A Mellin-Barnes Approach to Scattering in de Sitter

Charlotte Sleight

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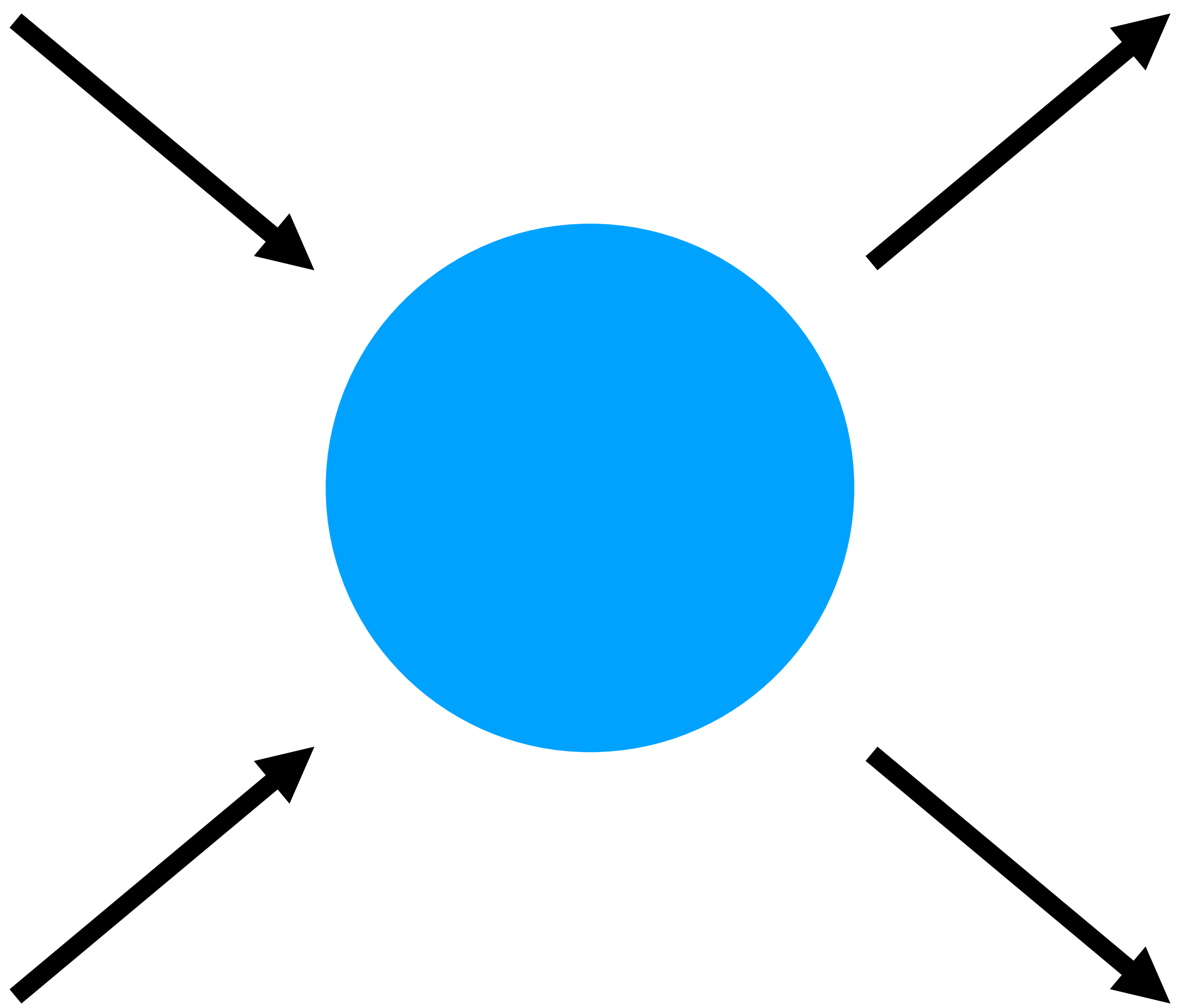
1906.12302 C.S.
1907.01143, 2007.09993 C.S. and M. Taronna
+ to appear.

Scattering Amplitudes



… are the bridge between theory and experiment.

Scattering Amplitudes

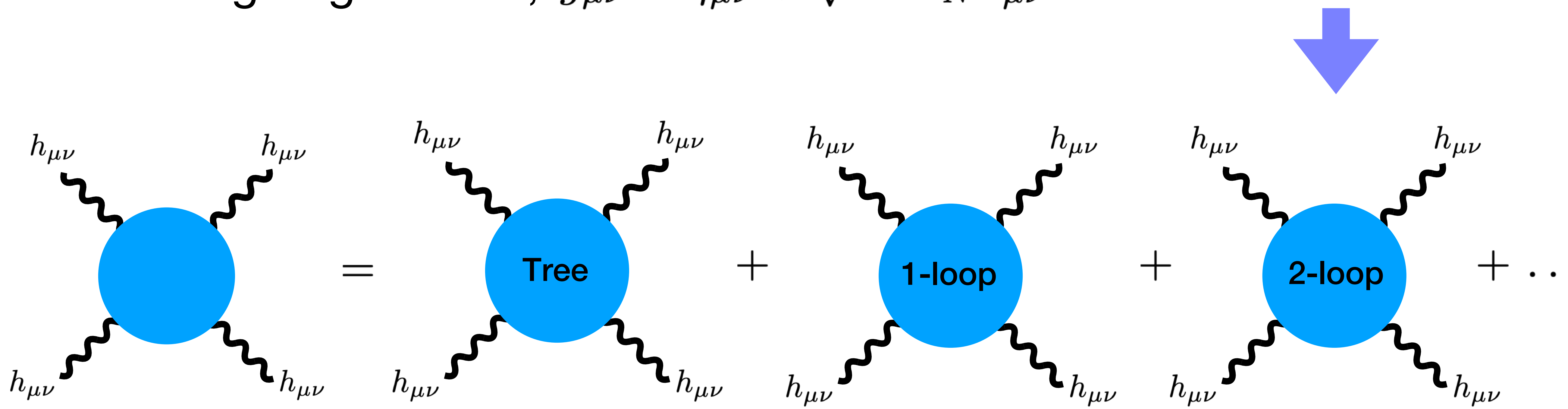
…provide a theoretical laboratory to test our theories.

General Relativity

$$\mathcal{L}_{EH}\left[g\right] = \frac{1}{16\pi G_N}\sqrt{-g}R$$

Scattering of gravitons, $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G_N}h_{\mu\nu}$:

UV divergent!



Scattering Amplitudes

Challenge: Quest for physics beyond the SM and GR

Access to theoretical and experimental laboratories is limited:

- Computing scattering amplitudes is **hard**.

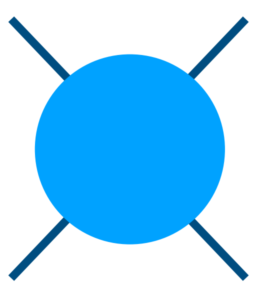
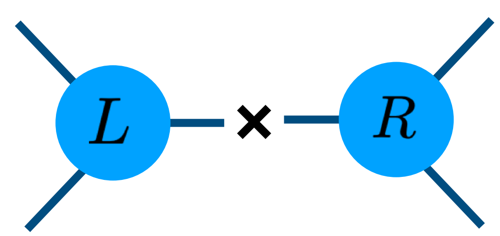
- At high energies we **lack** experimental data.

Challenge: Classify the set of consistent theories

Scattering Amplitudes: Bootstrap Approach

Challenge: Carve out space the of consistent theories

Collect theoretical data points by imposing basic physical criteria:

- Lorentz invariance
- Unitarity $SS^\dagger = 1$
- Locality  $\sim \frac{1}{s_{LR} - m^2}$ 

$\vdots$

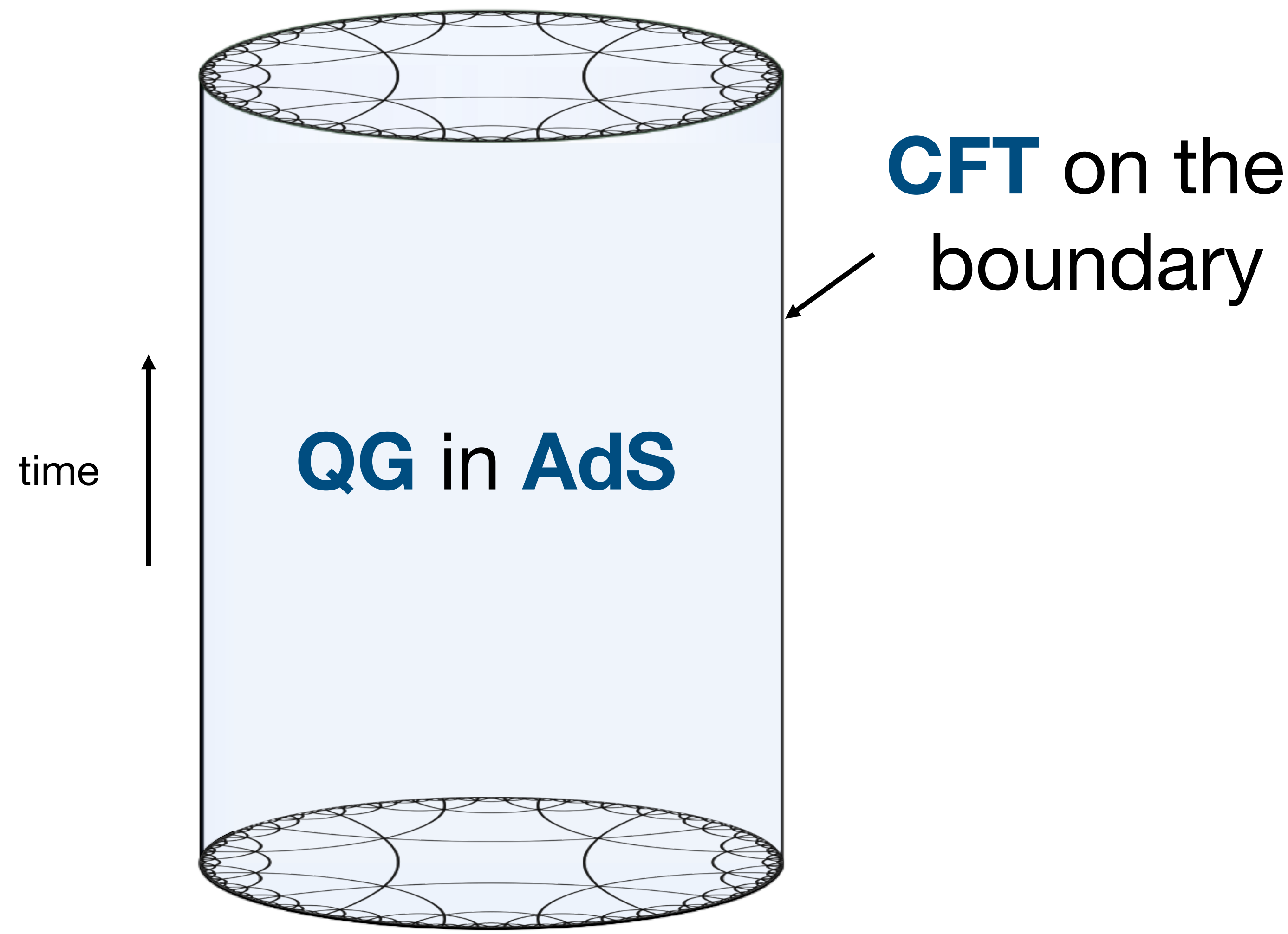
The AdS-CFT Correspondence

Maldacena 1997

Quantum **G**ravity
in **a**nti-**d**e **S**itter space

=

Conformal **F**ield **T**heory
on the boundary at infinity



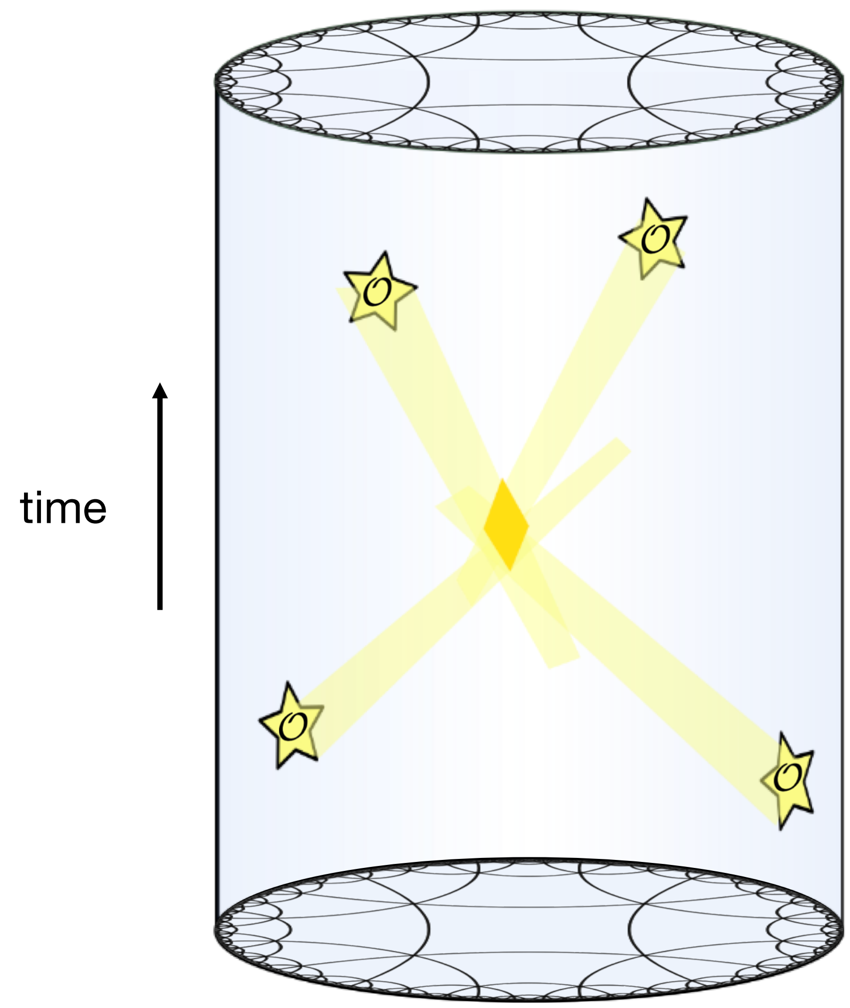
The AdS-CFT Correspondence

Maldacena 1997

Observables in
Quantum Gravity
in anti-de Sitter space

=

Correlation functions in the
Conformal Field Theory
on the boundary at infinity



Scattering in AdS

The AdS-CFT Correspondence

Maldacena 1997

Observables in
Quantum **G**ravity
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=

Correlation functions in the
Conformal **F**ield **T**heory
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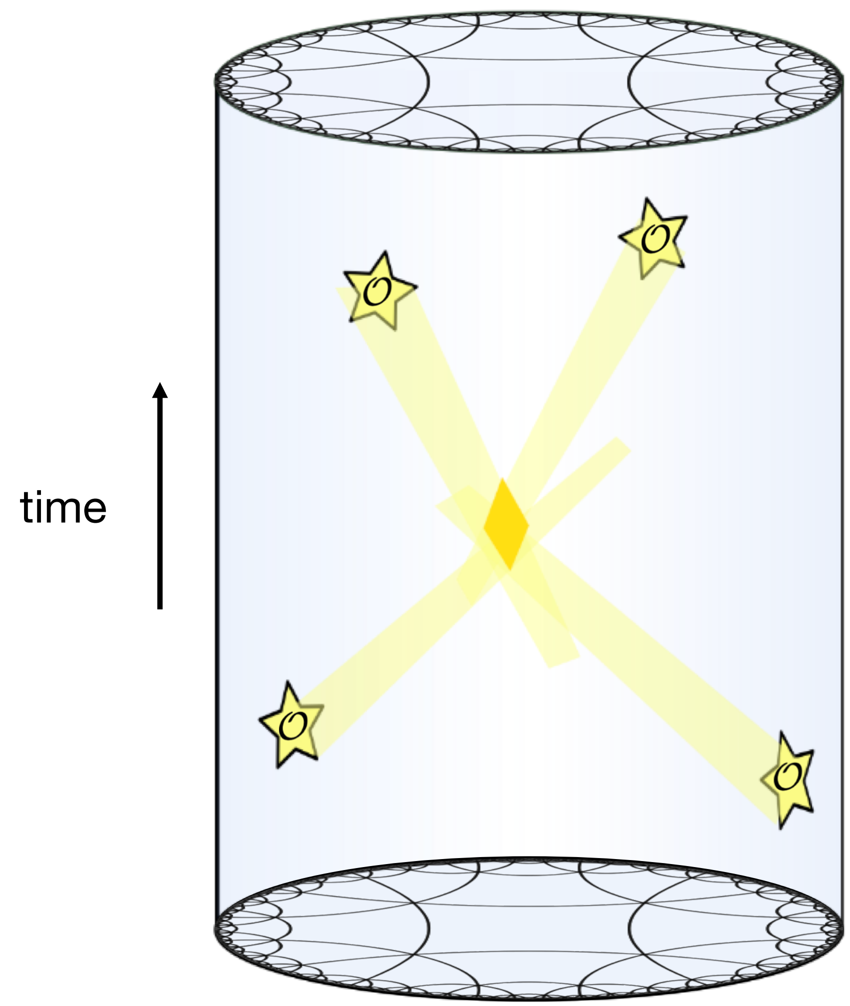
?!



