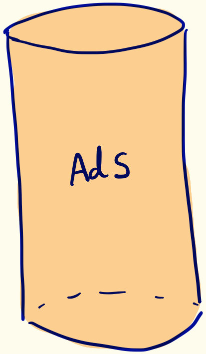
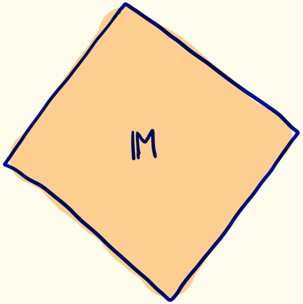
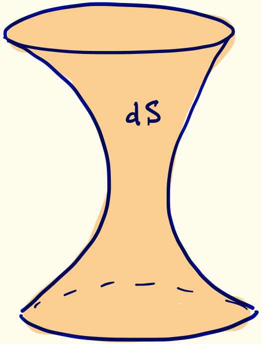
Towards the nonperturbative cosmological bootstrap







EPFL

João Penedones

HoloTube, 9/11/2021

Based on arXiv: 2107.13871 with:

Matthijs Hogervorst

Kamran Vaziri

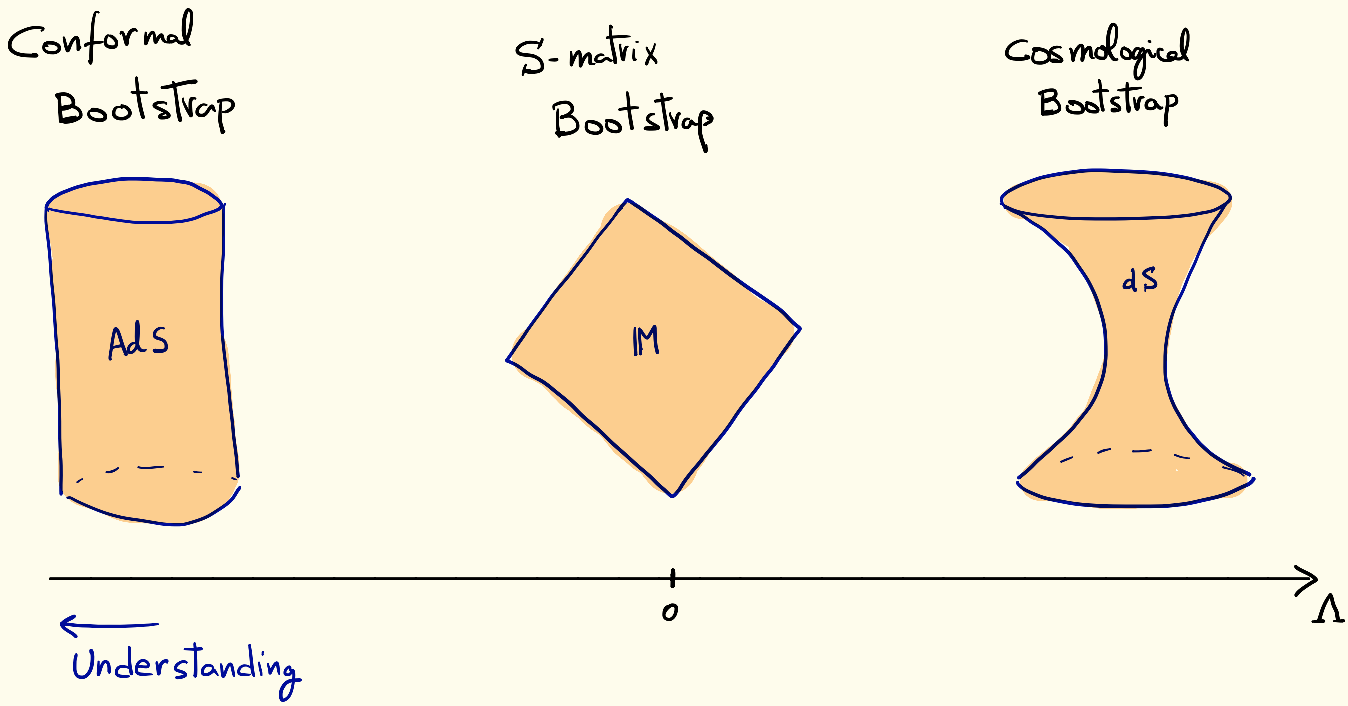
Introduction

The bootstrap is a nonperturbative approach to QFT and Quantum Gravity.

