Emergent times in holography

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HoloTube seminar

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based on work with Samuel Leutheusser

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Problem of time in quantum gravity

QM: time is absolute.

Gravity: most of the time time is meaningless as it can be changed by arbitrary gauge diffeomorphism transformations.

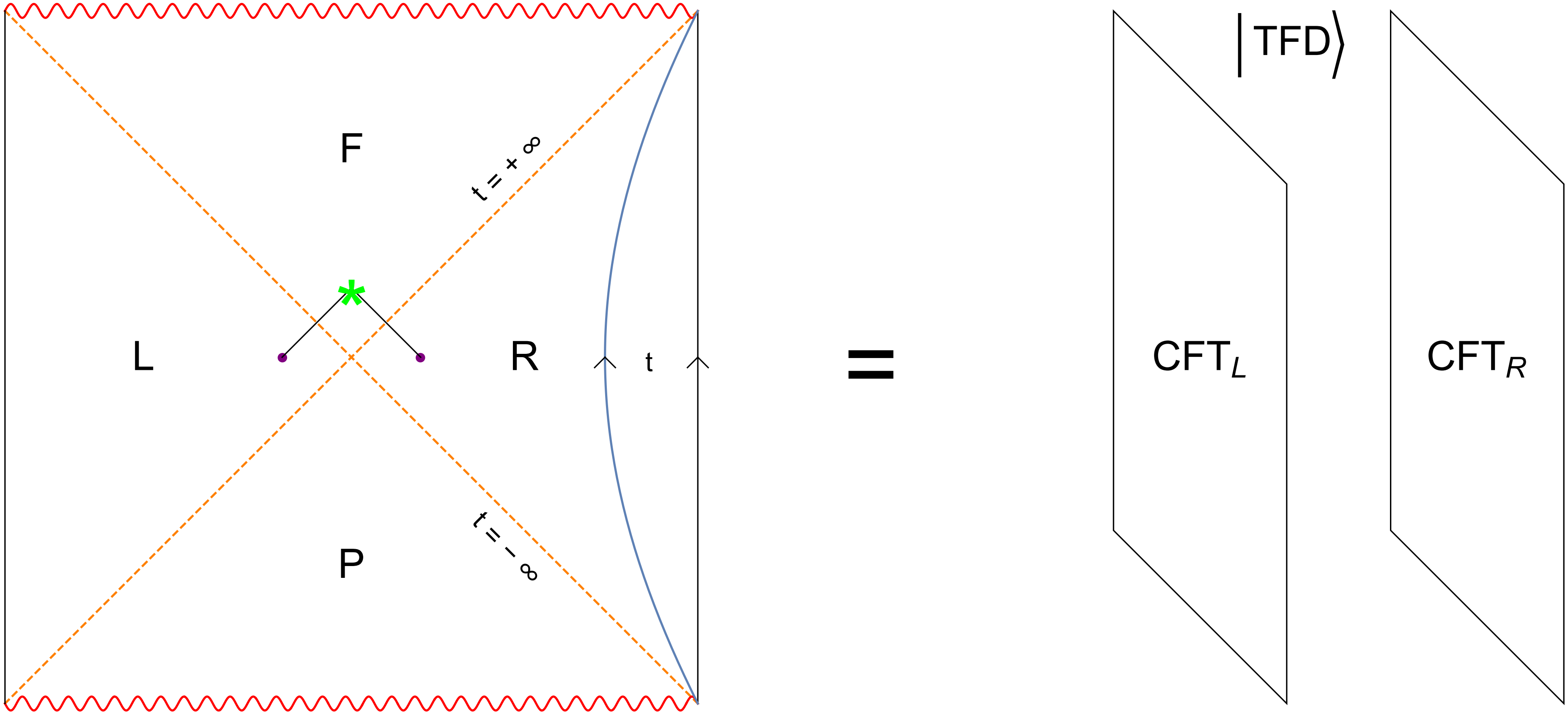
AdS: there is an absolute asymptotic time, which underlies AdS/CFT

QG in spacetimes of other asymptotics appears much more difficult.

In AdS/CFT: how to understand bulk time evolution?

When the bulk spacetime is time-translation invariant: yes.

General time-dependent cases: not clear



Maldacena, 2001

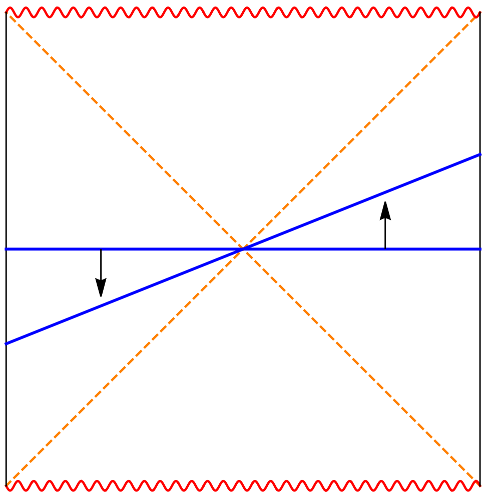
Time-like Killing vector outside the horizon

Many mysteries: F and P regions? Kruskal-like time?

horizons and associated causal structure ?

Interactions between R and L observers in the interior?

Boundary descriptions of bulk evolutions



$H_R - H_L$ $\qquad$ $H_R + H_L$ (not well defined) $\qquad$ Emergent?

Goals:

- Boundary constructions of Kruskal-like evolutions
- Boundary emergence of horizon and associated causal structure

Plan

1. Outline the main results

2. Entanglement structure in relativistic quantum field theory:
   type III_1 von Neumann algebra

   Half-sided modular inclusions/translations

3. Constructions of Kruskal-like time evolutions from boundary

4. Discussions

Outline of main results

A description of infalling observers

Will show that there exist "evolution operators" on the boundary

$$U(s) = e^{-iGs}, \; s \in \mathbb{R}, \; G \text{ hermitian}$$

an infalling time

Properties:

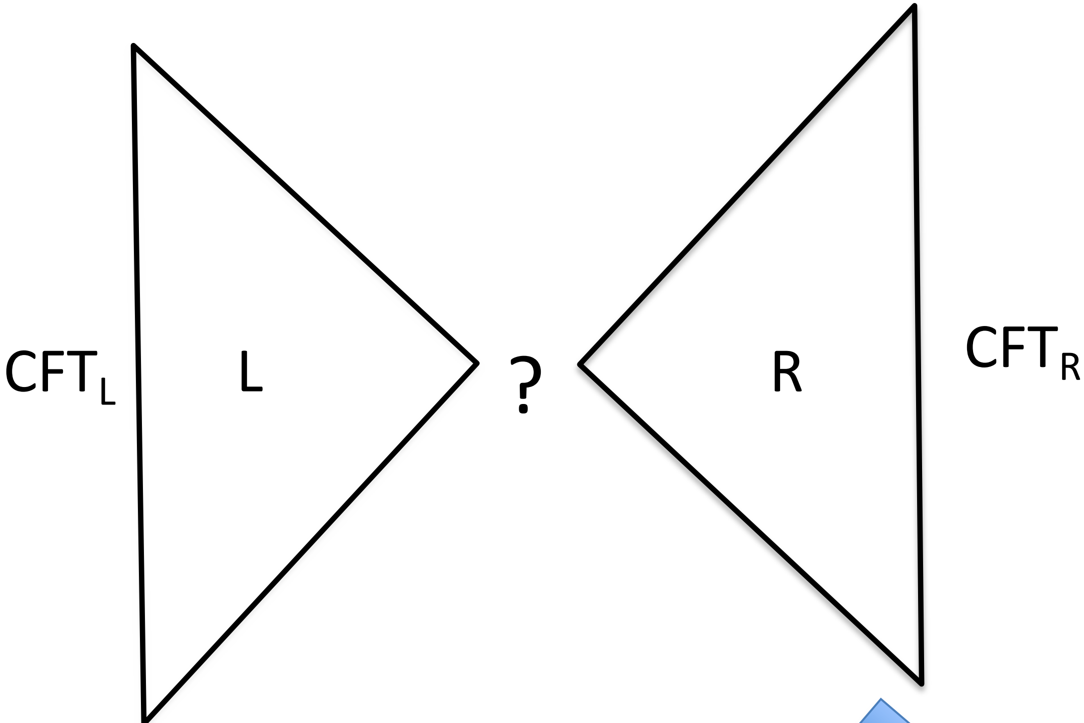
1. G involves both R and L degrees of freedom