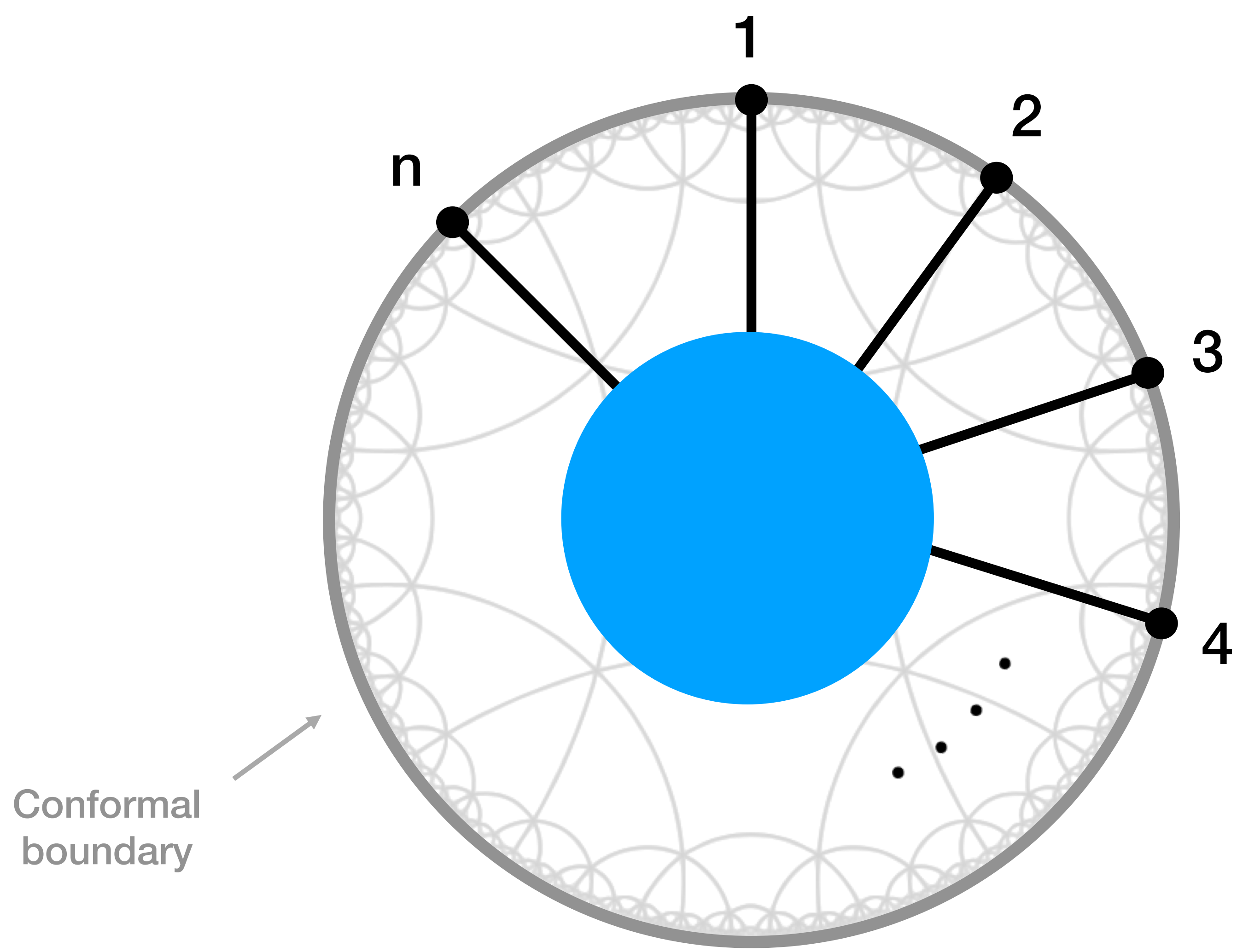
Scattering in AdS

Defined non-perturbatively by:

- Conformal symmetry
- Unitarity
- Consistent Operator Product Expansion

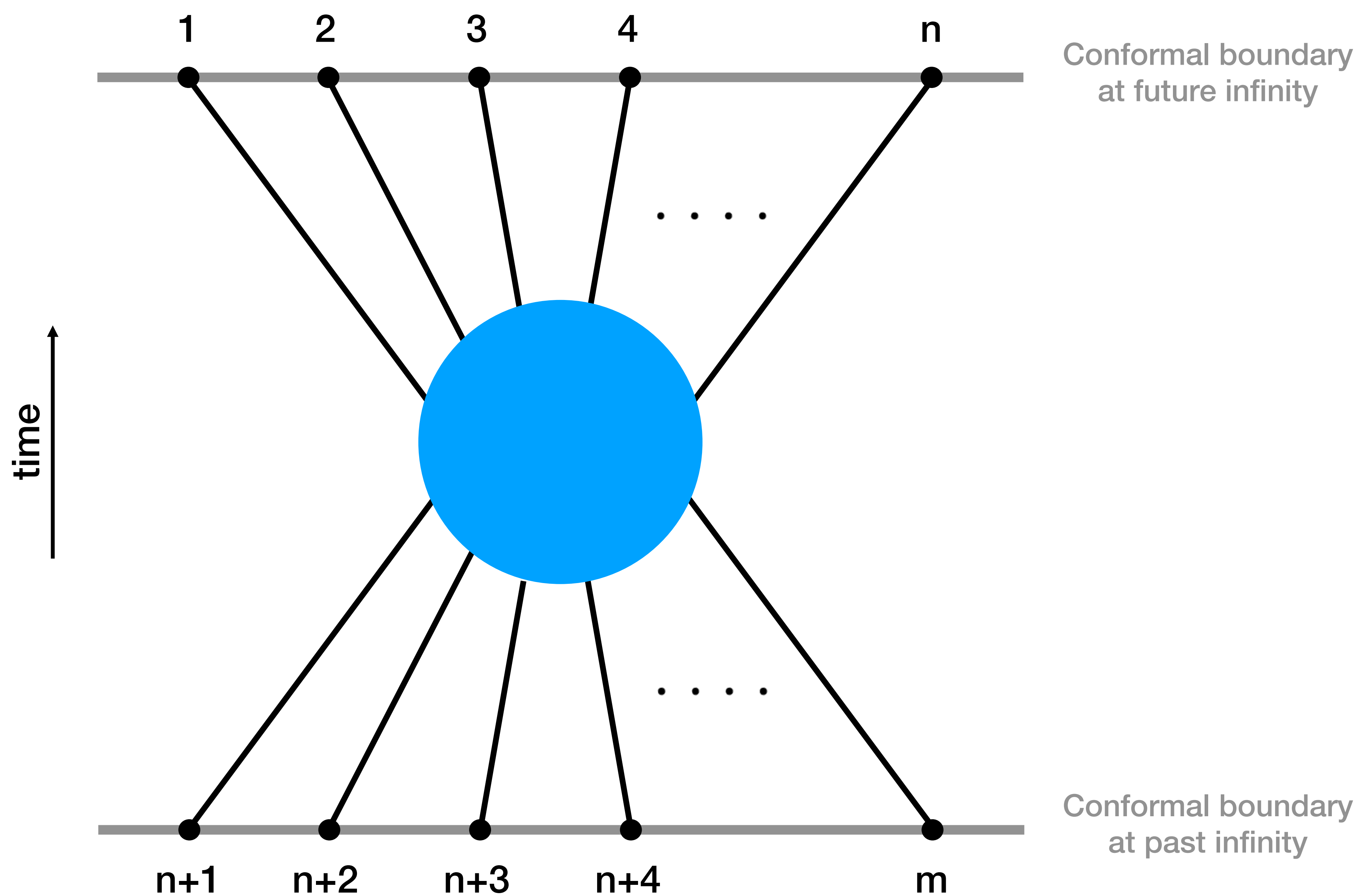
Scattering in anti-de Sitter

…in AdS we have a pretty good understanding.

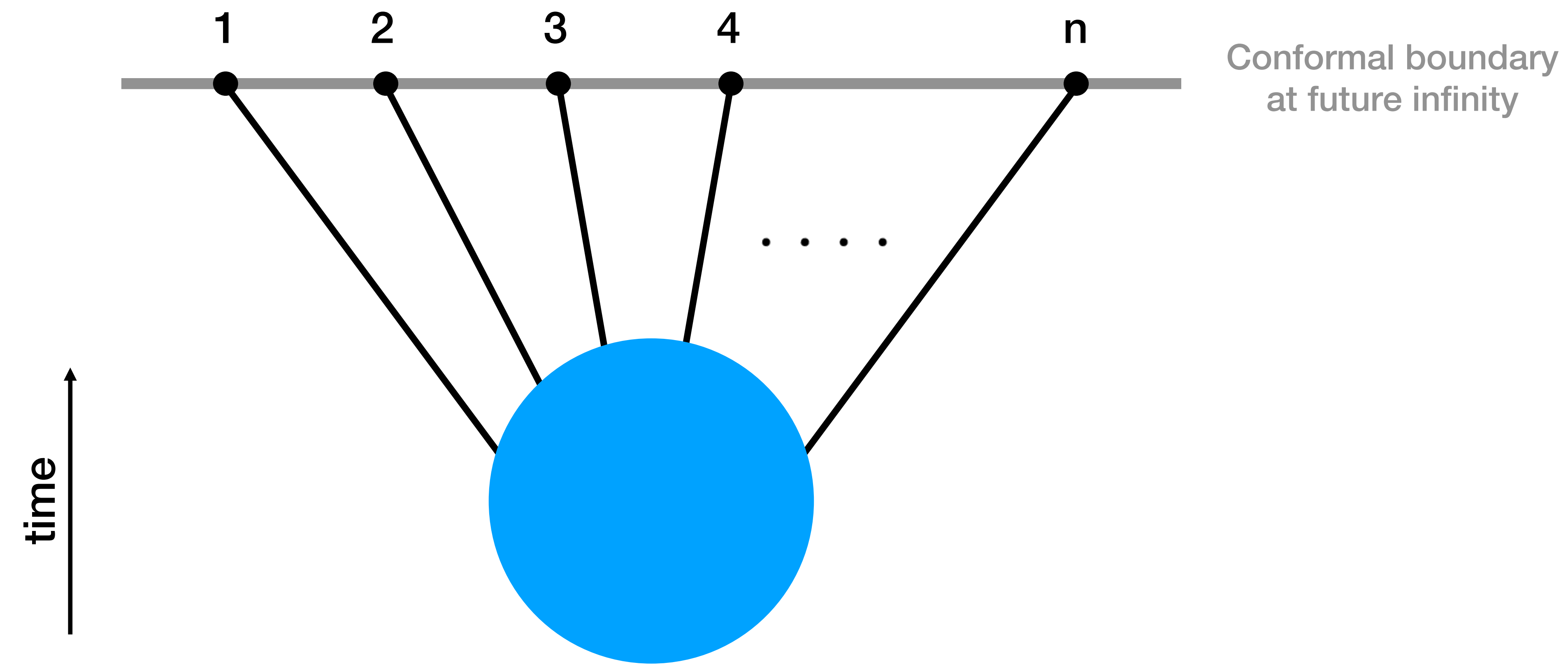


Can we adapt extend this understanding and techniques beyond the relative security of AdS space?

Scattering in de Sitter

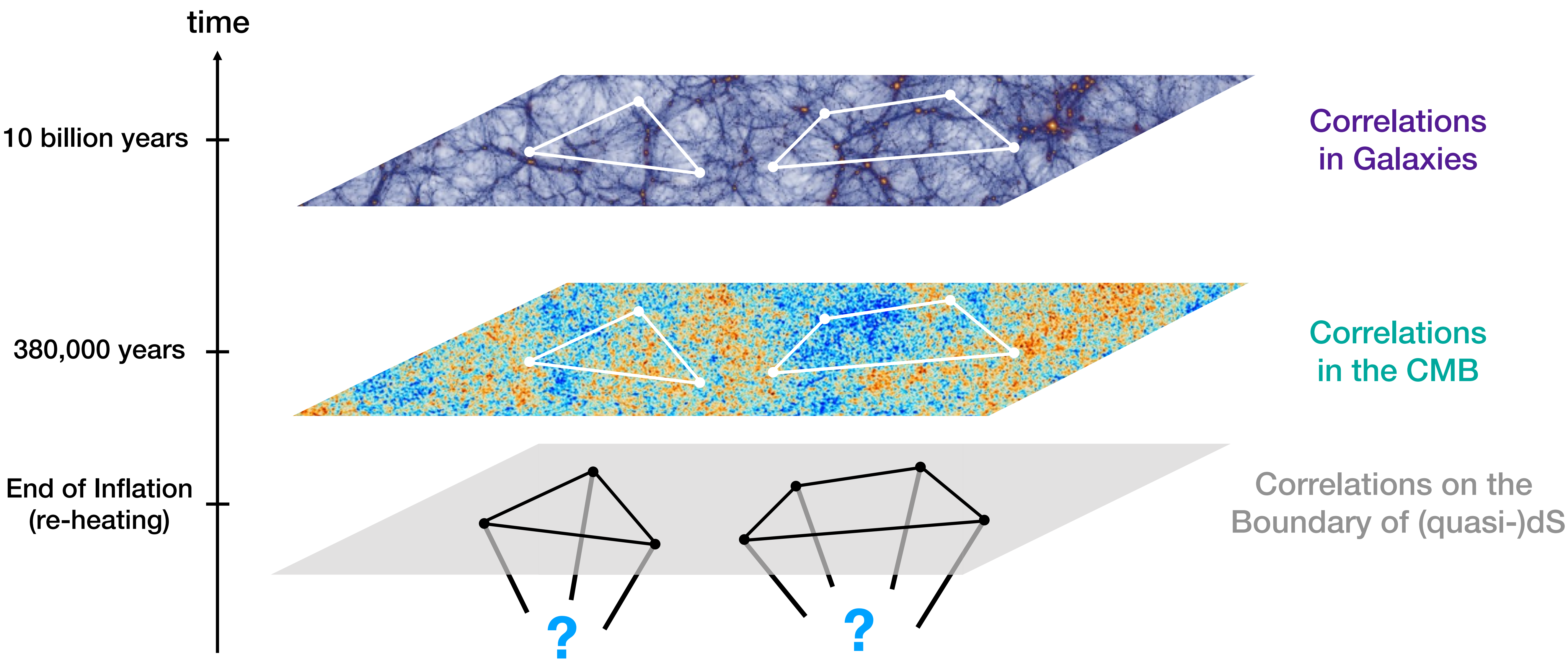


Scattering in de Sitter



Cosmological Collider Physics

Many groups, e.g.: Chen and Wang 2009, Baumann and Green 2011, Noumi, Yamaguchi and Yokoyama 2013, Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018, …

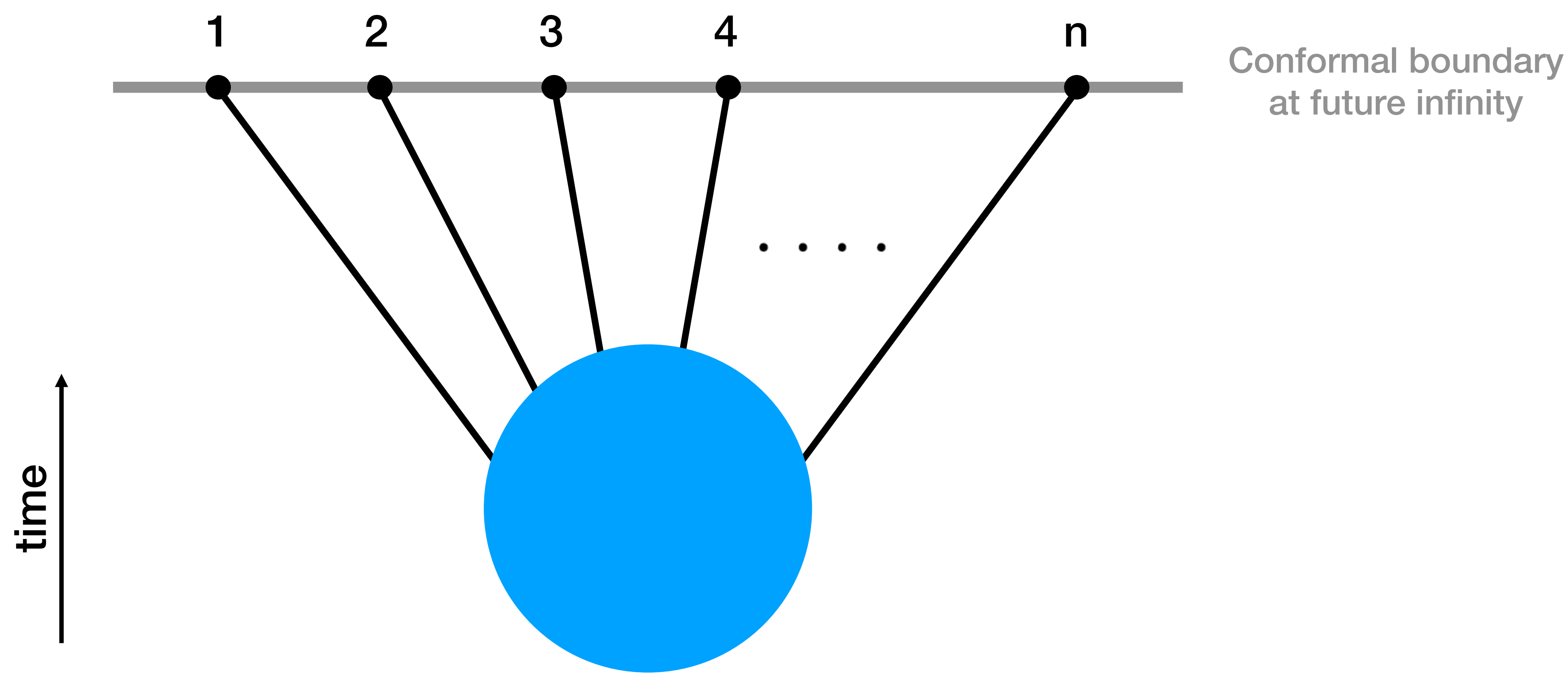


Task: Classify the imprints of new degrees of freedom

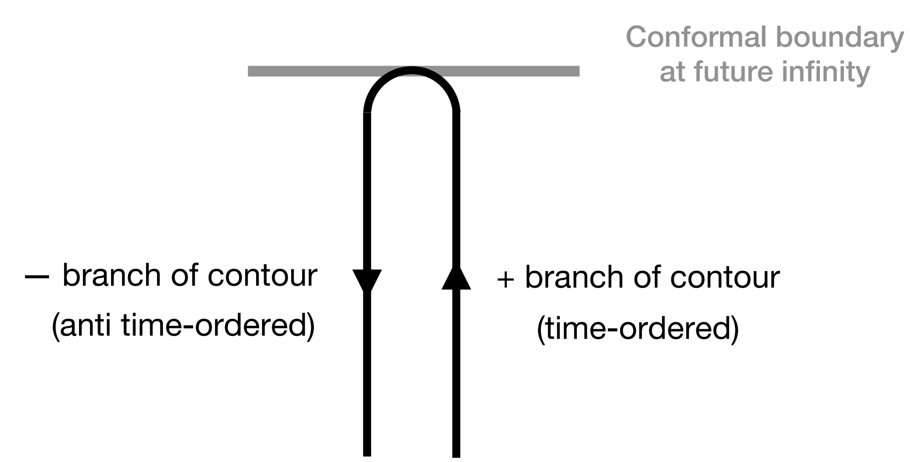
The Cosmological Bootstrap:

Maldacena 2015; Arkani-Hamed, Benincasa, Mcleod, Parisi, Postnikov, Vergu 2017-; Arkani-Hamed, Baumann, Duaso-Pueyo, J
en and Pajer 2020; Pajer, David Stefanyszyn, Jakub Supeł 2020; Goodhew, Jazayeri, Pajer 2020; Céspedes, Davis, Melville 202

Scattering in de Sitter



Computed within the in-in (Schwinger-Keldysh) formalism:



the bulk time integral
follows the in-in contour

Outline

Part I: Can we place boundary correlators in (A)dS on a similar footing?