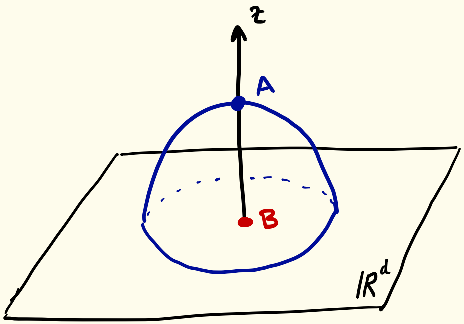
General Principles $\longrightarrow$ Bounds on Physical observables

It works in different backgrounds: AdS, $\mathbb{M}$, dS.

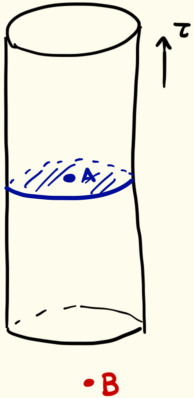
Introduction



QFT in AdS_{d+1}

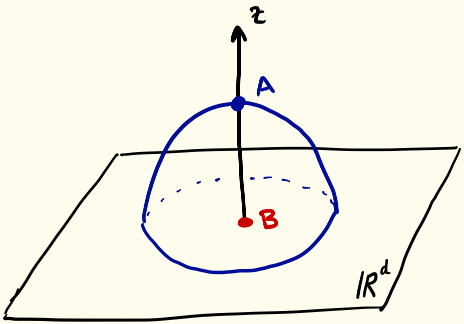


$\Longleftrightarrow$



Boundary operator / bulk state map

QFT in AdS_{d+1}



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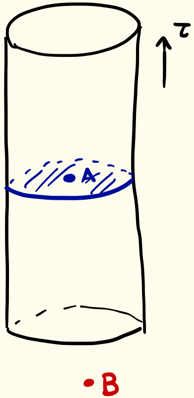


Boundary operator / bulk state map

bulk op.

bound. op.

$$\Rightarrow \begin{cases} \phi(z,x) = \sum_k b_{\phi k}\, z^{\Delta_k} \left[\mathcal{O}_k(x) + \text{desc.} \right] \\[2em] \mathcal{O}_i(x)\, \mathcal{O}_j(0) = \sum_k C_{ijk}\, |x|^{\Delta_k - \Delta_i - \Delta_j} \left[\mathcal{O}_k(0) + \text{desc} \right] \end{cases}$$

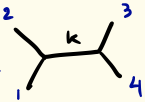
OPE coefficients

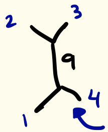
Convergent OPE

Boundary correlation functions obey the CFT_d axioms:

- Unitarity: $C_{ijk} \in \mathbb{R}$, $\Delta \geq \Delta_{min} = \begin{cases} \frac{d-2}{2} & \text{scalars} \\ d-2+\ell & \text{spin } \ell \end{cases}$

- OPE convergence and associativity

$\sum_k C_{12k} C_{34k}$ 

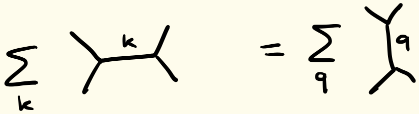
$= \sum_q C_{14q} C_{23q}$ 

conformal blocks

Boundary correlation functions obey the CFT$_d$ axioms:

- Unitarity: $\quad C_{ijk} \in \mathbb{R}$, $\quad \Delta \geq \Delta_{min} = \begin{cases} \frac{d-2}{2} & \text{scalars} \\ d-2+\ell & \text{spin } \ell \end{cases}$

- OPE convergence and associativity



- Stress tensor $T_{\mu\nu}$: $\quad \Delta = d$, Ward identities

With dynamical gravity in AdS

QFT in Minkowski space

Focus on scattering amplitudes (boundary observables) :

- Unitarity (partial waves)
- Lorentz invariance
- Causality (some analyticity)
- Crossing symmetry
- [$\exists$ massless spin 2 particle (graviton) : soft theorems (need $d > 3$ for $\mathbb{M}^{d+1}$ due to IR divergences)]

With dynamical gravity

What is the non-perturbative bootstrap formulation for QFT in de Sitter spacetime?

