Dissipation in Holographic Superfluids

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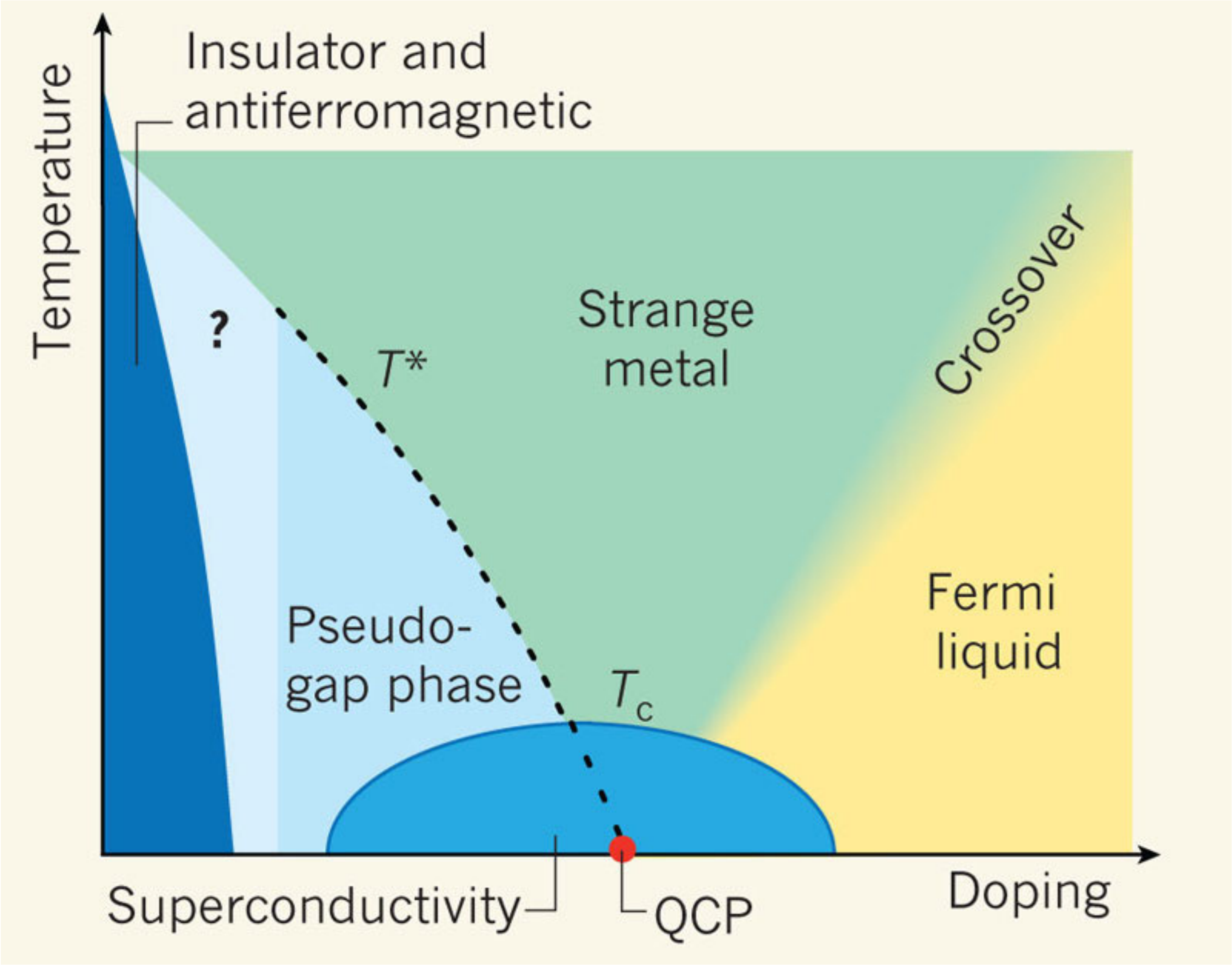
Outline

- Introduction / Generalities
  - Hydrodynamics of Conserved Currents in (Un)-Broken Phase
  - Pseudo-Spontaneous Breaking
- Holography
  - Superfluid at $\mu = 0$
  - The Symplectic Current as a Tool
  - Hydrodynamics

Motivation



- At strong coupling often no quasiparticles e.g. the cuprates
- Conserved charges and light Goldstone modes can dominate at long wavelengths
- Derivation of EFT with few/no symmetries
- Broken phase ground states

(Conformal) Field Theory Setup

- Relativistic field theory with global $U(1)$ at finite T and zero charge
- Charged operator $\mathcal{O}_\psi$ transforms as $\mathcal{O}_\psi \to e^{-iq\alpha}\,\mathcal{O}_\psi$
- Phase transition with $\langle \mathcal{O}_\psi \rangle \neq 0$ at $T < T_c$
- Couple to perturbative external gauge field A_μ and scalar source λ_ψ

$$\delta S = \int d^n x \left(J^\mu\,\delta A_\mu + \mathcal{O}_\psi^*\,\delta\lambda + \mathcal{O}_\psi\,\delta\lambda^* \right)$$

Toy Model

- Consider simple model with $U(1)$ global symmetry

$$S = \int d^n x \left(-\frac{1}{2} \left| \partial \psi \right|^2 - \frac{m^2}{2} |\psi|^2 - \frac{\lambda^2}{4} |\psi|^4 \right)$$

