Solitons and non-equilibrium physics in holography

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Discovery of solitons

- In 1843, John S. Russell observed a traveling soliton in the Union Canal in Scotland.
- Soliton solutions of the Korteweg–de Vries (KdV) equation

$$\partial_t \phi + \partial_x^3 \phi - 6\, \phi\, \partial_x \phi = 0$$

$$\phi(x,t) = -\frac{1}{2}\, c\, \mathrm{sech}^2 \left[\frac{\sqrt{c}}{2}(x - c\, t - a) \right]$$

Solitons in Bose-Einstein condensate (BEC)

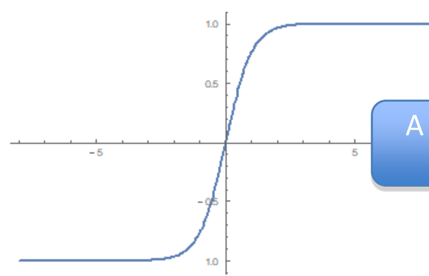
- Weakly coupled BEC described by the Gross–Pitaevskii (GP) equation (*m* = ½, without potential)

Chemical potential

$$i\partial_t \psi = -\nabla^2 \psi - (\mu - g|\psi|^2)\psi$$

- Soliton solutions of the GP equation for *g* > 0 (repulsive): black and gray solitons

$$\psi(t,x) = \sqrt{\frac{\mu}{g}}\left[i\beta + \sqrt{1-\beta^2}\tanh\left(\sqrt{\frac{\mu}{2}(1-\beta^2)}(x - \sqrt{2\mu}\beta t)\right)\right]$$



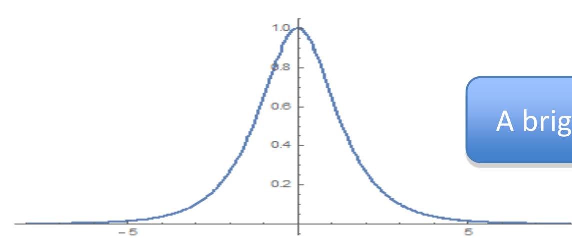
A black soliton (β=0)

Solitons in Bose-Einstein condensate (BEC)

- Soliton solutions of the GP equation for $g < 0$ (attractive): bright solitons

$$\psi(t,x) = \sqrt{\frac{2\mu}{g}}\operatorname{sech}(\sqrt{-\mu}x)$$

- Bright solitons can also travel (as gray solitons for the $g > 0$ case).



A bright soliton

Solitons in $\lambda\phi^4$ theory

- $\lambda\phi^4$ theory (real scalar field with double-well potential)

$$\partial_t^2\phi - \partial_x^2\phi = \frac{\lambda}{6}(a^2 - \phi^2)\phi$$

- Soliton solutions:

$$\psi(t,x) = a\tanh\left(\sqrt{\frac{\lambda}{12}}a\gamma(x-\beta t)\right), \qquad \gamma = (1-\beta^2)^{-1/2}$$

Why to study solitons?

- They are simple yet nontrivial.
- They play an important role in non-equilibrium physics (e.g. in superfluids).
- They have very interesting dynamics.

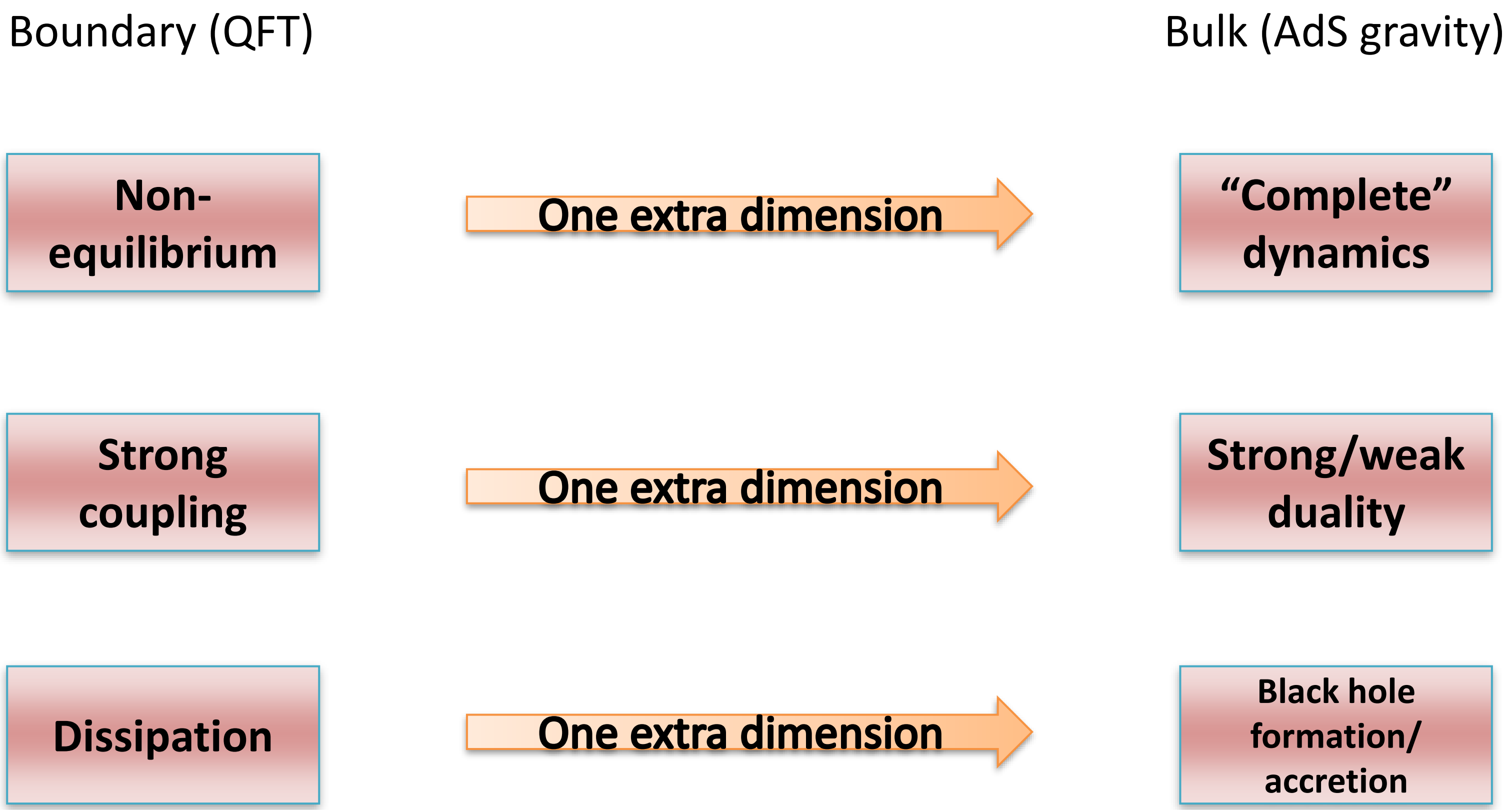
Why applied AdS/CFT (holography)?