QFT on:	AdS_{d+1}	$\mathbb{M}^{d+1}$	dS_{d+1}
Observable	boundary op. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{(x_{13}^2 x_{24}^2)^{\Delta_{\mathcal{O}}}} F(u,v)$ cross-ratios	${}_{out}\langle p_3, p_4 \mid p_1, p_2\rangle_{in}^{conn} = i\,\delta(\Sigma p_i)\, T(s,t)$ Mandelstam inv. $s+t+u=4m^2$	?
Crossing	$F(u,v) = F(v,u) = \frac{F(\frac{u}{v},\frac{1}{v})}{u^{\Delta_{\mathcal{O}}}}$	$T(s,t) = T(t,s) = T(\underbrace{4m^2-s-t}_{u},t)$	?
Decomposition in irreps	$F(u,v) = \sum_{\Delta,\ell} \lambda_{\Delta,\ell}\, g_{\Delta,\ell}(u,v)$ conf. blocks	$T \propto \sum_{\ell} f_\ell(s)\, P_\ell(\underbrace{\cos\theta}_{1-\frac{2t}{4m^2-s}})$	?
Unitarity	$\lambda_{\Delta,\ell} \geq 0$	$\lvert 1 + i f_\ell(s)\rvert^2 \leq 1$	?

Related works

- Cosmological Bootstrap : perturbative

  [Maldacena] [Arkani-Hamed, Maldacena]

  [Arkani-Hamed, Baumann, Chen, Duaso Pueyo, Joyce, Lee, Pimentel]

  [Goodhew, Jazayeri, Lee, Melville, Pajer, Stefanyszyn, Supel]

  [Hollands] [Marolf, Morrison] ...

- Dictionary between dS and EAdS

  [Sleight, Taronna]

  [Di Pietro, Gorbenko, Komatsu]

  ...

- dS / CFT

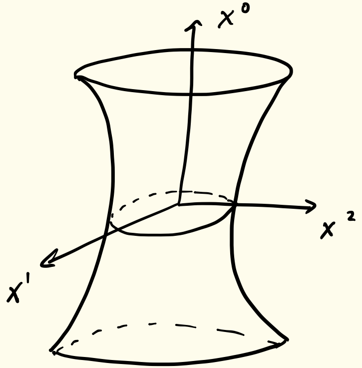
  [Strominger] [Witten] [Anninos, Hartman, Strominger] [Susskind]

  ...

QFT in dS

de Sitter spacetime

$$dS_{d+1} \subset \mathbb{M}^{d+2} \ni X^A \qquad A = 0, 1, \ldots, d+1$$



$$-(X^0)^2 + (X^1)^2 + \ldots + (X^{d+1})^2 = R^2$$

Isometry group $= SO(d+1,1) =$ conformal group in $\mathbb{R}^d$

de Sitter spacetime

$$dS_{d+1} \subset \mathbb{M}^{d+2} \ni X^A \qquad A = 0, 1, \ldots, d+1$$

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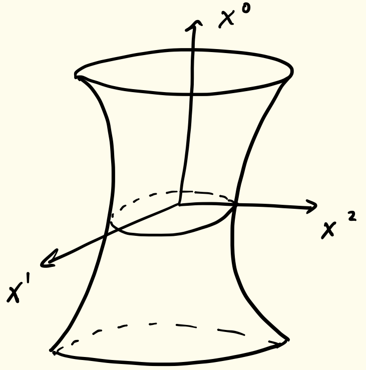
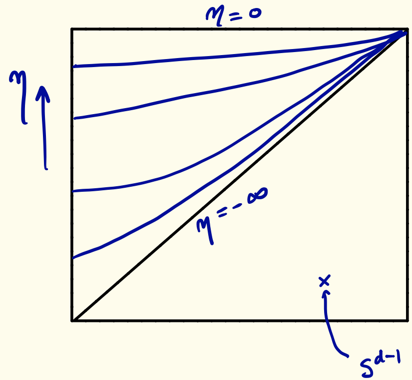