$\longrightarrow$ **Mellin-Barnes representation in momentum space**

cf. Mellin-Barnes representation of the Gauss Hypergeometric function

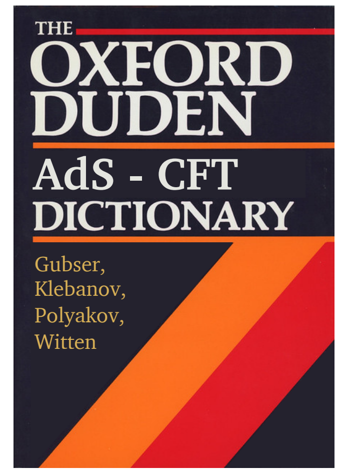
$$ {}_2F_1\left(a,b;c;z\right)=\frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\frac{\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(-s\right)}{\Gamma\left(c+s\right)}\left(-z\right)^{s} $$

Part II: Applications.

- Contact diagrams
- Exchanges
- Constraints on interactions of massless spinning particles

The AdS-CFT Dictionary

Maldacena 1997



Quantum **G**ravity
in **a**nti-**d**e **S**itter space

Conformal **F**ield **T**heory

boundary value

$$Z_{\text{QG AdS}}\left[\varphi \to \bar{\varphi}\right]$$

source

$$Z_{\text{CFT}}\left[\bar{\varphi}\right]$$

Elementary field φ

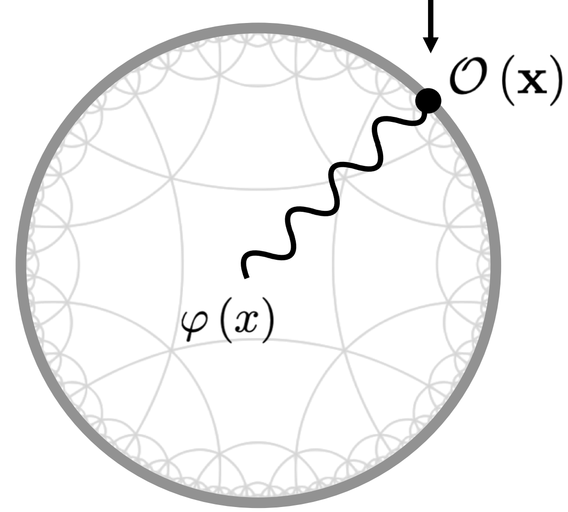
spin J, mass $m^2R^2 = -\left(\Delta_+\Delta_- + J\right)$

$$\Delta_+ + \Delta_- = d \ , \ \Delta_+ \geq \Delta_-$$

Local operator $\mathcal{O}$

spin J, scaling dimension Δ_+

$$\int d^d\mathbf{x}\, \bar{\varphi}\left(\mathbf{x}\right) \mathcal{O}\left(\mathbf{x}\right)$$



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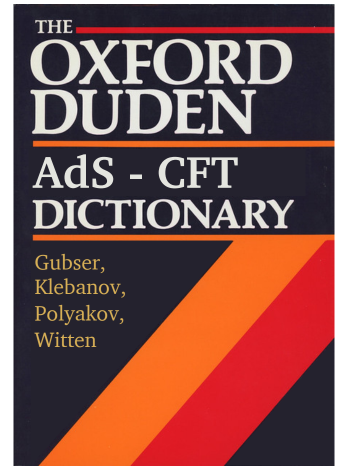

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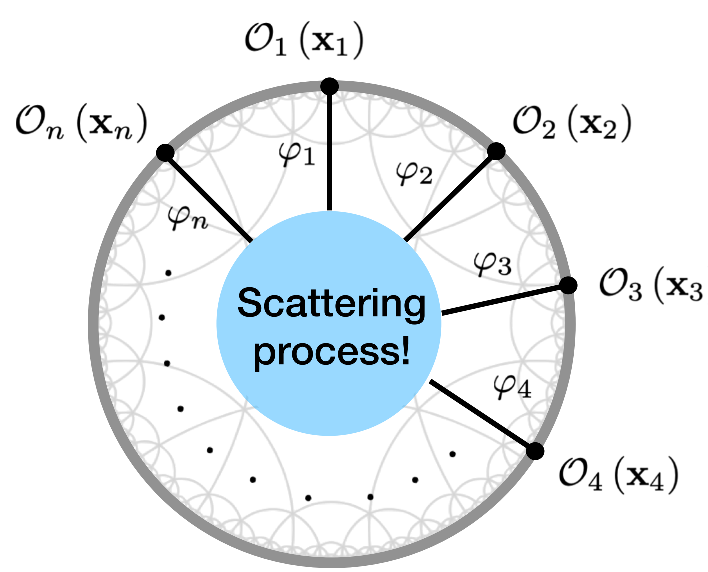
Local operator $\mathcal{O}$

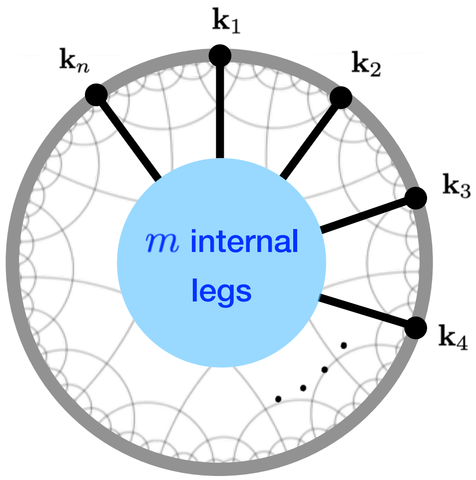
spin J, scaling dimension Δ_+

n-point scattering
of particles φ_i



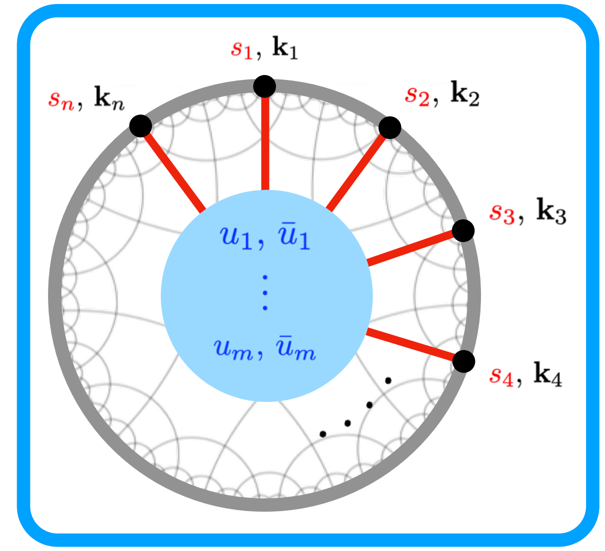
n-point correlator
of operators $\mathcal{O}_i$

Mellin-Barnes Representation



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs   internal legs

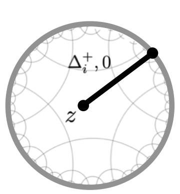


Mellin-Barnes representation

External leg, momentum $\mathbf{k}_i$ $(|\mathbf{k}_i|)^{-2s_i+\Delta_i^+-\frac{d}{2}}$ **external Mellin variable,** s_i

Poles from the Mellin-Barnes representation of the propagators for the external legs →

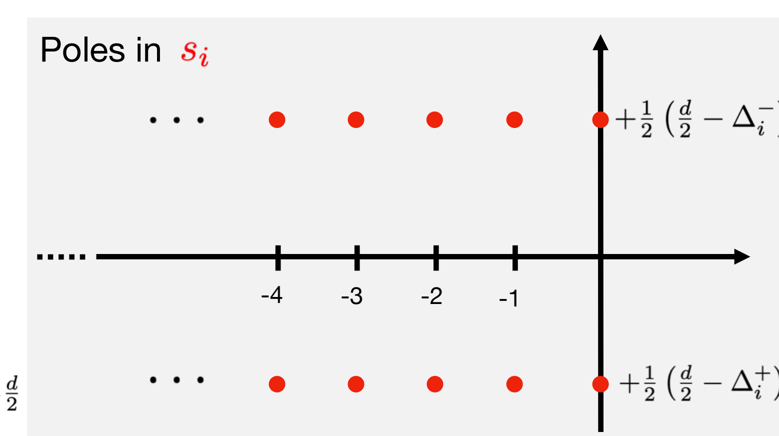
e.g. scalar field propagator:

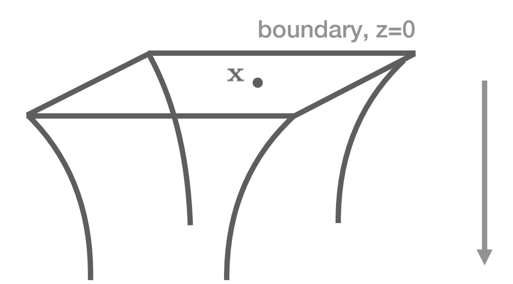


$$= z^{\frac{d}{2}} \left(\frac{|\mathbf{k}_i|}{2}\right)^{\Delta_i^+-\frac{d}{2}} K_{\Delta_i^+-\frac{d}{2}}\left(z|\mathbf{k}_i|\right)$$

Modified Bessel function of the second kind

$$= \int_{-i\infty}^{+i\infty} \frac{ds_i}{2\pi i} z^{\frac{d}{2}-2s_i}\, \Gamma\left(s_i - \tfrac{1}{2}\left(\tfrac{d}{2}-\Delta_i^-\right)\right) \Gamma\left(s_i - \tfrac{1}{2}\left(\tfrac{d}{2}-\Delta_i^+\right)\right) \left(\frac{|\mathbf{k}_i|}{2}\right)^{-2s_i+\Delta_i^+-\frac{d}{2}}$$

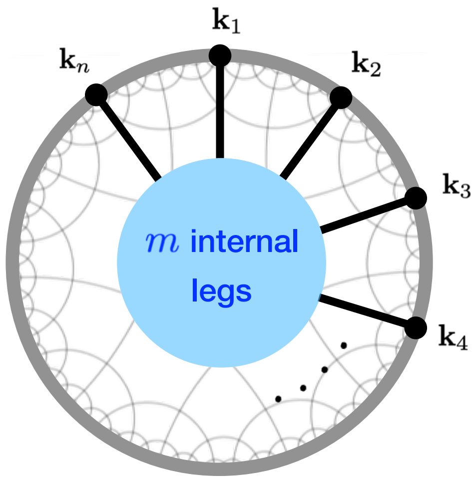




Euclidean AdS in Poincaré co-ordinates

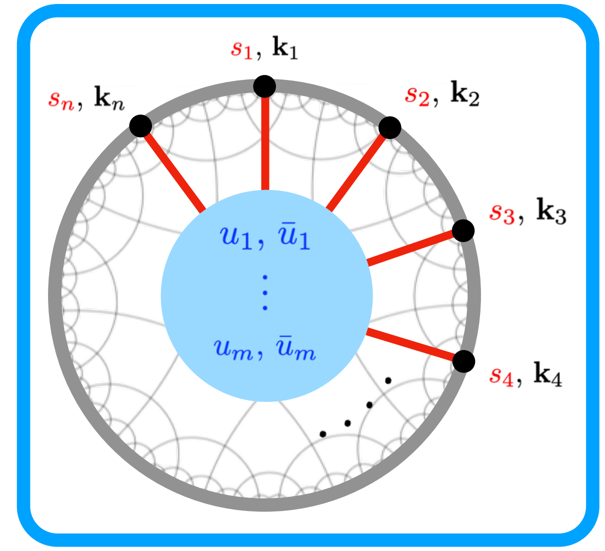
$$ds^2 = \left(\frac{R_{\mathrm{AdS}}}{z}\right)^2 \left(dz^2 + d\mathbf{x}^2\right)$$

Mellin-Barnes Representation



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs internal legs



Mellin-Barnes representation

External leg, momentum $\mathbf{k}_i$ $\quad (|\mathbf{k}_i|)^{-2s_i+\Delta_i^+-\frac{d}{2}}$ $\quad$ external Mellin variable, s_i

Translation invariance: $\mathbf{k}_1 + \ldots + \mathbf{k}_n = 0$

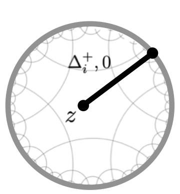
$$(2\pi)^d\, \delta^{(d)}\left(\mathbf{k}_1 + \ldots + \mathbf{k}_n\right) = \int d^d\mathbf{x}\, e^{i\mathbf{x}\cdot(\mathbf{k}_1+\ldots+\mathbf{k}_n)}$$

Dilatation Ward identities: $s_1 + \ldots + s_n = \text{const.}$

$$2\pi i\, \delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$

Poles from the Mellin-Barnes representation of the propagators for the external legs

e.g. scalar field propagator:



$$= z^{\frac{d}{2}} \left(\frac{|\mathbf{k}_i|}{2}\right)^{\Delta_i^+-\frac{d}{2}} K_{\Delta_i^+-\frac{d}{2}}\left(z|\mathbf{k}_i|\right)$$

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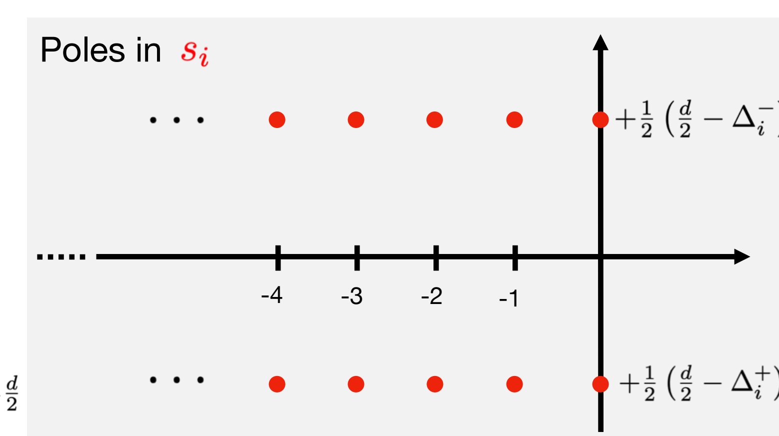

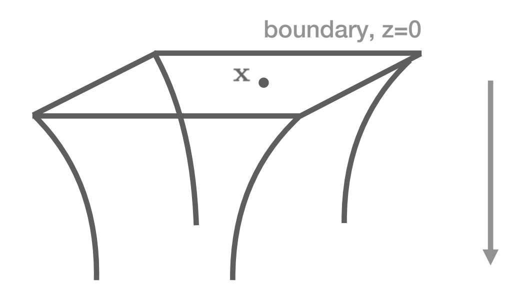

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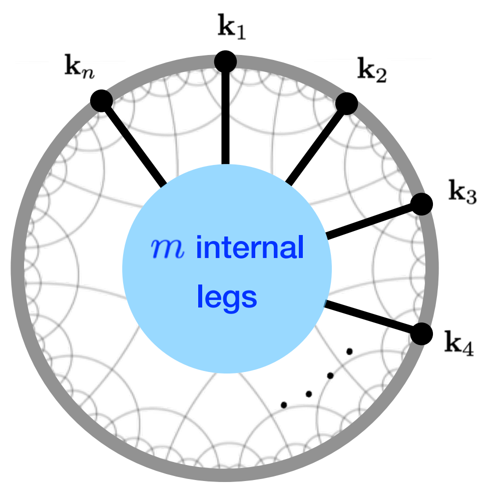
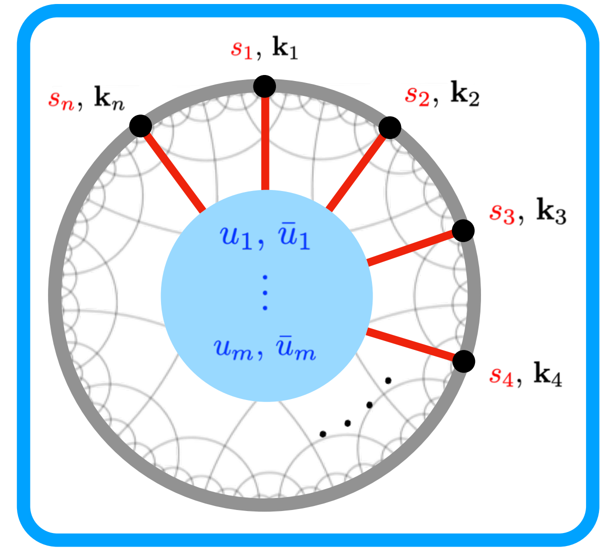
Mellin-Barnes Representation



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs internal legs



Mellin-Barnes representation

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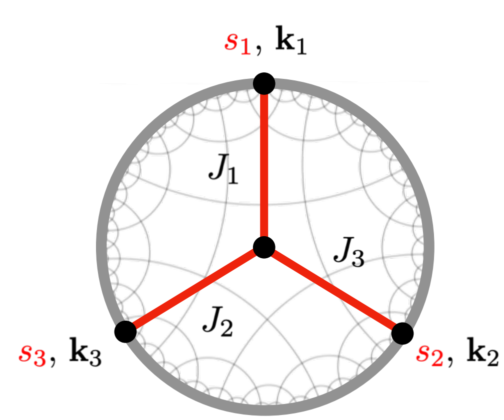
$$2\pi i\, \delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$