Three key problems to be solved for (quantum) many-body systems:

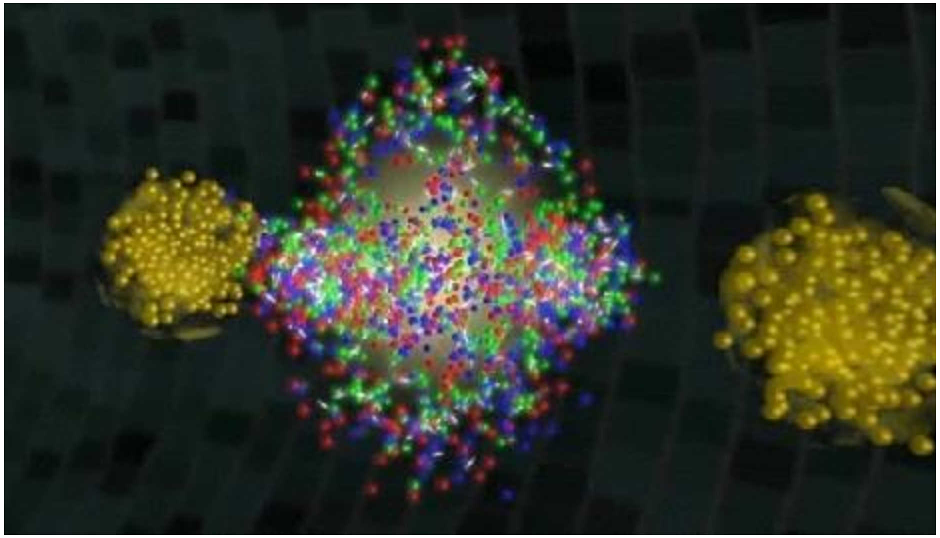
- No (local) equilibirum (hydrodynamics no longer valid, e.g. near the soliton center)
- Strong coupling (perturbative theories no longer valid)
- Dissipation (time irreversible)

Holography deals with these problems by constructing effective field theories with one extra dimension!

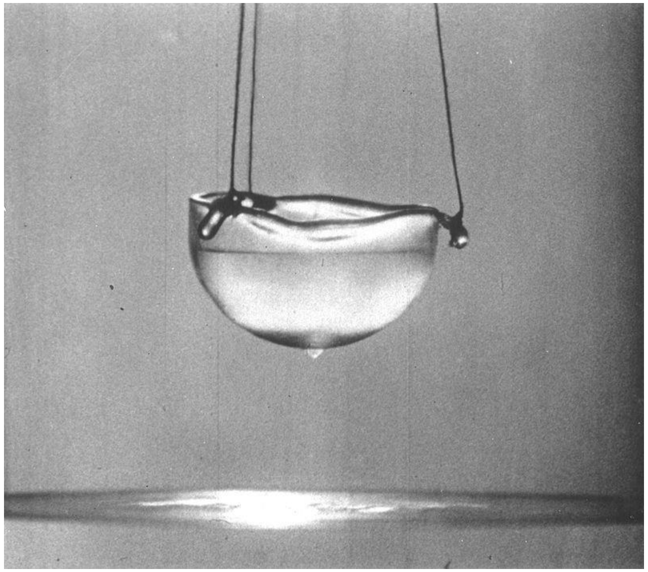
How does holography work?



