Holography and hydrodynamics of 2-group global symmetry

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Based on work 2010.00320 with Nabil Iqbal

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Quest for partition function of strongly coupled QFTs

* Object Z that contain (pretty much) all information: usually very difficult to compute

* Sometimes it can be expressed in a simple form e.g. in those with holographic dual

$$Z[g,a] = \exp\left[-S_{grav}(G,A)\right]_{G|_\partial \sim g, A|_\partial \sim a}$$

* For IR physics, one useful way is EFT: (educatedly) guess what is Z using *global* symmetry

➢ I will talk about one of such EFT — hydrodynamics

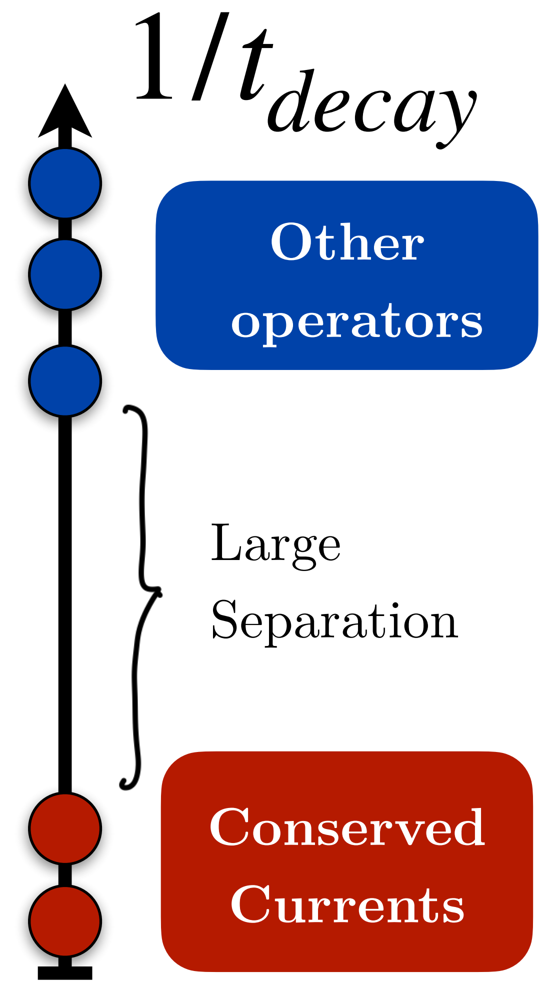
Heuristic explanation of hydro-limit

- ✻ All operators decays much faster than scale of interest (except conserved currents)

- ✻ No branch cut near at small ω, k or at late time
  All 1-pt functions at late time $\sim exp(-t/t_{decay})$

- Theories with same global symmetries. can be describe by the same equations!

- Intimately link to (bosonic sector of) holography



Policastro, Son & Starinets '01, '02; ... ; Bhattacharyya et. al '08,...

Hydrodynamics as a surprisingly good EFT

For a partition function which $Z[g,a] = Z[g + \mathscr{L}_\xi g, a + \mathscr{L}_\xi a + d\lambda]$

$$\partial_\mu j^\mu = 0$$

Consv. of "number"

$$\partial_\mu T^{\mu\nu} = 0$$

Consv. of energy and momentum

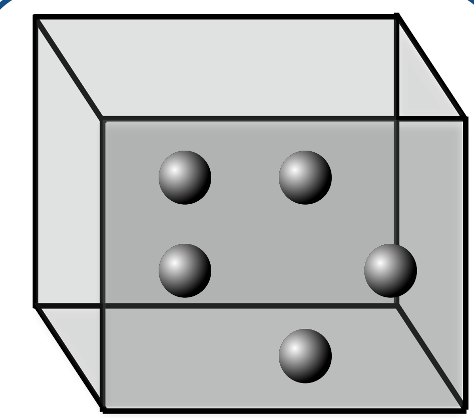
Write $T^{\mu\nu}$ and j^μ in terms of some "proxies" $\{T, u^\mu, \mu\}$ and background fields $\{g_{\mu\nu}, a_\mu\}$ then do gradient expansion

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + p\, g^{\mu\nu} + \mathcal{O}(\partial),$$
$$J^\mu = \rho\, u^\mu + \mathcal{O}(\partial)$$

Relativistic ideal fluid

- Only inputs: global symmetries, macroscopic constraints & gradient expansion
- Amazingly reliable for strongly interacting QFT
  e.g. quark-gluon plasma, graphene

Encapsulate symmetries of QFTs

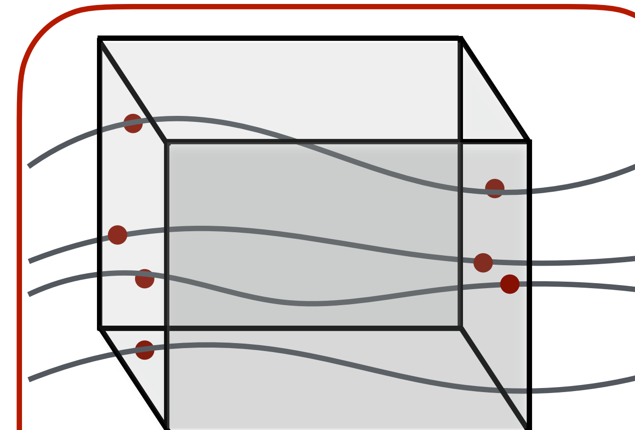


$\partial \langle j^{\mu} \rangle = 0$

- ✻ Ordinary symmetry : $Z[a_\mu] = Z[a_\mu + \partial_\mu \lambda]$

  Or when anomalous $Z[a_\mu + \partial_\mu \lambda] = Z[a] \times (\mathrm{U}(1)\mathrm{phase})$

- ✻ 1-form symmetry : $Z[b_{\mu\nu}] = Z[b_{\mu\nu} + 2\partial_{[\mu}\Lambda_{\nu]}]$



$\partial_\mu \langle J^{\mu\nu} \rangle = 0$

- ✻ (even) higher-form symmetry $Z[c_{\mu_1 \ldots \mu_p}]$

- Product between ordinary and higher-form
  Or extension of ordinary by higher-form symmetry

- Stuffs that aren't even obey group axioms

Dreams...