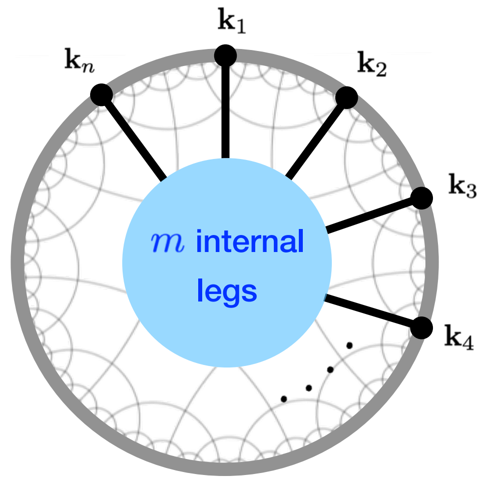
E.g. 3pt contact diagram, spins J1-J2-J3:

$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$$



$$s_1 + s_2 + s_3 = \frac{d + 2\left(J_1 + J_2 + J_3\right)}{4}$$

Mellin-Barnes Representation



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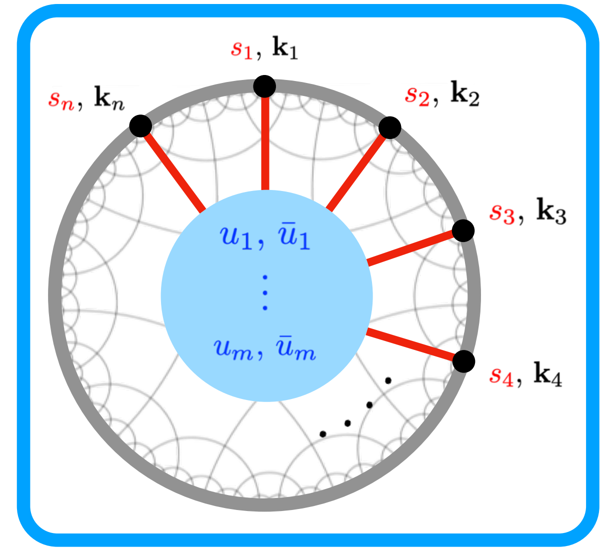
external legs   internal legs



Mellin-Barnes representation

External leg, momentum $\mathbf{k}_i$ $\quad (|\mathbf{k}_i|)^{-2s_i+\Delta_i^+-\frac{d}{2}}$ $\quad$ **external Mellin variable**, s_i

Translation invariance: $\mathbf{k}_1 + \ldots + \mathbf{k}_n = 0$

$$(2\pi)^d\,\delta^{(d)}\left(\mathbf{k}_1 + \ldots + \mathbf{k}_n\right) = \int d^d\mathbf{x}\, e^{i\mathbf{x}\cdot(\mathbf{k}_1+\ldots+\mathbf{k}_n)}$$

Dilatation Ward identities: $s_1 + \ldots + s_n = \text{const.}$

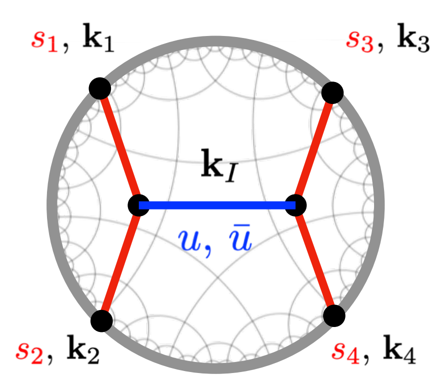
$$2\pi i\,\delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$

Internal leg, momentum $\mathbf{k}_I$ $\quad (|\mathbf{k}_I|)^{-2(u+\bar{u})}$ $\quad$ Two **internal Mellin variables**, $u,\, \bar{u}$

E.g. 4pt spin J exchange:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_I,$$

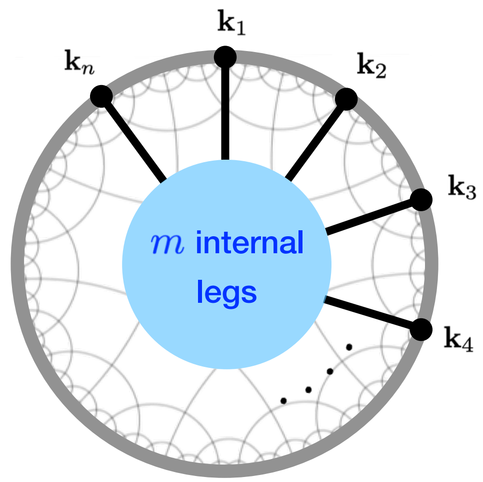
$$\mathbf{k}_3 + \mathbf{k}_4 = -\mathbf{k}_I$$



$$s_1 + s_2 + u = \frac{d + 2\left(J_1 + J_2 + J\right)}{4}$$

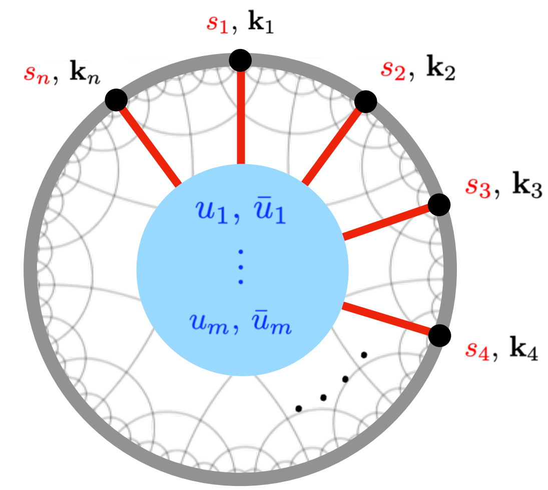
$$s_3 + s_4 + \bar{u} = \frac{d + 2\left(J + J_3 + J_4\right)}{4}$$

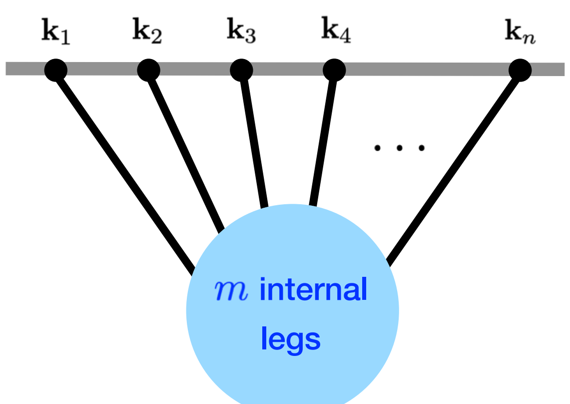
Bridging the Gap between EAdS and dS



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs internal legs





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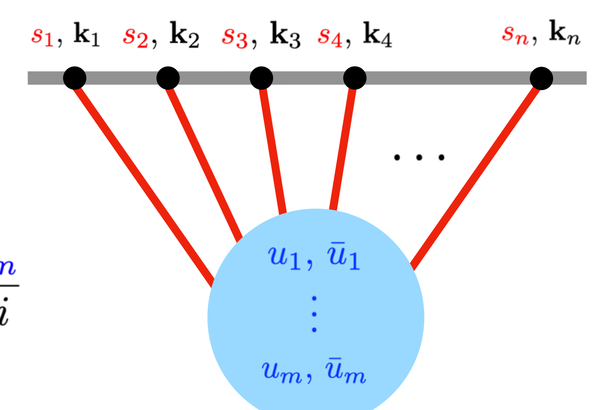
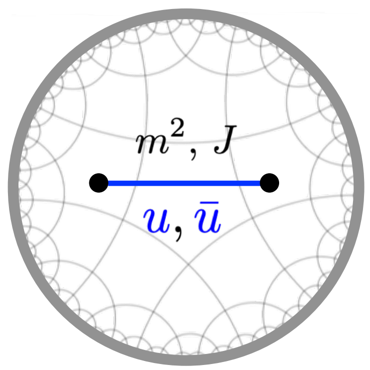


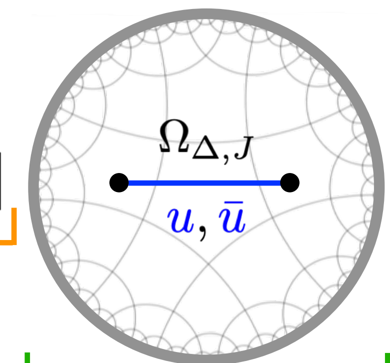
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In EAdS we have:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha\, \overset{\text{Dirichlet}}{\omega_D}\left(u,\bar{u}\right) + \beta\, \overset{\text{Neumann}}{\omega_N}\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

$\omega_{D/N}\left(u,\bar{u}\right)$project onto Dirichlet/Neumann boundary conditions:

$m^2R^2 = -\left(\Delta_+\Delta_- + J\right)$

$\Delta_+ + \Delta_- = d \ , \ \Delta_+ \geq \Delta_-$

$$\omega_{D/N}\left(u,\bar{u}\right) = \frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_{\mp}-\tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_{\mp}-\tfrac{d}{2}\right)\right)\right)$$

Recall the general solution to the wave equation near the boundary of EAdS, $z \to 0$:

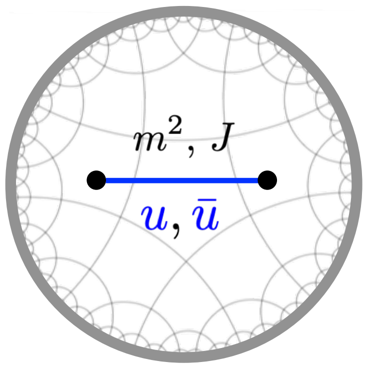
$$\varphi\left(z,\mathbf{k}\right) = \underbrace{\alpha\, z^{\Delta_+}\left[\mathcal{O}_{\Delta_+}\left(\mathbf{k}\right)+O\left(z^2\right)\right]}_{\text{Dirichlet boundary condition, selected by } \omega_D(u,\bar{u})} + \underbrace{\beta\, z^{\Delta_-}\left[\mathcal{O}_{\Delta_-}\left(\mathbf{k}\right)+O\left(z^2\right)\right]}_{\text{Neumann boundary condition, selected by } \omega_N(u,\bar{u})}$$

$$ds^2 = \left(\frac{R_{\text{AdS}}}{z}\right)^2\left(dz^2 + d\mathbf{x}^2\right)$$

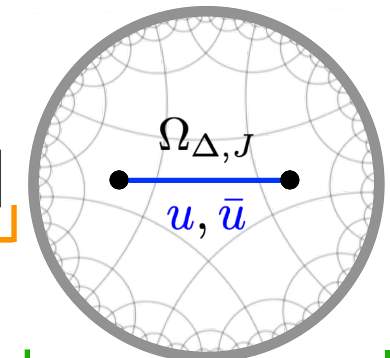
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In EAdS we have:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha\, \overset{\text{Dirichlet}}{\omega_D}\left(u,\bar{u}\right) + \beta\, \overset{\text{Neumann}}{\omega_N}\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



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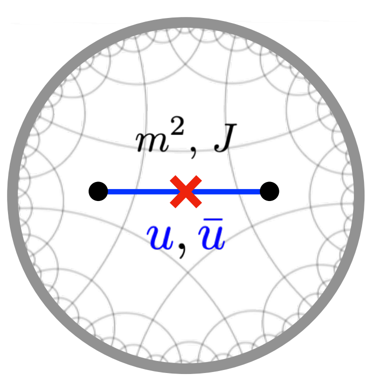
$\omega_{D/N}\left(u,\bar{u}\right)$ project onto Dirichlet/Neumann boundary conditions:

$m^2R^2 = -\left(\Delta_+\Delta_- + J\right)$

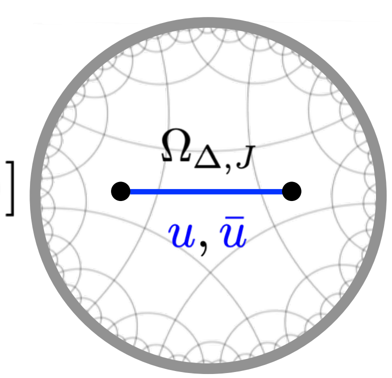
$\Delta_+ + \Delta_- = d\ ,\ \Delta_+ \geq \Delta_-$

$$\omega_{D/N}\left(u,\bar{u}\right) = \frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_{\mp}-\tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_{\mp}-\tfrac{d}{2}\right)\right)\right)$$

On shell, the factor $\csc\left(\pi\left(u+\bar{u}\right)\right)$ gets cancelled:



$$= \left[\alpha\, \omega_D\left(u,\bar{u}\right) + \beta\, \omega_N\left(u,\bar{u}\right)\right]$$

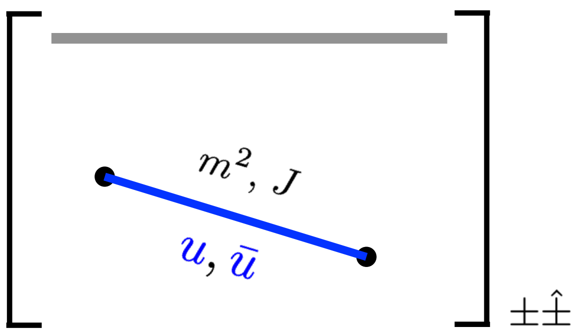


i.e. $\csc\left(\pi\left(u+\bar{u}\right)\right)$ is generated by the source term in the propagator equation.

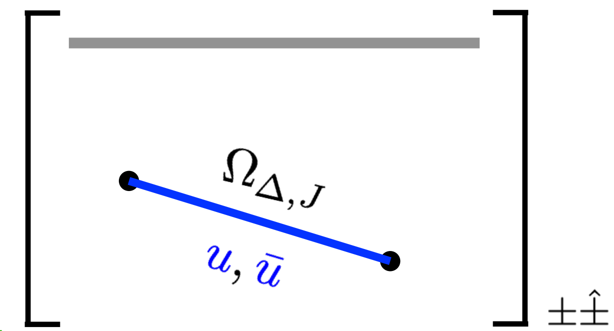
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In dS, for the $\pm\hat{\pm}$ branch of the in-in contour, we have:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



Harmonic function, $\left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0$

$m^2R_{\text{dS}}^2=\left(\Delta_+\Delta_-+J\right),\ \Delta_++\Delta_-=d$

Recall the general solution to the wave equation near the boundary of dS, $\eta\to 0$

$$\varphi\left(\eta,\mathbf{k}\right)=\alpha_{\pm\hat{\pm}}\underbrace{\left(-\eta\right)^{\Delta_+}\left[\mathcal{O}_{\Delta_+}\left(\mathbf{k}\right)+O\left(\eta^2\right)\right]}_{\text{Selected by } \omega_D\left(u,\bar{u}\right)}+\beta_{\pm\hat{\pm}}\underbrace{\left(-\eta\right)^{\Delta_-}\left[\mathcal{O}_{\Delta_-}\left(\mathbf{k}\right)+O\left(\eta^2\right)\right]}_{\text{Selected by } \omega_N\left(u,\bar{u}\right)}$$

$$ds^2=\left(\frac{R_{\text{dS}}}{\eta}\right)^2\left(-d\eta^2+d\mathbf{x}^2\right)$$

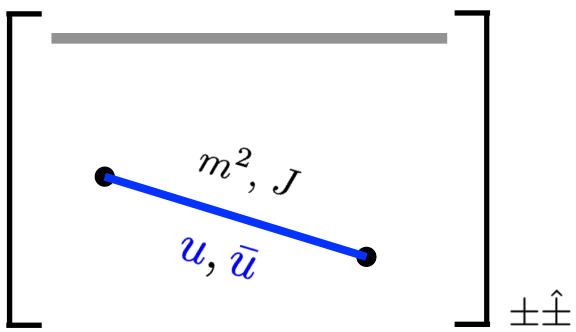
For the Bunch Davies (Euclidean) vacuum we have:

$$\alpha_{\pm\pm}=\beta_{\mp\mp}=\csc\left(\pi\left(\tfrac{d}{2}-\Delta_\pm\right)\right)\exp\left[-\left(\Delta_\pm-\tfrac{d}{2}\right)\pi i\right],\qquad \alpha_{\pm\mp}=-\beta_{\mp\pm}=\pm\csc\left(\pi\left(\tfrac{d}{2}-\Delta_\pm\right)\right)\exp\left[\left(\Delta_\pm-\tfrac{d}{2}\right)\pi i\right]$$

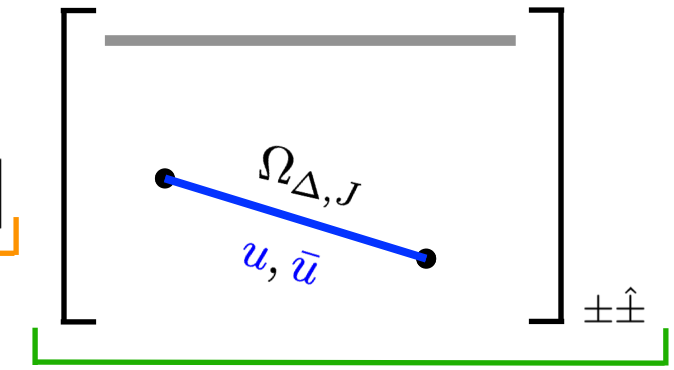
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In dS, for the $\pm\hat{\pm}$ branch of the in-in contour, we have:



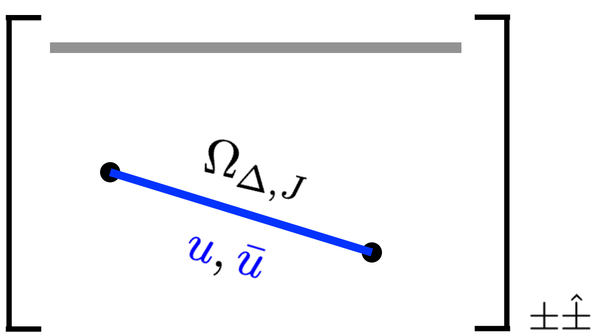
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