Isometry group $= SO(d+1,1) =$ conformal group in $\mathbb{R}^d$

Conformal coordinates

$$\boxed{ds^2 = R^2 \, \frac{d\eta^2 + dx^2}{\eta^2}}$$

$\eta < 0 \quad , \quad x \in \mathbb{R}^d$

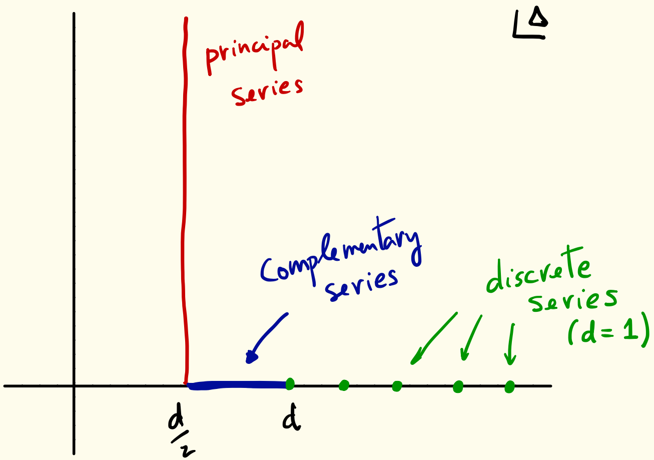


Penrose diagram

Hilbert space

$$\mathcal{H} = \bigoplus \left[\text{Unitarity irreps of } SO(d+1,1)\right]$$

$$\begin{array}{c} \cup \\ SO(1,1) \times SO(d) \\ \Delta \qquad\qquad \ell \end{array} \leftarrow \text{for simplicity}$$



$\ell = 0$

$\ell \geq 1$

Hilbert space

$$\mathcal{H} = \bigoplus \left[\text{Unitarity irreps of } SO(d+1,1) \right]$$

$$SO(d+1,1) \supset SO(1,1) \times SO(d)$$

Δ $\quad$ ℓ $\leftarrow$ for simplicity

Basis for irrep (Δ, ℓ) : $|\Delta, x\rangle^{(\mu_1 \cdots \mu_\ell)}$, $x \in \mathbb{R}^d$, $\mu_i \in \{1, \dots, d\}$

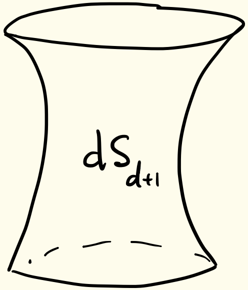
$\uparrow$ transforms like primary local op $\mathcal{O}^{\mu_1 \cdots \mu_\ell}(x)$

Projector $\quad \mathbb{P}_{\Delta,\ell} = \displaystyle\int d^d x \; |\Delta, x\rangle^{\mu_1 \cdots \mu_\ell} {}_{\mu_1 \cdots \mu_\ell}\langle \Delta, x|$

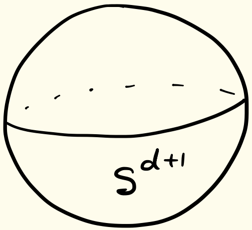
$$\boxed{\mathbb{1} = |0\rangle\langle 0| + \sum_\ell \int_{\frac{d}{2} - i\infty}^{\frac{d}{2} + i\infty} d\Delta \; \mathbb{P}_{\Delta,\ell} + \text{other irreps}}$$

Correlation functions

Correlators in the Bunch-Davies vacuum can be obtained by analytic continuation from the sphere.



$$ds^2 = R^2(-dt^2 + \cosh^2 t \, d\Omega_d^2)$$



$$ds^2 = R^2(d\theta^2 + \cos^2\theta \, d\Omega_d^2)$$

$$\langle 0|\phi_1(t_1,\Omega_1)\cdots\phi(t_n,\Omega_n)|0\rangle = \langle \phi(\theta_1 = \epsilon_1 + it_1, \Omega_1)\cdots\phi(\theta_n = \epsilon_n + it_n, \Omega_n)\rangle_{\text{sphere}}$$

BD vacuum

$0 < \epsilon_n < \dots < \epsilon_1 \longrightarrow 0$

Bulk Two-point function

$$\langle 0 | \phi(\eta, x)\, \phi(\eta', x') | 0 \rangle = G(\xi)$$

$$\xi = \frac{4\eta\eta'}{-(\eta-\eta')^2 + (x-x')^2} = \frac{4R^2}{(\underset{\mathbb{M}^{d+2}}{\underset{\Uparrow}{X}} - X')^2}$$

