Exploring continuum quantum field theories of fracton order

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HoloTube talk, November 16, 2021

in collaboration with Jacques Distler, Hao Geng, Murtaza Jafry, Shamit Kachru, Zhu-Xi Luo, Richard Nally, Brandon Rayhaun, Amir Raz, Ryan Spieler, and Haoyu Sun

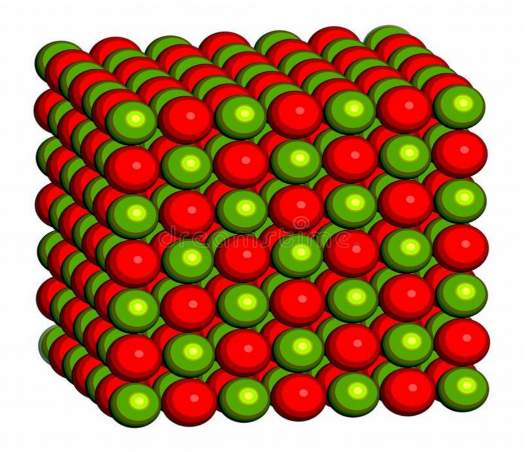
WARNING:

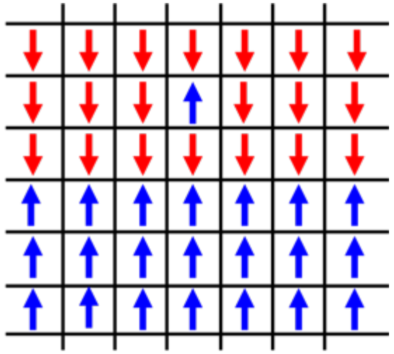
no holography in this talk
so why bother?
"wouldn't it be nice to redo this in holography"

Fractons and QFT

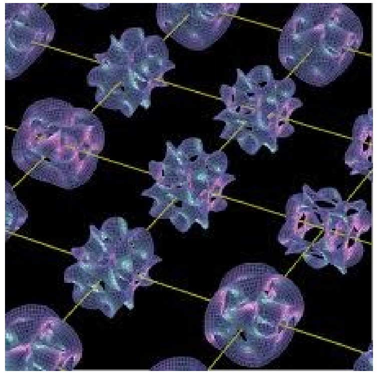
Fractons may force us to rethink
long held believes about quantum field theory.

