Spin hydrodynamics and Lambda polarization in heavy-ion collisions

Xu-Guang Huang

Fudan University, Shanghai

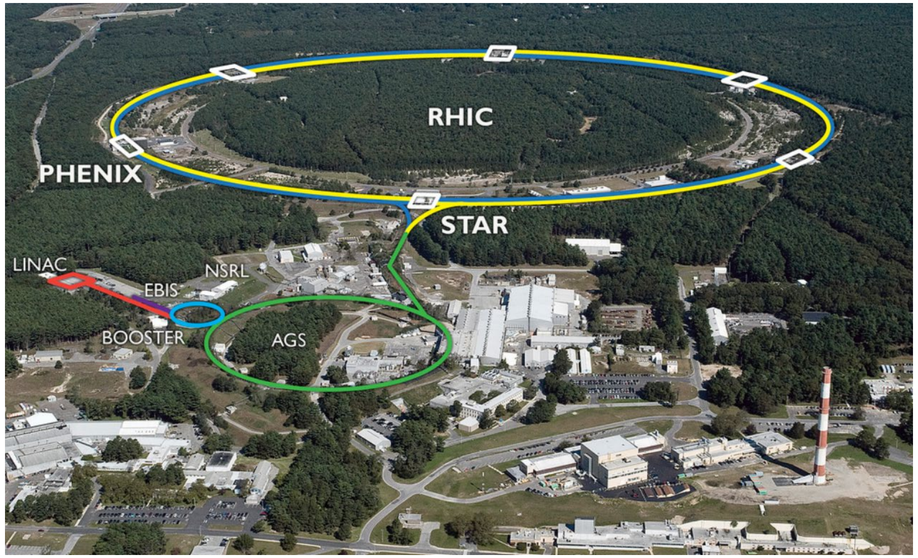
HoloTube Online Seminar Series
February 15, 2022

Content

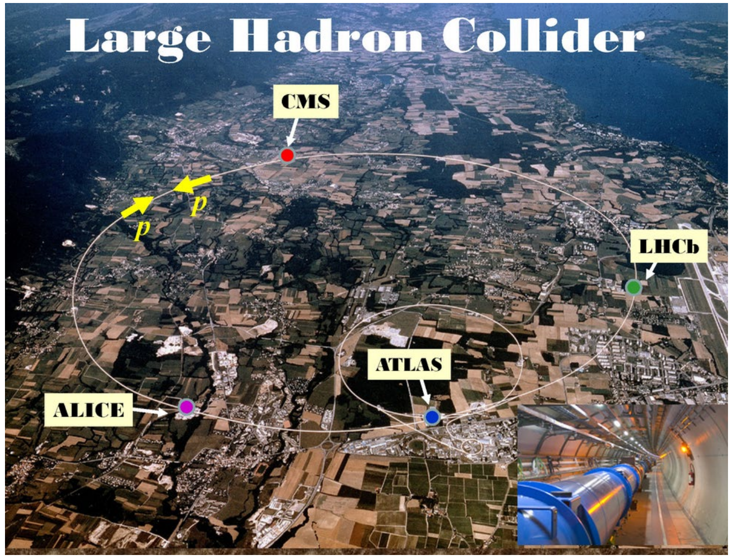
- Introduction: the probes to quark-gluon plasma
- Lambda polarization in heavy-ion collisions
- Spin hydrodynamics
- Heavy quark spin relaxation in perturbative regime
- Summary and outlook

Introduction

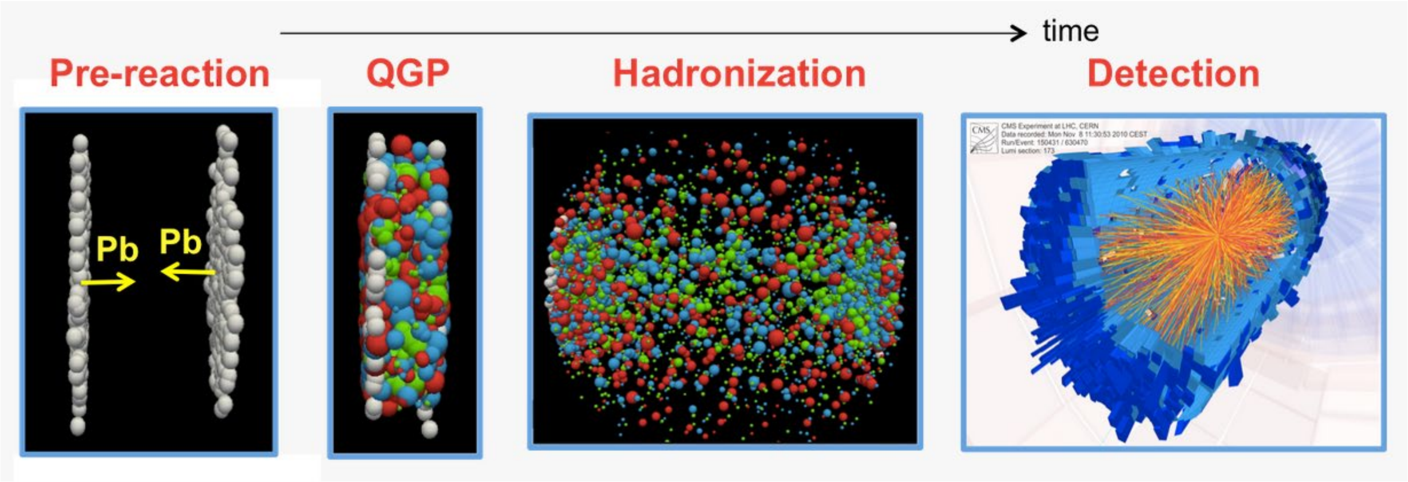
Heavy ion collisions and quark gluon plasma

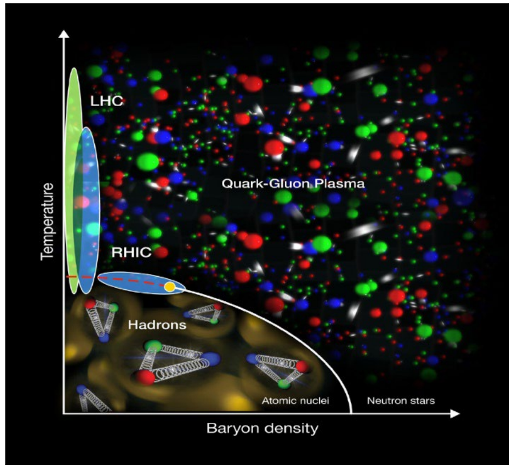


RHIC@BNL



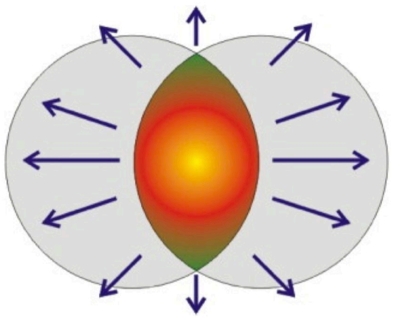
LHC@CERN

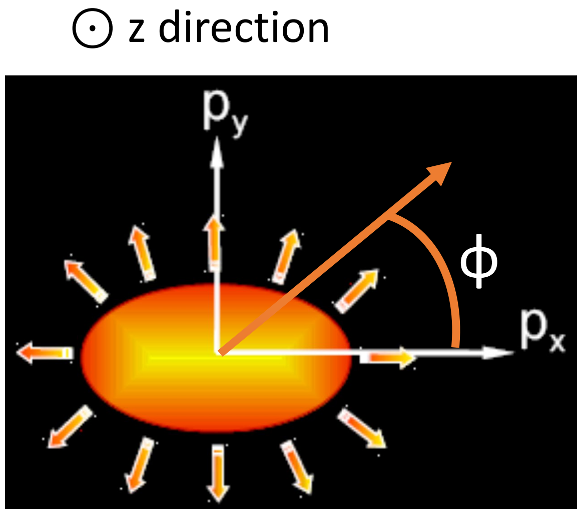


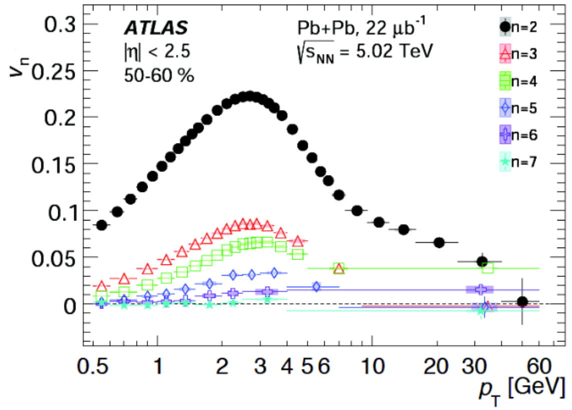


Probes of the quark gluon plasma

- Electric or flavor probes of quark gluon plasma (QGP)
- One example is the anisotropy in charged-hadron spectra: harmonic flow coefficients





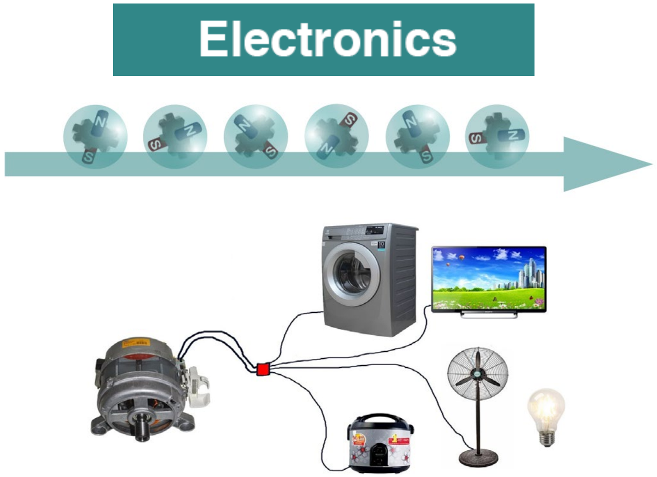


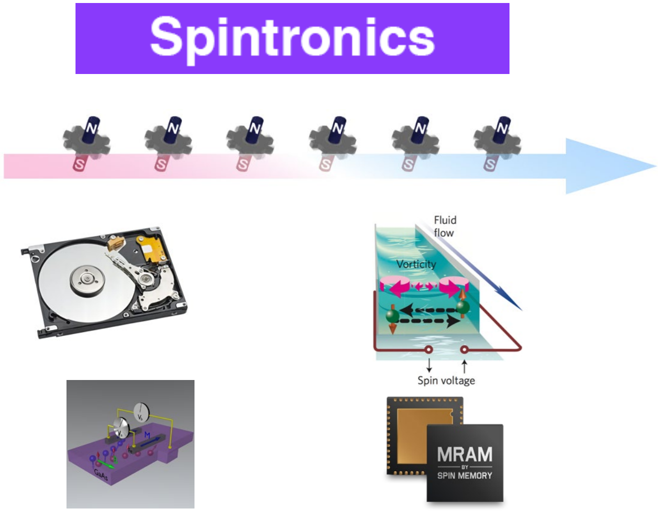
$$\frac{dN_{\rm ch}}{d\phi} \propto 1 + 2v_1 \cos(\phi - \Psi_1) + 2v_2 \cos[2(\phi - \Psi_2)] + \cdots$$

- This is the “electronics (flavortronics)” of QGP

Probes of the quark gluon plasma

- Comparing to what happened in condensed matter physics (and industry)
- Electronics vs. spintronics in condensed matter physics (and industry)



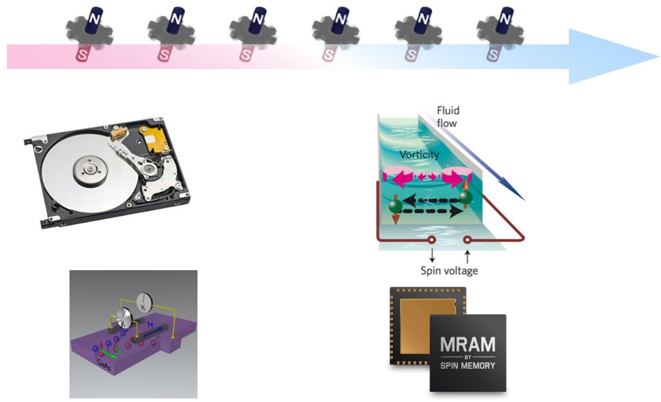


Probes of the quark gluon plasma

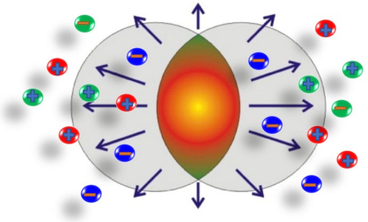
- Comparing to what happened in condensed matter physics (and industry)
- Electronics vs. spintronics in condensed matter physics (and industry)

