Space-dependent symmetries and fractons

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Based on some work with

K. Grosvenor (Leiden), F. Peña-Benitez and P. Surowka (Wraclaw)
2105.01084, 2112.00531

and

R. Argurio, D. Naegels (ULB), and D. Musso (UO)
2006.11047, 2107.03073

Outline

1. Fractons and symmetries
2. Hydrodynamics with "fractonic" symmetries
3. Broken translation invariance and emergent fractons
4. Outlook

Fractons and symmetries

What is a fracton?

In general, it is some excitation of restricted mobility
it is not allowed to move in all spatial directions

- Excitations in theoretical lattice models
(Haah's code Haah ('11), X-cube model Vijay, Haah, Fu ('16), Checkerboard model Shirley, Slagle, Chen ('18))

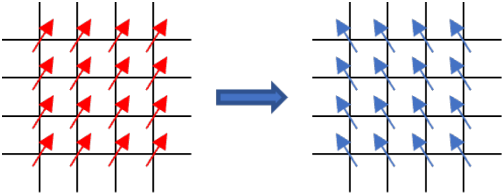
- Defects in solids: disclinations and dislocations

- Mode with momentum-independent dispersion relation

Related to symmetries dependent on spatial coordinates:
dipole, multipole, subsystem

Subsystem symmetries

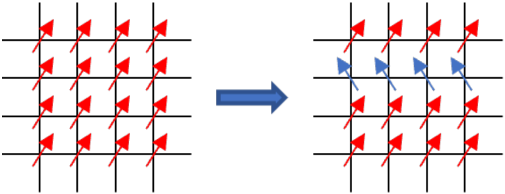
Global symmetry transformation



- Transformation parameters $e^{i\alpha}$
- Volume of space of vacua $=$ Vol(S^1)

Subsystem symmetries

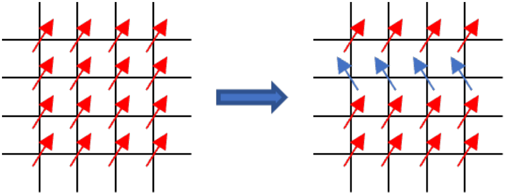
Subsystem symmetry transformation



- Transformation parameters $e^{i\alpha_i}$, $i = 1, \ldots, N_x$
- Volume of space of vacua $= (\mathrm{Vol}(S^1))^{N_x}$
  New phase of matter?

Subsystem symmetries

Subsystem symmetry transformation



- Transformation parameters $e^{i\alpha_i}$, $i = 1, \ldots, N_x$
- Volume of space of vacua $= (\text{Vol}(S^1))^{N_x}$
  New phase of matter?
- Continuum limit $\alpha_i \longrightarrow \alpha(x)$

Subsystem symmetries

- A simple field theory example (studied in detail by Seiberg, Shao ('20))

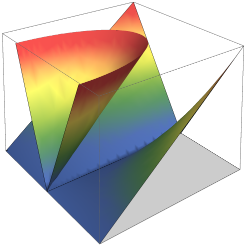
$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_x \partial_y \phi)^2$$

Shift symmetry

$$\phi \longrightarrow \phi + f(x) + g(y)$$

Dispersion relation

$$\omega^2 = q_x^2 q_y^2$$



Models with subsystem symmetries

A possible classification

- Exotic field theories (like the previous model)
- Foliated field theories
  Slagle, Aasen, Williamson ('19), Slagle ('20)
- Quiver theories
  Shirley, Slagle, Chen ('20), Ma, Shirley, Cheng, Levin, McGrevy, Chen ('20), Geng, Kachru, Karch, Nally, Rayhaun ('21), Razamat ('21), Franco, Rodriguez-Gomez ('22)

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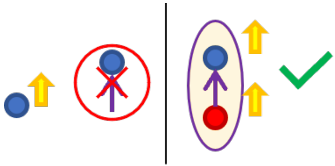
Can fractons appear in less symmetric theories?