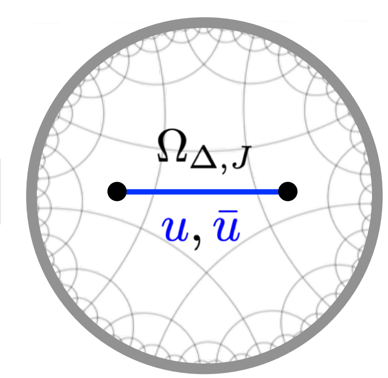
Harmonic function, $\left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0$

dS and EAdS Harmonic functions differ by a simple phase:

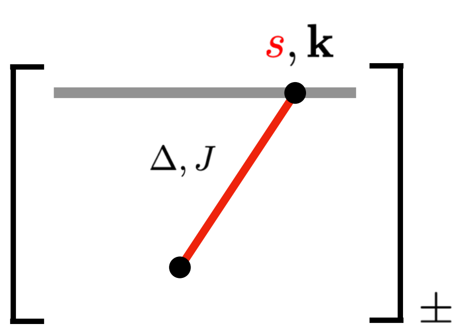
$m^2 R_{\rm dS}^2 = (\Delta_+\Delta_- + J)$



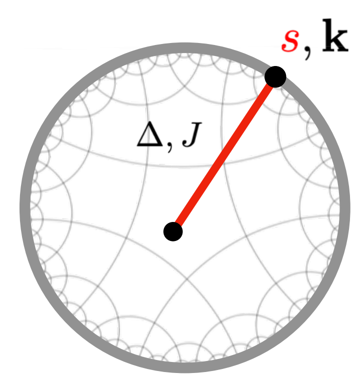
$$= \exp\left[\mp\left(u+\tfrac{1}{2}\left(\Delta_+-\tfrac{d}{2}\right)\right)\pi i\right]\exp\left[\hat{\mp}\left(\bar{u}+\tfrac{1}{2}\left(\Delta_--\tfrac{d}{2}\right)\right)\pi i\right]$$



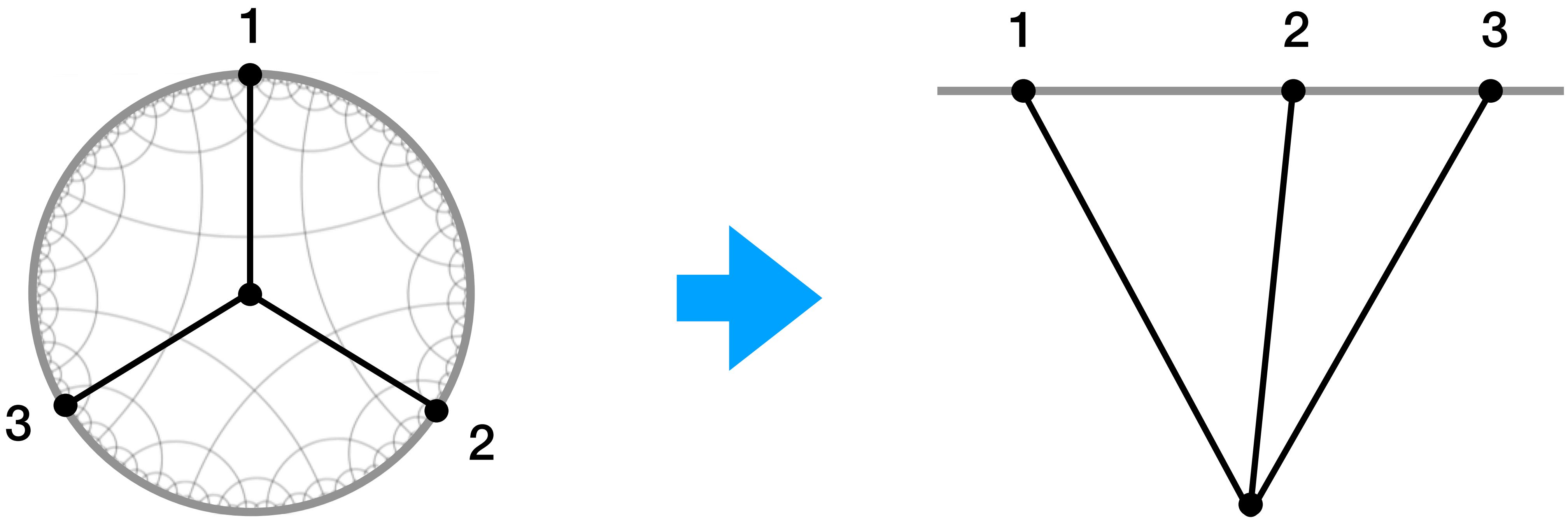
Also the bulk-boundary propagators:



$$= \exp\left[\mp\left(s+\tfrac{1}{2}\left(\Delta-\tfrac{d}{2}\right)\right)\pi i\right]$$

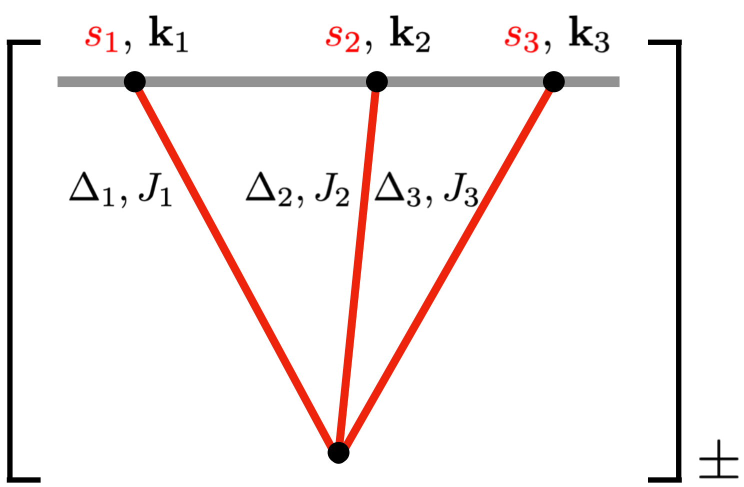


3pt Contact

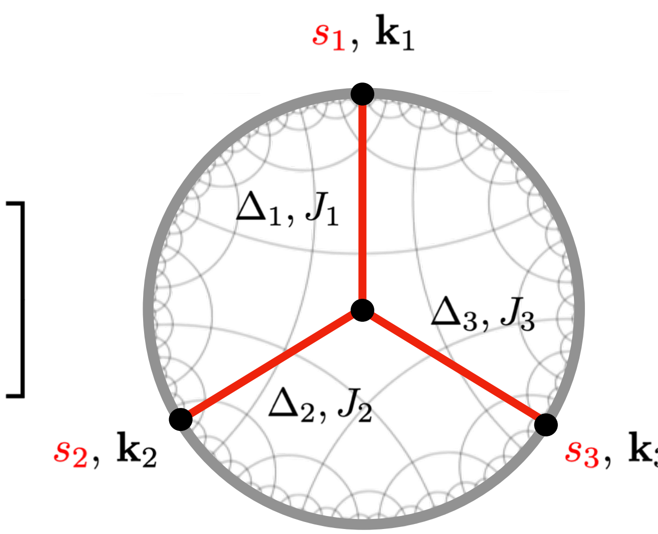


3pt Contact

Contact amplitudes in dS can be obtained directly from their EAdS counterparts:



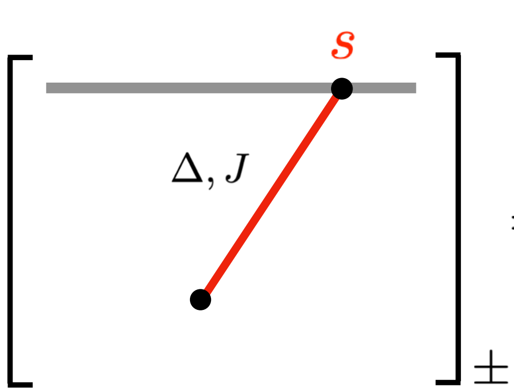
$$= \pm i \exp\left[\mp \pi i \sum_{j=1}^{3} \left(s_j + \frac{1}{2}\left(\Delta_j - \frac{d}{2}\right)\right)\right]$$



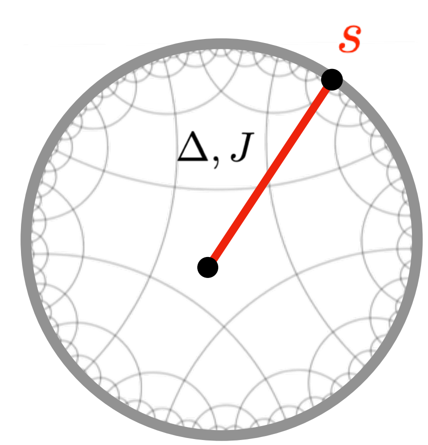
Overall phase is constant, as required by the Dilatation Ward identity, since:

$$s_1 + s_2 + s_3 = \frac{d + 2\left(J_1 + J_2 + J_3\right)}{4}$$

Above we simply used that:

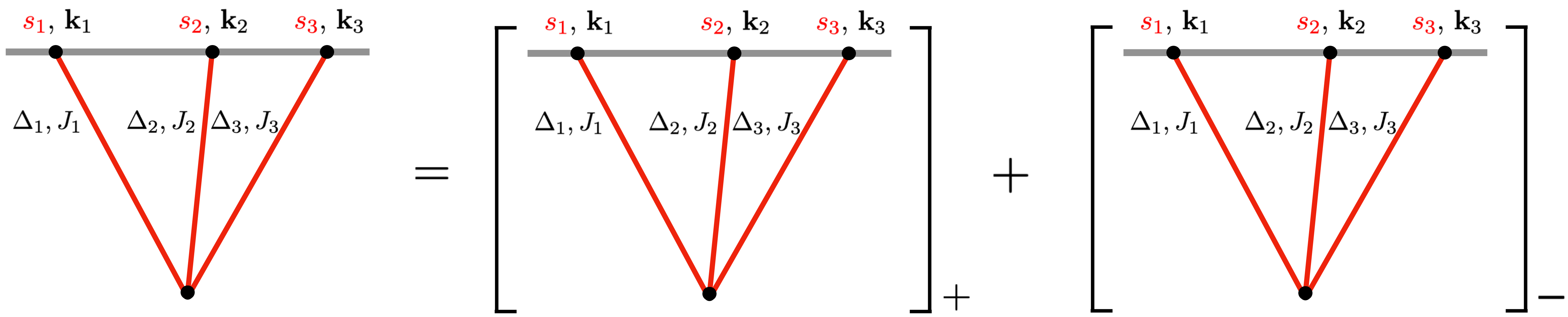


$$= \exp\left[\mp\left(s + \frac{1}{2}\left(\Delta - \frac{d}{2}\right)\right)\pi i\right]$$

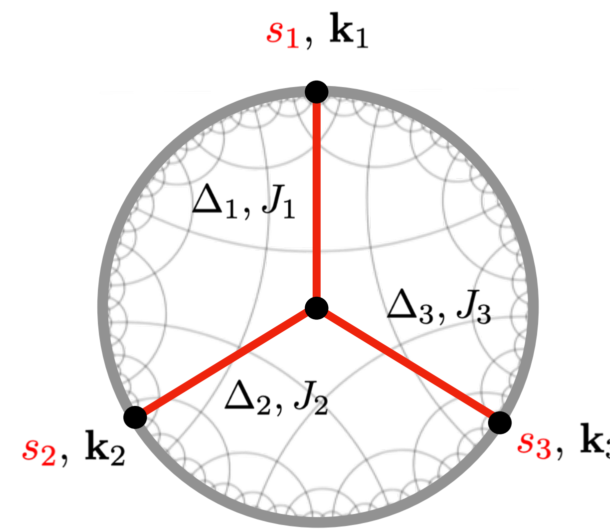


3pt Contact

The full de Sitter 3pt function is the sum from each branch of the in-in contour:

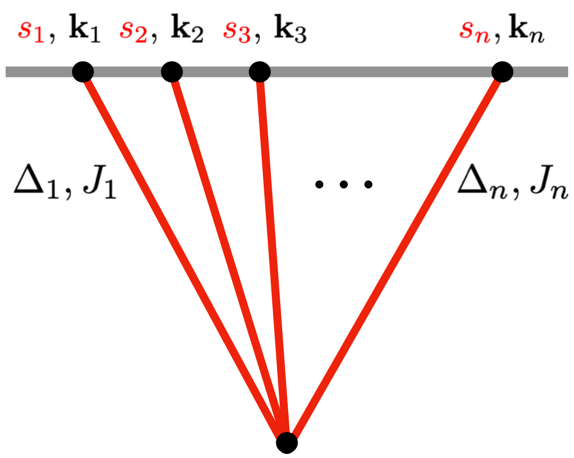


$$= \ \sin\left[\left(-d + \sum_{i=1}^{3} (\Delta_i + J_i)\right)\frac{\pi}{2}\right]$$

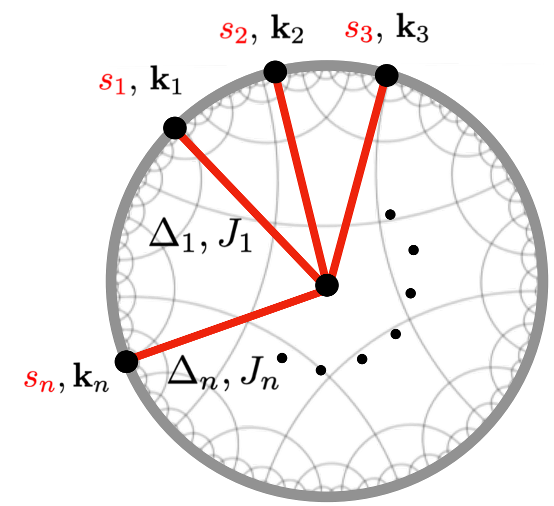


n ~~3~~pt Contact

The full de Sitter n ~~3~~pt function is the sum from each branch of the in-in contour:

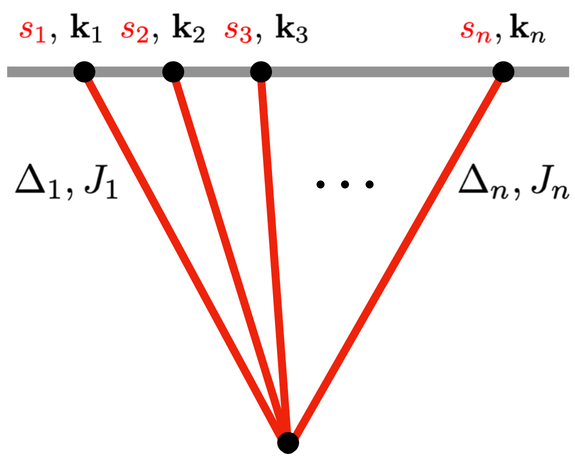


$$= \sin\left[\left(-d+\sum_{i=1}^{\cancel{3}\; n}(\Delta_i+J_i)\right)\frac{\pi}{2}\right]$$

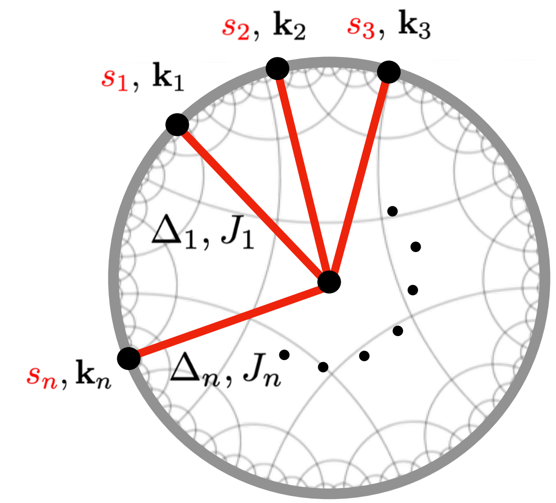


n~~3~~pt Contact

The full de Sitter n~~3~~pt function is the sum from each branch of the in-in contour:



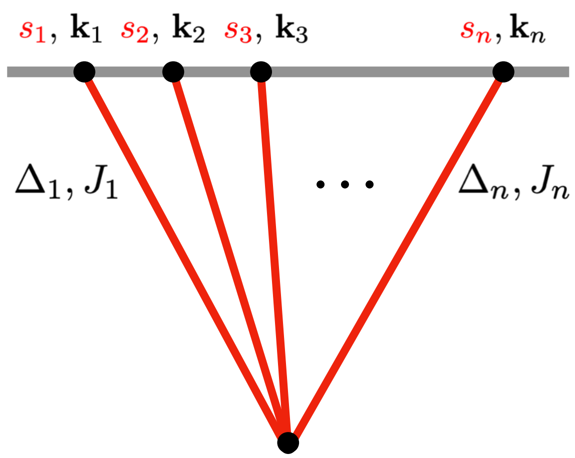
$$= \sin\left[\left(-d + \sum_{i=1}^{\cancel{3}\,n} (\Delta_i + J_i)\right)\frac{\pi}{2}\right]$$



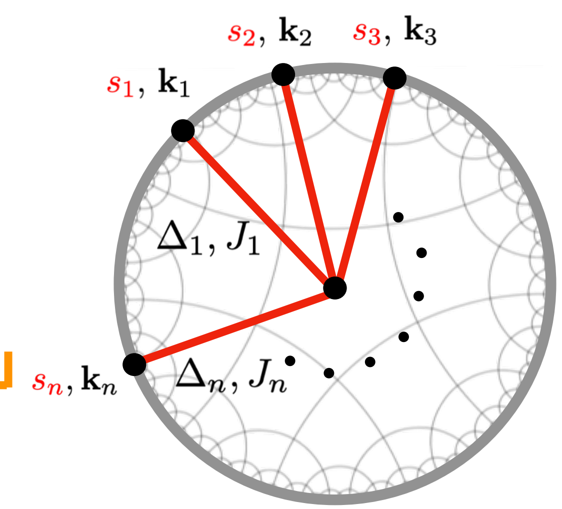
→ de Sitter contact diagrams can vanish when the sine factor has a zero!

n~~3~~pt Contact

The full de Sitter n~~3~~pt function is the sum from each branch of the in-in contour:

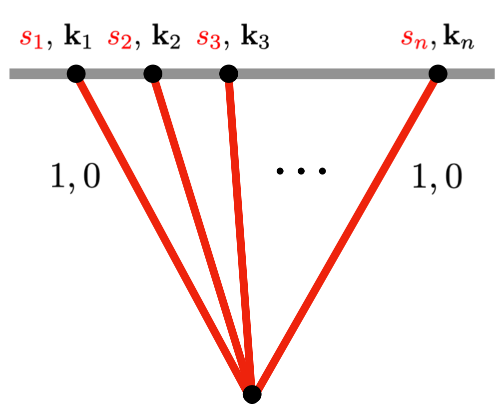


$$= \underbrace{\sin\left[\left(-d + \sum_{i=1}^{\cancel{3}\;n} (\Delta_i + J_i)\right)\frac{\pi}{2}\right]}_{\text{selected by unitarity}}$$

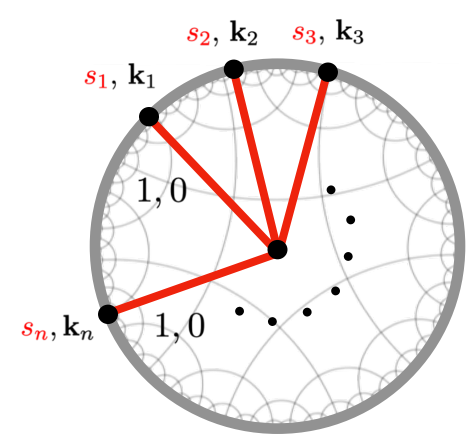


→ de Sitter contact diagrams can vanish when the sine factor has a zero!