Quantum many-body systems far from equilibrium



Quark-gluon plasma produced in LHC



He_4 superfluid

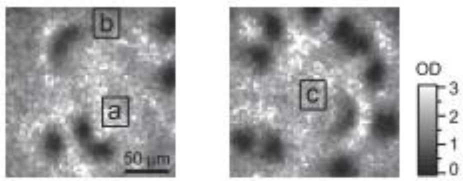
The simplest holographic model: black hole formation from collision of gravitational waves in the bulk

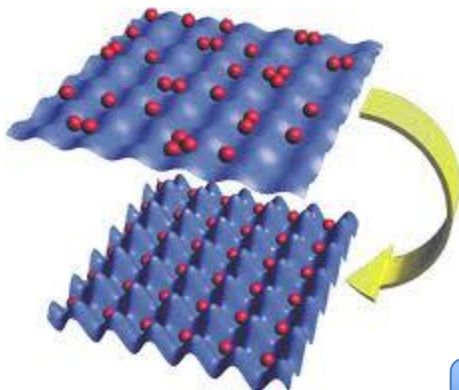
U(1) symmetry breaking + gravity in the bulk (beyond Landau's two-fluid model)

Significant progress made in experiments

Opportunities to test the holographic approach!

arXiv: 1212.0281, 1605.01193, 1904.05497, 2003.09423, 2011.12968, ……





arXiv:2109.09080

Superfluid vortices in BEC

Optical lattices

(Controllable manipulation)

Landau instability of the super flow

- Laudau's argument (by Galilean transformations): When the velocity of the super flow is greater than a critical value determined by the dispersion relation of elementary excitations, the superfluid becomes unstable because excitations can have negative energy.
- But what is the final state of the superfluid if the Landau instability occurs? Breakdown of the superfluidity?

Landau instability from holography

- In holography, elementary excitations correspond to linear perturbations on top of the bulk spacetime (and matter fields).
- A boundary system at finite temperature is dual to bulk AdS gravity with a black hole.
- Linear perturbations of the bulk black hole systems are described by the quasi-normal modes (QNM):

$$\delta\Psi(t,x,z) = e^{-i\omega t + ikx} p(z)$$

Effectively, only considering the dominant mode for stability analysis

Stable if all the quasi normal frequencies (for any *k*) have negative imaginary parts

The holographic model of a superfluid

[Hartnoll, Herzog and Horowitz, arXiv:0803.3295]

•The action:

$D = \partial - iA, \qquad q = 1$

$$S = \int_{\mathcal{M}} d^4x \sqrt{-g} \frac{1}{q^2} \left(-\frac{1}{4} F_{ab} F^{ab} - |D\Psi|^2 - m^2 |\Psi|^2 \right)$$

The most convenient choice: $m^2=-2/L^2$

•The background metric:

$$ds^2 = \frac{L^2}{z^2}[-f(z)dt^2 - 2dtdz + dx^2 + dy^2], \quad f(z) = 1 - \frac{z^3}{z_h^3}$$

•The heat bath temperature:

$$T = \frac{3}{4\pi z_h}$$

- Asymptotic behavior at the AdS boundary z=0

$$A_\nu = a_\nu + b_\nu z + o(z)$$

$$\Psi = \phi z + \psi z^2 + o(z^2)$$

For m^2=-2/L^2

- AdS/CFT dictionary

φ, ψ: source (turned off) & condensate (of superfluidity)

- Particle number density

J^ν: the only conserved current on the boundary

$$\rho \equiv \langle J^t \rangle = -\partial_z A_t|_{z\to 0} = -b_t$$

- Chemical potential (in equilibrium)

Radial gauge: A_z=0 all over the bulk

$$\mu = A_t|_{z\to 0} = a_t, \quad A_t(z_h) = 0$$

- Scaling symmetry

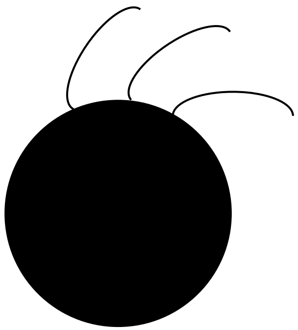
$$x^\mu \to \lambda x^\mu, \quad T \to \lambda^{-1} T, \quad \mu \to \lambda^{-1} \mu$$

Observable: μ/T
(scaling invariant)

- The normal fluid/superfluid phase transition
  - Lowering the temperature across T_c (or increasing μ)

Bald black hole (vanishing scalar field)	→	Hairy black hole (non-vanishing scalar)
Normal fluid (no condensate)	→	Superfluid (with condensate)
U(1) symmetry	→	SSB of U(1) (2^{nd} order PT)

Violation of the no-hair theorem in AdS)



- The super flow (along the *x* direction)

$\psi \sim e^{ikx}$ ⟷ Gauge equivalent ⟷ $a_x = -k$

- Velocity of the flow

$$j_\mu = \mathrm{Im}(\psi^* \nabla_\mu \psi) \Rightarrow v^i = \frac{j^i}{j^t} = -\frac{a_i - \partial_i \theta}{\mu - \partial_t \theta} \quad (\psi = |\psi| e^{i\theta})$$

Gauge invariant!