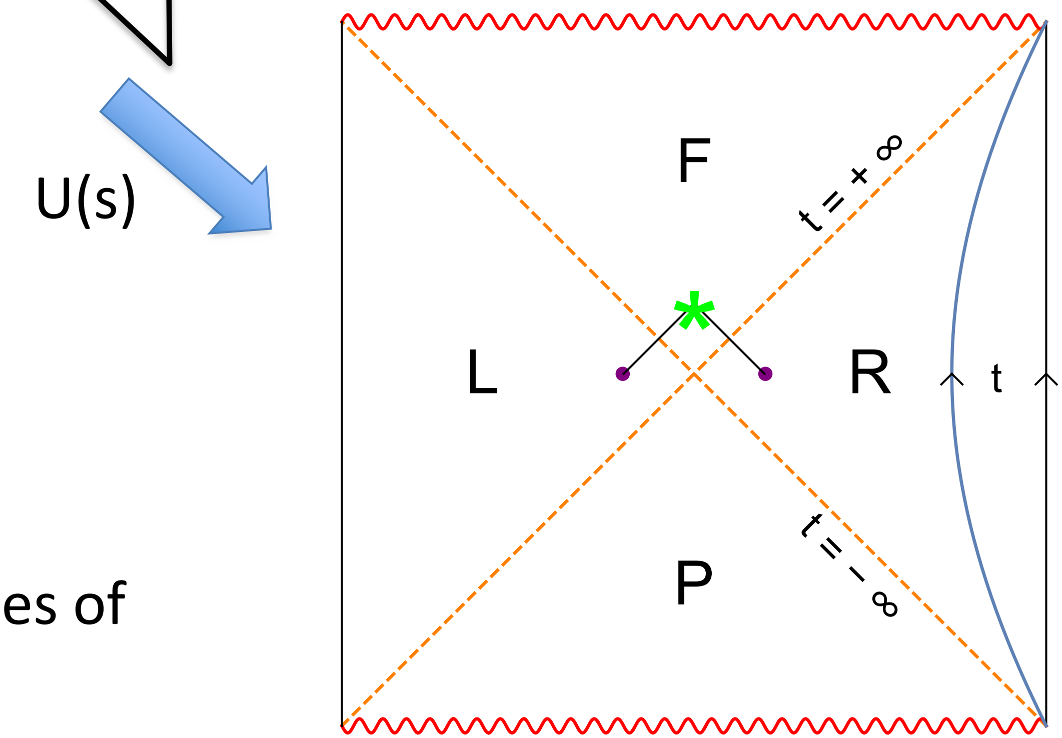
2. $$G \geq 0$$

3. $$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R \text{ region}$$

takes it inside the horizon for sufficiently large s.

Sharp signatures of horizon, "generate" F and P regions from R and L.





U(s) leads to a new
an emergent
infalling ”time”

Infinite number of choices of
such infalling times

An example for BTZ

For BTZ, such a U(s) can be worked out explicitly.

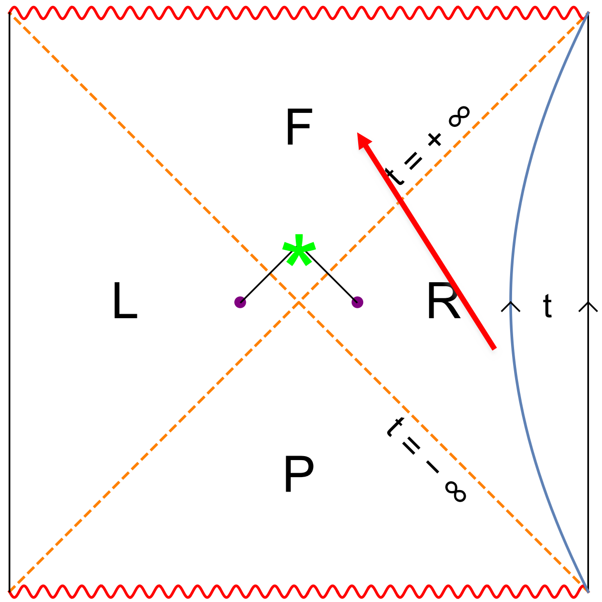
$$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R$$

There exists an $s_0 > 0$

$$s < s_0, \quad \Phi(X;s) \in \mathrm{CFT}_R$$

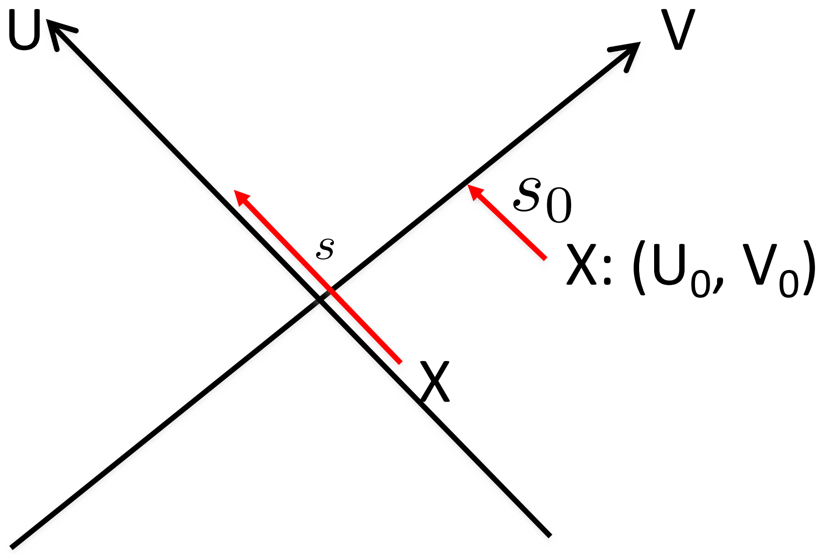
$$s > s_0, \quad \Phi(X;s) \in \mathrm{CFT}_R \otimes \mathrm{CFT}_L$$

signature of a sharp horizon.



For any two quantum systems, if such U(s) and s_0 exist, we say they are causally connectable.

$$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R$$



U, V: Kruskal null coordinates

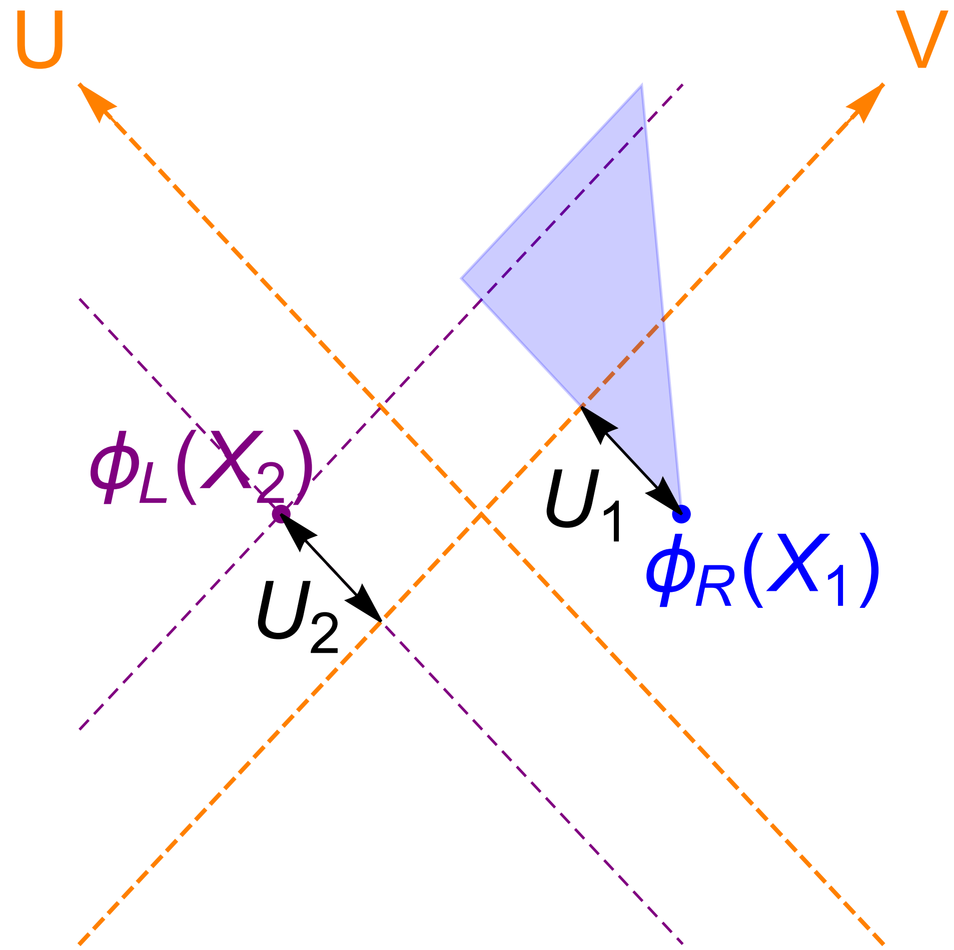
$$s_0 = -U_0$$

X near the horizon, local transformation: Kruskal null translation

General X: transformation is nonlocal, but respects the casual structure

Causal structure

$$[U^\dagger(s)\phi_R(X_1)U(s), \phi_L(X_2)] = \begin{cases} 0 & s < |U_1| + U_2 \\ \neq 0 & s > |U_1| + U_2 \end{cases}$$