QFT as the Universal Language

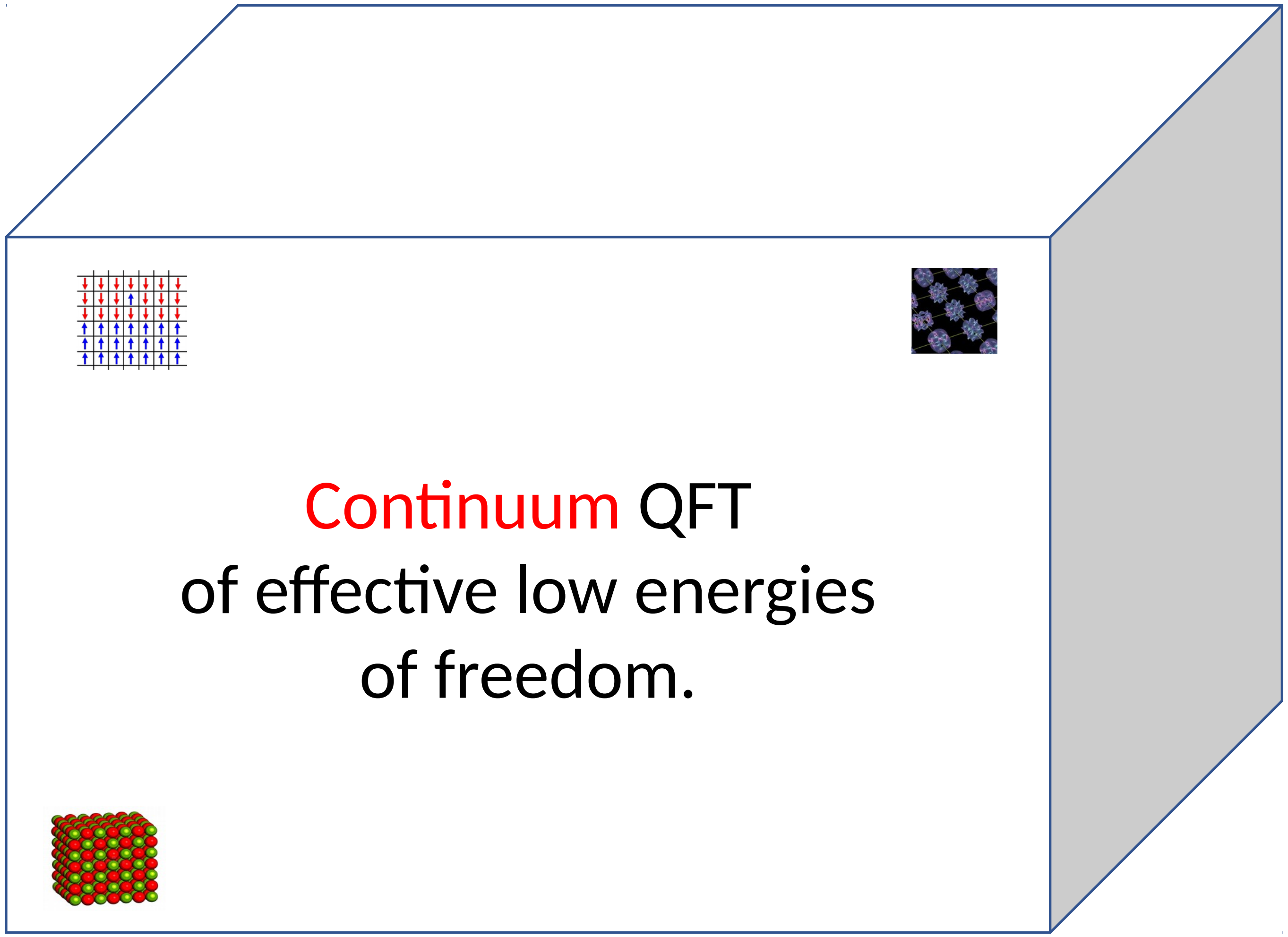




Zoom out

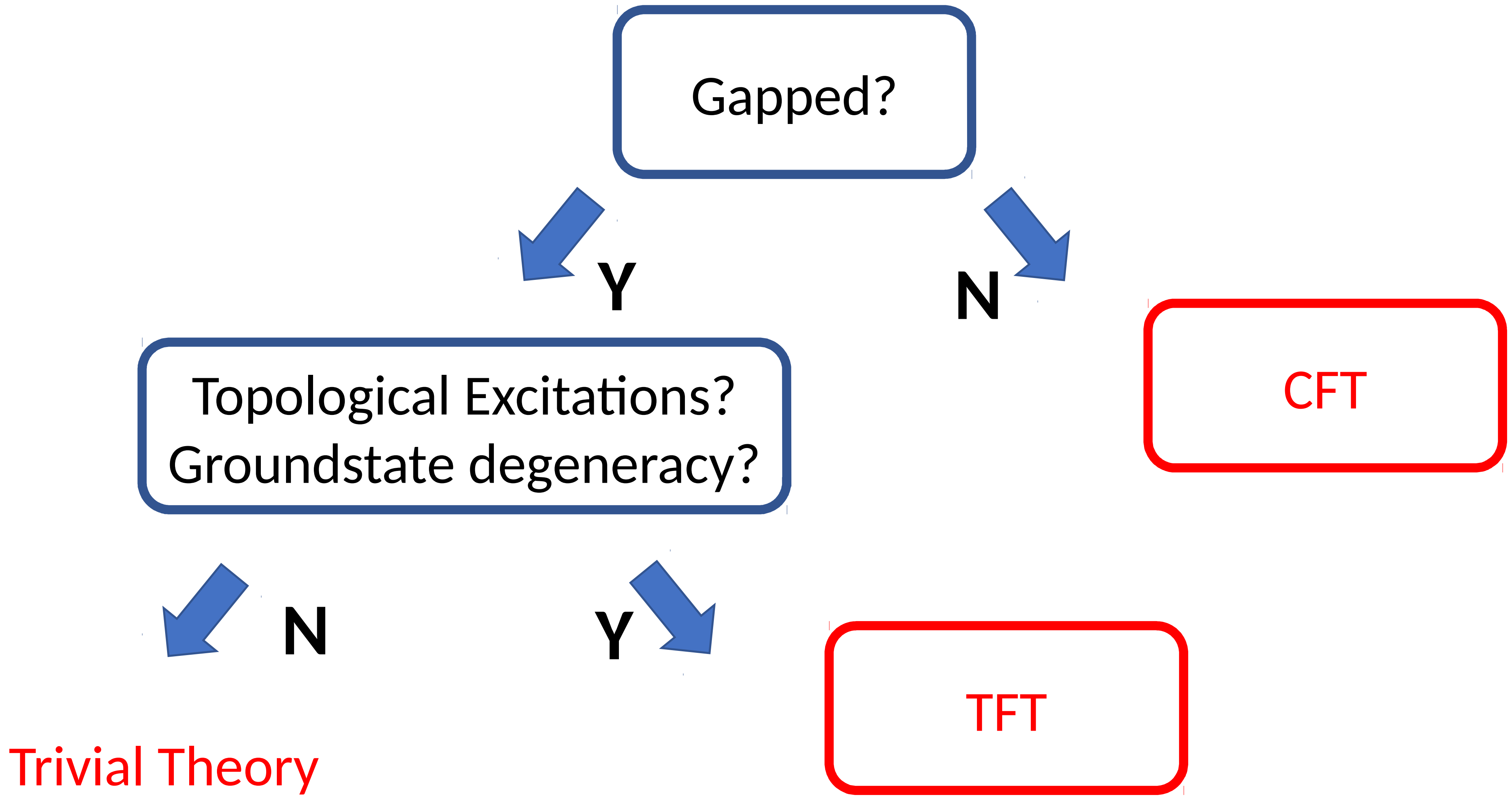


QFT as the Universal Language



Action uniquely
determined by just
a few low energy constants.

Very few possibilities



Wilsonian Paradigm

Classify all TFTs and CFTs

Including non-relativistic CFTs,
and maybe also scale invariant but not conformally invariant theories, which seem to exist if relax requirements of Lorentz invariance