* Ordinary, higher-form, higher-group global symmetry exist
  - Can one always build hydrodynamics of higher-form, higher-group or any symmetry as an IR EFTs for strong coupling?
  - Observable and universal prediction?
* Holographic dual ?
  - Guide & Testing ground for hydrodynamics' prediction
  - Template to understand global symmetry in top-down models

Dreams...and plan

- ✻ Simple examples of QFTs with 1-form U(1)
  and 2-group built out of 0-form U(1) and 1-form U(1)

- ✻ Building hydrodynamics for 2-group global symmetry

- ✻ Minimalist holographic dual to 2-group hydrodynamics

Example: 1-form U(1) symmetry in QED_4

- Maxwell theory has $U(1)_e \times U(1)_m$ *global* symmetry associated with Wilson and 't Hooft line

$$d \star F = 0$$

Consv. of Electric flux

$$d \star J = dF = 0$$

Consv. of Magnetic flux

- Coupled this to, say, electrically charged scalar

~~$$d \star F \neq 0$$~~

~~**Consv. of Electric flux**~~

$$d \star J = dF = 0$$

Consv. of Magnetic flux

- Partition function suitable for EFT is $Z[b] = Z[b + d\Lambda]$

$$Z[g_{\mu\nu}, b_{\mu\nu}] \sim \left\langle \exp\left[-\int d^4x \left(\frac{1}{2} T^{\mu\nu} g_{\mu\nu} + \frac{1}{2} J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

EXAMPLE: 1-FORM U(1) SYMMETRY IN QED_4

* You may want to think about this in terms of gauging

$$Z = \int \mathfrak{D}[A] Z_{U(1)}[g, A] \exp\left(\frac{i}{2e^2}\int dA \wedge \star dA\right)$$

* But this is inefficient in many ways
  - Maxwell Eqn break gradient expansion $d \star F[\partial^2] \sim \star j_{U(1)}[\partial^0]$
  - Classically: assume factorisation of partition function
  - Rely on gauge symmetry, which is not a real symmetry
    - Gauge symmetry not manifested in Hilbert space
    - Nobody would want to do this for SU(N) colour symmetry

Example: 2-group symmetry in QED_4

* Consider matter sector with one mixed anomaly between two U(1)s

$$d \star \langle j_1 \rangle = 0$$
$$d \star \langle j_2 \rangle = \kappa (da_1) \wedge (da_2)$$

$\Longrightarrow$ Guaging $U(1)_1$

~~$d \star \langle j_1 \rangle = 0$~~

$$d \star \langle j_2 \rangle = \kappa \langle \star J \rangle \wedge (da_2)$$
$$d \star J = 0$$

* After gauging, full theory is not anomalous.
  To get desired Ward identities, we need

$$Z[a_\mu, b_{\mu\nu}] \sim \left\langle \exp\left[- \int d^4x \left(j^\mu a_\mu + \frac{1}{2} J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

Invariant under

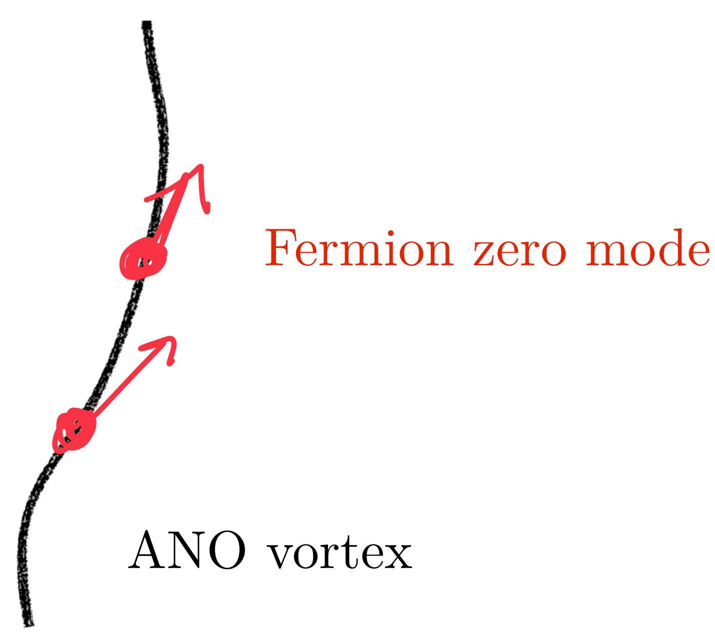
$$a \to a + d\lambda$$
$$b \to b + d\Lambda + \kappa \lambda (da)$$

