The p-spin glass model: a holographer’s perspective

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Based on **2106.03838** with **Tarek Anous (Amsterdam)**

Outline

- Review: SYK, chaos, glassiness
- The p-spin glass model
  - Replica symmetry breaking
  - Conformal limits
  - Quantum chaos (OTOCs)
- Holographic speculations
- Conclusion

Review:

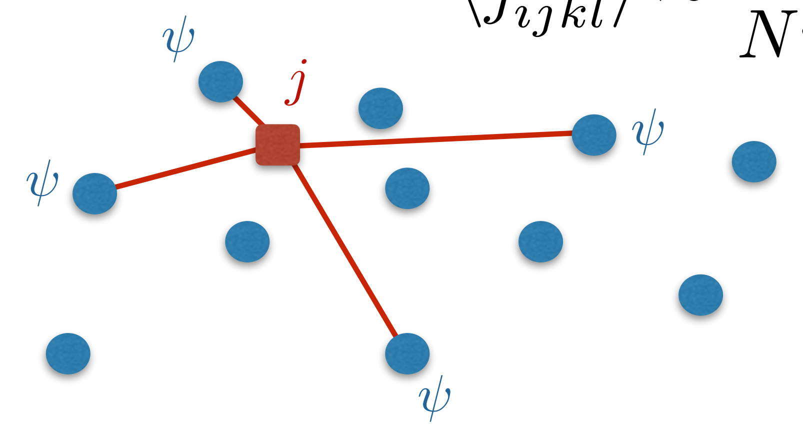
SYK, chaos, glassiness

Reminder: SYK model

$$H = -\sum_{ijkl}^{N} j_{ijkl}\, \psi_i \psi_j \psi_k \psi_l$$

- N Majorana fermions with random, Gaussian couplings

$$\langle j_{ijkl}^2 \rangle \sim \frac{\mathcal{J}^2}{N^3}$$



- Exactly solvable for $N \gg \beta J \gg 1$

- "Mean field" description at large N in terms of bilocal 2-point function

$$G(\tau,\tau') = \frac{1}{N}\sum_{i=1}^{N} \langle \psi_i(\tau)\psi_i(\tau')\rangle$$

- $\beta J \gg 1$: $S_{\text{eff}}[G]$ is approximately $\text{diff}(S^1)$ invariant, i.e., $\tau \to f(\tau)$

  - The saddle point solution breaks $\text{diff}(S^1) \to SL(2,\mathbb{R})$:

$$G_c(\tau-\tau') \propto \frac{1}{(\tau-\tau')^{2/q}}$$

[Sachdev-Ye '93] [Kitaev '15] [Maldacena-Stanford '16] ...

- The pseudo-Goldstone associated with reparametrizations $\tau \to f(\tau)$ has a 'Schwarzian' effective action:

$$I_{\text{Schw.}} \propto -\frac{N}{\mathcal{J}} \int d\tau \, \{f(\tau), \tau\}$$

- This action also describes the boundary degree of freedom associated with dilaton gravity in AdS_2 [Maldacena-Stanford-Yang '16]

- The symmetry breaking pattern also implies a near-extremal entropy of the form

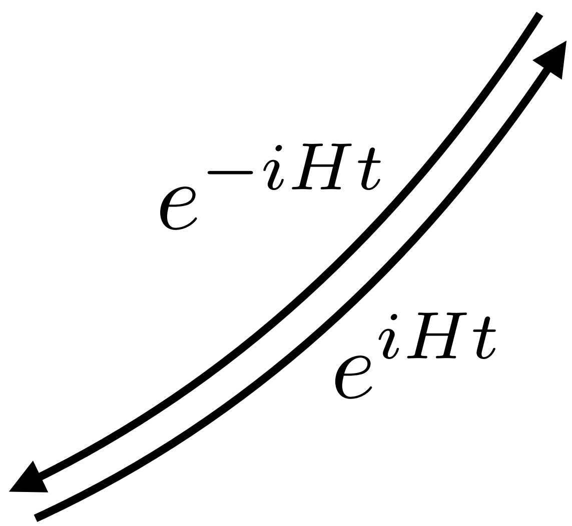
$$S = S_0 + \# \, \frac{N}{\beta \mathcal{J}}$$

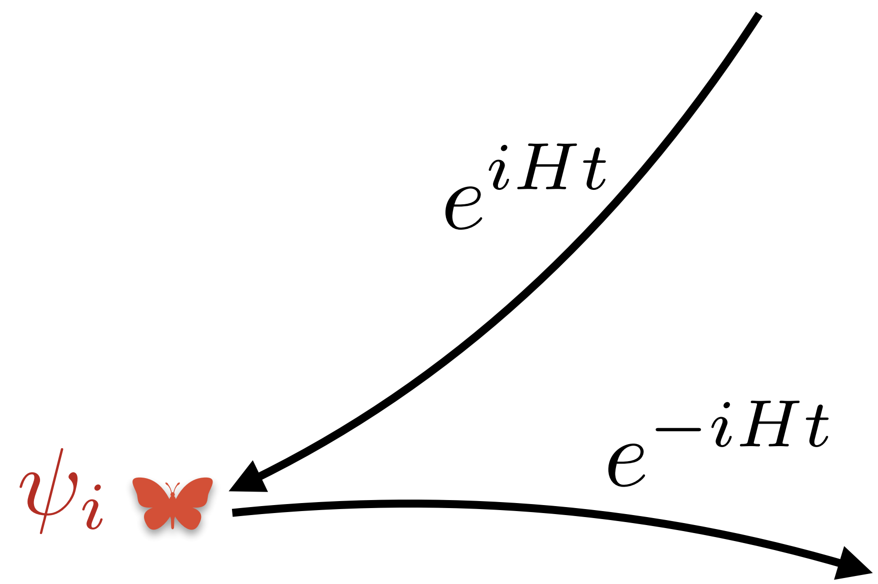
from Schwarzian

- Finally: the Schwarzian mode describes a universal contribution to out-of-time-order correlation functions (OTOCs)

Quantum butterfly effect

$\psi_i(t) = e^{iHt}\,\psi_i\,e^{-iHt}$ is 'complicated' even if ψ_i was 'simple'

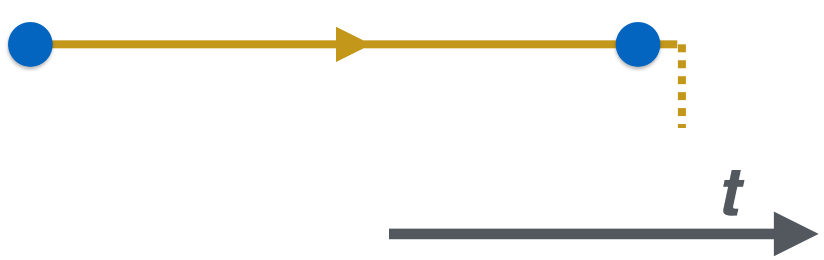




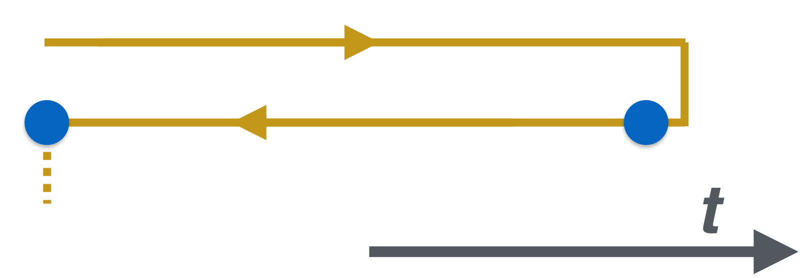
$$e^{iHt}\,\psi_i\,e^{-iHt} = \psi_i + it[H,\psi_i] - \frac{t^2}{2}[H,[H,\psi_i]] + \dots$$

