Damping of pseudo-Goldstone bosons

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Holotube

References and acknowledgments

- Ongoing work with Luca Delacrétaz (Chicago) and Vaios Ziogas (Ecole Polytechnique), to appear very soon.
- Wigner crystal hydro with pinning and/or *B*-field: [1612.04381],[1702.05104], [1904.04872] with Luca Delacrétaz, Sean Hartnoll and Anna Karlsson.
- Holographic phases breaking translations pseudo-spontaneously: [1812.08118], [1910.11330], with Andrea Amoretti, Daniel Areán, Daniele Musso.
- More holographic work by other authors (pseudo-spontaneous breaking of translations and of U(1)): [2107.03680], [1904.05785], [1905.09488], [1905.00398], [1906.03132], [2101.05343], [2107.00519], [2111.10305]
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Spontaneous symmetry breaking: Generalities

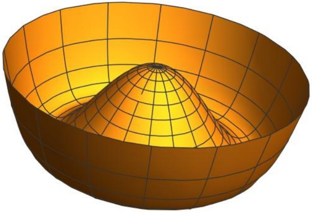
- A continous symmetry is said to be spontaneously broken when the ground state has a **lower amount of symmetry** than the Hamiltonian/Lagrangian/action of the system itself.
- This typically occurs when the system goes through a **phase transition** (thermal, quantum) from a disordered state to an order state: an 'order parameter' condenses $\equiv$ obtains a **nonzero vev**.
- This has a macroscopic manifestation: a new set of **gapless modes** appear, the Goldstone bosons. Their number depends on the number of broken generators and the type of broken symmetry (internal vs spacetime).

[Beekman, Rademaker & van Wezel, SciPost Lecture Notes 2019]

Baby example: SSB of U(1)

- Consider a complex scalar $\Phi = \phi e^{i\varphi}$ with the following Mexican hat potential:

[Schmitt, Intro to Superfluidity 2014]



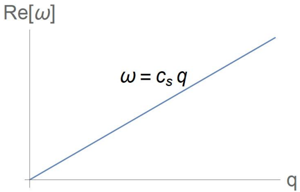
$$V(\Phi) = V_o - \frac{1}{2} m^2 \Phi\Phi^* + \frac{\lambda}{2} (\Phi\Phi^*)^2 \,,$$

$$\lambda, m^2 > 0$$

- The potential does not depend on the value of the phase: U(1) symmetry. Below a critical temperature, the system condenses in a state with a nonzero vev $\langle \phi \rangle = m/\sqrt{\lambda}$.

- Phase: gapless degree of freedom (flat direction)

$$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_t \varphi)^2 + \frac{c_s^2}{2}(\vec{\partial}\varphi)^2$$

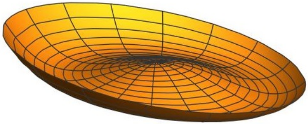


Baby example: pseudo-SSB of U(1)

- Turn on a small, phase-dependent deformation of the potential

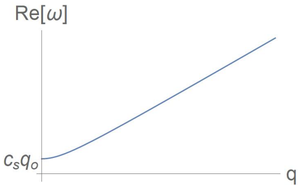
$$V(\Phi) = V(|\Phi|) + \delta V(|\Phi|, \varphi)$$

- **Breaks the U(1) symmetry explicitly**: gaps out the Goldstone.



- Acquires a small mass $q_o^2 \sim \partial_\varphi^2 \delta V$, related to the scale of explicit symmetry breaking (Gell-Mann-Oakes-Renner).

$$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_t \varphi)^2 + \frac{c_s^2}{2}\left((\vec{\partial}\varphi)^2 + q_o^2\varphi^2\right)$$

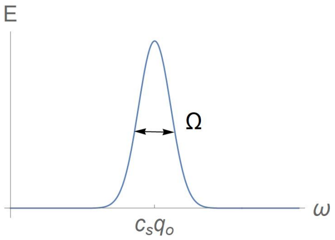


- **Symmetry breaking scale small compared to vev**: pseudo-Goldstone, light dof.

