Fingerprints of quantum criticality in locally resolved transport

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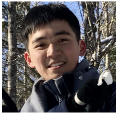

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GORDON AND BETTY MOORE FOUNDATION

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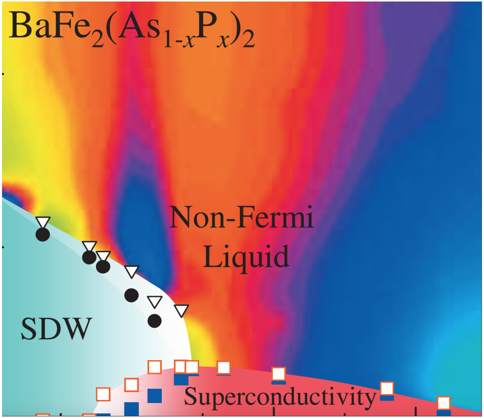
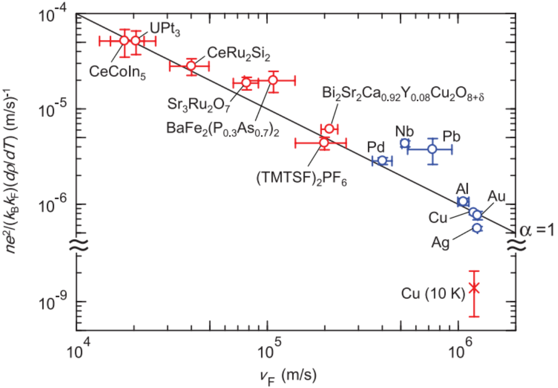
1 What are the dynamics of electrons in strange metals?



2

There's universal Planckian transport, with

$$\rho = \frac{m}{ne^2\tau}, \qquad \tau = \frac{\hbar}{k_B T}$$



[Bruin *et al* (2013)]

3

What is the mechanism for T-linear resistivity? Does it change from one material to the next?

- classical phonon scattering (above $T > T_{\mathrm{BG}}$?)
- Planckian non-quasiparticle transport?
- something else?

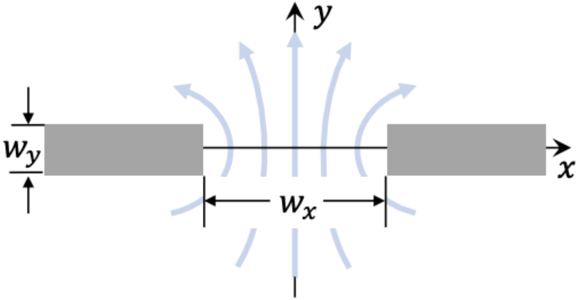
The bulk resistivity is given by

$$\frac{1}{\rho} = \sigma(k = 0, \omega = 0)$$

$$\sigma(k, \omega) = \frac{1}{\mathrm{i}\omega} G^{\mathrm{R}}_{J_y J_y}(k\hat{\mathbf{x}}, \omega)$$

Perhaps $\sigma(k, \omega)$ tells apart different mechanisms?

4 A constriction geometry is an indirect (though experimentally easy) way to probe the k-dependence in the conductivity.



We can either determine the conductance $G = 1/R$, or look at local current patterns.

5 A physically consistent flow pattern (with uncertain boundary conditions) can be found through the following algorithm:

$$J_i(x) = \int \mathrm{d}^2 y \sigma_{ij}(x-y) E_j(y),$$

$$\sigma_{ij}(x-y) = \text{F.T. of } \sigma(k, \omega = 0).$$

$$E_j(x) = E_j^{(0)} + E_j^{(\mathrm{ind})}(x)$$

$$E_j^{(\mathrm{ind})}(x) = -\int_{\text{inside}} \mathrm{d}^2 y \left[b\delta_{ij}\delta_{x,y} + \sigma_{ji}(x-y) \right]^{-1} \sigma_{ik}^{(0)} E_k^{(0)}, \quad (b \to 0).$$

This is numerically efficient to implement: only need to invert a matrix at grid points inside the constriction!

[Guo, Ilseven, Falkovich, Levitov; 1607.07269]

[Huang, Lucas; 2105.01075]

As a toy example, let's discuss what happens in a isotropic metal with a possible viscous hydrodynamic regime:

$$\sigma_{ij}(k) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \Sigma(k)$$

$$\Sigma(k) \sim \frac{1}{\ell_{\mathrm{mr}}^{-1} + \ell_{\mathrm{ee}}^{-1}(\sqrt{1+(\ell_{\mathrm{ee}}k)^2} - 1)}$$

- ohmic/diffusive: $k \ll (\ell_{\mathrm{ee}}\ell_{\mathrm{mr}})^{-1/2}$:

$$\Sigma(k) \sim k^0$$

- viscous: $(\ell_{\mathrm{ee}}\ell_{\mathrm{mr}})^{-1/2} \ll k \ll \ell_{\mathrm{ee}}^{-1}$:

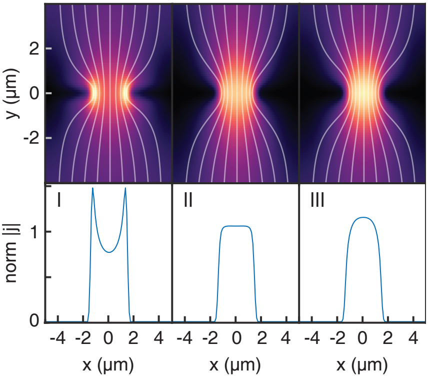
$$\Sigma(k) \sim k^{-2}$$

- ballistic: $\ell_{\mathrm{ee}}^{-1} \ll k$:

$$\Sigma(k) \sim k^{-1}$$

7

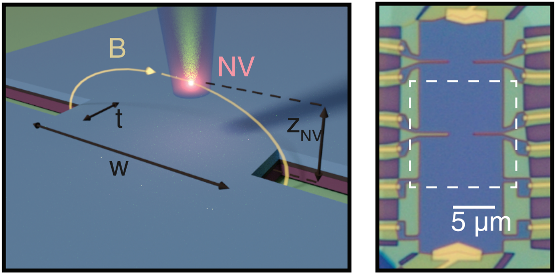
The spatial flow of current through the constriction illustrates the differing regimes.



8

NV center magnetometry can be used to probe current profiles.

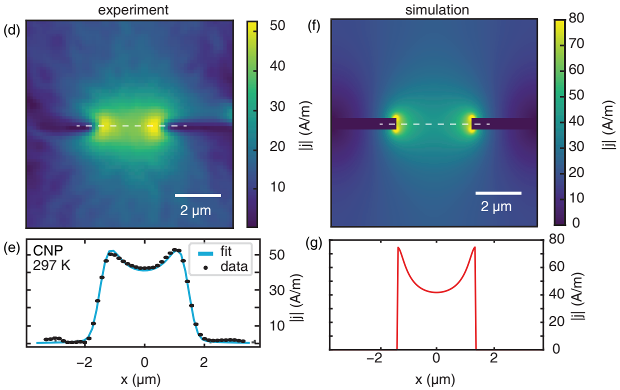
(a)



Inverting Biot-Savart we can infer **J** from **B**, with spatial resolution of ~ 150 nm.

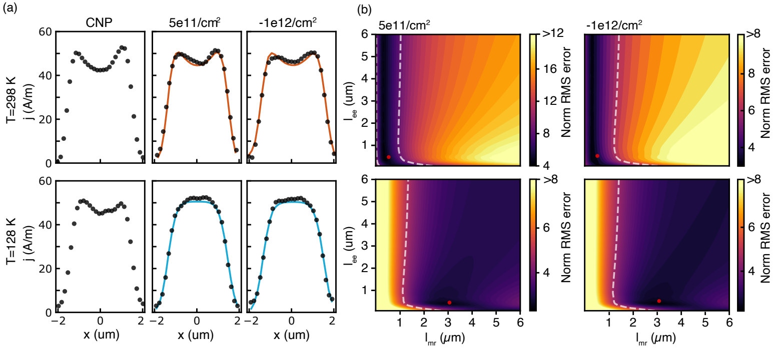
[Jenkins *et al*; 2002.05065]

9 We observe Ohmic transport at 298 K, imaging current flows through a constriction in monolayer graphene.



[Jenkins *et al*; 2002.05065]

10 At low temperatures, we appear to see the onset of viscous flow in monolayer graphene. But systematic errors make analysis of images subtle.



[Jenkins *et al*; 2002.05065]

What would happen in a quantum critical theory?

- hydrodynamic: $k \ll T$ (inverse Planckian length scale)
- quantum critical: $k \gg T$

What does (conformal, $z = 1$) criticality suggest about $\sigma(k)$?

$$G^{\mathrm{R}}_{J_y J_y}(k_x, \omega) = C_{JJ}\sqrt{k_x^2 - \omega^2},$$

so for $k > 0$,

$$\Sigma(k)_{\mathrm{CFT}} = 0.$$

Finite temperature resolution is important!

12 Holographic models suggest that

$$\Sigma(k) \sim \mathrm{e}^{-k/T}, \quad (k \gg T)$$

i.e. spectral weight is exponentially suppressed at high frequency. We therefore want to push “all” current through the smallest k mode we can, which suggests:

$$k_{\text{min}} = \frac{\pi}{w}$$

and we will see a sinusoidal current profile.

