Quantum Gravity and Statistical Physics

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Based on:
- -Alex Belin, JdB, arXiv:2006.05499
- -Alex Belin, JdB, Diego Liska, arXiv:2110.14649
- -Alex Belin, JdB, Pranyal Nayak, Julian Sonner, arXiv:2111.06373
- -Taren Anous, Alex Belin, JdB, Diego Liska, arXiv:2112.09143

Holotube
March 8, 2022

We normally view gravity as a low-energy effective field theory.

This would normally imply that gravity has no access to or possesses information about energies $E \gtrsim \Lambda_{\rm UV}$

But gravity is also very different from standard low-energy effective field theory.

It knows for example about

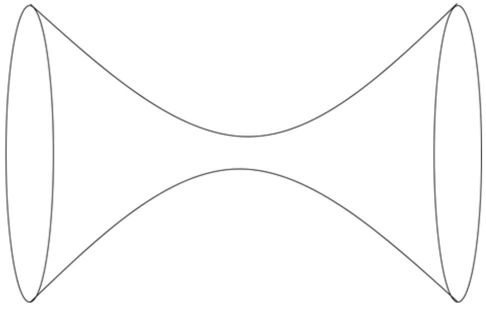
- Black hole entropy – the high temperature partition function
- The partition function on various Euclidean manifolds in AdS/CFT (eg finite temperature correlators)
- The page curve (using island/replica wormholes)

Penington ’19
Almheiri, Engelhardt, Marolf, Maxfield ‘19
Penington, Shenker, Stanford, Yang ‘19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ‘19

- Spectral correlations

Saad, Stanford, Shenker ‘19

- Apparent lack of factorization



None of these resolve *exact* information about the UV physics of the theory.

Rather, they provide *coarse grained* information about the UV physics

Example: the spectral density

From the entropy of a black hole we obtain an *approximate* expression for

$$Z(\beta) = \int dE \rho(E) e^{-\beta E}$$

Typically, we do not have the required exp(-S) accuracy to resolve the exact density of states

$$\rho(E) = \sum_i \delta(E - E_i)$$

We would then be able to see all the individual microstates of the black hole in LEEFT. Exceptions could be integrable models, topological theories, or BPS black holes.

What about

$$\langle Z(\beta_1)Z(\beta_2)\rangle_c \quad \text{or} \quad \langle \rho(E_1)\rho(E_2)\rangle_c$$

In a single microscopic theory such connected two point functions vanish. But the existence of wormholes suggests that LEEFT may yield a non-zero answer.

How can that be?

Consider a large set of N=e^S random phases $e^{i\phi_i}$

$$\left(\sum e^{i\phi_i}\right)\left(\sum e^{i\phi_i}\right)^* = \sum_{i=j} 1 + \sum_{i\neq j} e^{i(\phi_i - \phi_j)}$$