Key point of this talk

- At nonzero temperature and including dissipation, **locality** fixes the **damping rate** of pseudo-Goldstones in terms of the Goldstone **mass** q_o^2 and **diffusivity** D_φ, to leading order in the scale of explicit symmetry breaking:

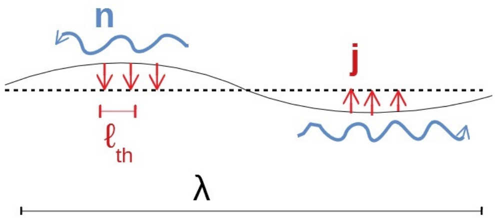
$$\boxed{\Omega = q_o^2 D_\varphi}$$



- This relation was first uncovered in holographic analyses of broken symmetry phases: applied holography success!

- This applies to a variety of cases and physical systems: U(1) (superfluid), translations (crystals, density waves), rotations (nematic phases), QCD in the chiral limit, and likely others we have not explored yet (pseudo-dilatons...).

Locality of hydrodynamic equations



- At scales large compared to the thermalization scales τ_{th}, ℓ_{th}, interacting systems can be described by hydrodynamics, ie by the **slow** relaxation of their **conserved densities**

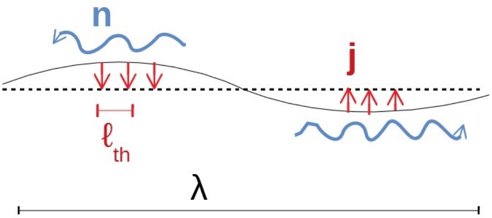
$$\partial_t n_a(t,x) + \nabla \cdot j_a(t,x) = 0$$

- At such scales, the j's are **fast** variables, which relax quickly in the bath of the n's. They are in local equilibrium and can be expanded locally in terms of the conserved densities

$$j_a = \alpha_{ab} n_b - D_{ab} \nabla n_b + O(\nabla^2)$$

[Kovtun'12, Glorioso & Liu'18]

Locality of hydrodynamic equations



- Plugging back in the conservation equations:

$$\partial_t n(t,q) + \boxed{M(q)} \cdot n(t,q) = 0$$

$\boxed{M(q)}$ is automatically **local** at scales large compared to ℓ_{th}:

$$M(q\ell_{th} \ll 1) = M_0 + M_1 \ell_{th} q + M_2 \ell_{th}^2 q^2 + O(\ell_{th}^3 q^3)$$

Locality of hydrodynamic equations: sources

- We now turn on external sources for the conserved densities

$$H_0 \mapsto H(t) = H_0 - \int d^d x \delta\mu^a(t,x) n_a(t,x)$$

- The equations of motion change as [Kadanoff & Martin, Chaikin-Lubenski]

$$\partial_t n(t,q) + M(q) \cdot \Big(n(t,q) - \chi(q) \cdot \delta\mu(t,q) \Big) = 0$$

- $\chi(q)$ is the matrix of static susceptibilities derived from the equilibrium thermal free energy $W(q)$

$$\chi_{ab}(q) = -\frac{\delta^2 W(q)}{\delta\mu_a(q)\delta\mu_b(-q)}\,, \quad W = -\frac{1}{\beta}\log \mathrm{Tr} e^{-\beta H}$$

Locality of hydrodynamic equations

$$\partial_t n(t,q) + M(q)\cdot n(t,q) - \boxed{M(q)\cdot\chi(q)}\cdot\delta\mu(t,q) = 0$$

- The static susceptibilities themselves are **local** at scales larger than the **thermal screening length**: $\boxed{M\cdot\chi}$ is also local.
- In hydrodynamic regime $t \gg \tau_{th}$, $x \gg \ell_{th}$, **integrating out the hydrodynamic modes only source of non-locality**.
- Eg in an effective action approach, integrating out the modes gives the generating function in terms of sources only, matrix-multiplied by the retarded Green's functions which are non-local due to the hydrodynamic modes [Crossley, Glorioso & Liu'15], [Haehl, Loganayagam & Rangamani '15].
- **The hydrodynamic equations of motion with sources on must be local.**

Locality of hydrodynamic equations: pinning

$$\partial_t n(t,q) + M(q) \cdot (n(t,q) - \chi(q) \cdot \delta\mu(t,q)) = 0$$

- When a Goldstone mode is present in the spectrum, and a small symmetry breaking term is turned on,

$$\chi_{\nabla\varphi\nabla\varphi} \sim \frac{q^2}{q^2 + q_o^2}$$

- Locality of $M \cdot \chi$ is no longer automatic – **restoring it will impose constraints on transport coefficients** (which generally do not follow from other constraints, eg Onsager relations or positivity of entropy production).

