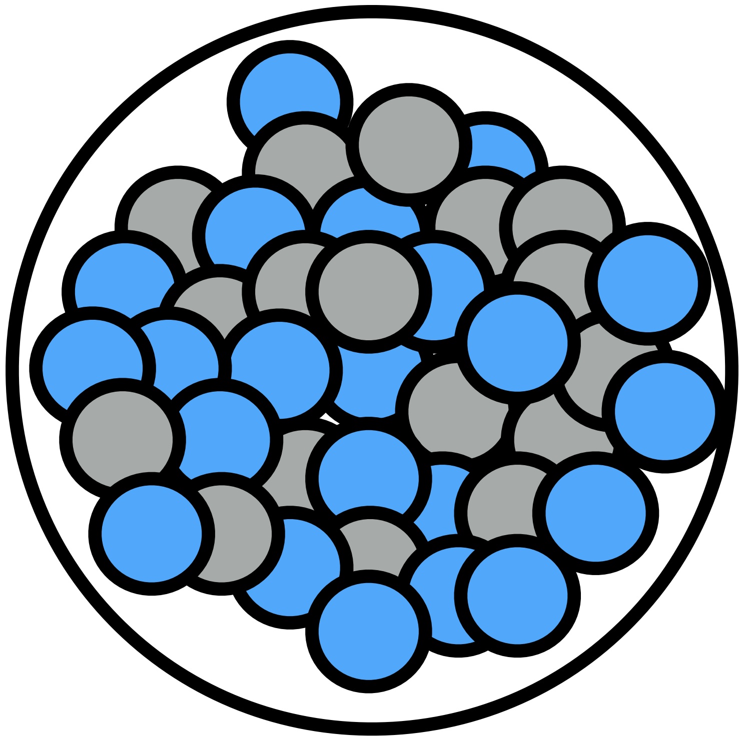
Hydrodynamics with spin

A. Yarom

Together with A. D. Gallegos and U. Gursoy

Motivation:



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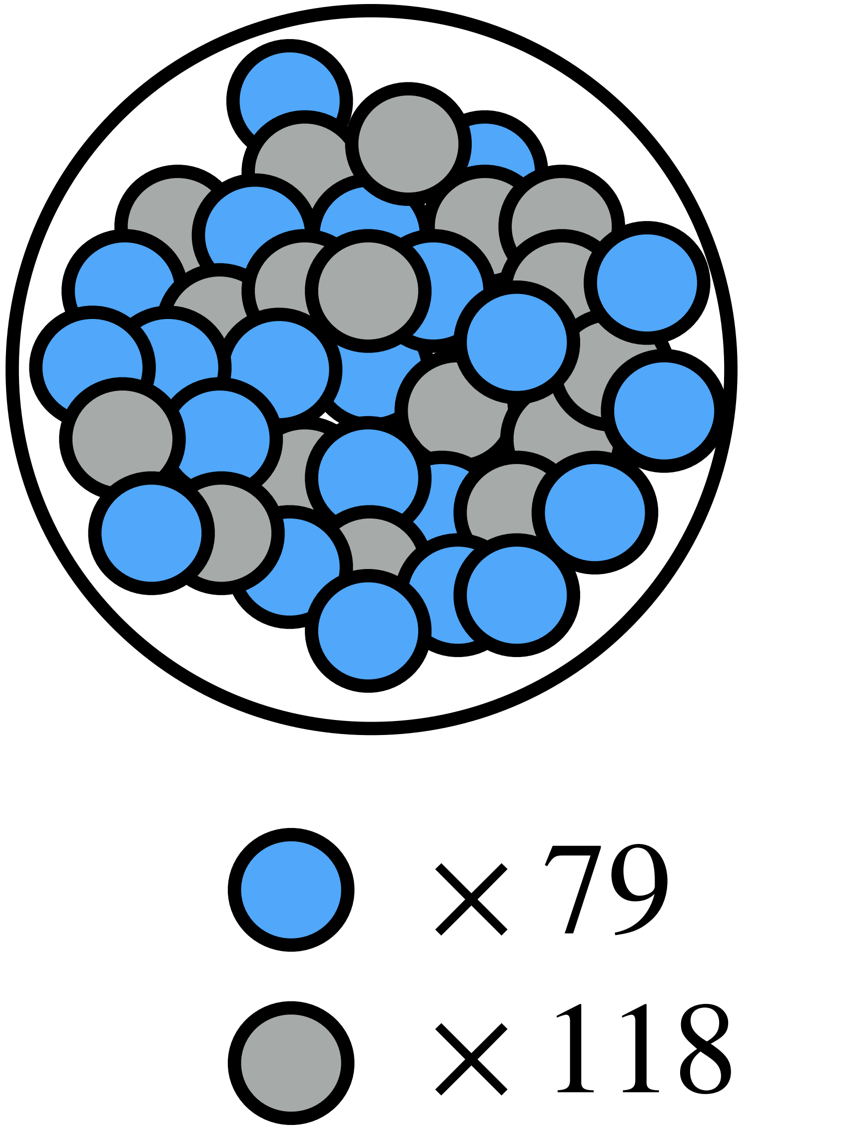

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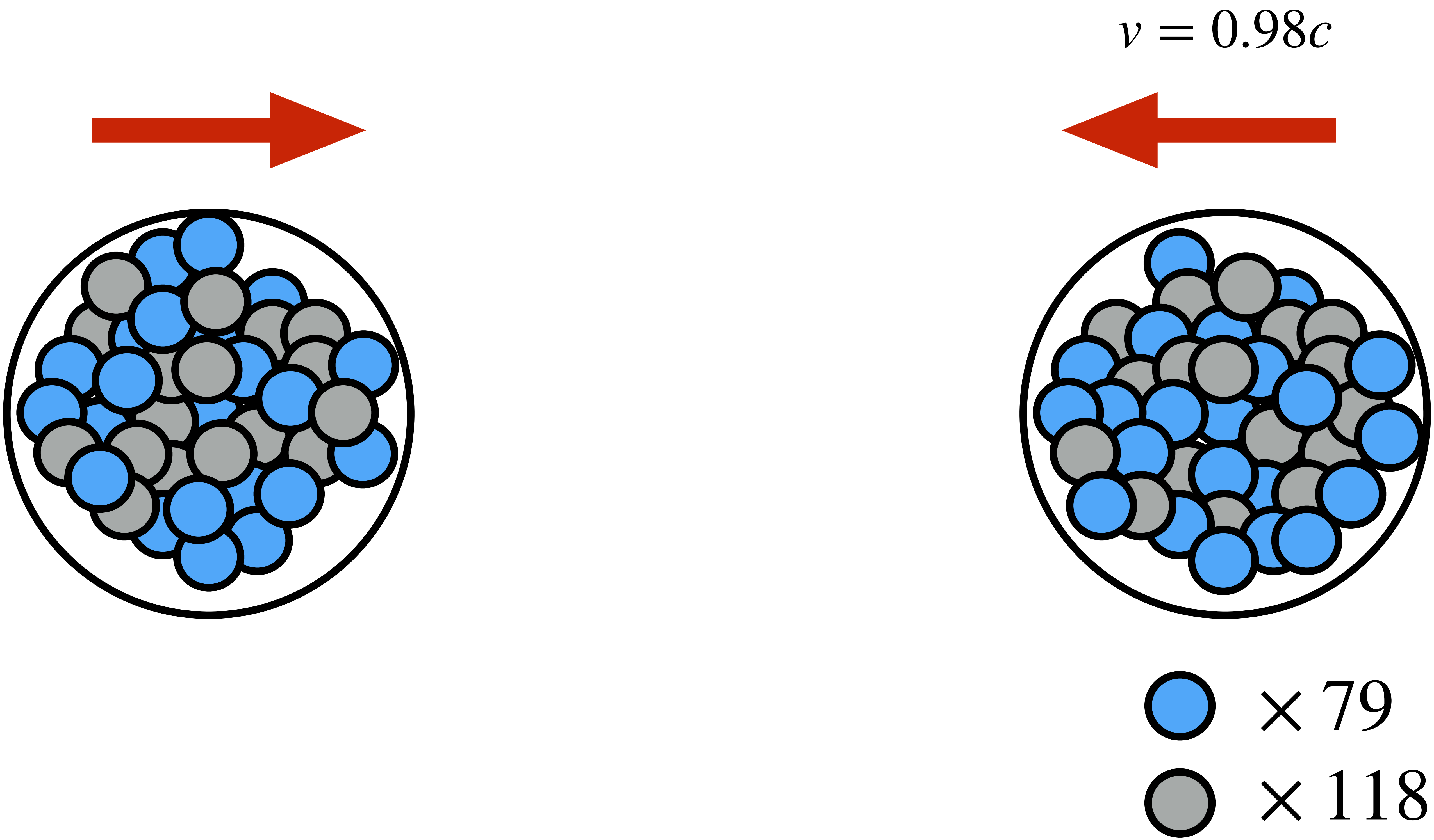

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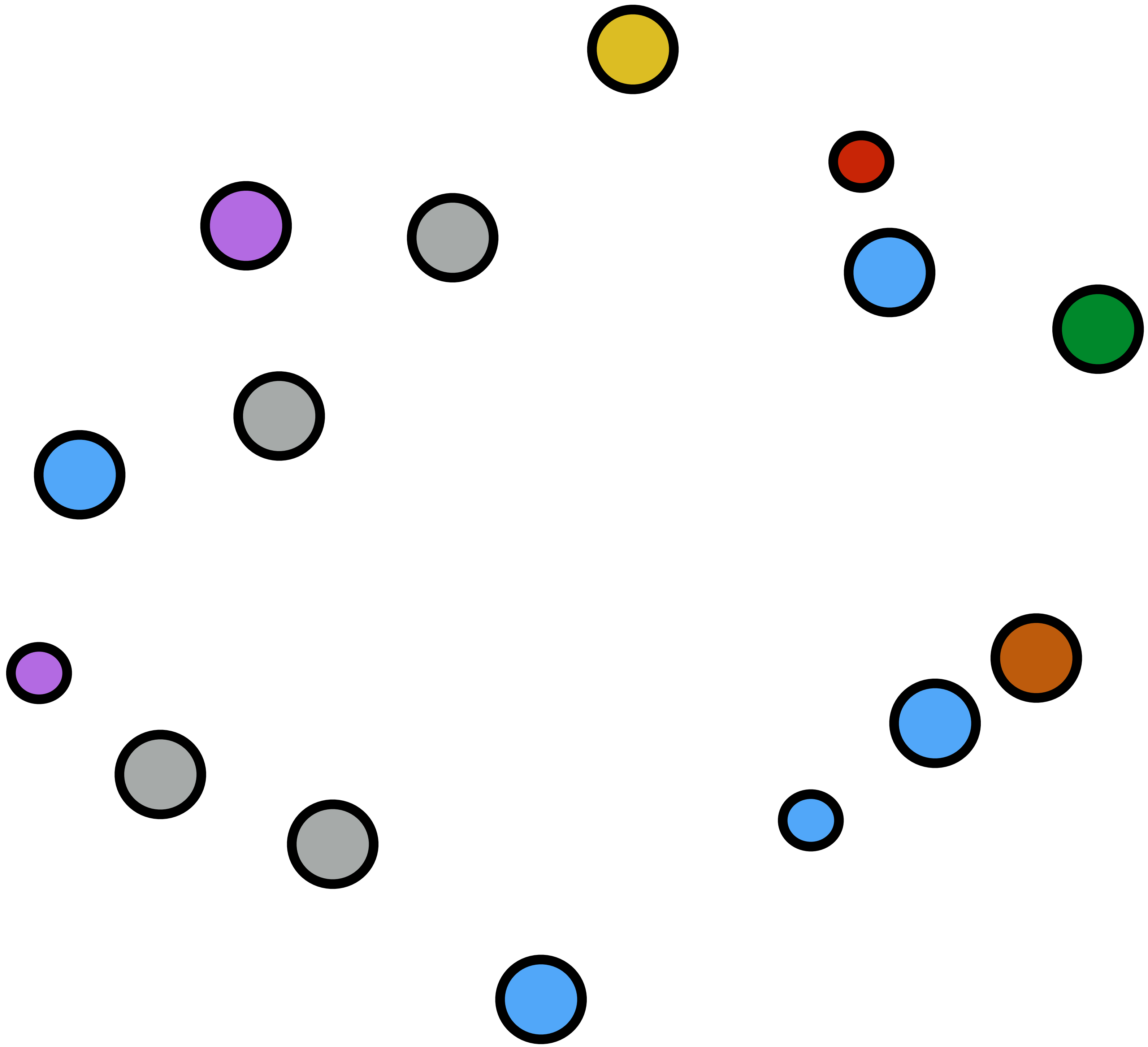

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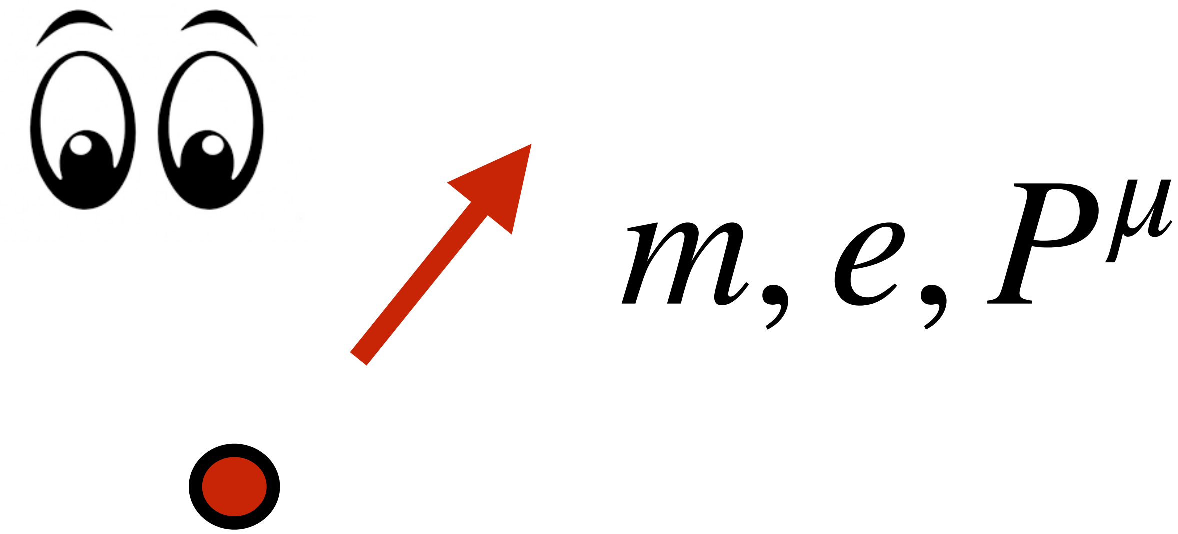

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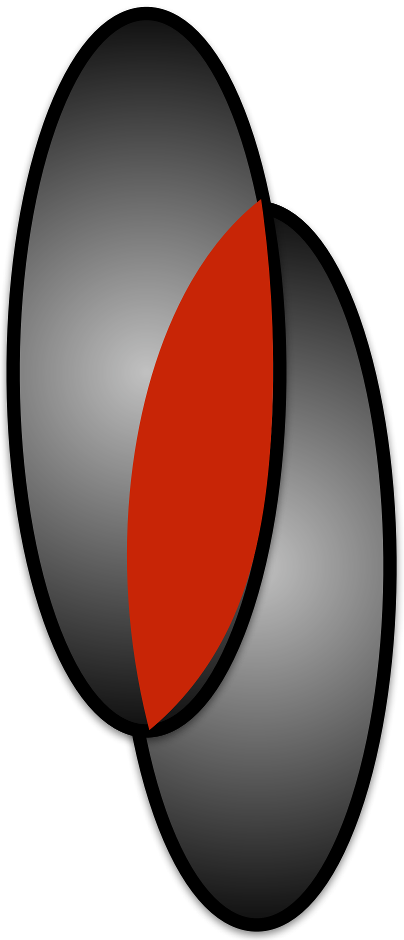

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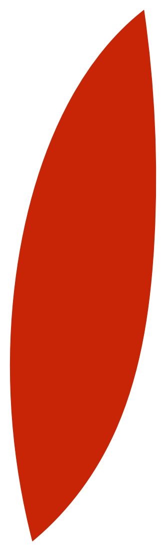

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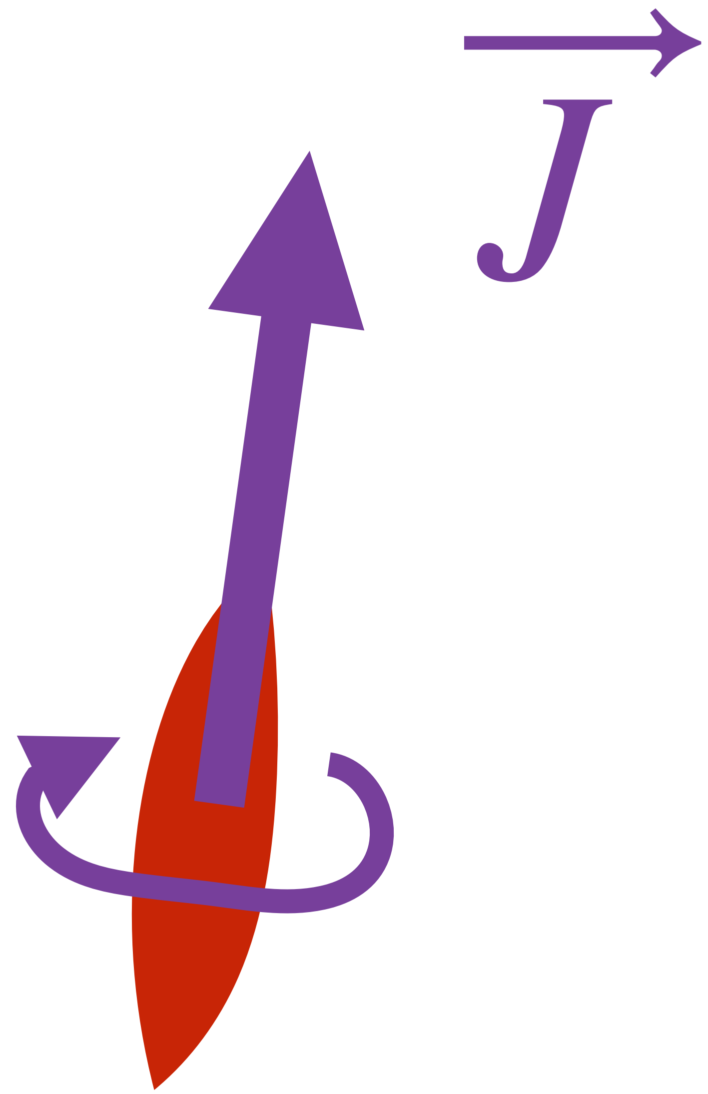