To quantify this, compare the following two states:

$$|\psi_i(t)\psi_j(0)\rangle$$



$$|\psi_j(0)\psi_i(t)\rangle$$



- The OTOC quantifies how much overlap these states have:

$$\text{OTOC} = \langle \psi_i(t)\psi_j(0) | \psi_i(t)\psi_j(0) \rangle$$

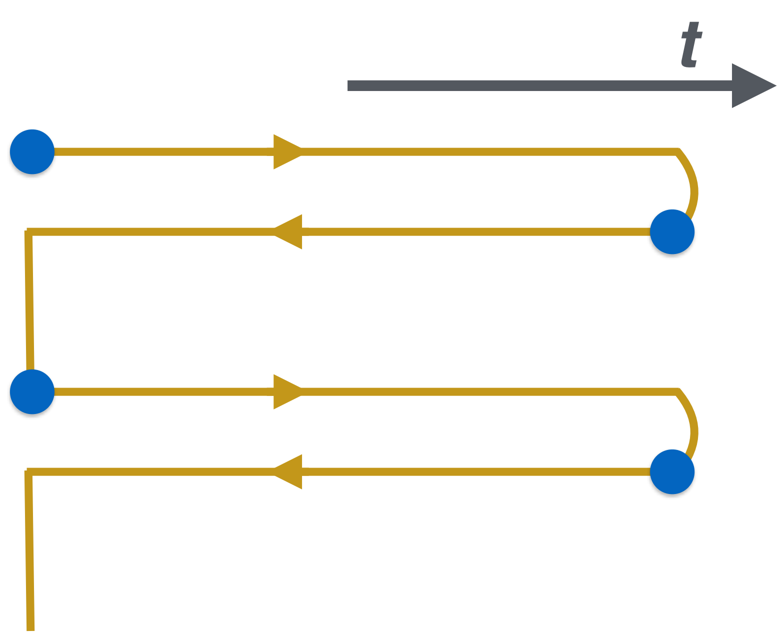
- The faster this overlap decays to 0, the more chaotic the system

[Larkin/Ovchinnikov '68] [Kitaev '14] [Shenker/Stanford '14]

- The soft mode contribution in SYK is maximally chaotic:

$$\text{OTOC}_{\text{SYK}} \sim a_0 - \frac{a_1}{N}\, e^{\lambda_L(\beta J)\, t} + \ldots$$

$$\lambda_L(\beta J \gg 1) = \lambda_L^{\max} \equiv \frac{2\pi}{\beta}$$



[Maldacena-Shenker-Stanford '15] [Kitaev '15]

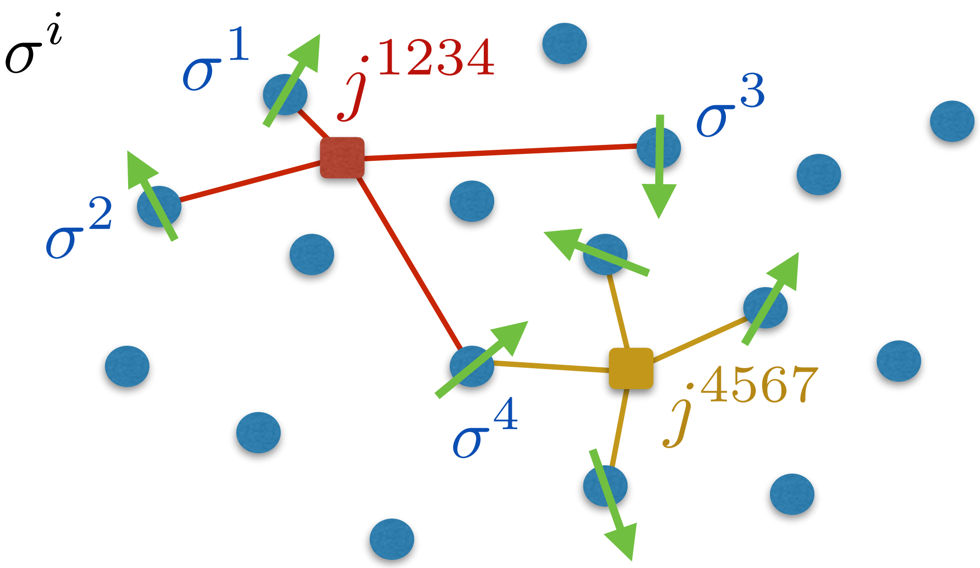
▸ Another signature of black hole physics

[Hayden/Preskill '07] [Sekino/Susskind '08] [Shenker/Stanford '13]

Spin glasses

- I will discuss a related model with some similar and some new features

- Roughly: replace Majorana fermions by bosonic spins σ^i constrained to live on an N-dimensional sphere
  [Crisanti/Sommers '92] [Cugliandolo/Grempel/da Silva Santos '01]



The model has <u>two</u> dimensionless couplings:

$$\beta J, \;\; MJ$$

~thermal fluctuations ~quantum fluctuations

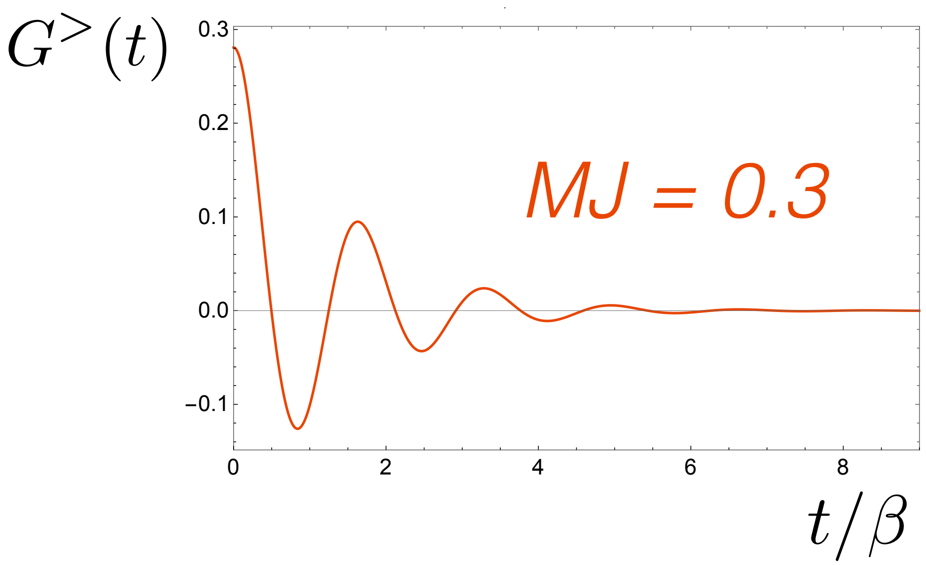
- Spin glass phase: if both thermal & quantum fluctuations are weak, the system gets "stuck" in one of many metastable states

- Useful order parameter: $$u \equiv \frac{1}{N} \sum_{i=1}^{N} \overline{\langle \sigma^i \rangle^2}$$ [Edwards/Anderson '75]

"paramagnetic" phase: $u = 0$

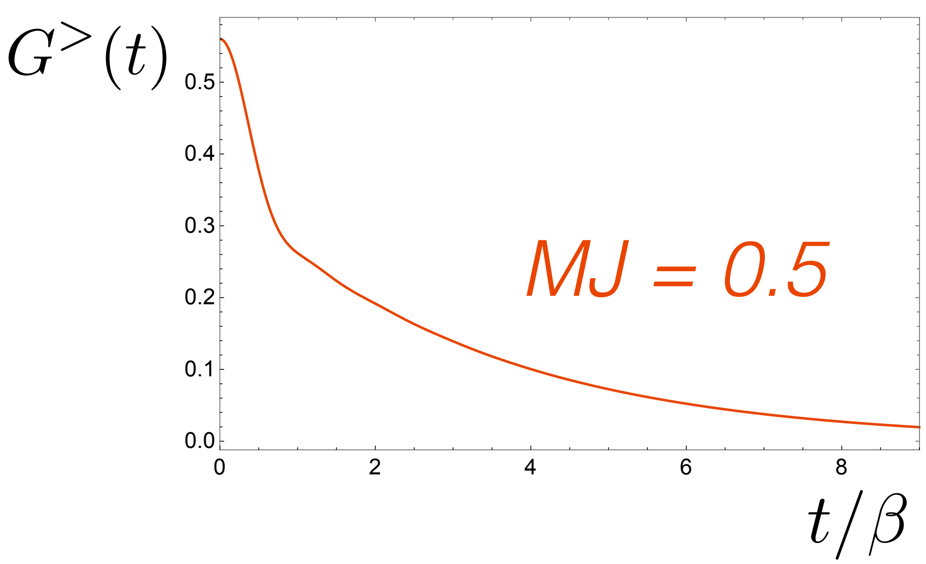
"spin glass" phase: $u > 0$

E.g.: two-point correlation function $G^{>}(t) = \frac{1}{N}\sum \overline{\langle \sigma^i(t)\sigma^i(0)\rangle}$ (fixed βJ):