Fractons from conservation of dipole charge



A single charge cannot move: fracton

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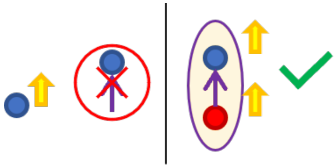

A single charge cannot move: fracton

- Conserved monopole and dipole charges

$$Q = \int d^d x \, \rho, \quad Q^i = \int d^d x \, x^i \, \rho$$

- Modified conservation equation

$$\boxed{\partial_t \rho + \partial_i \partial_j J^{ij} = 0}$$

Gauge fields coupled to fractons

- Coupling to tensor gauge fields Pretko ('16)

$$S = \int d^{d+1}x \left(A_0\, \rho + A_{ij} J^{ij}\right)$$

- Dipole gauge transformations

$$\delta A_0 = -\partial_0 \alpha, \quad \delta A_{ij} = \partial_i \partial_j \alpha$$

- Field strengths: $E_{ij} = \partial_0 A_{ij} + \partial_i \partial_j \phi$, $B_k = \epsilon^{ij} \partial_i A_{jk}$

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- Field strengths: $E_{ij} = \partial_0 A_{ij} + \partial_i\partial_j\phi$, $B_k = \epsilon^{ij}\partial_i A_{jk}$
- In $2+1$ dimensions tensor gauge fields are dual to elasticity
  Pretko, Radzihovsky ('17)

$$\begin{array}{ccc} \text{fractons} & \longleftrightarrow & \text{disclinations} \\ \text{dipoles} & \longleftrightarrow & \text{dislocations} \end{array}$$

- Has been generalized to other systems
  Cosserat elasticity Gromov, Surowka ('19), Radzihovsky, Hermele ('19), Hirono, Qi ('21), smetic phases Radzihovsky ('20), superfluid vortex lattices Nguyen, Gromov, Moroz ('20), quasicrystals Surowka ('21), moiré lattices Gaa, Palle, Fernandes, Schmalian ('21)

Realization of dipole symmetry

Two possibilities (scalar field):

- Linear realization:

$$\Phi \to e^{i\alpha + i\boldsymbol{\beta}\cdot\boldsymbol{x}}\Phi$$

- Non-linear realization:

$$\phi \to \phi + \alpha + \boldsymbol{\beta}\cdot\boldsymbol{x}$$

  Note that ϕ transforms as Goldstone for the broken phase $\langle\Phi\rangle \neq 0$

Dipole symmetry: non-linear realization

- Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\nabla^2 \phi)^2 + \cdots$$

- Dispersion relation:

$$\omega^2 = (\boldsymbol{q}^2)^2$$

  Propagating mode in all directions: there are no fractons

- Charge density and current

$$\rho = \partial_t \phi, \quad J^{ij} = \partial^i \partial^j \phi$$

$$\partial_t \rho + \partial_i \partial_j J^{ij} = 0$$

Dipole symmetry: linear realization

- Lagrangian: Pretko ('18)

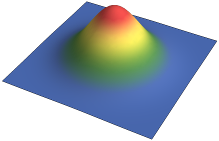
$$\mathcal{L} = \frac{1}{2}|\partial_t \Phi|^2 - c_1 \partial_i |\Phi|^2 \partial^i |\Phi|^2 - c_2 |\Phi \partial_i \partial_j \Phi - \partial_i \Phi \partial_j \Phi|^2$$
$$- c_3 \left[(\Phi^*)^2 \left(\Phi \partial^2 \Phi - \partial_i \Phi \partial_i \Phi \right) + h.c. \right]$$

- Expansion around $\Phi = 0$

$$\mathcal{L} \simeq \frac{1}{2}|\partial_t \delta\Phi|^2 + O(\delta\Phi^4)$$

- Dispersion relation: completely localized mode

$$\omega^2 = 0$$



Dipole symmetry: linear realization

- Lagrangian: Pretko ('18)

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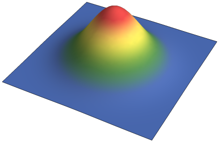
- Expansion around $\Phi = 0$

$$\mathcal{L} \simeq \frac{1}{2}|\partial_t\delta\Phi|^2 + O(\delta\Phi^4)$$

- Emergent subsystem symmetry: connected to unbroken dipole symmetry

$$\Phi \to \Phi + f(x,y)$$



Monopole-Dipole Momentum Algebra (MDMA)

- Generators: translations P_i, monopole charge Q, dipole charge Q^i
- Non-zero commutators Gromov ('18)

$$[P_i, Q^j] = \delta_i^j Q$$

  Heisenberg algebra on a sector of fixed charge Peña-Benitez ('21)

- It can be extended by adding spatial rotations
- Same as Galilean algebra ($c \to \infty$) with Q^j Galiean boosts and Q the mass
- Same as Carroll algebra ($c \to 0$) with Q^j Carrollian boosts and Q the Hamiltonian

Hydrodynamics with dipole symmetry

Why hydro?