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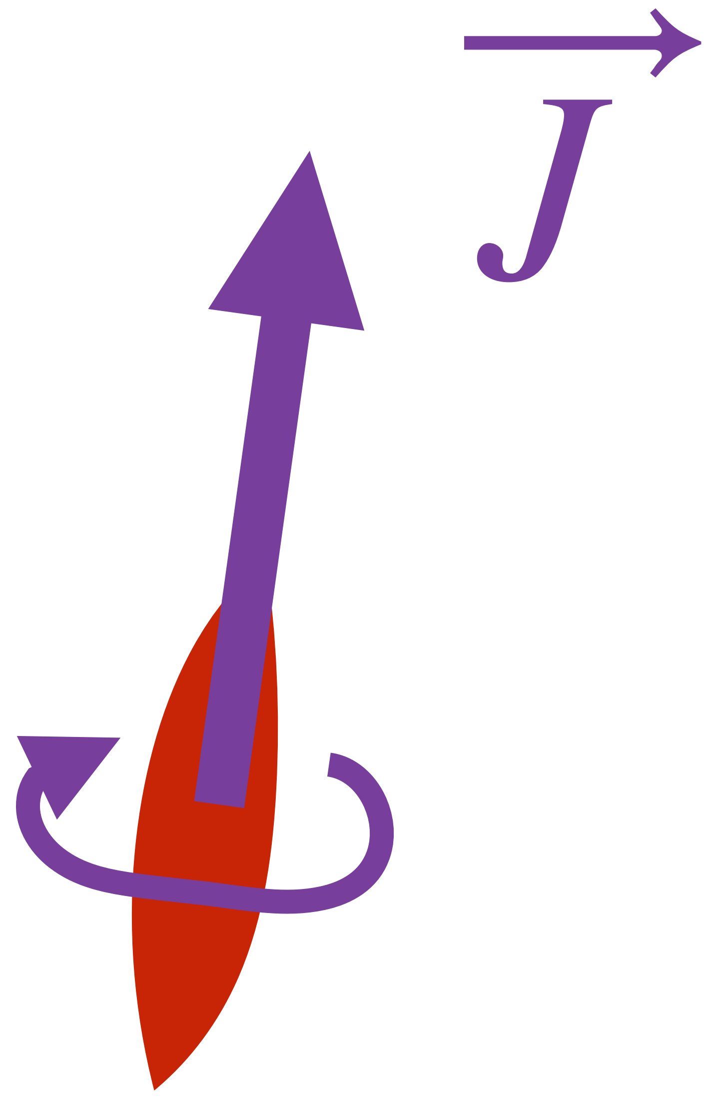

$$S \sim \int d^4x \ldots + C\,\vec{J}\cdot\vec{S} + \ldots$$

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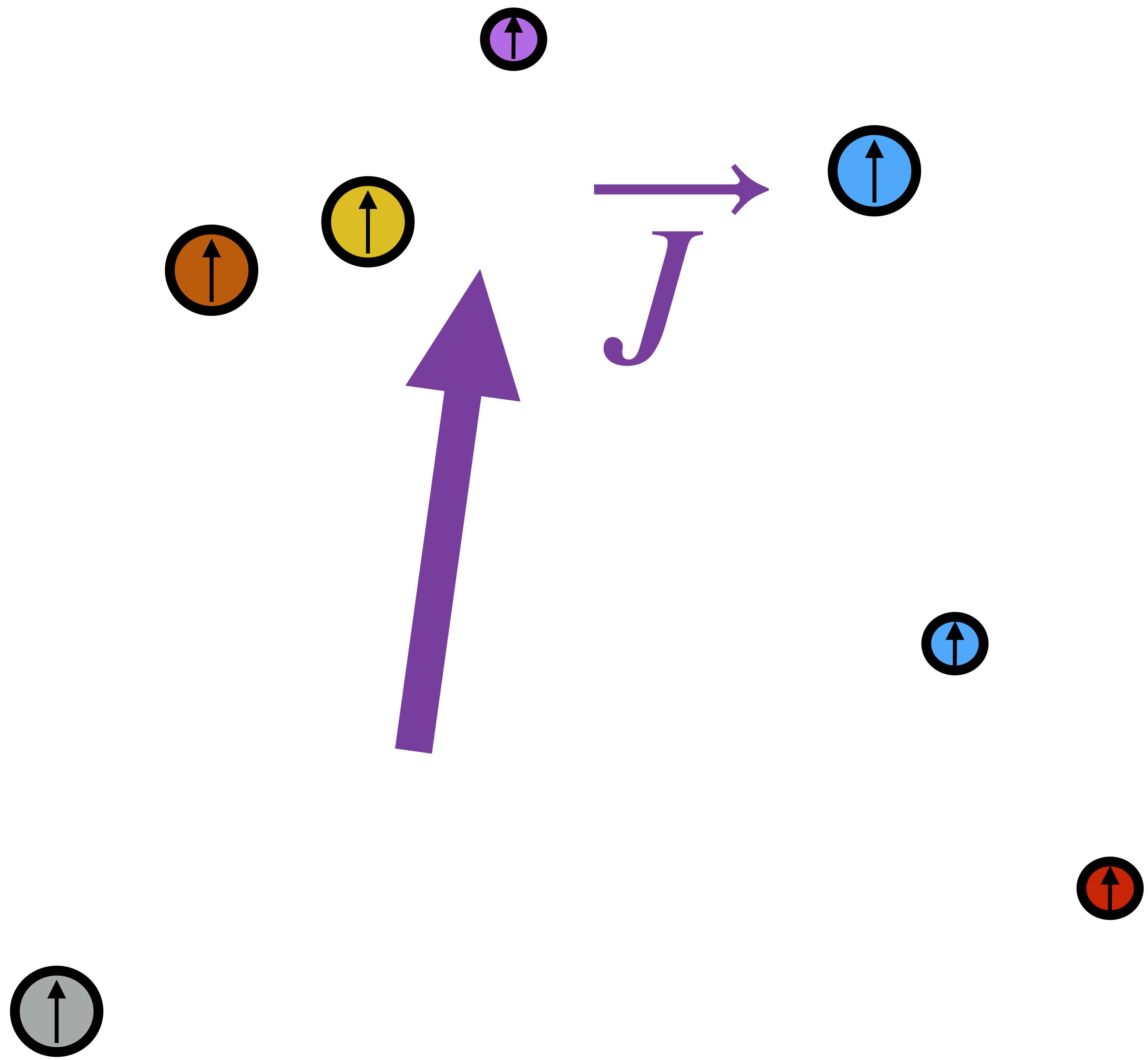

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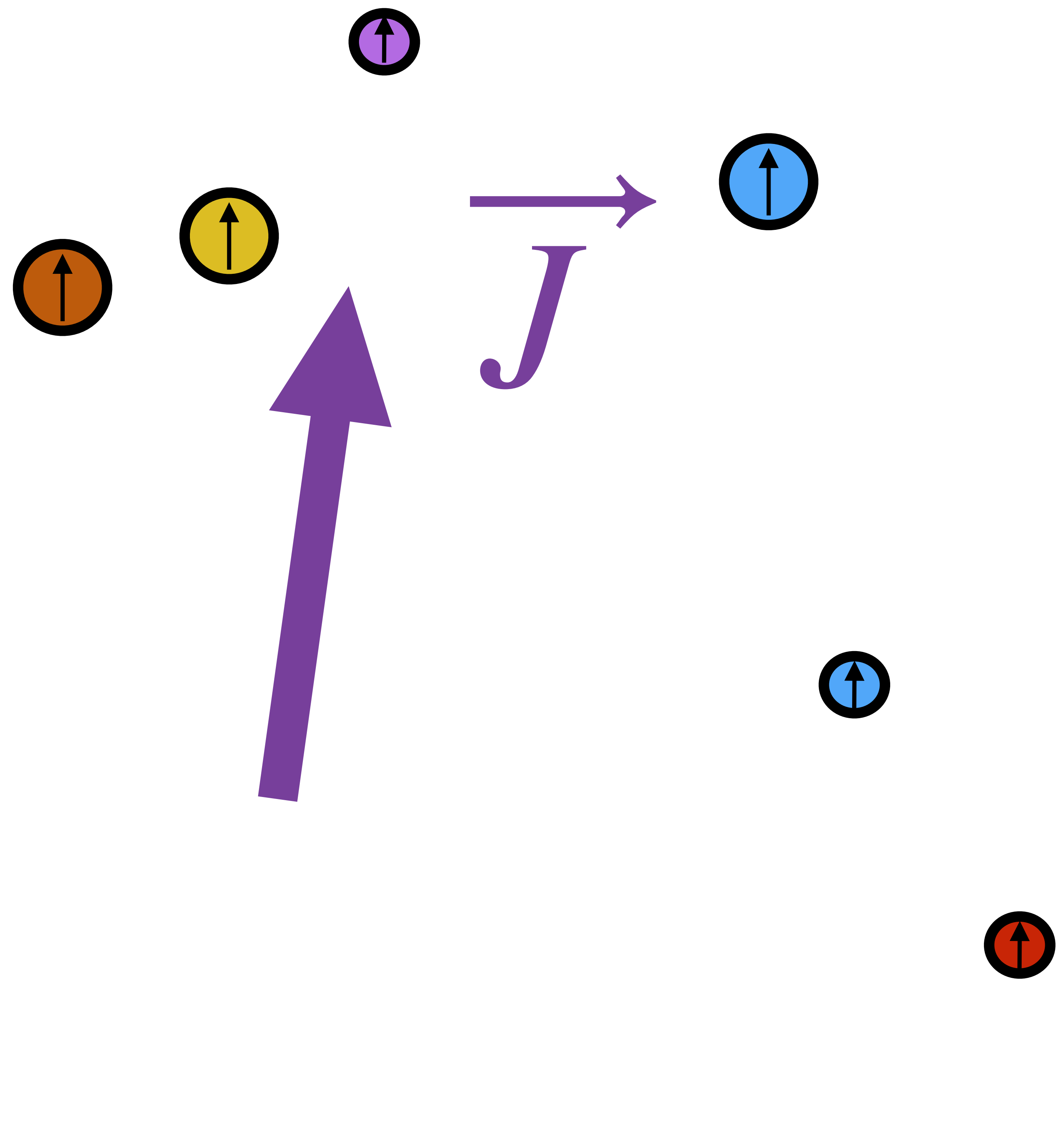

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(Becattini, Piccinini, Rizzo, 2008)

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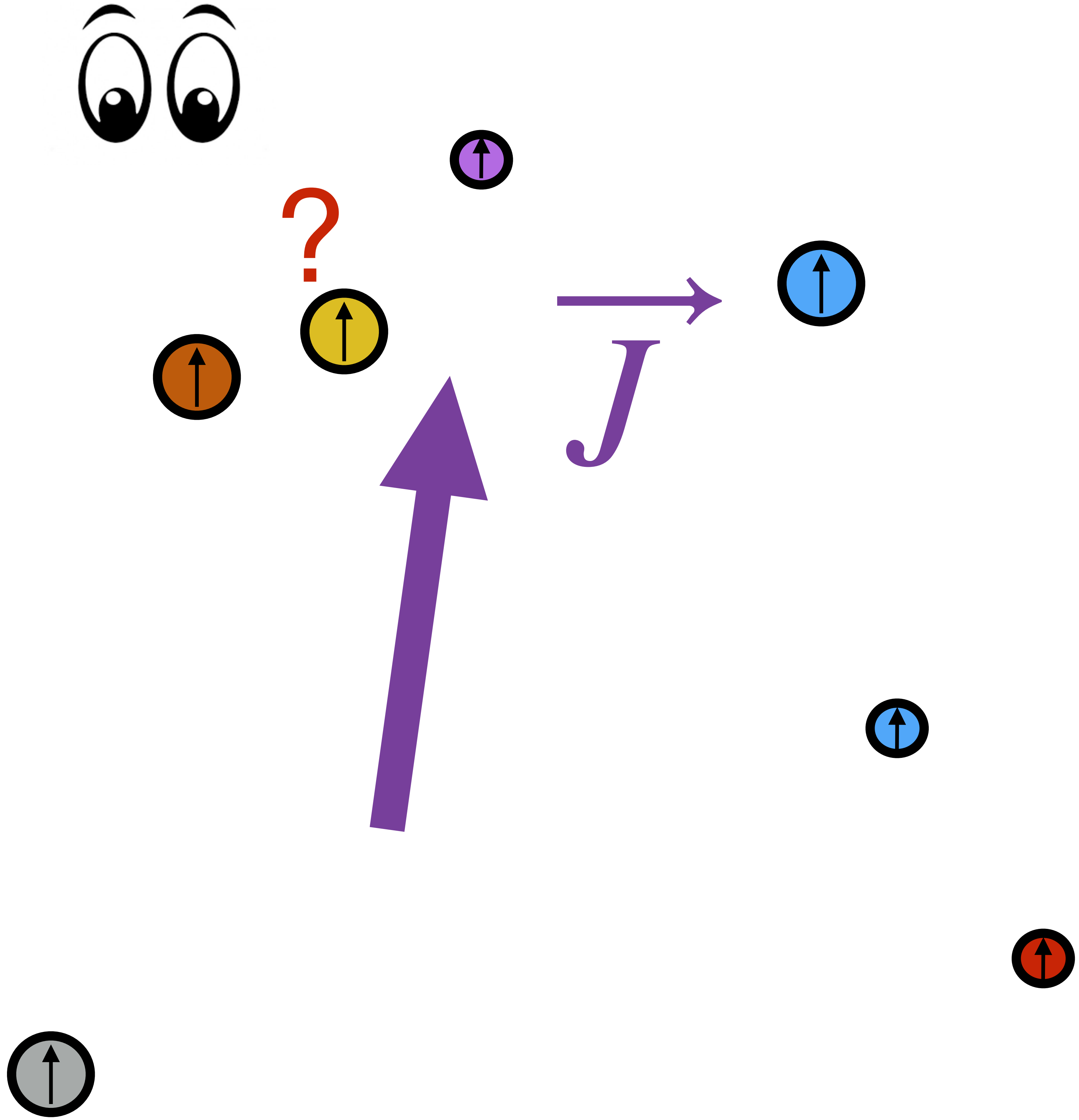

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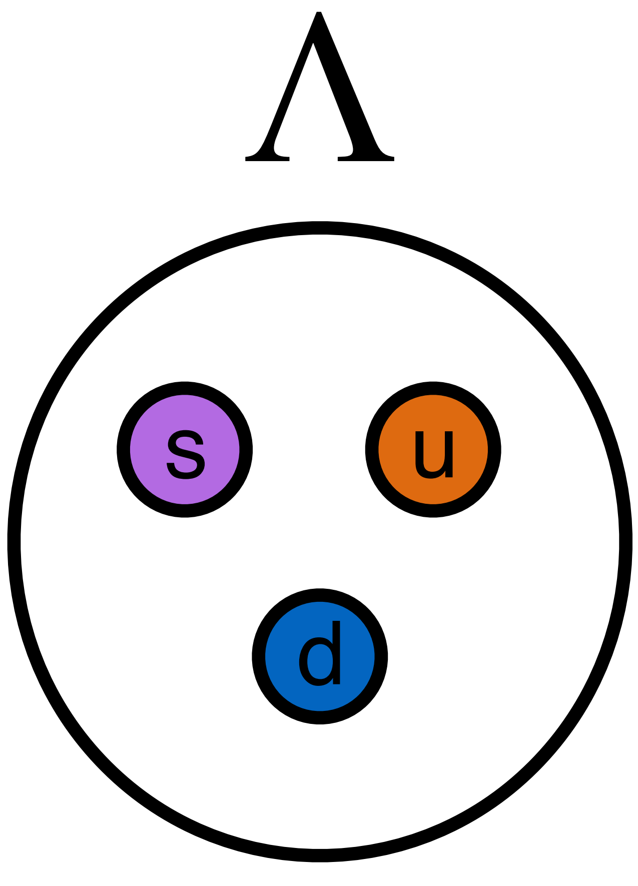

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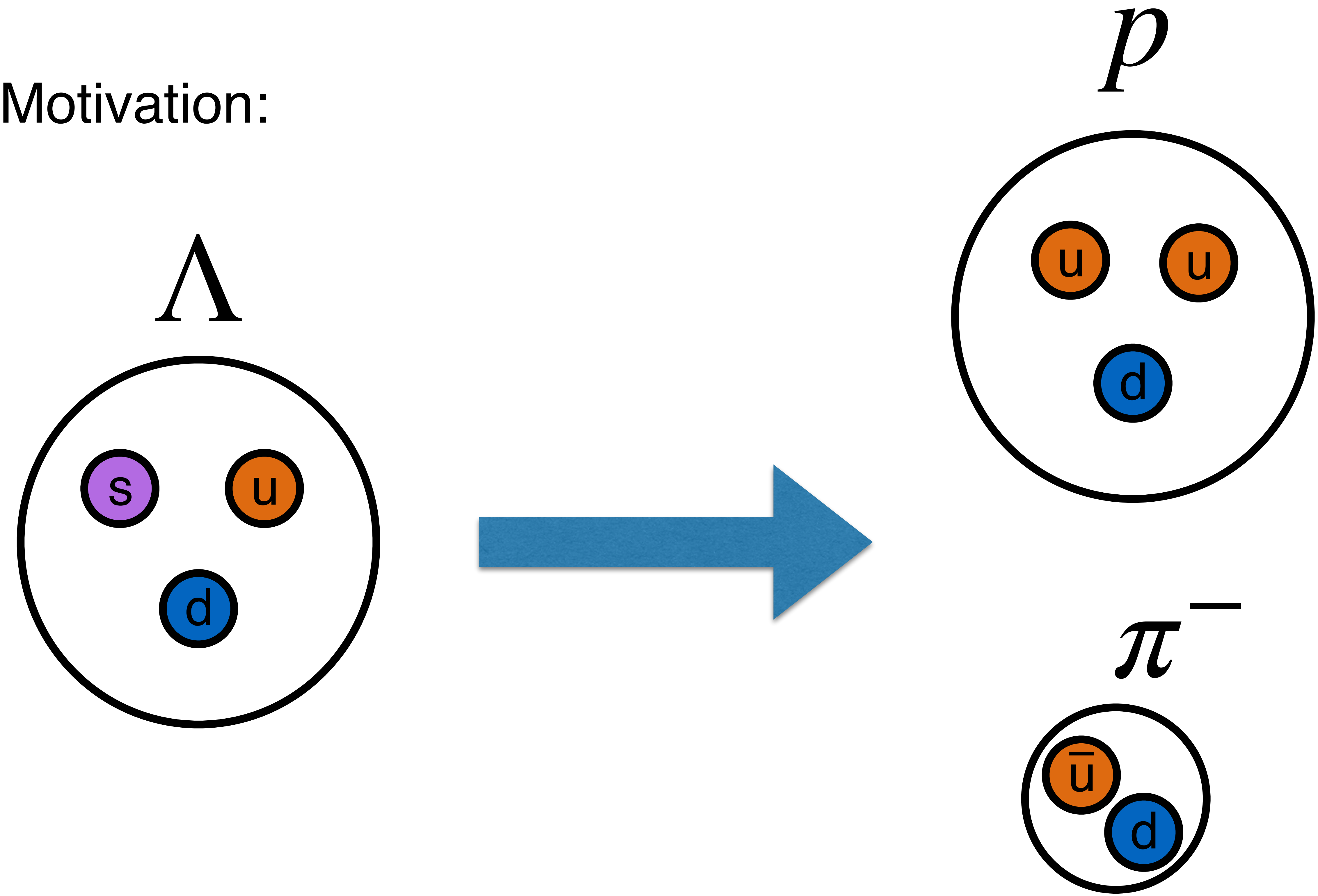