Bulk Two-point function

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Källén-Lehmann decomposition

$$\mathbb{1} = |0\rangle\langle 0| + \sum_{\ell} \int d\Delta \; \mathbb{P}_{\Delta,\ell} + \dots$$

$$\langle 0 | \phi(\eta, x)\, \phi(\eta', x') | 0 \rangle = \langle 0 | \phi | 0 \rangle^2 + \int_{\frac{d}{2}-i\infty}^{d/2+i\infty} d\Delta \; \rho_\phi(\Delta)\, G_\Delta^{free}(\xi)$$

positive spectral density

$$\rho_\phi(\Delta) = \rho_\phi(d-\Delta)$$

Free propagator

$$m^2 R^2 = \Delta(\Delta - d)$$

Bulk Two-point function

Continuation from the sphere:

$x = \cos\theta = X \cdot X'$

$-1 \leq x \leq 1$

$$\langle \phi(X) \phi(X') \rangle = G(x) = \sum_{J=0}^{\infty} a_J \, C_J^{d/2}(x)$$

$X \in S^{d+1} \subset \mathbb{R}^{d+2}$

$X^2 = 1 = X'^2$

Gegenbauer polynomials

Bulk Two-point function

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Gegenbauer polynomials

$$a_J = \begin{cases} K_J \displaystyle\int_{-1}^{1} dx \, (1-x^2)^{\frac{d-1}{2}} \, C_J^{d/2}(x) \, G(x) & \leftarrow \text{integer } J \\[2ex] \tilde{K}_J \displaystyle\int_{1}^{\infty} dx \; {}_2F_1\left(J+d, J+\frac{d}{2}+\frac{1}{2}, 2J+d+1, \frac{2}{1-x}\right) \frac{(x+1)^{\frac{d-1}{2}}}{(x-1)^{J+\frac{d}{2}+\frac{1}{2}}} \, \mathrm{Disc}\left[G(x)\right] & \leftarrow \text{complex } J \end{cases}$$

$$\boxed{\rho_\phi(\Delta) = \frac{2\pi^{d/2+1}}{\Gamma\left(\frac{d}{2}\right)} \left[a_{J=-\Delta} + a_{J=\Delta-d} \right]}$$

Bulk Two-point function - Examples

$$\langle 0|\phi(\eta,x)\,\phi(\eta',x')|0\rangle = \langle\phi\rangle + \int_{\frac{d}{2}-i\infty}^{d/2+i\infty} d\Delta\;\rho_\phi(\Delta)\;G_\Delta^{free}(\xi)$$

Massive free boson

$$m^2R^2 = \frac{d^2}{4}+\mu^2 \quad\Rightarrow\quad \rho_\phi\left(\Delta=\frac{d}{2}+i\nu\right) = \frac{1}{2}\left[\delta(\mu-\nu)+\delta(\mu+\nu)\right]$$

Bulk Two-point function - Examples

$$\langle 0 | \phi(\eta, x)\, \phi(\eta', x') | 0 \rangle = \langle \phi \rangle + \int_{\frac{d}{2} - i\infty}^{d/2 + i\infty} d\Delta \, \rho_\phi(\Delta)\, G_\Delta^{\text{free}}(\xi)$$

Massive free boson

$$m^2 R^2 = \frac{d^2}{4} + \mu^2 \quad \Rightarrow \quad \rho_\phi\left(\Delta = \tfrac{d}{2} + i\nu\right) = \tfrac{1}{2}\left[\delta(\mu - \nu) + \delta(\mu + \nu)\right]$$

Bulk CFT

bulk scaling dimension

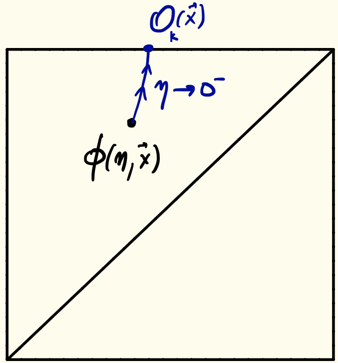
$$\langle \phi \phi \rangle = \xi^{\delta} \quad \Longrightarrow \quad \rho_\phi\left(\Delta = \tfrac{d}{2} + i\nu\right) \propto \nu \sinh(\pi\nu)\, \underbrace{\Gamma\left(\delta - \tfrac{d}{2} - i\nu\right)}_{\text{poles at } \Delta = \delta + \mathbb{N}_0}\, \Gamma\left(\delta - \tfrac{d}{2} + i\nu\right)$$

$\rightarrow$ Need complementary series if $\dfrac{d-1}{2} < \delta < \dfrac{d}{2}$.