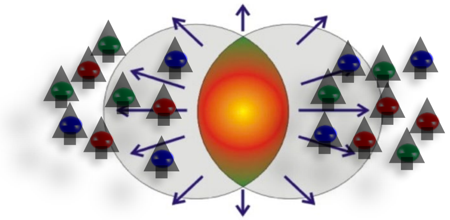
Electronics

Spintronics



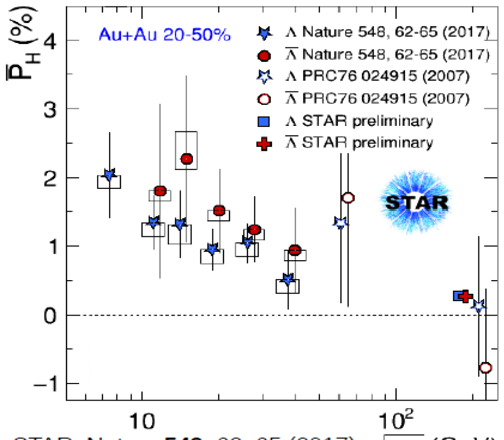
- "Electronics" vs. "spintronics" in heavy-ion collisions
  - Charged hadrons multiplicity N_{ch}
  - Harmonic flows of charges $v_1, v_2, \ldots$
  - Hyperon spin polarization P_{y}
  - Harmonic flows of spin $f_{2y,z}, g_{2y,z}, \ldots$





Spin probe of QGP: Global spin polarization

- First measurement of Λ polarization by STAR@RHIC *



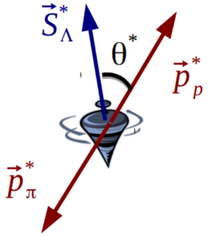
STAR, Nature **548**, 62–65 (2017)

parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

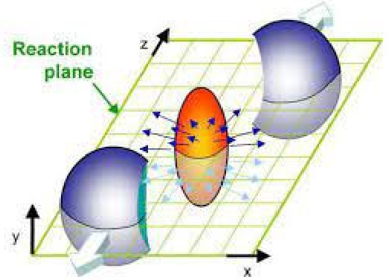
α: Λ decay parameter (α_Λ=0.732)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$$\Lambda \to p + \pi^+$$

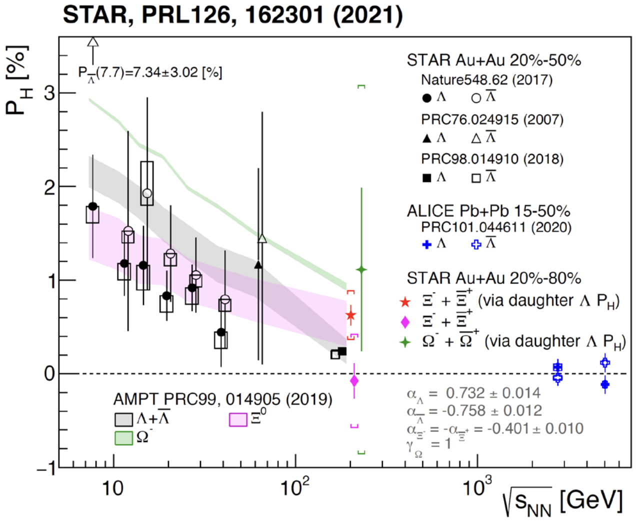
(BR: 63.9%, cτ~7.9 cm)

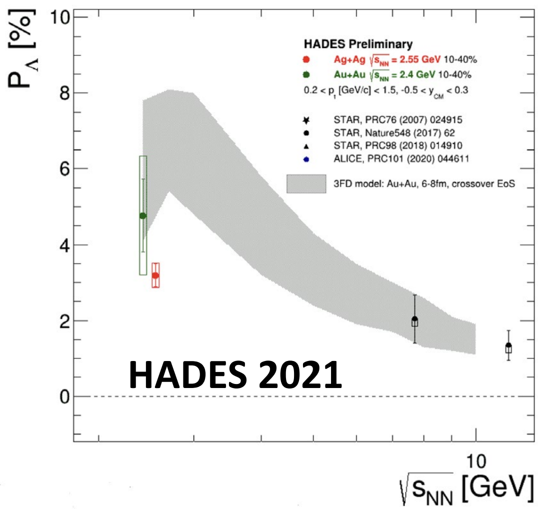
$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



- More recent measurements

hyperon	decay mode	α_H	magnetic moment μ_H	spin
Λ (uds)	$\Lambda \to p\pi^-$ (BR: 63.9%)	0.732	-0.613	1/2
Ξ^- (dss)	$\Xi^- \to \Lambda\pi^-$ (BR: 99.9%)	-0.401	-0.6507	1/2
Ω^- (sss)	$\Omega^- \to \Lambda K^-$ (BR: 67.8%)	0.0157	-2.02	3/2



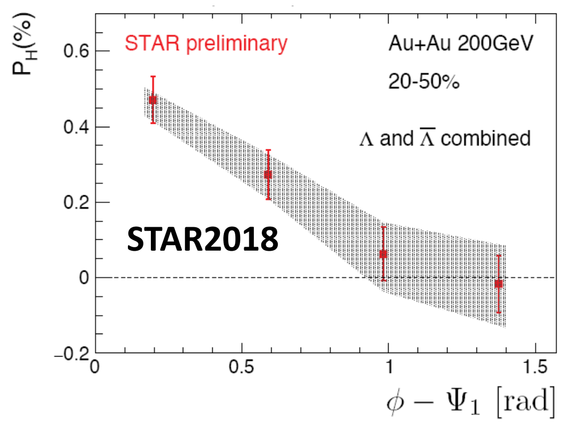


(* First theoretical proposal: Liang and Wang 2004, later by Voloshin 2004)

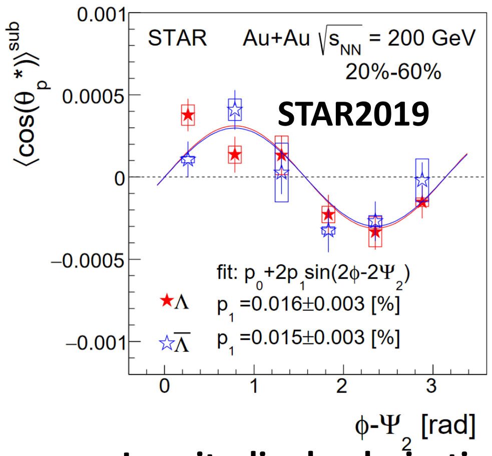
Spin probe of QGP: Local spin polarization

- How the spin polarization is distributed in different ϕ, p_T, η?

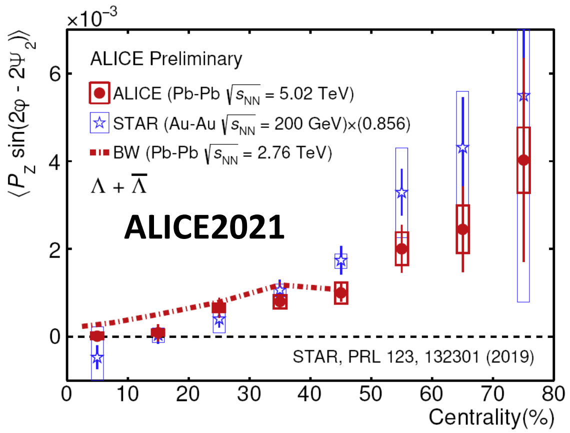
$$\mathrm{P}_{y,z}\,(\phi) = \frac{N_{y,z}(\phi) - N_{y,z}(\phi)}{N_{y,z}(\phi) + N_{y,z}(\phi)}$$



Transverse polarization



Longitudinal polarization



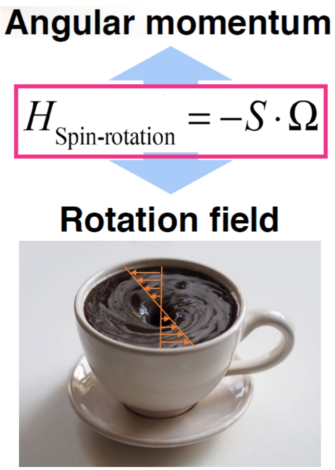
Longitudinal polarization

- Spin harmonic flow: $\frac{dP_{y,z}}{d\phi} = \frac{1}{2\pi}[P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \cdots]$

$$f_{2z}^{\mathrm{exp}} > 0 \qquad g_{2y}^{\mathrm{exp}} > 0$$

Spin polarization by vorticity

- What is probed by spin polarization observables?
- Quite naturally: Vorticity (local rotation)



(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \sim \frac{\omega}{2T}$$

Spin polarization by vorticity

- What is probed by spin polarization observables?
- Quite naturally: Vorticity (local rotation)

Angular momentum

$$H_{\text{Spin-rotation}} = -S \cdot \Omega$$

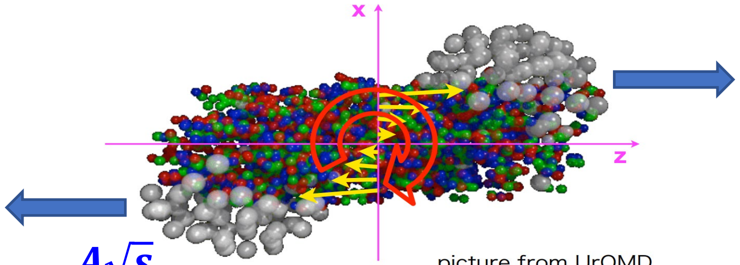
Rotation field

(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

- How vorticity emerges: Global angular momentum



picture from UrQMD

$$P_z \sim \frac{A\sqrt{s}}{2}$$

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)

$$\boldsymbol{J} = \int d^3x\, \boldsymbol{I}(\boldsymbol{x}) \boldsymbol{\omega}(\boldsymbol{x})$$

$$\boldsymbol{\omega} = \frac{1}{2} \nabla \times \boldsymbol{v}$$

(Angular velocity of fluid cell)

Vorticity by global angular momentum

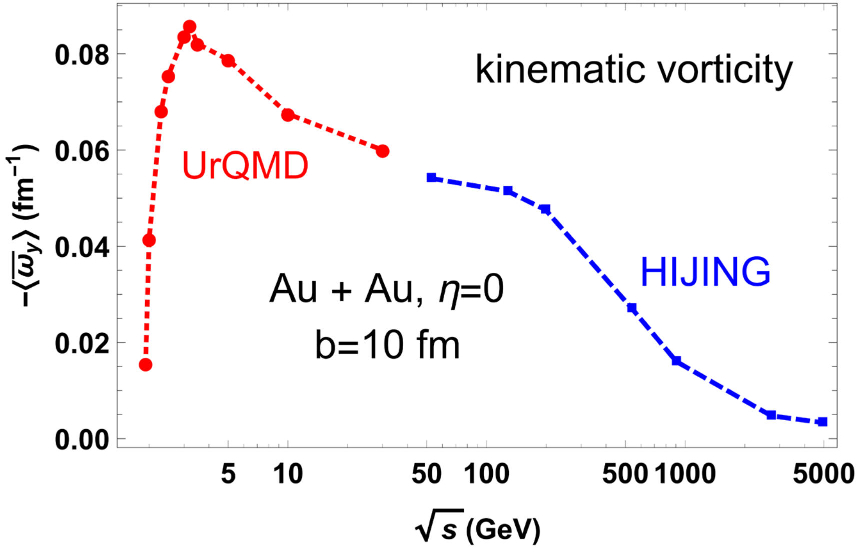
Global angular momentum

Local fluid vorticity

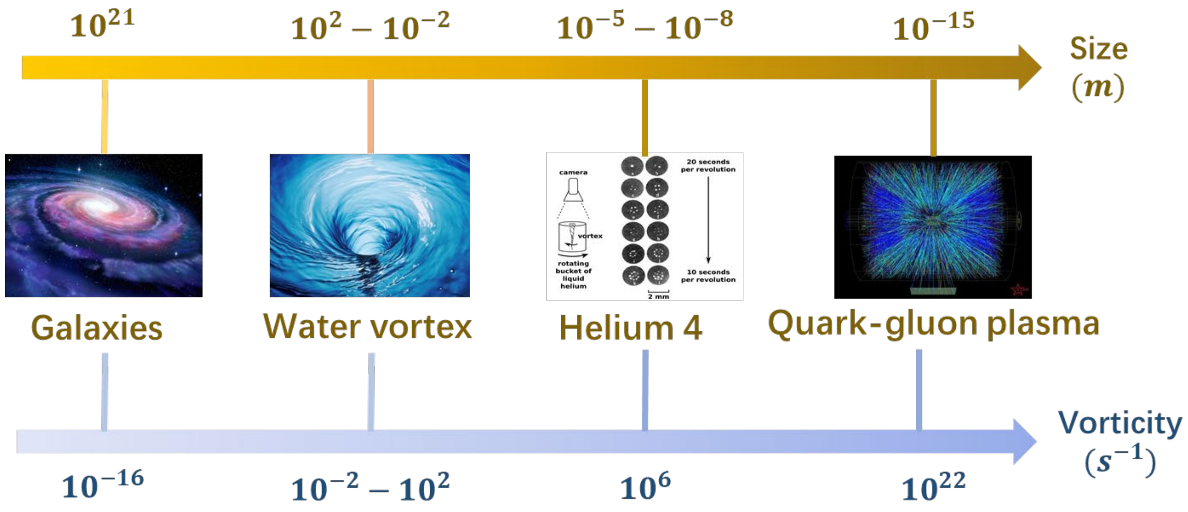
$$\omega = \frac{1}{2}\nabla \times v$$

(Angular velocity of fluid cell)