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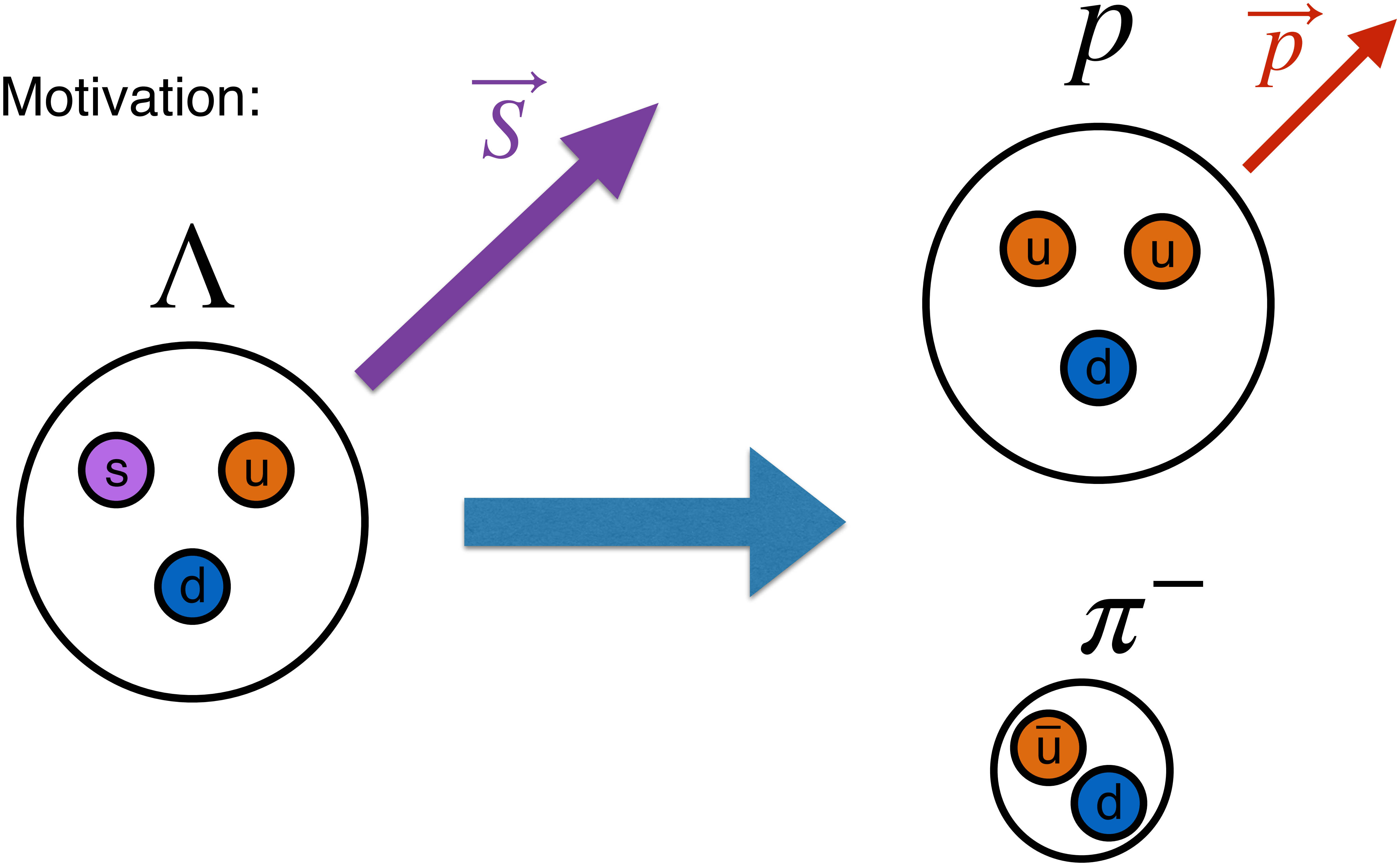

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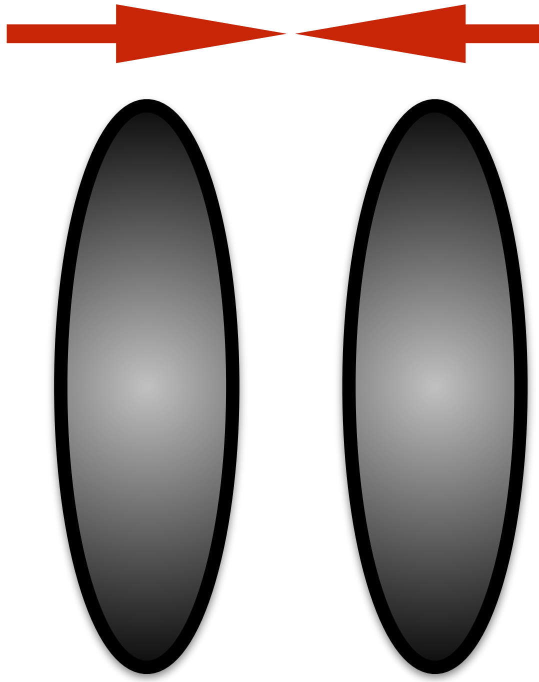

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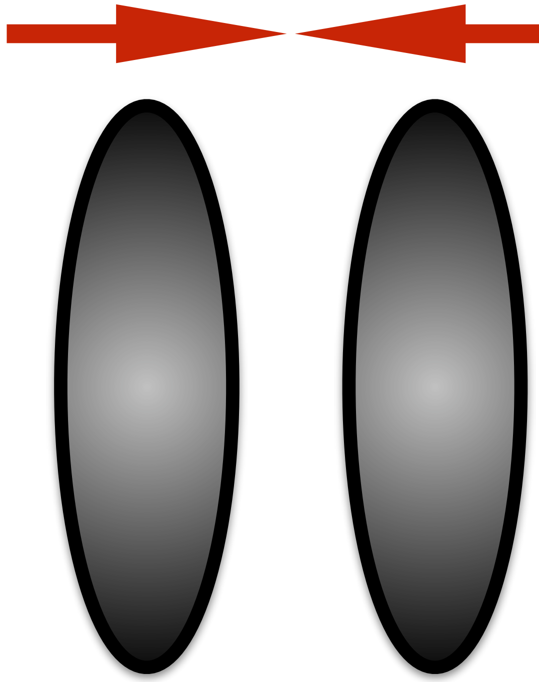

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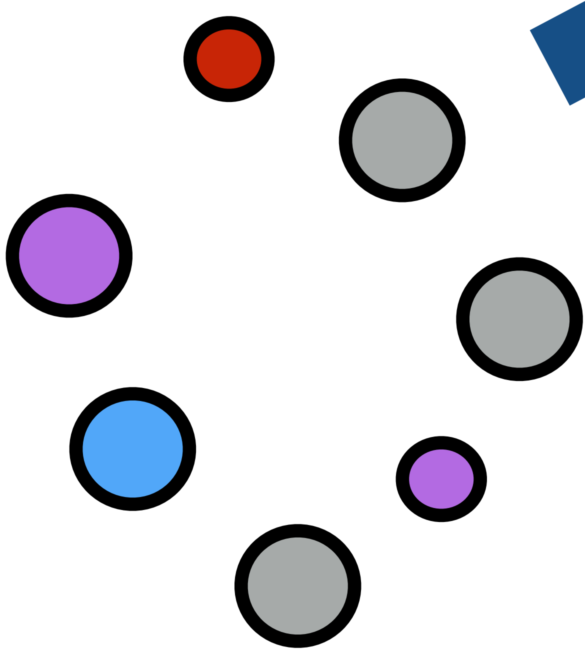
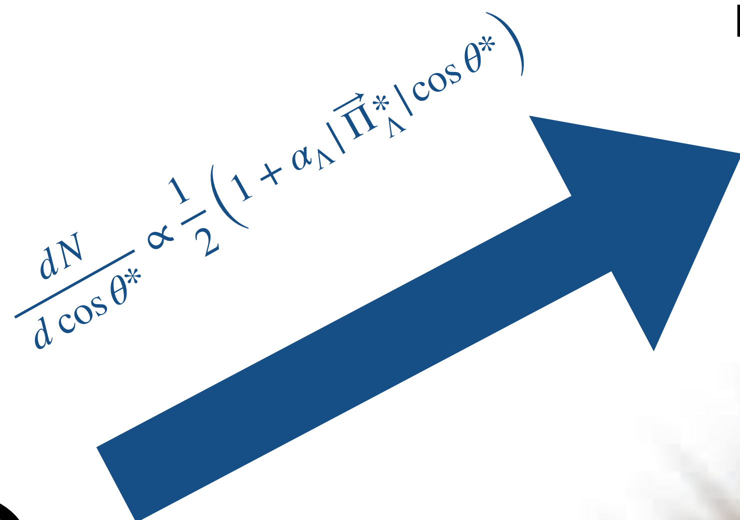
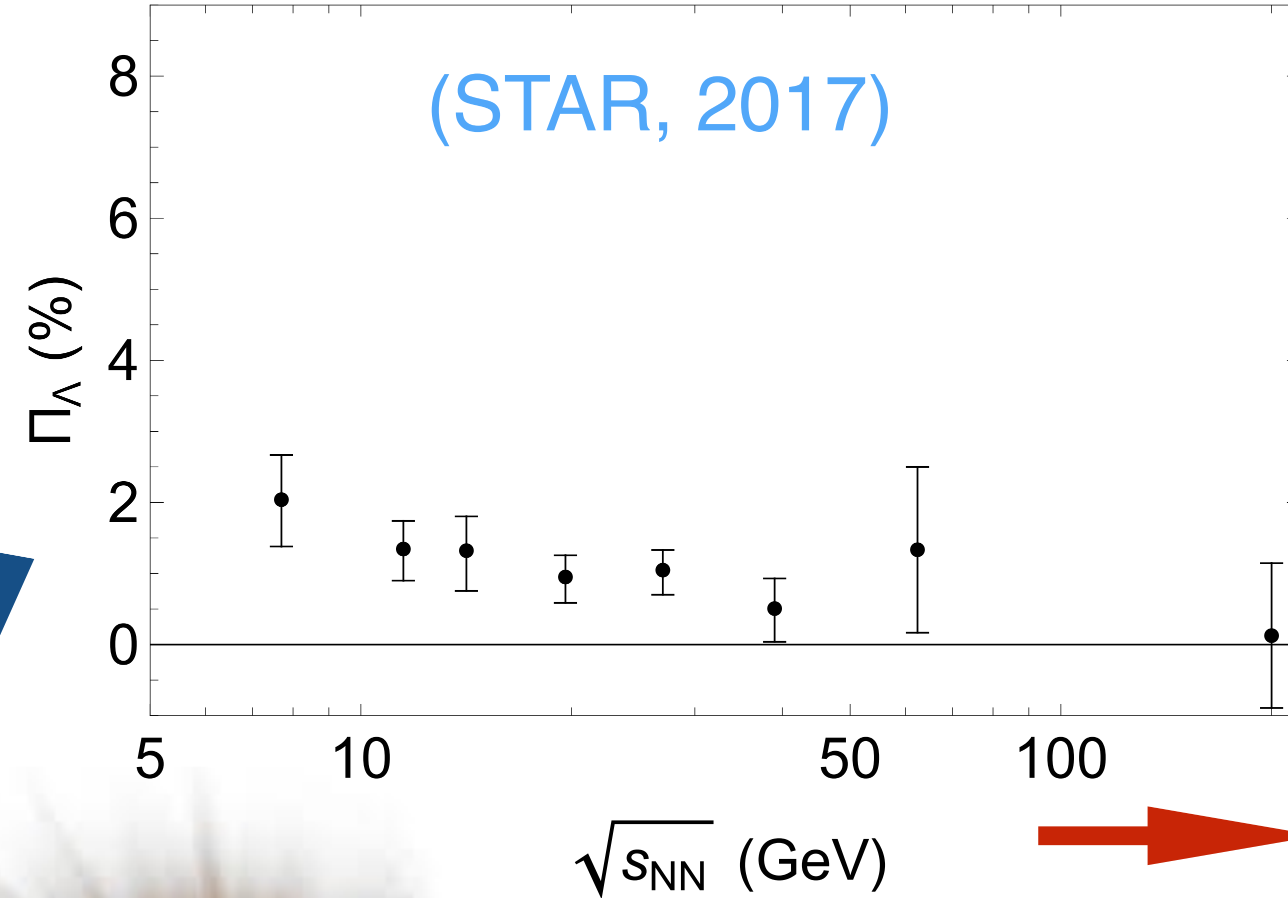
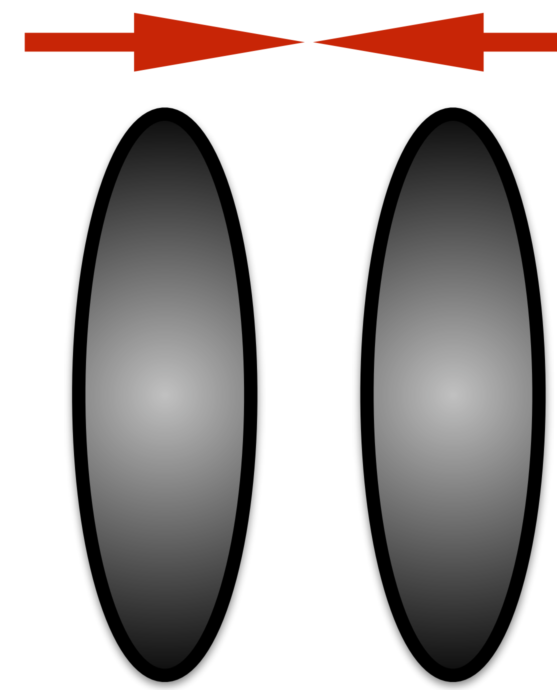



$\frac{dN}{d\cos\theta^*} \propto \frac{1}{2}\left(1 + \alpha_\Lambda |\vec{\Pi}^*_\Lambda| \cos\theta^*\right)$


(STAR, 2017)
Π_Λ (%)
$\sqrt{s_{NN}}$ (GeV)
0
2
4
6
8
5
10
50
100



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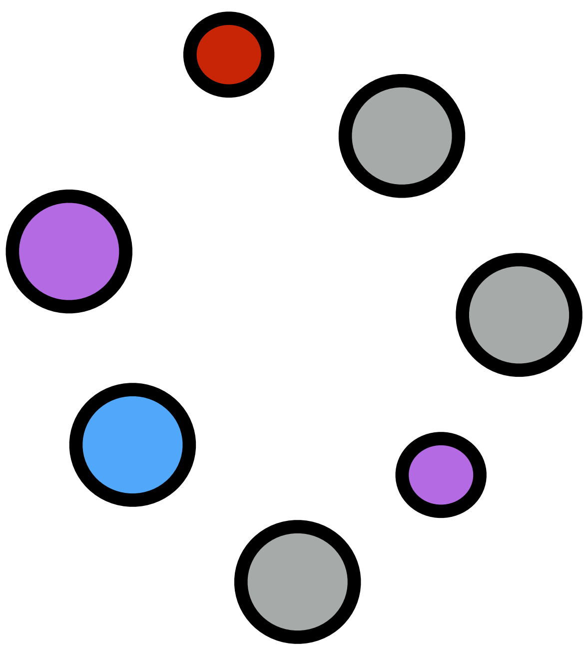
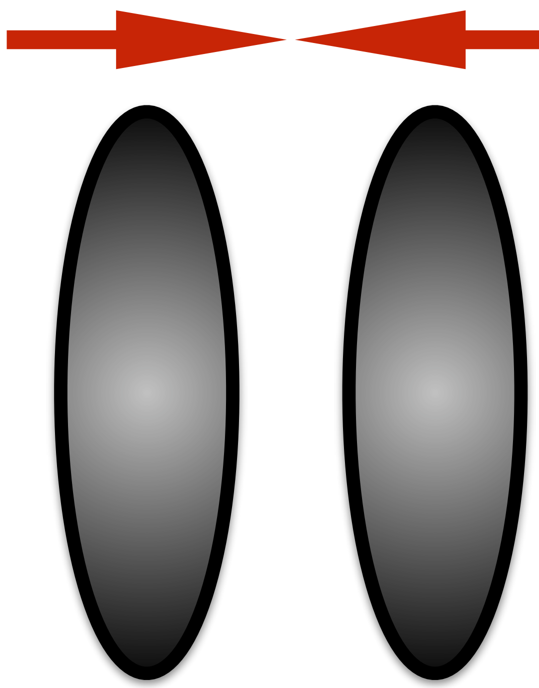


Hydrodynamics



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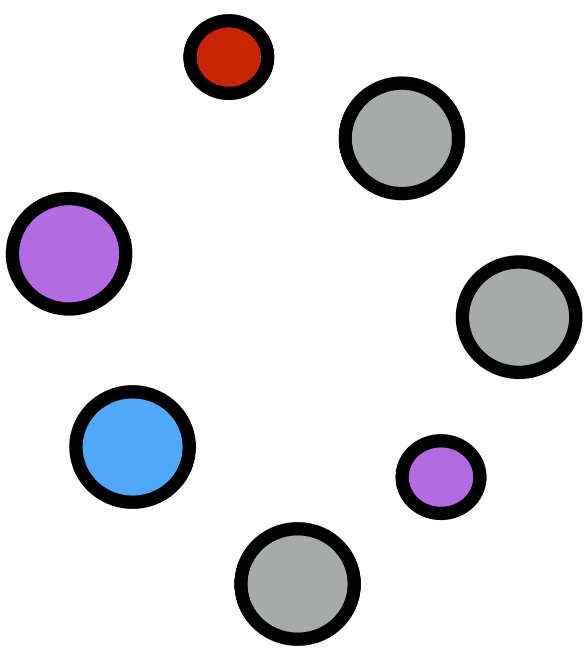

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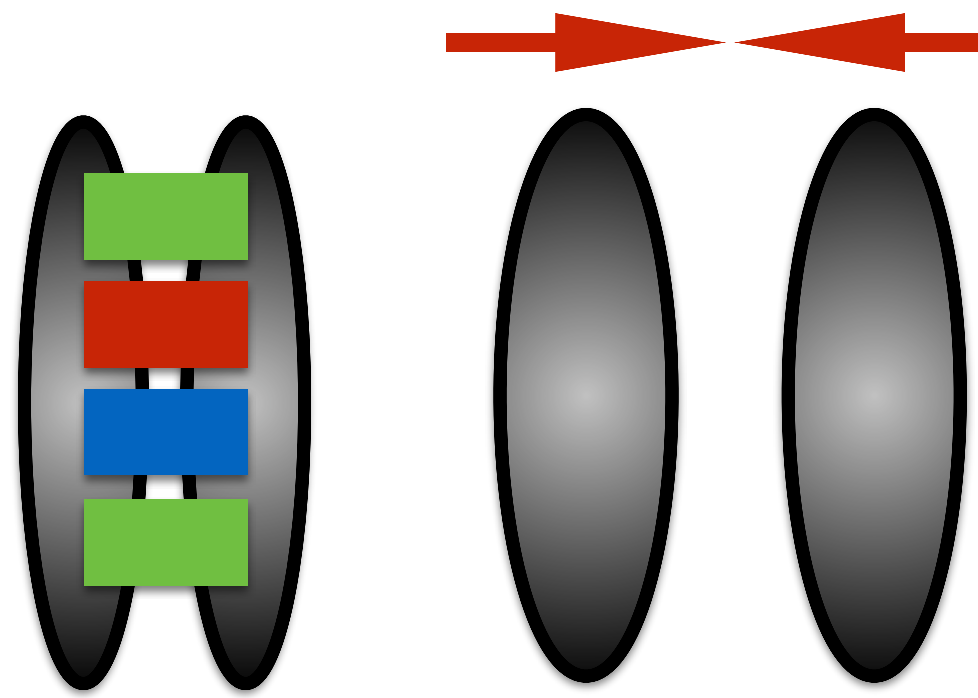

Pre equilibrium dynamics

Motivation:

Hydrodynamics

Hadronization

Pre equilibrium dynamics

Motivation:

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- What is hydrodynamics with a spin current?