E.g. conformally coupled scalars for d=3 $(\Delta_i = 1,\ J_i = 0)$:



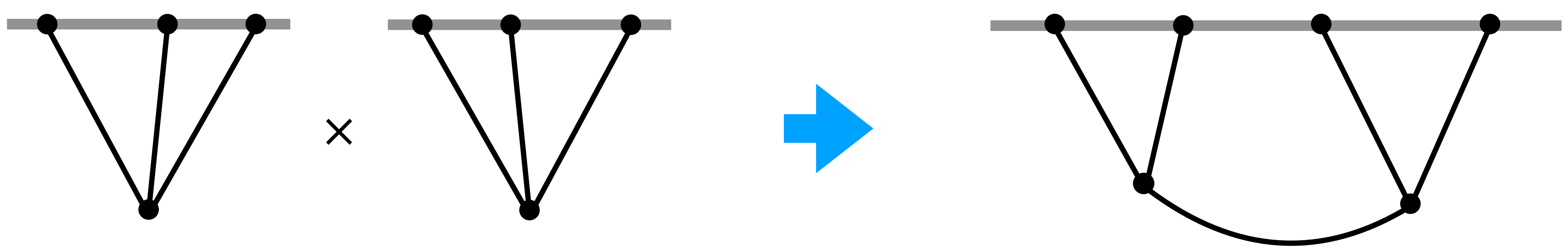
$$= \sin\left[\left(\frac{n-3}{2}\right)\pi\right]$$



$$= 0$$ for n odd!

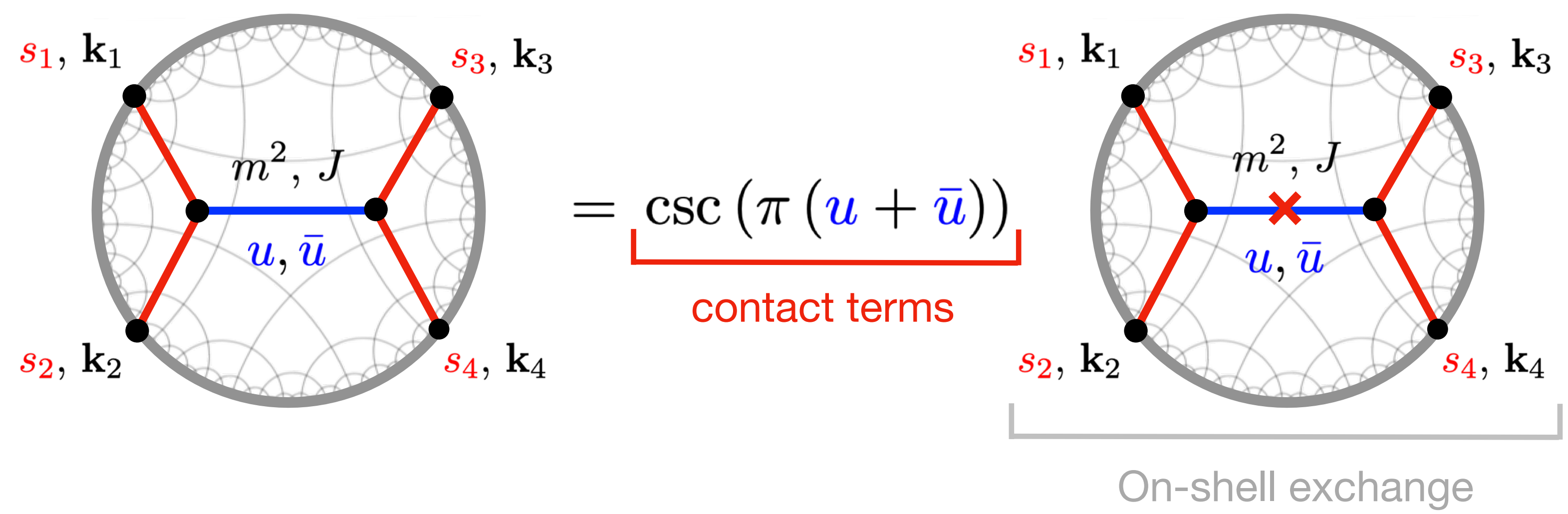
Shown to follow from unitarity by Goodhew, Jazayeri & Pajer - [Cosmological Optical Theorem, 2020]

Exchanges



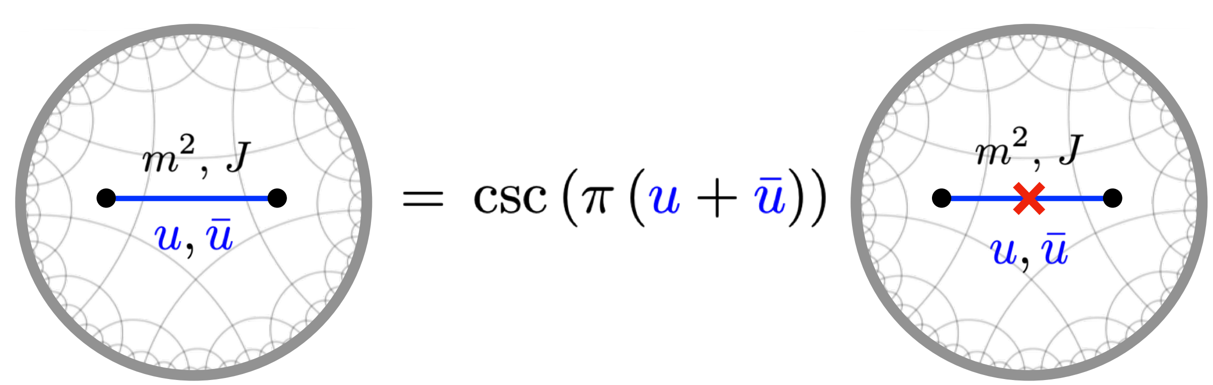
Exchanges in EAdS

Exchanges are straightforwardly reconstructed from their on-shell part:



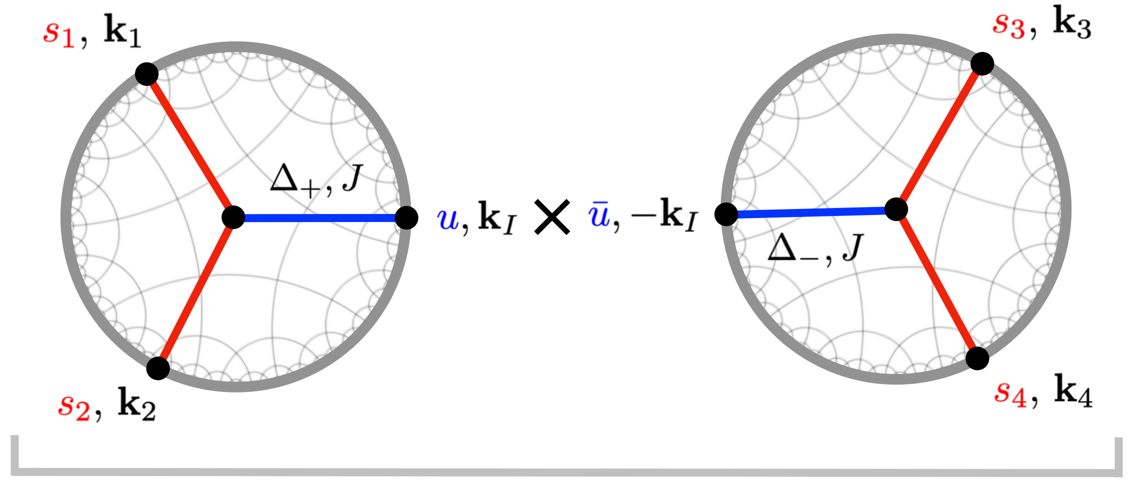
Constrained by:
- Factorisation
- Conformal Symmetry
- Boundary Conditions

Simply follows from:



Exchanges in EAdS

Factorisation and Conformal Symmetry:



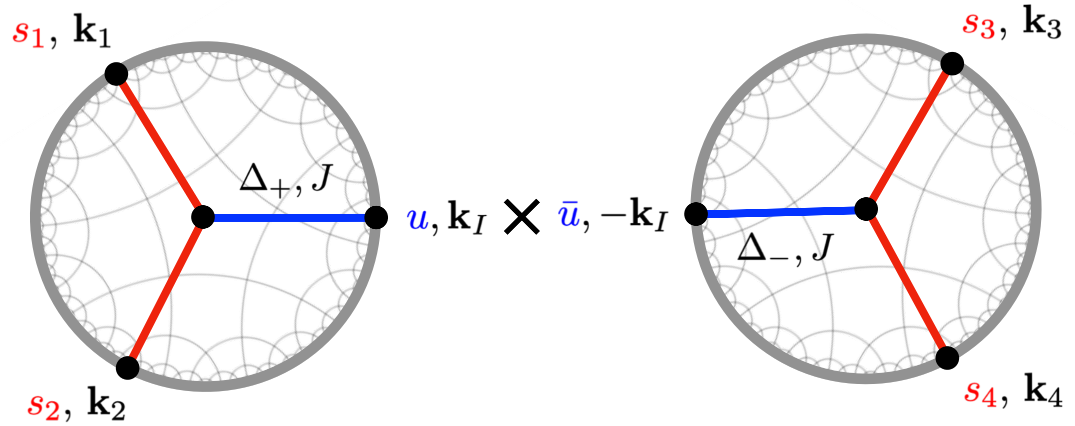
"Conformal Partial Wave",
single valued Eigenfunction of Conformal Casimirs

Mack, Dobrev, Petkova, Petrova, Todorov, 1974-7

$$m^2 R^2_{\text{AdS}} = -\left(\Delta_+ \Delta_- + J\right)$$

Exchanges in EAdS

Factorisation and Conformal Symmetry:



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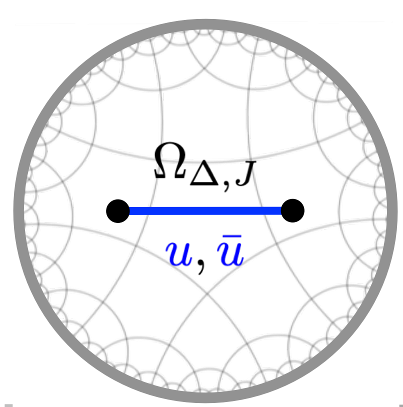
attach/remove external legs

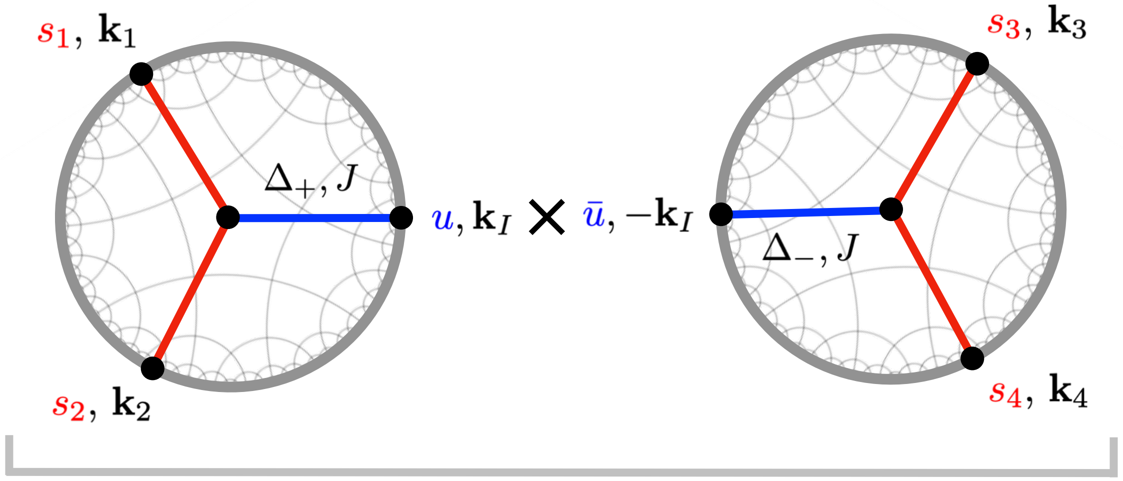


Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

e.g. Leonhardt, Manvelyan, Rühl 2003;
Costa, Gonçalves, Penedones 2014

Exchanges in EAdS

Factorisation and Conformal Symmetry:



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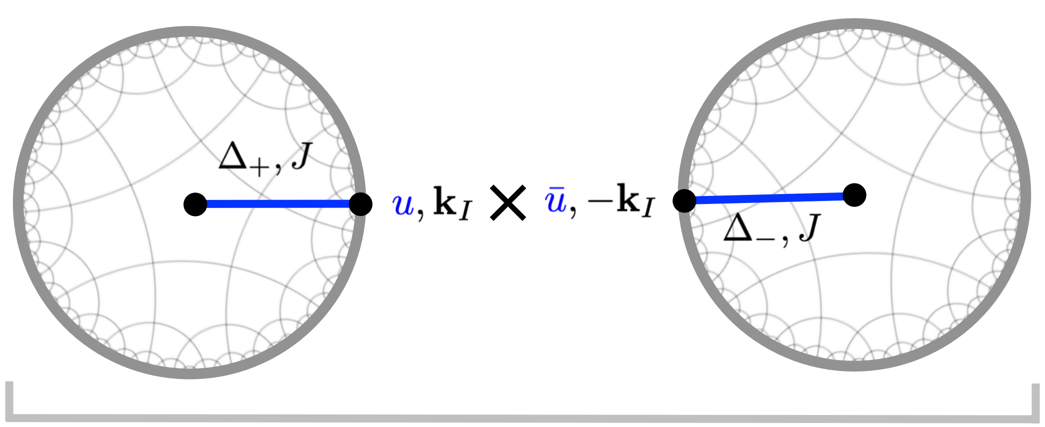


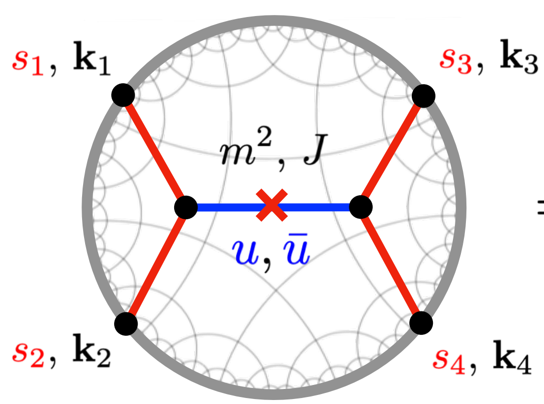
Harmonic function, $\left(\nabla^2 - m^2\right) \Omega_{\Delta, J} = 0$

e.g. Leonhardt, Manvelyan, Rühl 2003;
Costa, Gonçalves, Penedones 2014

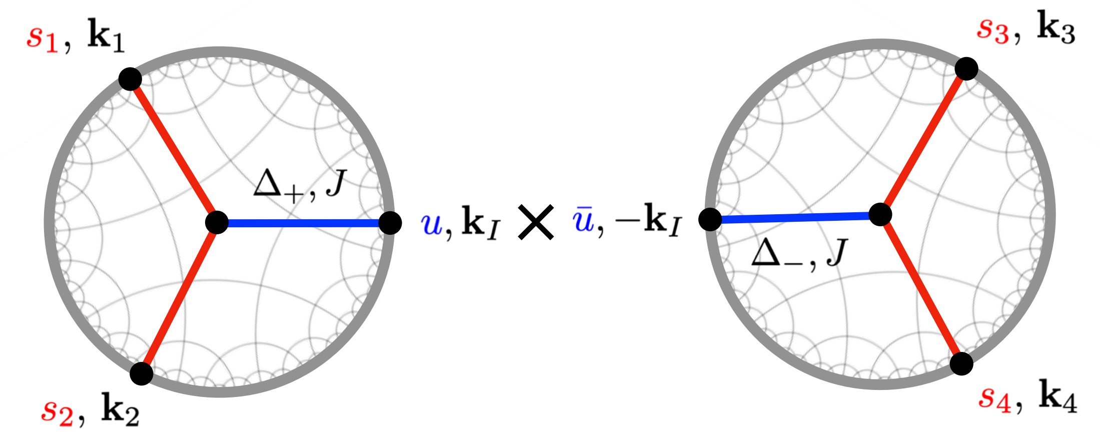
This duality is made manifest by the "split representation" of $\Omega_{\Delta,J}$

Exchanges in EAdS

Factorisation, Conformal Symmetry and boundary conditions:



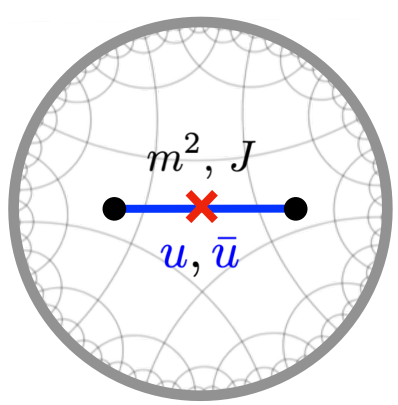
$$= \left[\alpha\, \omega_D\left(u, \bar{u}\right) + \beta\, \omega_N\left(u, \bar{u}\right)\right]$$