N=e^S
visible in
gravity

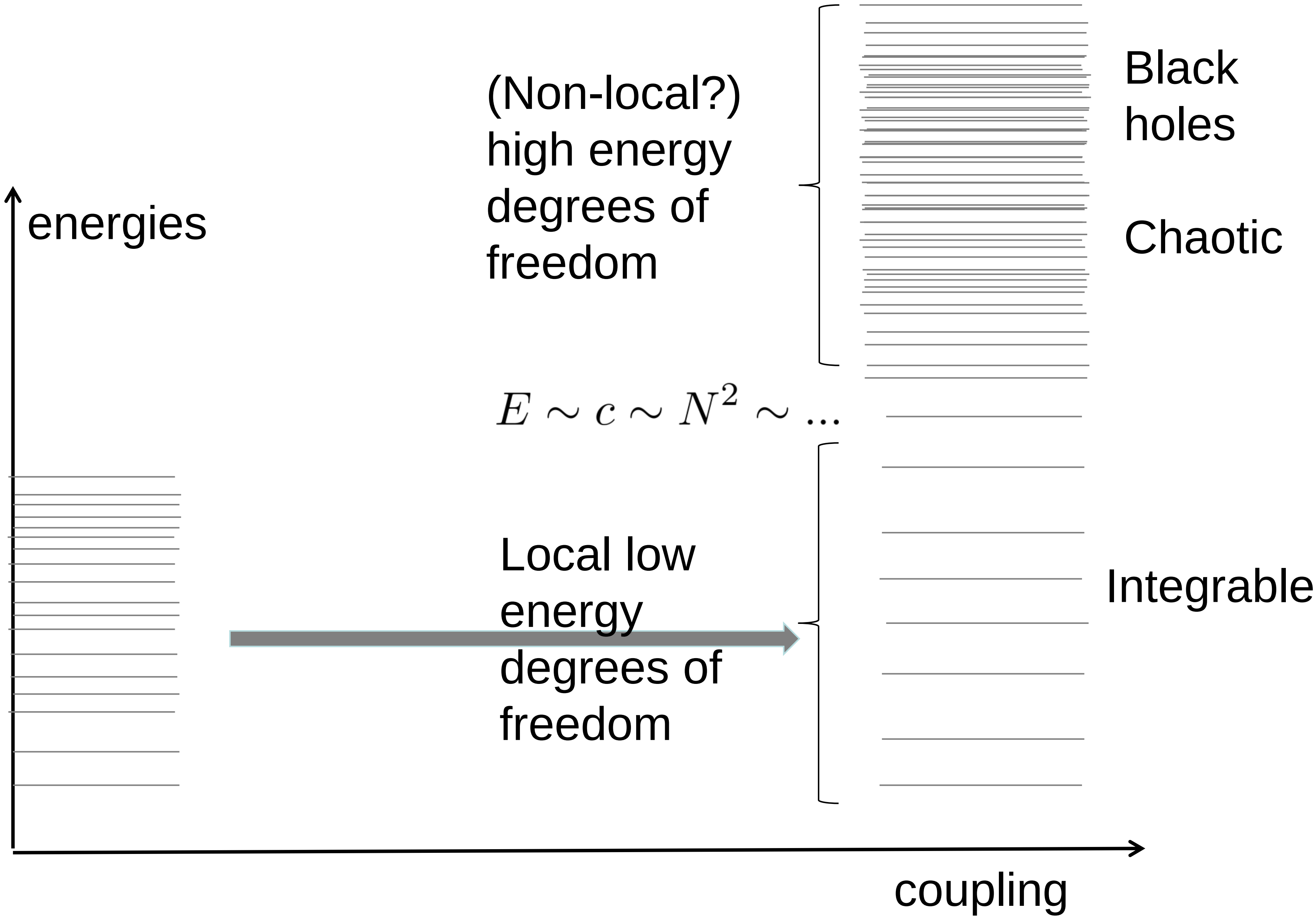
erratic
invisible in
gravity

So LEEFT is sensitive to the average size of fluctuations but not to the individual fluctuations themselves

$$\left(\sum e^{i\phi_i}\right)\left(\sum e^{i\phi_i}\right)^* = \sum_{i=j} 1 + \sum_{i\neq j} e^{i(\phi_i - \phi_j)}$$

What happens in the UV? Possibilities:

- The relevant gravitational solution (eg wormhole) is unstable and factorization is restored (but solution remains as off-shell configuration)
- UV physics adds the fluctuating contributions $\sum_{i\neq j} e^{i(\phi_i - \phi_j)}$ and factorization is restored
- The UV theory is an average of theories, averaging makes the fluctuating term exactly zero, and factorization is not restored



energies
(Non-local?) high energy degrees of freedom
Black holes
Chaotic
$E \sim c \sim N^2 \sim \ldots$
Local low energy degrees of freedom
Integrable
coupling

MAIN CLAIM:

- Semi-classical gravity is the theory of the statistics of the chaotic sector of the theory.
- It can probe (coarse-grained) higher moments of the relevant statistical distributions but not individual values.
- It cannot distinguish averaged from non-averaged theories as long as the averages yield the same moments of the statistical distribution (up to the accuracy of the low-energy effective field theory).

Is this a fundamental limitation on how much information low-energy observers can obtain?

Suppose e.g. that the chaotic sector of the theory is like the digits of π.

There is no coarse grained measurement of say a block of N digits of π which will distinguish it from an average over a uniform distribution of N digits (up to a “non-perturbative” error of order ~1/N)

Evidence from operator statistics.

OPE coefficients $C_{ijk} = \langle \mathcal{O}_i(\infty)\mathcal{O}_j(1)\mathcal{O}_k(0)\rangle$

Distinguish light (L) and heavy (H) operators depending on whether operator is in integrable or in chaotic sector of the theory.