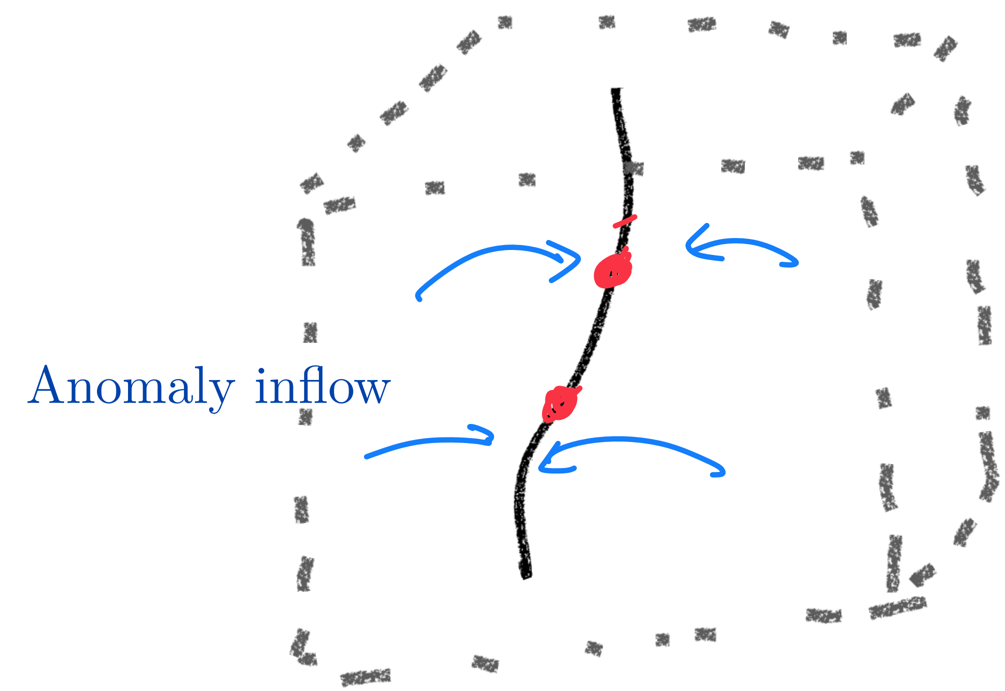
* Many more examples (mostly 1-form symmetry is (centre) $\mathbb{Z}_N$)

- *QED & QCD-ish : Cordova, Dumitrescu, Intriligator ‘18-…; Benini, Cordova & Hsin ‘18; Hsin & Lam ‘20*
- *6d: Saeman ‘19; Del Zotto & Ohmori ‘20,…*
- *Topological-ish: Kapustin & Thorngren ‘13; Barkeshli et. al ‘14; Tachikawa’ 17; Delcamp & Tiwari ‘18,…*

(Coherent) 2-group global symmetry
Physical example(s)

* Abrikosov-Nielsen-Olesen string with in superconductor QED with a certain anomaly





2d massless fermion is anomalous

Total system is anomaly free

* These zero modes affect macroscopic descriptions

(Seriously)oversimplified intro to 2-group

- Consider $Z[a_\mu, b_{\mu\nu}]$ where

$$a \to a + d\lambda$$
$$b \to b + d\Lambda + \kappa\lambda(da)$$

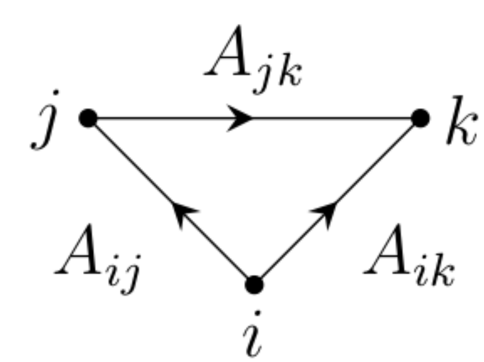
- This means that the "physical" flux is $H = db - \kappa a \wedge da$
or in group cohomology langauge

Where the class of
$\beta : G \times G \times G \to$ 1-form $\mathscr{A}$

Characterized by integer κ

$$A_{ij}A_{jk}A_{ik}^{-1} = 1$$

$$\int_{\mathscr{M}_2} F \in \mathbb{Z}$$

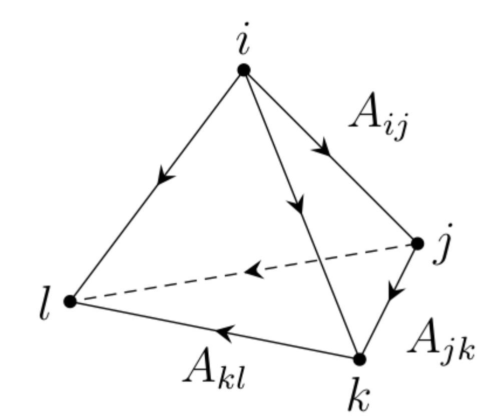


- For more see

- *Benini, Cordova & Hsin ' 18*
- *Cordova's talk at strings 2020*
- *Baez & Lauda '03; Baez & Huerta '10*

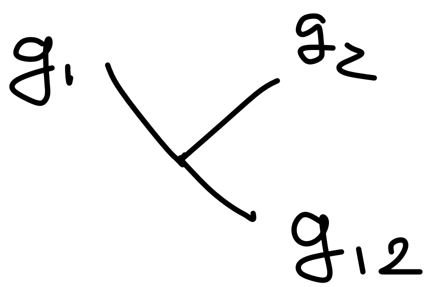
$$B_{jlk} - B_{ilk} + B_{ijl} + B_{ilk} = \beta(A_{ij}, A_{jk}, A_{kl})$$