- Vacua parametrised by $\psi_0 = \rho_0 \, e^{i\vartheta_0}$ with $0 \leq \theta_0 < 2\pi$
- Change variable $\psi = \rho \, e^{i\vartheta}$ to discover massless massless mode from $\vartheta \to \vartheta + c$

$$S = \int d^n x \left(-\frac{1}{2} \left(\partial \rho \right)^2 - \frac{1}{2} \rho^2 \, (\partial \vartheta)^2 - \frac{m^2}{2} \rho^2 - \frac{\lambda^2}{4} \rho^4 \right)$$

- Dispersion relation $\omega = \pm \, k$

Toy Model

- Explicitly deform to break the symmetry

$$S = \int d^n x \left(-\frac{1}{2} \left| \partial \psi \right|^2 - \frac{m^2}{2} |\psi|^2 - \frac{\lambda^2}{4} |\psi|^4 - \delta\lambda\, \psi^* - \delta\lambda^* \psi \right)$$

- Change variable $\psi = \rho\, e^{i\vartheta}$ and $\delta\lambda = \delta\rho_s\, e^{i\vartheta_s}$

$$S = \int d^n x \left(-\frac{1}{2} \left(\partial \rho\right)^2 - \frac{1}{2}\rho^2 \left(\partial\vartheta\right)^2 - \frac{m^2}{2}\rho^2 - \frac{\lambda^2}{4}\rho^4 - 2\,\rho\,\delta\rho_s \cos(\vartheta - \vartheta_s) \right)$$

- Two vacua with $\rho_0^2 = -\, m^2/\lambda^2 + \mathcal{O}(\delta\rho_s)$ and $\vartheta_0 = \vartheta_s$ or $\vartheta_0 = \vartheta_s + \pi$

Toy Model

- Fluctuations of angle acquires a mass
- Expanding around $\vartheta_0 = \vartheta_s$

$$\omega = \pm\sqrt{k^2 + 2\,\delta\rho_s/\rho_0}$$

- Expanding around $\vartheta_0 = \vartheta_s + \pi$

$$\omega = \pm\sqrt{k^2 - 2\,\delta\rho_s/\rho_0}$$

- Order parameter aligns with deformation parameter in the complex plane
- Expect dissipation to add a damping rate at $k = 0$. Also Delacretaz, Gouteraux, Ziogas

Toy Model

- Couple to external gauge field A_μ

$$S = \int d^n x \left(-\frac{1}{2} \left| D\psi \right|^2 - \frac{m^2}{2} |\psi|^2 - \frac{\lambda^2}{4} |\psi|^4 \right)$$

$$D_\mu \psi = \partial_\mu \psi + i\, q\, A_\mu\, \psi$$

- With the conserved current

$$J^\mu \approx \psi^* D^\mu \psi - \psi\, D^\mu \psi^* = \rho^2 \left(\partial^\mu \vartheta + q\, A^\mu \right)$$

- Non-zero current susceptibility

Finite Temperature

- Generating function $W[A_\mu, \lambda, \lambda^*]$ depends on external gauge field A_μ and complex source λ
- Functional differentiation gives

$$\langle J^\mu \rangle = i\frac{\delta W}{\delta A_\mu}, \qquad \langle \mathcal{O}_\psi \rangle = i\frac{\delta W}{\delta \lambda^*}$$

- Invariance under gauge transformations $\delta A_\mu \to \partial_\mu \delta\Lambda, \quad \delta\lambda = -\, i\, q\, \lambda\, \delta\Lambda$ yields the current (non)-conservation Ward identity

$$\nabla_\alpha \langle J^\alpha \rangle = iq \left(\langle \mathcal{O}_\psi \rangle \lambda^* - \langle \mathcal{O}_\psi^* \rangle \lambda \right)$$

Hydro

Parametrise massless collective dof:

- Normal fluid parametrised by local temperature T and fluid velocity v^μ
- Superfluid parametrised by phase ϑ of vev $\langle \mathcal{O}_\psi \rangle = \langle \mathcal{O}_\psi \rangle_b \, e^{i\vartheta}$
- Express $T_{\mu\nu}$ and J_μ as functions of the fluctuations $T, v^\mu, v_s^\mu = \partial^\mu \vartheta$
- Solve the closed system

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu + \lambda\, \mathcal{O}_\psi^* + \lambda^* \, \mathcal{O}_\psi$$

$$\nabla_\alpha \langle J^\alpha \rangle = iq \left(\langle \mathcal{O}_\psi \rangle \lambda^* - \langle \mathcal{O}_\psi^* \rangle \lambda \right)$$

Landau

Tisza

Israel

Khalatnikov, Lebedev

Bhattacharya, Bhattacharyya, Minawalla

Linearised Hydro

- At zero charge density the normal fluid and superfluid dofs decouple
- Only need to consider phase fluctuations of order parameter at long wavelengths

$$\delta\langle\mathcal{O}_\psi\rangle(x^\mu) = i\,\langle\mathcal{O}_\psi\rangle_b\,\delta c(x^\mu)$$

- Express $\delta\langle J^\mu\rangle$ as derivative expansion in $\delta c(x^\mu)$ with $\partial \sim O(\varepsilon)$