Landau instability from holography

- Linear analysis (QNM) of the Landau instability in holographic superfluid models
  [I. Amado et al, JHEP 1402 (2014) 063]

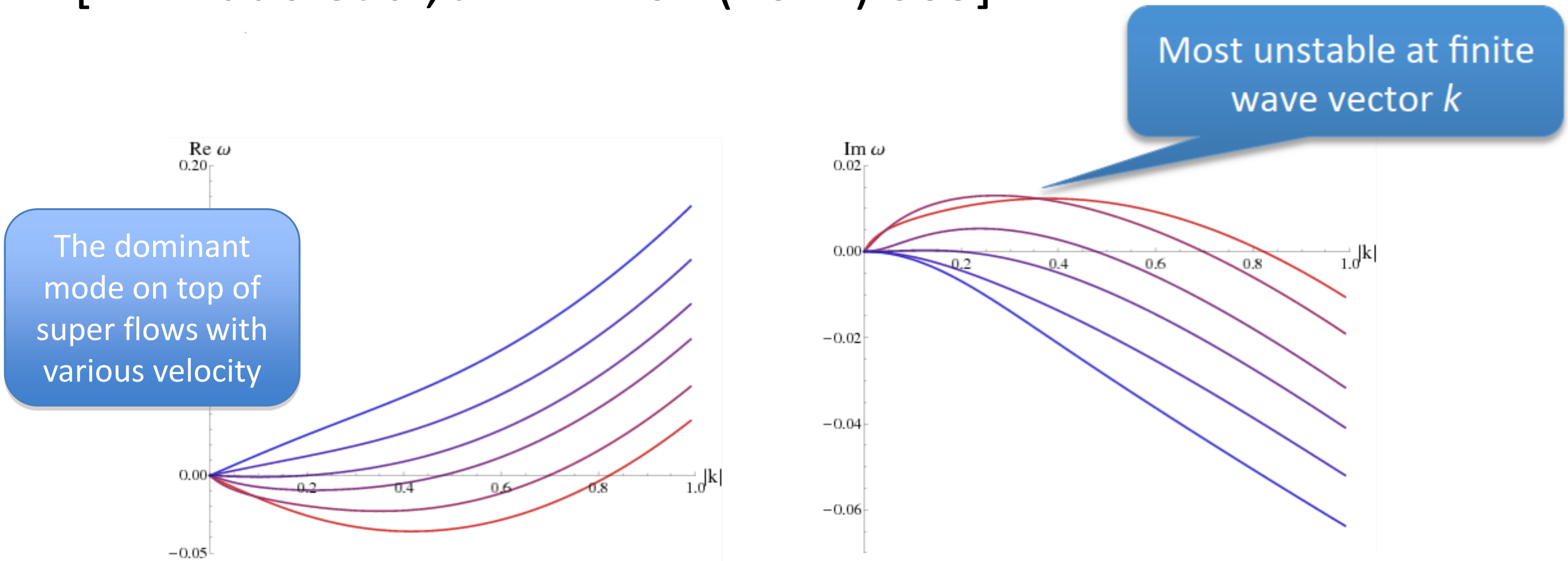


Figure: The left is for the real part of ω and the right for the imaginary part.

- Nonlinear dynamic evolution of the unstable super flow
  - A too fast super flow is unstable, then what will happen? What will be left eventually, a striped phase (conjectured by Amado *et al*) or something else?
    [S. Lan, H. Liu, YT & H. Zhang, arXiv:2010.06232]
  - 1D: soliton generation and 1D transient turbulence
    ➔ homogeneous super flow with a velocity lower than the critical value
  - 2D: soliton generation at the flow direction
    ➔ snake instability of the solitons (leading to vortex nucleation) and 2D transient turbulence
    ➔ homogeneous super flow with a velocity lower than the critical value

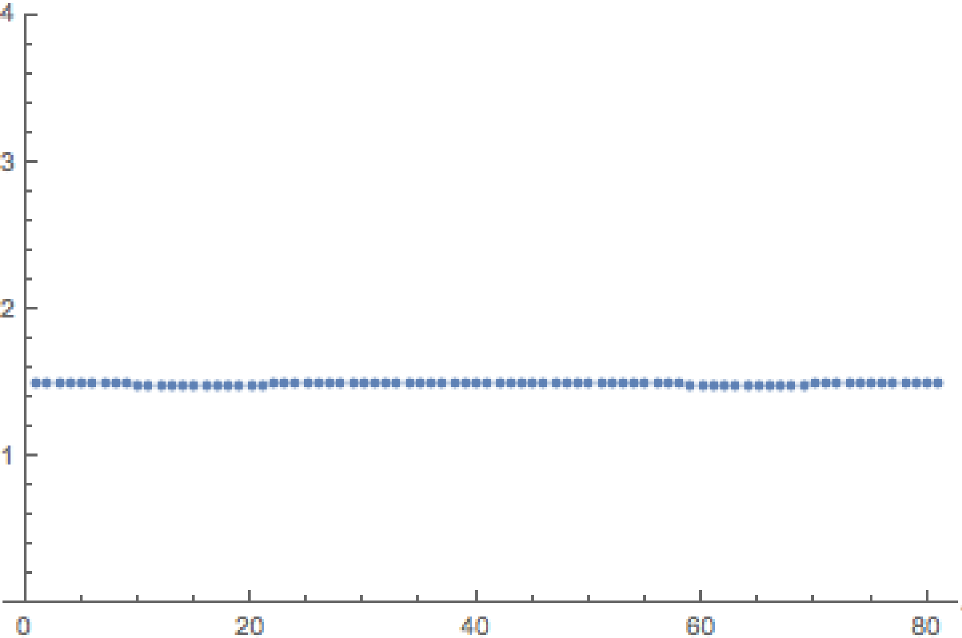
Time evolution of the Landau instability in holography