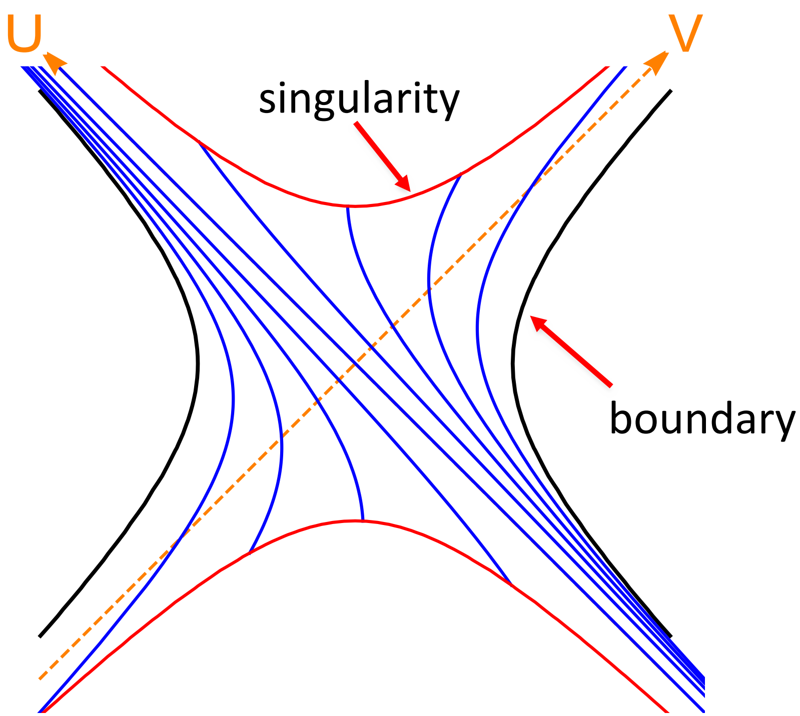


Flow pattern in the large mass limit

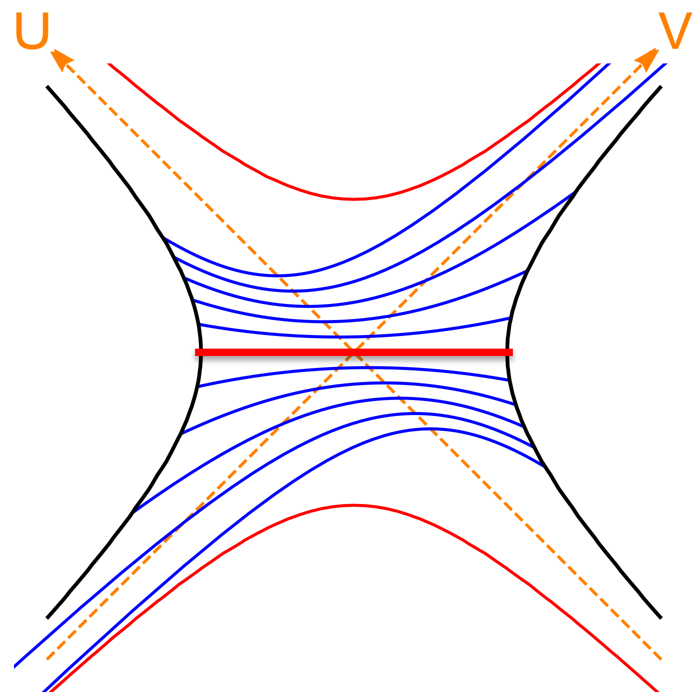
$$\Phi(X;s) \equiv U(-s)\phi(X)U(s) \propto \phi(X_s)$$

$$U_s = U_0 + s, \quad V_s = \frac{V_0}{1 - sV_0}$$

average over boundary spatial directions



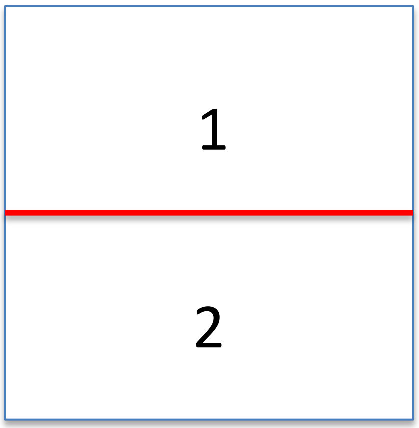
Family of trajectories



Constant-s slices

Entanglement in relativistic QFT: III_1 von Neumann algebra

Entanglement of a quantum system



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$\rho_1 = \mathrm{Tr}_2 |\Psi\rangle\langle\Psi|$$

$$S_1 = -\mathrm{Tr}_1 \rho_1 \log \rho_1$$

When ρ_1, ρ_2 are both full rank

Modular operator: $\Delta = \rho_2 \rho_1^{-1}$

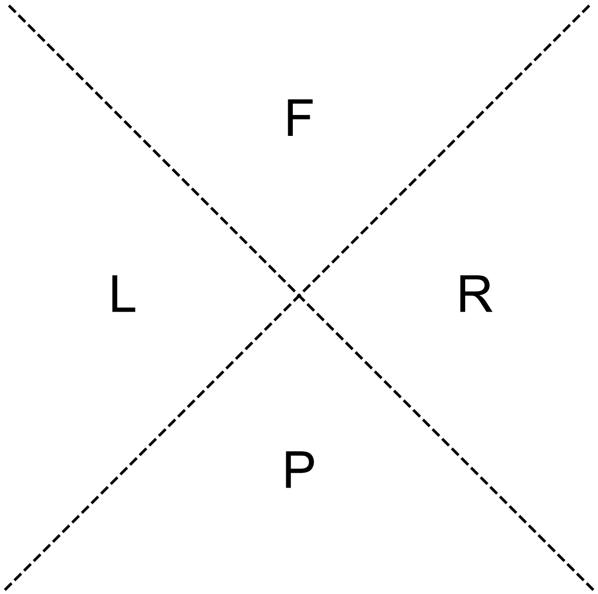
$$\Delta^{it} B(\mathcal{H}_1) \Delta^{-it} \in B(\mathcal{H}_1), \quad \Delta^{it} B(\mathcal{H}_2) \Delta^{-it} \in B(\mathcal{H}_2),$$

modular flow → highly entangled

Entanglement in QFT

Consider a QFT in Minkowski spacetime.

It is often said the Minkowski vacuum state can be interpreted as a thermal field double state for the R and L Rindler patches.



Strictly speaking, the statement is only correct in the discretized theory.

They are some **fundamental differences** between the discrete and continuum cases.

Discrete	Continuum
Local Hilbert spaces for L and R $$\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$$	no
Reduced density matrix	no
Finite entanglement entropy	Not defined (infinite)

modular operator and modular flows exist

Modular operator can be factorized	cannot
No sharp light cone	sharp light cone

Reasons: operator algebras in R region have different structures

Type I von Neumann algebra	Type III_1 von Neumann algebra

The story is general for relativistic QFTs:

For any local region, local operator algebra can be associated with a type III_1 vN algebra

Entanglement for any local region can be understood in terms of modular flows associated with such an algebraic structure.

Half-sided modular translation

Suppose $\mathcal{M}$ is a von Neumann algebra and the vector $|\Omega\rangle$ is cyclic and separating for $\mathcal{M}$

Suppose there exists a von Neumann subalgebra $\mathcal{N}$ of $\mathcal{M}$ with the properties:

$|\Omega\rangle$ is cyclic for $\mathcal{N}$

$$\Delta_{\mathcal{M}}^{-it} \mathcal{N} \Delta_{\mathcal{M}}^{it} \subset \mathcal{N}, \quad t \leq 0$$

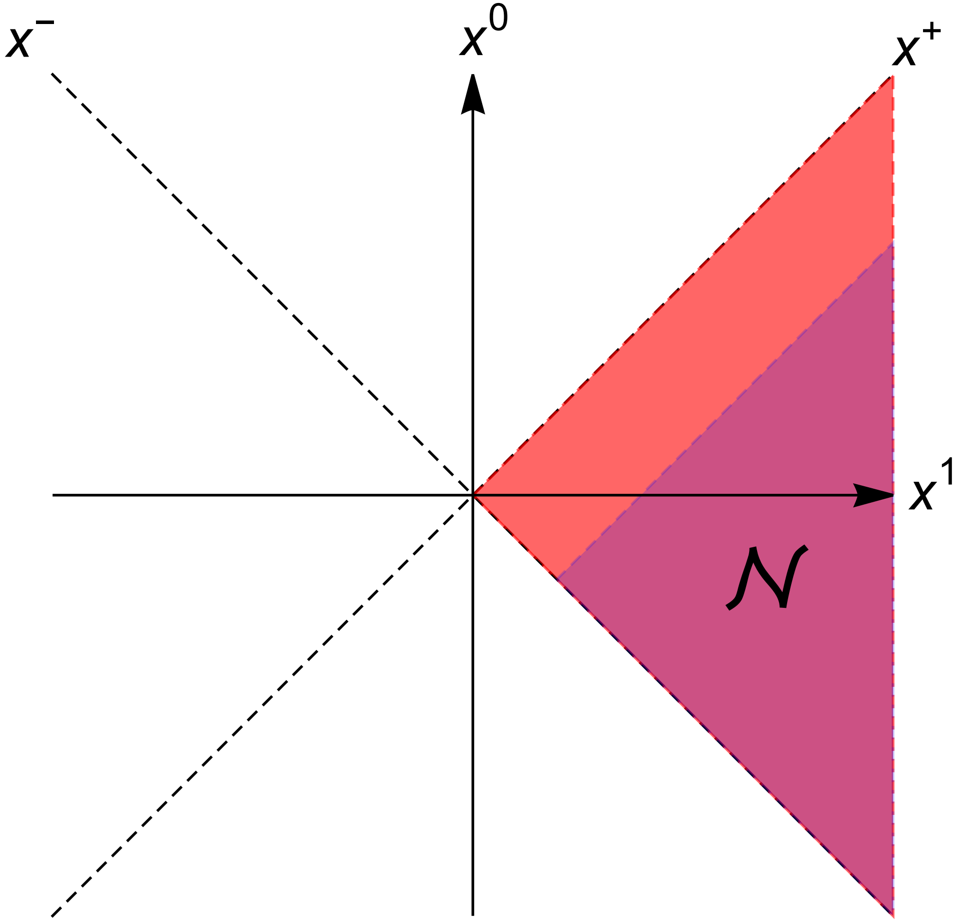
It can then be shown that for type III_1 there exists a unitary group U(s), with the following properties:

Borchers, Wiesbrock

$$U(s) = e^{-iGs}, \qquad G \geq 0$$

$$U(s)\Omega = \Omega, \quad \forall s \in \mathbb{R}$$

This can be used to generate “new” times!