Classify all Phases of Matter

Fractons

- Groundstate degeneracy but not topological

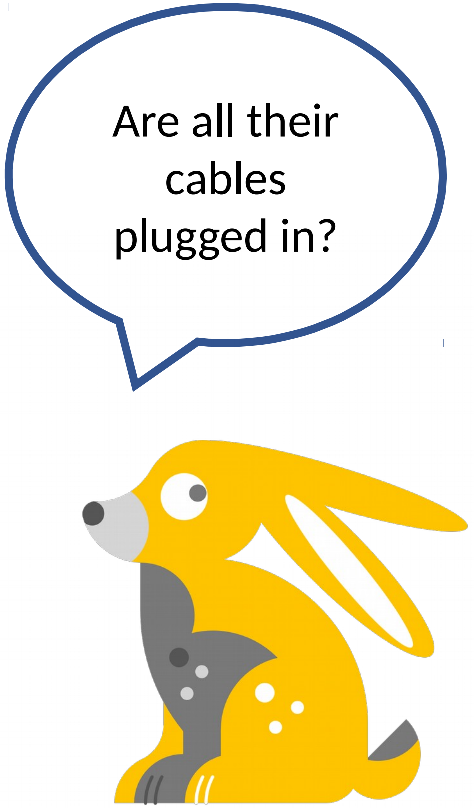
Number of GS grows with volume

- Excitations with limited mobility (for example moving along lines and/or planes)

- Subsystem symmetries

Surely an experimental error……

Whenever a dearly held theoretical believe
is challenged, our first impulse is to blame it on
the experimentalists……



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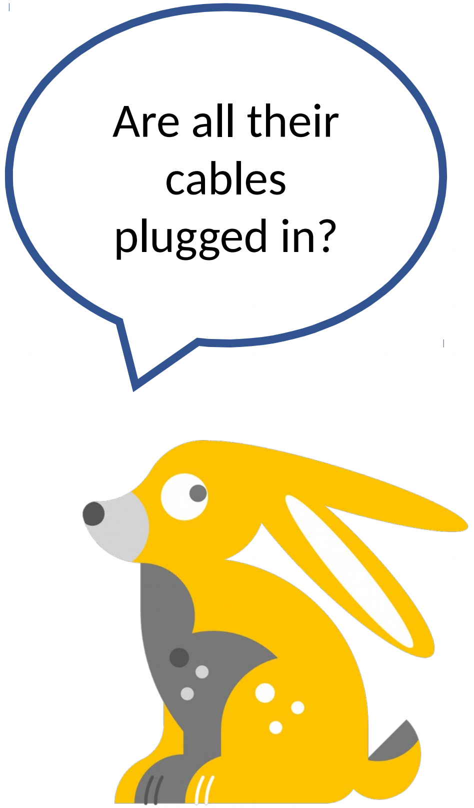


Problem: Fractons are a purely theoretical construct. In fact, at this stage we all can only wish they have some practical applications. Exactly solvable lattice models.

Simplest example: the X-cube model

(Vijay, Haah, Fu)

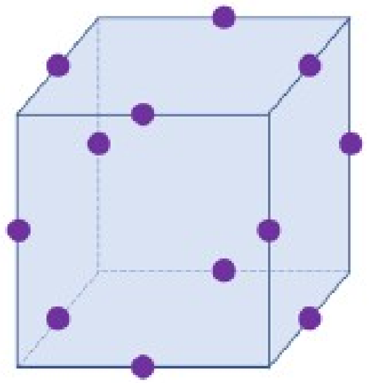
Lattice model with Ising spins on each lattice link.

Only interactions between near-by spins.

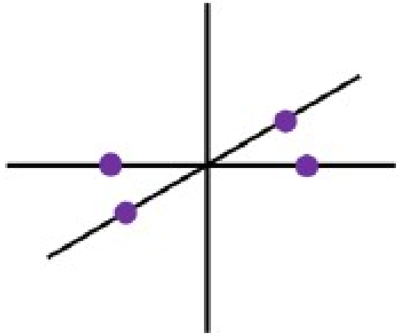
Generalization of Kitaev’s toric code
(“coupled layers of toric code”)

Simplest example: the X-cube model

(Vijay, Haah, Fu)



$$B_c = \prod_{\partial c} X$$



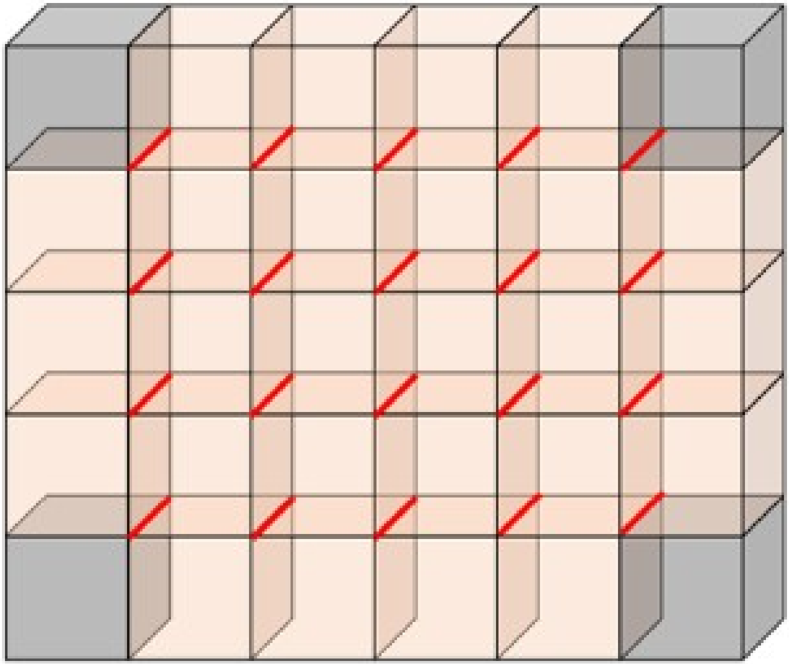
$$A_v^z = \prod_{+_{xy}} Z$$

Summed over all cubes, and over all crosses (in all orientations)