Then proposal is that semi-classical gravity has access to the statistics of C_{LLH}, C_{LHH} and C_{HHH} coarse grained over H indices but not to their individual values.

OPE randomness hypothesis (Belin, JdB ’20):

$$C_{LLH} \sim f(E_H) R_H$$

$$C_{LHH'} \sim f(\bar{E_H}, \Delta E_H) R_{HH'}$$

$$C_{HH'H''} \sim f(\bar{E_H}, \Delta E_H) R_{HH'H''}$$

Slowly varying function of arguments

- Pseudorandom
- Mean=0
- Variance=1
- Can have higher moments which are exponentially suppressed.

Particular case: the Eigenstate Thermalization Hypothesis (ETH) is a well-known hypothesis which connects precision low-energy data with statistical high-energy data:

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + \overbrace{e^{-S(\bar{E})/2} g_a(\bar{E},\Delta E) R^a_{ij}}^{C_{LHH}}$$

Deutsch '91
Srednicki '94
Foini, Kurchan '19

$f_a(\bar{E})$: one point functions of simple operators

$g_a(\bar{E},\Delta E)$: two point functions of simple operators

R^a_{ij} : Gaussian random variables

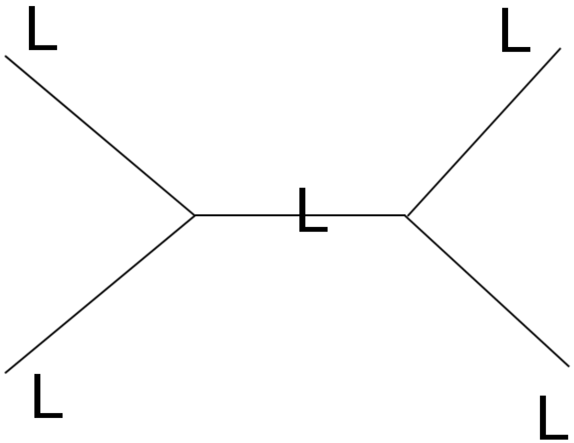
$$\langle R^a_{ij}\rangle = 0, \qquad \langle R^a_{ij} R^b_{kl}\rangle = \delta^{ab}\delta_{il}\delta_{jk}$$

ETH correctly reproduces the thermal one- and two-point functions and implies that typical states look thermal.

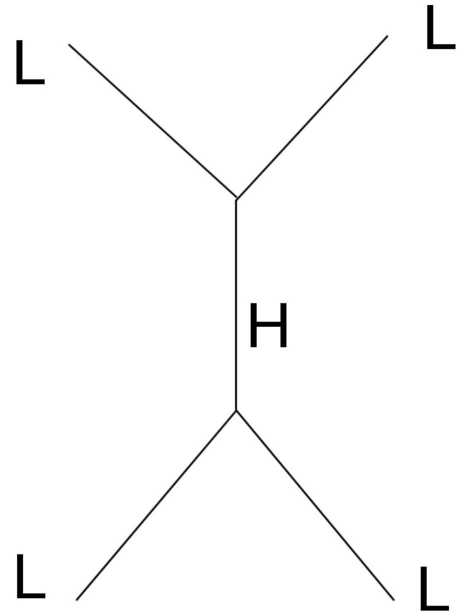
This is all information which is available to semi-classical gravity throught the AdS/CFT correspondence.

Note: this does *not* prove the validity of ETH, nor does ETH require more input that the thermal one- and two-point functions.

How does one test all of this?