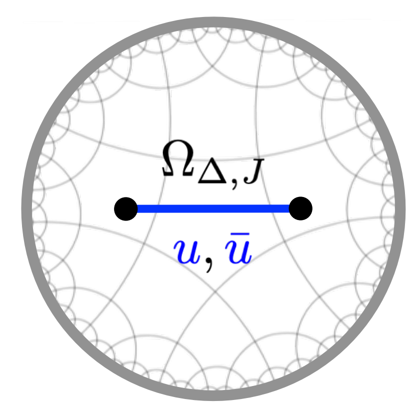
Factorisation and Conformal Symmetry

$$m^2 R^2_{\text{AdS}} = -\left(\Delta_+ \Delta_- + J\right)$$

attach/remove
external legs

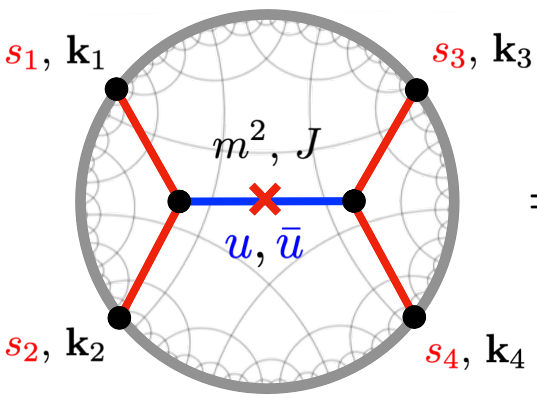


$$= \underbrace{\left[\alpha\, \overset{\text{Dirichlet}}{\omega_D}\left(u, \bar{u}\right) + \beta\, \overset{\text{Neumann}}{\omega_N}\left(u, \bar{u}\right)\right]}_{\text{boundary conditions}}$$

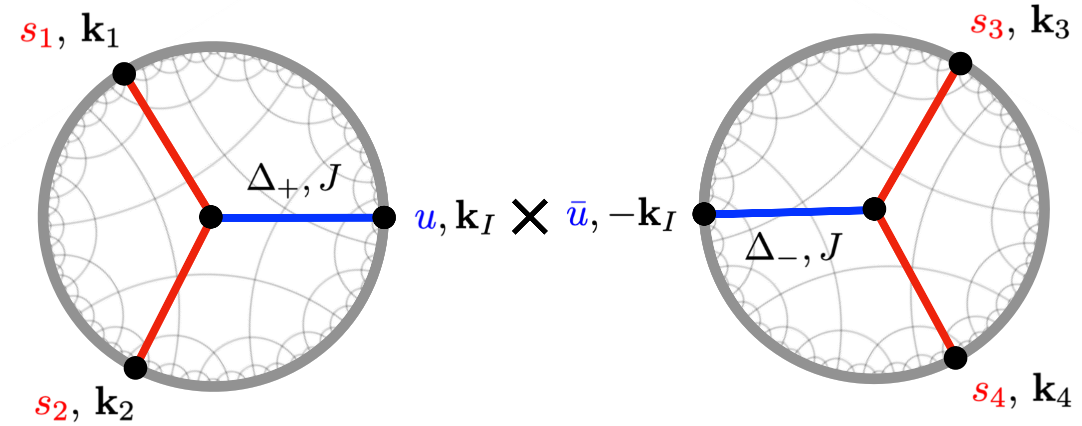


Exchanges in EAdS

Factorisation, Conformal Symmetry and boundary conditions:



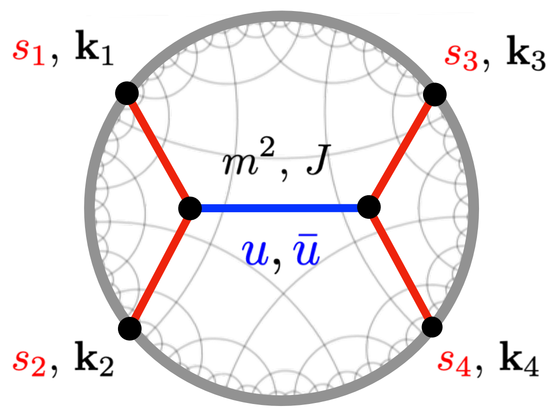
$$= \underbrace{\left[\alpha\, \omega_D\left(u, \bar{u}\right) + \beta\, \omega_N\left(u, \bar{u}\right)\right]}_{\text{boundary conditions}}$$



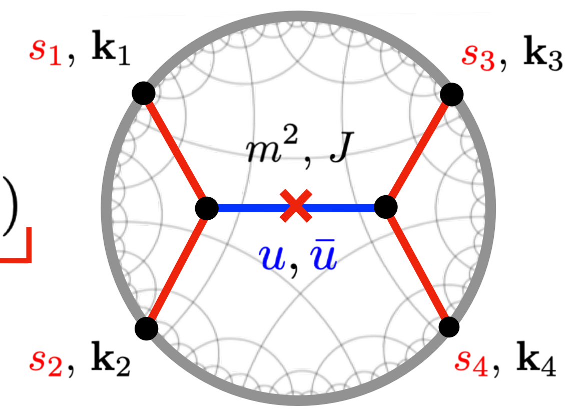
Factorisation and Conformal Symmetry

The full exchange is reconstructed via:

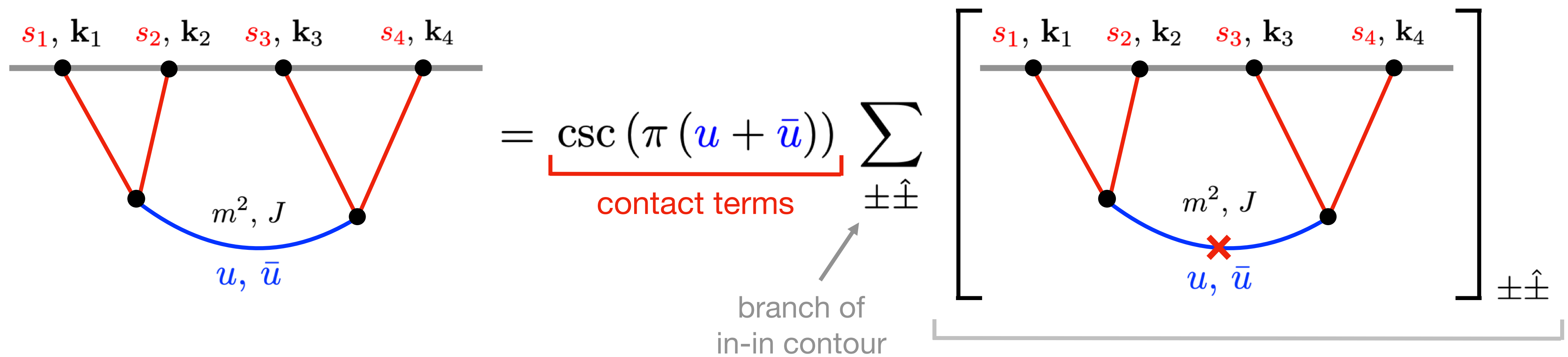
$m^2 R_{\rm AdS}^2 = -\left(\Delta_+\Delta_- + J\right)$



$$= \underbrace{\csc\left(\pi\left(u + \bar{u}\right)\right)}_{\text{contact terms}}$$



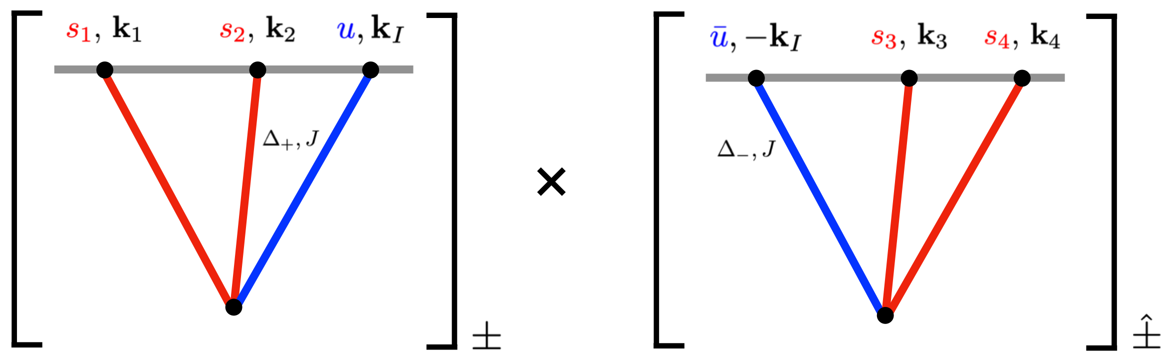
Exchanges in dS



Constrained by:
- Factorisation
- Conformal Symmetry
- Boundary Conditions

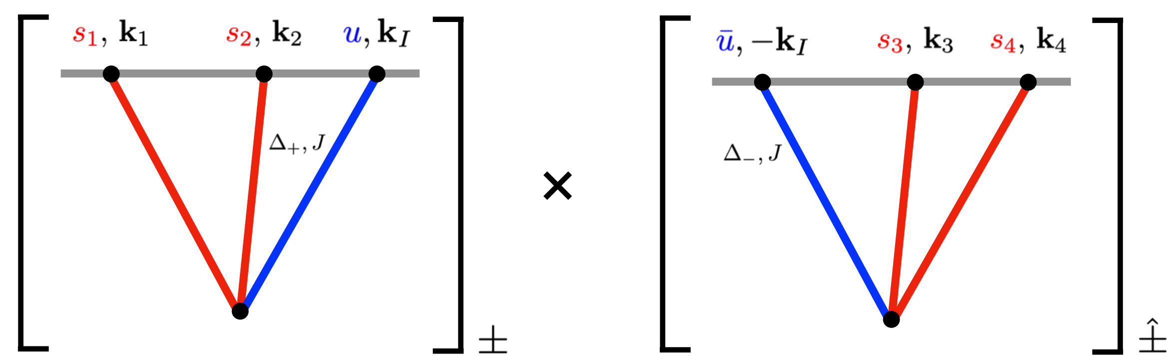
Exchanges in dS

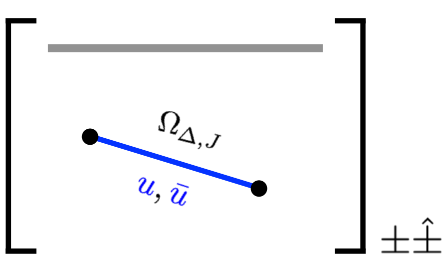
Factorisation and Conformal Symmetry:



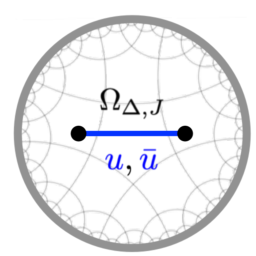
Exchanges in dS

Factorisation and Conformal Symmetry:

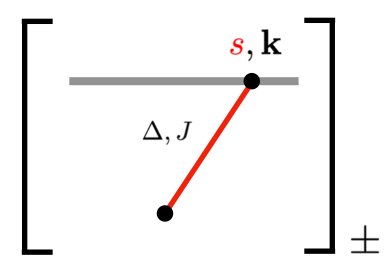




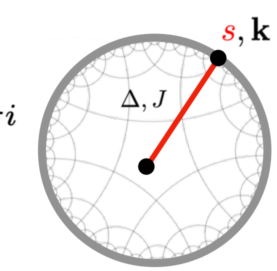
$= e^{\left[\mp\left(u+\frac{1}{2}\left(\Delta_+-\frac{d}{2}\right)\right)\hat{\mp}\left(\bar{u}+\frac{1}{2}\left(\Delta_--\frac{d}{2}\right)\right)\right]\pi i}$

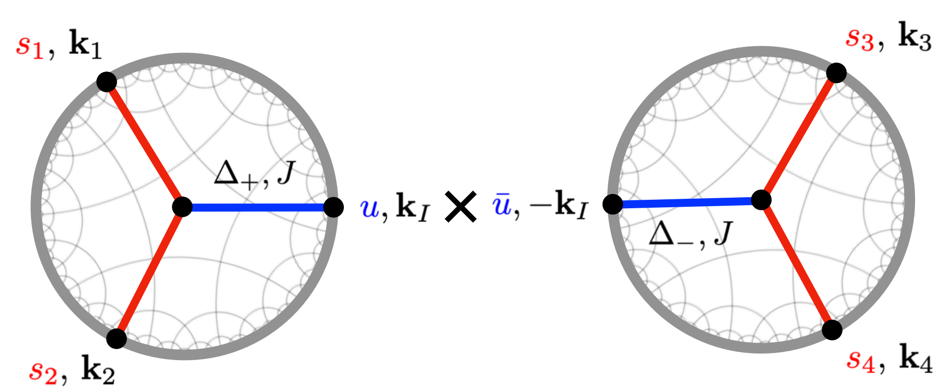


,



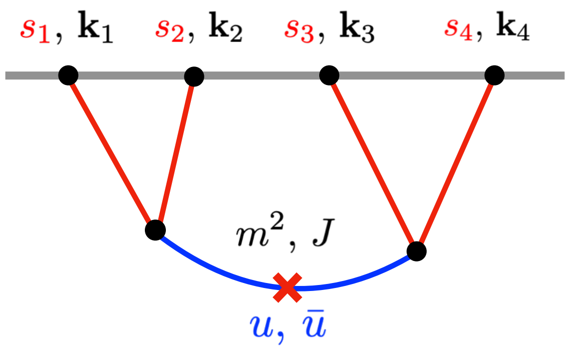
$= e^{\mp\left(s+\frac{1}{2}\left(\Delta-\frac{d}{2}\right)\right)\pi i}$



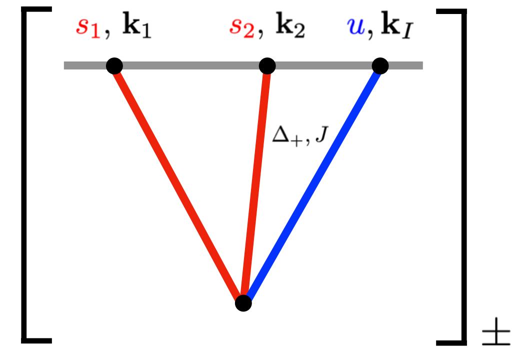


Exchanges in dS

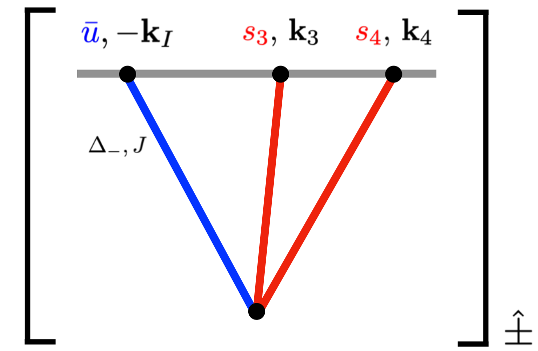
Factorisation, Conformal Symmetry and boundary conditions:



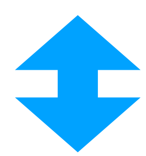
$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$

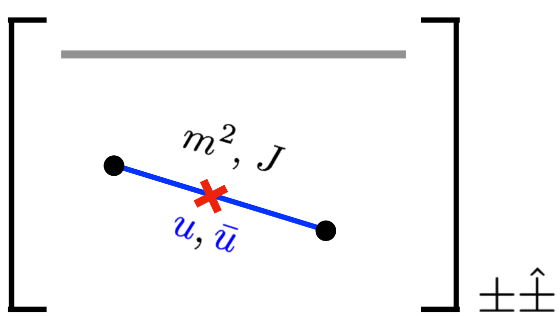


$\times$

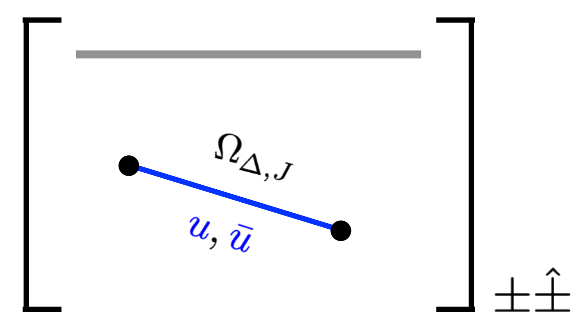


Factorisation + Conformal Symmetry





$$= \underbrace{\left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$

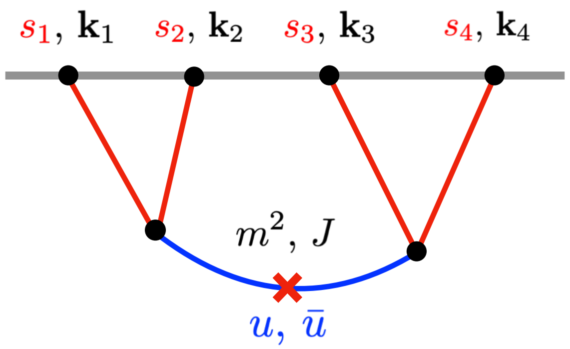


For the Bunch Davies (Euclidean) vacuum:

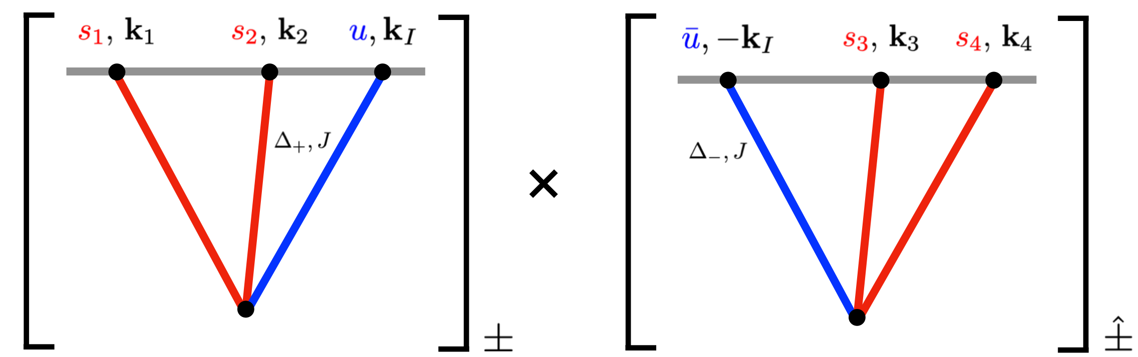
$$\alpha_{\pm\pm} = \beta_{\mp\mp} = \csc\left(\pi\left(\tfrac{d}{2}-\Delta_{\pm}\right)\right)\exp\left[-\left(\Delta_{\pm}-\tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp} = -\beta_{\mp\pm} = \pm\csc\left(\pi\left(\tfrac{d}{2}-\Delta_{\pm}\right)\right)\exp\left[\left(\Delta_{\pm}-\tfrac{d}{2}\right)\pi i\right]$$

