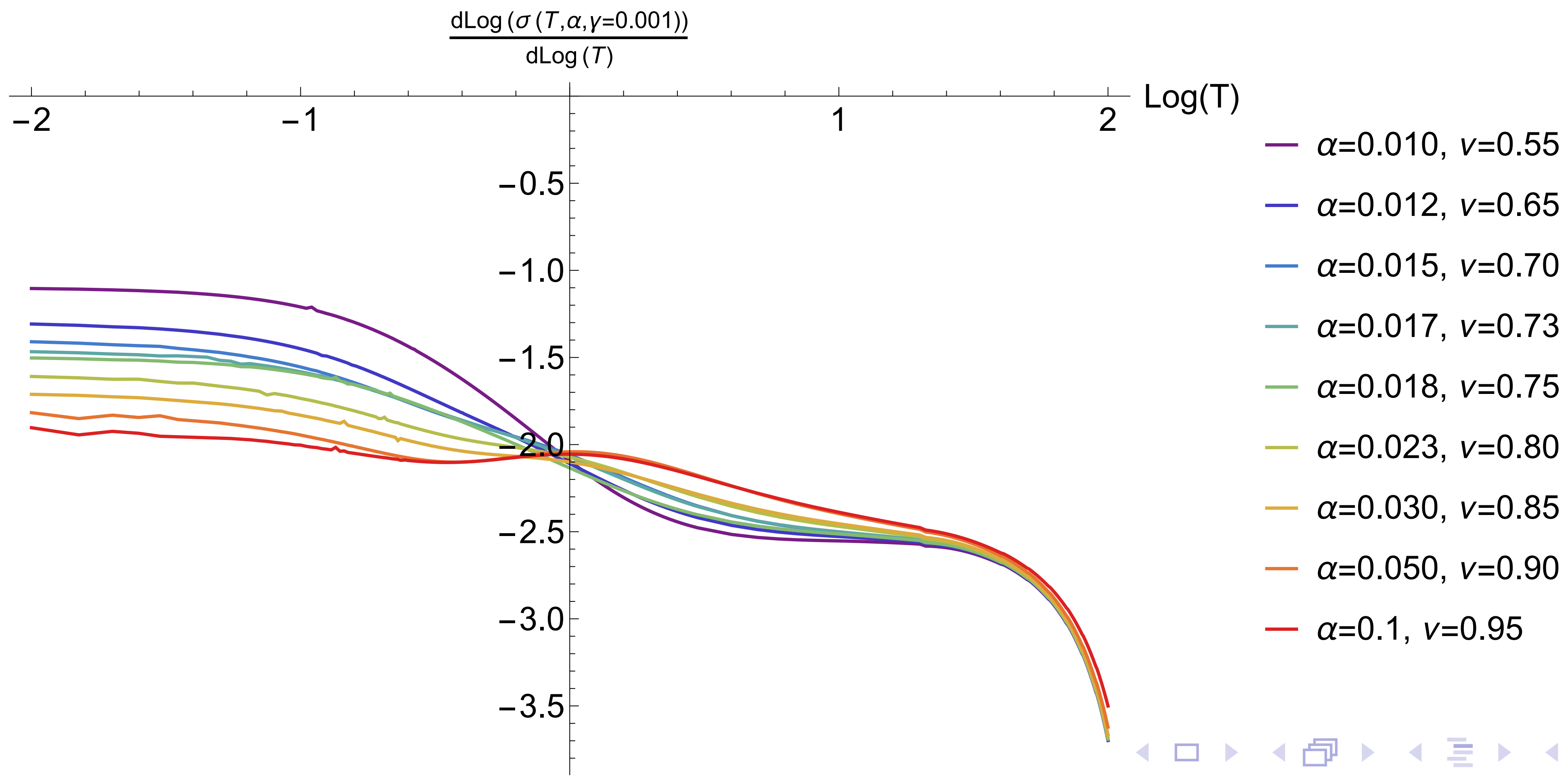


Hall conductivity

$$\sigma_{xy} = \int \frac{d\omega}{2\pi} \int \frac{d^2k}{4\pi^2}\, \rho(\omega, k, T) \frac{\partial\, \mathrm{Re} G_R(\omega, k)}{\partial k_x} \frac{\partial n_F(\omega, T)}{\partial \omega}$$