Energy dependence



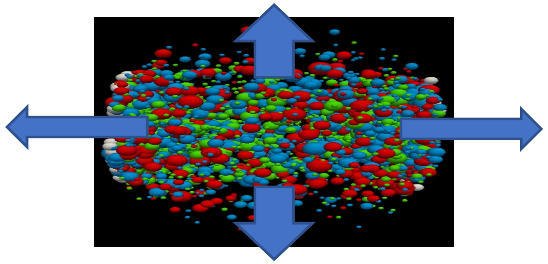
(Deng-XGH 2016; Deng-XGH-Ma-Zhang 2020)

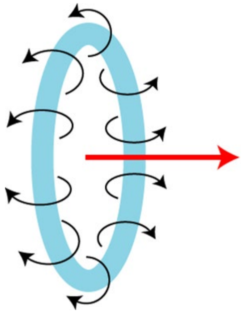


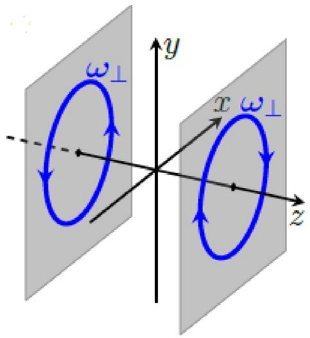
- The most vortical fluid: $\omega \sim 10^{20} - 10^{21} s^{-1}$
- Relativistic suppression at high energies

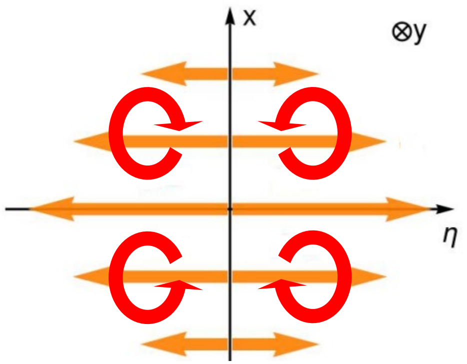
(See also: Jiang-Lin-Liao 2016; Becattini-Karpenko etal 2015,2016; Xie-Csernai etal 2014,2016,2019; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; Fu-Xu-XGH-Song 2020; Guo etal 2021; … …)

Vorticity by inhomogeneous expansion

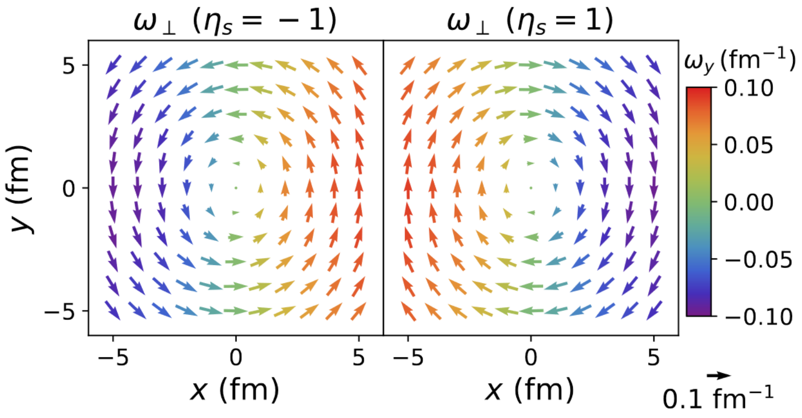




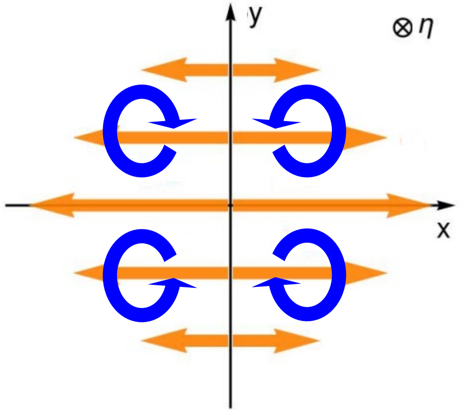




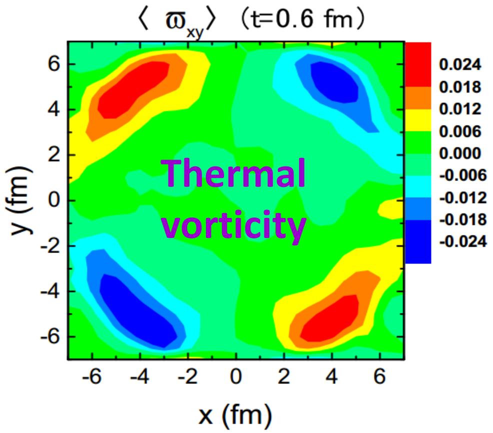
Transverse



(Xia-Li-Wang 2017)



Longitudinal



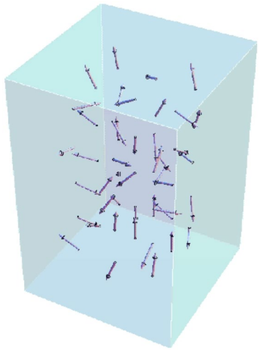
(Wei-Deng-XGH 2019)

(See also: Karpenko-Becattini 2017; Csernai etal 2014; Teryaev-Usubov 2015; Ivanov-Soldatov 2018; Fu etal 2020; Lei etal 2021; … …)

<u>Quantitative understanding of spin polarization</u>

Statistical mechanics description

- Consider a local Gibbs state for spin-1/2 fermions* (Zubarev etal 1979, Van Weert 1982, Becattini etal 2013)



Canonical stress tensor — Canonical spin tensor

$$\hat{\rho}_{\mathrm{LG}} = \frac{1}{Z_{\mathrm{LG}}} \exp\left\{ -\int_{\Xi} d\Xi_\mu(y) \left[\hat{\Theta}^{\mu\nu}(y)\beta_\nu(y) - \frac{1}{2}\hat{\Sigma}^{\mu\rho\sigma}(y)\mu_{\rho\sigma}(y) \right] \right\}$$

Thermal flow vector — Spin chemical potential

- The corresponding Wigner function

$$W(x,p) = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{LG}}\hat{W}(x,p)\right] = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{LG}} \int d^4 s e^{-ip\cdot s} \hat{\bar{\psi}}\left(x+\frac{s}{2}\right) \otimes \hat{\psi}\left(x-\frac{s}{2}\right)\right]$$

- The canonical spin vector in phase space

$$S^\mu(x,p) = -\frac{1}{6}\epsilon^{\mu\nu\rho\sigma}\Sigma_{\nu\rho\sigma}(x,p) = -\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}_{\mathrm{D}}\left[\{\gamma_\nu, \Sigma_{\rho\sigma}\} W(x,p)\right]$$

---

* Obtained by maximizing Von Neumann entropy under local constraints of stress and angular momentum tensors:

$s = -\mathrm{Tr}\left(\hat{\rho}\ln\hat{\rho}\right)$ with $n_\mu \mathrm{Tr}(\hat{\rho}\,\hat{\Theta}^{\mu\nu}) = n_\mu \Theta^{\mu\nu}$ and $n_\mu \mathrm{Tr}(\hat{\rho}\,\hat{\Sigma}^{\mu\rho\sigma}) = n_\mu \Sigma^{\mu\rho\sigma}$

Spin Cooper-Frye formula

- Mean spin vector (on-shell, for particle branch) (Liu-XGH 2021; Buzzegoli 2021)

$$S^\mu(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_\nu\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda} + \Delta\mu_{\nu\lambda}\right)n_\beta p_\alpha p^\lambda\right] n_F(1-n_F) + O(\mu_{\rho\sigma}^2, \partial^2)$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$: Thermal shear tensor

$$\Delta\mu_{\rho\sigma} = \mu_{\rho\sigma} - \varpi_{\rho\sigma} \quad \text{with} \quad \varpi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma\right)$$: Thermal vorticity tensor

- Suppose Ξ is the hypersurface on which both spin and particle number freeze out

$$P^\mu(p) = \frac{\int d\Xi_\nu(x)\frac{p^\nu}{E_p}S^\mu(x,p)}{\int d\Xi_\nu(x)\frac{p^\nu}{E_p}n_F(x,p)}$$

- Temperature and fluid velocity can be well simulated via hydro or transports models but so-far no knowledge is known for spin chemical potential
- At which conditions, the spin chemical potential is known?

Spin Cooper-Frye formula

- Global equilibrium

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad \mu_{\rho\sigma} = \varpi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma \beta_\rho - \partial_\rho \beta_\sigma\right)$$

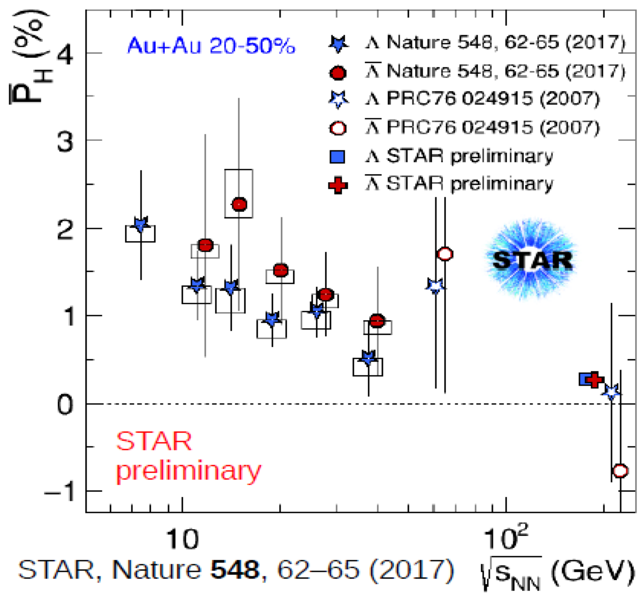
- The above mean spin vector becomes (Becattini etal 2013; Fang etal 2016; Liu etal 2020)

$$S^\mu(x,p) = -\frac{1}{4E_p}\epsilon^{\mu\nu\alpha\beta} p_\nu \varpi_{\alpha\beta} n_F(1-n_F) + O(\partial^2)$$

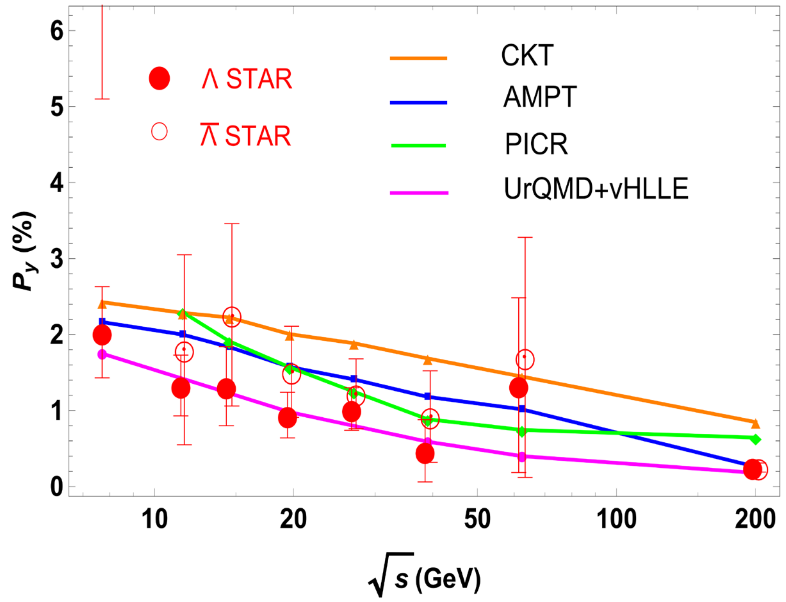
  - Valid at global equilibrium.
  - Spin polarization is enslaved to thermal vorticity, not dynamical
  - Friendly for numerical simulation
  - Surprisingly good in describing global spin polarization

Global spin polarization: Theories

Λ hyperons: Experiment = Theory



STAR, Nature **548**, 62–65 (2017)



(Li-Pang-Wang-Xia 2017; Sun-Ko 2017; Wei-Deng-XGH 2019; Xie-Wang-Csernai 2017; Karpenko-Becattini 2016; Shi etal 2017; … …)

Though with big error bar, a difference between $P_y(\Lambda)$ and $P_y(\bar{\Lambda})$ is seen. Magnetic field?

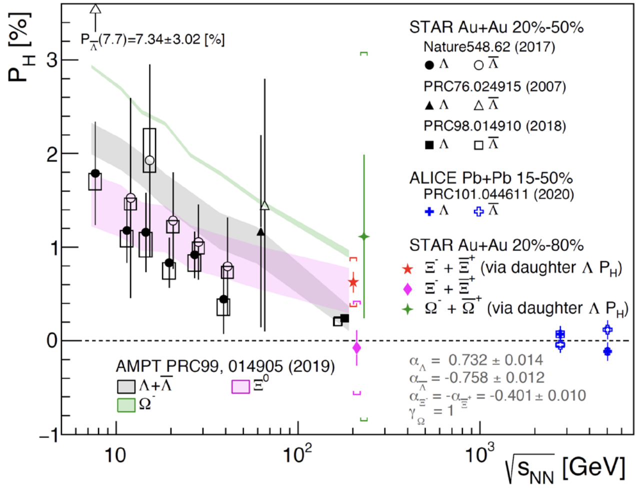
$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S} - \boldsymbol{m} \cdot \boldsymbol{B}$$

Vorticity interpretation of global spin polarization works well!