- Finite temperature states in systems with fracton excitations
  An example: population of defects in crystalline solids or superfluid vortex lattices
- Caveats: mobility constraints can produce very long equilibration times Prem, Haah, Nandkishore ('17) or even localization in subspaces Pai, Pretko, Nandkishor ('19), Khemani and Rahul Nandkishore ('19), Sala, Rakovszky, Verresen, Knap,Pollmann ('20)
- For a generic system and long enough times we expect local thermal equilibrium to be reached

Hydrodynamic theories according to symmetries

- P_i and Q: business as usual. Note $[P_i, Q] = 0$

$$\partial_t \rho + \partial_i J^i = 0, \quad \partial_t T^{ti} + \partial_j T^{ji} = 0.$$

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- Q and Q^i: dipole current. Note $[Q^i, Q] = 0$

$$\partial_t \rho + \partial_i \partial_j J^{ij} = 0$$

Subdiffusive modes Gromov, Lucas, Nandkishore ('20)

$$J_{ij} = D\partial_i \partial_j \rho + \cdots \quad \rightarrow \quad \omega = -iD(\boldsymbol{q}^2)^2$$

Width of a distribution increases as $\Delta x \sim t^{1/4} \ll t^{1/2}$ of ordinary diffusion

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- P_i, Q^i and Q: non-trivial algebra $[P_i, Q^j] = \delta_i^j Q$

How does this affect to the hydrodynamic equations?

Poisson bracket formalism

- Poisson brackets of conserved charges: depend **only** on the symmetry

  Landau ('41),Dzyaloshinskii Volovick ('80), Son ('19)

$$\{p_i(\boldsymbol{x}), \rho(\boldsymbol{y})\} = -\rho(\boldsymbol{x})\partial_{x^i}\delta(\boldsymbol{x}-\boldsymbol{y})\,,$$
$$\{p_i(\boldsymbol{x}), p_j(\boldsymbol{y})\} = -[p_j(\boldsymbol{x})\partial_{x^i} + p_i(\boldsymbol{y})\partial_{x^j}]\delta(\boldsymbol{x}-\boldsymbol{y})$$

- Hydrodynamic equations are obtained from a Hamiltonian h depending on the conserved charges

$$\partial_t\rho = \{\rho, h\}, \quad \partial_t p_i = \{p_i, h\}$$

Hydrodynamic equations

- We introduce the chemical potential and velocity as conjugate variables

$$h = h(\rho, \boldsymbol{p}), \quad dh = \mu d\rho + v^i dp_i$$

- The pressure is (minus) the Legendre transform

$$p = \mu\rho + p_i v^i - h, \quad dp = \rho d\mu + p_i dv^i$$

- Universal form of the hydrodynamic equations

$$\partial_t \rho = \{\rho, h\} = -\partial_i(\rho v^i)\,,$$
$$\partial_t p_i = \{p_i, h\} = -\rho\,\partial_i \mu - \partial_j(v^j p_i) - p_j \partial_i v^j$$

- Using the definition of the pressure

$$\partial_t p_i = -\partial_j(v^j p_i + \delta_i^j p) \equiv -\partial_j T^j{}_i$$

An example

- All other physical information is in the constitutive relations derived from the Hamiltonian. For instance

$$h = \frac{p_i^2}{2m\rho} + \varepsilon(\rho)$$

$$v^i = \frac{\partial h}{\partial p_i} = \frac{p_i}{m\rho} \;\Rightarrow\; p_i = m\rho v_i$$

- This satisfies the Ward identity for a Galilean invariant theory $p_i = mJ_i$
- We recover the usual ideal Navier-Stokes and continuity equations

$$\partial_t \rho + \partial_i(\rho v^i) = 0, \quad \partial_t(m\rho v_i) + \partial_j(m\rho v_i v^j + \delta_i^j p) = 0$$

MDMA hydrodynamics

- A dipole transformation changes the momentum density

$$\delta_\beta p_i = \left\{p_i, \beta_j Q^j\right\} = \beta_i \rho.$$

- We introduce a velocity field that is shifted by a constant

$$V_i = \frac{1}{\rho} p_i, \quad \delta_\beta V_i = \beta_i$$

- Since this is a symmetry, the Hamiltonian should be

$$h = h(\rho, \partial_i V_j)$$

- The velocity is

$$v^i = \frac{1}{\rho}\partial_j j^{ji}, \quad j^{ji} = -\frac{\partial h}{\partial(\partial_j V_i)}$$