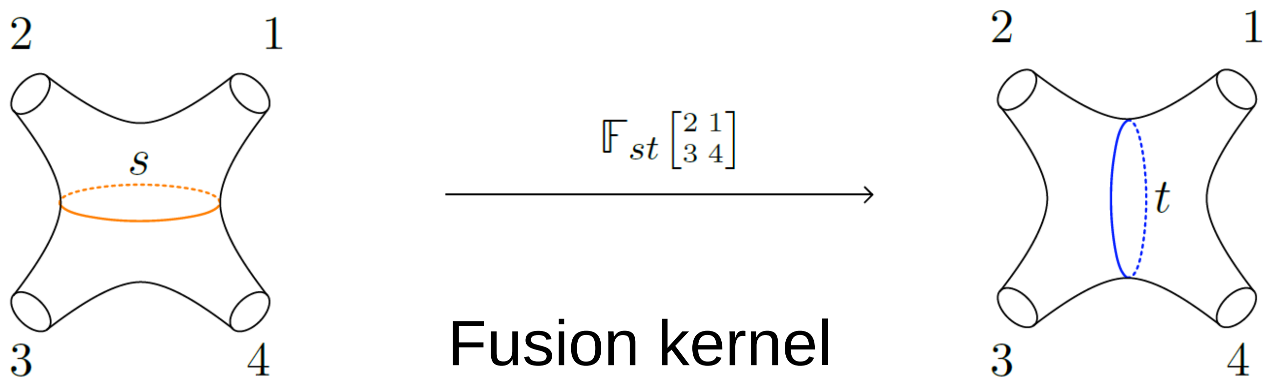
$$= \sum_H$$



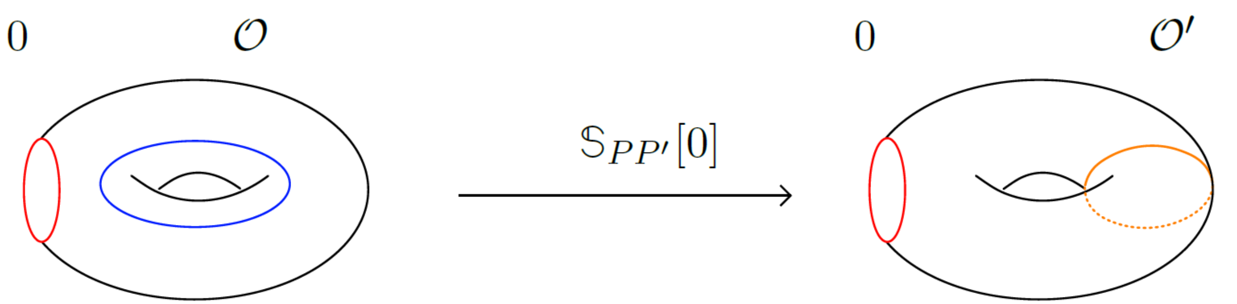
Crossing symmetry

Pappadopulo, Rychkov, Espin, Ratazzi ‘12

In d=2:



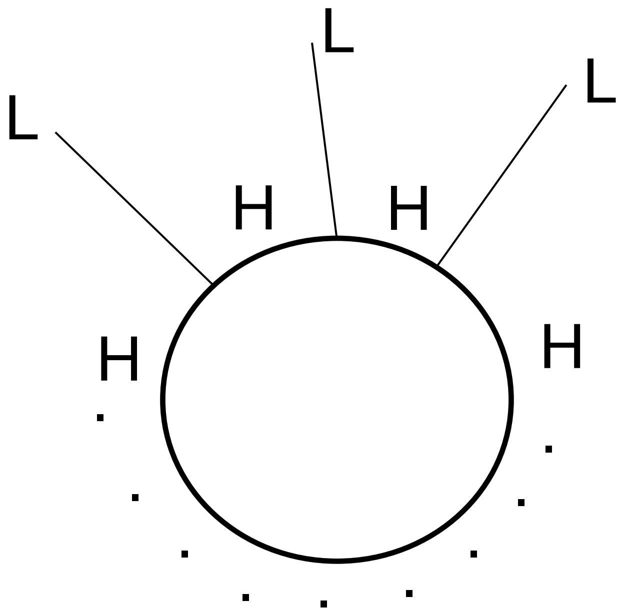
Fusion kernel



Modular kernel

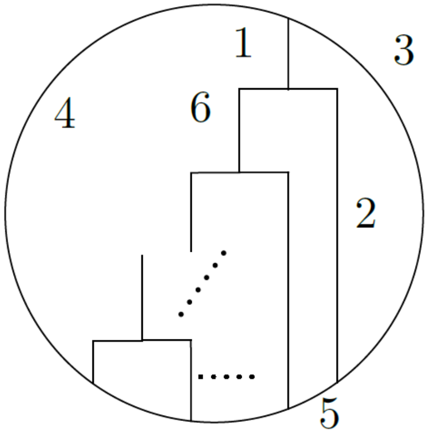
Ponsot, Teschner ’99, ‘00
Teschner ‘03
Collier, Maloney, Maxfield, Tsiaris ‘19

Results:



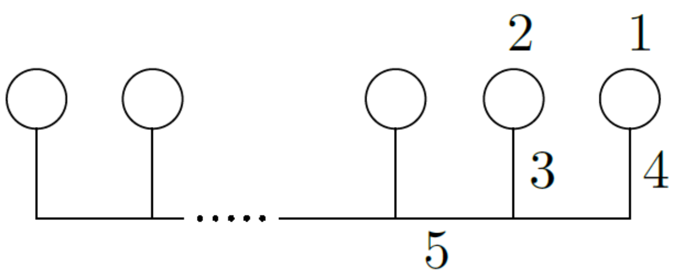
$$\langle C^k_{LHH} \rangle \sim e^{-(k-1)S}$$

Foini, Kurchan '19



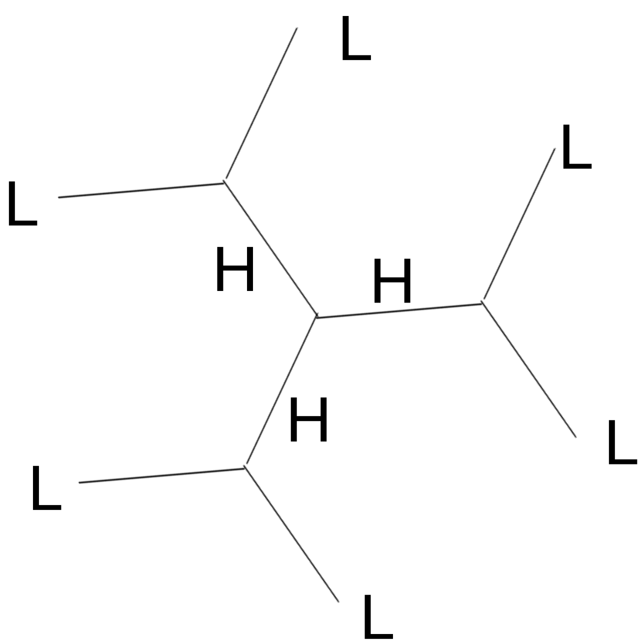
$$\langle C^k_{HHH} \rangle \sim e^{-\frac{5k-4}{4}S}$$

Belin, JdB, Liska '21



$$\langle C^k_{HHH}\rangle \sim e^{-\frac{9k-6}{8}S}$$

Belin, JdB, Liska '21



$$\overline{C^3_{LLH}C_{HHH}}\Big|_{d\geq 2,\ \text{all op}} \sim \frac{\Delta_H^{6\Delta_L-3}}{\rho(\Delta_H)^3}$$

$$\overline{C^3_{LLH}C_{HHH}}\Big|_{d=2,\ \text{quasi-prim}} \sim \frac{\Delta_H^{6\Delta_L-6}}{\rho(\Delta_H)^3}$$

$$\overline{C^3_{LLH}C_{HHH}}\Big|_{d=2,\ \text{Vir-prim}} \sim \left(\frac{3\sqrt{3}}{16}\right)^{3\Delta_H}\frac{\Delta_H^{6\Delta_L-\frac{19+11c}{36}}}{\rho_{\text{vip}}(\Delta_H)^{\frac{9}{4}}}\,.$$

$$\langle C^3_{LLH}C_{HHH}\rangle_{\text{all}} \sim e^{-3S}$$

$$\langle C^3_{LLH}C_{HHH}\rangle_{d=2\,\text{quasi}-\text{prim}} \sim e^{-3S}$$

$$\langle C^3_{LLH}C_{HHH}\rangle_{d=2\,\text{prim}} \sim e^{-\frac{9}{4}S}$$

Anous, Belin, JdB, Liska '21

More complicated tree-level diagram in d=2

$$\langle C^{m+2}_{LLH} C^{m}_{HHH} C^{\eta}_{HHL} \rangle_{d=2\,\mathrm{prim}} \sim e^{-\frac{2+4\eta+7m}{4}S}$$

Anous, Belin, JdB, Liska '21

General lesson:

Connected higher-point functions are exponentially suppressed

Results apply in any CFT, not necessarily chaotic – size of window one needs to average over depends on theory.