Boundary Operators

$$\phi(\eta,\vec{x}) = \sum_{k} b_{\phi k}(-\eta)^{\Delta_k}\left[\mathcal{O}_k(\vec{x}) + \text{desc.}\right]$$

primary



$$\phi^\dagger = \phi \quad \Rightarrow \begin{cases} \mathcal{O}_k^\dagger = \mathcal{O}_k \quad , \quad \Delta_k \in \mathbb{R} \quad , \quad b_{\phi k} \in \mathbb{R} \\ \mathcal{O}_k^\dagger = \mathcal{O}_{k'} \quad , \quad \Delta_k^* = \Delta_{k'} \quad , \quad b_{\phi k}^* = b_{\phi k'} \end{cases}$$

Boundary Operators

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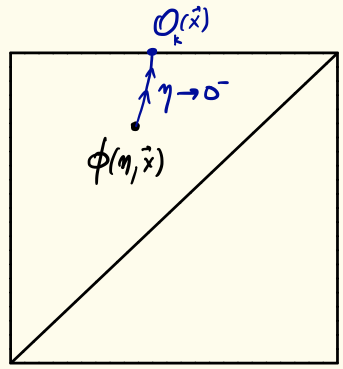
↳ primary



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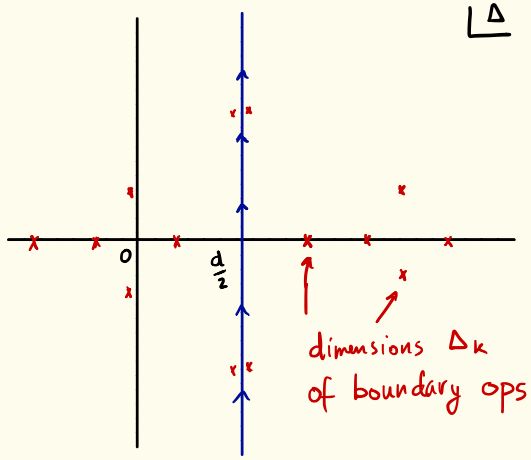
$$\langle \phi(\eta_1,\vec{x}_1)\,\phi(\eta_2,\vec{x}_2)\rangle = \int_{\frac{d}{2}-i\infty}^{d/2+i\infty} d\Delta\; \rho_\phi(\Delta)\; \underbrace{G_\Delta^{free}(\xi)}$$

$\downarrow \xi \to 0 \iff \eta_1 \to 0$

$\xi^\Delta(\ldots) + \xi^{d-\Delta}(\ldots)$



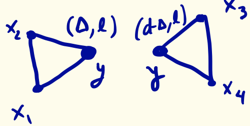
No state-operator map!

Boundary 4pt - function

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \sum_{\ell}\int d\Delta\; I_{\Delta,\ell}\; \underbrace{\Psi^{12,34}_{\Delta,\ell}(x_1,x_2,x_3,x_4)}_{\text{Conf. partial wave}} \;+\; \langle \mathcal{O}_1\mathcal{O}_2\rangle\langle \mathcal{O}_3\mathcal{O}_4\rangle$$

$$\mathbb{1} = \sum_{\ell}\int d\Delta \int d^d y\, |\Delta, y\rangle\langle \Delta, y| + \dots$$