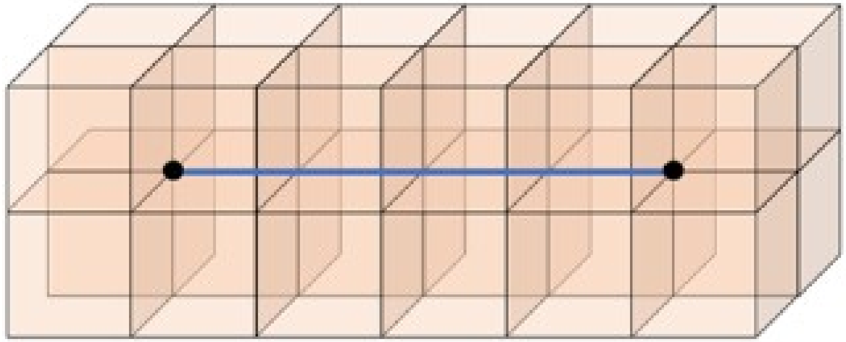
Commuting Projector Hamiltonian:

Groundstates simultaneous eigenstates of all A's and B's with eigenvalue -1. Spectrum gapped!

Fractons in the X-cube model

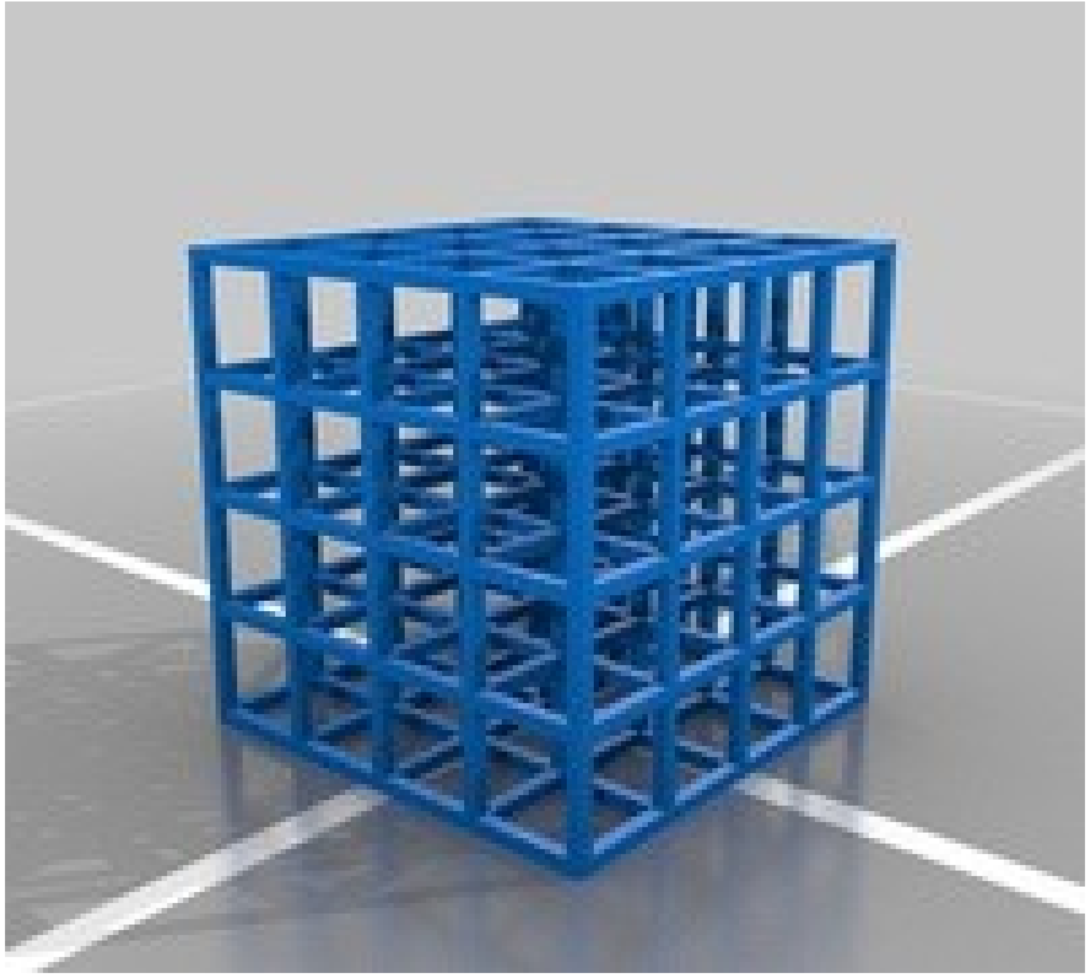


Z



X

Subsystem symmetry



Standard Ising spins:

Flipping all spins is a symmetry

X-cube:

Flipping all spins on any given plane is a symmetry

Subsystem symmetry

In fact, every plane has two Z_2 symmetries.

These symmetries take ground states to ground states

$$\#_{GS} = 4^{\#planes} = 2^{2L_x + 2L_y + 2L_z - 3}$$

QFTs for Fractons?

Fractons do NOT allow a field theory description in terms of "standard" QFTs.

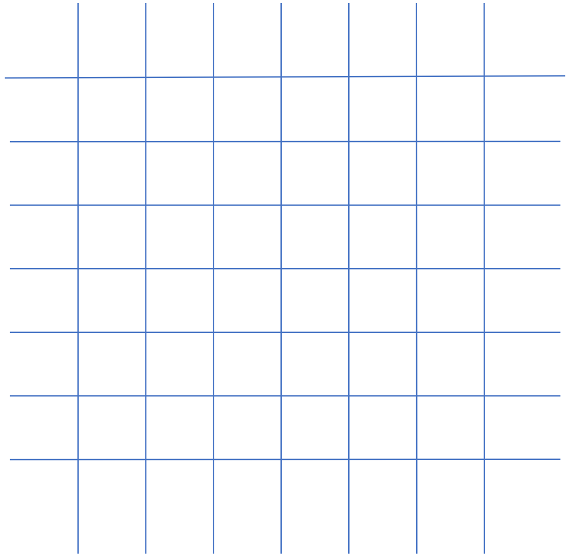
To describe their low-energy physics we need to extend our notion of what a QFT "is".