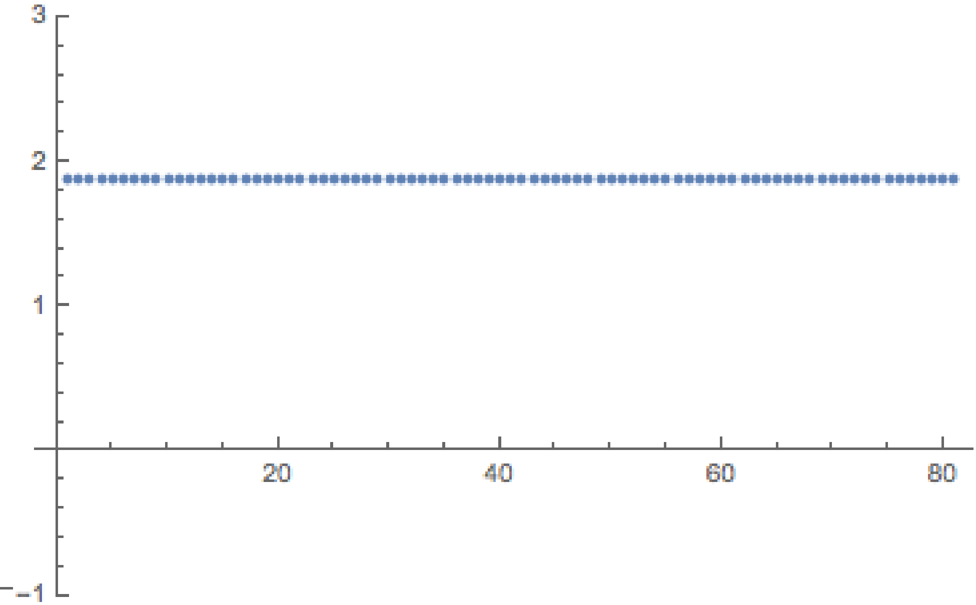
The equations of motion (for numerics):

$$
\begin{aligned}
\partial_z^2 A_t &= \partial_z \partial \cdot \mathbf{A} + i(\bar{\Phi}\partial_z \Phi - \Phi \partial_z \bar{\Phi}), \quad \Phi \equiv \Psi/z \\
\partial_t \partial_z \Phi &= i A_t \partial_z \Phi + \frac{1}{2}[i \partial_z A_t \Phi + f \partial_z^2 \Phi + f' \partial_z \Phi \\
&\quad + (\partial - i\mathbf{A})^2 \Phi - z\Phi] \\
\partial_t \partial_z \mathbf{A} &= \frac{1}{2}[\partial_z(\partial A_t + f \partial_z \mathbf{A}) + (\partial^2 \mathbf{A} - \partial \partial \cdot \mathbf{A}) \\
&\quad - i(\bar{\Phi}\partial \Phi - \Phi \partial \bar{\Phi})] - \mathbf{A}\bar{\Phi}\Phi \\
\partial_t \partial_z A_t &= \partial^2 A_t + f \partial_z \partial \cdot \mathbf{A} - \partial_t \partial \cdot \mathbf{A} - 2A_t \bar{\Phi}\Phi \\
&\quad + i f(\bar{\Phi}\partial_z \Phi - \Phi \partial_z \bar{\Phi}) - i(\bar{\Phi}\partial_t \Phi - \Phi \partial_t \bar{\Phi})
\end{aligned}
$$

$$\mathbf{A} \equiv (A_x, A_y), \qquad \partial \equiv (\partial_x, \partial_y)$$

- Landau instability of the super flow
  - Movies to illustrate the Landau instability in 1D



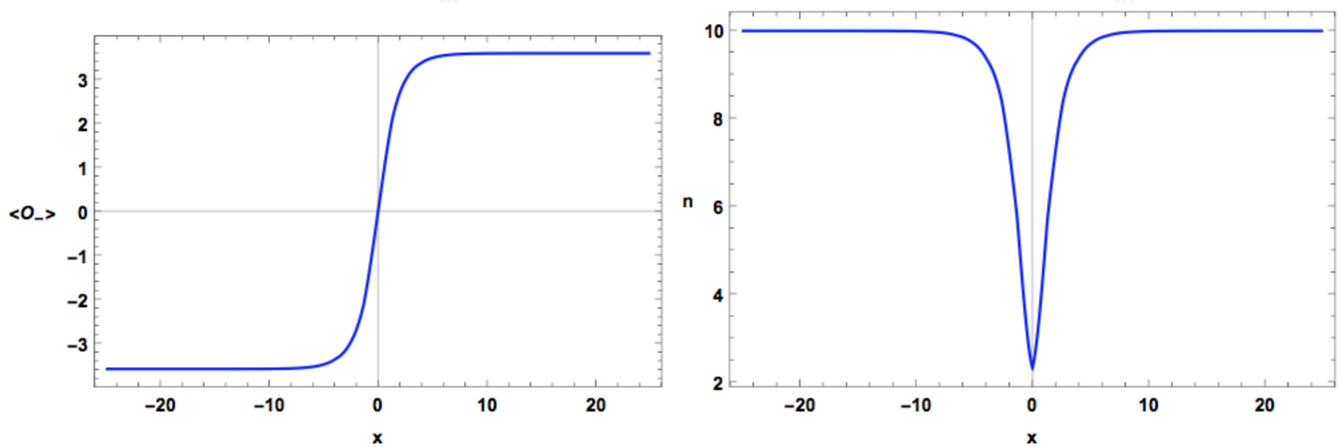


Evolution of the absolute value of the wave function (and generation of solitons)

