Non-equilibrium coset construction

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December 20, 2021

Overview

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Why bother with non-equilibrium coset constructions?

- Non-equilibrium EFTs only exist for a small number of physical systems (until now)
- Can classify many condensed matter systems by symmetry-breaking pattern alone; coset construction is very useful for formulating Goldstone EFTs
- The standard coset construction only works for zero-temperature systems
- Can combine two powerful tools to construct non-equilibrium EFTs for novel states of matter

What is the coset construction?

- Let $\bar{P}_\mu =$ unbroken translations, $T_A =$ unbroken generators,
- $\tau_\alpha =$ broken generators

$$\gamma(x) = e^{ix^\mu \bar{P}_\mu} e^{i\pi^\alpha(x)\tau_\alpha}, \tag{1}$$

- Maurer-Cartan form

$$\gamma^{-1}\partial_\mu \gamma = iE_\mu^\nu (\bar{P}_\nu + \nabla_\nu \pi^\alpha \tau_\alpha + \mathcal{A}_\nu^A T_A), \tag{2}$$

- E_μ^ν is the vierbein, $\nabla_\mu \pi^\alpha$ is the covariant derivative of π^α, and $\mathcal{A}_\mu^A$ transforms as a connection. For higher-order covariant derivatives, use

$$\nabla_\mu^H = (E^{-1})_\mu^\nu (\partial_\nu + i\mathcal{A}_\nu^A T_A) \tag{3}$$

- Inverse Higgs: Can sometimes remove extraneous Goldstones

Non-equilibrium EFTs: highlight reel

- Defined on Schwinger-Keldysh (SK) contour $\implies$ doubled field content: $\varphi_r \equiv \frac{1}{2}(\varphi_1 + \varphi_2)$ and $\varphi_a = \varphi_1 - \varphi_2$
- Defined on fluid worldvolume with coordinates ϕ^M, $M = 0, 1, 2, 3$ with embedding into spacetime $X_s^\mu(\phi)$ for $s = 1, 2$.
- KMS symmetry (classical)

$$\begin{aligned} \varphi_r &\to \Theta\varphi_r \\ \varphi_a &\to \Theta\varphi_a + \Theta i\beta_0\partial_0\varphi_r, \end{aligned} \tag{4}$$

for UV time-reversing symmetry Θ

Emergent gauge symmetries

- Fluid symmetry:
$$\phi^M \to \phi^M + \xi^M(\phi^I), \tag{5}$$
for $I = 1, 2, 3$.
- Generalized chemical-shift symmetry: Let ϵ_s^A (s=1,2) be Goldstones of unbroken generators T_A. Then we have a symmetry
$$e^{i\epsilon_s^A(\phi)T_A} \to e^{i\epsilon_s^A(\phi)T_A} e^{i\xi_T^A(\phi^I)T_A}. \tag{6}$$

Non-equilibrium coset construction

- $\bar{P}_\mu$ = unbroken translations, T_A = other unbroken generators
- τ_α = broken generators
- All symmetries have associated Goldstones so parameterize the full symmetry group $g_s(\phi) = e^{iX_s^\mu(\phi)\bar{P}_\mu} e^{i\pi_s^\alpha(\phi)\tau_\alpha} e^{i\epsilon_s^A(\phi)T_A}$, $s = 1, 2$.
- Maurer-Cartan forms: $g_s^{-1}\partial_M g_s$, for $\partial_M \equiv \frac{\partial}{\partial\phi^M}$.
- Expand MC forms in terms of symmetry generators

$$g_s^{-1}\partial_M g_s = iE_{sM}^\mu(\bar{P}_\mu + \nabla_{s\mu}\pi_s^\alpha\tau_\alpha) + i\mathcal{B}_{sM}^A T_A, \quad (7)$$

- Covariant building-blocks: $\nabla_{s\mu}\pi^\alpha$, E_{s0}^μ, $\mathcal{B}_{s0}^A$, $\mathcal{B}_{aM}^A \equiv \mathcal{B}_{1M}^A - \mathcal{B}_{2M}^A$, etc.
- Higher-order derivatives:

$$\partial_0 \equiv \frac{\partial}{\partial\phi^0} \quad \text{or} \quad \nabla_{s\mu}^H \equiv (E_s^{-1})_\mu^M(\partial_M + i\delta_M^I\mathcal{B}_{sI}^A T_A). \quad (8)$$

- Often, KMS symmetry requires $I_{\text{EFT}}[\varphi_1, \varphi_2] = S[\varphi_1] - S[\varphi_2] + \mathcal{O}(\varphi_a^3)$ at leading order in derivatives

Non-equilibrium inverse Higgs (IH)

- Standard IH constraints: removes a broken Goldstone in favor of another broken Goldstones $[\bar{P}, \tau'] \supset \tau$
- Thermal inverse Higgs: removes a broken Goldstone in favor of an unbroken Goldstone $[\bar{P}, \tau] \supset \bar{P}_0$
- Unbroken inverse Higgs: removes an unbroken Goldstone in favor of an unbroken Goldstone $[\bar{P}, T] \supset \bar{P}_i$
- Appears we cannot remove a broken Goldstone in favor of a broken Goldstone because $[\bar{P}, T] \supset \tau$ is impossible; however non-algebraic IH constraints exist in specific circumstances