Linearised Hydro

- Due to $U(1)$ symmetry, no current when $\partial_\mu \delta c = 0$

- At non-dissipative order, gauge invariance of sources yields

$$\delta J_t = -\chi_{QQ} \left(\partial_t \delta c + \delta A_t\right) + O(\partial^2) \qquad \delta J_x = -\chi_{JJ} \left(\partial_x \delta c + \delta A_x\right) + O(\partial^2)$$

Susceptibilities

- The current divergence gives $\partial_\mu \delta J^\mu \sim O(\epsilon^2)$

➡ Include scalar sources $\delta\lambda$ of $O(\epsilon^2)$

- In our frame $\mu = \partial_t \delta c$

Scalar Sources and Phase Pinning

- Assume thermal state with real vev $\langle \mathcal{O}_\psi \rangle$
- Include complex scalar source $\delta\lambda = \delta\rho_{(s)} + i\,\delta s_\psi$

Time independent pinning parameter

Time dependent phase source

- Expanding the current conservation Ward identity yields

$$\partial_\mu \delta\langle J^\mu \rangle = -2\,q\,|\langle \mathcal{O}_\psi \rangle_b|\,\delta\rho_{(s)}\,\delta c + 2\,q\,|\langle \mathcal{O}_\psi \rangle_b|\,\delta s_\psi$$

Second Sound

- Solve the source free system

$$\partial_\mu \delta \langle J^\mu \rangle = -2\, q\, |\langle \mathcal{O}_\psi \rangle_b|\, \delta\rho_{(s)}\, \delta c \Rightarrow$$

$$\chi_{QQ}\, \partial_t^2 \delta c - \chi_{JJ}\, \partial_x^2 \delta c = -2\, q\, |\langle \mathcal{O}_\psi \rangle_b|\, \delta\rho_{(s)}\, \delta c$$

- To obtain the non-dissipative dispersion relation

$$\omega^2(k) = c_s^2\, k^2 + m_s^2$$

- With

$$c_s^2 = \chi_{JJ}/\chi_{QQ} \qquad m_s^2 = \frac{2\, q\, |\langle \mathcal{O}_\psi \rangle_b|\, \delta\rho_{(s)}}{\chi_{QQ}}$$

Second Sound

$$c_s^2 = \chi_{JJ}/\chi_{QQ} \qquad\qquad m_s^2 = \frac{2\,q\,|\langle \mathcal{O}_\psi \rangle_b|\,\delta\rho_{(s)}}{\chi_{QQ}}$$

- Speed of second sound goes to zero close to the transition where $\chi_{JJ} \to 0$
- Instability when $\delta\rho_{(s)} < 0$, similar to toy model

Dissipative Corrections

- Dissipation captured by higher derivative terms in constitutive relations Khalatnikov, Lebedev

$$\delta\langle J_t\rangle = -\chi_{QQ}\left(\delta s_t + \partial_t \delta c\right) + \Xi\, \partial_t \left(\delta s_t + \partial_t \delta c\right)$$

$$\delta\langle J_x\rangle = -\chi_{JJ}\left(\delta s_x + \partial_x \delta c\right) - \sigma_d\, \partial_t \left(\delta s_x + \partial_x \delta c\right)$$

- In the normal phase with $\chi_{JJ} = 0$ and $\delta\mu \sim O(\epsilon)$

$$\delta\langle J_t\rangle = -\chi_{QQ}\,\delta\mu \qquad \delta\langle J_x\rangle = -\sigma_d\,\partial_x \delta\mu$$

- Where σ_d the incoherent conductivity

Retarded Green's Functions

- Solve Ward identity for phase fluctuation δc in terms of sources δA_μ and δs_ψ
- Express VEVs $\delta\langle J^\mu \rangle$ and $\delta\langle \mathcal{O}_\psi \rangle$ as linear combination of sources
- Read off retarded Green's functions from $\delta\langle \mathcal{O}_A \rangle = G_{AB}(\omega, k)\, \delta s_B$ for all $\{A, B\} = \{J^t, J^x, \mathcal{O}_Y\}$
- Determined in terms of susceptibilities χ_{QQ}, χ_{JJ} and coefficients Ξ and σ_d
- Kubo formulae

$$\sigma_d = \lim_{\omega\to 0}\lim_{k\to 0} \frac{\mathrm{Im}\, G_{J^x J^x}}{\omega}, \qquad \Xi = \lim_{k\to 0}\lim_{\omega\to 0} \frac{\mathrm{Im}\, G_{J^t J^t}}{\omega}$$

Retarded Green's Functions

- At $k = 0$ the transport current J^x and the "phase" operator $\mathcal{O}_Y$ decouple

- Persistent current exists after explicit breaking of $U(1)$

$$\sigma_{AC}(\omega) = \frac{G_{J^x J^x}(\omega, k = 0)}{i\,\omega} = \frac{i\,\chi_{JJ}}{\omega} + \sigma_d$$

- Need to include vortices for current to relax   Davidon, Delacretaz, Gouteraux, Hartnoll

- All $G_{AB}(\omega, k)$ have a pair of simple poles related to the pseudo-Goldstone mode