$$\int_{\mathscr{M}_3} (db - \kappa a \wedge da) \in \mathbb{Z}$$

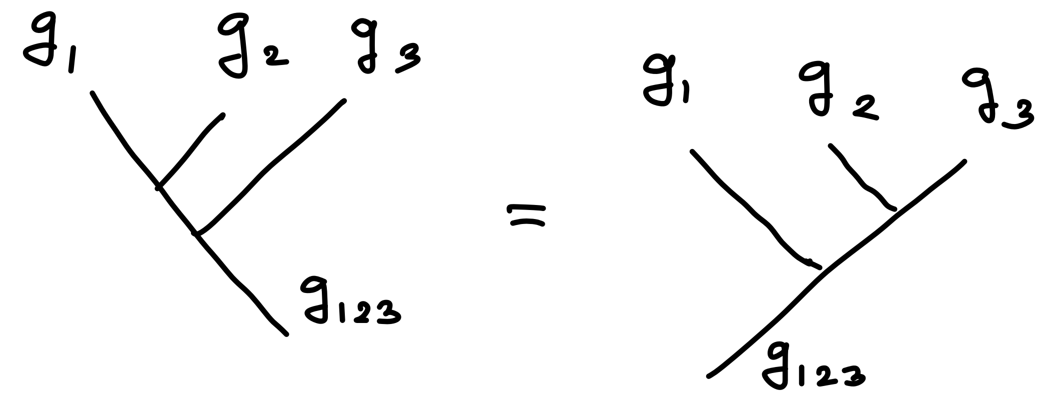


(Another)oversimplified intro to 2-group

* In the usual charge $Q[g] = \int dV_\mu j^\mu[g]$ we have $U[g] = \exp(iQ[g])$

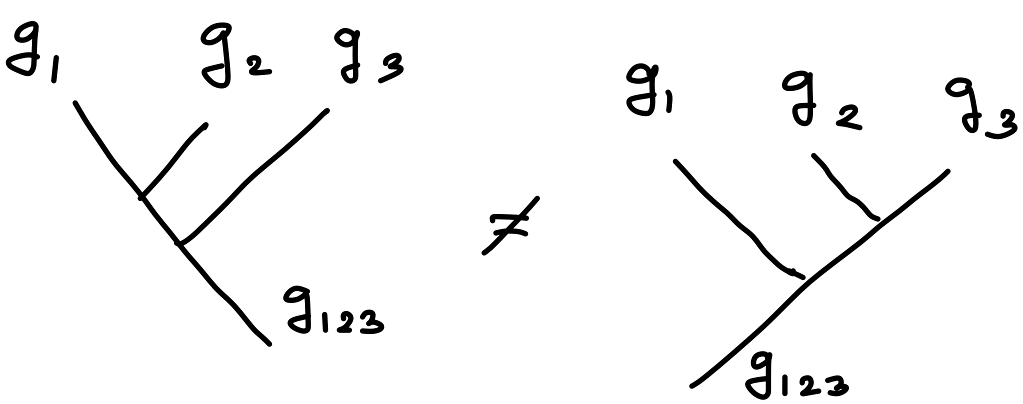


$U[g_1]U[g_2] = U[g_{12}]$



Associativity

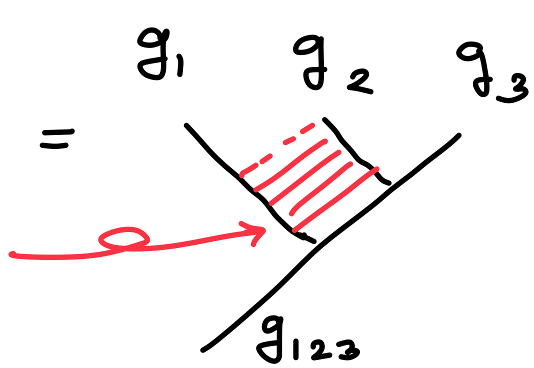
* But this doesn't happen in 2-group



$U[g_{12}]U[g_3] \neq U[g_1]U[g_{23}]$

$= \beta[g_1, g_2, g_3]\ U[g_1]U[g_{23}]$

Codim 2 surface



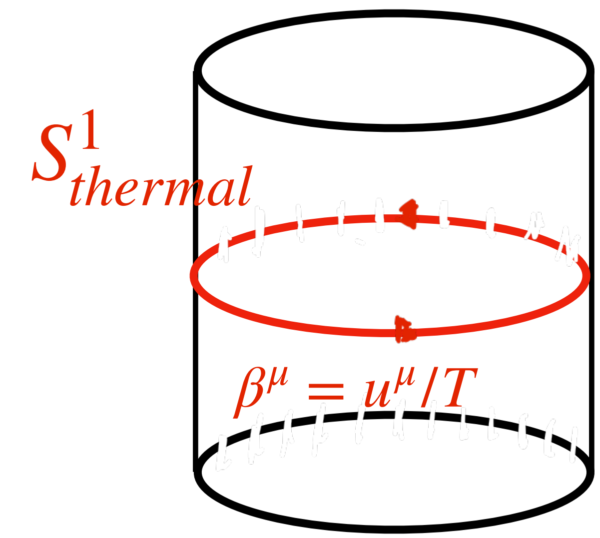
Building hydrodynamics -1

- For ordinary symmetry $\log Z = p(T,\mu) + \mathcal{O}(\partial)$ which gives

$$\langle j^\mu \rangle = \rho_a u^\mu + \mathcal{O}(\partial)$$
$$\langle T^{\mu\nu} \rangle = (\varepsilon + p) u^\mu u^\nu + p g^{\mu\nu} + \mathcal{O}(\partial)$$