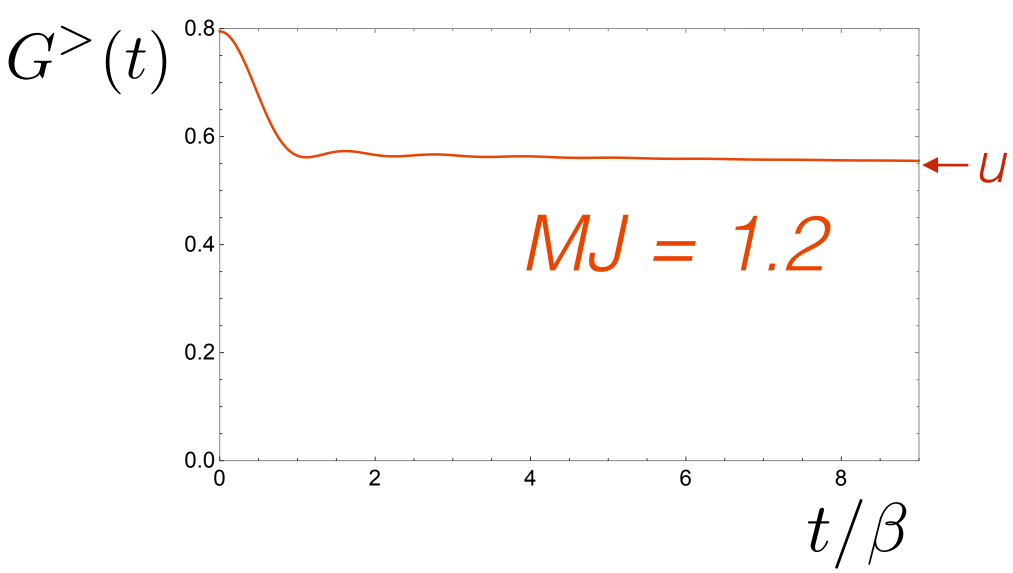
❖ <u>Far above SG transition:</u>

- Strong quantum fluctuations
- Relatively fast decay



❖ <u>Near SG transition:</u>

- Competition between thermal & quantum effects
- Slow decay (“two-step” relaxation)



❖ <u>Below SG transition:</u>

- Correlator “freezes”
- Decays to asymptotic value:
  EA order parameter *u>0*

Goals

- Characteristic features of SG phase: slow dynamics, many metastable states, inability to reach equilibrium, loss of ergodicity, …
  - Universal features of the low temperature thermodynamics?
  - Interplay with other chaos characteristics such as OTOCs?
  - Emergent reparametrization symmetry?
  - Can we incorporate this in the $nAdS_2/nCFT_2$ paradigm?

The p-spin glass model

The p-spin model

$$P(J_{i_1\ldots i_p}) \propto \exp\left[-\frac{N^{p-1}}{p!}\frac{J^2_{i_1\ldots i_p}}{J^2}\right]$$

$$Z[J_{i_1\ldots i_p}] = \int D\sigma_i Dz\ \exp\left\{-\int_0^\beta d\tau\left[\frac{M}{2}\dot\sigma_i(\tau)\dot\sigma_i(\tau) + \sum_{i_1<\ldots<i_p} J_{i_1\ldots i_p}\sigma_{i_1}(\tau)\ldots\sigma_{i_p}(\tau)\right]\right.$$

$$\left. + i\int_0^\beta d\tau\, z(\tau)\left(\sum_{i=1}^N \sigma_i(\tau)\sigma_i(\tau) - N\right)\right\}$$

"spherical constraint"

- Dimensionless parameters: $\beta J,\ MJ$
- Nonlinear sigma-model with fixed size spherical target space
- Spherical constraint will be crucial for stability of such a bosonic model

[Crisanti/Sommers '92] [Cugliandolo/Grempel/da Silva Santos '01]

- First goal: compute disorder averaged ("quenched") free energy

$$\beta\overline{F} = -\int dJ_{i_1\ldots i_p} P(J_{i_1\ldots i_p}) \log Z[J_{i_1\ldots i_p}]$$

- Strategy: use *replica trick*

$$\log Z = \lim_{n\to 0} \partial_n Z^n$$

$$\beta\overline{F} = -\lim_{n\to 0} \partial_n \overline{Z^n}$$

$$\overline{Z^n} = \int dJ_{i_1\ldots i_p} P(J_{i_1\ldots i_p}) \int D\sigma_i^a Dz^a \, \exp\Bigg\{ -\int_0^\beta d\tau \Bigg[\frac{M}{2}\dot{\sigma}_i^a(\tau)\dot{\sigma}_i^a(\tau) + \sum_{i_1<\ldots<i_p} J_{i_1\ldots i_p}\sigma_{i_1}^a(\tau)\ldots\sigma_{i_p}^a(\tau) \Bigg]$$
$$+ i\int_0^\beta d\tau \, z^a(\tau) \left(\sum_{i=1}^N \sigma_i^a(\tau)\sigma_i^a(\tau) - N \right) \Bigg\}$$

- Index *a = 1,…,n* labels the replica copy

- Introduce collective bilocal field with replica indices:

$$Q_{ab}(\tau,\tau') = \frac{1}{N}\sum_{i=1}^{N}\sigma_i^a(\tau)\sigma_i^b(\tau')$$

- After integrating out the disorder and the spins, we get an effective action for Q_{ab}. Schwinger-Dyson equation:

$$-\delta_{ab}\left[\frac{M}{2}\partial_\tau^2 + iz^a(\tau)\right]Q_{ab}(\tau,\tau') - \frac{pJ^2}{4}\int_0^\beta d\tau''\, Q_{ac}^{p-1}(\tau,\tau'')Q_{cb}(\tau'',\tau') = \frac{1}{2}\delta_{ab}\,\delta(\tau-\tau')$$

- Some notable features:
  - $Q_{a\neq b}(\tau,\tau') \to \frac{1}{N}\sum_i \overline{\langle\sigma_i^a(\tau)\rangle\langle\sigma_i^b(\tau')\rangle}$ can be non-zero
  - Two-derivative kinetic term with tunable coefficient M
  - Lagrange multiplier field $z^a(\tau)$

$$-\delta_{ab}\left[\frac{M}{2}\partial_\tau^2 + iz^a(\tau)\right] Q_{ab}(\tau,\tau') - \frac{pJ^2}{4}\int_0^\beta d\tau''\, Q_{ac}^{p-1}(\tau,\tau'')Q_{cb}(\tau'',\tau') = \frac{1}{2}\delta_{ab}\,\delta(\tau-\tau')$$

- "1-step replica symmetry breaking" ansatz (Parisi):

$$Q_{ab}(\tau,\tau') = q_r(\tau,\tau')\,\delta_{ab} + \underbrace{\begin{pmatrix} \overbrace{\begin{matrix} u & u & u \\ u & u & u \\ u & u & u \end{matrix}}^{m\times m} & s & \cdots \\ s & \begin{matrix} u & u & u \\ u & u & u \\ u & u & u \end{matrix} & \\ \vdots & & \ddots \end{pmatrix}}_{n\times n}$$

- Diagonal: $q(\tau,\tau') \equiv q_r(\tau,\tau') + u$ subject to $q(\tau,\tau) = 1$

$$\left.\begin{array}{l} \bullet\ \textit{u:}\ \text{overlap of replicas} \\ \\ \bullet\ \textit{m:}\ \text{block size parameter} \end{array}\right\}\ \text{both may remain finite as } n \longrightarrow 0$$

- *s:* can be consistently set to 0

- SD equation for Q_{ab} gives equation of motion for $q(\tau,\tau')$ & u
  - After some rewritings, the equation of motion for $\hat{q}_r(k \neq 0)$ is:

$$\frac{1}{\hat{q}_r(k)} - \frac{1}{\hat{q}_r(0)} = M\left(\frac{2\pi k}{\beta}\right)^2 - J^2\left(\hat{\Lambda}_r(k) - \hat{\Lambda}_r(0)\right)$$