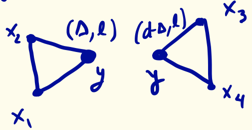
$$\Psi = \int d^d y$$



$$\text{Unitarity} \Rightarrow \boxed{I_{\Delta,\ell} \geq 0}$$

Boundary 4pt-function

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2) \underset{\mathbb{1}}{\uparrow} \mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \sum_{\ell} \int d\Delta \; I_{\Delta,\ell} \; \underbrace{\Psi^{12,34}_{\Delta,\ell}(x_1,x_2,x_3,x_4)}_{\text{Conf. partial wave}} + \langle \mathcal{O}_1\mathcal{O}_2\rangle\langle \mathcal{O}_3\mathcal{O}_4\rangle$$

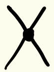
$\Psi = \int d^d y$ 

Unitarity $\Rightarrow$ $\boxed{I_{\Delta,\ell} \geq 0}$

Example: free scalar field in dS_{d+1} : $\Delta = \frac{d}{2} + i\mu$, $\mu \in \mathbb{R}$

$$\langle \mathcal{O}(x_1)\,\mathcal{O}^\dagger(x_2)\,\mathcal{O}^\dagger(x_3)\,\mathcal{O}(x_4)\rangle = \frac{1}{(x_{14}^2)^{\Delta}(x_{23}^2)^{\Delta^*}} + c\,\delta(x_{13})\,\delta(x_{24}) + c\,\delta(x_{12})\,\delta(x_{34})$$

$\Rightarrow I_{\Delta,\ell} \geq 0$ ✓

$\lambda\phi^4$  $\rightarrow$ $I_{\Delta,\ell} = I^{(0)}_{\Delta,\ell} + \lambda\, I^{(1)}_{\Delta,\ell} + \mathcal{O}(\lambda^2)$

<u>Crossing</u> + <u>Unitarity</u> $\rightarrow$ <u>Bootstrap</u>

$\boxed{d=1}$ QFT in dS_2

unique cross ratio

$$\langle \mathcal{O}(x_1)\,\mathcal{O}^\dagger(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}^\dagger(x_4)\rangle = \frac{1}{\left(x_{12}^2\, x_{34}^2\right)^{\Delta_{re}}} \left|\frac{x_{24}}{x_{13}}\right|^{2i\Delta_{im}} \mathcal{G}\left(z \equiv \frac{x_{12}\, x_{34}}{x_{13}\, x_{24}}\right)$$

$\Delta_{\mathcal{O}} = \Delta_{re} + i\Delta_{im}$

$\boxed{d=1}$ QFT in dS_2

$$\langle \underset{\uparrow}{\mathcal{O}}(x_1)\,\mathcal{O}^\dagger(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}^\dagger(x_4)\rangle = \frac{1}{(x_{12}^2\, x_{34}^2)^{\Delta_{re}}}\left|\frac{x_{24}}{x_{13}}\right|^{2i\Delta_{im}} \mathcal{G}\left(z \equiv \frac{x_{12}\,x_{34}}{x_{13}\,x_{24}}\right)$$

$\Delta_{\mathcal{O}} = \Delta_{re} + i\Delta_{im}$

Crossing $x_1 \leftrightarrow x_3$:
$$\boxed{(1-z)^{2\Delta_{re}}\,\mathcal{G}(z) = z^{2\Delta_{re}}\,\mathcal{G}(1-z)}$$

positive

$$\boxed{\mathcal{G}(z) = \underbrace{\sum_{\ell=0,1}\int_0^\infty \frac{d\nu}{2\pi}\, I_{\frac{1}{2}+i\nu,\ell}\;\Psi_{\frac{1}{2}+i\nu,\ell}(z)}_{\text{principal series}} + \underbrace{\sum_{\substack{n\in\mathbb{N}\\ \ell=0,1}} \tilde{I}_{n,\ell}\;\Psi_{n,\ell}(z)}_{\text{discrete series}}}$$

$\boxed{d=1}$ QFT in dS_2

$$\langle \underset{\uparrow}{\mathcal{O}}(x_1)\mathcal{O}^\dagger(x_2)\mathcal{O}(x_3)\mathcal{O}^\dagger(x_4)\rangle = \frac{1}{(x_{12}^2\, x_{34}^2)^{\Delta_{re}}}\left|\frac{x_{24}}{x_{13}}\right|^{2i\Delta_{im}} \mathcal{G}\left(z \equiv \frac{x_{12}\, x_{34}}{x_{13}\, x_{24}}\right)$$

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Problem: Not absolutely convergent.