Evolution of the super flow velocity

Black solitons in holographic superfluids at finite temperature

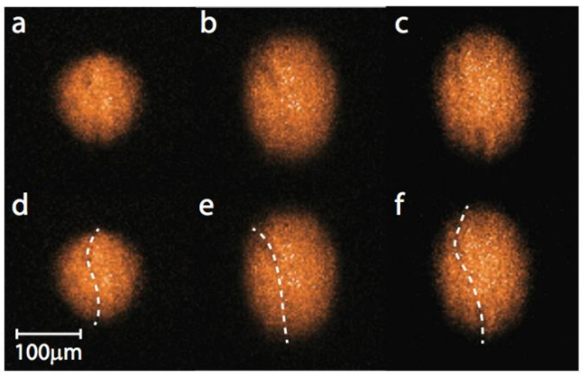
- Construction of holographic black solitons
  [Keranen, Keski-Vakkuri, Nowling and Yogendran, 2009; ...]



Profile of a holographic black soliton. Left: the superfluid condensate (wave function); Right: the particle number density.

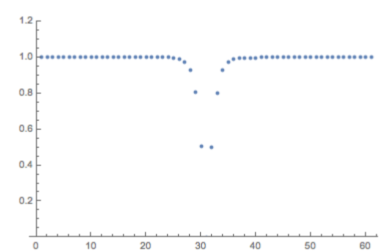
Soliton instability and dynamic phase transition

- Snake instability: Black solitons are unstable in D>1 (has at least one transverse direction) under bending.



Observation of the snake instability in a fermionic superfluid [T. Yefsah *et al,* Nature 499 (2013) 426].

- Self-acceleration: Black solitons are unstable even without bending (or without transverse direction), in particular at finite temperature.



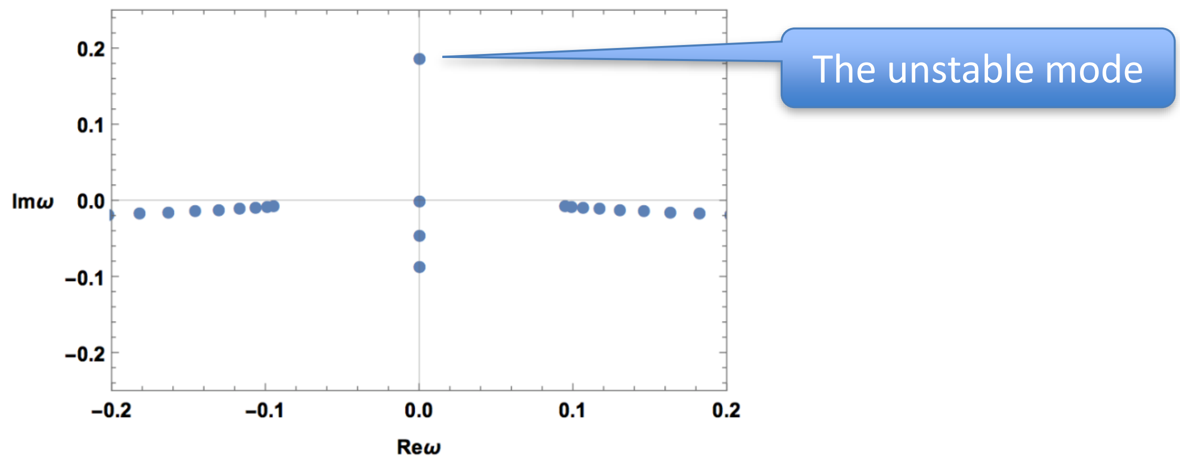
Soliton instability and dynamic phase transition

Linear instability of black solitons from QNM of the inhomogeneous black hole configuration by holography

[M. Guo, E. Keski-Vakkuri, H. Liu, YT & H. Zhang, PRL 124 (2020) 031601]

Transverse wave vector

$$e^{-i\omega t+iqy}$$



A typical plot of the quasi-normal modes of the holographic black soliton configuration.