What does a "QFT for fractons" look like?

Two basic approaches

Lattice →

(Slagle, ….)



Foliated Field theory

each layer: 2+1d QFT,
subsystem symmetry acts on layers

→

Continuum QFT with discrete rotation symmetry

(Paramekanti, Balents, Fisher)

(Seiberg-Shao, …..)

retain some sensitivity to lattice scale!

The simplest Seiberg-Shao theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

- This is a 2+1 dimensional theory, gapless
- continuous translations
- 90 degree rotations: x→y, y→-x
- more bells and whistles needed for X-cube

"The XY model"

(Paramekanti, Balents, Fisher)

The simplest Seiberg-Shao theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

Subsystem symmetry:

$$\phi(t,x,y) \to \phi(t,x,y) + c^x(x) + c^y(y)$$

x labels lines. Shifting the field at every point along a line of fixed x is a symmetry.

The simplest Seiberg-Shao theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

The theory is free!

- For a single scalar, all interactions consistent with symmetries are irrelevant!
- Interacting lattice fractons → free field theory
- Interacting theories possible if we couple SeSh scalar to other fields (more later)

Dispersion relation

$$\omega^2 = \frac{1}{\mu\mu_0} k_x^2 k_y^2 \,.$$

- k_x=0 gives ω=0 for any k_y

UV/IR mixing!

- These would-be zero modes are lifted by quantum effects; mass set by lattice scale

New field theories = new opportunities!

Now that we changed the rules of QFT,
we discovered a brand new **playground!**

Let's go play .....

A vast new realm of possibilities….

Today:

- Subsystem conformal symmetry
- Spontaneous breaking of subsystem symmetry
- Perturbative Interactions
- String Theory embedding
- Fracton QFTs with boundaries

A vast new realm of possibilities....

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Subsystem Conformal Symmetries

(AK, Amir Raz)

Do SeSh-type theories allow conformal symmetries?
What’s their structure?

Subsystem Conformal Symmetries

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

Simplest 2+1 SeSh scalar

Two independent scale symmetries:

$$D_x : t \to \lambda t \,,\ x \to \lambda x$$

$$D_y : t \to \lambda t \,,\ y \to \lambda y$$

Can they be extended to full conformal symmetries?

Yes! Subsystem Conformal Symmetries

Fourier transform in y:

$$\mathcal{L} = \left|\partial_t \phi_{k_y}\right|^2 + k_y^2 \left|\partial_x \phi_{k_y}\right|^2 .$$

standard 1+1 dimensional relativistic scalar, k_y^{-1} = "speed of light"

→ D_x part of standard relativistic 1+1 CFT in x and t

and similar for D_y

Full subsystem conformal algebra

- D_x and D_y do not commute
- Infinitely many additional generators needed to close algebra
- simpler models exist in which we can work out the full algebra

Spontaneous Breaking of subsystem symmetry

(Jacques Distler, AK, Amir Raz)

Subsystem Symmetry Breaking

If subsystem symmetries are such a defining feature of these novel field theories, what can we say about their breaking?

Can subsystem symmetries be spontaneously broken?

What are good diagnostics? Examples?

Coleman-Mermin-Wagner for subsystems:

Batista/Nussinov:

A subsystem symmetry acting on d+1 dimensional defects obeys the same theorems as a QFT in d+1 dimensions.

In particular, Mermin/Wagner tells us we need at least 2+1 dimensional subsystem symmetries to break continuous symmetries.

Examples?

$$\mathcal{L} = |\partial_t \phi|^2 + |\partial_x \partial_y \phi|^2 \, .$$

(2+1 Seiberg Shao)

Subsystem symmetry acting on lines.
Can not be broken.

$$\mathcal{L}_{3'} = \frac{\mu_0}{2}(\partial_0 \theta)^2 - \frac{1}{2\mu}(\partial_x \partial_y \partial_z \theta)^2.$$

(3+1 generalization)

Subsystem symmetry acting on lines.
Can not be broken.

$$\theta \to \theta + c_x(y,z) + c_y(z,x) + c_z(x,y)$$

A pair (x,y) labels a line in the x-direction. This is a subsystem symmetry acting on lines labelled by (x,y)

Examples?

$$\mathcal{L}_3 = \frac{\mu_0}{2}(\partial_0\theta)^2 - \frac{1}{2\mu}\left[(\partial_x\partial_y\theta)^2 + (\partial_z\partial_x\theta)^2 + (\partial_y\partial_z\theta)^2\right]$$

$$\theta \to \theta + c_x(x) + c_y(y) + c_z(z)$$

(Alternate 3+1 theory)

Subsystem symmetry acting on planes.
Can be broken.

Alas …… **it isn't!**

(Seiberg, Shao)

Symmetry Breaking and the Spectrum

Broken symmetry should give rise to Goldstone Bosons.

Can we just look for E=0 modes protected by shift symmetry.?

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

$$\phi(t,x,y) \to \phi(t,x,y) + c^x(x) + c^y(y)$$

$$\omega^2 = \frac{1}{\mu\mu_0}k_x^2k_y^2\,.$$

Naively this IS already theory of Goldstone bosons!