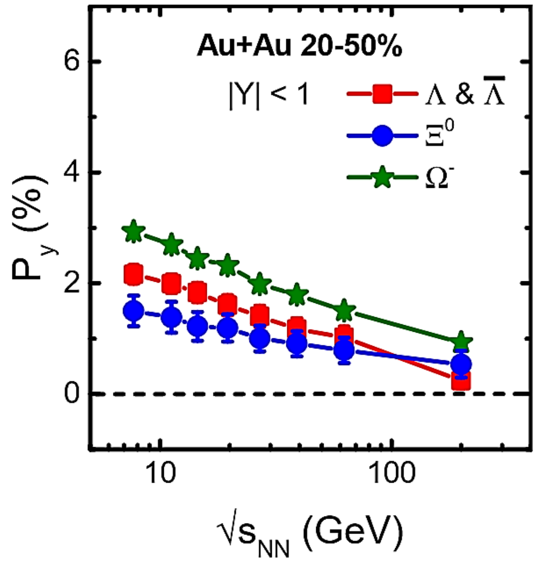
Global spin polarization: Theories

Ξ, Ω hyperons: Experiment = Theory

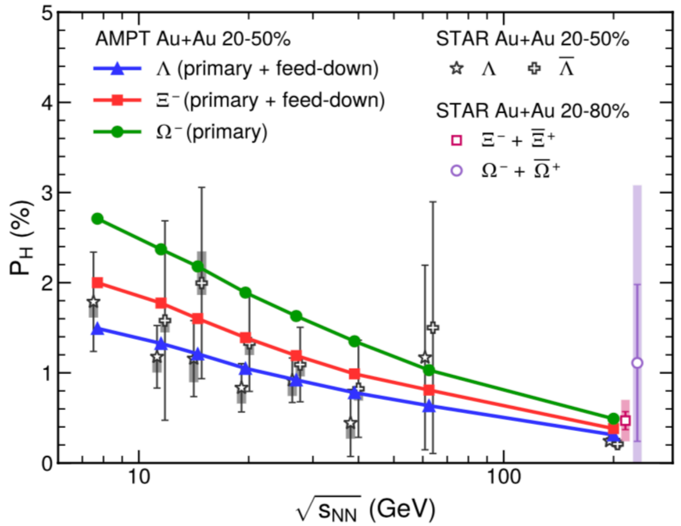
STAR, PRL126, 162301 (2021)



(Primary: Wei-Deng-XGH 2019)



(Feed-down: Li-Xia-XGH-Huang 2021)



Global AM--- vorticity ---global spin polarization

Vorticity interpretation of global spin polarization works well!

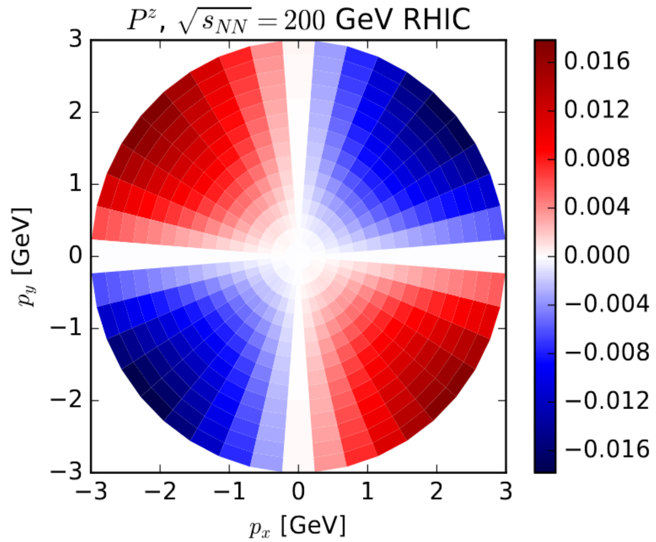
Local Λ spin polarization

The global Λ polarization reflects the total amount of angular momentum retained in the mid-rapidity region. How about local polarization?

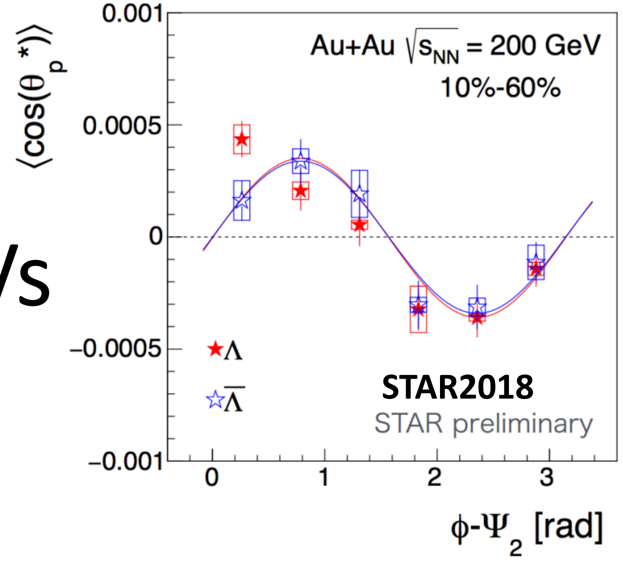
- Spin harmonic flow: $$\frac{dP_{y,z}}{d\phi} = \frac{1}{2\pi}\left[P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \cdots\right]$$

1) longitudinal polarization vs ϕ

(Becattini-Karpenko 2018)



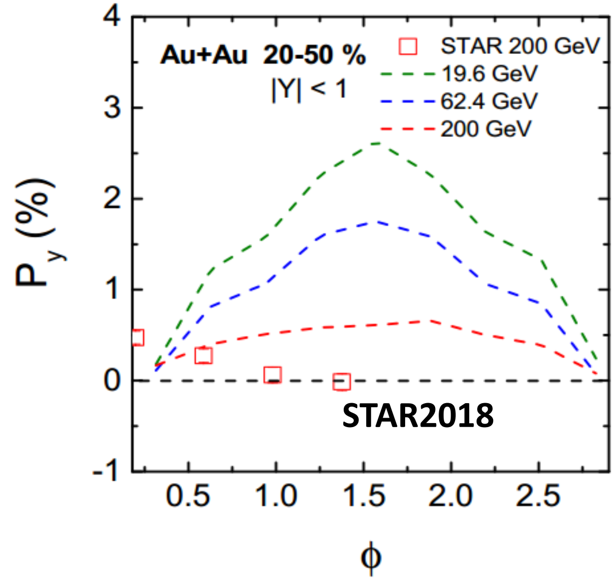
Vs



$f_{2z}^{\text{ther}} < 0$ $f_{2z}^{\text{exp}} > 0$

2) Transverse polarization vs ϕ

(Wei-Deng-XGH 2019)



$g_{2y}^{\text{ther}} < 0, g_{2y}^{\text{exp}} > 0$

We have a spin "sign problem"!

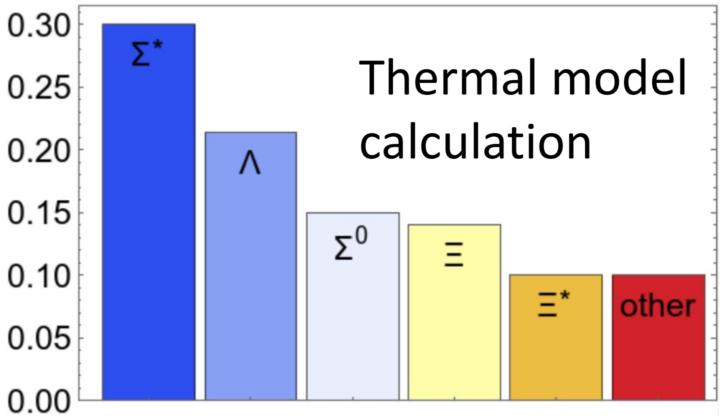
How to resolve the local spin polarization puzzles

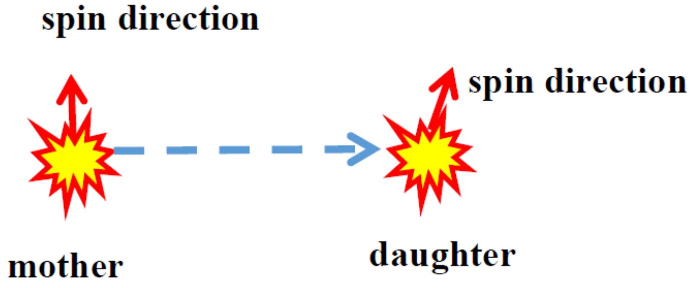
Attack the spin sign problem from theory side:

- Understand the vorticity (☺)
- Effect of feed-down decays (☺) (Xia-Li-XGH-Huang 2019; Becattini-Cao-Speranza 2019)
  (Measured Λ may come from decays of heavier particles)
- Go beyond equilibrium treatment (spin as a dynamic d.o.f)
  spin hydrodynamics
  spin kinetic theory
- Initial condition
  (Initial polarization, initial flow, … …)
- Other possibilities
  (chiral vortical effect (Liu-Sun-Ko 2019), mesonic mean-field(Csernai-Kapusta-Welle 2019), choice of spin chemical potential (Wu-Pang-XGH-Wang 2019; Florkowski etal 2019), contribution from shear flow (Becattini etal 2021; Fu-Liu-Pang-Song-Yin 2021; Yi-Pu-Yang 2021; Florkowski etal 2021), contribution from gluons, … …)

The feed-down effects

About 80% of final Λ's are from decays of higher-lying particles





Spin polarization transfer (Xia-Li-XGH-Huang 2019, Becattini-Cao-Speranza 2019)

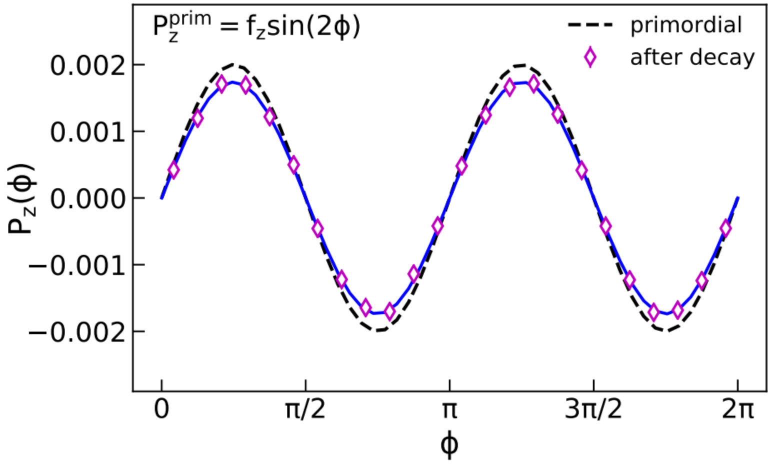
	spin and parity	$(1/N)dN/d\Omega^*$	$\mathbf{P}_D$	$\langle\mathbf{P}_D\rangle/\mathbf{P}_P$
strong decay	$1/2^+ \to 1/2^+ 0^-$	$1/(4\pi)$	$2\left(\mathbf{P}_P\cdot\hat{\mathbf{p}}^*\right)\hat{\mathbf{p}}^* - \mathbf{P}_P$	-1/3
strong decay	$1/2^- \to 1/2^+ 0^-$	$1/(4\pi)$	$\mathbf{P}_P$	1
strong decay	$3/2^+ \to 1/2^+ 0^-$	$3\left[1-2\Delta/3-(1-2\Delta)\cos^2\theta^*\right]/(8\pi)$	Too long to be shown; see ref.	1
strong decay	$3/2^- \to 1/2^+ 0^-$	$3\left[1-2\Delta/3-(1-2\Delta)\cos^2\theta^*\right]/(8\pi)$		-3/5
weak decay	$1/2 \to 1/2\ 0$	$(1+\alpha P_P\cos\theta^*)/(4\pi)$		$(2\gamma+1)/3$
EM decay	$1/2^+ \to 1/2^+ 1^-$	$1/(4\pi)$	$-\left(\mathbf{P}_P\cdot\hat{\mathbf{p}}^*\right)\hat{\mathbf{p}}^*$	-1/3

Some decay channels can lead to spin-polarization flip!