Consistent with T^{-2} scaling

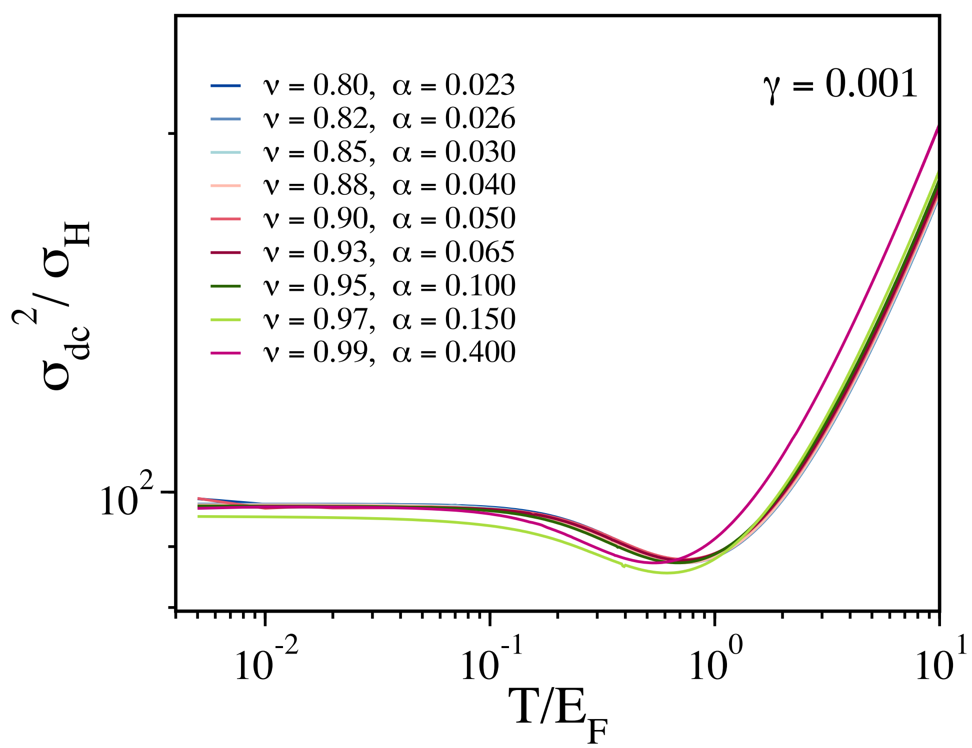


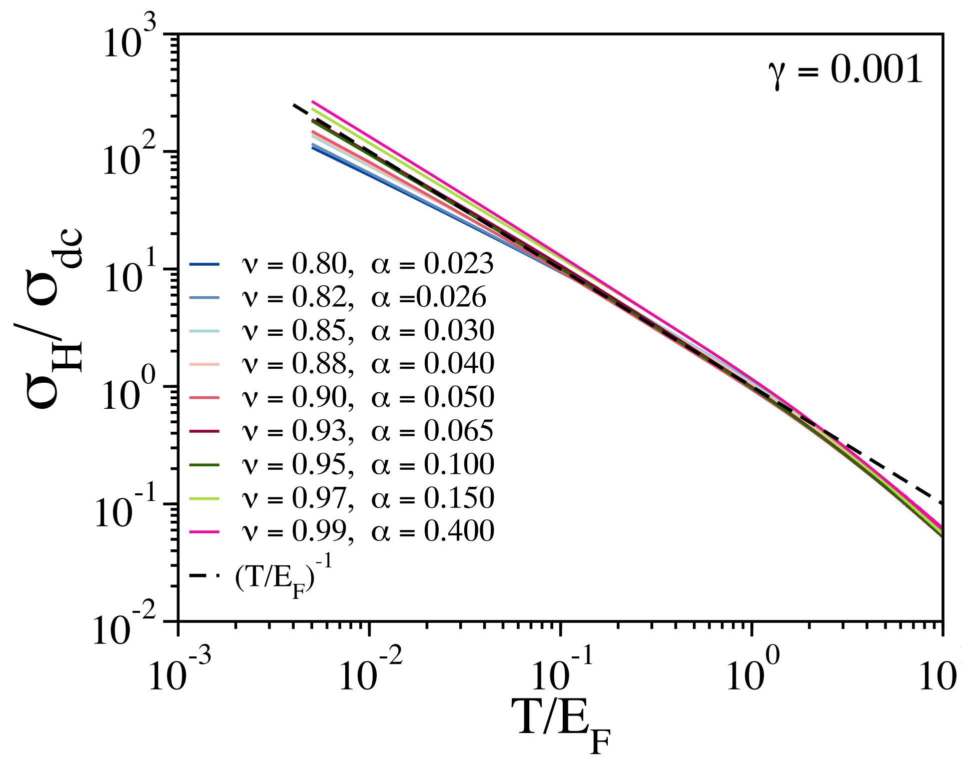
dLog (σ (T,α,γ=0.001))
dLog (T)
Log(T)
−2
−1
1
2
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
α=0.010, v=0.55
α=0.012, v=0.65
α=0.015, v=0.70
α=0.017, v=0.73
α=0.018, v=0.75
α=0.023, v=0.80
α=0.030, v=0.85
α=0.050, v=0.90
α=0.1, v=0.95

Using Drude model phenomenology we can estimate the carrier density and relaxation time (to be interpreted with caution , see 2205.13382)

$$\sigma_{dc} = \frac{ne^2}{m}\tau \qquad \sigma_H = \frac{ne^2}{m}\omega_c \tau^2$$

$$n \propto \frac{\sigma_{dc}^2}{\sigma_H} \sim \text{const} \qquad \tau \propto \frac{\sigma_H}{\sigma_{dc}} \sim T^{-1}$$





$$f = \frac{\tau}{\tau_P} \sim 1 \div 10$$

Summary

We considered a phenomenological semiholographic model that combines the holographic IR sector with a UV given by band structure

Not (yet) a realistic model of strange metals (no lattice, no pseudogap)

Plasmonic excitations at intermediate energy

Linear resistivity at intermediate temperatures without tuning the exponent

Future directions

Fully characterize the phenomenology
(Magnetoresistance, optical conductivity)

Connection between doping and the model's parameters

Superconducting instability

Interaction with AF magnetic order, anisotropy

Effect of adding disorder

Connection with SYK-type models (coupling to a lattice of near-AdS2)