And automatically gives

$$dp = s dT + \rho_a d\mu_a \,,$$
$$\varepsilon + p = sT + \mu_a \rho_a$$



$1/T \sim$ **size of thermal cycle**

$\mu_a / T \sim \ln \mathcal{P} e^{i \int_{S^1} a} \sim \int a_\mu u^\mu d\tau$

$\sim$ **background U(1) holonomy**

- Alternatively, demand no entropy production

$$s^\mu = p u^\mu - \left(\frac{u_\nu}{T}\right) T^{\mu\nu} - \left(\frac{\mu_a}{T}\right) j^\mu$$
$$= s u^\mu$$

Get same constraints when $\partial_\mu s^\mu = 0$

JKKMRY '12

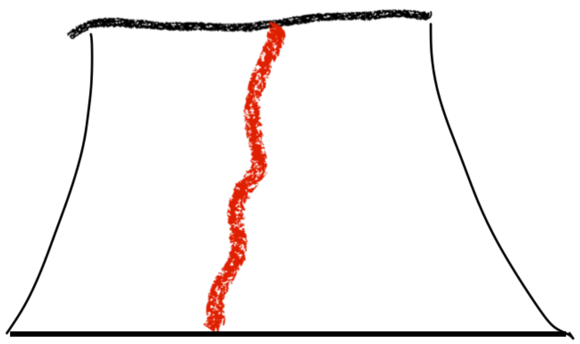
BBBJMS '12

Haehl, Loganayagam& Rangamani '15

Deconstructing hydrodynamics -1

* Take $S_{bulk} = \frac{1}{4}\int \sqrt{-G} F_{ab}F^{ab}$ with $A_a = (A_\mu, A_r)$ and take

$$A_\mu(r \to \infty, x) = a_\mu(x) \qquad \phi(r,x) = \int_r^\infty dr' A_r(r',x)$$



* Choosing radial gauge

$$A_\mu \to \mathscr{A}_\mu = A_\mu + \partial_\mu \phi\,,$$
$$A_r \to \mathscr{A}_r = 0$$

* Regularity condition $\mathscr{A}_t(r_h, x) = 0$ left residual gauge transformation

$$\phi \to \phi + c(x^i)$$

Nickel & Son '10

de Boer, Heller, Pinzani-Forkeeva '15, '18

Glorioso, Crossley & Liu '18

The only invariant quantity is $\mu_a = \mathscr{A}_t(r \to \infty)$

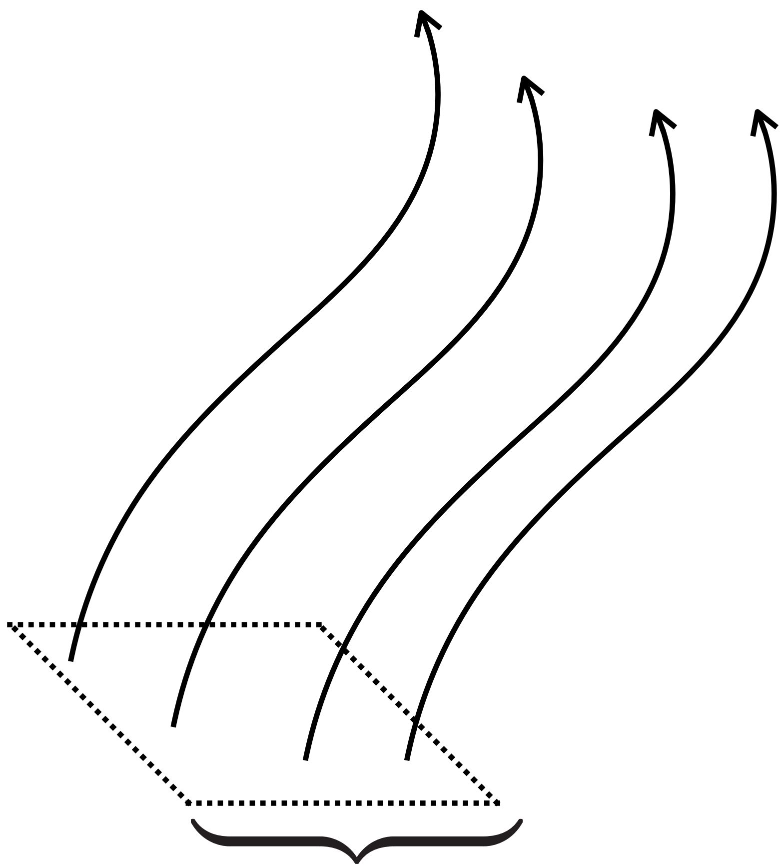
Building hydrodynamics -2

* For 1-form symmetry $\log Z = p(T, \mu_b) + \mathcal{O}(\partial)$ which gives

$$\langle J^{\mu\nu} \rangle = \rho_b (u^\mu h^\nu - u^\nu h^\mu) + \mathcal{O}(\partial)$$
$$\langle T^{\mu\nu} \rangle = (\varepsilon + p) u^\mu u^\nu + p g^{\mu\nu} - \mu_b \rho_b h^\mu h^\nu + \mathcal{O}(\partial)$$

And automatically gives

$$dp = s dT + \rho_b d\mu_b \,,$$
$$\varepsilon + p = sT + \mu_a \rho_b$$



ρ lines per unit area each with chem. pot. μ

$1/T \sim$ **size of thermal cycle**

$$\mu_b \sim \ln \mathscr{P} \exp\left(i \int_{T^2} b \right) \sim \int (b_{\mu\nu} u^\mu h^\nu) d^2\sigma$$

$\sim$ **Wilson surface**

* Consistent with (no) entropy production constraint

Anile '89

Emparan et. al '09

Schubring '14;