M: operator algebra in R region

$$U(s) = e^{-iGs}, \qquad G \geq 0$$

G generates translations along x⁻ direction

$$U^\dagger(s)\mathcal{M}U(s) \subseteq \mathcal{M}, \quad \forall s \leq 0\ . \qquad \mathcal{N} = U^\dagger(-1)\mathcal{M}U(-1)$$

s> 0, “generate” F and P regions from the algebras of R and L regions

Starting with Rindler time in L and R, we obtain the Minkowski time!

Key: a type III_1 vN algebra and appropriately chosen subalgebras lead to new emergent times

Boundary constructions of emergent infalling times

Emergent type III_1 vN algebras

BH is described by CFT_R x CFT_L in the thermal field double state

At finite N, the (bounded) operator algebra of CFT_R or CFT_L is type I.

We argue there is an emergent type III_1 vN algebra in the large N limit which leads to the emergence of a sharp horizon and the interior.

$\mathcal{A}_R$: algebra generated by single-trace operators of CFT_R

In the large N limit, there is another Hilbert space $\mathcal{H}_{\rm GNS}$: Hilbert space of small excitations around the thermal field double state.

$\mathcal{M}_R$: action of $\mathcal{A}_R$ in $\mathcal{H}_{\rm GNS}$

Conjecture: $\mathcal{M}_R$ and $\mathcal{M}_L$ are **type III$_1$ vN algebras**

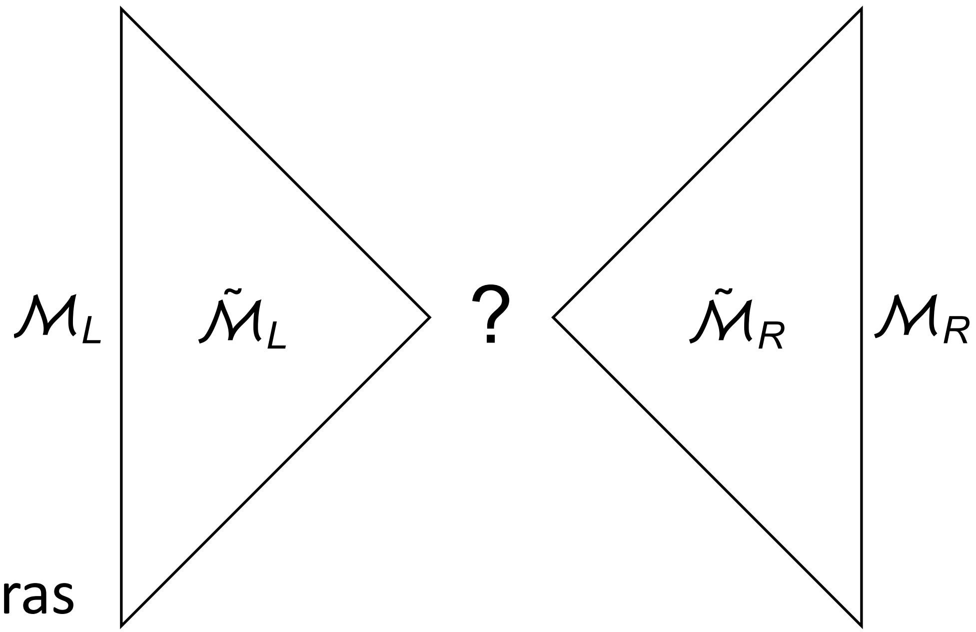
Supports:
- Thermal spectral functions of single-trace operators
- Half-sided modular inclusion/translation structure
- Duality with bulk

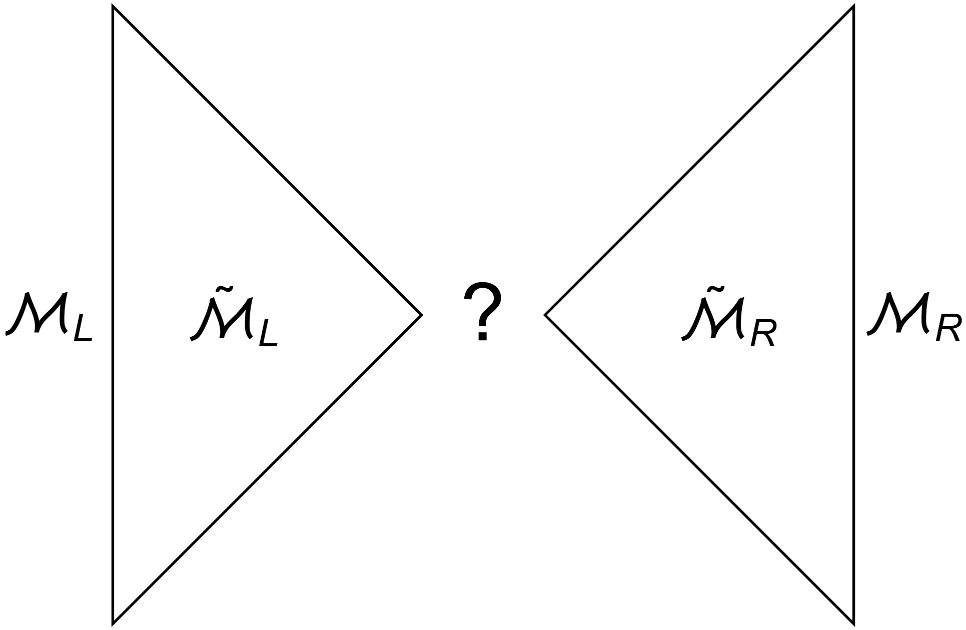
In the bulk: $\mathcal{H}_{\rm BH}^{(\rm Fock)}, \quad |HH\rangle, \quad \widetilde{\mathcal{M}}_R, \quad \widetilde{\mathcal{M}}_L$

Duality:

$$\mathcal{H}_{\rm GNS} = \mathcal{H}_{\rm BH}^{(\rm Fock)},$$
$$|\Omega\rangle = |HH\rangle,$$
$$\mathcal{M}_R = \widetilde{\mathcal{M}}_R, \quad \mathcal{M}_L = \widetilde{\mathcal{M}}_L$$

$\widetilde{\mathcal{M}}_R, \widetilde{\mathcal{M}}_L$ are type III$_1$ vN algebras



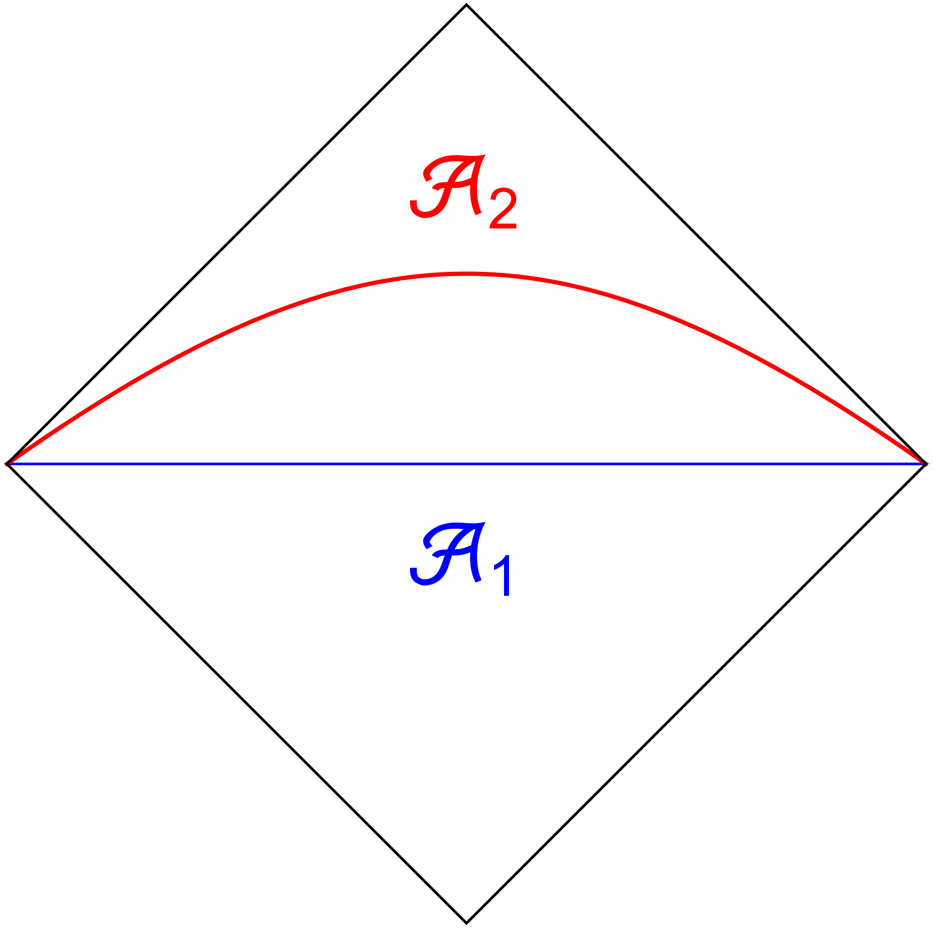


finding a U(s) boils down to finding an appropriate von Neumann subalgebra of $\mathcal{M}_R$

While theorems of half-sided modular translations ensure existence of a U(s), finding it explicitly in general is very difficult.

Here in the large N limit, algebra of single-trace operators in $\mathcal{H}_{\mathrm{GNS}}$ can be described by that of a generalized free theory.