MDMA hydrodynamics

- The hydrodynamic equations are the naïve generalization

$$\partial_t \rho + \partial_i \partial_j J^{ij} = 0, \quad \partial_t(\rho V_i) + \partial_j T^j_{\ i} = 0$$

- With a dipole current and stress tensor

$$J^{ij} = j^{(ij)}, \quad T^j_{\ i} = \partial_k j^{kj} V_i + \delta^j_i p$$

- Let us consider up to second order in derivatives and rotational invariance

$$h = \varepsilon_0(\rho) + \frac{M(\rho)}{2}\delta^{ij}\partial_i\rho\partial_j\rho + \frac{G(\rho)}{2}\partial_i V_j \partial^i V^j + \frac{\tilde{G}(\rho)}{2}\partial_i V_j \partial^j V^i + \frac{K(\rho)}{2}(\partial^i V_i)^2$$

- We will expand to linear order in $\rho = \rho_0 + \delta\rho$ and V_i

$$j^{ji} = -(G_0 \partial^j V^i + \tilde{G}_0 \partial^i V^j + K_0 \delta^{ij} \partial^k V_k), \quad p = p_0 + \frac{\partial p_0}{\partial \rho_0}\delta\rho$$

MDMA hydrodynamics

- Linearized hydrodynamic equations

$$\partial_t \delta\rho - (G_0 + \tilde{G}_0 + K_0)\partial^2(\partial^i V_i) = 0, \quad \rho_0 \partial_t V_i + \frac{\partial p_0}{\partial \rho_0}\partial_i \delta\rho = 0.$$

- This is equivalent to

$$\partial_t^2 \delta\rho + \frac{G_0 + \tilde{G}_0 + K_0}{\rho_0}\frac{\partial p_0}{\partial \rho_0}(\partial_i^2)^2 \delta\rho = 0$$

- Sound mode with dispersion relation (subpropagating?)

$$\omega^2 = \frac{G_0 + \tilde{G}_0 + K_0}{\rho_0}\frac{\partial p_0}{\partial \rho_0}(\boldsymbol{q}^2)^2$$

- Breaking rotational invariance leads to a general dispersion relation

$$\omega^2 = C^{ijkl} q_i q_j q_k q_l$$

- Extension to non-ideal hydrodynamics qualitatively similar

  Glorioso, Guo, Rodriguez-Nieva, Lucas ('21)

Effect of dipole symmetry in hydrodynamics

- Conservation equations are the same, constitutive relations are modified
- Continuity equation is modified to dipole current equation $J^i = \partial_j J^{ji}$
- Dispersion relations are $\omega \sim q^2$ for propagating and diffusive modes
- Breaking of rotational invariance allows fractonic dispersion relations (to lower order)

Higher multipole symmetries are expected to be similar, with dispersion relations involving higher powers of spatial momentum

Emergent subsystem symmetries

What is needed to have emergent subsystem symmetries?

Examples so far:

- Linearly realized dipole symmetry in the unbroken phase (Pretko's model)
- Broken rotational invariance in hydrodynamics and non-linearly realized dipole symmetry

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We found more examples in systems with spontaneously broken spatial symmetries

The original questions

We were interested in the
spontaneous breaking of scale **and** translation invariance:

- Some examples found in holographic models of Q-lattices and Chiral Density Waves

  Amoretti, Areán, Argurio, Musso, Pando Zayas ('16),Amoretti, Areán, Goutéraux, Musso ('17)

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- Counting theorems when Poincaré invariance is broken:
  - Non-relativistic

    Nielsen, Chadha ('76) Schäfer, Son, Stephanov, Toublan, Verbaarchot ('01) Watanbe, Murayama ('12), Hidaka ('12)
  - Chemical potential (gapped Goldstones)

    Nicolis, Piazza 1204.1570 Watanabe, Brauner, Murayama 1303.1527
  - Spacetime symmetries

    Low and Manohar ('02) Volkov ('73), Ivanov and Ogievetsky ('75) Watanabe and Murayama, ('13)