Solution: Act with linear functional: $\omega[f] = \int_0^1 dz\, z^{\gamma}(1-z)^{\sigma} f(z)$

Regularized Crossing Equations

$$\sum_{\ell=0,1} \int_0^\infty \frac{d\nu}{2\pi} \, I_{\frac{1}{2}+i\nu,\ell} \, \tilde{F}_{\frac{1}{2}+i\nu,\ell}(\gamma,\sigma) + \sum_{\ell=0,1} \sum_{n=1}^{\infty} \tilde{I}_{n,\ell} \, \tilde{F}_{n,\ell}(\gamma,\sigma) = 0 \qquad \forall \gamma,\sigma > -1$$

Example of bound: Take $\mathcal{O} = \mathcal{O}^{+}$ with $\Delta_{\mathcal{O}} = \frac{1}{2} + \frac{1}{8}$ $\Rightarrow$ $\ell = 0$ & n even

disconnected term

$$\int_0^\infty \frac{d\nu}{2\pi} \, I_{\frac{1}{2}+i\nu,0} \, \tilde{F}_{\frac{1}{2}+i\nu,0}(\gamma,\sigma) + \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \tilde{I}_{n,0} \, \tilde{F}_{n,0}(\gamma,\sigma) + D(\gamma,\sigma) = 0$$

If $I_{\frac{1}{2}+i\nu,0} = 0$ for $\nu < 8.53$ then $\tilde{I}_{2,0} < 5.7$

QFT on:	AdS_{d+1}	$\mathbb{M}^{d+1}$	dS_{d+1}
Observable	boundary op. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{(x_{13}^2 x_{24}^2)^{\Delta_{\mathcal{O}}}} F(u,v)$ cross-ratios	${}_{out}\langle p_3,p_4 \| p_1,p_2\rangle_{in}^{conn} = i\,\delta(\Sigma p_i)\, T(s,t)$ Mandelstam inv. $s+t+u=4m^2$	boundary op. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{(x_{13}^2 x_{24}^2)^{\Delta_{\mathcal{O}}}} F(u,v)$ cross-ratios
Crossing	$F(u,v) = F(v,u) = \frac{F(\frac{u}{v},\frac{1}{v})}{u^{\Delta_{\mathcal{O}}}}$	$T(s,t) = T(t,s) = T(\underbrace{4m^2-s-t}_{u},t)$	$F(u,v) = F(v,u) = \frac{F(\frac{u}{v},\frac{1}{v})}{u^{\Delta_{\mathcal{O}}}}$
Decomposition in irreps	$F(u,v) = \sum_{\Delta,\ell} \lambda_{\Delta,\ell}\, g_{\Delta,\ell}(u,v)$ conf. blocks	$T \propto \sum_\ell f_\ell(s)\, P_\ell(\underbrace{\cos\theta}_{1-\frac{2t}{4m^2-s}})$	$F(u,v) = \sum_\ell \int d\Delta\, I_{\Delta,\ell}\, \Psi_{\Delta,\ell}(u,v)$ CPWs
Unitarity	$\lambda_{\Delta,\ell} \geq 0$	$\|1+i f_\ell(s)\|^2 \leq 1$	$I_{\Delta,\ell} \geq 0$

Open Questions

Open Questions

For a generic QFT in dS, what is the:

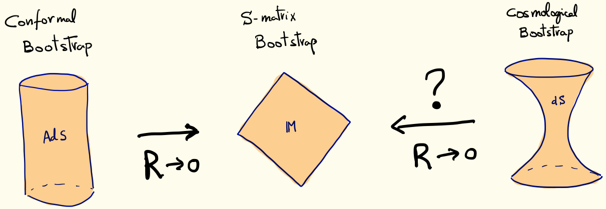
- Hilbert space?
- Spectrum of boundary ops.?

$\Big\}$ Computable examples: $\begin{cases} \text{free QFT} \\ \text{bulk CFT} \end{cases}$ + small deformations

What are the relevant quantities to bound?

Better numerical bootstrap methods? " $I_{\Delta,\ell} = \sum_{\ell'} \int d\Delta' \, J(\Delta,\Delta',\ell,\ell') \, I_{\Delta',\ell'}$ "

Flat space limit?



Can we include dynamical gravity?

Thank you!