Dispersion Relations

- All $G_{AB}(\omega, k)$ have a pair of simple poles related to the pseudo-Goldstone mode
- Dispersion relation

$$\omega = \pm\sqrt{\frac{w + k^2 \chi_{JJ}}{\chi_{QQ}}} - \frac{i}{2\chi_{QQ}^2}\left(w\,\Xi + k^2\left(\Xi\chi_{JJ} + \sigma_d\,\chi_{QQ}\right)\right)$$

$$w = 2q^2\,|\langle \mathcal{O}_\psi \rangle_b|\,\delta\rho_{(s)}$$

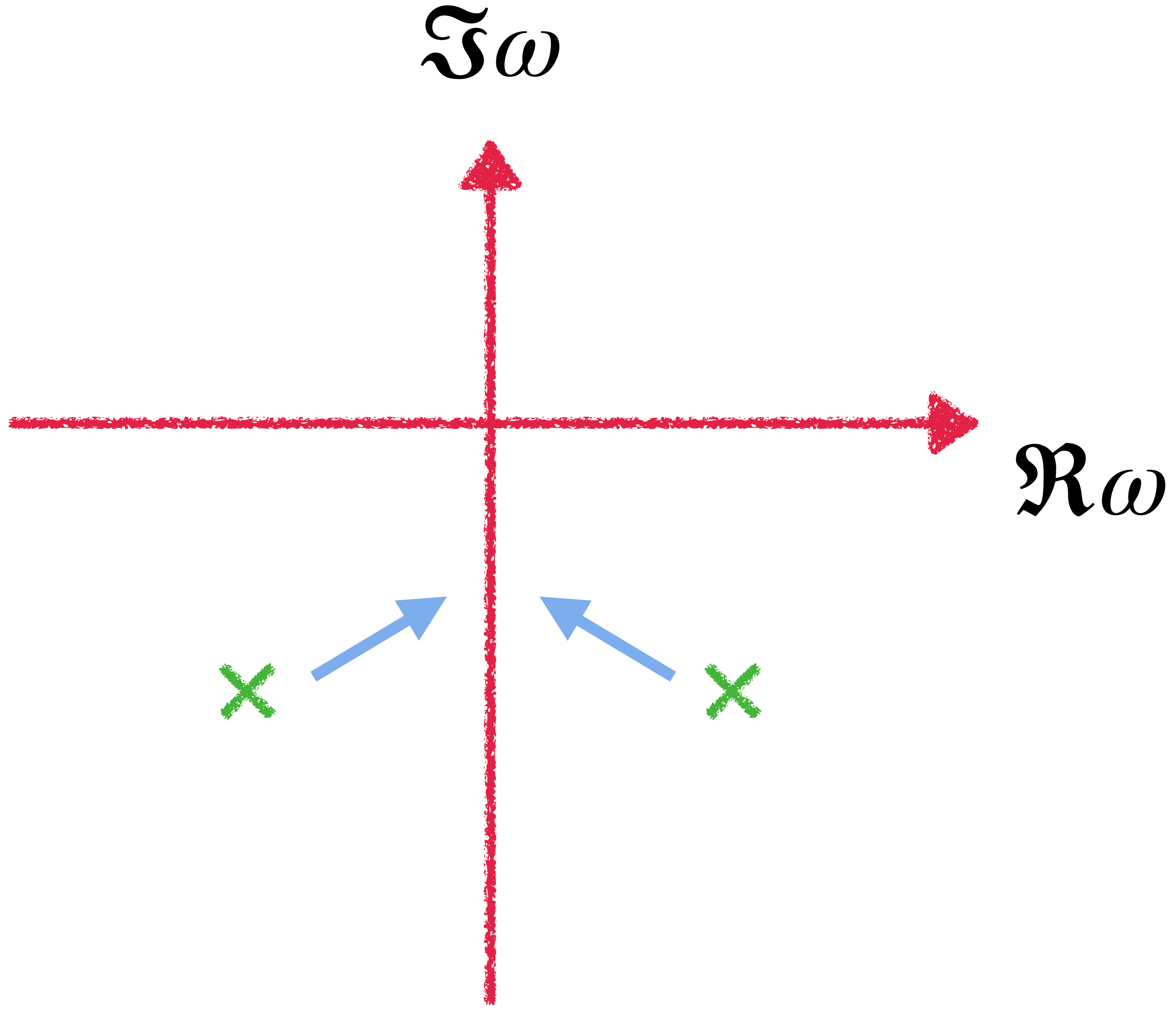
- Dissipative effects will introduce a damping rate determined by Ξ