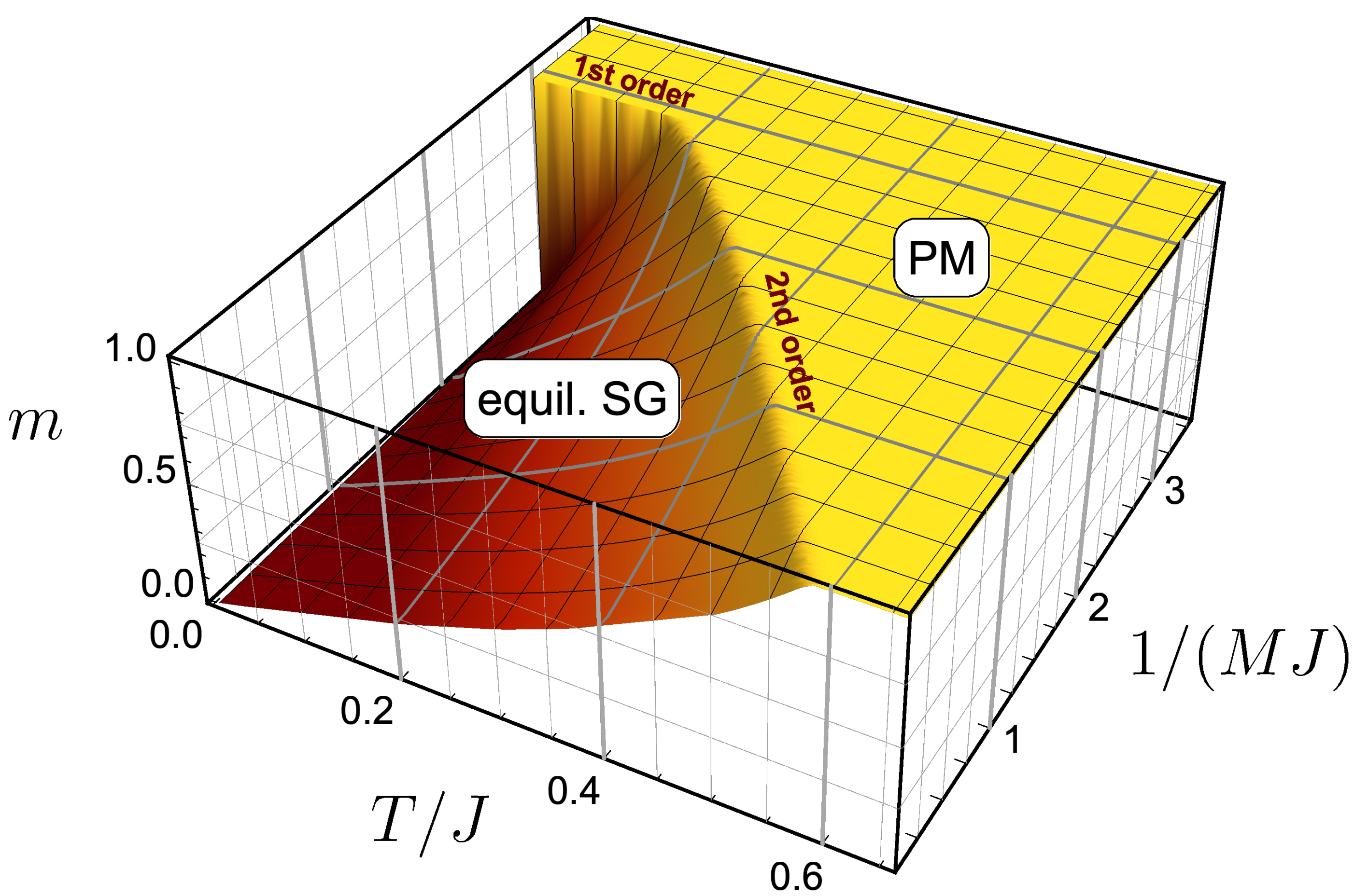
$$\Lambda_r(\tau) = \frac{p}{2}\left[(q_r(\tau)+u)^{p-1} - u^{p-1}\right]$$

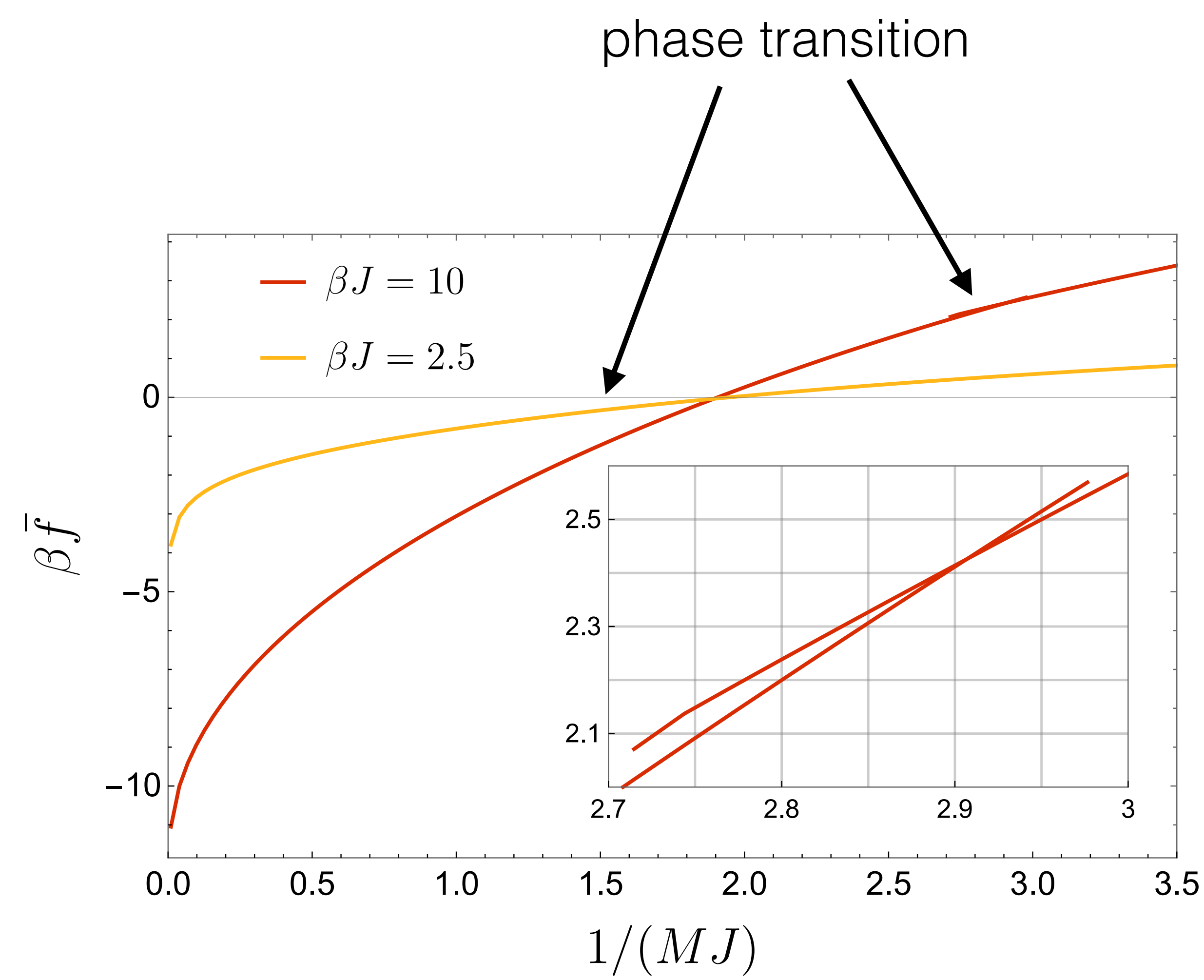
  - In addition: 2 algebraic equations for $\hat{q}_r(0)$ and u

- What about *m* ?
  - For replica symmetric solutions, *m* plays no role.
  - In spin glass, we could impose $\frac{\delta S_{eff}}{\delta m} = 0$ —> *"equilibrium spin glass"*
    - This turns out to be <u>not quite the right condition</u>
      (recall: the essence of glassy physics is an inability to reach equilibrium)

Phase diagram

- Solving the “equilibrium spin glass” equations numerically gives:
  - Small βJ , MJ : paramagnetic phase ($u = 0,\ m = 1$)
  - Large βJ , MJ : spin glass (RSB: $0 < u, m < 1$)





Conformal paramagnetic solution

- For unbroken replica symmetry *(u=0, m=1):* seemingly similar to SYK

  - At strong coupling $\beta J \gg 1$, small frequencies:

$$\delta(\tau,\tau') \approx -J^2 \int_0^\beta d\tau'' \Lambda_r(\tau,\tau'')\, q_r(\tau'',\tau')\,, \qquad \Lambda_r(\tau,\tau') = \frac{p}{2}\, q_r(\tau,\tau')^{p-1}$$

  - Reparametrization invariance!
    —> spontaneously broken by the conformal solution:

$$q^c(\tau,\tau') \sim \left[\frac{\pi}{\beta \sin\left(\frac{\pi(\tau-\tau')}{\beta}\right)}\right]^{\frac{2}{p}}$$

  - Leads to Schwarzian action etc.