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- What is hydrodynamics with a spin current?
- Is it relevant to heavy ion collisions?

Hydrodynamics with a spin current

Hydrodynamics is a theory of conserved currents:

$$\nabla_\mu T^{\mu\nu} = 0$$

supplemented with a set of constitutive relations:

$$T^{\mu\nu} = \epsilon(T) u^\mu u^\nu + P(T) \left(\eta^{\mu\nu} + u^\mu u^\nu \right) + \ldots$$

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Conservation of angular momentum reads

$$\nabla_\mu J^{\mu\nu\rho} = 0$$

where

$$J^{\mu\nu\rho} = -x^\nu T^{\mu\rho} + x^\rho T^{\mu\nu} + S^{\mu\nu\rho}$$

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$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu S^{\mu\nu\rho} = T^{\nu\rho} - T^{\rho\nu}$$

But then

$$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\nabla_\lambda\left(\phi^{\mu\nu\lambda} + \phi^{\nu\mu\lambda} - \phi^{\lambda\nu\mu}\right)$$

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So one can always choose $\phi^{\lambda\mu\nu} = -S^{\lambda\mu\nu}$ and obtain:

$$\nabla_\mu T'^{\mu\nu} = 0 \qquad 0 = T'^{\nu\rho} - T'^{\rho\nu}$$

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One usually further defines

$$T''^{\mu\nu} = T'^{\mu\nu} - \frac{1}{2}\left(T'^{\mu\nu} - T'^{\nu\mu}\right)$$

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Hydrodynamics with a spin current

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This is the hydrodynamic theory we want

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This is what we're used to

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It should reduce to

$$\nabla_\mu T''^{\mu\nu} = 0$$

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Constitutive relations will involve

u^μ - velocity field

T - temperature

$\mu^{\alpha\beta}$ - spin chemical potential

Hydrostatics with a spin current

Consider a hydrostatically equilibrated configuration

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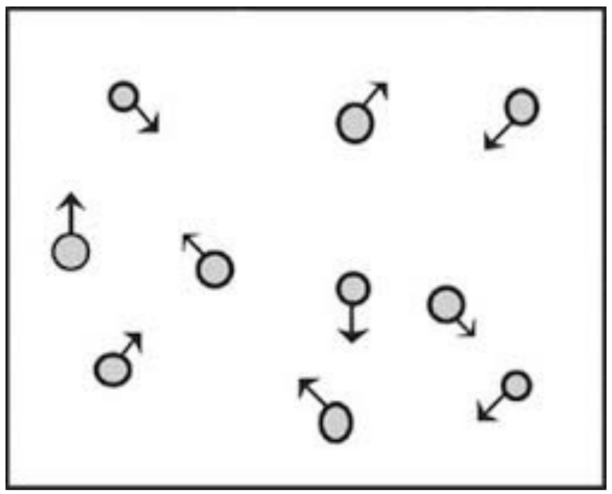

$$P = P_0$$

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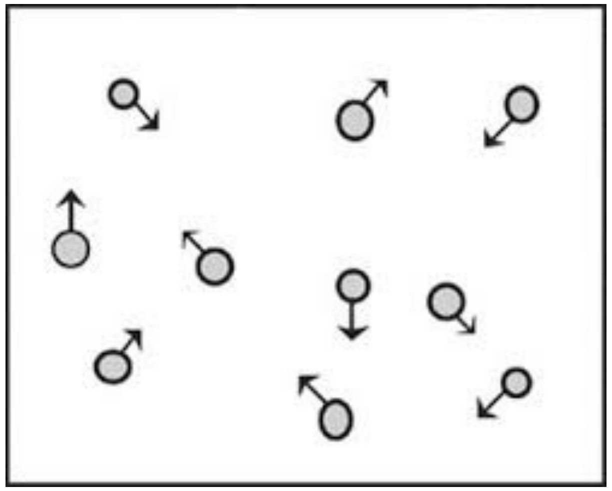

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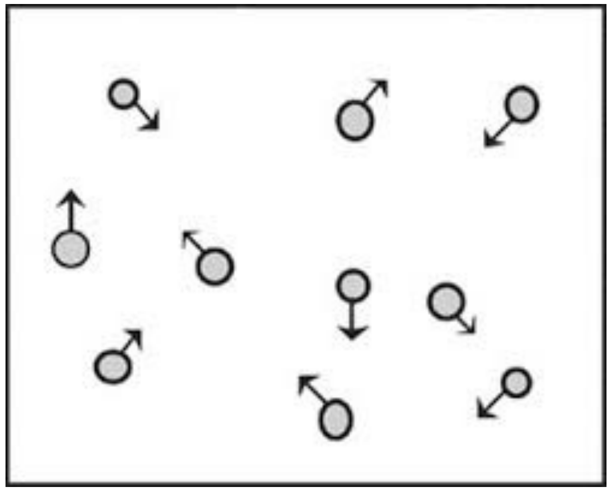

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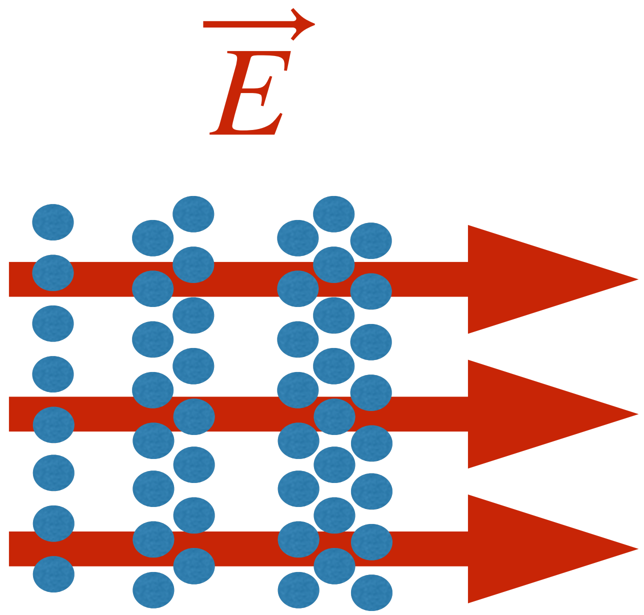

$$T \vec{\nabla} \frac{\mu}{T} = \vec{E}$$

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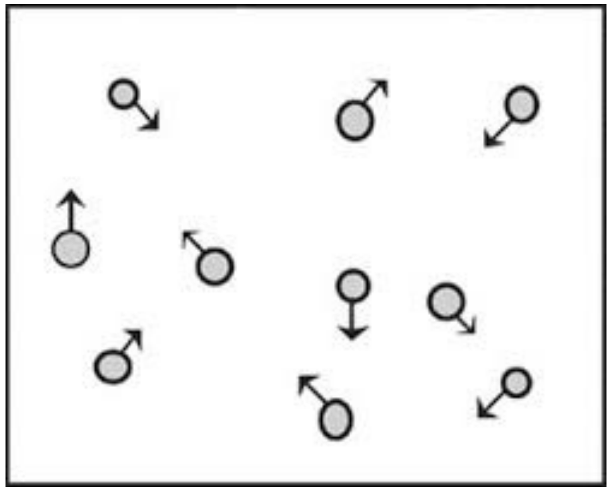

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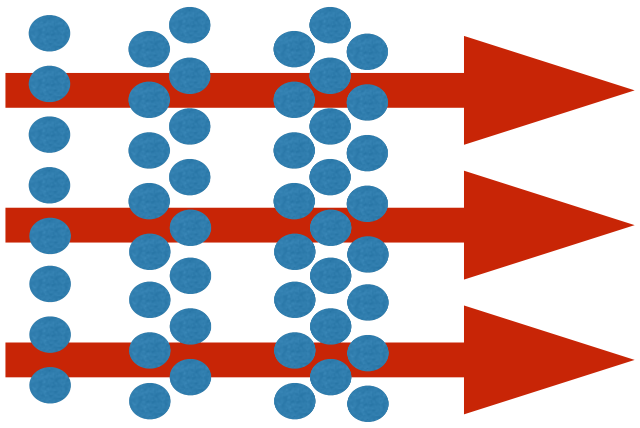
$\vec{E}$



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There is a general technique for obtaining the constitutive relations in hydrostatic equilibrium by coupling the fluid to background fields

(Jensen et. al. 2012, Banerjee et. al. 2012)

Hydrostatics with a spin current

Consider a hydrostatically equilibrated configuration

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E.g.,

$$\delta S = \int d^4x \frac{1}{2}\sqrt{g}\,T^{\mu\nu}\delta g_{\mu\nu}$$

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will be constrained by the requirement that

$$\frac{\partial^2 \ln Z}{\partial g^{\alpha\beta}\partial g^{\gamma\delta}} \sim \mathrm{Tr}\left(e^{-\beta H+\beta\mu Q} T_{\alpha\beta}(x) T_{\gamma\delta}(0)\right) \sim e^{-\xi|x|}$$

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In our case

$$\delta S = \int d^4x\,|e|\left(T^{\mu}{}_{a}\delta e^{a}{}_{\mu} + \frac{1}{2}S^{\lambda}{}_{ab}\delta\omega_{\lambda}{}^{ab}\right)$$

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The resulting equations of motion are

$$\nabla_{\mu}T^{\mu\nu} = \frac{1}{2}R^{\rho\sigma\nu\lambda}S_{\rho\lambda\sigma} - T_{\rho\sigma}K^{\nu ab}e^{\rho}{}_{a}e^{\sigma}{}_{b}$$

$$\nabla_{\lambda}S^{\lambda}{}_{\mu\nu} = 2T_{[\mu\nu]} - 2S^{\lambda}{}_{\rho[\mu}e_{\nu]}{}^{a}e_{\rho}{}^{b}K_{\lambda ab}$$

Hydrostatics with a spin current

In our case

$$\delta S = \int d^4x\,|e|\left(T^{\mu}{}_{a}\delta e^{a}{}_{\mu} + \frac{1}{2}S^{\lambda}{}_{ab}\delta\omega_{\lambda}{}^{ab}\right)$$

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Hydrostatics with a spin current

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Hydrostatics with a spin current

Consider a hydrostatically equilibrated configuration

There is a general technique for obtaining the constitutive relations in hydrostatic equilibrium by coupling the fluid to background fields

(Jensen et. al. 2012, Banerjee et. al. 2012)

In our case

$$\delta S = \int d^4x \, |e| \left(T^{\mu}{}_{a} \delta e^{a}{}_{\mu} + \frac{1}{2} S^{\lambda}{}_{ab} \delta \omega_{\lambda}{}^{ab} \right)$$

Hydrostatics with a spin current

One finds, for an "ideal" fluid:

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) - 2\left(\frac{\partial P}{\partial m^2} + 4\frac{\partial P}{\partial M^2}\right) u^\alpha M^{\beta\gamma} m_\gamma + \ldots$$

$$S^\lambda{}_{\alpha\beta} = u^\lambda \rho_{\alpha\beta}$$

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Hydrostatics with a spin current

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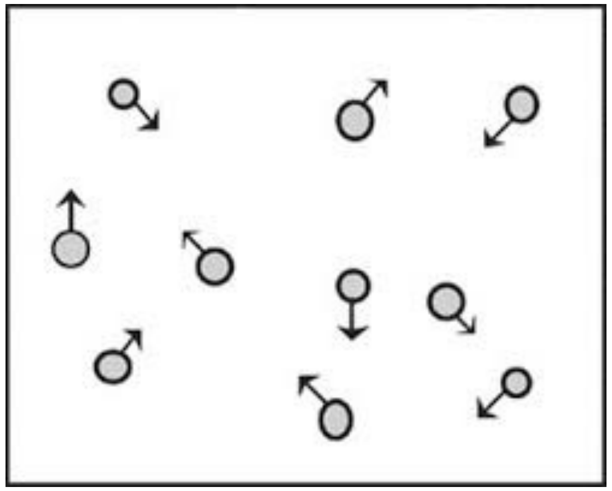
Hydrostatics with a spin current

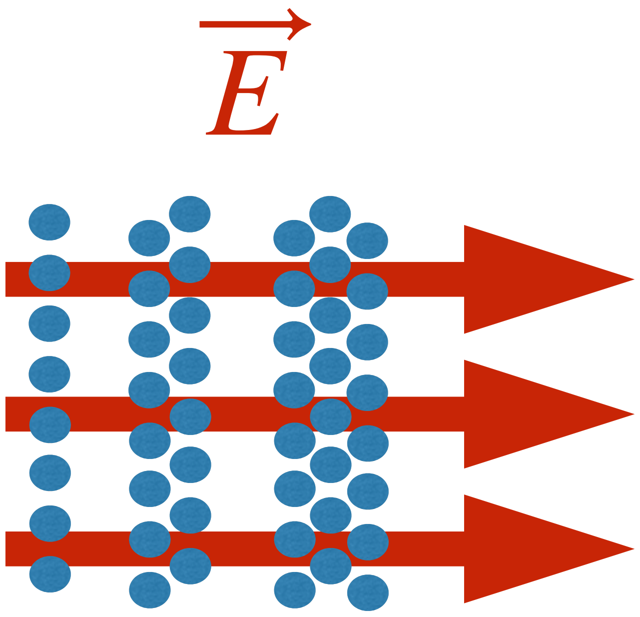
Further:



$$P = P_0$$

$$u^\mu = (1,0)$$

$$P = \rho g z$$

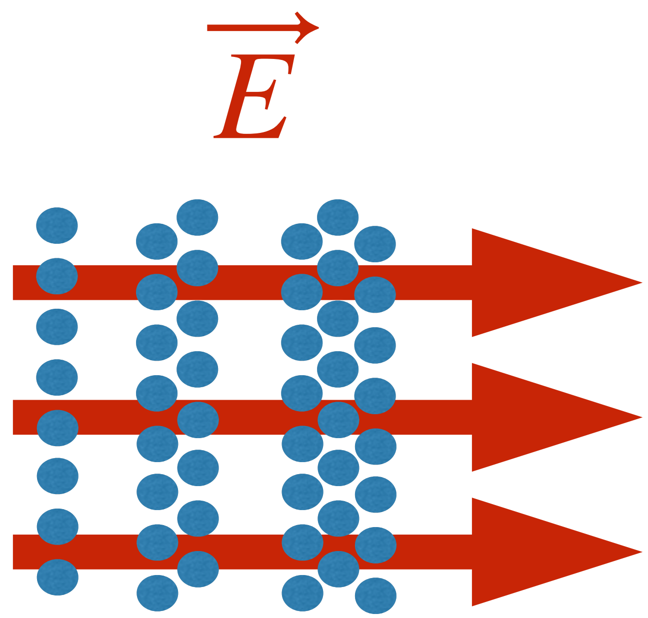


$$T \vec{\nabla} \frac{\mu}{T} = \vec{E}$$

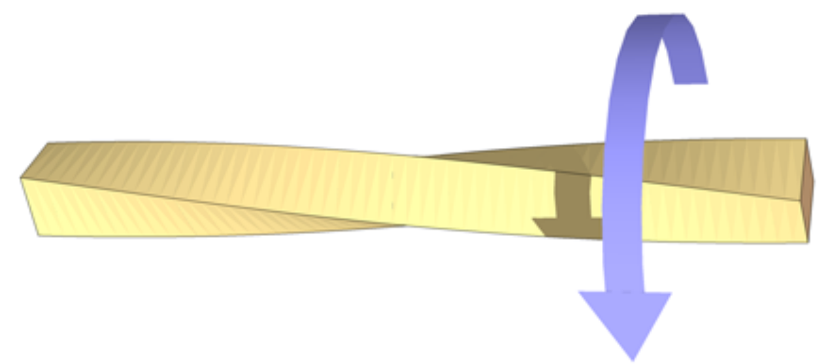
Hydrostatics with a spin current

Further:

$$P = \rho g z$$



$$T \overrightarrow{\nabla} \frac{\mu}{T} = \overrightarrow{E}$$



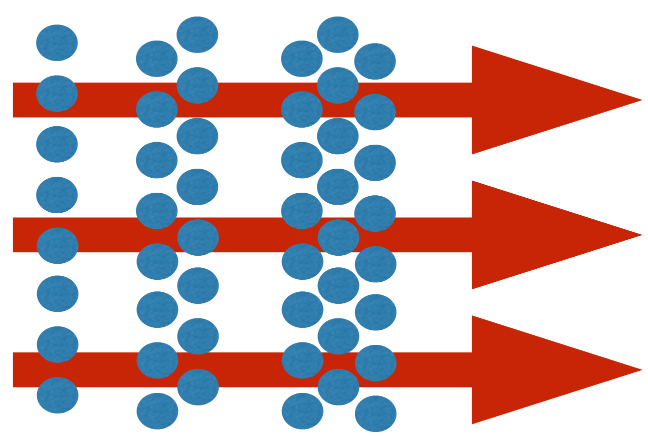
$$u^{\mu} K_{\mu}{}^{ab} = \mu^{ab} + e^{a}{}_{\mu} e^{b}{}_{\nu} \left(\Omega^{\mu\nu} - 2 u^{[\mu} a^{\nu]} \right)$$