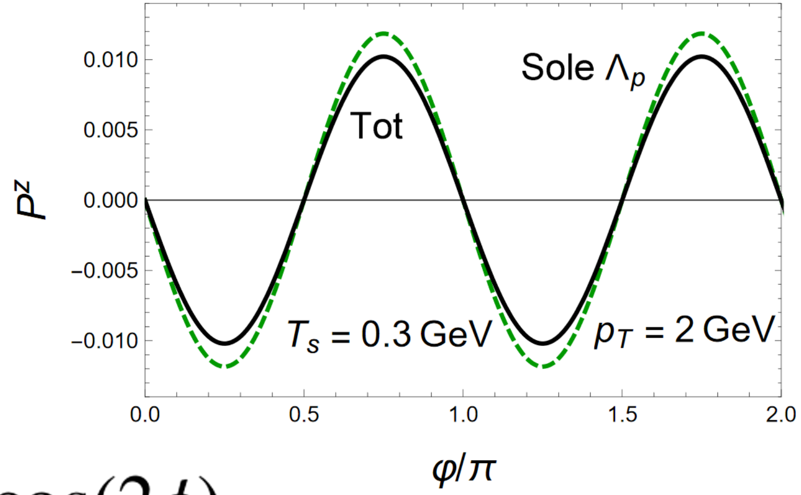
The feed-down effects

- Longitudinal polarization $P_z = f_z \sin(2\phi)$

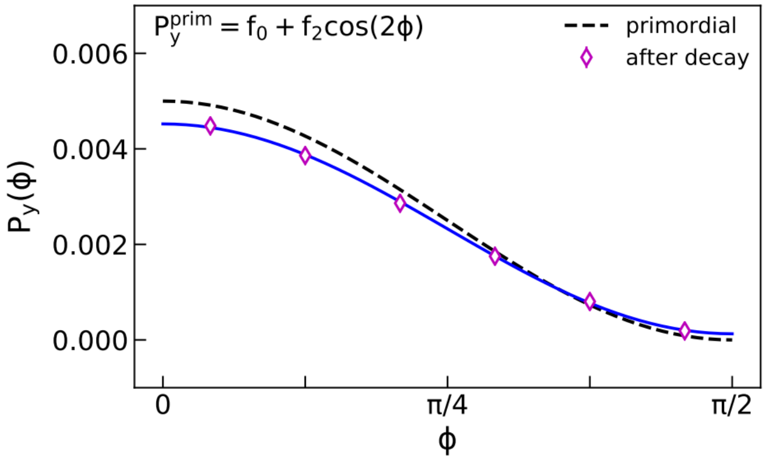
(Xia-Li-XGH-Huang 2019)



(Becattini-Cao-Speranza 2019)



- Transverse polarization $P_y = f_0 + f_2 \cos(2\phi)$



Conclusion:

- Feed-down effects suppress ~10% Λ primordial spin polarization
- Do not solve the spin sign problem

Temperature vorticity as spin chemical potential

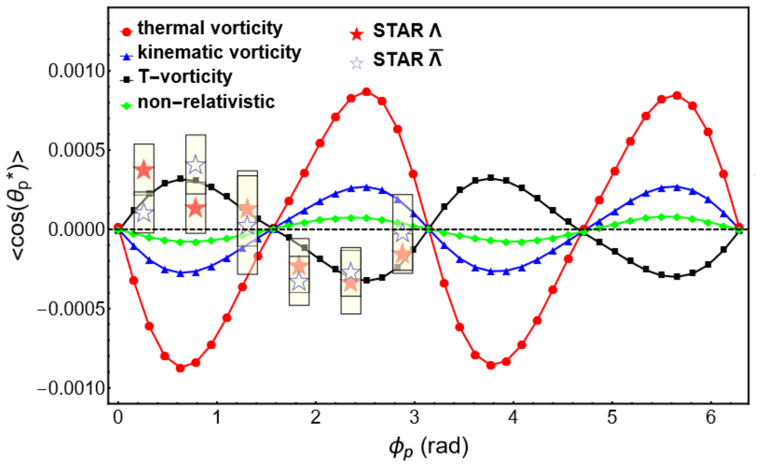
- Recall

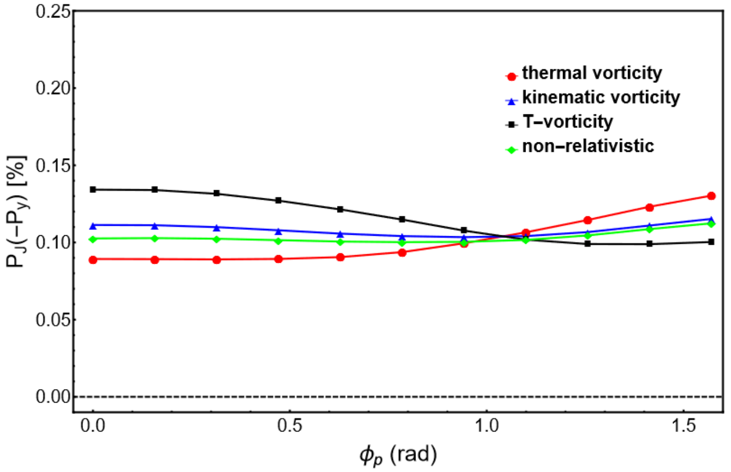
$$S^{\mu}(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_{\nu}\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda} + \Delta\mu_{\nu\lambda}\right)n_{\beta}p_{\alpha}p^{\lambda}\right]n_F(1-n_F) + O(\mu_{\rho\sigma}^2, \partial^2)$$

- Relax the global equilibrium condition (1)

$$\partial_{\mu}\beta_{\nu} + \partial_{\nu}\beta_{\mu} = 0$$

$$\mu_{\rho\sigma} = \frac{1}{2T^2}\left[\partial_{\sigma}(Tu_{\rho}) - \partial_{\rho}(Tu_{\sigma})\right]$$





(Wu-Pang-XGH-Wang 2019)

Shear tensor contribution

- Recall

$$S^\mu(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_\nu\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda}+\Delta\mu_{\nu\lambda}\right)n_\beta p_\alpha p^\lambda\right]n_F(1-n_F) + O(\mu_{\rho\sigma}^2,\partial^2)$$

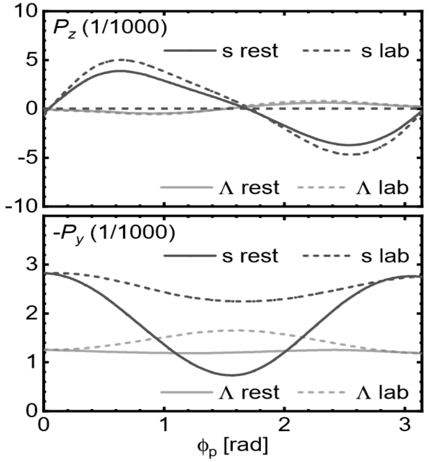
- Relax the global equilibrium condition (2)* (Becattini-Buzzegoli-Palermo 2021, Liu-Yin 2021)

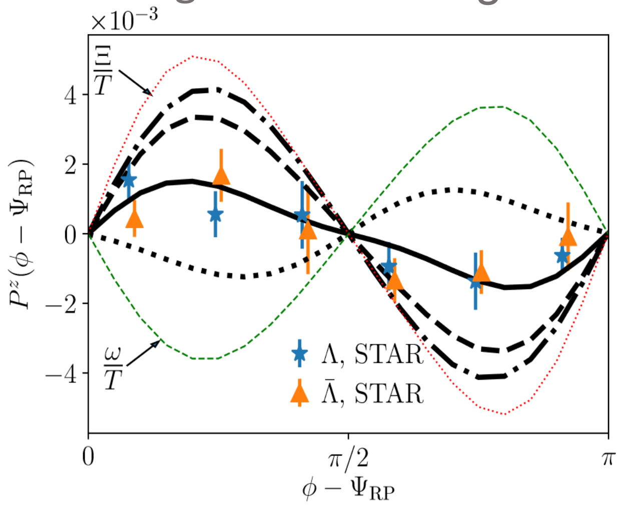
$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu \neq 0$$

(Fu-Liu-Song-Yin 2021)

$$\mu_{\rho\sigma} = \varpi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma\right)$$

(Becattini-Buzzegoli-Palermo-Inghirami-Karpenko 2021)





(See also Yi-Pu-Yang 2021; Florkowski-Kumar-Mazeliauskas-Ryblewski 2021)

* Can also be considered as being derived using a local Gibbs state with vanishing spin tensor (Belinfante gauge)

Local spin polarization puzzle

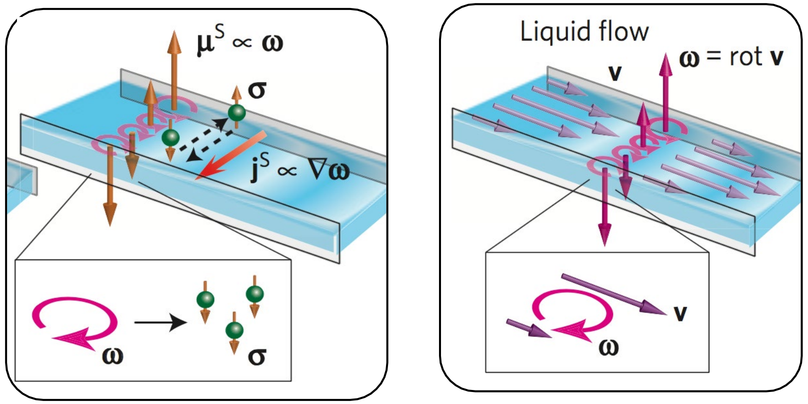
- Spin chemical potential is very essential!
- We need a dynamical theory for it!

Spin hydrodynamics

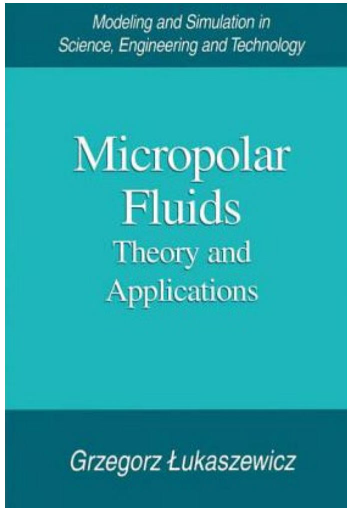
Spin hydrodynamics

Framework for collective spin dynamics. Spin as a (quasi-)hydrodynamic variable

- Widely used in non-relativistic spintronics, micropolar fluid, … …



(Takahashi etal 2016)



- Hydrodynamics: low-energy effective theory for conserved quantities
  - Hydro modes relax at $\tau_{\rm hydro} = 1/\omega_{\rm hydro}(k) \to \infty$ when $k \to 0$
  - Hydro is constructed by gradient expansion
  - Typical hydro modes: energy density, momentum density, baryon density, …

Ideal spin hydrodynamics?

- If spin current is conserved, hydro equations would be

$$\text{Charge conservation}: \quad \partial_\mu J^\mu(x) = 0,$$
$$\text{Energy} - \text{momentum conservation}: \quad \partial_\mu \Theta^{\mu\nu}(x) = 0,$$
$$\text{Spin conservation}: \quad \partial_\mu \Sigma^{\mu\nu\rho}(x) = 0,$$

with J^μ, $\Theta^{\mu\nu}$, and $\Sigma^{\mu\nu\rho}$ expanded order by order in gradient giving constitutive relations

$$J^\mu = nu^\mu + O(\partial),$$
$$\Theta^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} + O(\partial),$$
$$\Sigma^{\mu\nu\rho} = \sigma^{\nu\rho}u^\mu + O(\partial)$$

where $O(1)$ terms usually correspond to ideal hydrodynamics (Florkowski etal 2018)

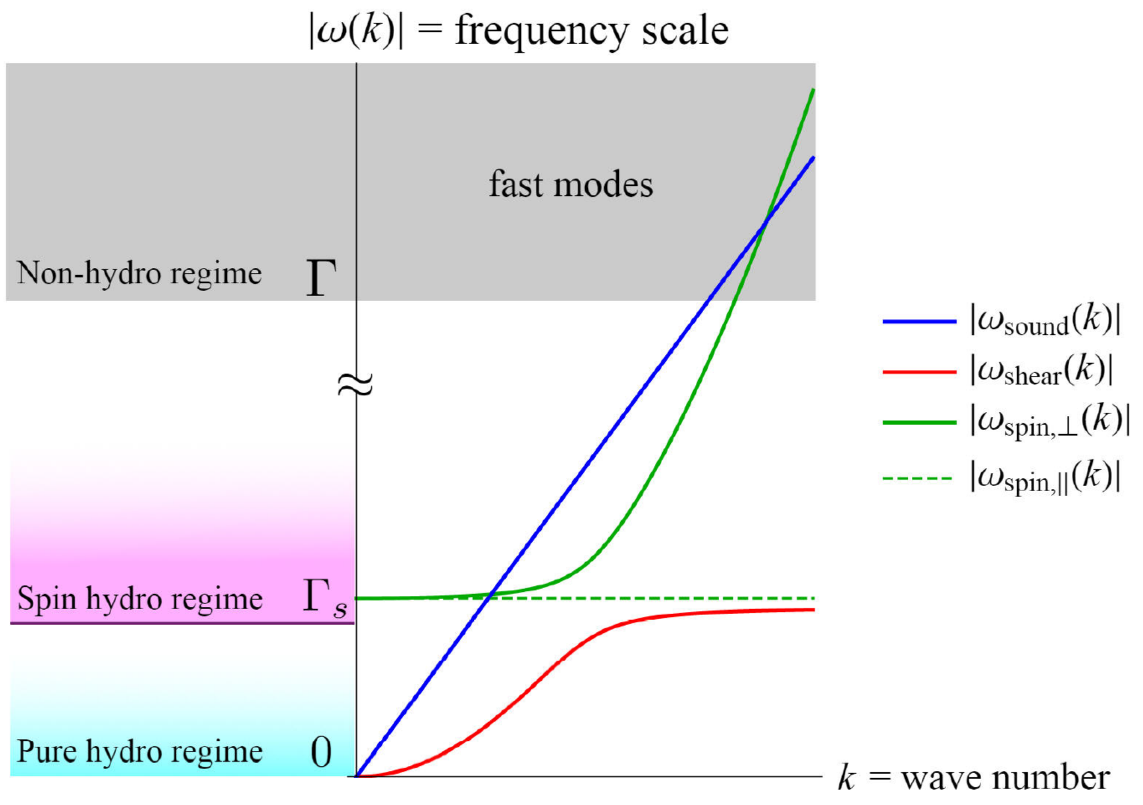
- But spin is not conserved in general (and thus not strict hydro mode)

$$\partial_\mu J^{\mu\nu\rho} = 0, \quad J^{\mu\nu\rho} = x^\nu \Theta^{\mu\rho} - x^\rho \Theta^{\mu\nu} + \Sigma^{\mu\nu\rho} \quad \Rightarrow \quad \partial_\mu \Sigma^{\mu\nu\rho}(x) = \Theta^{\rho\nu} - \Theta^{\nu\rho}$$