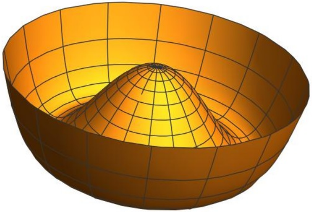
- We have discussed it at the level of eoms, but it is naturally implemented in the Schwinger-Keldysh construction of effective actions for hydrodynamics [Crossley, Glorioso & Liu'15],[Hael, Loganayagam & Rangamani'15].

Example 1: U(1) symmetry breaking (superfluid)

- Consider a system with a global U(1):

$$\partial_t n + \nabla \cdot j = 0$$

- If the U(1) is spontaneously broken, a gapless mode φ appears in the spectrum. The free energy density becomes



$$f = \frac{n_s}{2} \nabla\varphi^2 - \delta s_\varphi \varphi - \frac{1}{2}\chi_{nn}\delta\mu^2$$

- The static susceptibilities are

$$\chi(q) \simeq \begin{pmatrix} \chi_{nn} & 0 \\ 0 & \frac{1}{n_s q^2} \end{pmatrix}.$$

$\chi_{\varphi\varphi}$ diverges as $q \to 0$: long-range order.

Example 1: U(1) symmetry breaking (superfluid)

- The j constitutive relation and Josephson relation are

$$j = n_s \nabla \varphi - D_n \nabla n + \cdots \,, \qquad \dot{\varphi} = -\frac{1}{\chi_{nn}} n + D_\phi \nabla^2 \varphi + \cdots \,,$$

so that both matrices of interest are **local**

$$M(q) \simeq \begin{pmatrix} D_n q^2 & -n_s q^2 \\ \frac{1}{\chi_{nn}} & D_\varphi q^2 \end{pmatrix} \,, \quad M(q)\chi(q) \simeq \begin{pmatrix} \chi_{nn} D_n q^2 & -1 \\ 1 & \frac{D_\varphi}{n_s} \end{pmatrix} .$$

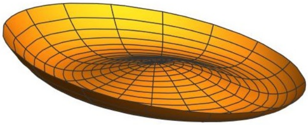
- The hydrodynamic modes are the well-known **second sound**:

$$\omega = \pm c_s q - \frac{i}{2}(D_n + D_\varphi) q^2$$

- Positivity of dissipation imposes $D_n, D_\varphi > 0$.

Example 1: U(1) symmetry breaking (superfluid)

- Now consider breaking the symmetry explicitly. The free energy density becomes



$$f = \frac{n_s}{2}\left(\nabla\varphi^2 + q_o^2\varphi^2\right) - \delta s_\varphi \varphi - \frac{1}{2}\chi_{nn}\delta\mu^2$$

  - The static susceptibilities are

$$\chi(q) \simeq \begin{pmatrix} \chi_{nn} & 0 \\ 0 & \frac{1}{n_s(q^2 + q_o^2)} \end{pmatrix}.$$

$\chi_{\varphi\varphi}$ no longer diverges as $q \to 0$.

Example 1: U(1) symmetry breaking (superfluid)

$$H \mapsto H_o + \delta H\,, \quad [H_o, n] = 0\,, \quad \delta H = g \int d^d x\, O\varphi\,, \quad i[n, O] = O$$

- Integrating out O, the conservation equation changes to

$$\boxed{\partial_t n + \nabla \cdot j = -\Gamma\, n + n_s q_o^2 \varphi}$$

- Γ is given by the imaginary part of G^R_{OO}

$$\Gamma = \frac{1}{\chi_{nn}} \lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{\partial_t n \partial_t n} = \frac{g^2}{\chi_{nn}} \lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{OO}$$

- The second term is given by the real part of G^R_{OO}

$$n_s q_o^2 = \lim_{\omega\to 0} \mathrm{Re}\, G^R_{\partial_t n \partial_t n} = g^2 \lim_{\omega\to 0} \mathrm{Re}\, G^R_{OO}$$

- This can all be made precise in the SK formalism.