[Huang, Lucas; 2105.01075]

13

We have numerically computed $\Sigma(k)$ in the AdS_4-Einstein-Maxwell theory:

$$S = \int \mathrm{d}^4x \left(R + 6 - \frac{1}{4}F^2 \right),$$

$$\mathrm{d}s^2 = \frac{1}{r^2}\left(-f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + \mathrm{d}x^2 + \mathrm{d}y^2 \right), \quad A = g(r)\mathrm{d}t.$$

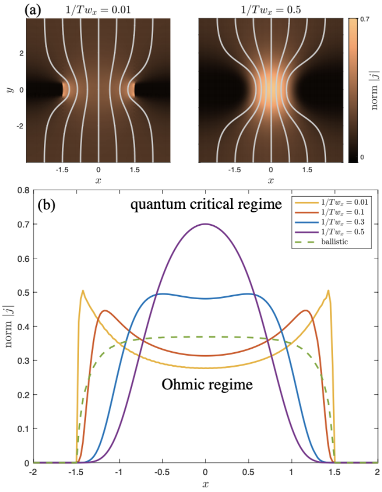
The computation of $\Sigma(k)$ requires looking for solutions of the form

$$\delta A_y \sim a_y(r)\mathrm{e}^{\mathrm{i}kx - \mathrm{i}\omega t}, \quad \delta g_{ty} = h_{ty}(r)\mathrm{e}^{\mathrm{i}kx - \mathrm{i}\omega t}$$

which can be handled using master fields.

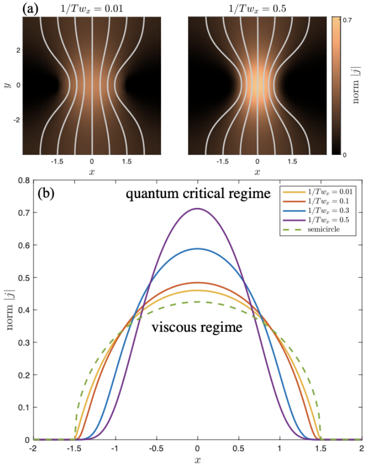
[Edalati, Jottar, Leigh; 1001.0779]

14 At charge neutrality, we see an ohmic-to-quantum critical crossover:



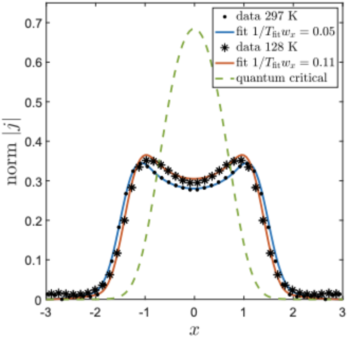
[Huang, Lucas; 2105.01075]

At finite density, we see a viscous-to-quantum critical crossover.



[Huang, Lucas; 2105.01075]

The charge neutral quantum critical crossover is consistent with the experimental data from graphene.

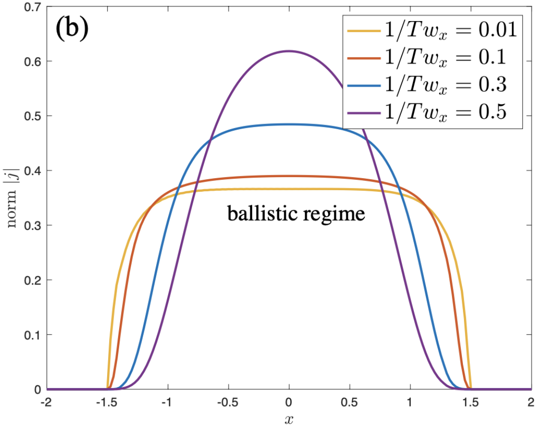


We used an effective Planckian length scale

$$\ell_{\rm Pl} \approx \frac{C\hbar v_{\rm F}}{k_{\rm B}T}.$$

with $C \approx 5$ our sole fit parameter, which is compatible with another experiment. [Gallagher *et al* (2019)]

17 Our quantum critical prediction also holds for free Dirac fermions, suggesting it is a robust prediction of criticality and is not sensitive to holography:



[Huang, Lucas; 2105.01075]

18 In the absence of imaging probes, we can also measure the conductance of the slit:

$$G_{\text{slit}} \sim \Sigma(k \sim w^{-1}).$$

This heuristic argument predicts:

$$G_{\text{slit}} \sim \begin{cases} w^0 & \text{ohmic} \\ w^1 & \text{ballistic} \\ w^2 & \text{viscous} \\ \exp[-\ell_{\text{Pl}}/w] & \text{quantum critical} \end{cases}$$

and is (up to logarithms) compatible with numerics.

[Huang, Lucas; 2105.01075]

Take home messages:

- New experiments can image *local transport phenomena.* Can these distinguish between theories of T-linear resistivity?
- Quantum criticality seems to have a clear signature in local transport.
- Our formalism can easily be generalized to other geometries and (more importantly) other models (holographic, field theoretic, kinetic, etc...).