- The conversion between spin and orbital AM is dissipative in general

Spin hydrodynamic regime

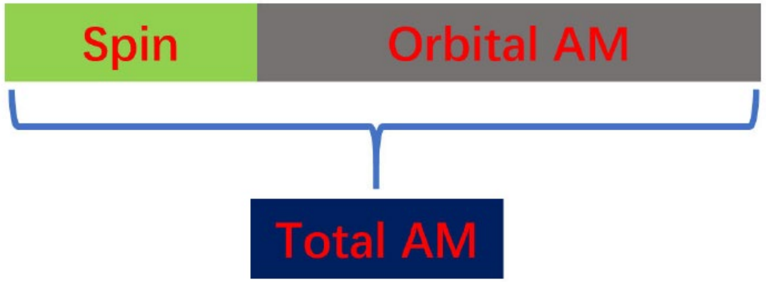
- Even though spin is not conserved, when spin relaxation rate is much smaller than other non-hydro modes, we could formulate a hydro+ for spin: Relativistic dissipative spin hydrodynamics



(Hongo-XGH-Kaminski-Stephanov-Yee 2021)

Ambiguity in definition of spin current

- The definition of spin current is ambiguous



- The pseudo-gauge transformation: preserves total conserved charges and conservation law (Becattini-Florkowski-Speranza 2018)

$$\Sigma^{\mu\nu\rho} \to \Sigma^{\mu\nu\rho} - \Phi^{\mu\nu\rho},$$

$$\Theta^{\mu\nu} \to \Theta^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)$$

- Formulation of spin hydro depends on the pseudo-gauge choice

  (Florkowski etal 2017; Montenegro etal 2017; Hattori etal 2019; Gallegos etal 2020; Bhadury etal 2020; Li-Stephanov-Yee 2020; Fukushima-Pu 2020; She etal 2021; … …)

- Fix the pseudo-gauge by coupling spin to torsion (or spin connection)

  (Hongo-XGH-Kaminski-Stephanov-Yee 2021; Gallegos etal 2020)

Stress tensor and spin current

- The stress tensor and spin current

$$\Theta^{\mu}{}_{a}(x) \equiv \frac{1}{e(x)} \frac{\delta S}{\delta e_{\mu}{}^{a}(x)}\bigg|_{\omega}, \quad \Sigma^{\mu}{}_{ab}(x) \equiv -\frac{2}{e(x)} \frac{\delta S}{\delta \omega_{\mu}{}^{ab}(x)}\bigg|_{e}$$

- For QCD

$$\Theta^{\mu}{}_{a} = \frac{1}{2}\bar{q}\left(\gamma^{\mu}\overrightarrow{D}_{a} - \overleftarrow{D}_{a}\gamma^{\mu}\right)q + 2\mathrm{tr}\left(G^{\mu\rho}G_{a\rho}\right) + \mathcal{L}_{\mathrm{QCD}}e_{a}{}^{\mu},$$

$$\Sigma^{\mu}{}_{ab} = -\frac{i}{2}\bar{q}e^{\mu}{}_{c}\{\gamma^{c}, \Sigma_{ab}\}q$$

- Equations of motion (Ward-Takahashi identities for diffeomorphisim and local Lorentz invariance)($G_{\mu} = T^{\nu}{}_{\nu\mu}$)

$$(D_{\mu} - G_{\mu})\Theta^{\mu}{}_{a} = -\Theta^{\mu}{}_{b}T^{b}{}_{\mu a} + \frac{1}{2}\Sigma^{\mu}{}_{b}{}^{c}R^{b}{}_{c\mu a} + F_{a\mu}J^{\mu},$$

$$(D_{\mu} - G_{\mu})\Sigma^{\mu}{}_{ab} = -(\Theta_{ab} - \Theta_{ba})$$

Construction of spin hydrodynamics

- Step 1: Identify (quasi-)hydro modes

  - Eight (quasi-)hydro variables: $\epsilon, n, u^a, \sigma_{ab}$ (or $\sigma_a = \varepsilon^{abcd} u_b \sigma_{cd}/2$) with constraints $u^2 = -1$, $\sigma^a u_a = \sigma_{ab} u^b = 0$.
  - Local first law of thermodynamics: $s = \beta(\epsilon + P - \mu n - \mu_{ab}\sigma^{ab}/2)$ and $T ds = d\epsilon - \mu dn - \mu^{ab} d\sigma_{ab}/2$.
  - Conjugate variables: inverse temperature $\beta \equiv \frac{\partial s}{\partial \epsilon}$, chemical potentials $\mu = \frac{\partial s}{\partial n}$, $\mu^{ab} = -\frac{T}{2}\frac{\partial s}{\partial \sigma_{ab}}$.
  - Power counting scheme

$$\{\beta, n, u^a, e_\mu{}^a\} = O(\partial^0) \quad \text{and} \quad \{\mu^{ab}, \sigma_{ab}, \omega_\mu{}^{ab}\} = O(\partial)$$

- Step 2: Tensor decomposition

$$\Theta^\mu{}_a = \epsilon u^\mu u_a + p\Delta^\mu_a + u^\mu \delta q_a - \delta q^\mu u_a + \delta\Theta^\mu{}_a,$$
$$\Sigma^\mu{}_{ab} = \varepsilon^\mu{}_{abc}(\sigma^c + \delta\sigma u^c)$$

Construction of spin hydrodynamics

- Step 3: Calculate the entropy production rate

$$(\nabla_\mu - G_\mu)s^\mu = (\nabla_\mu - G_\mu)(\delta s^\mu + \beta\mu\delta J^\mu) - \delta\Theta^\mu{}_a\big|_{(s)}(D_\mu\beta^a - T^a{}_{\mu b}\beta^b)$$
$$- \delta\Theta^\mu{}_a\big|_{(a)}(D_\mu\beta^a - T^a{}_{\mu b}\beta^b - \beta\mu_\mu{}^a) - \delta J^\mu[\nabla_\mu(\beta\mu) - F_{\mu\nu}\beta^\nu] + O(\partial^3)$$

- Step 4: Second law of local thermodynamics $(\nabla_\mu - G_\mu)s^\mu \geq 0$

$$\delta\Theta^\mu{}_a\big|_{(s)} = -\eta^{\mu}{}_a{}^{\nu}{}_b(D_\nu u^b - T^b{}_{\nu c}u^c),$$

(Hongo-XGH-Kaminski-Stephanov-Yee 2021)

$$\delta\Theta^\mu{}_a\big|_{(a)} = -(\eta_s)^{\mu}{}_a{}^{\nu}{}_b(D_\nu u^b - T^b{}_{\nu c}u^c - \mu_\nu{}^b)$$

$$\eta^{\mu}{}_a{}^{\nu}{}_b = 2\eta\left(\frac{1}{2}(\Delta^{\mu\nu}\Delta_{ab} + \Delta^\mu_b\Delta^\nu_a) - \frac{1}{3}\Delta^\mu_a\Delta^\nu_b\right) + \zeta\Delta^\mu_a\Delta^\nu_b,$$

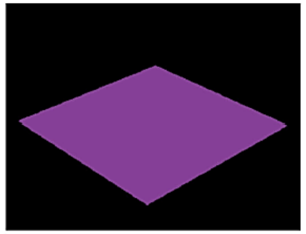
$$(\eta_s)^{\mu}{}_a{}^{\nu}{}_b = \frac{1}{2}\eta_s(\Delta^{\mu\nu}\Delta_{ab} - \Delta^\mu_b\Delta^\nu_a).$$

with $\eta \geq 0$ shear, $\zeta \geq 0$ bulk, and $\eta_s \geq 0$ rotational viscosities.

- With equation of state $p = p(\epsilon, n, \sigma_{ab})$, the equations are closed

(Quasi-)hydro modes

Perturbation about global static thermal equilibrium

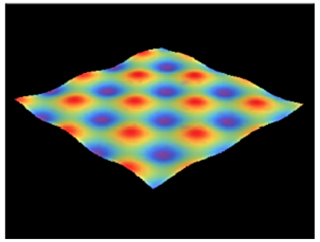


$\epsilon = \epsilon_0$

$u^\mu = (1, \mathbf{0})$

$\sigma^a = 0$

perturb



$\epsilon = \epsilon_0 + \delta\epsilon$

$u^\mu = (1, \mathbf{0}) + \delta u^\mu$

$\sigma^a = 0 + \delta\sigma^a$

$$\begin{cases} \bullet \text{ One pair of sound modes : } \omega_{\text{sound}}(\boldsymbol{k}) = \pm c_s |\boldsymbol{k}| - \frac{i}{2}\gamma_{\|}\boldsymbol{k}^2 + O(\boldsymbol{k}^3), \\ \bullet \text{ One longitudinal spin mode : } \omega_{\text{spin},\|}(\boldsymbol{k}) = -i\Gamma_s, \\ \bullet \text{ Two shear modes : } \omega_{\text{shear}}(\boldsymbol{k}) = -i\gamma_\perp \boldsymbol{k}^2 + O(\boldsymbol{k}^4), \\ \bullet \text{ Two transverse spin modes : } \omega_{\text{spin},\perp}(\boldsymbol{k}) = -i\Gamma_s - i\gamma_s \boldsymbol{k}^2 + O(\boldsymbol{k}^4). \end{cases}$$

where we introduced a set of static/kinetic coefficients as

$$c_s^2 \equiv \frac{\partial p}{\partial \epsilon}, \quad \gamma_{\|} \equiv \frac{1}{\epsilon_0 + p_0}\left(\zeta + \frac{4}{3}\eta\right), \quad \gamma_\perp \equiv \frac{\eta}{\epsilon_0 + p_0}$$

$$\chi_s \delta_{ij} \equiv \frac{\partial \sigma_i}{\partial \mu^j}, \quad \gamma_s \equiv \frac{\eta_s}{2(\epsilon_0 + p_0)}, \quad \boxed{\Gamma_s \equiv \frac{2\eta_s}{\chi_s}}$$

Spin relaxation rate

(Hongo-XGH-Kaminski-Stephanov-Yee 2021)

Spin relaxation rate

When spin relaxes slowly

- Spin relaxes slowly when the spin-orbit coupling is suppressed
- One situation for this to happen is when heavy fermions exist in the system

$$H_{\mathrm{SOC}} = -\frac{g}{2M}\psi^\dagger(\boldsymbol{B}\cdot\boldsymbol{\sigma})\psi$$

- Let us consider the heavy-quark limit of QCD

$$\mathcal{L} = -M\psi^\dagger\psi + \mathrm{i}\psi^\dagger D_0\psi - \frac{1}{2M}(\boldsymbol{D}\psi)^\dagger\cdot\boldsymbol{D}\psi + \frac{g}{2M}\psi^\dagger(\boldsymbol{B}\cdot\boldsymbol{\sigma})\psi + \mathcal{L}_{\mathrm{gluon}} + \mathcal{O}(1/M^2)$$

- Equations of motion for spin density (note that, to distinguish with the relativistic case, we change the notations)

$$\partial_0 J^0_{\mathrm{a}} + \boldsymbol{\nabla}\cdot\boldsymbol{J}_{\mathrm{a}} = \Theta_{\mathrm{a}} \quad (\mathrm{a}=1,2,3)$$

$$J^\mu_{\mathrm{a}} = \begin{pmatrix} \frac{1}{2}\psi^\dagger\sigma_{\mathrm{a}}\psi \\ -\frac{\mathrm{i}}{4M}\left[\psi^\dagger\sigma_{\mathrm{a}}(\boldsymbol{D}\psi) - (\boldsymbol{D}\psi)^\dagger\sigma_{\mathrm{a}}\psi\right] \end{pmatrix}, \quad \Theta_{\mathrm{a}} \equiv -\frac{g}{2M}\epsilon_{\mathrm{abc}}\psi^\dagger B^{\mathrm{b}}\sigma^{\mathrm{c}}\psi$$

Kubo formulas for spin relaxation

- Constitutive relations at homogeneous limit

$$\Theta_{\rm a} = -\lambda_s \left(\mu_{\rm a} - b_{\rm a}\right)$$

λ_s is rotational viscosity (i.e., η_s in the relativistic case)