Symmetry broken?

Quantum mechanically energies pushed to lattice scale!

Symmetry restored!

Diagnosing symmetry breaking

A more reliable diagnostic of symmetry breaking are correlators:

$$\langle \mathcal{O}^\dagger(\vec{r})\mathcal{O}(0)\rangle \xrightarrow{r\to\infty} \begin{cases} = 0 & \textbf{symmetry unbroken} \\ \neq 0 & \textbf{symmetry broken} \end{cases}$$

O is operator charged under symmetry! Correlator itself charge neutral.

$$\langle \mathcal{O}^\dagger(\vec{r})\mathcal{O}(0)\rangle \to \langle \mathcal{O}^\dagger(\vec{r})\rangle \langle \mathcal{O}(0)\rangle \quad \text{by locality}$$

Non-vanishing correlators indicates non-vanishing vev.

Correlators for Subsystem Symmetry Breaking

$$\left\langle e^{i[\theta(0,0,0)-\theta(0,x_1,0)-\theta(0,0,x_2)+\theta(0,x_1,x_2)]}\right\rangle = e^{-2K_d(x_1,x_2)}$$

- The simplest neutral correlators of charged operators is 4-pt function of exponentials
- Theory is Gaussian – we can calculate this by Wick contractions!
- Long distance behavior determines symmetry breaking.
- Unbroken symmetry = diverging, positive K

K in 2+1d Seiberg-Shao theory

(Seiberg, Shao; Distler, Raz, AK)

$$K_2(x,y) = \frac{\sqrt{\mu\mu_0}}{2\pi^2}\int_0^{\Lambda_x} dk_1 \frac{(1-\cos(k_1 x))}{k_1}\int_0^{\Lambda_y} dk_2 \frac{(1-\cos(k_2 y))}{k_2}$$
$$= \frac{\sqrt{\mu\mu_0}}{2\pi^2}\left(\gamma + \log(x\Lambda_x) + O(1/\Lambda_x)\right)\left(\gamma + \log(y\Lambda_y) + O(1/\Lambda_y)\right).$$

- μ and μ_0 are coefficients of the kinetic terms
- Λ_x and Λ_y are cutoffs on x and y momenta

The fact that cutoff appears with position dependent term prohibits standard UV regularization!

Reflects the fact that the lightest charged excitations get pushed to lattice scale

Symmetry restoration

- K intrinsically divergent
- no meaningful finite value can be assigned
- Symmetry restored
- Significant UV/IR mixing; UV cutoff depends on IR properties
- Same results hold in other dimensions. No symmetry breaking, even when allowed.

A modified model with symmetry breaking!

$$L = \frac{1}{2g} \sum_{1 \leq i < j \leq d} \partial^i \partial^j \phi \partial_i \partial_j \phi.$$

"Classical" XY-model

NO time derivative ---

same lattice theory, but viewed as statistical mechanics model

We can Wick rotate one dimension to obtain theory with unusual kinetic term and subsystem symmetry that allows shifts of the field that depend on time!

Subsystem symmetry living on codimension 1 spacetimes manifolds!
Breaking allowed when d>3

Correlators in classical XY model

$$\left\langle e^{i[\phi(0,0)-\phi(x_1,0)-\phi(0,x_2)+\phi(x_1,x_2)]}\right\rangle = e^{-2gI_d(x_1,x_2)},$$

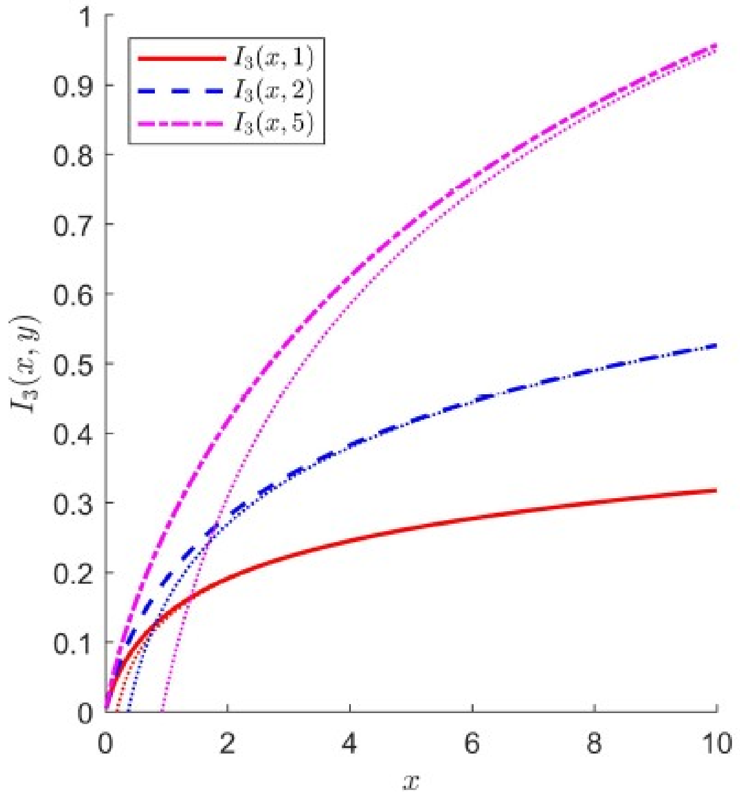
$$I_d(x,y) = \int \frac{d^dk}{(2\pi)^d}\frac{(1-\cos(k_1x))(1-\cos(k_2y))}{\sum_{i\neq j}k_i^2k_j^2}.$$