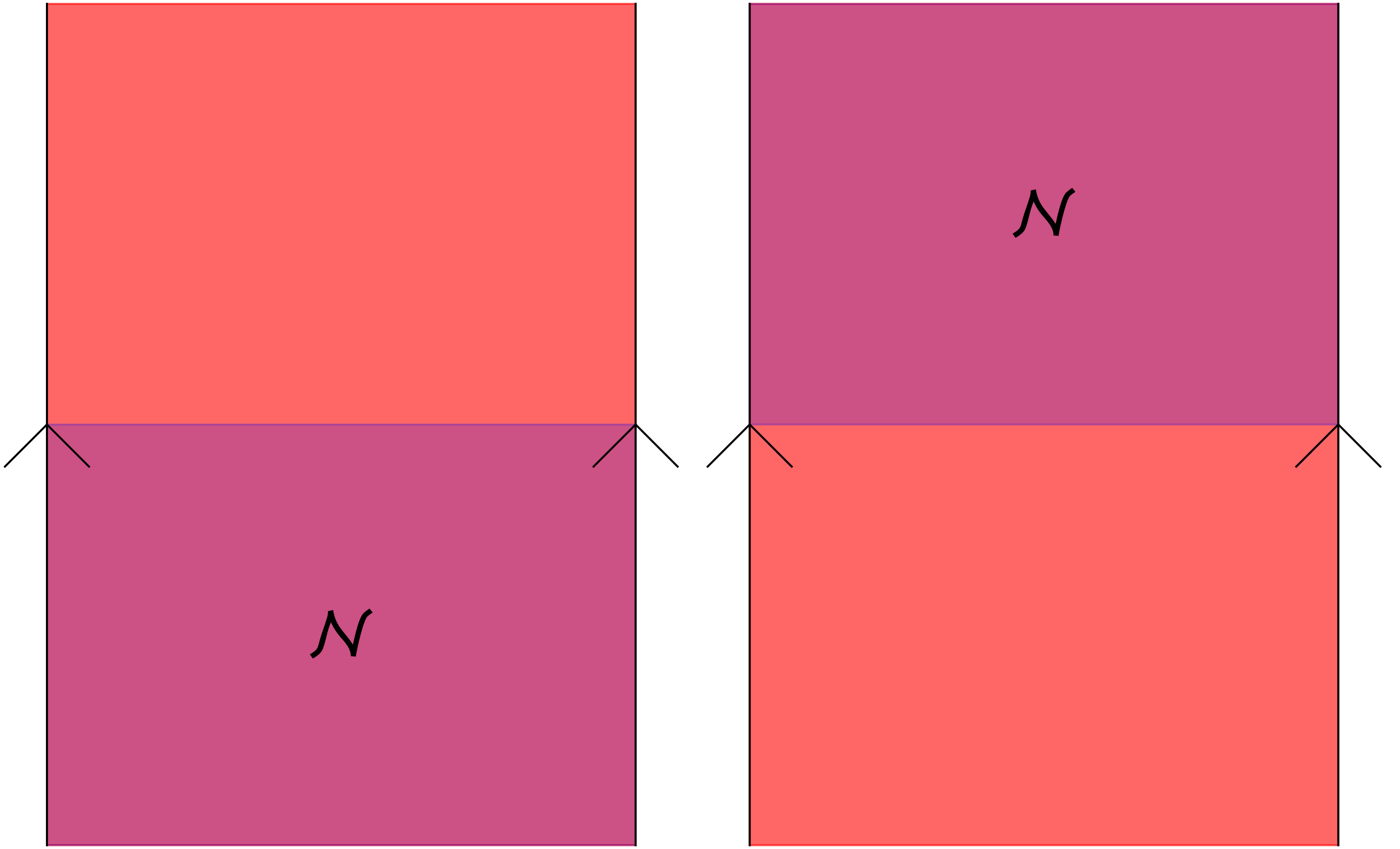
In this case, the expression of U(s) has a universal form up to a phase factor which depends on specific algebras.