- Solving the EOM at linear order in external field $b_{\rm a}$ gives retarded Green functions

$$G_R^{J^0_{\rm a} J^0_{\rm b}}(\omega) = \frac{{\rm i}\chi_s \Gamma_s}{\omega + {\rm i}\Gamma_s}\delta_{\rm ab} = \left[\chi_s + i\omega\frac{\chi_s}{\Gamma_s} \quad + \mathcal{O}(\omega/\Gamma_s)^2\right]\delta_{\rm ab} \qquad \text{at} \quad \omega \ll \Gamma_s$$

$$G_R^{\Theta_{\rm a}\Theta_{\rm b}}(\omega) = \frac{{\rm i}\omega^2\chi_s \Gamma_s}{\omega + {\rm i}\Gamma_s}\delta_{\rm ab} = \left[i\omega\chi_s\Gamma_s + \chi_s\Gamma_s^2 + \mathcal{O}(\Gamma_s/\omega)\right]\delta_{\rm ab} \qquad \text{at} \quad \Gamma_s \ll \omega \ll \Gamma$$

- Kubo-type formulas

$$\Gamma_s = \frac{\delta_{\rm ab}}{3\chi_s}\lim_{\Gamma_s \ll \omega \ll \Gamma}\frac{1}{\omega}{\rm Im}G_R^{\Theta_{\rm a}\Theta_{\rm b}}(\omega) = \frac{\delta_{\rm ab}}{6T\chi_s}\lim_{\Gamma_s \ll \omega \ll \Gamma} G_{12}^{\Theta_{\rm a}\Theta_{\rm b}}(\omega)$$

Source-source correlator

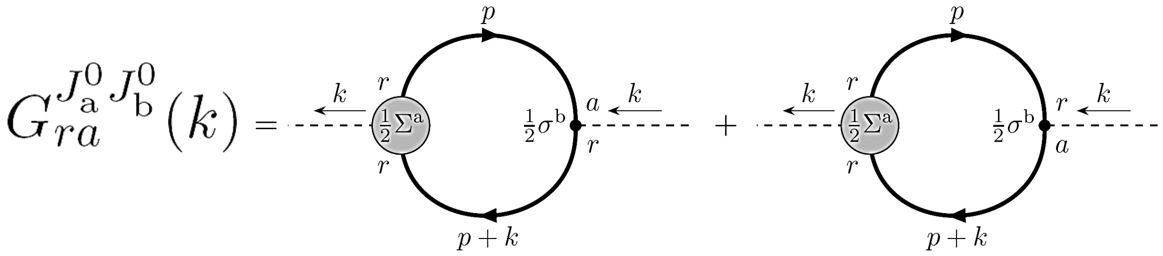
$$\Gamma_s^{-1} = \frac{\delta_{\rm ab}}{3\chi_s}\lim_{\omega \to 0}\frac{1}{\omega}{\rm Im}G_R^{J^0_{\rm a} J^0_{\rm b}}(\omega) = \frac{\delta_{\rm ab}}{3\chi_s}\lim_{\omega \to 0}\frac{1}{\omega}{\rm Re}G_{ra}^{J^0_{\rm a} J^0_{\rm b}}(\omega)$$

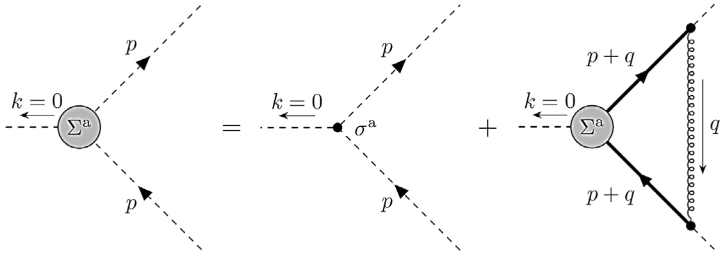
Spin-spin correlator

(Hongo-XGH-Kaminski-Stephanov-Yee 2022)

Leading-log results for QCD

- For the spin-spin correlator at strict hydro limit





- The heavy quark spin relaxation at leading-log approximation:

(Hongo-XGH-Kaminski-Stephanov-Yee 2022)

$$\Gamma_s = C_2(F)\frac{g^2 m_D^2 T}{6\pi M^2}\log(1/g)$$

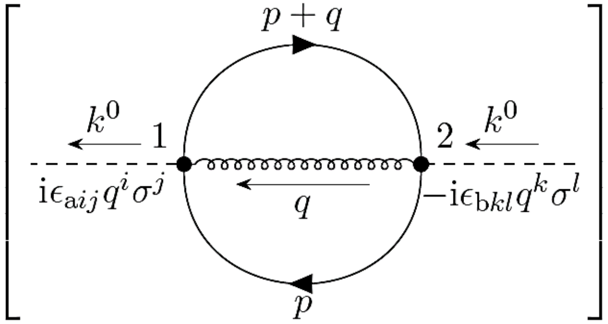
- Also checked by solving the linearized Boltzmann equation for heavy quark

(Li-Yee 2019; Hongo-XGH-Kaminski-Stephanov-Yee 2022)

Leading-log results for QCD

- Above calculation at strict hydro limit is very tedious
- Interestingly, calculating the source-source correlator is much simpler

$$\chi_s\Gamma_s = \frac{1}{6T}\delta^{\mathrm{ab}} G_{12}^{\Theta_{\mathrm{a}}\Theta_{\mathrm{b}}}(\Gamma_s \ll k^0 \ll \Gamma) = \frac{g^2}{12M^2T}\delta^{\mathrm{ab}} \lim_{k^0\to 0} \mathrm{Tr}$$



- There is no pinching singularity and it is straightforward to get

$$\Gamma_s = C_2(F)\frac{g^2 m_D^2 T}{6\pi M^2}\log(1/g)$$

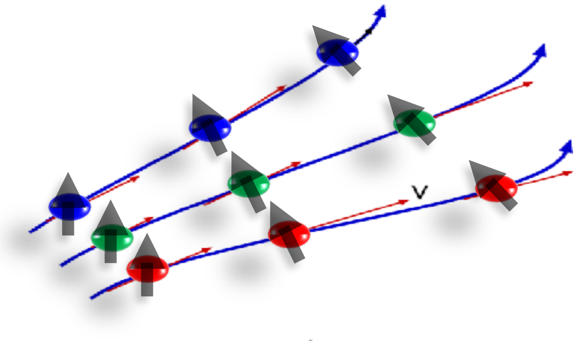
(Hongo-XGH-Kaminski-Stephanov-Yee 2022)

- All the calculations are at weakly coupled limit. Hope holography provides a calculation at strongly coupled limit.

Summary

- Spin polarization of hyperons are observed at heavy-ion collision experiments
- It seems a dynamic theory for spin d.o.f is necessary to have a full understanding of the data: spin hydrodynamics + spin Cooper-Frye formula

- Causal and stable (e.g. Israel-Stewart) 2nd order spin hydrodynamics
- Calculation of rotational viscosity using e.g. holographic methods
- Formulate spin hydrodynamics with magnetic field and anomaly
- Spin hydrodynamics above a globally rotating equilibrium state
- Derive spin hydrodynamics from kinetic theory
  (Shi-Jeon-Gale 2020; Peng-Zhang-Sheng-Wang 2021)
- Application: numerical spin hydrodynamics for e.g. Λ polarization



Thank you

Back up

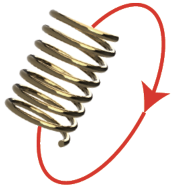
Spintronics

- How to manipulate spin?

Magnetic moment

$$H_{\text{Zeeman}} = -\gamma \boldsymbol{S} \cdot \boldsymbol{B}$$

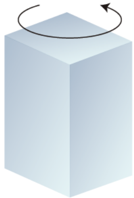
Magnetic field



Angular momentum

$$H_{\text{Spin-rotation}} = -S \cdot \Omega$$

Rotation field



Spin orbit coupling

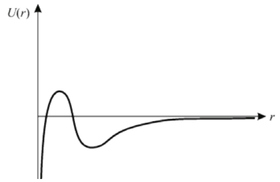
$$H_{\text{SOC-E}} = -\lambda \boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{E})$$

Electric field

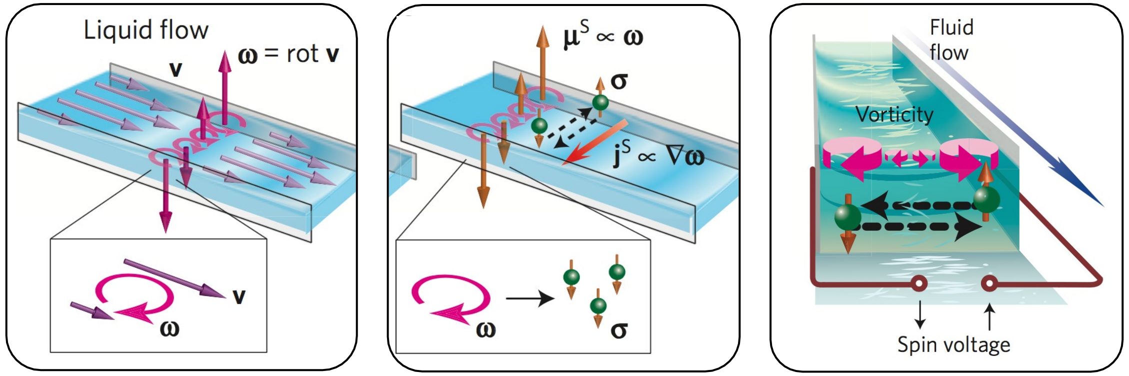
Spin orbit coupling

$$H_{\text{SOC-U}} = \eta \frac{1}{r} \frac{\partial U}{\partial r} \boldsymbol{S} \cdot \boldsymbol{L}$$

External potential



- An interesting example

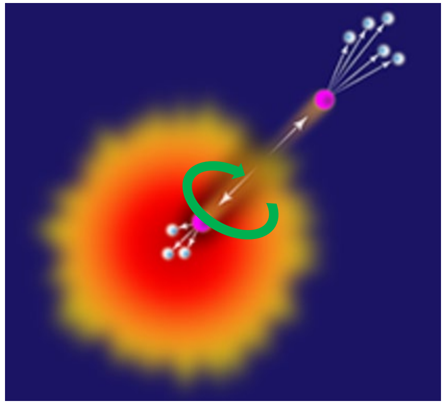


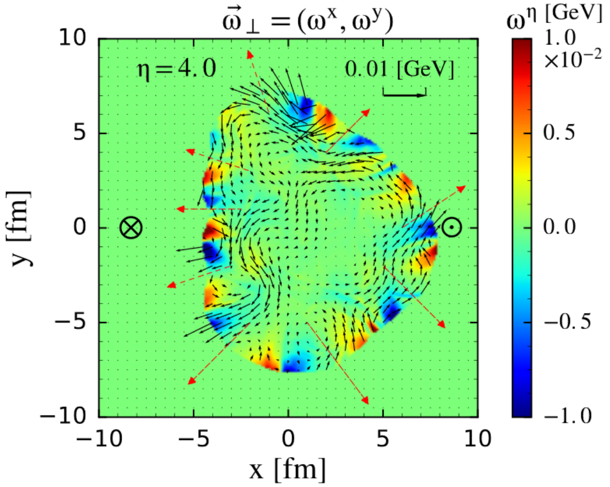
Takahashi et al. 2016

To realize spintronics in quark gluon plasma (QGP):
Rotation, Magnetic field, …… , in heavy-ion collisions?

Other sources of vorticity

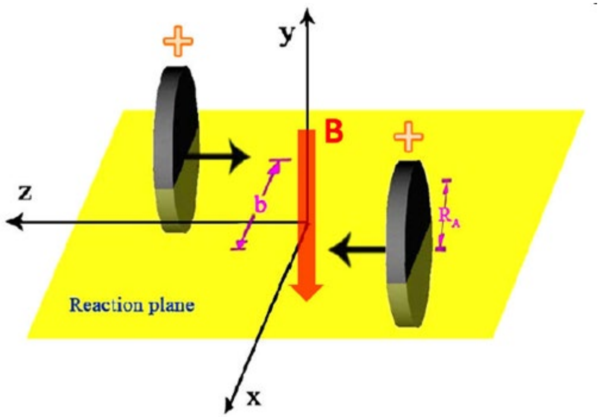
1) Jet

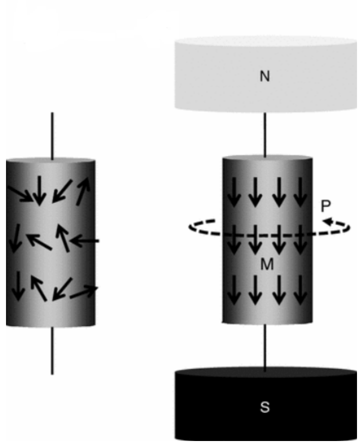




(Pang-Peterson-Wang-Wang 2016)

2) Magnetic field





Einstein-de-Haas effect

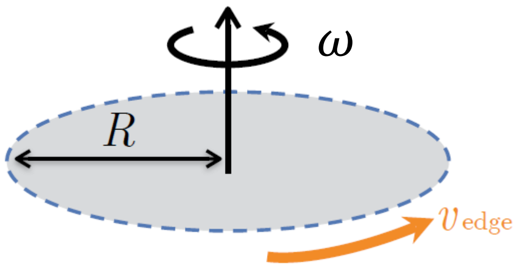
Phase structure under rotation

Rotation induced phase transitions

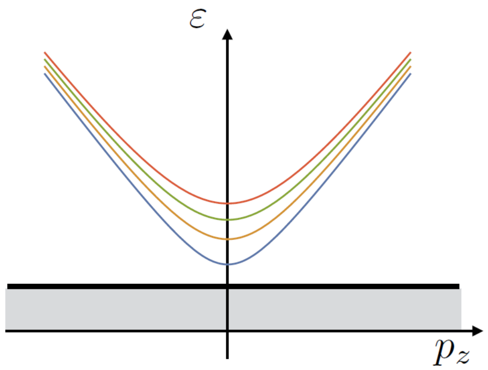
Analogy and difference between rotation and density

$$H_{\mathrm{rot}} = H - \omega J_z \qquad\qquad H_{\mu} = H - \mu N$$

- This indicates ωJ_z plays similar role as chemical potential term μN. However … …
- Uniformly rotating system must be finite!