Soliton instability and dynamic phase transition

QNM on top of inhomogeneous backgrounds

- Technically complicated, so better to use the evolution-like approach (under the Eddington-Finkelstein coordinates)
  arXiv:1511.07179 (homogeneous)
  arXiv:1904.05497 (inhomogeneous)
- After discretization along the inhomogeneous directions, becomes a very large (generalized) eigenvalue problem

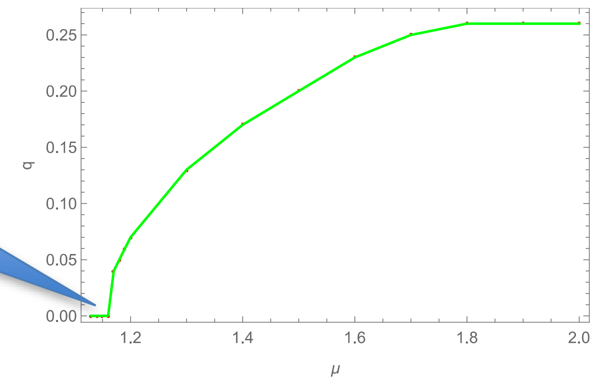
$$\delta\Psi = p(z,x)e^{-i\omega t+iqy} + \bar{p}(z,x)e^{i\omega^* t-iqy} \Rightarrow L_q(\omega)\begin{pmatrix} p \\ \bar{p}^* \end{pmatrix} = 0$$

z and *x* discretized

Soliton instability and dynamic phase transition

- Prediction of a novel dynamical phase transition (DPT)

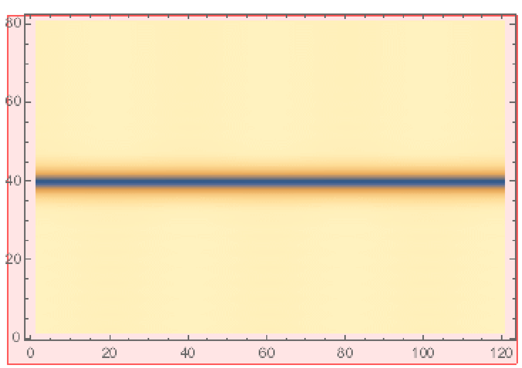
DPT at temperature $T_d \approx 0.9 T_c$ (alternative)

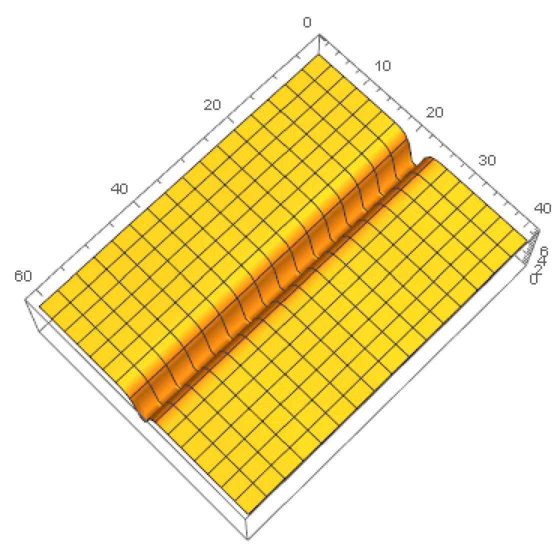


Plot of the transverse wave vector q of the most unstable mode with respect to the chemical potential μ (or temperature T).

$q \neq 0$: snake instability
$q = 0$: acceleration instability

- Full nonlinear evolution





Soliton instability and dynamic phase transition

Characterization of the DPT [M. Guo, E. Keski-Vakkuri, H. Liu, YT & H. Zhang, PRL 2020]

- Critical behavior of the DPT as $T \rightarrow T_d$ from below:

$$q_{\max} \propto (T_d - T)^{\gamma}$$

- The wave number q_{max} determines the vortex number density n_v as the result of snake instability in the real time evolution:

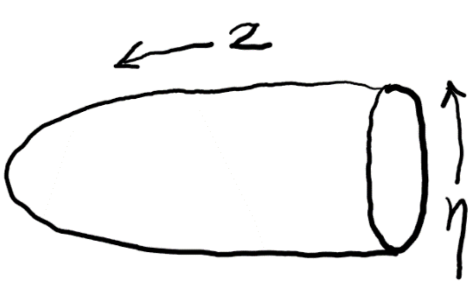
$$n_v \propto q_{\max}$$

  n_v can be measured by experiments.
- In various cases, γ is well fit by ½ and a Ginzburg-Landau-like argument is proposed to describe the DPT and obtain γ=½ theoretically.