Example 1: U(1) symmetry breaking (superfluid)

- New symmetry breaking term allowed in Josephson equation

$$\dot{\varphi} = -\Omega\varphi - \frac{1}{\chi_{nn}} n + D_\phi \nabla^2 \varphi + \cdots ,$$

- The matrices appearing in the eoms are now

$$M(q) \simeq \begin{pmatrix} \Gamma + D_n q^2 & -n_s(q^2 + q_o^2) \\ \frac{1}{\chi_{nn}} & \Omega + D_\varphi q^2 \end{pmatrix} , \quad \Rightarrow \quad \text{local}$$

$$M(q)\chi(q) \simeq \begin{pmatrix} \chi_{nn}(\Gamma + D_n q^2) & -1 \\ 1 & \frac{\Omega + D_\varphi q^2}{n_s(q^2 + q_o^2)} \end{pmatrix} .$$

local **only if** $\boxed{\Omega = q_o^2 D_\varphi}$.

- Demanding locality is not optional: we did not integrate out the massive mode.

Example 1: U(1) symmetry breaking (superfluid)

- In general hydro frames, locality will still need to be restored by fixing Ω.
- One specific frame choice is helpful (passing over to relativistic notation):

$$u^\mu \partial_\mu \varphi = -\mu$$

  (no dissipative corrections to the Josephson relation)

- Then, it suffices to write dissipative corrections to the current:

$$J^\mu = n u^\mu + \frac{n_s}{\mu} P^{\mu\nu} \partial_\nu \varphi - \chi_{nn} D_n P^{\mu\nu} \partial_\nu \mu + \chi_{nn}^2 \frac{\mu}{n_s} D_\varphi u^\mu u^\lambda \partial_\lambda \mu$$

  together with

$$\partial_\mu J^\mu = \frac{n_s}{\mu} q_o^2 \varphi - \Gamma J^t$$

- This is the natural frame we land on in the Schwinger-Keldysh formulation, which is local by construction all the way to the hydro cut-off instead of q_o.

Example 1: U(1) symmetry breaking (superfluid)

- Ω: damping of the Goldstone due to weak explicit breaking

$$\omega = \pm q_o c_s - \frac{i}{2}\left(\Gamma + \Omega\right) + O(q_o^3)$$

Fixed by locality in terms of the Goldstone mass and diffusivity

$$\boxed{\Omega = q_o^2 D_\varphi}$$

It has nothing to do with phase relaxation by vortices.

- This relation was recently derived in a holographic superfluid with weak explicit breaking by direct computation [Donos, Kailidis & Pantelidou, arXiv: 2107.03680]. Also verified independently by [Ammon, Areán, Baggioli, Gray & Grieninger, arXiv: 2111.10305].

Example 2: QCD in the chiral limit

- In QCD, chiral symmetry is broken by the quark mass, which gives a small gap to the pions.

- A hydrodynamic theory can be written for these soft pions [Son & Stephanov, arXiv: hep-ph/0204226]. The non-Abelian $U(1) \times SU_L(2) \times SU_R(2)$ is broken to $U(1) \times SU_V(2)$ pseudo-spontaneously.

- Locality fixes the damping rate of the axial pion in terms of the pion mass and the pion diffusivity as before (the non-Abelian character plays no role in linear response).

- [Grossi, Soloviev, Teaney & Yan, arXiv: 2005.02885] used positivity of entropy production to derive this result. This argument naively seems to fail in the Abelian case due to total derivative ambiguities. But properly constructing the entropy current can be subtle.

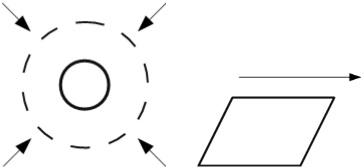
Example 3: Phases breaking translations spontaneously

- The same type of relation also appeared in earlier holographic works describing the interplay of spontaneous and explicit breaking of spatial translations, [Amoretti, Areán, Goutéraux & Musso, arXiv: 1812.08118], [Donos, Martin, Pantelidou & Ziogas, arXiv: 1905.00398] (see also [Ammon, Baggioli, Jiménez-Alba, arXiv: 1904.05785], [Baggioli and Grieninger, arXiv: 1905.09488], [Donos, Martin, Pantelidou & Ziogas, arXiv: 1906.03132]).

- In isotropic 2d crystals, there are two Goldstone modes

$$f_\varphi = \frac{1}{2}(B+G)(\partial_i \varphi^i)^2 + \frac{G}{2}(\epsilon^{ij}\partial_i \varphi_j)^2 + \frac{G}{2} q_o^2 \varphi_i^2 \,,$$

where B and G are the bulk and shear moduli respectively.