Causality: $v_{\mathrm{edge}} = \omega R < 1$



- Excitation gap due to finite size: J_z/R
- Effective chemical potential: $\omega J_z < J_z/R$
- Pure uniform rotation does not excite any modes

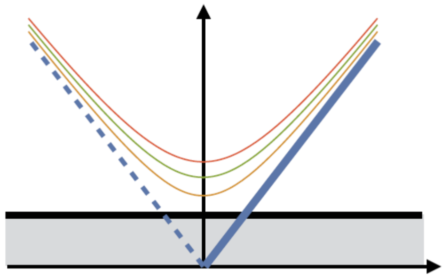
(Chen-Fukushima-XGH-Mameda 2015, Ebihara-Fukushima-Mameda 2017)

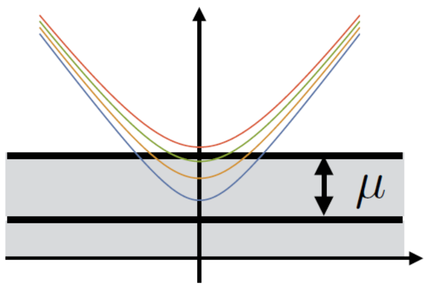
Figures drawn by Mameda

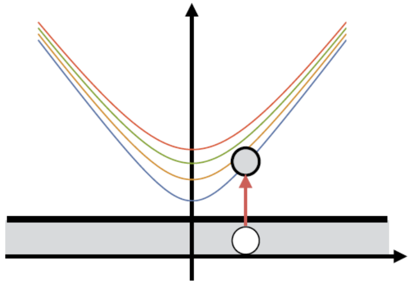
Rotation induced phase transitions

To see uniform rotation effect, we need $\boldsymbol{T}$, $\boldsymbol{\mu}$, $\boldsymbol{B}$, ...

Figures drawn by Mameda





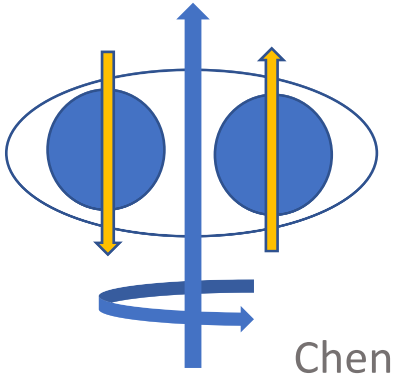


B: Chen etal 2015, Liu-Zahed 2017,Chen-Mameda-XGH 2019, Cao-He 2019, Tabatabaee etal 2021…

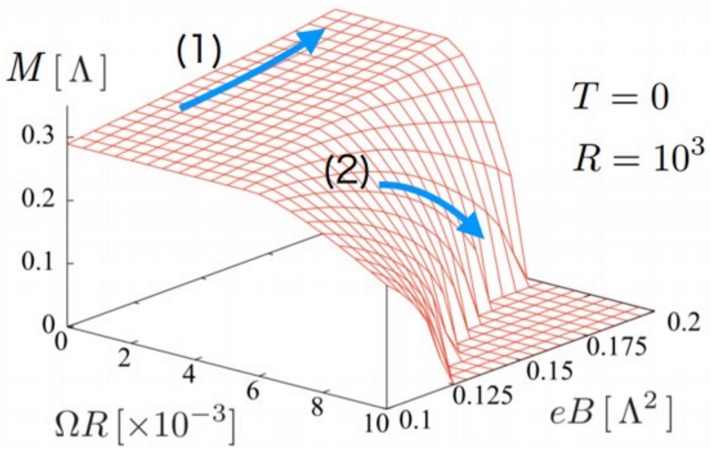
μ: XGH-Nishimura-Yamamoto 2017, Zhang-Hou-Liao 2018, Huang etal 2018, Nishimura etal 2020,2021…

T: Jiang-Liao 2016, Chernodub-Gongyo 2017, Wang etal 2019, Luo etal 2020, Jiang 2021, …

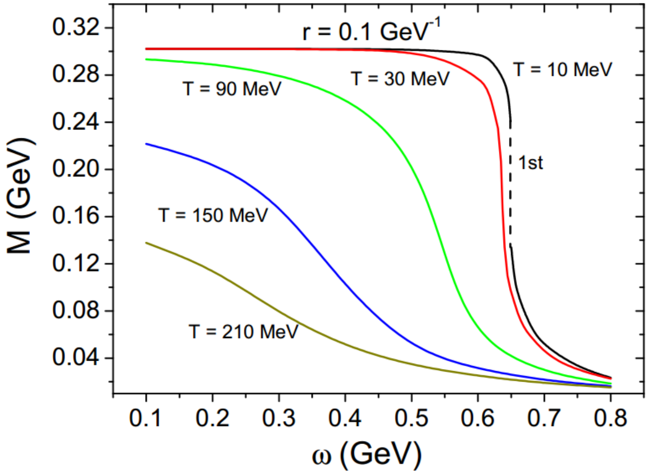
Rotation disfavors spin-0 condensates, e.g., chiral condensate



Chen etal 2015



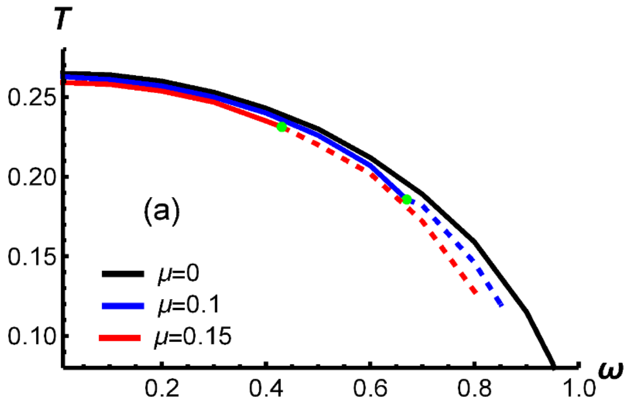
Jiang-Liao 2016



Does rotation influence confinement?

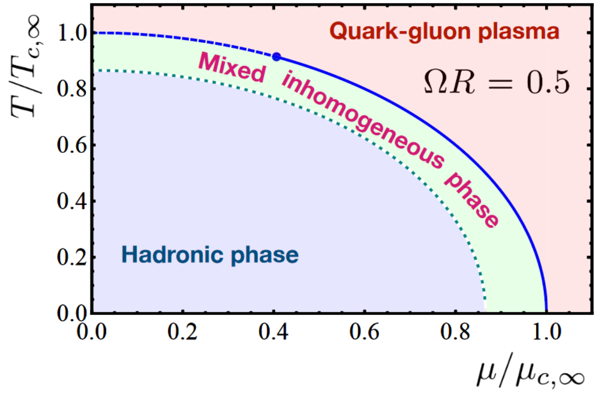
Does strong rotation catalyze deconfinement?

Yes, in model studies



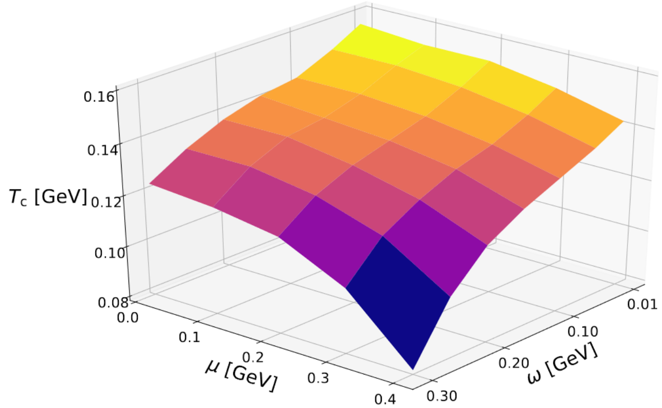
Holography

(Chen-Zhang-Li-Hou-Huang 2020)



Compact 2D QED

(Chernodub 2020)

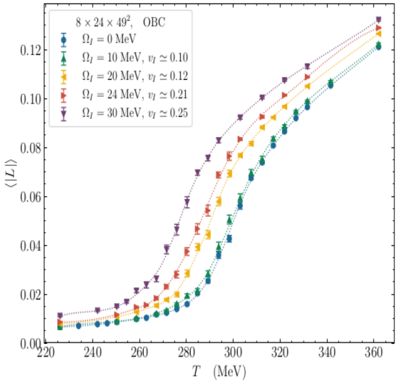


Hadron resonance gas model

(Fujimoto-Fukushima-Hidaka 2021)

No, in lattice study for pure gluons

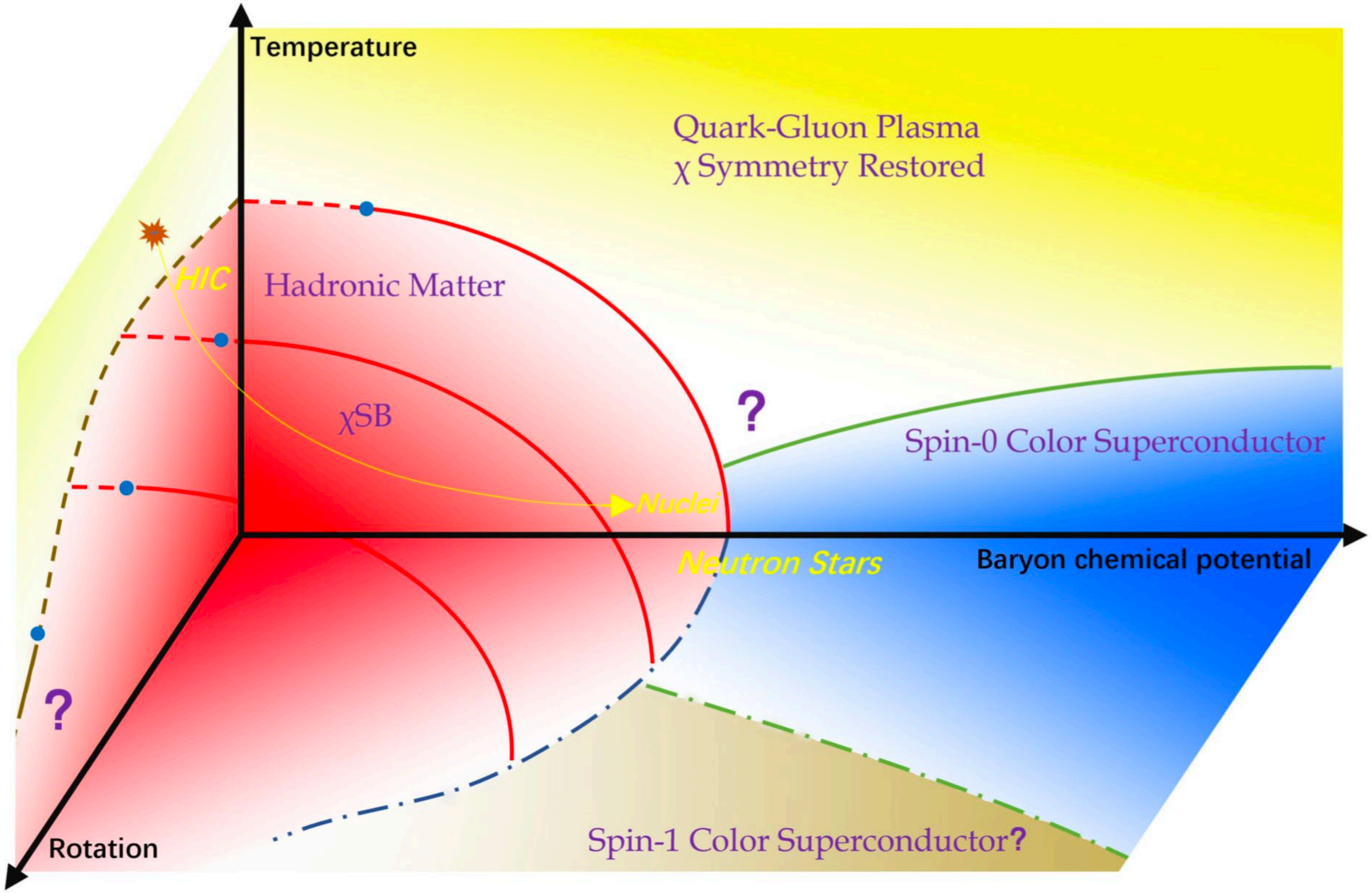
Note that lattice simulation works for imaginary rotation.



(Braguta etal 2021)

Rotation induced phase transitions

A possible phase diagram of QCD matter



Chen-XGH-Liao 2021