$$\omega_{gap} = \frac{w\,\Xi}{2\,\chi_{QQ}^2}$$

Continuity (?) Across Transition

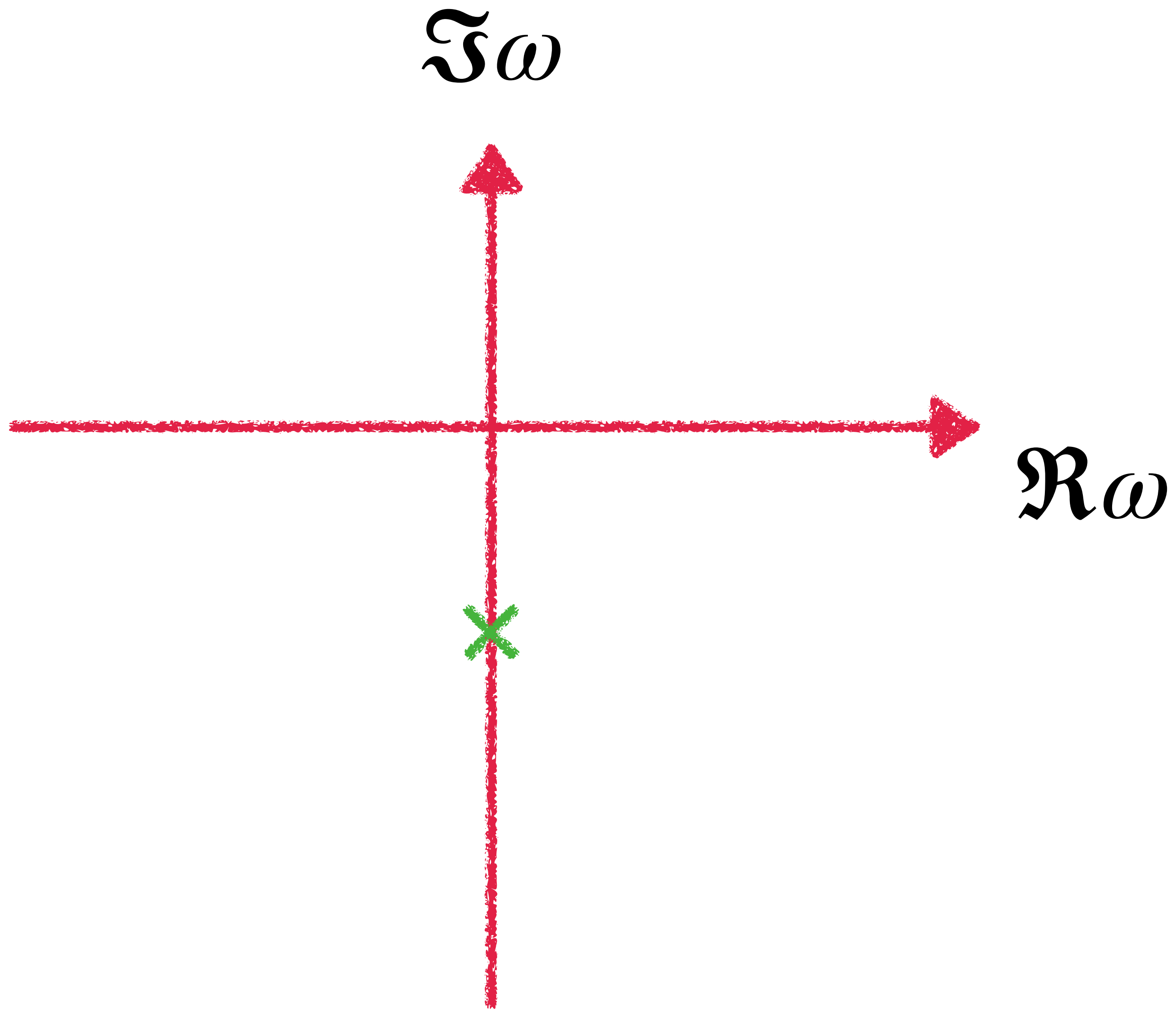
Undeformed Superfluid ($T < T_c$)

$$\omega_\pm = \pm\sqrt{\frac{\chi_{JJ}}{\chi_{QQ}}}\,k^2 - \frac{i}{2\chi_{QQ}^2}\left(\Xi\,\chi_{JJ} + \sigma_d\,\chi_{QQ}\right)k^2$$



Normal Fluid ($T > T_c$)