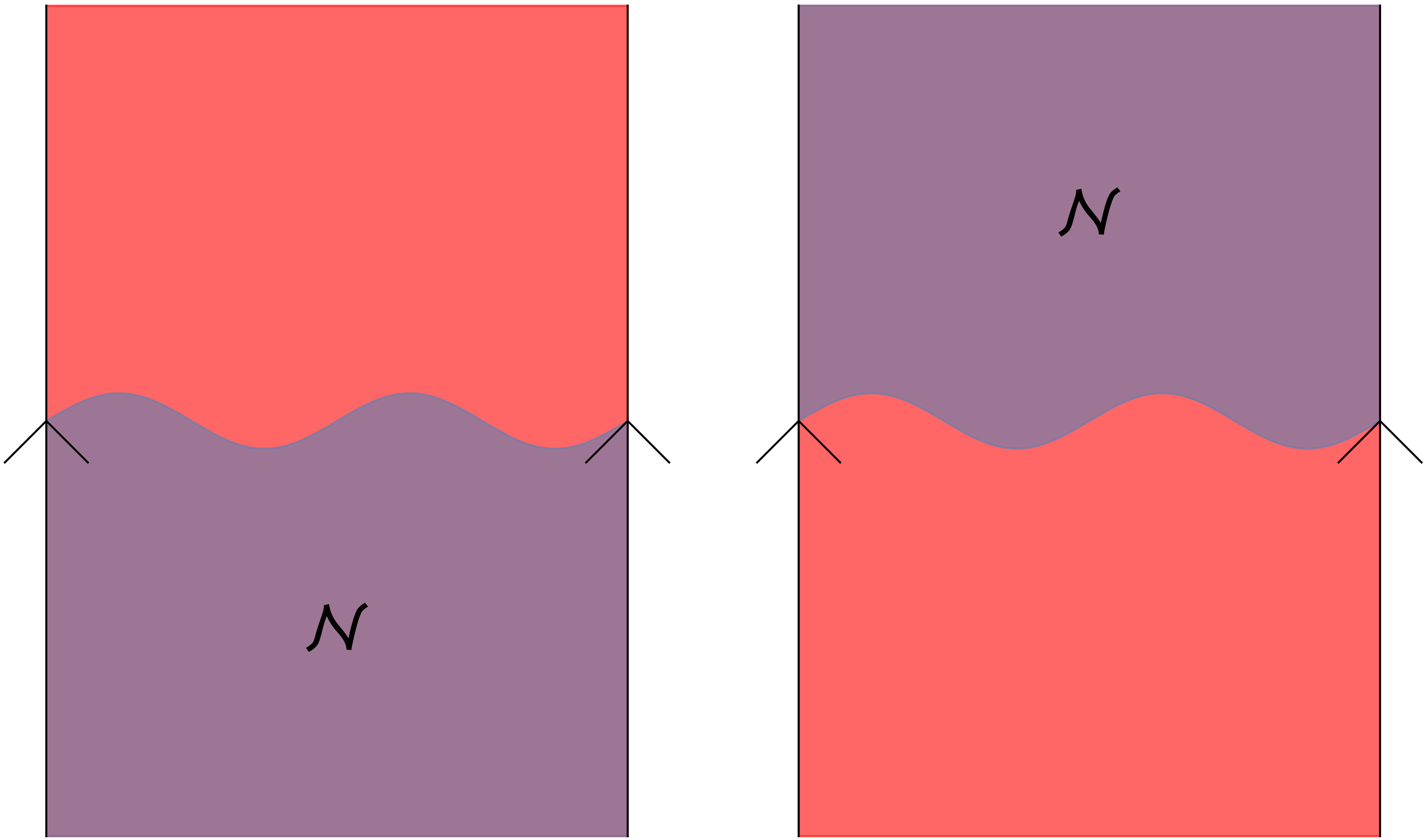


In a QFT, $\mathcal{A}_1 = \mathcal{A}_2$

But for algebras of single-trace operator, $\mathcal{A}_1 \neq \mathcal{A}_2$

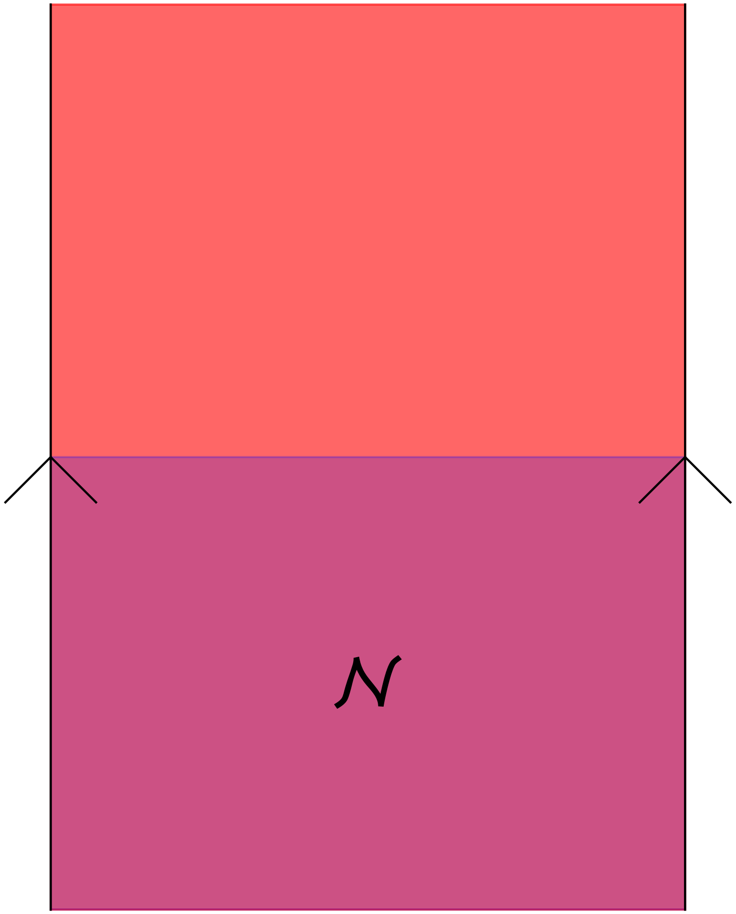
Emergent boundary times



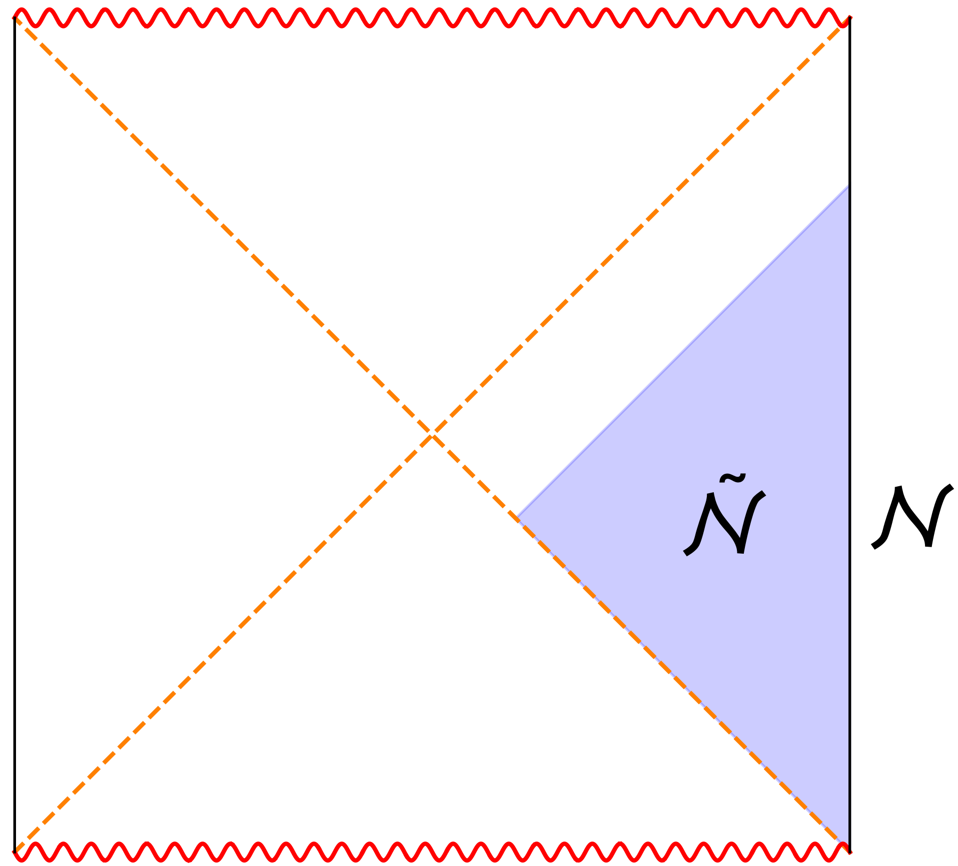


An infinite number of emergent times

Entanglement wedge of a subalgebra

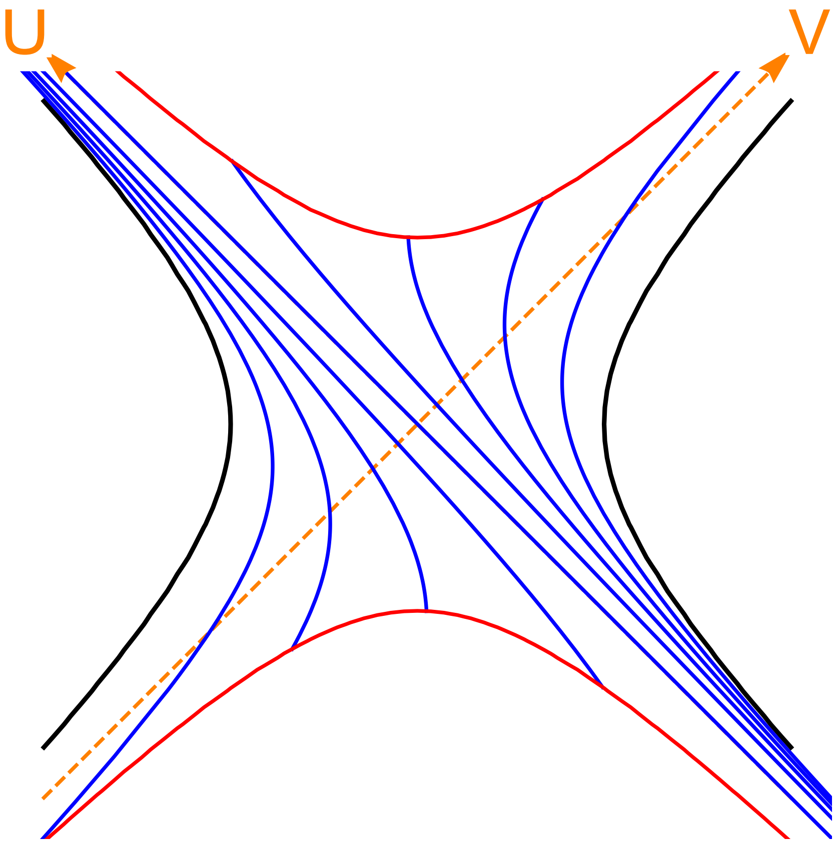


Finding U(s) is now a strongly coupled problem