Hydrostatics with a spin current

Further:

$$P = \rho g z$$

$\vec{E}$



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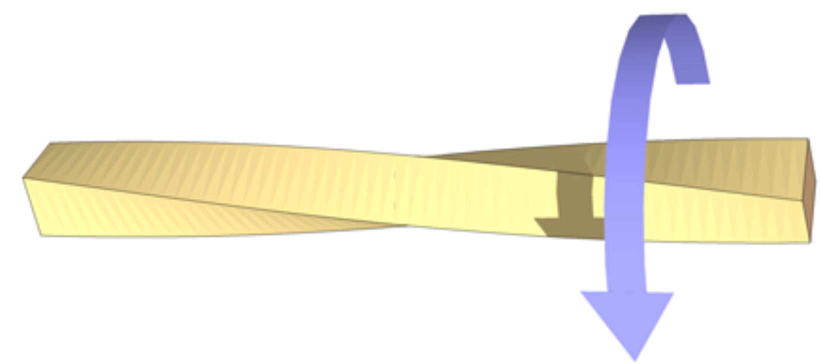

$$u^{\mu} K_{\mu}{}^{ab} = \mu^{ab} + e^{a}{}_{\mu} e^{b}{}_{\nu} (\Omega^{\mu\nu} - 2u^{[\mu}a^{\nu]})$$

$$\Omega^{\mu\nu} = \Delta^{\mu\alpha} \Delta^{\nu\beta} \nabla_{[\alpha} u_{\beta]}$$

$$\Delta^{\alpha\beta} = \eta^{\alpha\beta} + u^{\alpha} u^{\beta}$$

Hydrostatics with a spin current

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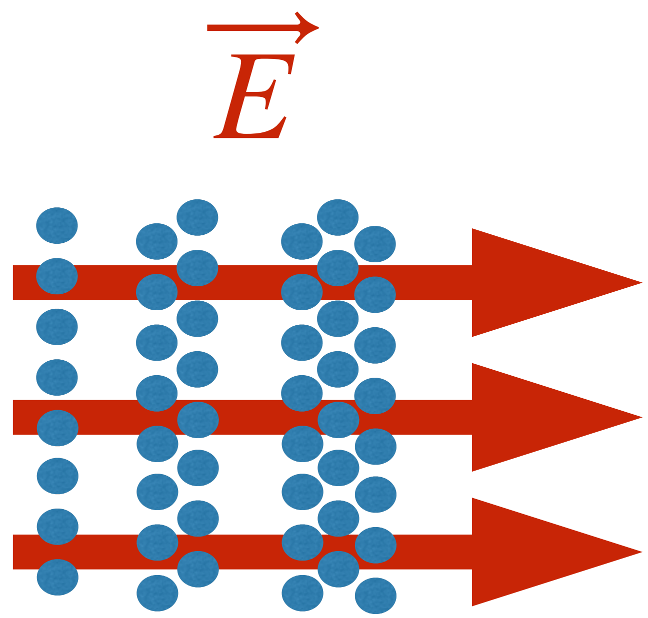
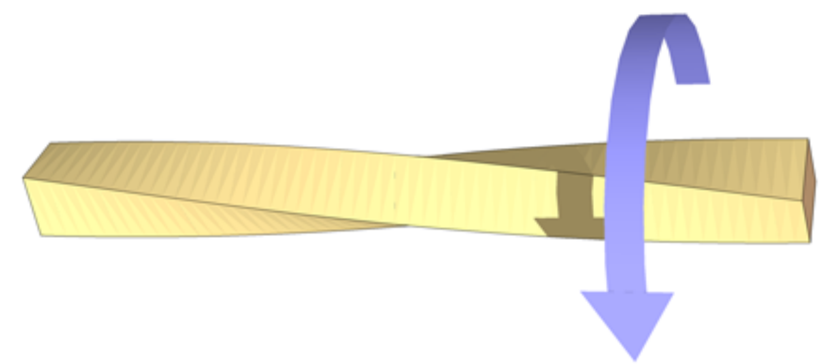
$$P = \rho g z$$



$$T \vec{\nabla} \frac{\mu}{T} = \vec{E}$$



$$u^\mu K_\mu{}^{ab} = \mu^{ab} + e^a{}_\mu e^b{}_\nu \Omega^{\mu\nu} - 2u^{[\mu} a^{\nu]}$$

$$\Omega^{\mu\nu} = \Delta^{\mu\alpha} \Delta^{\nu\beta} \nabla_{[\alpha} u_{\beta]}$$

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$$a_\mu = u^\alpha \nabla_\alpha u_\mu$$

Hydrostatics with a spin current

Further:

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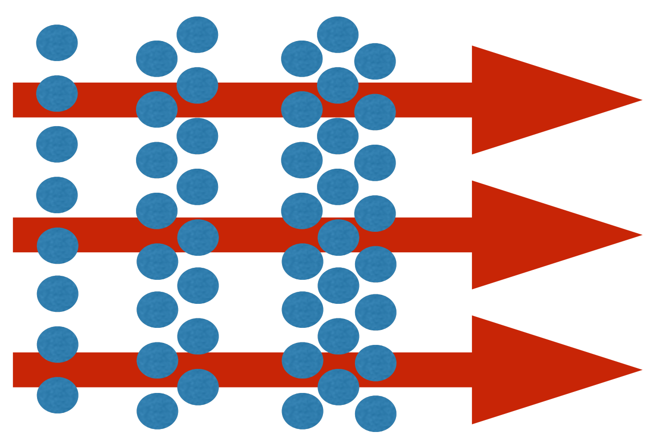

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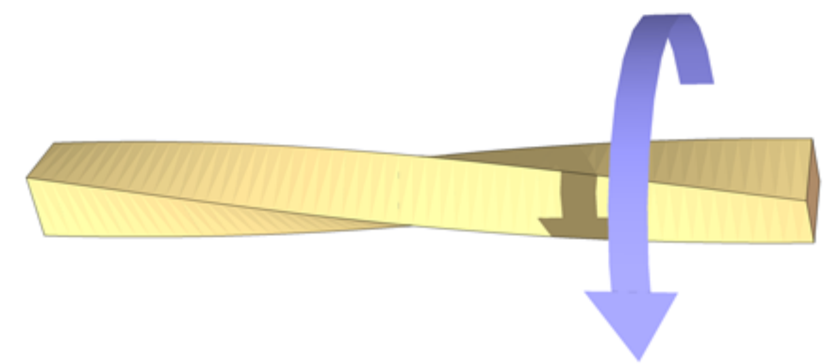

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Hydrostatics with a spin current

One finds, for an "ideal" fluid:

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) - 2\left(\frac{\partial P}{\partial m^2} + 4\frac{\partial P}{\partial M^2}\right) u^\alpha M^{\beta\gamma} m_\gamma + \dots$$

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$$\rho_{\alpha\beta} = 8\frac{\partial P}{\partial M^2}M_{\alpha\beta} + \dots$$

Hydrostatics with a spin current

One finds, for an "ideal" fluid:

$\mathcal{O}(\partial^0)$

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Hydrostatics with a spin current

One finds, for an "ideal" fluid:

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Hydrodynamics with a spin current

An "ideal" fluid:

$\mathcal{O}(\partial^0)$ $\qquad$ $\mathcal{O}(\partial^2)$

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$\mathcal{O}(\partial^1)$

Expanding to 1st order we have

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

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Hydrodynamics with a spin current

Expanding to 1st order we have

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$$S^\lambda{}_{\alpha\beta} = u^\lambda \rho_{\alpha\beta} + \mathcal{O}(\partial^1)$$

But recall that

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\lambda S^\lambda{}_{\mu\nu} = 2T_{[\mu\nu]}$$

Hydrodynamics with a spin current

Expanding to 1st order we have

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

$$T^{[\alpha\beta]} = \mathcal{O}(\partial^1) + \mathcal{O}(\partial^2)$$

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Hydrodynamics with a spin current

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Hydrodynamics with a spin current

Expanding to 1st order we have

$\times$ 3 terms

$$T^{(\alpha\beta)} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

$$T^{[\alpha\beta]} = \mathcal{O}(\partial^1) + \mathcal{O}(\partial^2)$$

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Hydrodynamics with a spin current

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$\times$ 3 terms

$$T^{(\alpha\beta)} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

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Hydrodynamics with a spin current

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$$S^\lambda{}_{\alpha\beta} = u^\lambda \rho_{\alpha\beta} + \mathcal{O}(\partial^1)$$

where

$$\hat{m}^\mu = m^\mu - a^\mu$$

$$\hat{M}^{\mu\nu} = M^{\mu\nu} + \Omega^{\mu\nu}$$

Hydrodynamics with a spin current

Expanding to 1st order we have

× 3 terms

$$T^{(\alpha\beta)} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

× 2 terms

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

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Hydrodynamics with a spin current

Expanding to 1st order we have

× 3 terms

$$T^{(\alpha\beta)} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

× 2 terms   × 4 terms

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

$$S^\lambda{}_{\alpha\beta} = u^\lambda \rho_{\alpha\beta} + \mathcal{O}(\partial^1)$$

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Hydrodynamics with a spin current

Expanding to 1st order we have

$\times$ 3 terms

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$\times$ 2 terms $\times$ 4 terms

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

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Hydrodynamics with a spin current

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$\times$ 2 terms $\times$ 4 terms

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

$\times$ 1 terms

$$S^\lambda{}_{\alpha\beta} = \mathcal{O}(\partial^0) + u^\lambda \rho_{\alpha\beta} + \mathcal{O}(\partial^1)$$

where

$$\hat{m}^\mu = m^\mu - a^\mu$$

$$\hat{M}^{\mu\nu} = M^{\mu\nu} + \Omega^{\mu\nu}$$

Hydrodynamics with a spin current

Expanding to 1st order we have

$\times$ 3 terms

$$T^{(\alpha\beta)} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

$\times$ 2 terms $\times$ 4 terms

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

$\times$ 1 terms $\times$ 4 terms

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where

$$\hat{m}^\mu = m^\mu - a^\mu$$

$$\hat{M}^{\mu\nu} = M^{\mu\nu} + \Omega^{\mu\nu}$$

Hydrodynamics with a spin current

Expanding to 1st order we have

× 3 terms

$$T^{(\alpha\beta)} = \epsilon u^\alpha u^\beta + P(g^{\alpha\beta} + u^\alpha u^\beta) + \mathcal{O}(\partial^1)$$

× 2 terms × 4 terms

$$T^{[\alpha\beta]} = \sigma_m u^{[\mu}\hat{m}^{\nu]} + \sigma_M \hat{M}^{\mu\nu} + \ldots + \mathcal{O}(\partial^2)$$

× 1 terms × 4 terms

$$S^\lambda{}_{\alpha\beta} = \mathcal{O}(\partial^0) + u^\lambda \rho_{\alpha\beta} + \mathcal{O}(\partial^1)$$

Recall

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu S^{\mu\nu\rho} = T^{\nu\rho} - T^{\rho\nu}$$

$$\Updownarrow \quad T'^{\mu\nu} = \frac{1}{2}\nabla_\lambda\left(\phi^{\mu\nu\lambda} + \phi^{\nu\mu\lambda} - \phi^{\lambda\nu\mu}\right), \quad S'^{\lambda\mu\nu} = S^{\lambda\mu\nu} + \phi^{\lambda\mu\nu}$$

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Hydrodynamics with a spin current

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$$\nabla_\mu T''^{\mu\nu} = 0 \qquad (T''^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta\nabla_\alpha u^\alpha \Delta^{\mu\nu})$$

Hydrodynamics with a spin current

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Hydrodynamics with a spin current

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Hydrodynamics with a spin current

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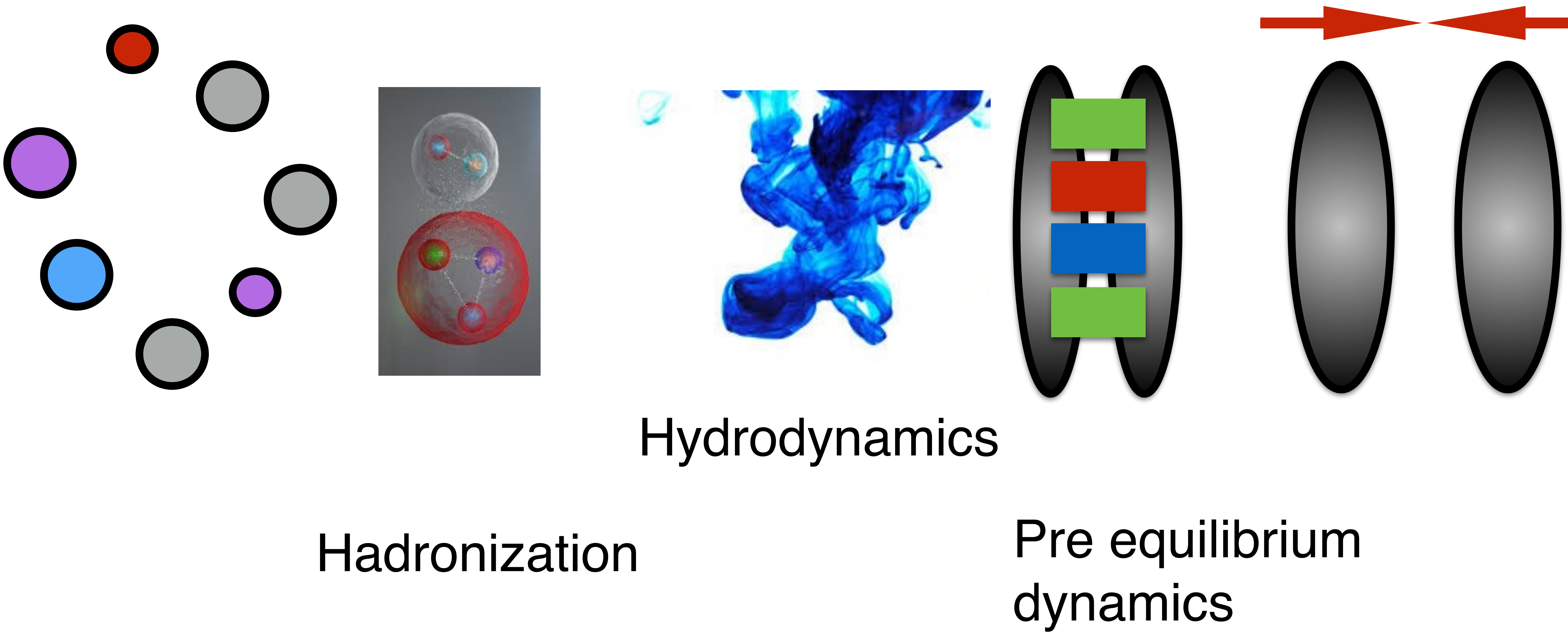
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu S^{\mu\nu\rho} = T^{\nu\rho} - T^{\rho\nu}$$