$$\omega_D = -\,i\frac{\sigma_d}{\chi_{QQ}}\,k^2$$



- $\chi_{JJ} \to 0$ but need to know more about Ξ

Why holography

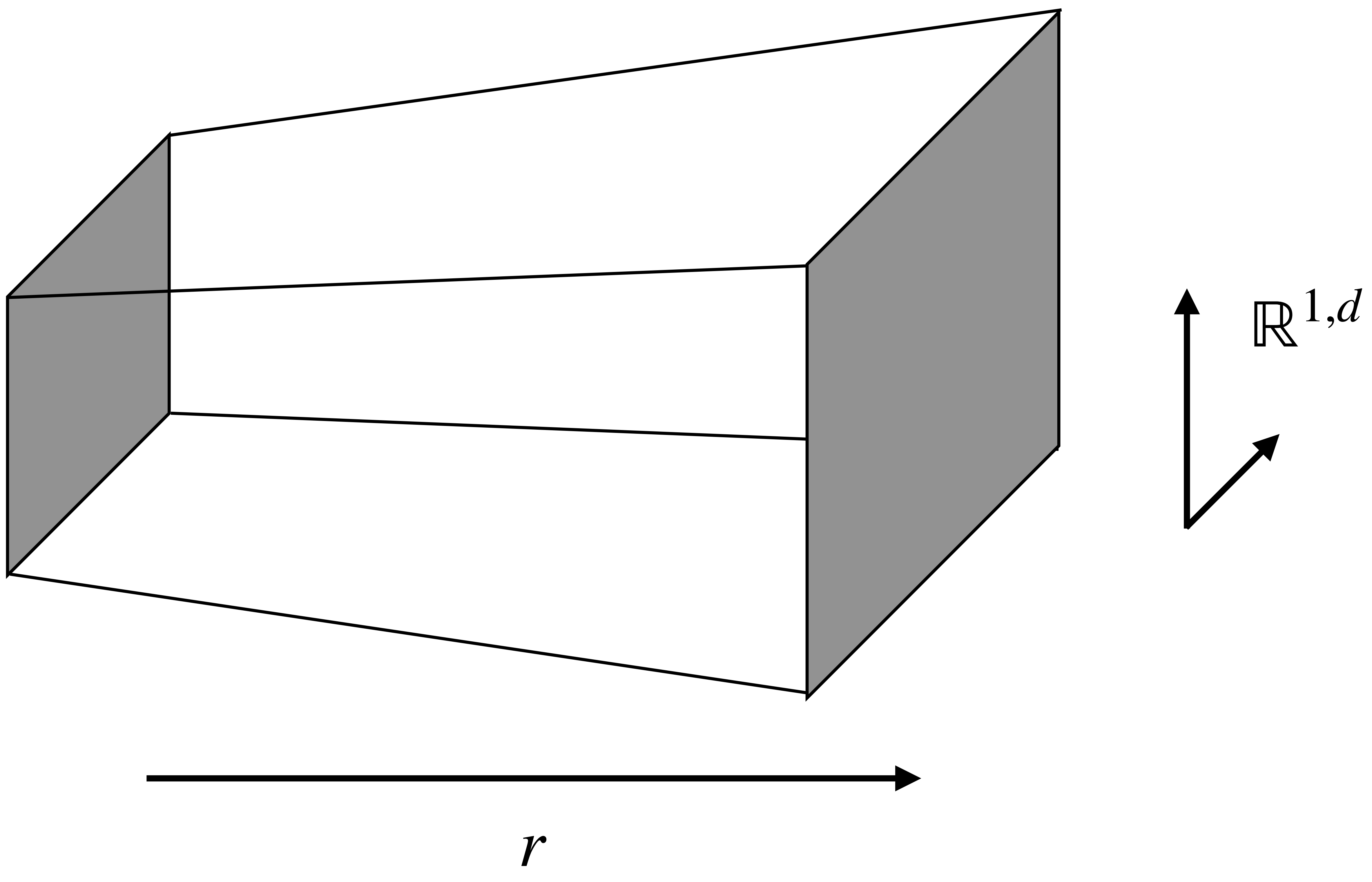
- Microscopic derivation $\Rightarrow$ Compute transport coefficients
- Valid away from $T_c \Rightarrow$ Ground states
- Implement disorder without assumptions
- Real time dynamics

CFT Setup

Model superfluid transitions at $\mu = 0$:

- CFT with a global $U(1)$ and charged operator $\mathcal{O}_\psi$
- Finite temperature T
- Deform by neutral relevant operator $\mathcal{O}_\phi$ to break scaling symmetry
- Phase transition at T_c

AdS/CFT



The vacuum of $CFT_{1,d}$ is modelled by AdS_{d+2}

$$ds^2 = r^2(-dt^2 + d\mathbf{x}_d^2) + \frac{dr^2}{r^2}$$

Holographic Setup

Boundary conditions of bulk fields correspond to sources in CFT:

- Metric ➜ Source for the stress tensor

$$ds^2 = r^2\left(-dt^2 + d\mathbf{x}_d^2 + \delta g_{\mu\nu}(\mathbf{x})dx^\mu dx^\nu\right) + \frac{dr^2}{r^2} + \cdots$$

- Gauge Field ➜ Source for $U(1)$ current

$$A = a_\mu(\mathbf{x})\, dx^\mu + \cdots$$

- Massive Scalar ➜ Source for scalar boundary with dimension Δ_ϕ

$$\psi(r, \mathbf{x}) = \frac{\psi_s(\mathbf{x})}{r^{d+1-\Delta_\phi}} + \cdots$$

Holographic Setup

- Boundary theory gets deformed to

$$S[\phi_s, a_\mu, \delta g_{\mu\nu}] = S_{CFT} + \int d^{d+1}x \left(\phi_s(x)\,\mathcal{O}(x) + a_\mu(x)\,J^\mu(x) + \frac{1}{2}\delta g_{\mu\nu}(x)T^{\mu\nu}(x) \right)$$

- Holographic conjecture relates partition functions

$$Z_{CFT}[\phi_s, a_\mu, \delta g_{\mu\nu}] = Z_{bulk}[\phi_s, a_\mu, \delta g_{\mu\nu}] \approx e^{iS_{bulk}[\phi_s, a_\mu, \delta g_{\mu\nu}]}$$

- Powerful tool to extract VEVs of operators

$$\langle \mathcal{O}(x) \rangle = \frac{1}{i}\frac{\delta}{\delta\phi_s(x)} \ln Z_{CFT}[\phi_s, a_\mu, \delta g_{\mu\nu}] \approx \frac{\delta}{\delta\phi_s(x)} S_{bulk}[\phi_s, a_\mu, \delta g_{\mu\nu}]$$

Symplectic Current

- Cast the bulk action in terms of first derivatives

$$S_{bulk} = \int_M d^{d+1}x\, \mathscr{L}(\partial\phi, \phi) + counterterms$$

- Vary with respect to bulk field to find

$$\langle \mathcal{O}(x) \rangle = \int_{\partial M} d^d x\, \delta\phi\, \frac{\delta\mathscr{L}}{\delta\partial_r\phi} + \cdots$$

- Non-trivial information from knowing on shell value of $\dfrac{\delta\mathscr{L}}{\delta\partial_r\phi}$ close to the boundary
- Useful to think of it as momentum density