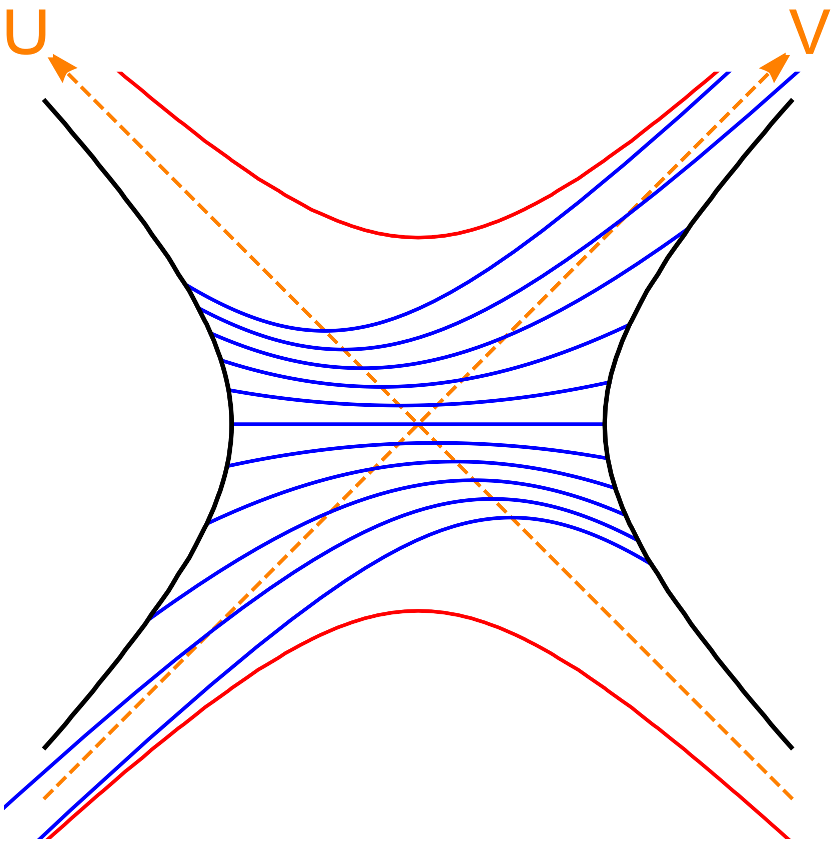


U(s) determined by phase shifts at the horizon

Flow pattern in the large mass limit

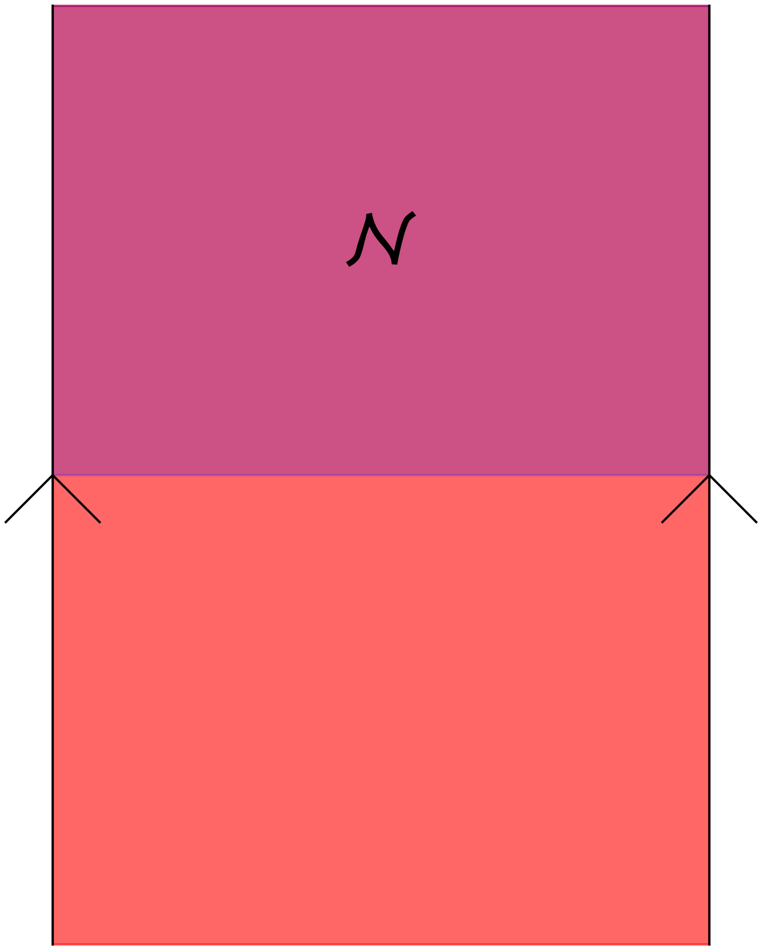


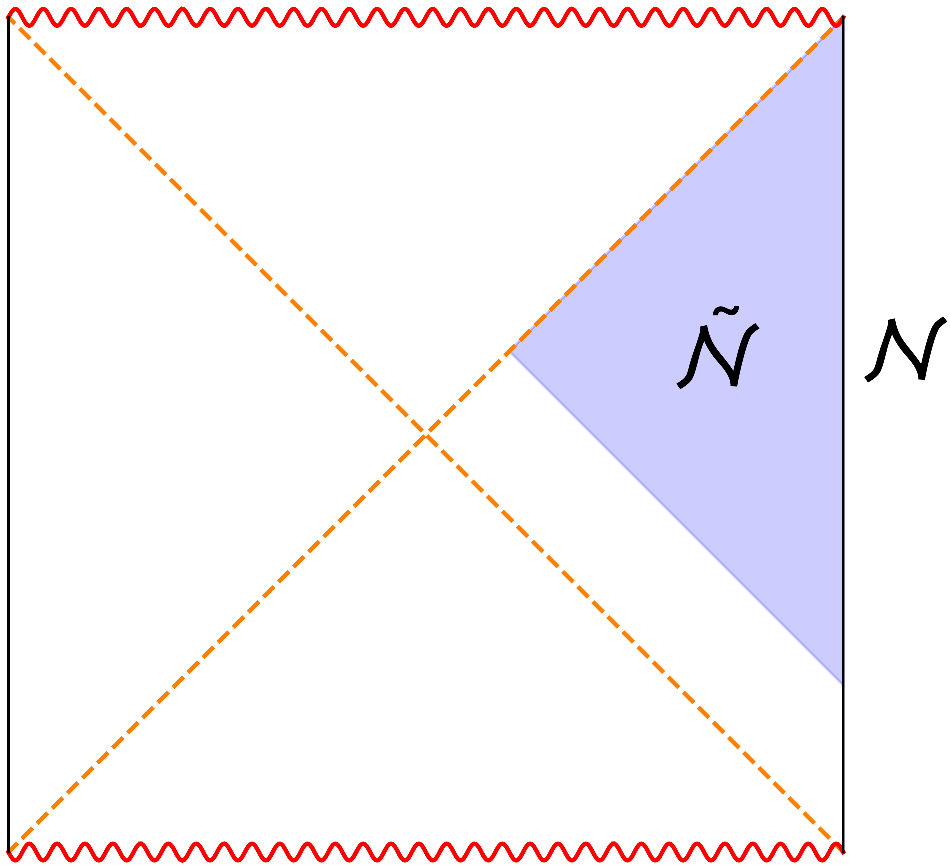
Family of trajectories

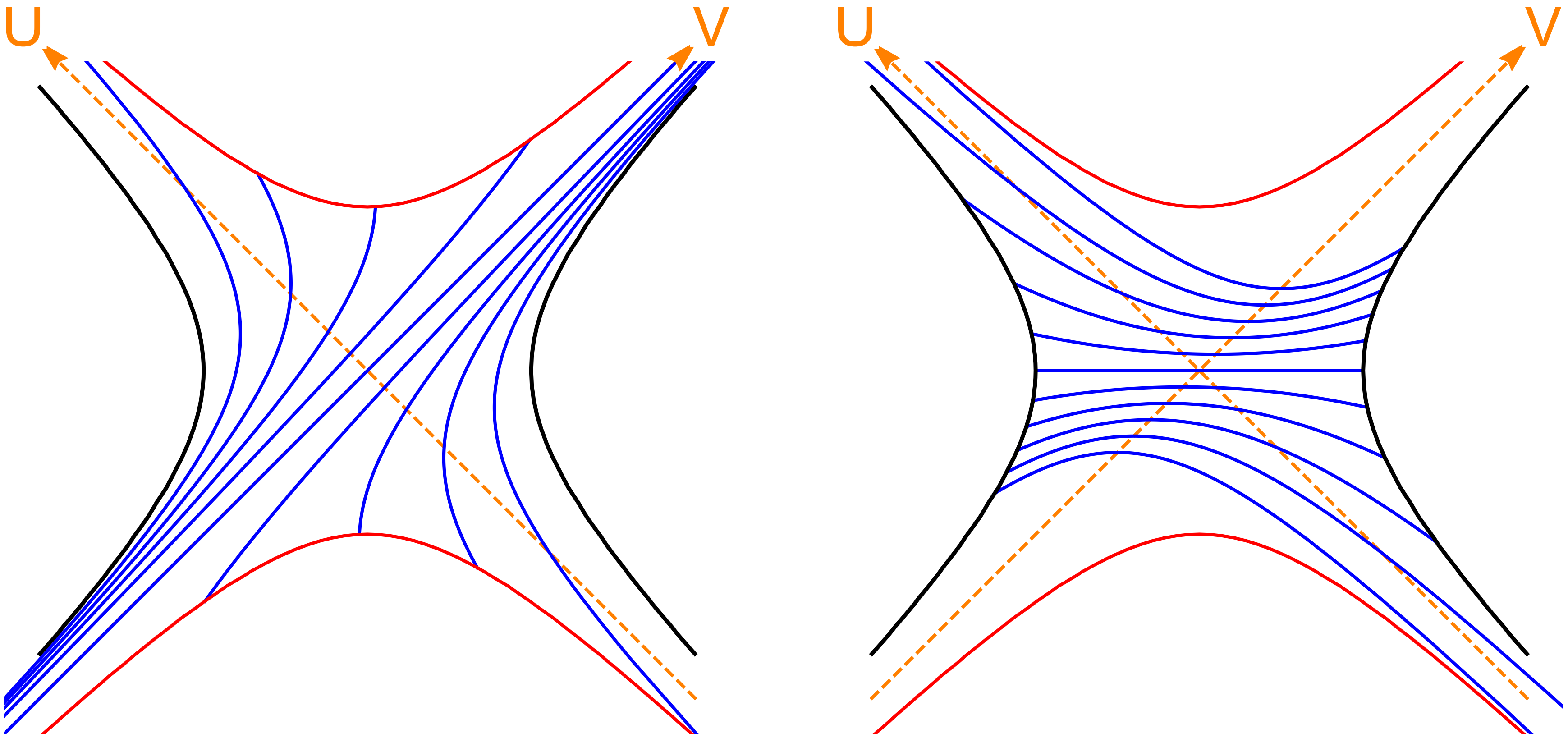


Constant-s slices

Generation of Kruskal-like V-flow





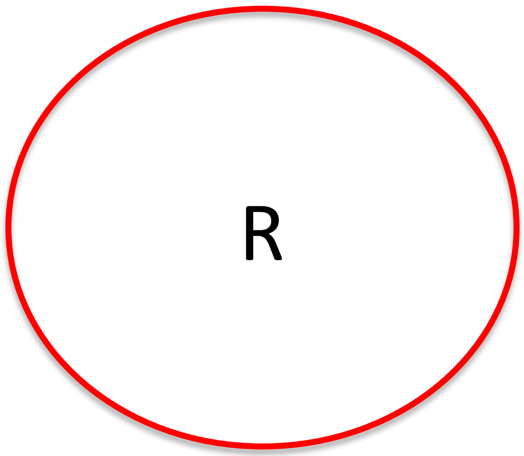


U
V
U
V

We can also consider compositions of such Kruskal-like U-type and V-type flows.