Mixing of dilatations and translations

- Assume spontaneously broken charge Q, and translations P_a
- Unbroken P_i and $\tilde{P}_a = P_a - k_a Q$, homogenous state

$$P_i|0\rangle = 0 \quad \tilde{P}_a|0\rangle = 0,$$

$$P_a|0\rangle = k_a Q|0\rangle \neq |0\rangle$$

- Dilatations must be spontaneously broken

$$P_i D|0\rangle = [P_i, D]|0\rangle = iP_i|0\rangle = 0,$$

$$\tilde{P}_a D|0\rangle = [\tilde{P}_a, D]|0\rangle = iP_a|0\rangle = ik_a Q|0\rangle \neq |0\rangle$$

$$\Rightarrow D|0\rangle \neq |0\rangle$$

- We expect a mixing of $U(1)$ NG mode with the dilaton $\sim k_a$

Spontaneous breaking of translation in mean field theory

- Ginzburg-Landau model

$$\mathcal{L} = \frac{1}{2}|\partial_t \Phi|^2 + A|\nabla\Phi|^2 - V(|\Phi|) + \text{higher derivatives}$$

- Usual sign of the kinetic term $A < 0$: energy increases for $\nabla\Phi \neq 0$
  no breaking
- Opposite sign $A > 0$: breaking is possible, higher derivative terms needed for stability
- Mean field models of inhomogeneous phases in QCD and condensed matter Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superconductor Buzdin, Kachkachi ('96), Huang, Ting, Zhu, Lin ('21) , Chiral Spiral Pisarski, Skokov, Tsvelik ('22)

Mexican hat model for spatial derivatives

$$\mathcal{L} = \partial_t \Phi^* \partial_t \Phi + \frac{1}{2} \partial_\mu \Xi \partial^\mu \Xi - \frac{B}{\Xi^6} \left(\partial_i \Phi^* \partial_i \Phi - \frac{A}{2B} \Xi^6 \right)^2 - \lambda^2 (\Phi^* \Phi)^3$$

- "wrong" sign for the kinetic term $\mathcal{L} \sim A \partial_i \Phi^* \partial_i \Phi$, $A > 0$
- Scale invariant (in $2+1$ dimensions)
- If $\lambda = 0$ flat direction $\Xi^6 = v^6 = \frac{2B}{A} \partial_i \Phi^* \partial_i \Phi$

Ground states

Ground states have zero energy density
and homogeneous action for fluctuations

- **Helical superfluid**

$$\Xi = v, \quad \Phi = \rho e^{ikx}, \quad \xi = \frac{k^2\rho^2}{v^6} = \frac{A}{2B}$$

Charge/Chiral density wave, Q-lattice

- **Meta-fluid**

$$\Xi = v, \quad \Phi = b(x+iy), \quad \frac{|b|^2}{v^6} = \frac{A}{4B}$$

EFTs of solids and supersolids

Leutwyler ('97), Son ('05), Nicolis, Penco, Piazza, Rattazzi ('15)

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Symmetries

$$\mathcal{L} = \partial_t\Phi^*\partial_t\Phi + \frac{1}{2}\partial_\mu\Xi\partial^\mu\Xi - \frac{B}{\Xi^6}\left(\partial_i\Phi^*\partial_i\Phi - \frac{A}{2B}\Xi^6\right)^2 - \lambda^2(\Phi^*\Phi)^3$$

- $U(1)$ global symmetry $\Phi \to e^{i\alpha}\Phi$
- Complex shift symmetry (for $\lambda = 0$) $\Phi \to \Phi + a_R + i a_I$
- Scale transformations

$$\Phi \to e^{\eta/2}\Phi, \quad \Xi \to e^{\eta/2}\Xi, \quad x^\mu \to e^{-\eta}x^\mu$$

- Spatial rotations
- Time and spatial translations

$U(1)$ and complex shifts not independent

Symmetry breaking:

$$\Xi = v, \quad \Phi = \rho e^{ikx}, \quad \xi = \frac{k^2\rho^2}{v^6} = \frac{A}{2B}$$

Four broken symmetries:

- $U(1)$ global symmetry + translations along $x \longrightarrow$ diagonal
- Real shifts
- Scale transformations
- Spatial rotations

Two gapless modes and one gapped mode

Quadratic effective action

$$\delta\Phi = \rho e^{ikx}\left(\sigma(t,\boldsymbol{x}) + i\chi(t,\boldsymbol{x})\right), \quad \delta\Xi = v\tau(t,\boldsymbol{x})$$

$$\mathcal{L} \simeq \frac{v^2}{2}\partial_\mu\tau\partial^\mu\tau + \rho^2(\partial_t\chi)^2 + \rho^2(\partial_t\sigma)^2 - 2A\rho^2\left[k(\sigma - 3\tau) + \partial_x\chi\right]^2$$