Qualitative behavior can also be obtained from

- Microcanonical unitary averages
- Altland-Sonner sigma model for chaos

Altland, Sonner ‘21
Belin, JdB, Nayak, Sonner ‘21

Results suggests statistics of OPE coefficients can be represented by a “generating functional” which captures the higher moments of the probability distributions

$$\mathcal{Z}(J_{abc}) = \exp\Big[f_1(\Delta) J_{abc} J^{abc} + f_2(\Delta) J^{a}{}_{ab} J^{bc}{}_{c} + \sum_{i=1}^{5} g_i(\Delta) JJJJ\big|_{\text{i-type contraction}} + \dots \Big]$$

so that

$$\langle C...C \rangle = \frac{\delta}{\delta J} ... \frac{\delta}{\delta J} \mathcal{Z}(J)\Big|_{J=0}$$

Open questions:

- What index contractions appear?
- Is single-sided information sufficient to construct Z?

Notice that we have *not* proven the OPE randomness hypothesis. It is presumably only valid in chaotic CFT's, and we have not proven that strongly coupled CFT's with weakly coupled gravitational duals are chaotic either (though there are strong indications they are). e.g. Maldacena, Shenker, Stanford '15

But we can test its consistency by comparing with gravitational computations.

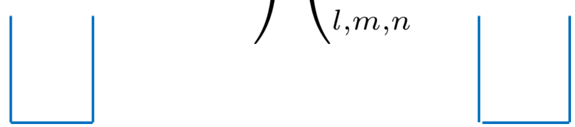
For example, one implication is that typical states are exponentially difficult to distinguish from a thermal state. The corresponding statement in gravity is that all typical states look like the same black hole to low-energy observers.

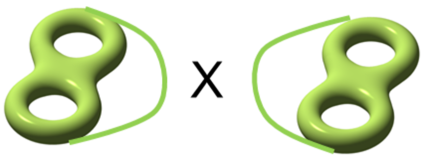
The OPE randomness hypothesis (in the form of the generating functional) can also be used to try to make predictions for disconnected correlators – OPE coefficients are connected by propagators and vertices just as in Feynman diagrams.

Example: the square of the high-temperature genus two partition function (dominated by high-energy states)

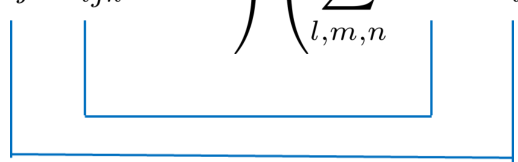
$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta} \right) \right\rangle$$

$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta} \right) \right\rangle$$





$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta} \right) \right\rangle$$



In this example, the wormhole exists (Maldacena, Maoz ‘04) and agrees with the above prediction from “perturbation theory”.

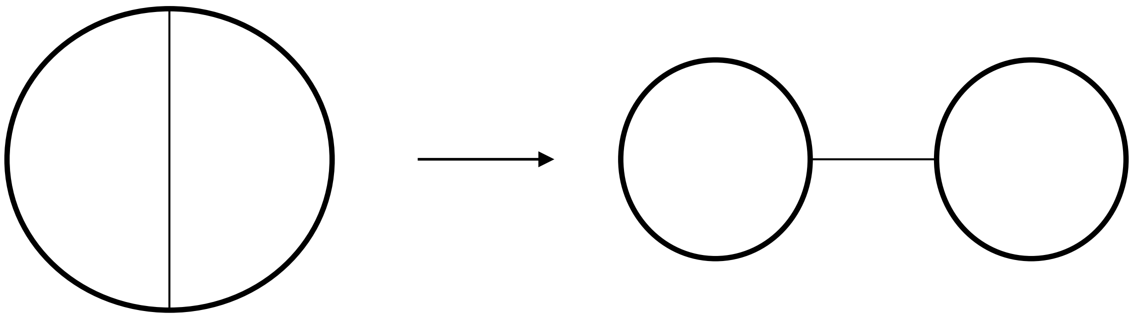
Belin, JdB ‘20

Any disagreement could in principle be fixed by adding additional quartic vertices in which the same index appears more than twice – open question whether such vertices appear or not.

Two wormhole "predictions"

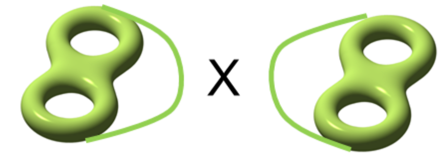
1. Take the high-temperature genus two partition function in a different corner of moduli space

$$\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \Longrightarrow \sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta}$$

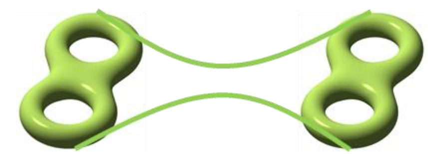


We can repeat the previous computation for the square of this partition function

$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{lmn} C_{llm} C^*_{mmn} e^{-3\beta\Delta} \right) \right\rangle$$



$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{llm} C^*_{mnn} e^{-3\beta\Delta} \right) \right\rangle$$



$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{llm} C^*_{mnn} e^{-3\beta\Delta} \right) \right\rangle$$

????