Grozdanov, Hofman & Iqbal '16

Armas & Jain '17-...; Glorioso & Son '18

Relations to ideal MHD

- To related to traditional MHD, take $u^\mu = (1, V^i),\ h^\mu = (0, B^i/\rho_b)$
- The electric field is $E^i = \frac{1}{2}\epsilon^{ijk}J_{jk} = -(V \times B)^i$ ← Ideal Ohm's law $E + V \times B = \frac{j}{\sigma} \to 0$
- Ward identity $\partial_\mu J^{\mu\nu} = 0$, encodes

$$\partial_i B^i = 0$$

Gauss' law

$$\partial_t B^i + (\nabla \times E)^i = 0$$

Faraday's law

- Assuming $p = \ldots + \frac{1}{2}\mu_b^2$ so that $\mu_b/\rho_b \sim \text{const}$

$$\varepsilon(\partial_t + V \cdot \partial)V^i = -\partial^i p + \big((\partial \times B) \times B\big)^i$$

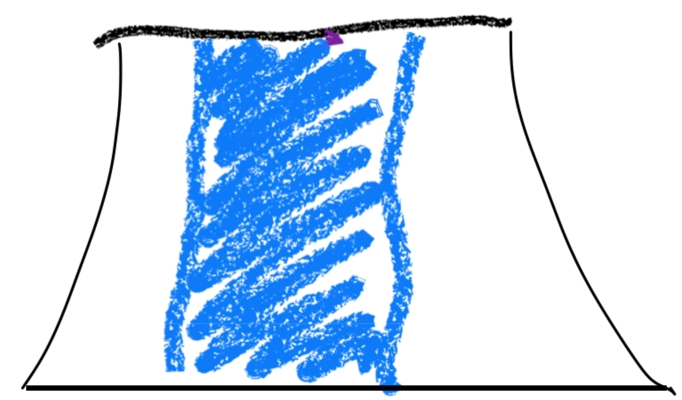
Euler + Lorentz Force + Ampere's law $j = \nabla \times B - \partial_t E$ ($\partial_t E \approx 0$)

Deconstructing hydrodynamics -2

Hofman & Iqbal '18
Grozdanov & NP '18-'19
Grozdanov, Lucas & NP '19

✻ Take $S_{bulk} = \frac{1}{4}\int \sqrt{-G}(dB)_{abc}(dB)^{abc}$
with $B_{ab} = (B_{r\mu}, B_{\mu\nu})$ and take

$$B_{\mu\nu}(r \to \Lambda_{cutoff}, x) \sim b_{\mu\nu}(x) \qquad \varphi_\mu(r,x) = \int_r^\infty dr' B_{\mu r}(r', x)$$



$$\int dx^\mu \varphi_\mu \sim \int dx^\mu \int dr\, \mathscr{B}_{r\mu}$$

✻ Choosing radial gauge

$$B_{\mu\nu} \to \mathscr{B}_{\mu\nu} = B_{\mu\nu} + (d\varphi)_{\mu\nu}\,,$$
$$B_{r\mu} \to \mathscr{B}_{r\mu} = 0$$

✻ Regularity condition $\mathscr{B}_{0\mu}(r_h, x) = 0$ left residual gauge transformation

$$\varphi_0 \to \varphi_0\,, \qquad \varphi_i \to \varphi_i + C_i(x^j)$$

The only invariant quantity is $\mu_b h_\mu \sim \mathscr{B}_{0\mu}(r \to \Lambda_{cutoff}, x)$

Glorioso & Son '19
Landry '20

Building hydrodynamics for 2-group

* For **2-group symmetry** $\log Z = p(T, \mu_a, \mu_b) + \mathcal{O}(\partial)$. But there is a problem

$$\mu_b \sim \int_{T^2} b \qquad \text{But now} \qquad \int_{T^2} b \to \int_{T^2} b + \hat{\kappa} \int_{T^2} \lambda(da)$$

Invariant under $\begin{array}{c} a \to a + d\lambda \\ b \to b + d\Lambda \end{array}$

chem pot transform like anomalous partition function

* To allow finite "string" density

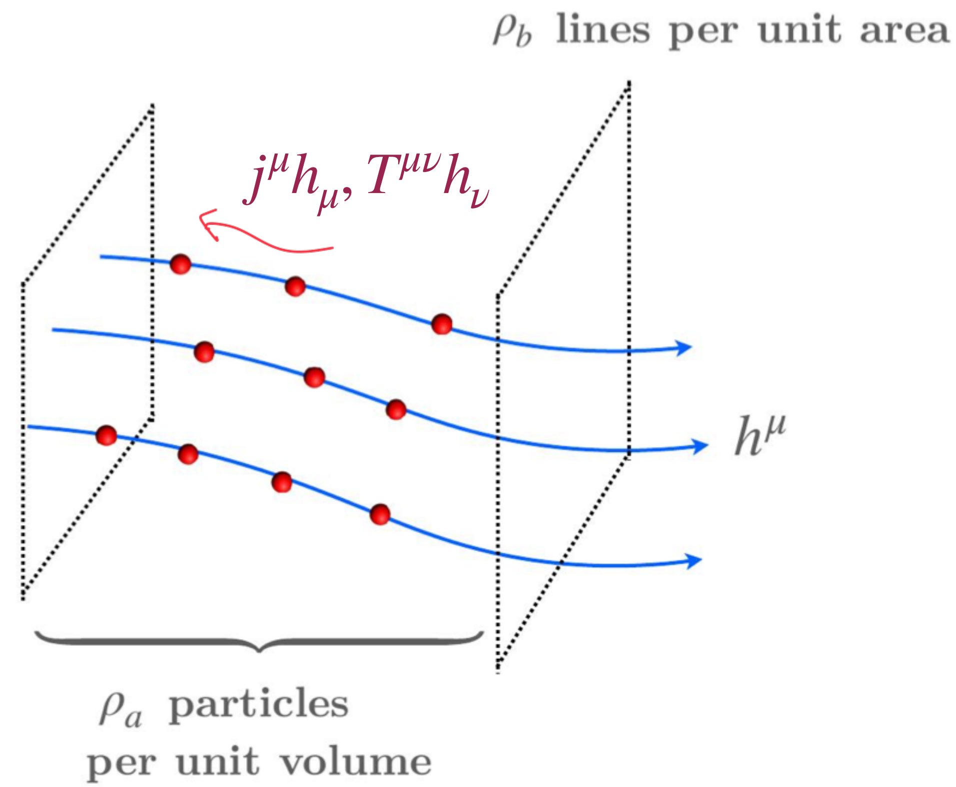
$$\mu_b \sim \int_{T^2} b + S_{WZ} \qquad S_{WZ} \sim \kappa \int d^2x \left[\mu(\epsilon^{\alpha\beta} u_\alpha a_\beta) \right]$$