- Emergent dipole and subsystem symmetries

$$\delta\tau = \delta + \gamma_i x^i,\ \delta\chi = \alpha(y) + \beta(x,y),\ \delta\sigma = 3\delta + 3\gamma_i x^i - \frac{1}{k}\partial_x\beta(x,y)$$

- There is a fracton with dispersion relation $\omega^2 = 0$
- $k \to 0$ is mostly σ, and χ is a 'lineon' $\omega^2 \simeq 2Aq_x^2$
- $k \to \infty$ is mostly χ

Derivative Mexican hat at finite density

$$\mathcal{L} = \partial_t \Phi^* \partial_t \Phi + \frac{1}{2}\partial_\mu \Xi \partial^\mu \Xi - \frac{B}{\Xi^6}\left(\partial_i \Phi^* \partial_i \Phi - \frac{A}{2B}\Xi^6\right)^2 - \lambda^2 (\Phi^* \Phi)^3$$

- $\lambda \neq 0$ is necessary to have a ground state
- Only helical superfluid has homogeneous effective action

$$\Xi = v, \quad \Phi = \rho e^{i(\mu t + kx)}$$

$$\xi = \frac{k^2 \rho^2}{v^6} = \frac{A}{2B}, \quad \mu^2 = 3\rho^2 \lambda^2$$

- Fracton dispersion relation is modified

$$\omega^2(q_x = 0) = 0, \quad \omega^2 \sim q_x^2 \boldsymbol{q}^2 \ (|\boldsymbol{q}| \ll \mu), \quad \omega^2 \simeq 4\mu^2 \ (q_x \gg \mu, q_y)$$

Deformed action

$$\mathcal{L} = \partial_t\Phi^*\partial_t\Phi + A\partial_i\Phi^*\partial_i\Phi + \frac{1}{2}\partial_t\Xi\partial_t\Xi - \frac{1}{2}\partial_i\Xi\partial_i\Xi - \lambda^2(\Phi^*\Phi)^3 + \frac{1}{\Xi^6}\left[-B\left(\partial_i\Phi^*\partial_i\Phi\right)^2 + G\,\partial_i\Phi^*\partial_i\Phi^*\partial_j\Phi\partial_j\Phi\right] - H\Xi^6\ ,$$

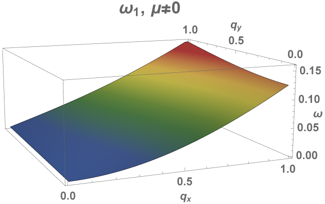
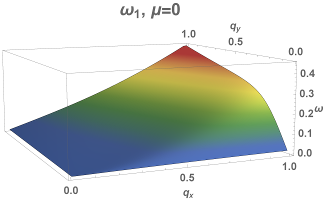
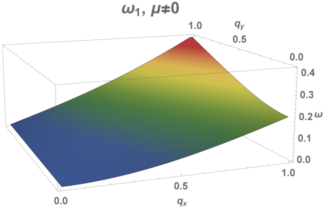
- New term in the quadratic action for the helical superfluid

$$\mathcal{L}_G = -4G\xi(\partial_y\sigma)^2$$

- Dispersion relation of the fracton becomes non-analytic, at low momentum

$$\omega^2 \sim \frac{q_x^2 q_y^2 \boldsymbol{q}^2}{aGq_y^2 + \boldsymbol{q}^2} \sim O(q^4)$$

- We can also add a chemical potential

	$\mu = 0$	$\mu \neq 0$
$G = 0$	$\omega_1 = 0$	
$G \neq 0$		

Summarizing

- The low energy EFT of a system with spontaneously broken translation invariance can show multipolar or subsystem symmetries, IR rather than UV as in lattice models
- Exact dipole or subsystem symmetries are not necessary for fractonic dispersion relations
- However, this is all mean field, fluctuations may alter this picture, as IR divergences may be strong

Outlook

Outlook

- Strongly coupled theories: holographic models dual to theories with tensor currents

  Ganesan, Lucas ('20)

- Spontaneous breaking of multipolar symmetries: number of Goldstones, dispersion relations?

- Fracton-elasticity duality: is there an effective description of defects as fractonic charges?