from Wick contractions

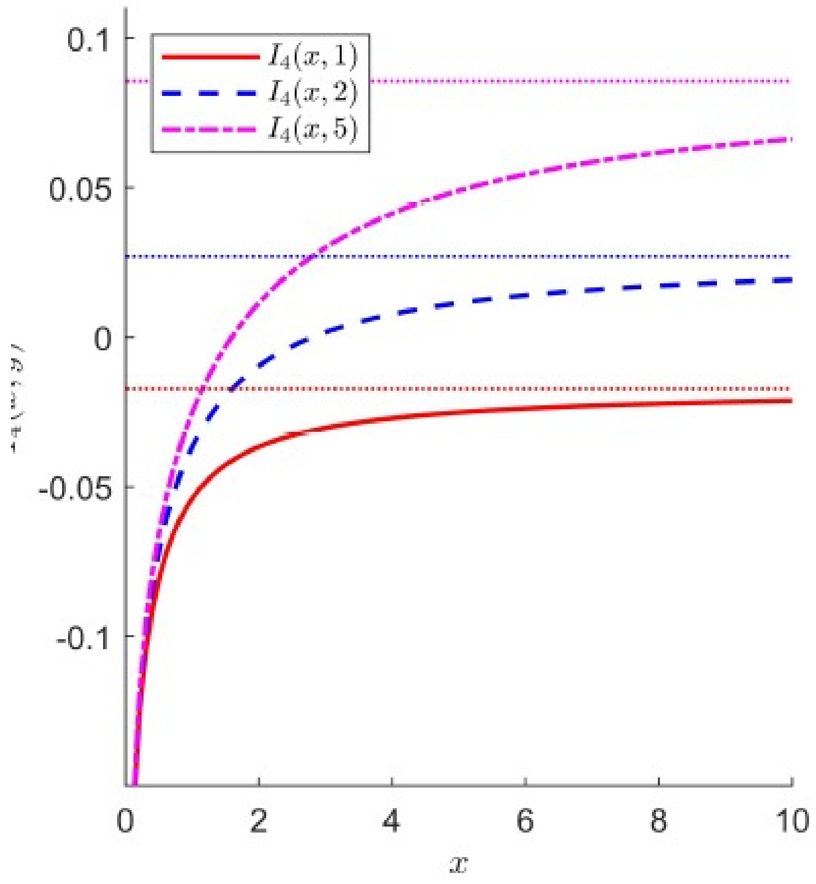
$$I_2(x,y) = \frac{|xy|}{4}.$$

symmetry restored

Correlators in Classical XY model



d=3: symmetry restored



d=4: symmetry broken

Generalizations

Can write down models with subsystem symmetries on higher codimension sub-manifolds:

$$L = \frac{1}{2g} \sum_{i_1 < i_2 < \ldots < i_m = 1}^{d} \partial^{i_1} \partial^{i_2} \ldots \partial^{i_m} \phi \, \partial^{i_1} \partial^{i_2} \ldots \partial^{i_m} \phi.$$

$$\phi \to \phi + f_{i_1, i_2, \ldots, i_{m-1}}(x_{i_1}, x_{i_2}, \ldots, x_{i_{m-1}})$$

Symmetry breaking happens whenever allowed (d>m+1)

Rich structure emerges already at the level of free theories!!

Perturbative Interactions

(Jacques Distler, Murtaza Jafry, Amir Raz, AK)

Perturbative Interactions

SeSh scalar by itself allows no marginal or relevant interactions.

SeSh scalar + Fermion does allow interesting interactions!
(so does theory of a complex scalar)

Simplest interacting QFT with subsystem symmetry?

SeSh theory with perturbative control

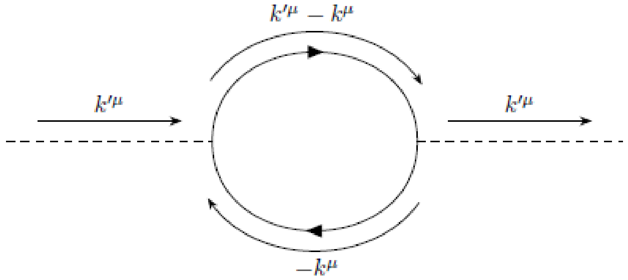
$$\mathcal{L}_{scal} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

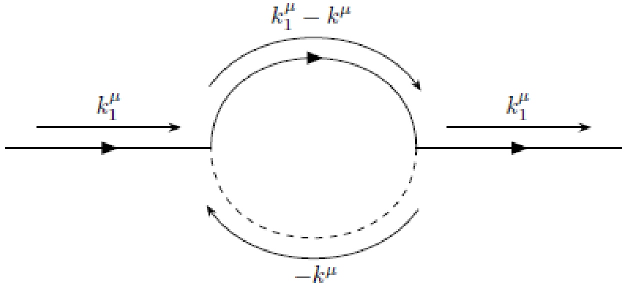
$$\mathcal{L}_{fer} = \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2m}\right)\psi$$

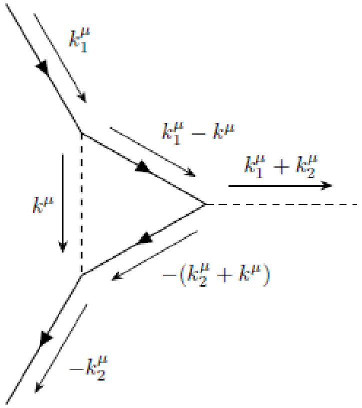
$$\mathcal{L}_{int} = \lambda\psi^\dagger\psi\partial_x\partial_y\phi$$

- **Scalar transforms non-trivial under Z4**
- **[t]=-2, [x]=-1, [φ]=0, [ψ]=1**
- **Interaction Marginal!**

Textbook QFT!







$$\beta(\lambda) = \Lambda\frac{\partial}{\partial\Lambda}\left(-\delta_\lambda + \frac{1}{2}\lambda(2\delta_{Z_\psi})\right) = 0,$$
$$\beta(1/m) = \Lambda\frac{\partial}{\partial\Lambda}\left(-\delta_{1/m} + \frac{1}{2}\frac{1}{m}(2\delta_{Z_\psi})\right) = \Xi_2,$$
$$\beta(1/\mu) = \Lambda\frac{\partial}{\partial\Lambda}\left(-\delta_{1/\mu}\right) = \frac{m\lambda^2}{2\pi}.$$

One-loop beta function for coupling vanishes!!