Dubovsky, Hui & Nicolis '11;
Jensen, Loganayagam & Yarom '12
Haehl, Logan & Rangamani '13
Delacretaz & Glorioso '20

* Generated terms that ensure no entropy production

$$\langle T^{\mu\nu} \rangle = \ldots - 2\kappa\mu_a^2 \rho_b u^{(\mu} h^{\nu)}, \qquad \langle j^\mu \rangle = \ldots - 2\kappa \mu_a \rho_b h^\mu$$

What does this means



- New terms implies that there is equilibrium currents

$$\langle j^\mu \rangle h_\mu = -2\kappa \mu_a \rho_b$$

  - Unlike usual anomalous hydro, it occurs at ideal level (dimension independent)
  - Related to fermion zero modes in microscopic picture

- New modes and modified speed of sound

$$\omega_\perp = \left(-\frac{\hat{\kappa}\mu_a^2 \rho_b}{\varepsilon + p} \pm \sqrt{\mathcal{V}_A^2 + \left(\frac{\hat{\kappa}\mu_a^2 \rho_b}{\varepsilon + p} \right)^2} \right) q_z \,, \qquad \mathcal{V}_A^2 = \frac{\mu_b \rho_b}{\varepsilon + p}$$

$$\omega_\parallel = -\frac{\hat{\kappa}\rho_b}{\chi_{aa}} q_z \,.$$

  - No need to perform gauging in hydrodynamic setup (break gradient expansion)
  - 2-group can exist in any dimensions

Holographic dual

Cordova, Dumitrescu & Intrilligator '18

* Simplest holographic action $S_{grav} = \int d^5X\sqrt{-G}\Big((dA)^2 + (dB - \kappa A \wedge dA)^2\Big)$

$$A_\mu(r \to \infty, x) = a_\mu(x)$$
$$\phi(r,x) = \int_r^\infty dr' A_r(r',x)$$

$$B_{\mu\nu}(r \to \Lambda_{cutoff}, x) \sim b_{\mu\nu}(x)$$
$$\varphi_\mu(r,x) = \int_r^\infty dr' \left[B_{\mu r}(r',x) + \phi(dA)_{\mu r}\right]$$

* Choosing radial gauge $A \to A + d\lambda \quad B \to B + d\Lambda + \kappa\lambda(dA)$

$$A_\mu \to \mathscr{A}_\mu = A_\mu + \partial_\mu\phi\,,$$
$$B_{\mu\nu} \to \mathscr{B}_{\mu\nu} = B_{\mu\nu} + \phi(dA)_{\mu\nu} + (d\varphi)_{\mu\nu}$$

* After regularity $\mathscr{A}_0(r_h) = 0$, $\mathscr{B}_{0\mu}(r_h) = 0$, there is still residual

$$\phi \to \phi + c(x^i)$$
$$\varphi_\mu \to \varphi_\mu + \delta_{\mu i}C_\mu(x^j) - \hat{\kappa}c(x^j)\mathscr{A}_\mu$$

$\mathscr{A}_0$ and $\mathscr{B}_{0i} + \hat{\kappa}\mathscr{A}_0\mathscr{A}_i$

0-form and 1-form chem pot.

Holographic dual

* Simplest holographic action $S_{grav} = \int d^5X\sqrt{-G}\Big((dA)^2 + (dB - \kappa A \wedge dA)^2\Big)$

  * Even in equilibrium and probe limit $\mu_a, \sqrt{\mu_b} \ll T$ around $\langle J^{\mu\nu}\rangle = \rho_b \delta^{tz}$

$$\sqrt{-G}\,(d\mathscr{A})^{rt} - 2\hat{\kappa}\left(\frac{\rho_b}{\sqrt{-G}}\right)\mathscr{A}_z = \textsc{const}$$

$$\sqrt{-G}\,(d\mathscr{A})^{rz} - 2\hat{\kappa}\left(\frac{\rho_b}{\sqrt{-G}}\right)\mathscr{A}_t = 0$$

$$A_z \neq 0\,, \quad \textsc{when} \quad A_t \neq 0 \quad \Longrightarrow \quad \boxed{\langle j^\mu\rangle h_\mu = -\,2\kappa\mu_a\rho_b}$$

More on Ward identities and holographic dictionary
see DeWolfe & Higginbotham 2010.06594