Conformal paramagnetic solution

- However, this solution is actually unstable

- Can compute the spectrum of operators appearing in the $\sigma^i \sigma^i$ OPE as determined by conformal symmetry

[Kitaev '15] [Gross/Rosenhaus '16]
[Tikhanovskaya/Guo/Sachdev/Tarnopolsky '20]

Find a tower of allowed dimensions. E.g. for *p=3:*

$$h_0 = 2, \qquad h_{n=1,2,3,\ldots} = 4.303,\ 6.404,\ 8.456,\ 10.489,\ 12.511,\ \ldots$$

... and: an operator with complex dimension $h = \frac{1}{2} \pm 1.560\, i$

c.f. [Giombi/Klebanov/Tarnopolsky '17]

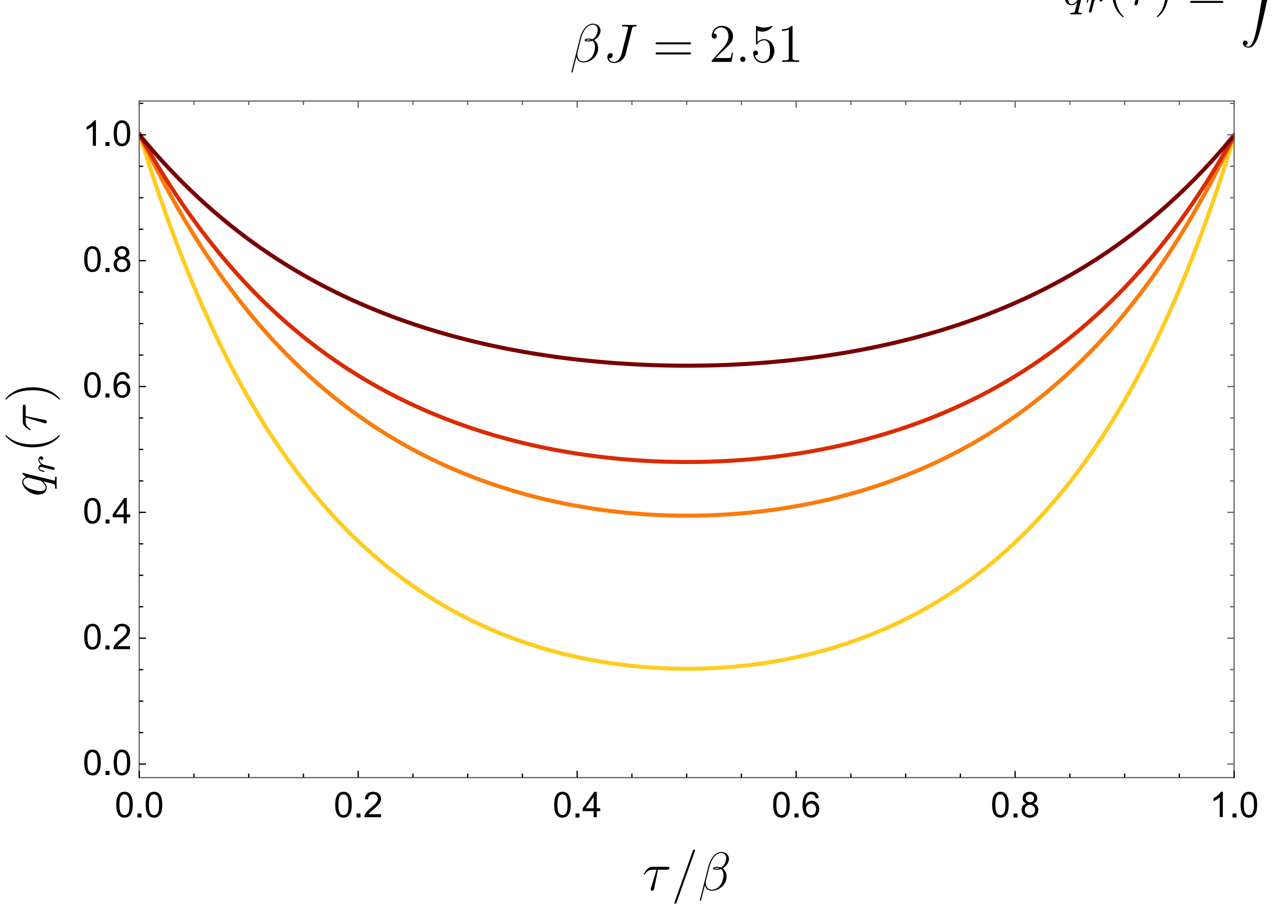
- Indeed, there exists another paramagnetic solution, which has no conformal limit

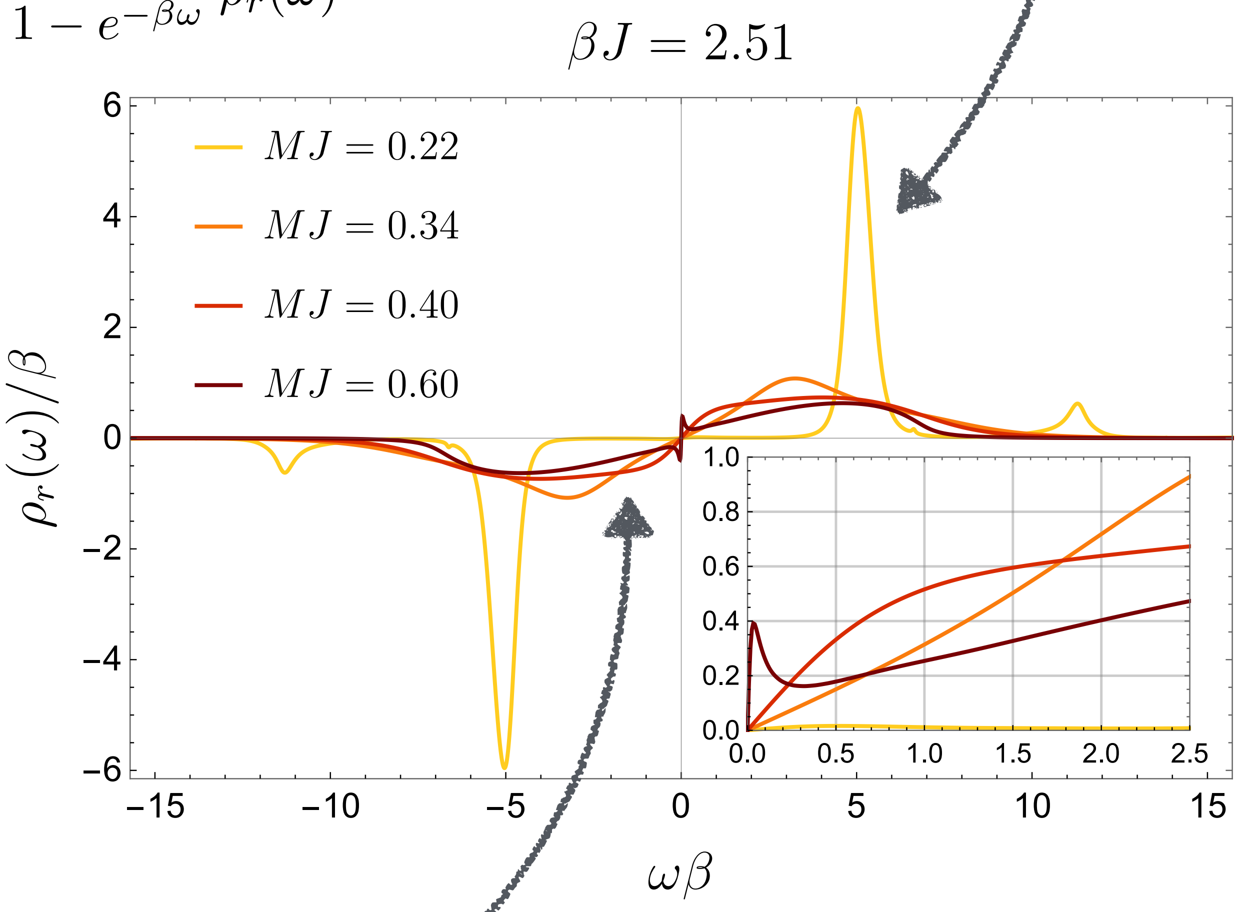
—> construct numerically

Physical paramagnetic solution

far from SG transition

$$q_r(\tau) \equiv \int \frac{d\omega}{2\pi} \frac{e^{-\omega\tau}}{1-e^{-\beta\omega}} \rho_r(\omega)$$





close to SG transition

- Gap closes near spin glass transition
- Is there a (physical) conformal solution in the SG phase?