A vanishing beta function?

$$\mathcal{L}_{fer} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi \qquad\qquad \mathcal{L}_{int} = \lambda \psi^\dagger \psi \partial_x \partial_y \phi$$

Novel symmetry: $\psi \to e^{i\alpha t}\psi \, , \; \phi \to \frac{\alpha}{\lambda} x \, y$

This symmetry links anomalous dimensions of fermion to vertex correction to all orders in perturbation theory!

String Theory Embeddings

(Hao Geng, Shamit Kachru, AK, Richard Nally, Brandon Rayhaun)

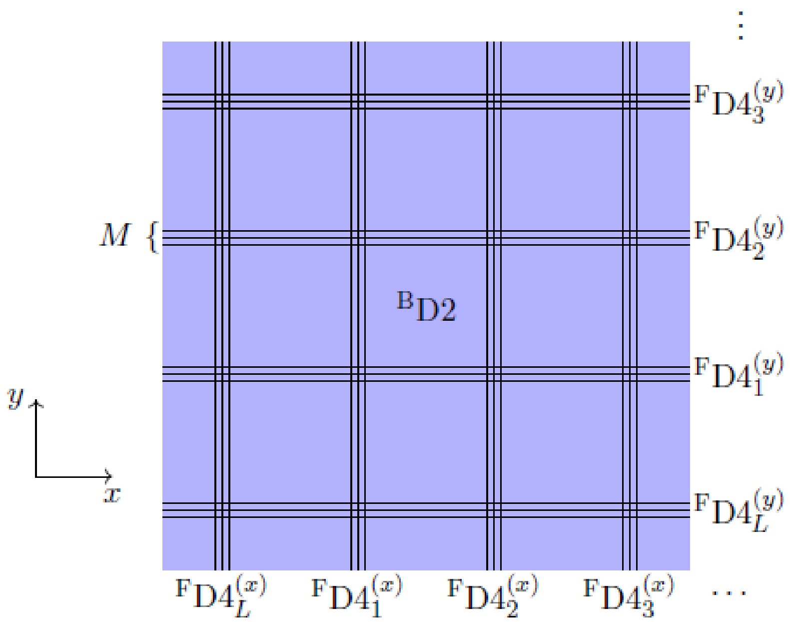
Hanany-Witten and beyond....

For supersymmetric field theories, many structures could be "geometrized" by embedding them in string theory.

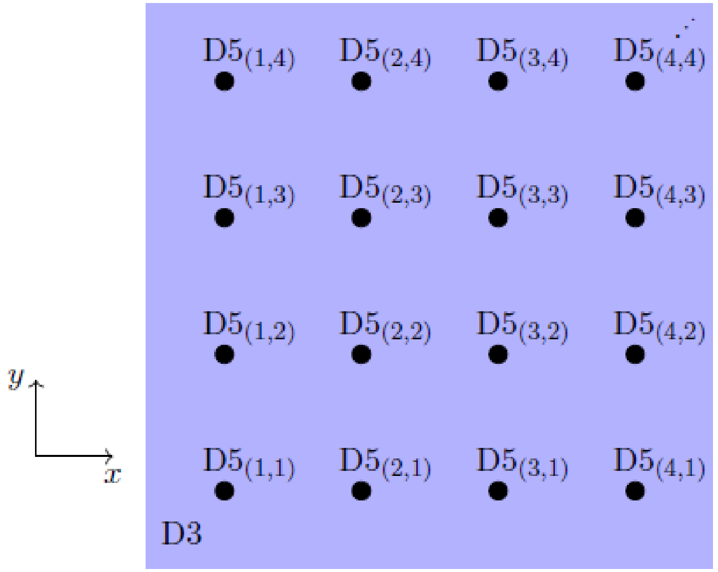
Gauge theory realized on worldvolume of D-branes.

This can be done very naturally for both foliated field theories and exotic field theories (Seiberg-Shao type theories).

String theory fractons



Foliated



Exotic

Fracton QFTs with Boundaries

(Zhu-Xi Luo, Ryan Spieler, Haoyu Sun, AK)

Boundaries

For topological phases of matter, much of the interesting physics gets only revealed in the presence of boundaries

How about fractons?

We use both continuum QFT and lattice methods to count groundstate degeneracies (for gapped boundaries) on T^2 x interval

Boundary vs Bulk

Interestingly, the boundary conditions affect even the extensive part of the GSD!

$$GSD_{T^2\times I} = N^{L_x+L_y+2L_z-1}$$

xz and yz plane degeneracies change!

z-direction on interval

$$GSD_{T^3} = N^{2L_x+2L_y+2L_z-3}$$

The End.

That’s all I have for today.

We just scratched the surface on these novel field theories.

Holography?

Lots more to come.

Remains to be seen if any of this is useful for anything.