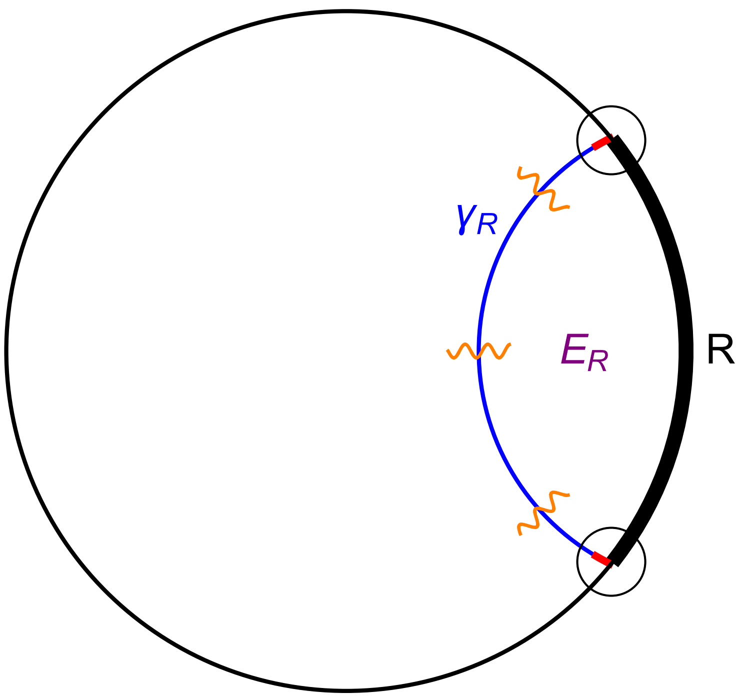
Generalizations

Emergent type III_1 is ubiquitous



Consider the boundary CFT in the vacuum state and a region R

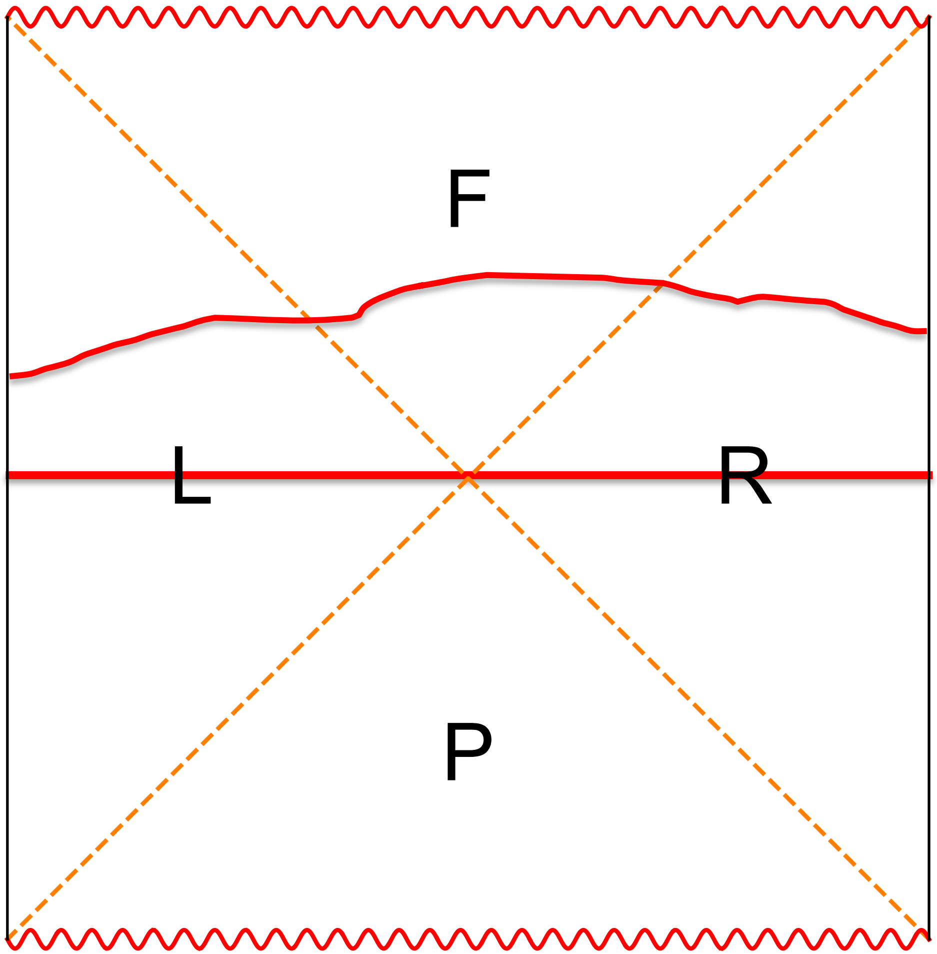
Full operator algebra in R is type III_1



The completion of single-trace operator algebra in R is an emergent type III_1 in the large N limit.

These two type III_1 algebras lead to two different types of divergences in the holographic entanglement entropy

Minimal description of infalling evolution



Properties:

1. G involves both R and L degrees of freedom

2. $$G \geq 0$$

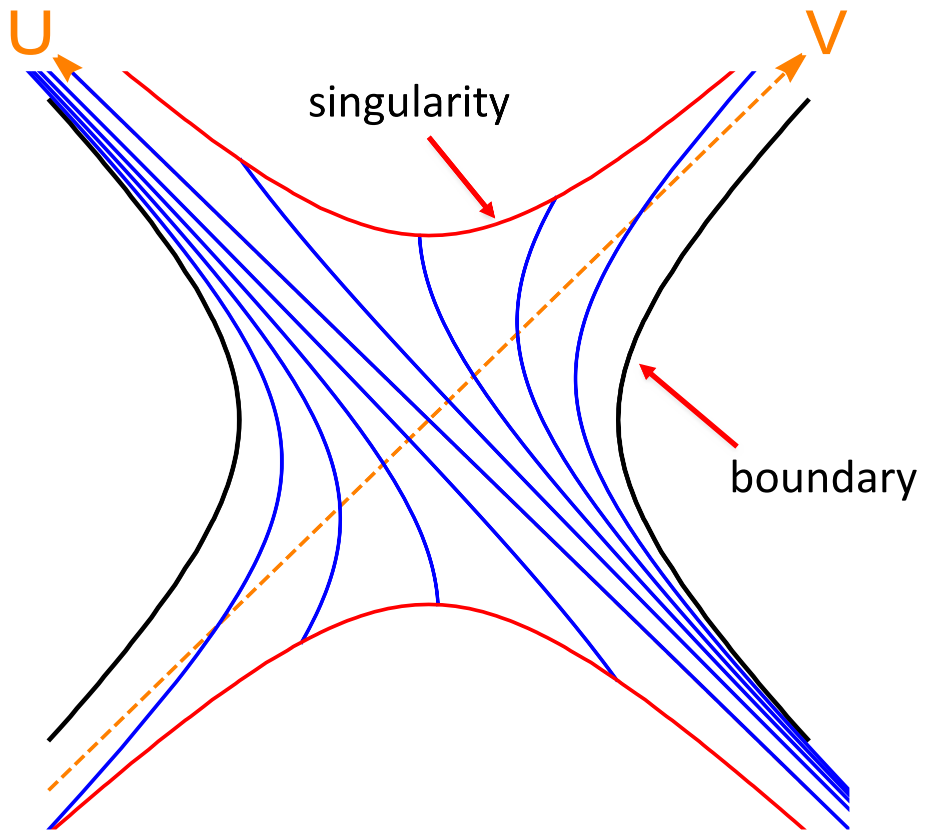
$$U(s) = e^{-iGs},\ s \in \mathbb{R},\quad G \text{ hermitian}$$

It can then be shown that a sharp horizon cannot exist at finite N.

This is very similar to the situation a discretized QFT cannot have a sharp lightcone.

Discussions

Discussion (I): black hole singularities



Singularities signal that the emergent type III_1 cannot be extended arbitrarily.

Discussion (II): bulk UV divergences

In the Rindler case, what distinguishes the discrete and continuum limit are UV divergences.

Similarly, this emergent III_1 structure can be understood as origin of bulk UV divergences.

Very different from the standard interpretation, and could survive in the full string theory.

Some future directions

- Better understanding of emergence of the type III_1 structure in the infinite N limit, including emergence of singularities and their resolution.

- Emergent symmetries: SYK, horizon symmetries, …..

- Single-sided black holes

- Maybe hints for emergence of "time" in cosmological spacetimes like de Sitter.

……

Thank you!

Signature of a sharp horizon

$$|\Psi_0\rangle = e^{iA_L}|\mathrm{TFD}\rangle$$

Now consider the probability of p(s) of an in-falling observer from R region to observe e^{iA_L}.

A **sharp** horizon means that it must have the form:

$$p(s) = \begin{cases} 0 & s < s_0 \\ \neq 0 & s > s_0 \end{cases},$$

Non-smoothness of p(s) reflects a sharp causal structure and a sharp horizon.

A no-go argument

But such a sharp signature of horizon appears not allowed by QM:

$$p(s) = \langle \Psi_0 | M(s) | \Psi_0 \rangle, \quad M(s) = U^\dagger(s) P_R U(s)$$

$$p(s) = \langle \phi(s) | \phi(s) \rangle, \quad |\phi(s)\rangle = P_R e^{-iGs} |\Psi_0\rangle$$

$|\phi(s)\rangle$ is a vector-valued analytic function of s in the lower half complex s-plane

If it zero for a finite interval s_0, then it is identically zero.

Thus p(s):

zero at isolated points (always in causal contact)

always zero (never in causal contact)

The argument is very general, independent of specific states and quantum systems.

We appear to find a puzzle here: a sharp bulk horizon is incompatible with general rules of QM on the boundary.

There are an infinite number of $|\Omega\rangle$ and an infinite number of $\mathcal{N}$ to choose from, leading to an infinite number of choices of U(s).

Discrete	Continuum
Local Hilbert spaces for L and R	no
Finite entanglement entropy	infinite
Modular operator can be factorized	cannot
No sharp light cone (no-go argument applies)	sharp light cone (no-go argument must fail)
Type I von Neumann algebra	Type III_1 von Neumann algebra
Finite projectors, can make local measurement using projectors	All projectors infinite, cannot make local measurement using projectors

So far:

- Many boundary observables can probe regions behind the horizon

  correlation functions, entanglement entropies, complexity ....

  but **not** directly the casual structure or sharp signatures of the horizon.

- ER=EPR type of arguments also largely a single time slice, not causal structure

- HKLL reconstruction can be continued beyond the horizon (need to use bulk evolution or analytic structure around the horizon)