Example 3: Phases breaking translations spontaneously

- For the sake of simplicity in this talk, let's freeze charge and energy fluctuations and keep only $\pi_\parallel$, $\pi_\perp$.
- Then the relevant eoms are

$$\partial_t \varphi_i = -\Omega_{ij}\varphi_j + v^i + D_\varphi \partial_j^2 \varphi_i + \tilde{D}_\varphi \partial_i \partial^j \varphi_j \,,$$

- The same locality argument as before fixes

$$\boxed{G\tilde{D}_\varphi = BD_\varphi \,, \qquad \Omega_{ij} = q_o^2 D_\varphi \delta_{ij}}$$

consistent with the holographic results.

Example 3: Phases breaking translations spontaneously

- Restoring charge and energy fluctuations does not affect this result (new terms appear in the currents constitutive relations) but allows to identify an important phenomenological consequence.

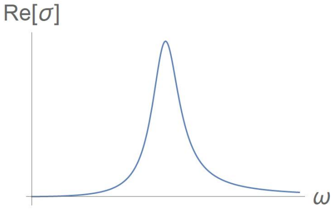
- Consider a Galilean-invariant state.

- Without Ω, the ac conductivity

$$\sigma(\omega) = \left(\frac{ne^2}{m}\right) \frac{-i\omega}{-i\omega(\Gamma - i\omega) + \omega_o^2},$$

$$\omega_o^2 = \frac{G}{mn} q_o^2 = c_s^2 q_o^2$$

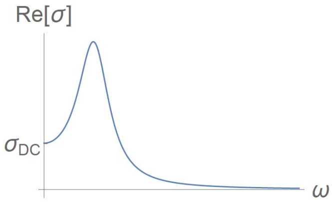
which vanishes at $\omega = 0$: insulator.



Example 3: Phases breaking translations spontaneously

- Taking into account damping $\Omega = q_o^2 D_\varphi$

$$\sigma(\omega) = \left(\frac{ne^2}{m}\right) \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$



$$\sigma_{dc} = \frac{ne^2}{m} \frac{1}{\Gamma + \frac{c_s^2}{D_\varphi}}$$

- The second contribution to the scattering rate is universal and does not depend on the strength of explicit breaking.

- Potentially interesting [Delacrétaz, B.G., Hartnoll & Karlsson'16], [Amoretti, Areán, B.G. & Musso'18] for the physics of strange metals where charge density fluctuations now observed [Arpaia & Ghiringhelli'21].

Example 3: Phases breaking translations spontaneously

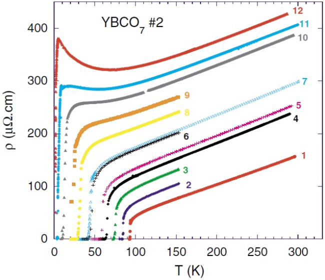
$$\rho_{dc} = \frac{m}{ne^2}\left(\Gamma + \frac{c_s^2}{D_\varphi}\right)$$

- Assume now that the diffusivity saturates a Planckian bound [Hartnoll'14]:

$$D_\varphi \simeq \frac{c_s^2}{\alpha}\frac{\hbar}{k_B T}$$

- The resistivity receives a T-linear contribution with a disorder-independent slope.

$$\rho_{dc} = \frac{m}{ne^2}\left(\Gamma + \alpha\frac{k_B T}{\hbar}\right)$$



ion-irradiated $YBCO_7$, [Rullier-Albenque et al, PRL'03] increases disorder without changing doping.

Other examples

- Wigner crystals in the presence of a large magnetic field (2DES, GaAs heterostructures). Holographic results in [Donos, Pantelidou, Ziogas'21], [Amoretti, Areán, Brattan & Martinoia'21].
- Nematic phases (spontaneously broken rotations): these are observed eg in high T_c superconductors. These materials often also have some intrinsic anisotropy.
- Ferromagnets.
- Others (eg pseudo-dilatons)?

Summary

- In the hydrodynamic regime, all non-localities are pushed beyond the hydrodynamic cut-off by construction.
- When light degrees of freedom are retained in the effective description (pseudo-Goldstones), locality is not automatic in the hydrodynamic equations, depending on the choice of frame. If so, it must be restored, and this imposes constraints on transport coefficients.
- In this regard, the Schwinger-Keldysh construction is particularly powerful as locality is automatically built-in.

Thanks!