Gray solitons in holographic superfluids at zero temperature

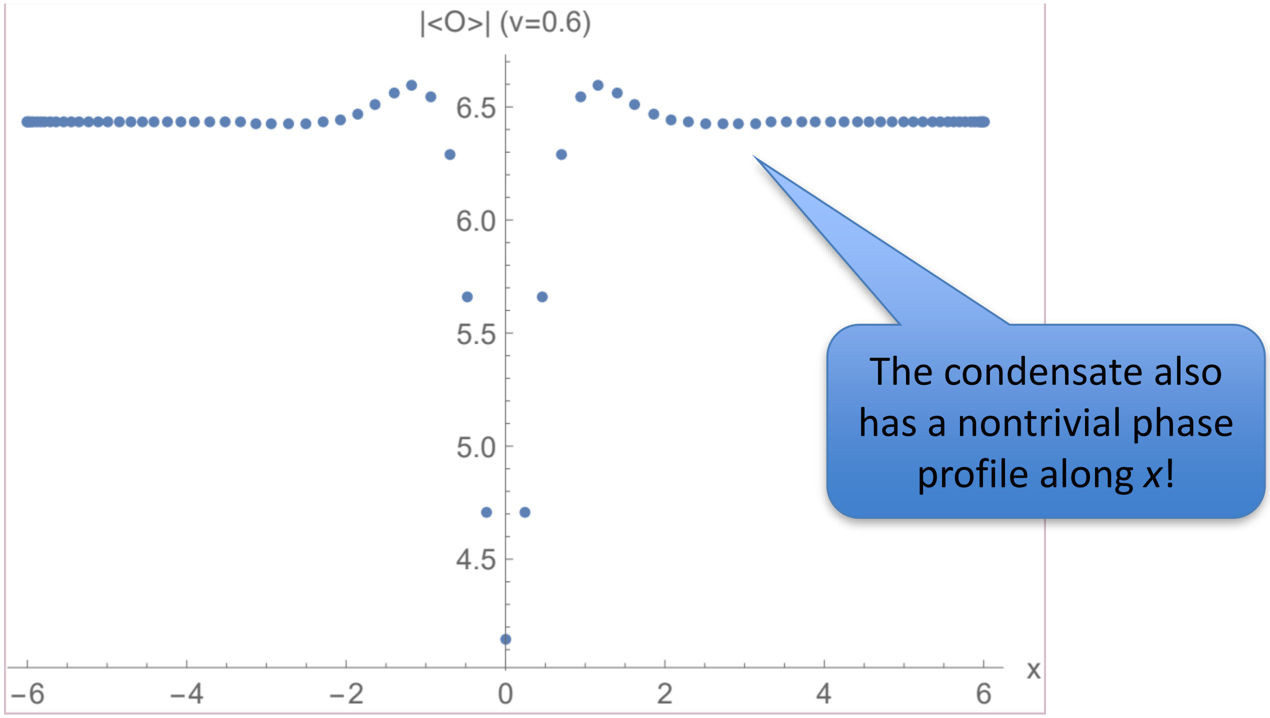
- No (holographic) gray solitons at finite temperature because of dissipation
  [A. Adams, P.M. Chesler and H. Liu, Science 2012]
- Construction of holographic gray solitons at zero temperature (AdS soliton background)
  [YT *et al*, JHEP 1905 (2019) 167]

$$ds^2 = \frac{L^2}{z^2}[-dt^2 + \frac{dz^2}{f(z)} + dx^2 + f(z)d\eta^2], \quad f(z) = 1 - \frac{z^3}{z_0^3}.$$



Periodic to avoid the conical singularity at the tip $z = z_0$

Gray solitons in holographic superfluids at zero temperature



Profile of a holographic gray soliton traveling at the velocity v=0.6.

Bright solitons in holographic superfluids

Conclusion

- Solitons play an important role in superfluid instability and dynamics, including quantum turbulence.
- A (superfluid) soliton is itself unstable, which leads to interesting non-equilibrium physics, like dynamic phase transition.

Open questions

- How to characterize 1D quantum turbulence, where solitons are randomly generated?
- Bright solitons in holography?
- Gray soliton dynamics?

Thanks for your attention!

Local equilibrium and non-equilibrium

The validity of hydrodynamics depends on local equilibrium, which means that the system is

- evolving in time slowly enough
- varying in space slowly enough

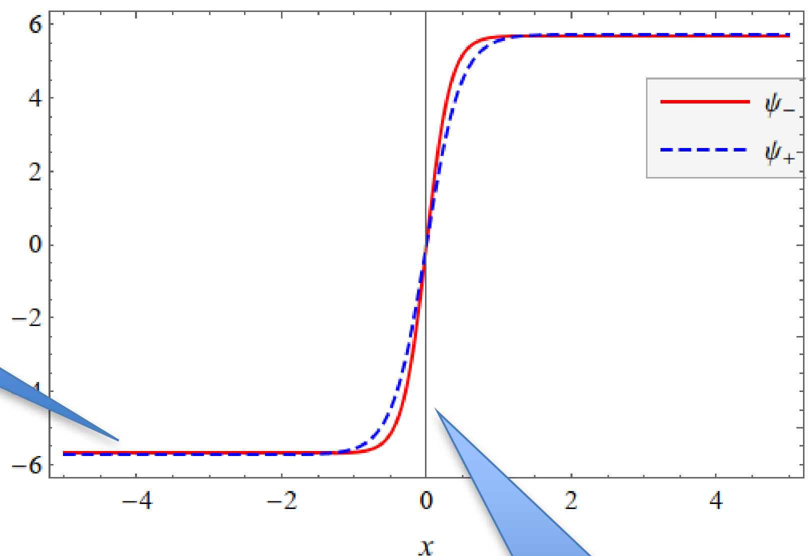
with respect to all the characteristic (microscopic) time and space scales. In other words, hydrodynamics is the low energy, long wavelength effective theory of quantum many body systems.

For a CFT without any conserved quantities other than energy, the only characteristic scale is the temperature T, so the validity of hydrodynamics is determined by T.

Local equilibrium and non-equilibrium

So, even a stable, static system can be in non-equilibrium, if there are local structures, like solitons, vortices, domain walls, etc.

Restoration of thermodynamics



Non-equilibrium region

Holographic superfluid solitons (with backreaction). Blue: standard quantization; Red: alternative quantization. [Z. Xu et al, arXiv:1910.09253]