Conformal spin glass

Approximate analytical solution

$$\frac{1}{\hat{q}_r(k)} - \frac{1}{\hat{q}_r(0)} = M\left(\frac{2\pi k}{\beta}\right)^2 - J^2\left(\hat{\Lambda}_r(k) - \hat{\Lambda}_r(0)\right)$$

- To get a feeling for the spin glass equations, start with an analytical analysis at strong coupling ("deep spin glass")
- Recall $q(\tau) \equiv q_r(\tau) + u$ and expand self-energy for $q_r(\tau) \ll u$:

$$\Lambda_r(\tau) = \frac{p}{2}\left[(q_r(\tau)+u)^{p-1} - u^{p-1}\right] = \frac{p(p-1)}{2}\, q_r(\tau)\, u^{p-2} + \ldots$$

- At first non-trivial order we can solve e.o.m. analytically
  - This gives an approximate solution:

$$\boxed{\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 + 2\gamma^2\omega^2 - 2\sqrt{\gamma^2\omega^2 + \gamma^4\omega^4}} \qquad \gamma \equiv \sqrt{\frac{M\hat{q}_r(0)}{4}} \qquad \omega = \frac{2\pi k}{\beta}$$

c.f. [Read/Sachdev/Ye '95] [Cugliandolo/Grempel/da Silva Santos '01] ...

$$\boxed{\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 + 2\gamma^2\omega^2 - 2\sqrt{\gamma^2\omega^2 + \gamma^4\omega^4}}$$

$$\gamma \equiv \sqrt{\frac{M\hat{q}_r(0)}{4}} \qquad \omega = \frac{2\pi k}{\beta}$$

- Low frequency limit: $$\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 - 2\gamma|\omega| + \ldots$$

zero mode

conformal term: $q_r^c(\tau) \equiv \dfrac{8\gamma^3}{M\pi}\dfrac{1}{\tau^2}$

—> conformal (dimension $\Delta = 1$)

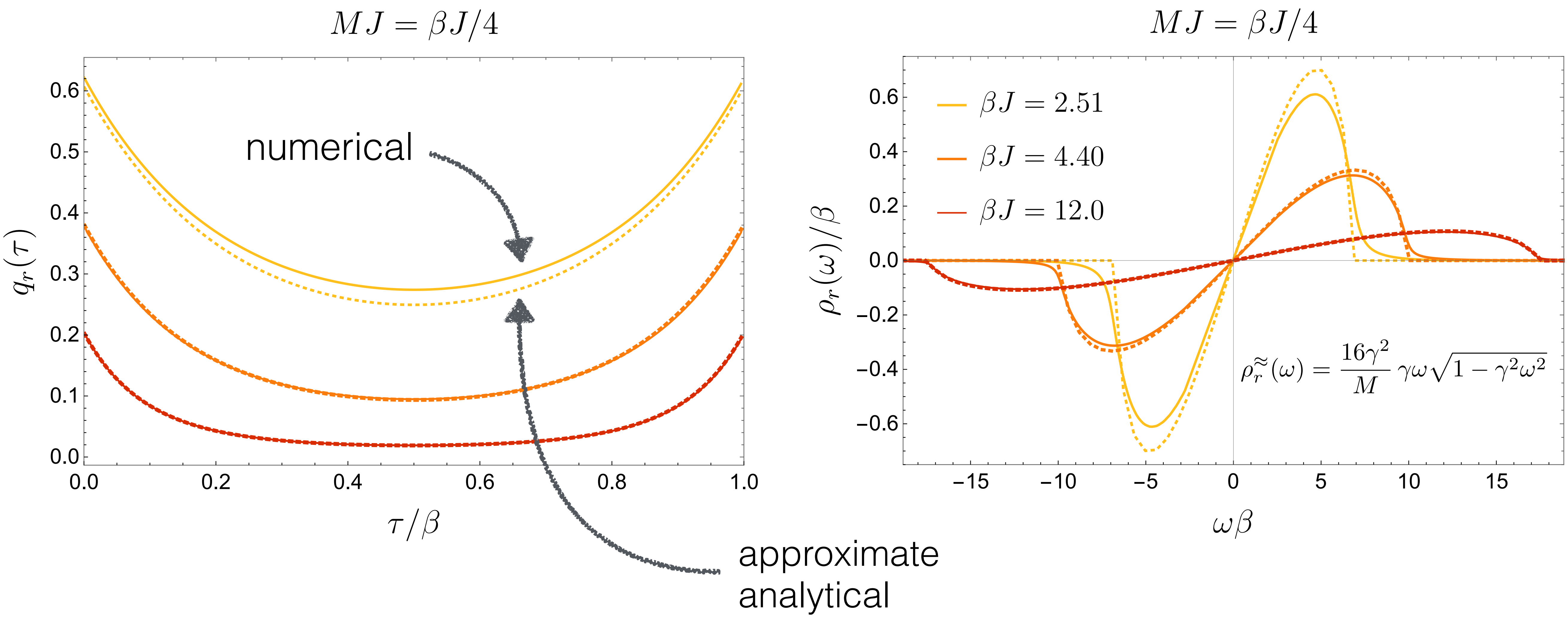
- High frequency limit: $$\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = \frac{1}{4\omega^2} + \ldots$$

—> finite (can consistently impose $q_r^{\approx}(\tau = 0) = 1 - u$)

- The approximate solution is well-behaved at long and short distances

$$\boxed{\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 + 2\gamma^2\omega^2 - 2\sqrt{\gamma^2\omega^2 + \gamma^4\omega^4}}$$

$$\gamma \equiv \sqrt{\frac{M\hat{q}_r(0)}{4}} \qquad \omega = \frac{2\pi k}{\beta}$$



- Spin glass physics for small temperatures is governed by conformal properties: power law scaling, gapless spectrum, …

Subtlety: the value of *m*

- Conformal solution fixes *m*: doesn't extremize free energy!

  —> useful to think of *m* as an external thermodynamic parameter (like *T*)

  —> tune *m* to the value required for this solution to exist

  [Monasson '95]

  [Mezard '99]

- Consider a thermodynamic ensemble where we consider *m* physical copies of the system & replica symmetry is explicitly broken:

$$\frac{S_{\text{eff}}(Q^*)}{Nn} = \beta m \Phi$$

$$\Sigma = -\partial_{1/m}(\beta\Phi) \qquad F = \partial_m(m\Phi)$$

'complexity': counts metastable states

$$\Sigma(Q^*) = \frac{1}{2}\log(p-1) - \frac{p-2}{p}$$

c.f. usual thermodynamics:

$$\frac{S_{\text{eff}}(Q^*)}{Nn} = \beta F$$

$$S = -\partial_T F \qquad E = \partial_\beta(\beta F)$$

Marginal stability criterion

- Another way to determine *m:*

    - Consider fluctuations:

$$Q_{ab} = Q^*_{ab} + \delta Q_{ab} \longrightarrow \delta^{(2)} S_{eff}[Q] = N \int d\tau d\tau' \, \delta Q_{ab}(\tau) \, G_{ab,cd}(\tau,\tau') \, \delta Q_{cd}(\tau')$$

    - For physically sensible solutions: $G_{ab,cd}$ should have eigenvalues > *0*

    - Determine *m* by demanding existence of a vanishing eigenvalue

—> “Condition of marginal stability”: $\mathcal{J}^2 u^{p-2} = (\hat{q}_r(0))^{-2}$

—> Coincides with the value in the conformal solution!

[Cugliandolo/Kurchan ’93]

[Georges/Parcollet/Sachdev ’00]

Recap: signs of gravity?

- Spin glass phase has emergent conformal symmetry at strong coupling. Marginal mode on top of self-overlap:

$$q(\tau, \tau') = u + q_r^c(\tau, \tau') + \dots$$

  - Reparametrization symmetry, gapless spectrum, …

- An extensive number of nearby states is counted by the entropy-like density

$$m\bar{s} + \Sigma = \left[\frac{1}{2}\log(p-1) - \frac{p-2}{p}\right] + \dots$$

Let us now consider the quantum Lyapunov exponent as another diagnostic of gravitational physics…

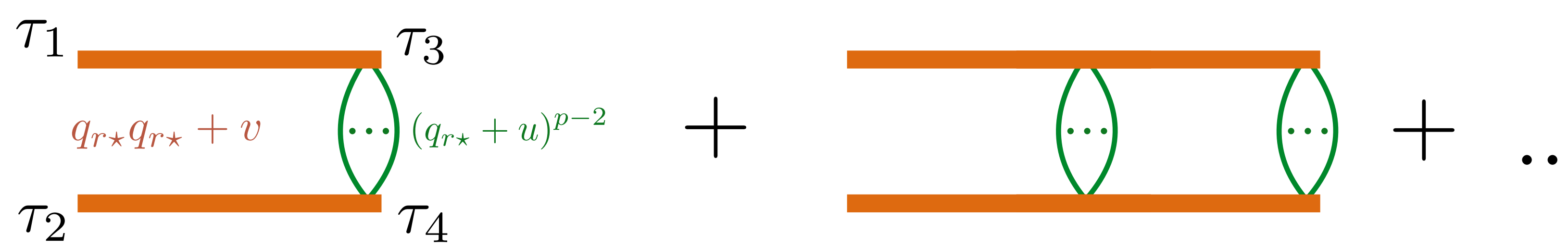
Quantum chaos

Euclidean 4-point function

- Consider $\mathcal{F}(\tau_1,\tau_2,\tau_3,\tau_4) \equiv \frac{1}{N^2}\sum_{i,j}\big\langle \sigma_i(\tau_1)\sigma_i(\tau_2)\sigma_j(\tau_3)\sigma_j(\tau_4)\big\rangle \equiv 1 + \frac{1}{N}\mathcal{F}_{\text{conn.}}$