Vertex computed independently from the genus 3 partition function

The quartic vertex dominates over the second wormhole contribution.

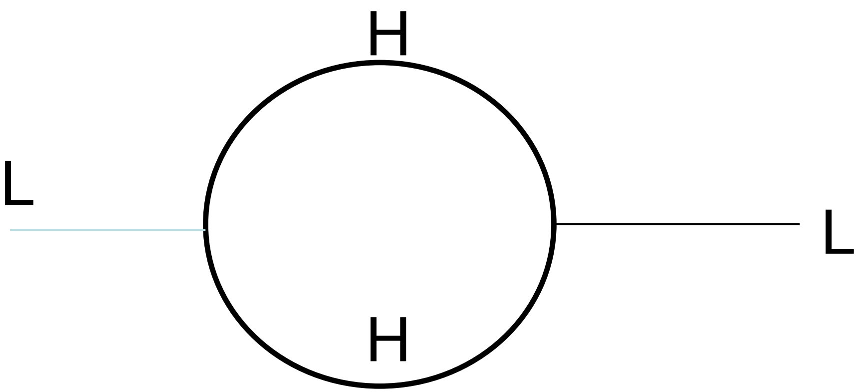
Suggests that there exists a new wormhole connecting two genus two Riemann surfaces with action

$$Z = e^{\frac{25c - 360\Delta_\chi}{288}\frac{\pi^2}{\beta}}$$

lightest scalar

Result suggest that this is a wormhole supported by matter fields. Would be interesting to construct it explicitly.

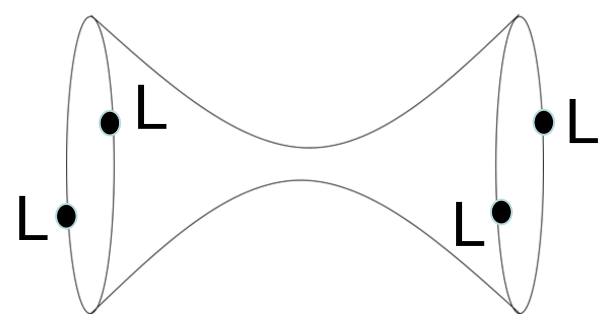
2. Consider the product of two finite temperature two-point functions $\langle (C_{LHH}C_{LHH})_1 (C_{LHH}C_{LHH})_2 \rangle$



$\sim C^2_{LHH}$

Connected Wick contraction "predicts" a new wormhole

$\langle (C_{LHH}C_{LHH})_1 (C_{LHH}C_{LHH})_2 \rangle$



Such wormholes seem to exist as complexified solutions..