Papadimitriou

Symplectic Current

- Within linear response need to know variation $\delta\left(\frac{\delta\mathscr{L}}{\delta\partial_r\phi}\right)$ against specific bulk perturbations
- For any two perturbations $\delta_1\phi_A$ and $\delta_2\phi_A$ define the symplectic current density

$$P^\mu = \delta_1\phi_A\,\delta_2\left(\frac{\delta\mathscr{L}}{\delta\partial_\mu\phi_A}\right) - \delta_2\phi_A\,\delta_1\left(\frac{\delta\mathscr{L}}{\delta\partial_\mu\phi_A}\right)$$

- Divergence free when evaluated on-shell

$$\partial_\mu P^\mu = 0$$

- Component P^r interesting in holography

Symplectic current

- Useful in a hydro/derivative expansion
- Suppose $\delta_1\phi_A^{(s)}$ is a set of static solutions e.g. thermodynamic/zero mode perturbation
- Construct hydro perturbation in derivative expansion

$$\delta_2\phi_A = e^{-i\epsilon\,\omega\,t+i\epsilon\,k\,x}(\delta\phi_A^{(s)} + \epsilon\,\delta\phi_A^{(s)(1)} + O(\epsilon^2))$$

First dissipative corrections

- Construct P^μ out of $\delta_1\phi_A^{(s)}$ and $\delta_2\phi_A$
- Expand conservation of P^μ in ϵ, integrate along radial direction to study corrections $\delta\phi_A^{(s)(1)}$

Setup

The minimum bulk action includes a complex scalar ψ in four dims

$$\mathscr{L} = R - V(\phi, |\psi|^2) - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - (D_\mu\psi)(D^\mu\psi)^* - \frac{1}{4}\tau(\phi, |\psi|^2)\,F^{\mu\nu}F_{\mu\nu}$$

$$D_\mu\psi = \nabla_\mu\psi + i\,q\,A_\mu\,\psi$$

- Invariant under $\psi \to e^{-iq\Lambda}\psi,\, A_\mu \to A_\mu + \partial_\mu\Lambda$

- Scenario were we deform by neutral scalar ϕ and ψ breaks $U(1)$ below T_c

$$V \approx -6 + \frac{1}{2}m_\phi^2\phi^2 + m_\psi^2|\psi|^2 + \cdots$$

- UV dimensions Δ_ϕ and Δ_ψ of dual operators fixed by m_ϕ^2 and m_ψ^2

Setup

- Deform the theory by source $\phi_{(s)}$ of operator dual to ϕ
- Trigger non-trivial RG flow described by black brane solutions with

$$ds^2 = -\,U(r)\,dt^2 + \frac{dr^2}{U(r)} + e^{2g(r)}\left(dx^2 + dy^2\right)$$

$$\phi = \phi(r), \qquad \psi = \psi(r), \qquad A = 0$$

- Complex scalar ψ non-trivial below T_c

Setup

- Horizon at $r = 0$ with

$$U(r) \approx \ 4\pi T\, r + \mathcal{O}(r^2), \qquad g(r) \approx g^{(0)} + \mathcal{O}(r)$$

$$\phi(r) \approx \phi^{(0)} + \mathcal{O}(r), \qquad \rho(r) \approx \rho^{(0)} + \mathcal{O}(r)$$

- UV boundary at $r = \infty$ with

$$U(r) \approx (r+R)^2 + \cdots + g_{(v)}\,(r+R)^{-1} + \cdots \qquad g(r) \approx \ln(r+R) + \ldots$$

$$\phi(r) \approx \phi_{(s)}\,(r+R)^{\Delta_\phi - 3} + \cdots + \phi_{(v)}\,(r+R)^{-\Delta_\phi} + \cdots$$

$$\rho(r) \approx \rho_{(s)}\,(r+R)^{\Delta_\psi - 3} + \cdots \rho_{(v)}\,(r+R)^{-\Delta_\psi} + \cdots$$

- Set $\rho_{(s)} = 0$

Setup

- In the broken phase useful to redefine $\psi = \rho e^{i\theta}$ and $B_\mu = \partial_\mu \theta + q A_\mu$
- The Goldstone mode packaged in a gauge invariant
- Asymptotics close to UV boundary

Current

$$B_\alpha = \partial_\alpha \theta_{(s)} \, r^{2\Delta_\psi - 3} + \cdots + (\partial_\alpha \delta c + q \, \delta s_\alpha) + \cdots + \frac{q \, j_\alpha}{r} + \cdots$$

Source for phase of complex scalar

Gauge invariant combination of phase and source vector

Hydro Perturbations

$$B_\alpha = \partial_\alpha \theta_{(s)}\, r^{2\Delta_\psi - 3} + \cdots + (\partial_\alpha \delta c + q\,\delta s_\alpha) + \cdots + \frac{q\, j_\alpha}{r} + \cdots$$

- At zero frequency/infinite wavelength solutions for B_α purely thermodynamic
- Change in chemical potential $\delta\mu_t$ and constant external vector field $\delta\mu_x$
- Near the UV boundary

$$\delta B_t^{(t)} = q\,\delta\mu_t - q\,\frac{\chi_{QQ}}{r+R}\,\delta\mu_t + \cdots \qquad \delta B_x^{(x)} = q\,\delta\mu_x - q\,\frac{\chi_{JJ}}{r+R}\,\delta\mu_x + \cdots$$