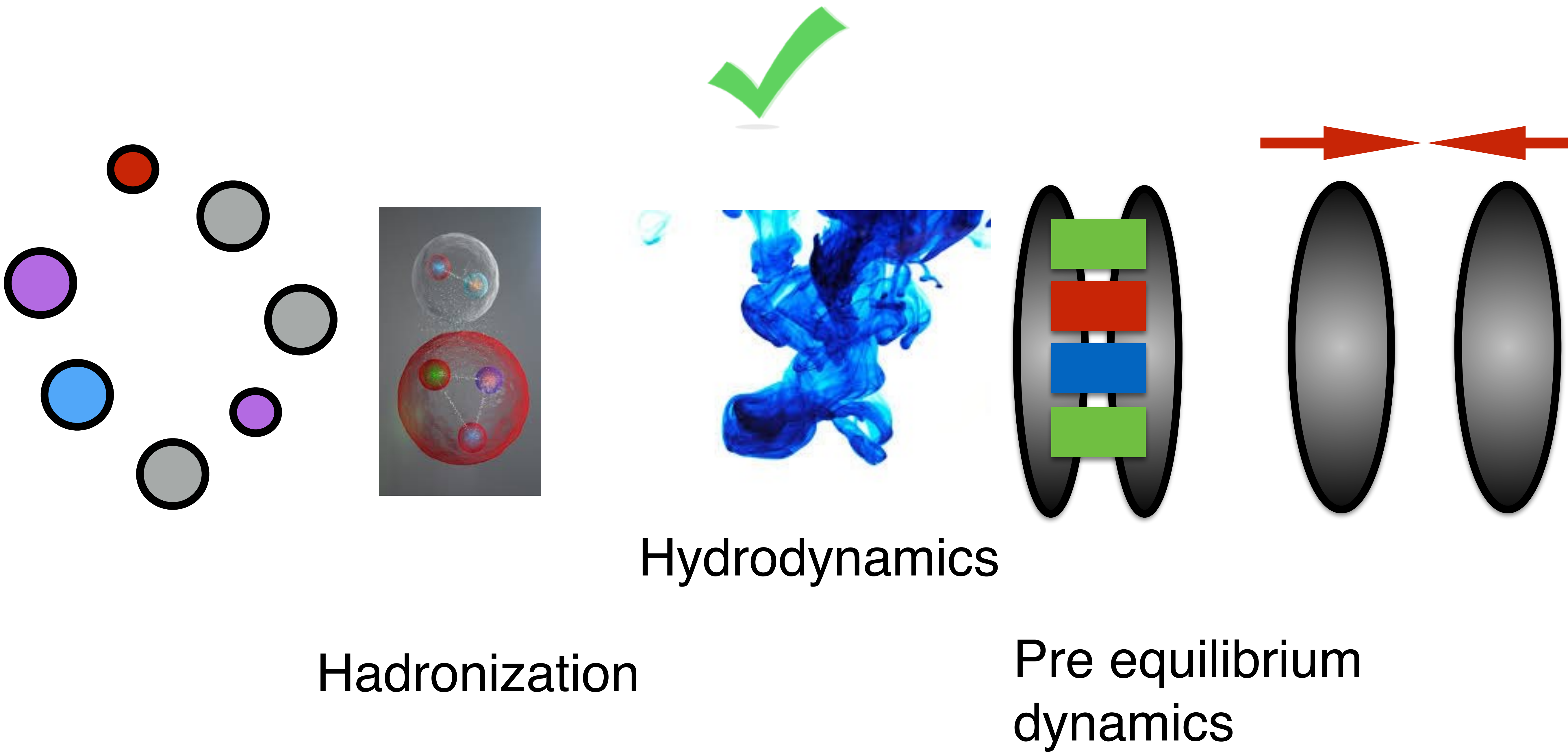
$$\Downarrow \qquad T''^{\mu\nu} = T'^{\mu\nu} - \frac{1}{2}\left(T'^{\mu\nu} - T'^{\nu\mu}\right)$$

$$\nabla_\mu T''^{\mu\nu} = 0$$

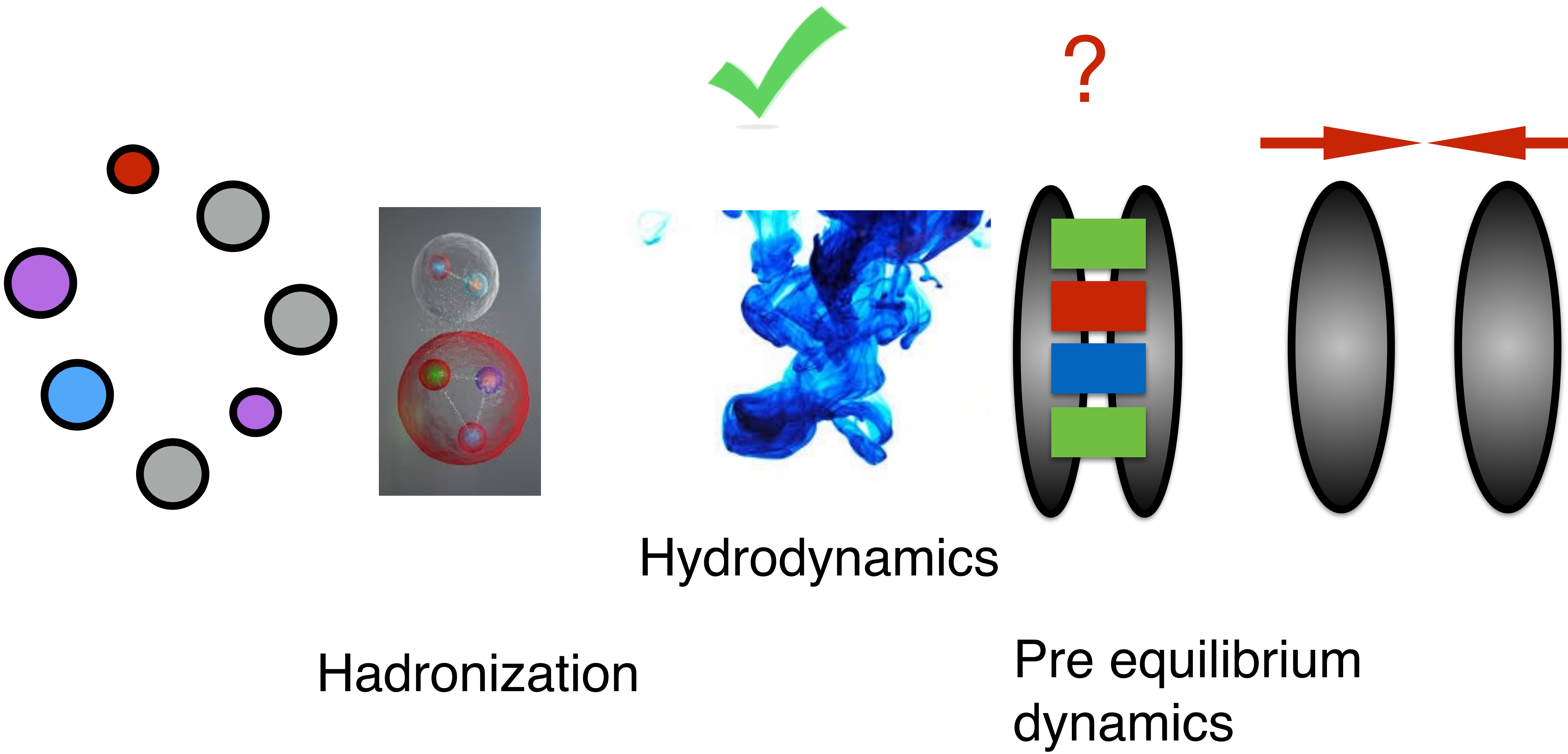
Back to Λ polarization



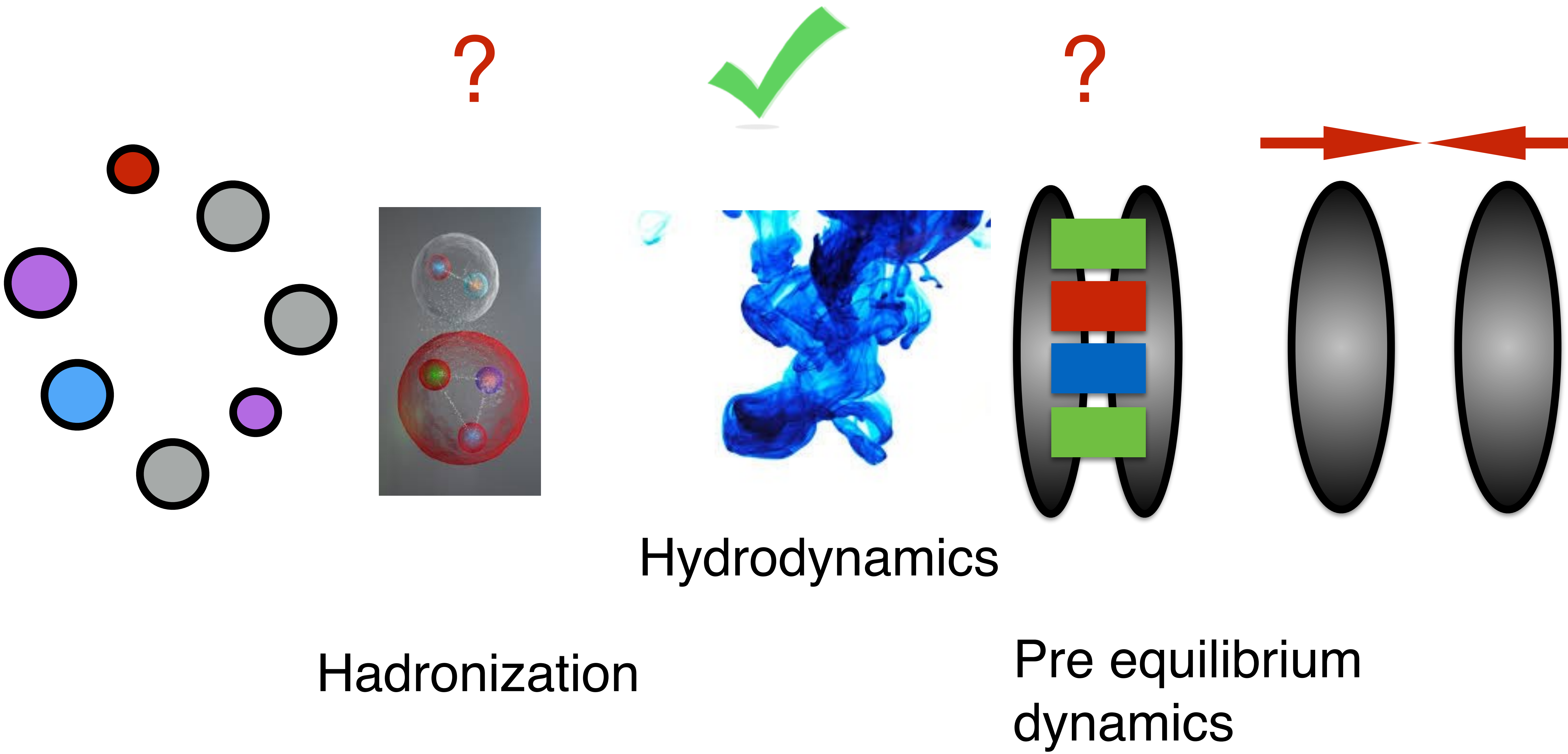
Back to Λ polarization



Back to Λ polarization



Back to Λ polarization



Solving the equations

$$\nabla_\mu T''^{\mu\nu} = 0 \qquad (T''^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta\nabla_\alpha u^\alpha \Delta^{\mu\nu})$$

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Use an ansatz (Bjorken flow)

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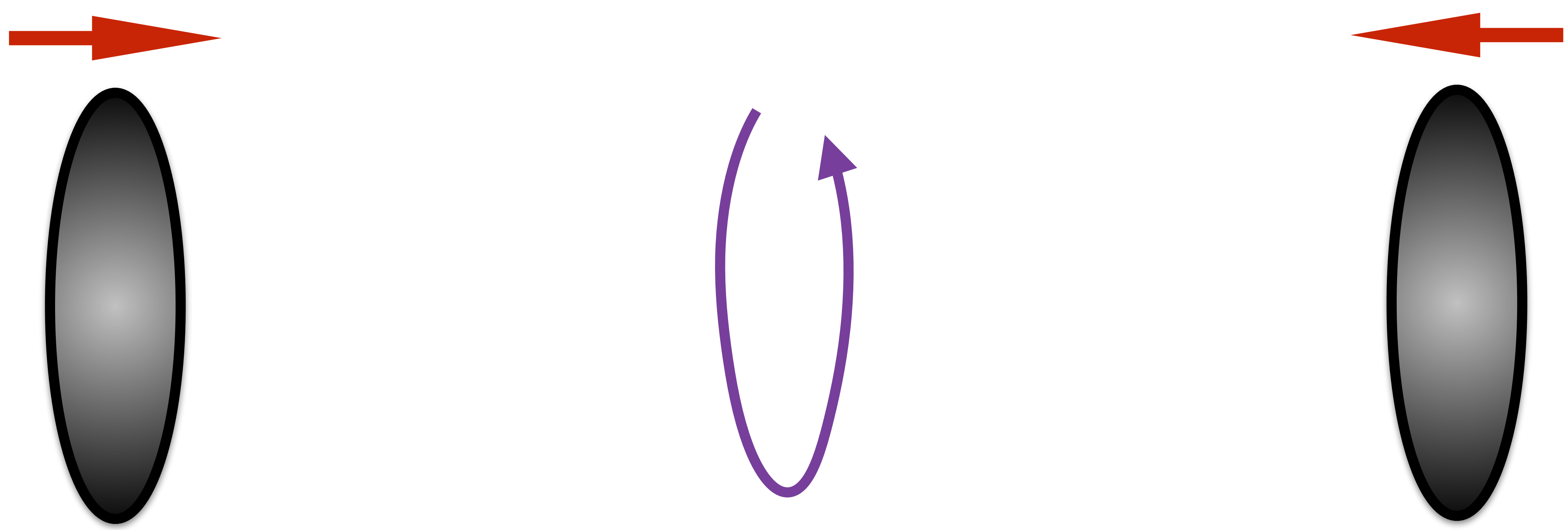

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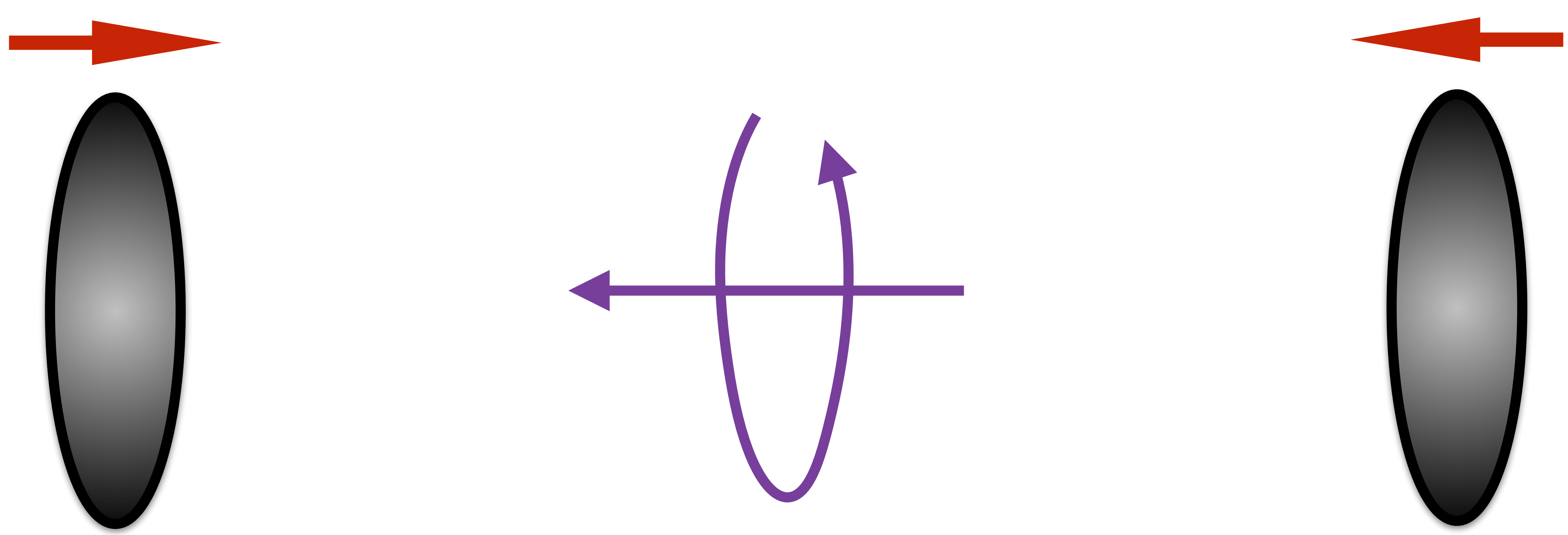

Solving the equations

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$$ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx^2 + dy^2$$

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with linearized perturbations

$$T \to T + \int d^2q \delta T e^{i(q_x x + q_y y)}$$

$$u^\mu \to u^\mu + \int d^2q \delta u^\mu e^{i(q_x x + q_y y)}$$

$$\mu^{ab} \to \mu^{ab} + \int d^2q \delta \mu^{ab} e^{i(q_x x + q_y y)}$$

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and initial conditions

$$\delta u^\eta(\tau_0) \propto \overrightarrow{b} \cdot \overrightarrow{q}$$

Solving the equations

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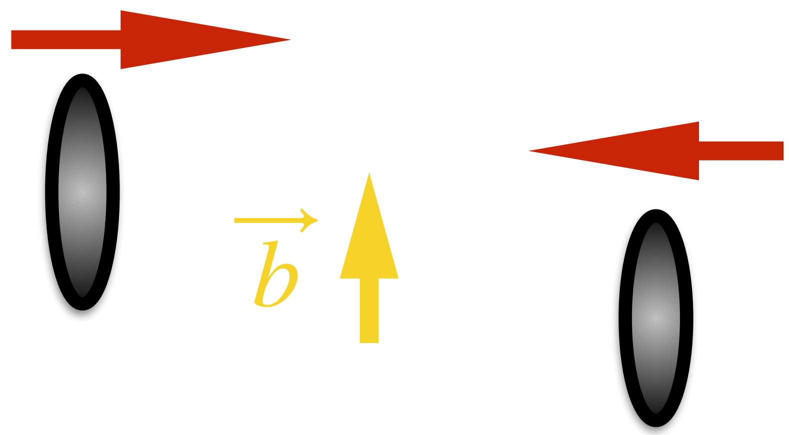

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one finds an analytic solution

$$\delta u^\eta = i u_0\, b\, q_x\, \tau^{-\frac{5}{3}} e^{-\frac{9q^2\eta_0\tau_0}{16T_0\epsilon_0}\left(\frac{\tau}{\tau_0}\right)^{\frac{4}{3}}}$$

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Back to Λ polarization

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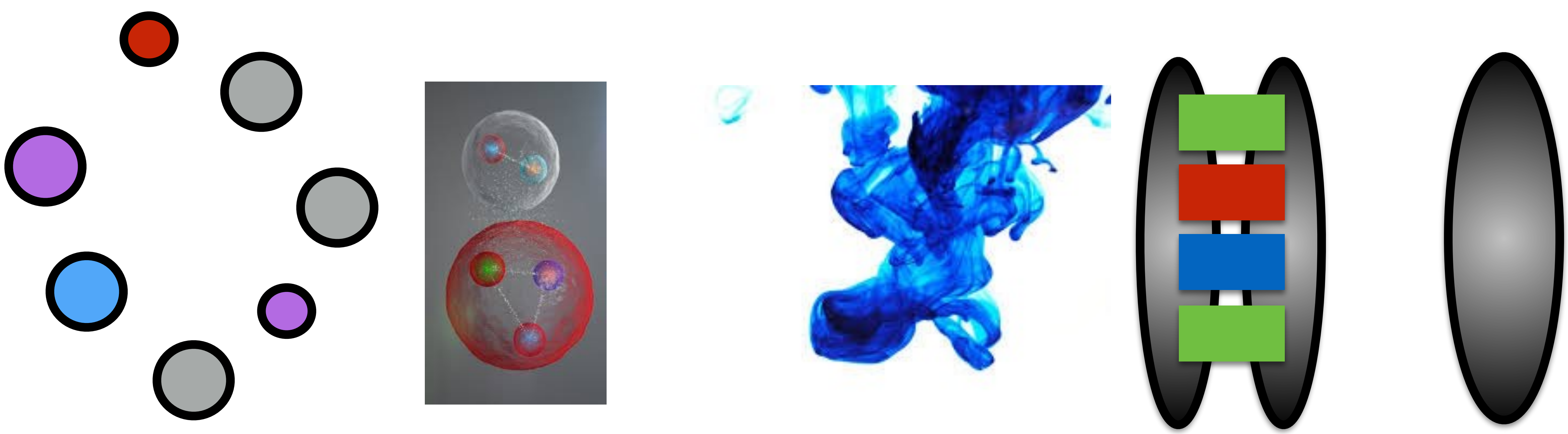