Constructing solids

- Unbroken: $\bar{P}_0 = P_0$, $\bar{P}_i = P_i + Q_i$, Broken: K_i, Q_i, J_i.
- Q_i are emergent $U(1)$ symmetries that only exist in the IR

$$g(\phi) = e^{iX^\mu(\phi)P_\mu} e^{i\pi^i(\phi)Q_i} e^{i\eta^i(\phi)K_i} e^{i\theta^i(\phi)J_i}. \tag{9}$$

- MC form

$$g^{-1}\partial_M g = iE^\mu_M(P_\mu + \nabla_\mu \pi^i Q_i + \nabla_\mu \eta^i K_i) + i\Omega^i_M J_i, \tag{10}$$

such that

$$\begin{aligned} E^\mu_M &= \partial_M X^\nu [\Lambda R]_\nu{}^\mu, \\ \nabla_\mu \pi^i &= (E^{-1})^M_\mu \partial_M \psi^i - \delta^i_\mu, \\ \nabla_\mu \eta^i &= (E^{-1})^M_\mu [R^{-1}\Lambda^{-1}\partial_M(\Lambda R)]^{0i}, \\ \Omega^i_M &= \frac{1}{2}\epsilon^{ijk}[R^{-1}\Lambda^{-1}\partial_M(\Lambda R)]^{jk}, \end{aligned} \tag{11}$$

For $\psi^i \equiv X^i + \pi^i$.

Inverse Higgs

- Impose the covariant IH constraint equations $\nabla_t \pi^i = \epsilon^{ijk} \nabla_j \pi_= 0$.
- The first constraint is solved by

$$\frac{\eta^i}{\eta} \tanh \eta = -(e^{-1})_t^M \partial_M \psi^j (a^{-1})^{ij},$$

where $e_M^\mu = \partial_M X^\mu$ and $a^{ij} \equiv (e^{-1})_i^M \partial_M \psi^j$.
- The second constraint tells us that

$$\nabla^{(i}\pi^{j)} = (Y^{1/2})^{ij} - \delta^{ij}, \qquad Y^{ij} = G^{MN} \partial_M \psi^i \partial_N \psi^j,$$

where G^{MN} is the inverse of the metric $G_{MN} = \partial_M X^\mu \eta_{\mu\nu} \partial_N X^\nu$.

The solid EFT

- Leading-order invariant building-blocks: G_{00}, Y^{ij} (from previous slide)
- Also have $E_0^\mu \nabla^i_\mu = (\delta^{ij} - (Y^{1/2})^{ij})\partial_0 \psi^j$, from which we extract: $Z^i = \partial_0 \psi^i$.
- Solid EFT at leading order: $S = \int d^4\phi \sqrt{-G}\ P[Y^{ij}, Z^i, G_{00}]$.
- Local temperature $T = 1/\beta = 1/\sqrt{-G_{00}}$
- Y^{ij} describes motion of the lattice—shear and pressure waves
- Z^i measures the discrepancy between temperature rest-frame and lattice rest-frame. Allows propagation of second-sound mode; is closely related to the conservation of emergent charges Q_i.

Second sound and its removal (I)

- Must work in retarded-advanced basis now with doubled field content and formulate action on physical spacetime.

$$I_0 = S_1 - S_2 + \mathcal{O}(a^3) = \int d^4x T^{\mu\nu}\partial_\mu X_{a\nu} + J^{i\mu}\partial_\mu \psi^i_a.$$

- If Q_i are not conserved, the Goldstones need not be Stückelberg fields $\implies$ ψ_a with no derivatives is a legitimate building-block. New action:

$$I = \int d^4x T^{\mu\nu}\partial_\mu X_{a\nu} + J^{i\mu}\partial_\mu \psi^i_a + iM^{ij}\psi^i_a(\psi^j_a + i\beta_0\partial_0\psi^j).$$

- Integrating out ψ^i in the large-M limit (i.e. in the deep IR) leads to

$$\partial_0\psi^i = \psi_a = 0.$$

- Gauge-fix $\phi^i = \psi^i$. Now the field contents are just $X^\mu(\phi)$ and the diff symmetries are $\phi^0 \to \phi^0 + f(\phi^i)$ and rigid spatial translations $\phi^i \to \phi^i + c^i$.

Second sound and its removal (II)

- Now that ψ^i are removed, work with a single-copy of the action again.
- Think of $\partial_0 \psi^i = \psi^i_a = 0$ as a kind of non-algebraic IH constraint; the building-blocks become G_{00}, $Y^{ij} \to G^{ij}$, $Z^i \to 0$.
- The new solid action is $S = \int d^4\phi \sqrt{-G}\ P[G^{ij}, G_{00}]$, and there is no second sound.

Conclusion

- All symmetries are broken by microstates $\implies$ Goldstones for all symmetries
- Unbroken Goldstones enjoy time-independent gauge symmetries—fluid diffs and generalized chemical shift symmetries
- Three types of algebraic IH constraints + one non-algebraic IH constraint
- Non-equilibrium coset construction was successfully used to construct both known and previously unknown non-equilibrium EFTs for condensed matter systems including: fluids, superfluids, solids, supersolids, nematic liquid crystals, smectic liquid crystals in phases A,B,C, plasmas etc.
- Further reading: arXiv:1912.12301, arXiv:2008.11725, arXiv:2101.02210