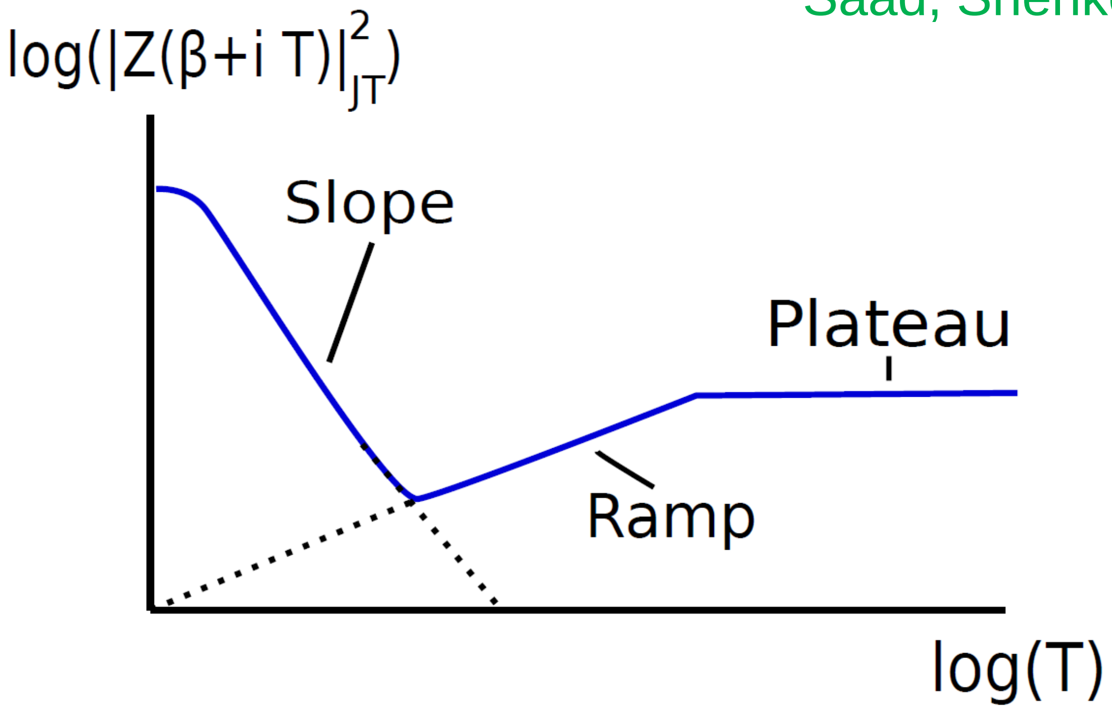
Besides operator statistics there is also spectral statistics related to e.g. $\langle \rho(E)\rho(E')\rangle$

This can be studied explicitly e.g. in

- JT gravity (Saad Shenker Stanford ‘19)
- Pure 3d gravity (Cotler, Jensen ‘20)
- Using a sigma model based on symmetry breaking (Altland, Sonner, ’21)

More precisely, using wormholes in JT gravity, one can find the following picture for the spectral form factor

Saad, Shenker, Stanford, ‘19



Knows about discrete features of the spectrum.. But spoils factorization

The ramp is related to a particular wormhole configuration known as the “double-cone”. It has been studied in d>3 e.g. by Mahajan, Marolf, Santos ’21; Cotler, Jensen ’21

The plateau requires a non-perturbative resummation of wormholes.

It is not clear to what extent the ramp and especially the plateau are part of “semi-classical gravity” as they involve very late time physics of order $t \sim e^S$.

To “prove” the chaotic, random matrix nature of the high-energy sector of a strongly coupled CFT would presumably require finding a precise reduction of late time gravitational physics to a 2d JT sector? Does the topological string perspective of Van der Heijden, Post, Verlinde ‘22 help?

A new statistical framework?

It is an interesting question whether there is a single general formalism which captures both operator and spectral statistics.

Since we are trying to parametrize our ignorance about the chaotic high-energy sector of the theory, it is tempting to think about a suitable generalization of statistical physics.

The similarity with averaged theories, and the relation of wormholes to superselection sectors and "alpha-vacua" (Marolf, Maxfield '20 '21) suggest to allow for a probability distribution on the space of density matrices

$$\rho_{\text{micro}} \Longrightarrow \rho_{\text{LEEFT}} = \int \mu[\rho] d\rho$$

In statistical physics, the thermal state arises by requiring (i) maximal entropy and (ii) the right expectation value of the energy.

What replaces those notions for ensembles of density matrices? If one replaces (i) by a combination of classical and quantum entropy and keeps (ii) one arises at the following measure on the space of density matrices

$$\mu[\rho] = \mathcal{N} e^{-S(\rho|\rho_\beta)}$$

which has several nice properties

Work in progress:
Arav, Chapman, JdB
JdB, Liska, Post, Sasieta

CONCLUSIONS

Semi-classical gravity is the theory of the statistics of the high-energy, chaotic sector of the theory.

Picture is consistent with known wormhole solutions and predicts new wormhole solutions.

Picture is consistent with observation that wormholes only correct 2d low-energy observables non-perturbatively (Schlenker, Witten '22)

What is the right overarching statistical framework?

Can we get more insight from bootstrap approaches?

Is single-sided information enough or do wormholes yield genuine new information?

Can one prove the chaotic random matrix theory nature of the high-energy sector of a strongly coupled CFT?

Or there any deep lessons for other chaotic systems in nature?

It seems very difficult to probe interesting aspects of quantum gravity using semi-classical gravity alone.

Semi-classical gravity is “averaging agnostic”.