- Near the horizon

$$\delta B_t^{(t)} = q\,\delta\mu_t\, a_t^{(0)}\, r + \mathcal{O}(r^2) \qquad \delta B_x^{(x)} = q\,\delta\mu_x\, a_x^{(0)} + \mathcal{O}(r)$$

Hydro Perturbations

- Leading term in hydro expansion simply by replacing

$$\delta\mu_t \to \partial_t \delta c + \delta s_t \qquad \delta\mu_x \to \partial_x \delta c + \delta s_x$$

- At leading order

$$\delta j_t = -\chi_{QQ}\left(\partial_t \delta c + \delta s_t\right) \qquad \delta j_x = -\chi_{JJ}\left(\partial_x \delta c + \delta s_x\right)$$

Hydro Perturbations

- The symplectic current yields the expected dissipative corrections

$$\delta j_t = -\chi_{QQ}\left(\delta s_t + \partial_t \delta c\right) + \Xi\, \partial_t \left(\delta s_t + \partial_t \delta c\right)$$

$$\delta j_x = -\chi_{JJ}\left(\delta s_x + \partial_x \delta c\right) - \sigma_d\, \partial_t \left(\delta s_x + \partial_x \delta c\right)$$

- With specific transport coefficients determined by the black hole horizon

$$\sigma_d = \tau^{(0)}\,(a_x^{(0)})^2$$

$$\Xi = e^{2g^{(0)}} \frac{(\tau^{(0)} a_t^{(0)})^2}{2q^2(\rho^{(0)})^2}$$

Near T_c

- Horizon expressions allow us to determine

$$\Xi \sim (T_c - T)^{-1} \qquad \chi_{JJ} \sim T_c - T$$

$$\Xi \chi_{JJ} \sim \text{finite}$$

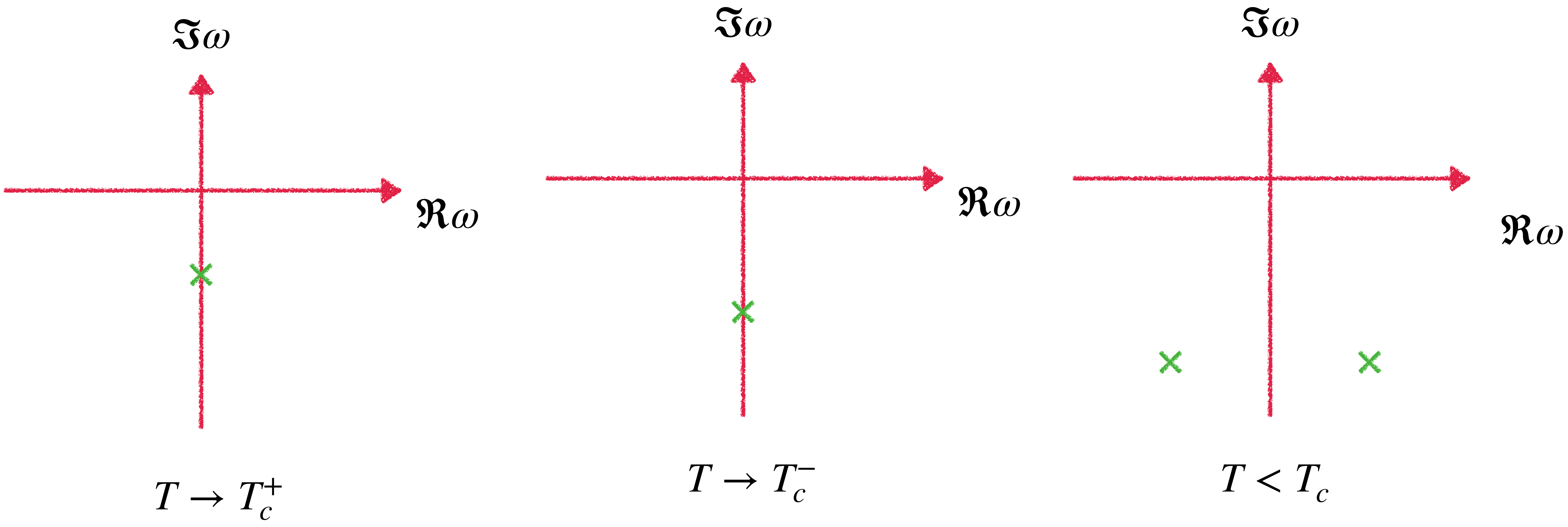
- Near the transition the phase δc becomes not well defined
- Hydro expansion naively breaks down

$$\omega_{\pm} = \pm \sqrt{\frac{\chi_{JJ}}{\chi_{QQ}}}\, k^2 - \frac{i}{2\chi_{QQ}^2} \left(\Xi \chi_{JJ} + \sigma_d \, \chi_{QQ} \right) k^2$$

- The mode is well behaved

Near T_c

- Diffusive mode of normal phase “jumps” at the phase transition



- In more complicate setup Arean, Baggioli, Grieninger, Landsteiner

Outlook

- Powerful holographic techniques to extract transport coefficients
- Understand better the hydro convergence at the phase transition
- Study inhomogeneous superfluids
- Finite chemical potential
- Dissipation at $T \to 0$