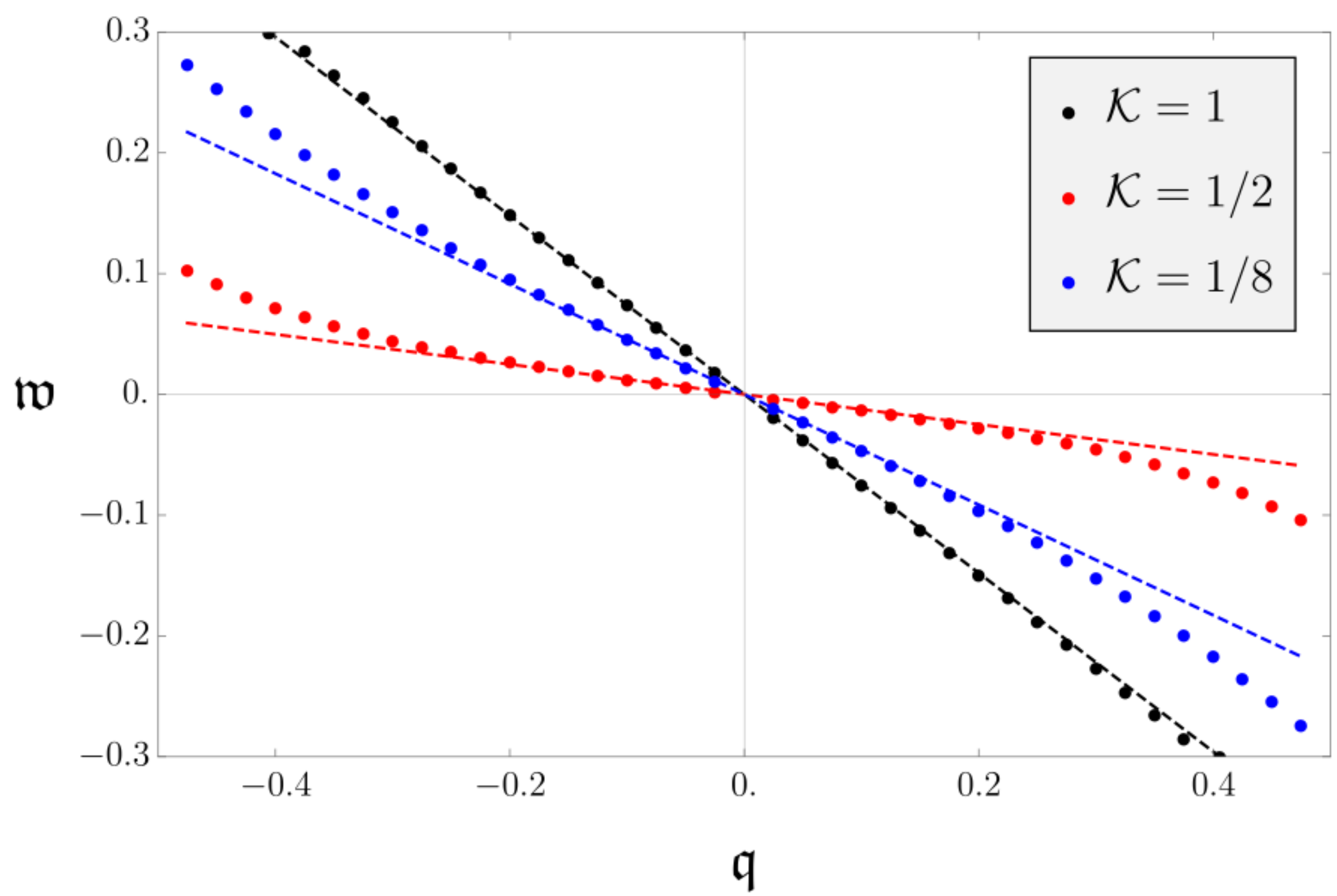
Holographic dual

* Simplest holographic action $S_{grav} = \int d^5X\sqrt{-G}\left((dA)^2 + (dB - \kappa A \wedge dA)^2\right)$
   - Compute Longitudinal QNM

Quainormal modes = poles in $\langle j^\mu j^\nu \rangle = \dfrac{\delta^2 S_{grav}}{\delta a_\mu \delta a_\nu}$

$\mathcal{K} := \hat{\kappa}\rho_b / r_h^2$

$$\omega_{||} = -\frac{\hat{\kappa}\rho_b}{\chi_{aa}} q_z \, .$$



Comparison with Anomalous fluid

✻ Something obvious $\langle T^{\mu\nu}\rangle = \ldots - 2\kappa\mu_a^2\rho_b u^{(\mu}h^{\nu)}\,, \qquad \langle j^\mu\rangle = \ldots - 2\kappa\mu_a\rho_b h^\mu$

- Anomaly induced transport occur at $n-1$ derivative of 2n-dim QFT
- Magnetic field treated perturbatively as a (1st derivative) source
- 1 sound + 2 diffusion in higher dim
  1 sound +1 diffusion + 1 (chiral)sound in 1+1d
- For some equation of states: can coupled to Maxwell fields at a cost of gradient expansions

- 2-group transport always occur at 0th order
- Magnetic field is at 0th order in derivative expansion and is dynamical
- 2 sounds (longitudinal + transverse) +1 (chiral) sound + 1 diffusion
- No assumptions on EoS

Summary and Outlook

- Expanding hydrodynamic framework to (at least one of)higher-structure
  - Hydrodynamics of everything(gapless)?
- New phenomena similar to anomaly induced transports
  - Gauging of others mixed anomaly?
  - Bypass process of gauging?
- More higher-group/structure?

Recently see: Hidaka, Nitta & Yonekura '20;
Brennan & Cordova '20;
Tanizaki & Ünsal '19, Brauner '20,...

Thank you very much!