Hydrodynamics

Hadronization

Pre equilibrium dynamics

Back to Λ polarization

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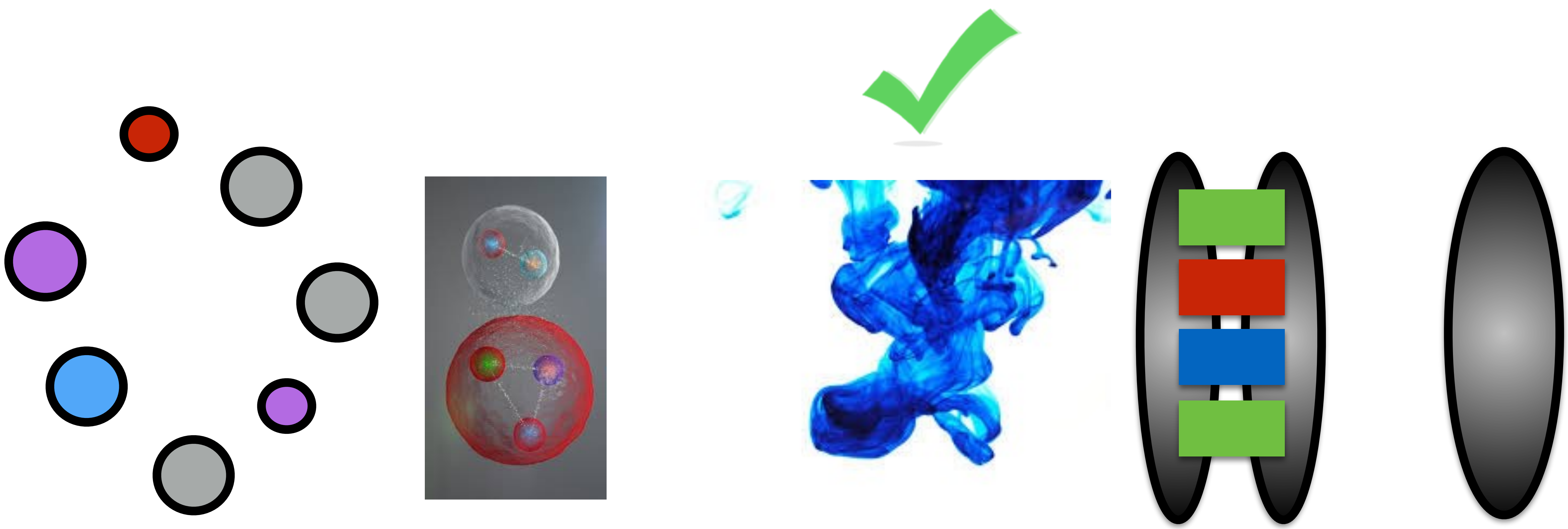

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To obtain the polarization we use a Cooper Frye prescription:

$$\Pi_\alpha(p) = -\frac{1}{4}\epsilon_{\alpha\rho\sigma\beta}\frac{p^\beta}{m}\frac{\int d\Sigma_\lambda p^\lambda B\mu^{\rho\sigma}}{2\int d\Sigma_\lambda p^\lambda n_F}$$

Hadronization

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with linearized perturbations and initial conditions

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To obtain the polarization we use a Cooper Frye prescription. After some massaging

$$\Pi = \alpha \frac{\exp\left(-\frac{5.1}{\left(\frac{s_{NN}}{\mathrm{GeV}^2}\right)^{\frac{3}{8}}}\right) \mathrm{Erf}\left(\frac{5.5}{\left(\frac{s_{NN}}{\mathrm{GeV}^2}\right)^{\frac{3}{16}}}\right)}{\left(\frac{s_{NN}}{\mathrm{GeV}^2}\right)^{\frac{13}{16}}}$$

Hadronization

Use an ansatz (Bjorken flow)

with linearized perturbations and initial conditions

and some approximations (Floerchinger-Wiedemann, 2011)

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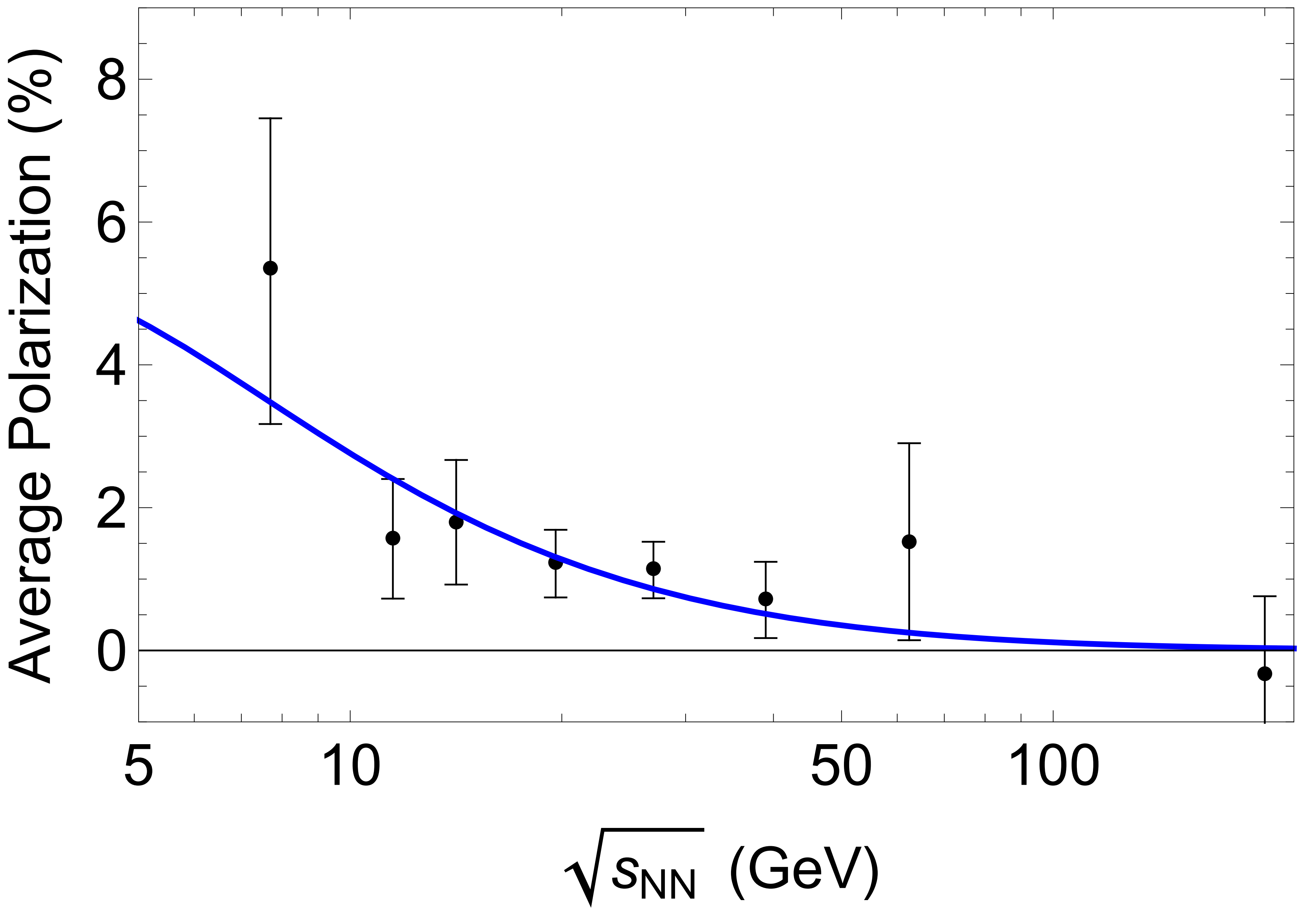
$$\delta u^\eta = i u_0\, b\, q_x\, \tau^{-\frac{5}{3}} e^{-\frac{9 q^2 \eta_0 \tau_0}{16 T_0 \epsilon_0}\left(\frac{\tau}{\tau_0}\right)^{\frac{4}{3}}} \quad \ldots$$

To obtain the polarization we use a Cooper Frye prescription. After some massaging

$$\Pi = \alpha \frac{\exp\left(-\frac{5.1}{\left(\frac{s_{NN}}{\mathrm{GeV}^2}\right)^{\frac{3}{8}}}\right) \mathrm{Erf}\left(\frac{5.5}{\left(\frac{s_{NN}}{\mathrm{GeV}^2}\right)^{\frac{3}{16}}}\right)}{\left(\frac{s_{NN}}{\mathrm{GeV}^2}\right)^{\frac{13}{16}}}$$

Fit to data



Summary

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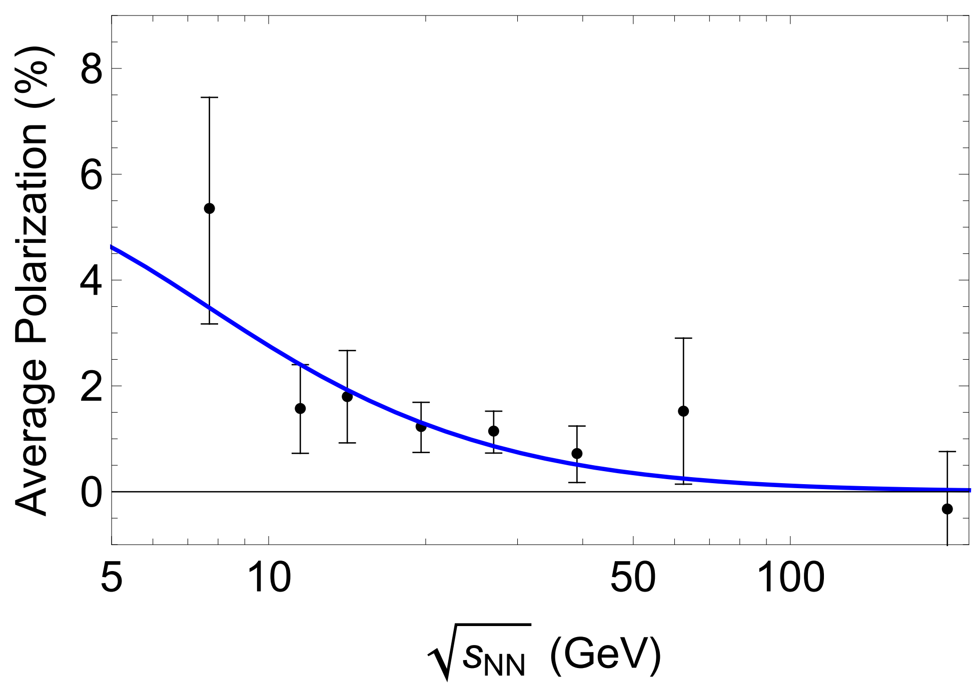
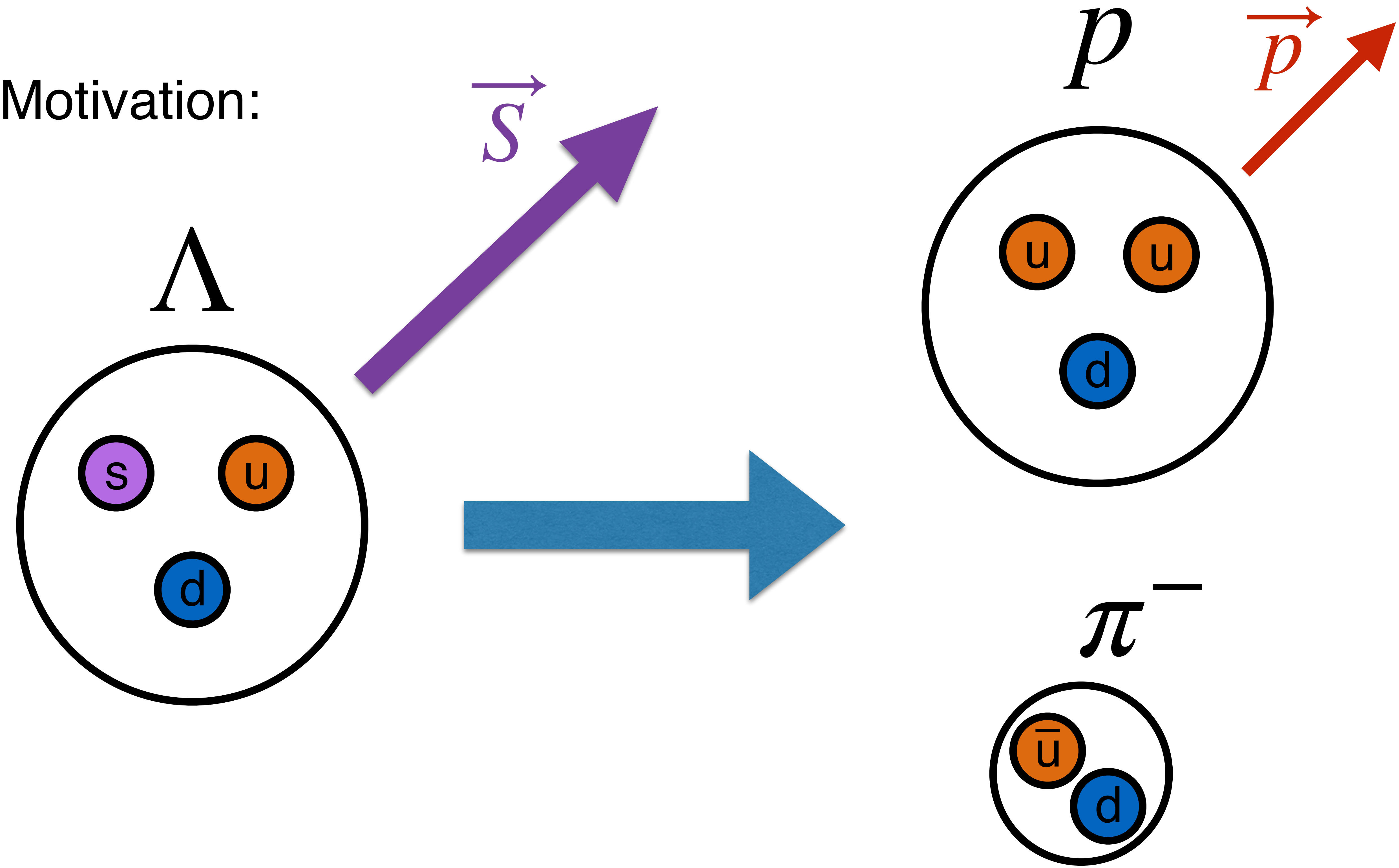


Thank you

Motivation:



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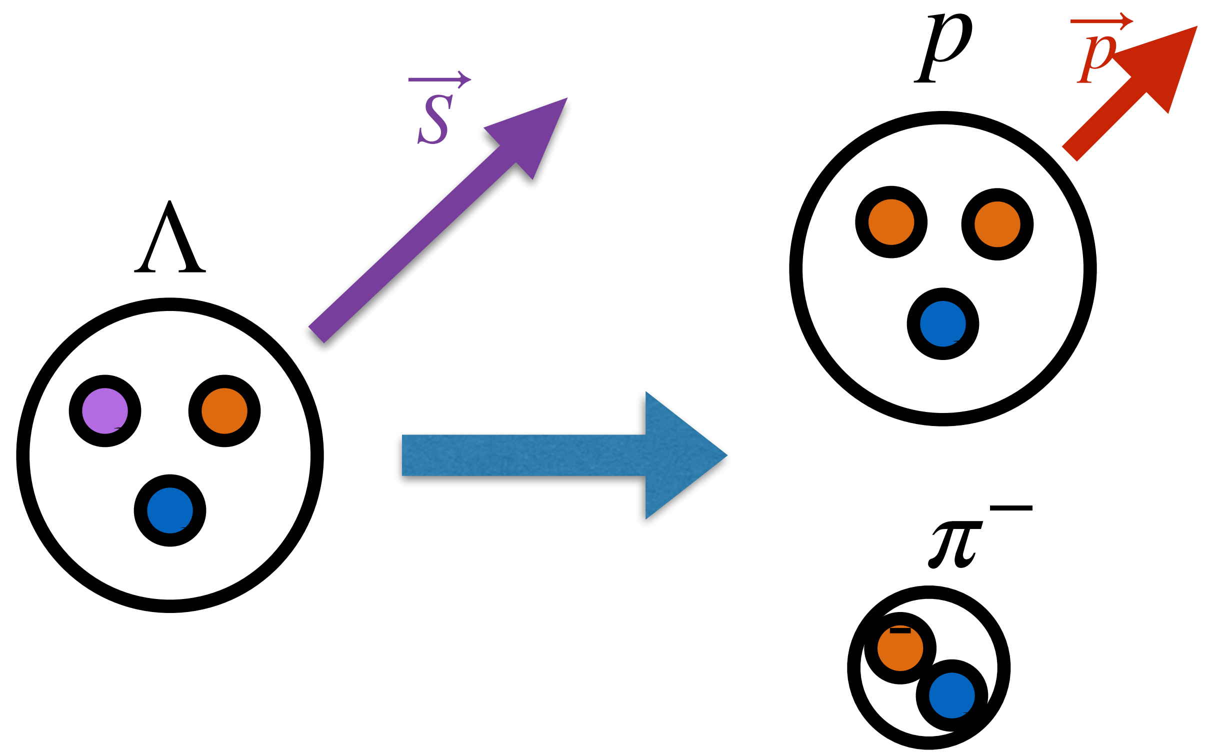

$$\frac{dN}{d\cos\theta^*} \propto \left(1 + \alpha_\Lambda \,|\, \vec{\Pi}^*_\Lambda \,|\, \cos\theta^*\right)$$