  - Connected piece is built recursively from ‘ladder diagrams’:

$$\mathcal{F}_{\text{conn.}} = \frac{1}{(\beta\mathcal{J})^2}\sum_{n\geq 1}\tilde{K}^n =$$



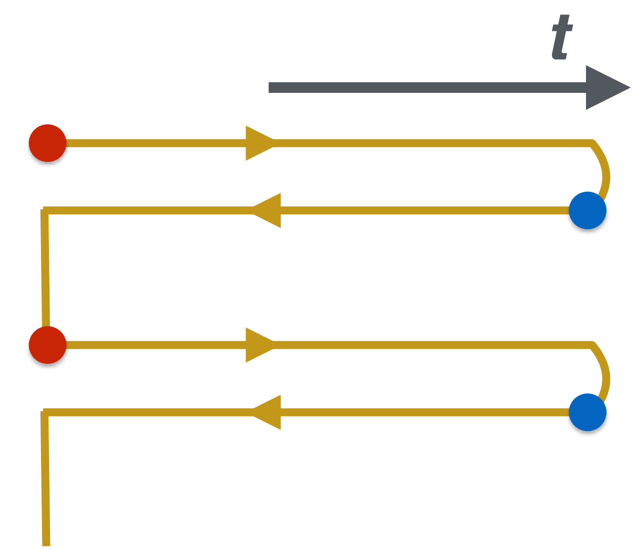
$$\tilde{K}(\tau_1,\tau_2;\tau_3,\tau_4) = (\beta\mathcal{J})^2\,[q_{r\star}(\tau_{13})\,q_{r\star}(\tau_{24})+v]\,[q_{r\star}(\tau_{34})+u]^{p-2}$$

‘rails’ offset by 4-point version of Edwards-Anderson parameter:

$$v = \frac{p}{p-2}\,\frac{m^2u^2}{m(p-1)^2-p(p-2)}$$

c.f. [Kitaev ’15][Maldacena/Stanford ’16] ... [Cheng/Swingle ’19][Mao/Chowdhury/Senthil ’19]

Out-of-time-order correlator



- We wish to compute the OTOC $(t_1 \approx t_2 \gg t_3 \approx t_4)$:

$$\mathcal{F}(t_1, t_2, t_3, t_4) \equiv \frac{1}{N^2} \sum_{i,j} \left\langle \sigma_i(t_1)\, \sigma_j(t_3)\, \rho_\beta^{1/2}\, \sigma_i(t_2)\, \sigma_j(t_4)\, \rho_\beta^{1/2} \right\rangle$$

  - Analytically continue the Euclidean result. Retarded ladder kernel:

$$\tilde{K}_{\mathrm{ret}}(t_1, t_2; t_3, t_4) = (\beta \mathcal{J})^2\, q^R_{r\star}(t_{13})\, q^R_{r\star}(t_{24})\, \left[q^>_{r\star}(t_{34} - i\beta/2) + u\right]^{p-2}$$

- Condition for exponential growth of the OTOC:

$$\mathcal{F}_{\mathrm{conn.}}(t_1, t_2; t_3, t_4) = \frac{1}{\beta^2} \int dt dt'\, \tilde{K}_{\mathrm{ret}}(t_1, t_2; t, t')\, \mathcal{F}_{\mathrm{conn.}}(t, t'; t_3, t_4)$$

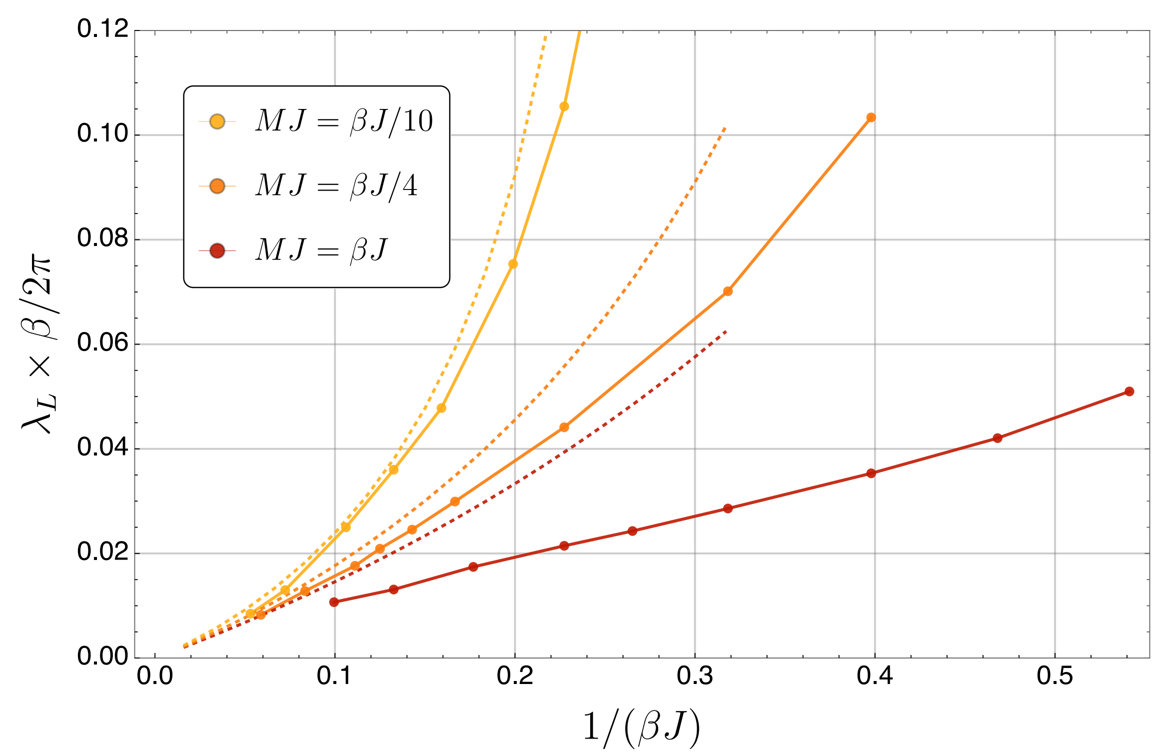
  - Two ways to solve this eigenvalue problem:
    (1) perturbatively in the conformal limit
    (2) numerically

Perturbative analysis (conformal SG)

- Exponential growth ansatz: $\mathcal{F}_{\text{conn.}}(t_1, t_2, 0, 0) \sim f(t_1 - t_2)\, e^{\lambda_L (t_1+t_2)/2}$
- For $\beta J \sim MJ \gg 1$: conformal perturbation theory around $q^{\approx}$ turns exponential growth condition into a 'Pöschl-Teller' Schrödinger problem:

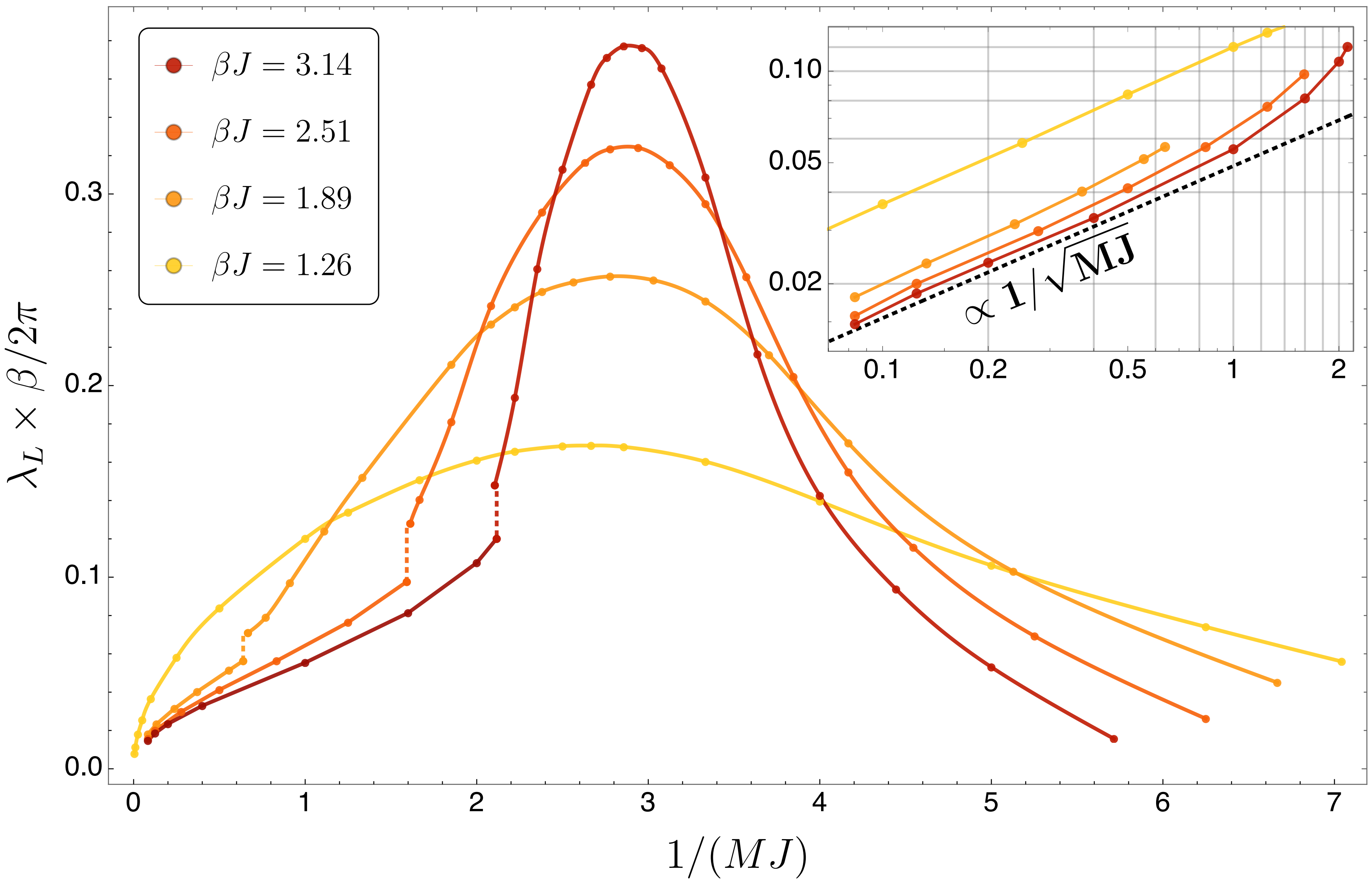
$$-\frac{1}{2} f''(x) - \frac{6}{\cosh^2 x}\, f(x) = -\left(2 + \frac{3Mu\beta^2\lambda_L}{2(p-2)\pi\gamma^2}\right) f(x)$$

—> unique positive solution: $$\frac{\beta}{2\pi}\lambda_L \approx \frac{5m}{24} = 5(p-2)\left(\frac{1}{24\beta\mathcal{J}} + \frac{p}{36\pi\beta\mathcal{J}\sqrt{M\mathcal{J}}} + \dots\right)$$



similar results: [Bera/Lokesh/Banerjee '21]

Lyapunov exponents: numerics



- Paramagnetic phase: strong, non-monotonic dependence on *MJ*
  - λ_L "peaks" a finite distance above the SG transition
- Spin glass phase: both thermal and quantum fluctuations increase λ_L

similar results: [Bera-Lokesh-Banerjee '21]

Holographic speculations

Signatures of gravity?

- p-spin glass has features reminiscent of gravity:
  - emergent conformal symmetry (appearance of $\Delta = 1$ mode)…

    … 'on top of' constant background value *u*
  - extensive number of (metastable) states…

    … contributing to the complexity
  - non-zero quantum Lyapunov exponent…

    … which vanishes at strong coupling

- Can we accommodate for these effects in $nAdS_2/nCFT_2$?

- At first sight: have learnt a lot in recent years about the replica trick in gravity to compute entropies, free energies, etc.

  [Lewkowycz/Maldacena] [Saad/Shenker/Stanford] [Penington/Shenker/Stanford/Yang] [Almheiri/Hartman/Maldacena/Shaghoulian/Tajdini] [Engelhardt/Fischetti/Maloney] …..

  …but it seems like we need a bit more to incorporate the above

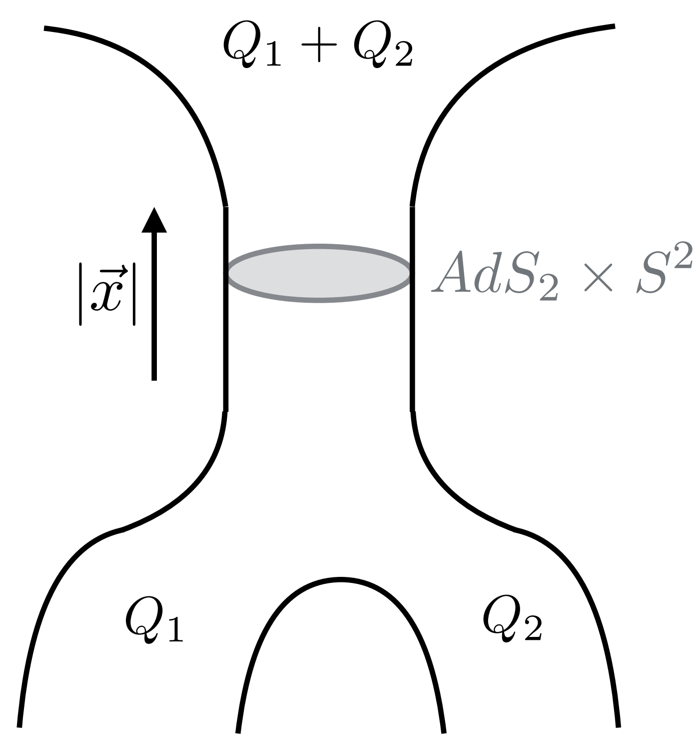
AdS2 fragmentation

- Disclaimer: the following is a suggestion. We have not yet worked out any detailed picture.
- Near horizon of 4d extremal two-RN black holes with fixed charge:

$$ds^2 = -V^{-2}dt^2 + V^2 d\vec{x}^2$$

$$\star F = dt \wedge dV^{-1}$$

$$V = \frac{Q_1}{|\vec{x} - \vec{x}_1|} + \frac{Q_2}{|\vec{x} - \vec{x}_2|}$$



[Majumdar/Papapetrou '47]

[Maldacena/Michelson/Strominger '98]

- Large $|\vec{x}|$: same as geometry with a single throat with charge $Q_1 + Q_2$
- For $\vec{x} \to \vec{x}_{1,2}$: fragmentation into two (or more) *AdS2* regions

AdS$_2$ fragmentation

- Some features of AdS2 fragmentation are reminiscent of the spin glass phase we discussed:
  - Large moduli space of geometries which locally minimize free energy (variational parameters $\vec{x}_i,\ Q_i$)

    —> counted by something like the complexity Σ?
  - "Microscopic" fragmentation can be detected asymptotically as a nonzero average dipole moment

    —> similar to u in the p-spin model?
  - Symmetries

String theory constructions

- *N=2* SUGRA in *d=4* has multi-horizon (“fragmented”) black hole solutions:
  - Exponentially many bound states of black holes
  - Exhibit slow relaxation etc.

[Denef ’00]
[Cardoso et al. ’00]

- Certain brane constructions in string theory lead quiver quantum mechanics similar to the p-spin model:
  - Chiral and vector multiplets
  - SUSY fixes much of the Lagrangian —> bosonic potential:

$$V = \sum_{i,a} \left| \underbrace{\frac{\partial W(\phi)}{\partial \phi_i^a}}_{\text{superpotential}} \right|^2 + \frac{1}{g_{\text{YM}}^2} \sum_a \left(\underbrace{\theta_a}_{\text{FY parameters}} - \sum_i |\phi_i^a|^2 \right)^2$$

$$W(\phi) = \Omega_{ijk} \phi_i^1 \phi_j^2 \phi_k^3 + \dots$$

[Douglas/Moore ’96]
[Denef/Moore ’07]

Summary

Summary

- Paramagnetic phase (similar to SYK) vs. spin glass phase
- Emergent reparametrization symmetry, gapless, nonzero quantum Lyapunov exponent, …
  - More systematic conformal perturbation theory?
  - Holographic interpretation in terms of AdS_2 fragmentation?
  - Connect to string theory constructions?