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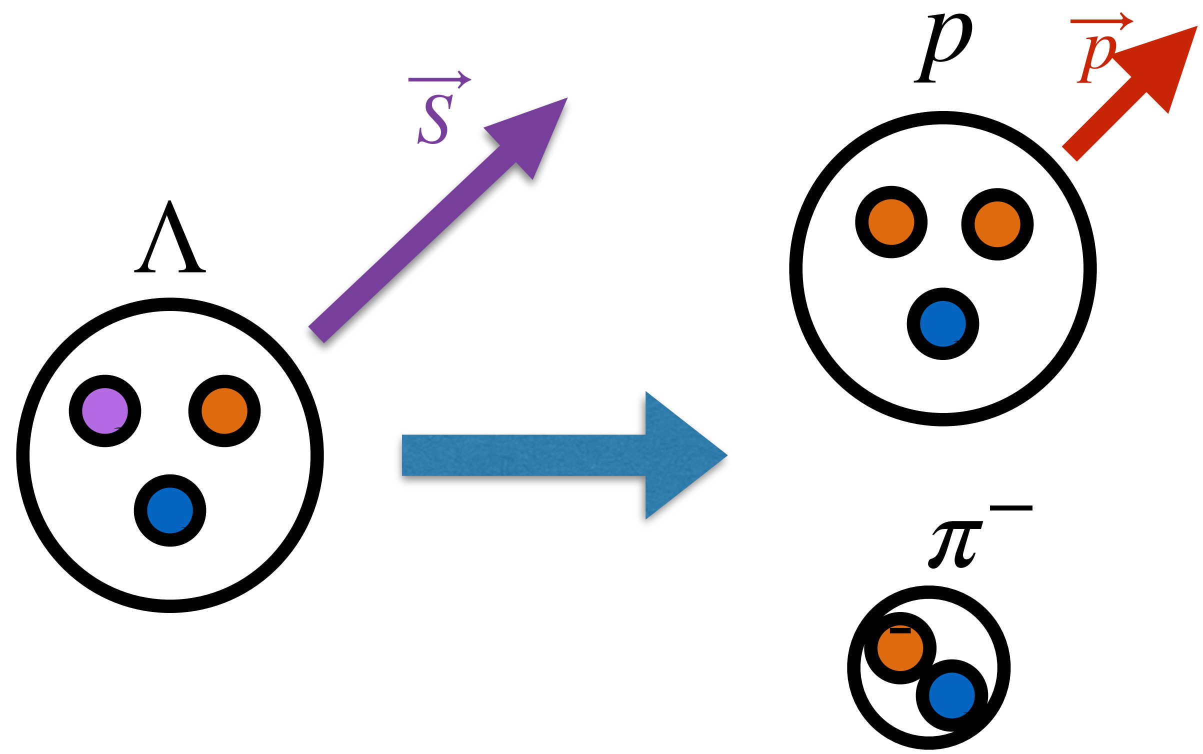

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$$\Pi^\mu_\Lambda = -\frac{1}{2M_\Lambda}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}P_\sigma$$

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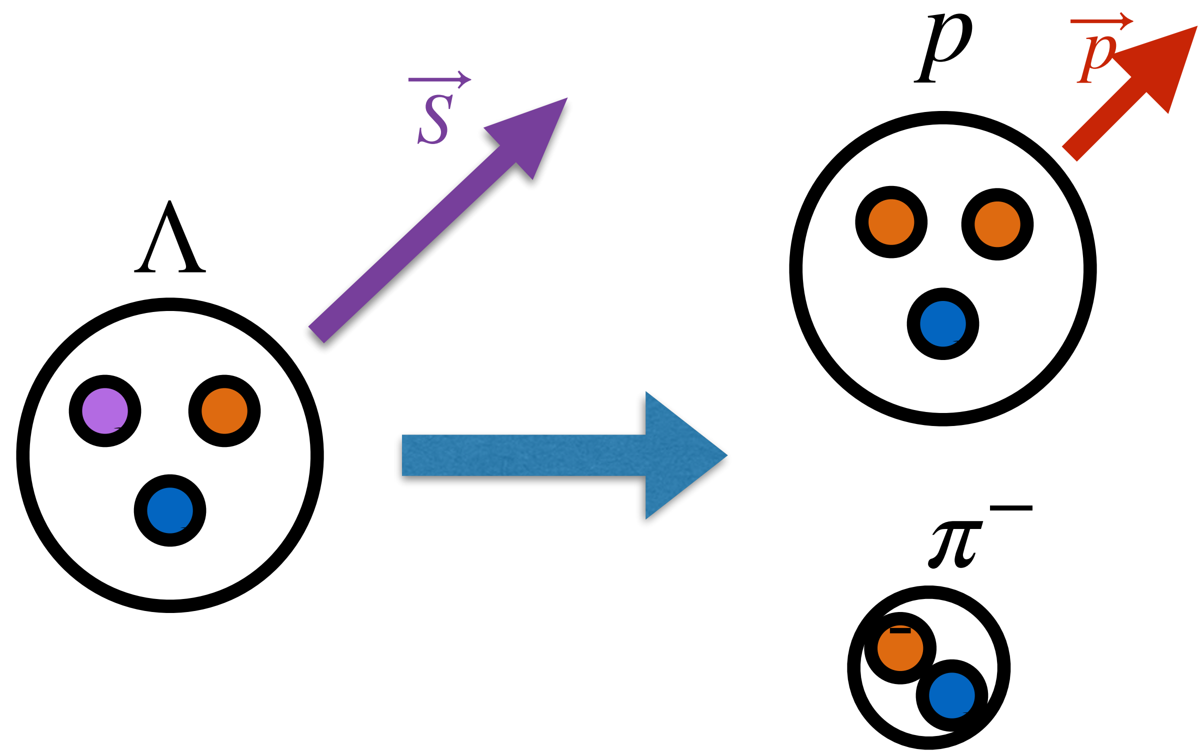

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$$P^*_\sigma = \left(-m_\Lambda, 0, 0, 0\right)$$

$$J^*_{\nu\rho} = S^*_{\nu\rho}$$

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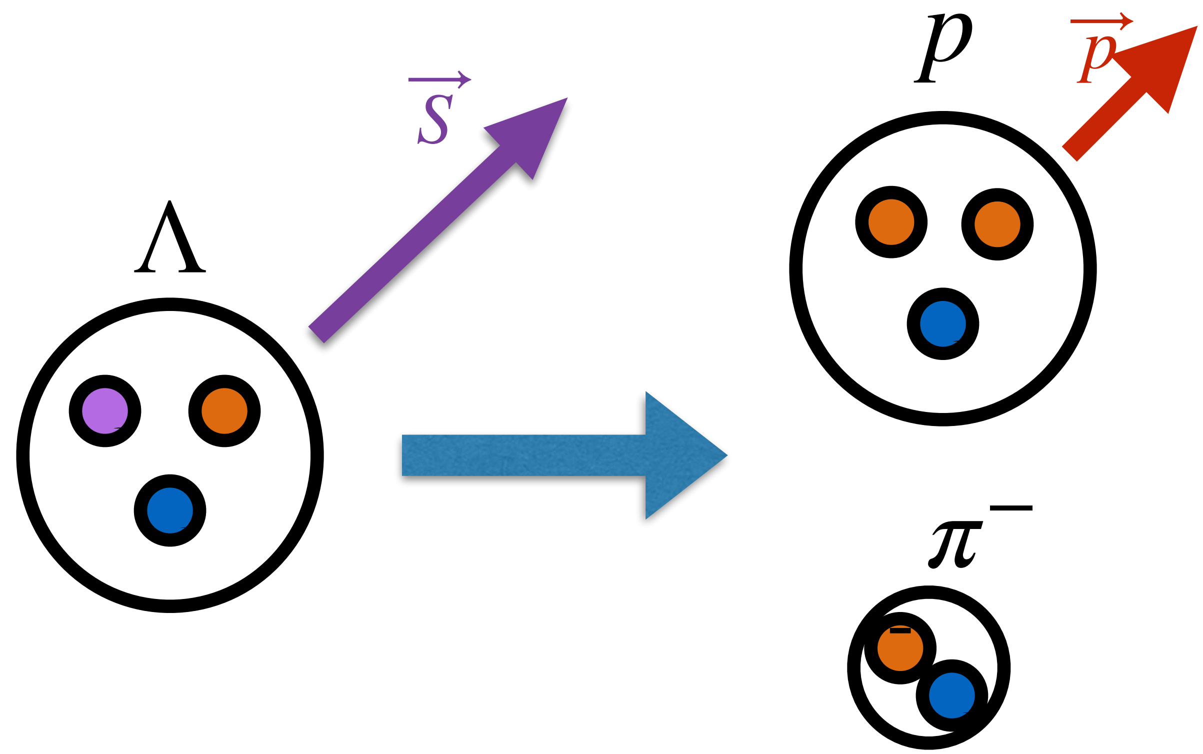
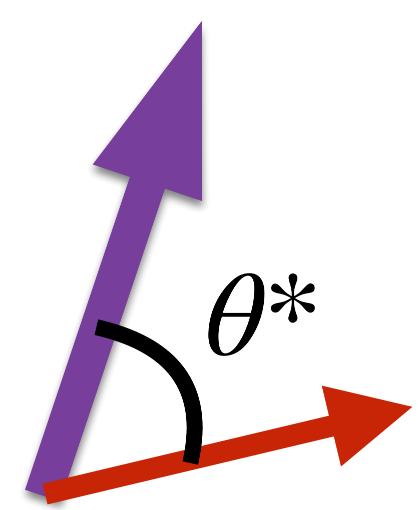


$$\frac{dN}{d\cos\theta^*} \propto \frac{1}{2}\left(1 + \alpha_\Lambda |\vec{\Pi}^*_\Lambda| \cos\theta^*\right)$$



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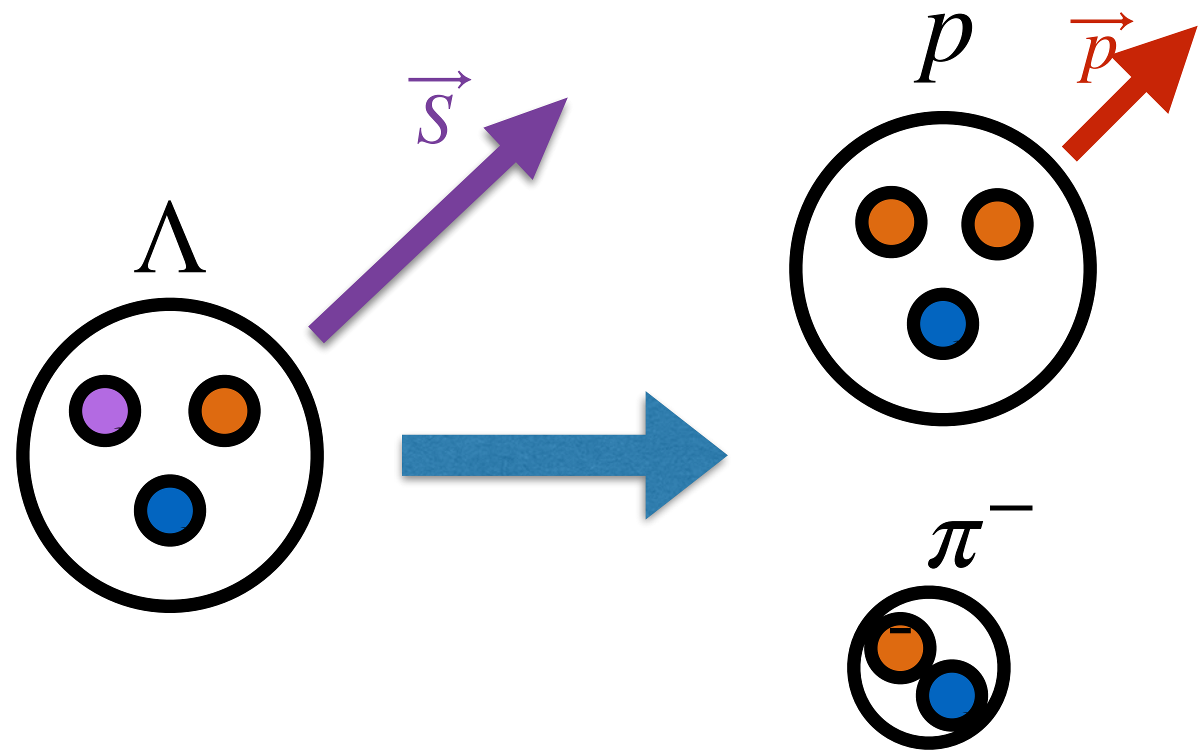

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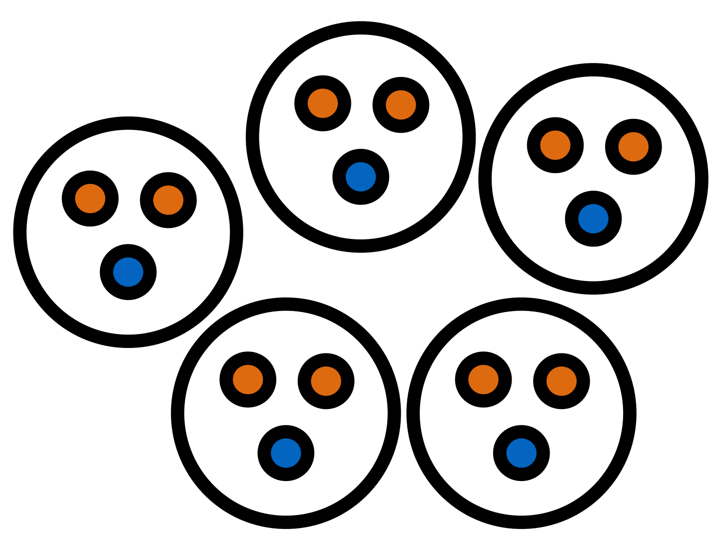

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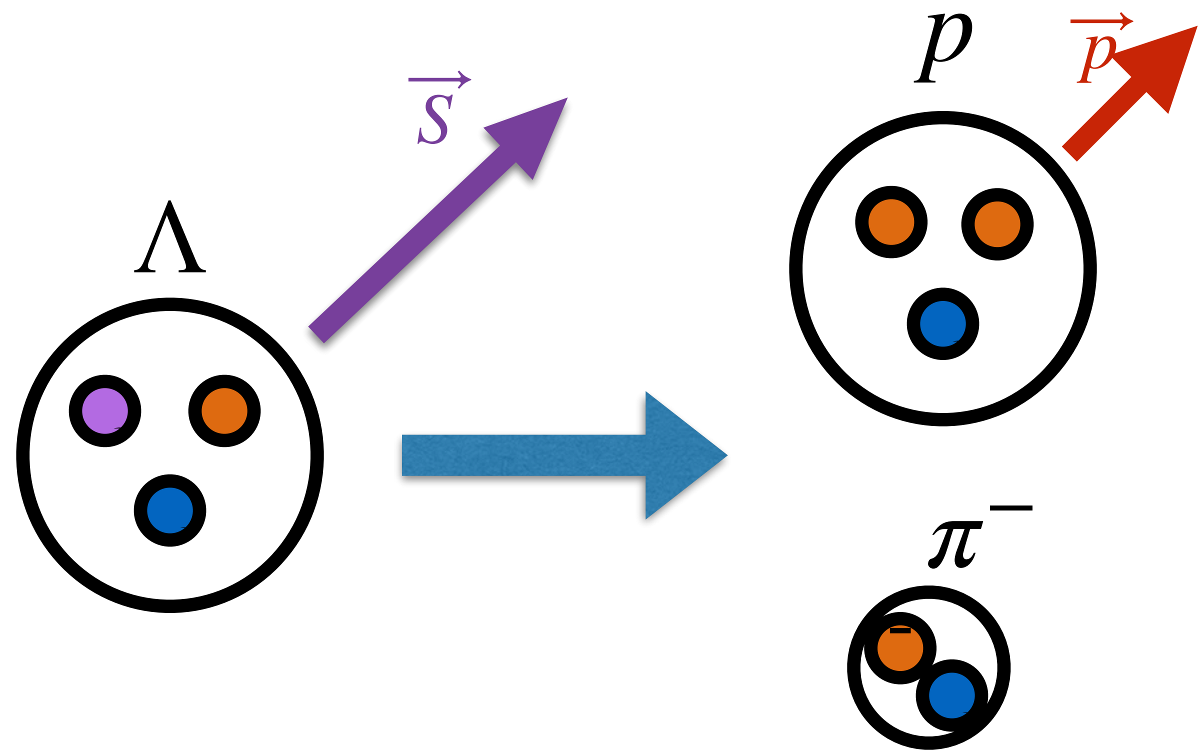

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$$\alpha_\Lambda = 0.642 \pm 0.013$$

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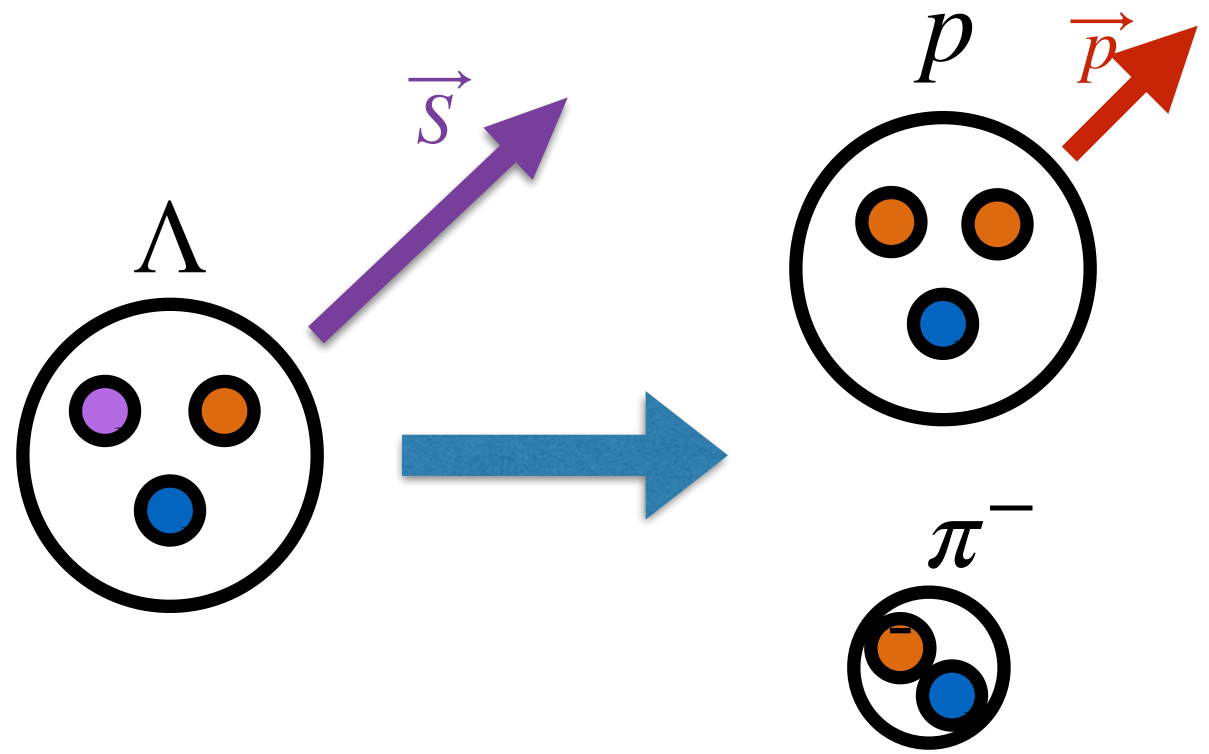

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