Exchanges in dS

Factorisation, Conformal Symmetry and boundary conditions:

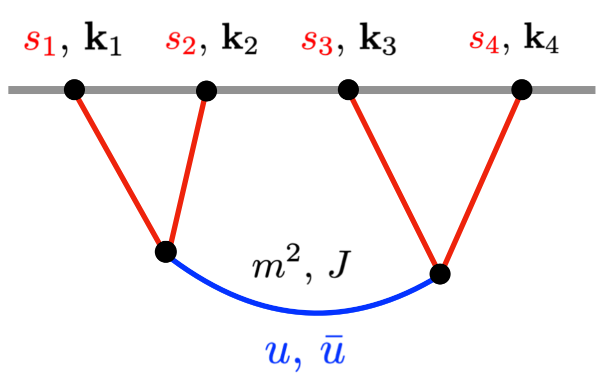


$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u, \bar{u}\right) + \beta_{\pm\hat{\pm}}\,\omega_N\left(u, \bar{u}\right)\right]$$

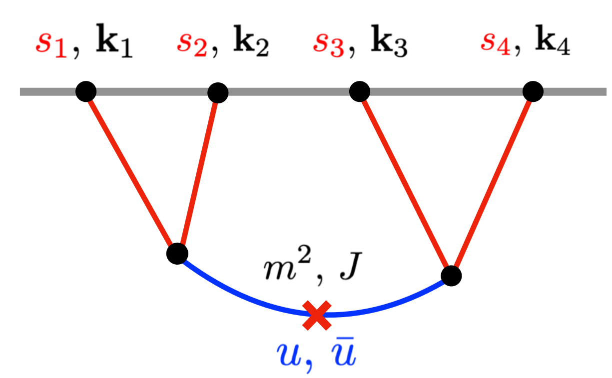


Factorisation + Conformal Symmetry

The full exchange is reconstructed via:



$$= \underbrace{\csc\left(\pi\left(u + \bar{u}\right)\right)}_{\text{contact terms}}$$



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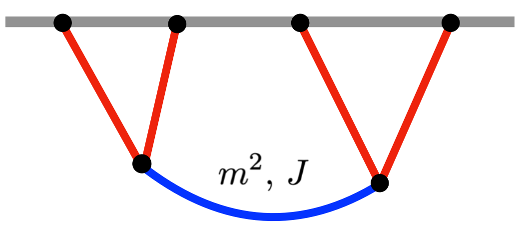
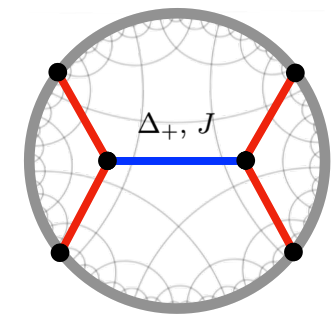
$$\alpha_{\pm\pm} = \beta_{\mp\mp} = \csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[-\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp} = -\beta_{\mp\pm} = \pm\csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right]$$

Exchanges in (A)dS

dS exchange in the Bunch-Davies vacuum is a linear combination of AdS exchanges:



$$= \sin\left(\tfrac{\pi}{2}\left(-d+\Delta_+ + J + \sum_{k=1}^{2}(\Delta_k + J_k)\right)\right)\sin\left(\tfrac{\pi}{2}\left(-d+\Delta_+ + J + \sum_{k=3}^{4}(\Delta_k + J_k)\right)\right)$$

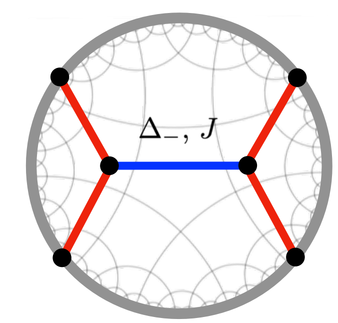


Dirichlet

$\omega_D\,(u, \bar{u})$

Change in 3pt function (OPE) coefficient, selected by unitarity

$$+\ \sin\left(\tfrac{\pi}{2}\left(-d+\Delta_- + J + \sum_{k=1}^{2}(\Delta_k + J_k)\right)\right)\sin\left(\tfrac{\pi}{2}\left(-d+\Delta_- + J + \sum_{k=3}^{4}(\Delta_k + J_k)\right)\right)$$

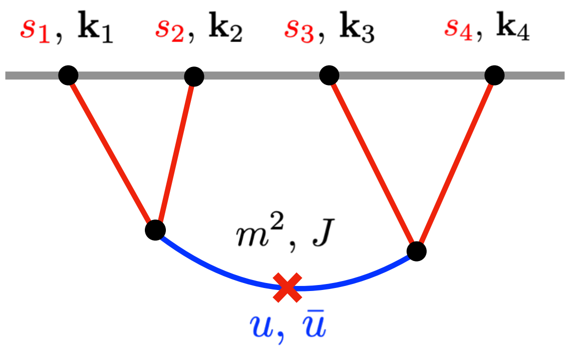


Neumann

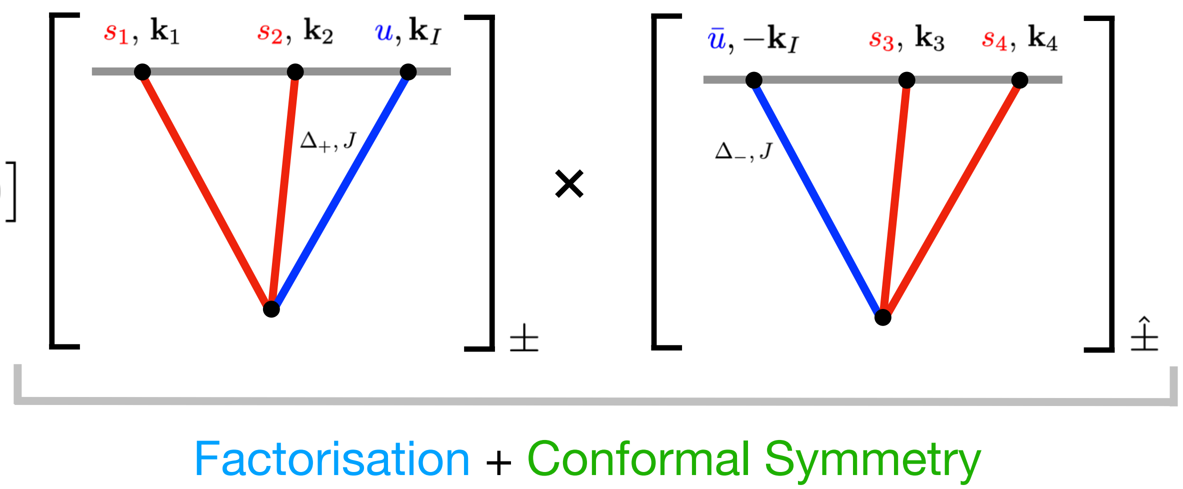
$\omega_N\,(u, \bar{u})$

Exchanges in (A)dS

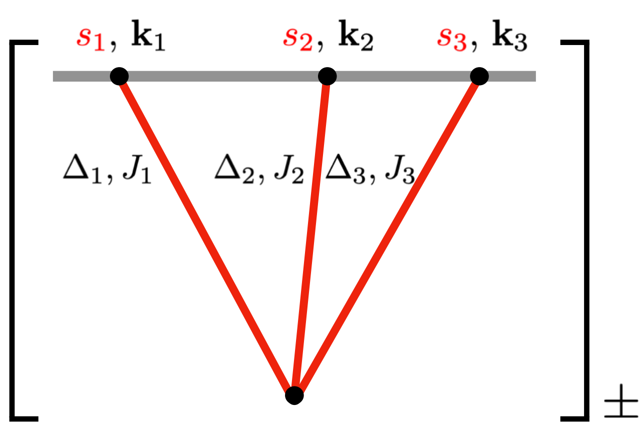
Factorisation, Conformal Symmetry and boundary conditions:



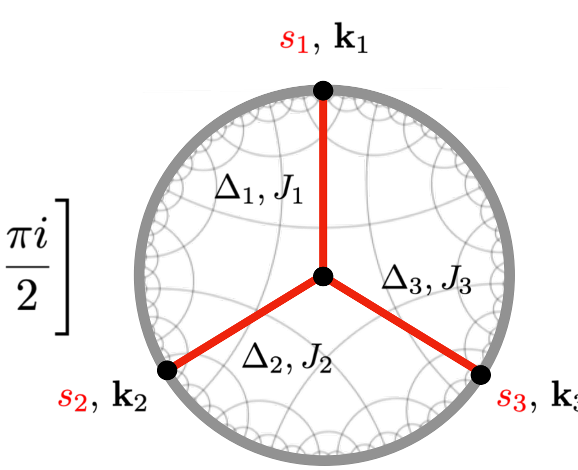
$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u, \bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u, \bar{u}\right)\right]$$



The bridge to the EAdS exchanges is via:



$$= \pm i \exp\left[\mp\left(-d + \sum_{k=1}^{3}\left(\Delta_k + J_k\right)\right)\frac{\pi i}{2}\right]$$



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Exchanges in (A)dS

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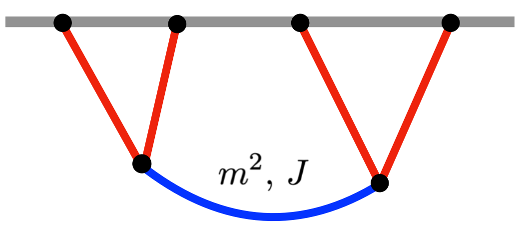
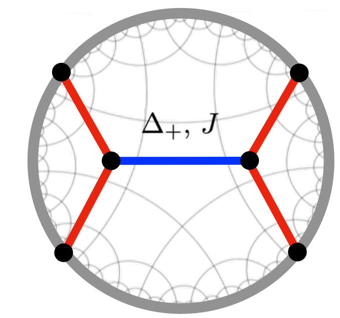


$$= \underbrace{\sin\left(\frac{\pi}{2}\left(-d+\Delta_+ + J + \sum_{k=1}^{2}(\Delta_k+J_k)\right)\right)\sin\left(\frac{\pi}{2}\left(-d+\Delta_+ + J+\sum_{k=3}^{4}(\Delta_k+J_k)\right)\right)}$$

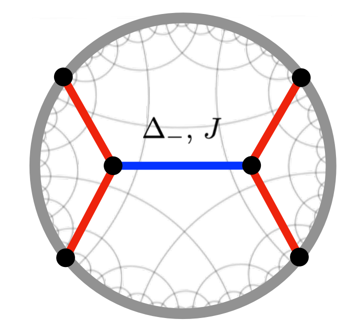


Dirichlet

$\omega_D\,(u,\bar{u})$

Change in 3pt function (OPE) coefficient, selected by unitarity

$$+\ \overbrace{\sin\left(\frac{\pi}{2}\left(-d+\Delta_- + J + \sum_{k=1}^{2}(\Delta_k+J_k)\right)\right)\sin\left(\frac{\pi}{2}\left(-d+\Delta_- + J+\sum_{k=3}^{4}(\Delta_k+J_k)\right)\right)}$$



Neumann

$\omega_N\,(u,\bar{u})$

This identity can be used to directly import techniques and results from AdS to dS!

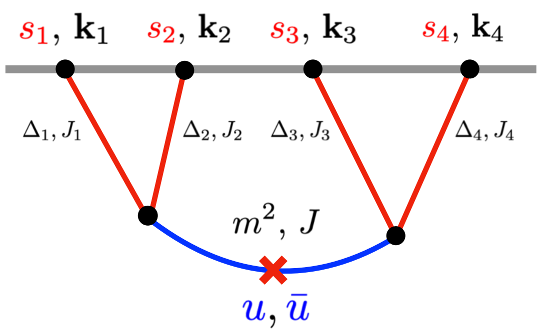
Some small steps in 2007.09993 [hep-th]:

- AdS exchanges are basic solutions to the crossing equation (associativity of operator algebra)

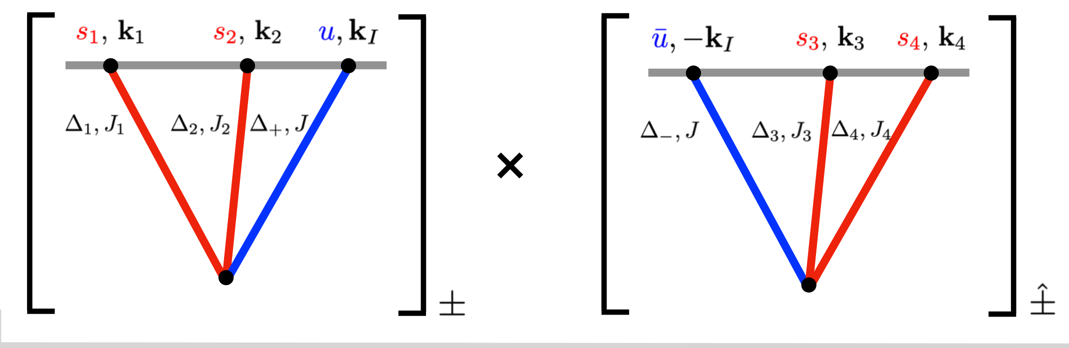
$\longrightarrow$ **dS exchanges are also solutions to crossing.** Their decomposition into conformal blocks (in all channels) is inherited from those of AdS exchanges (which are known)

- **Mellin amplitudes for dS correlators.** For AdS, Mellin amplitudes have been an instrumental tool owing to striking parallels with scattering amplitudes — Mack 2009, Penedones 2010

Exchanges in dS



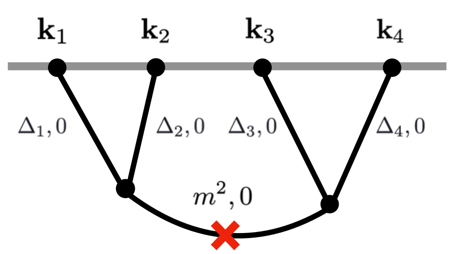
$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\,(u, \bar{u}) + \beta_{\pm\hat{\pm}}\, \omega_N\,(u, \bar{u})\right]$$



$\sim (|\mathbf{k}_I|)^{-2(u+\bar{u})}$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)

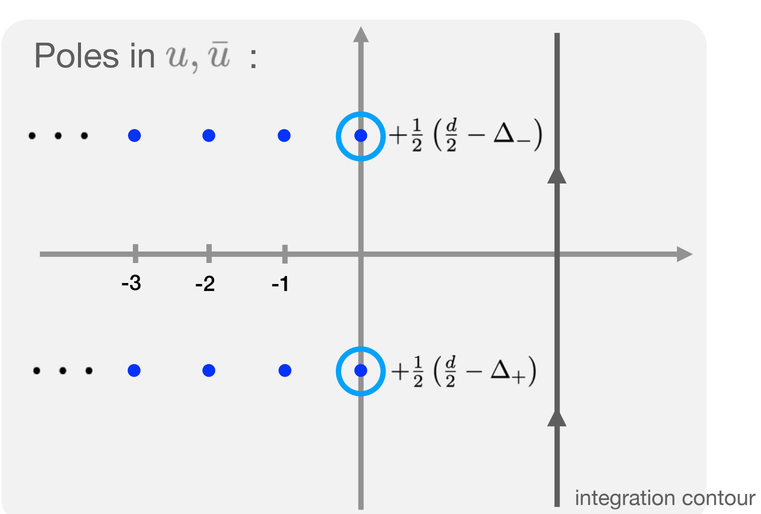
The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If all the fields are **scalars**:



non-analytic

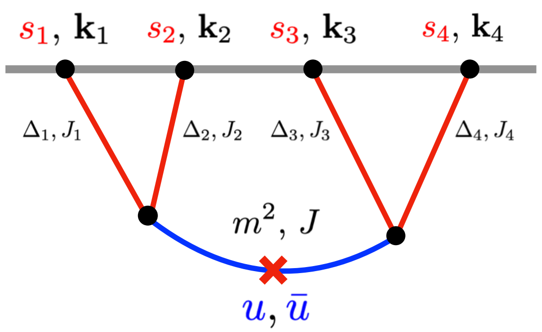
$$\underset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}\left[1+\mathcal{O}\left(k_I^2\right)\right]$$

$$+\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\left[1+\mathcal{O}\left(k_I^2\right)\right] + \ldots$$

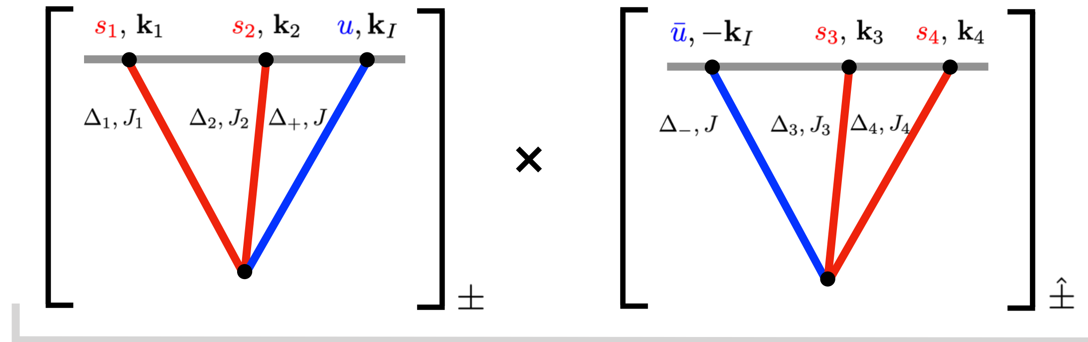


External conformally coupled/massless scalars:
Arkani-Hamed and Maldacena 2015;
Arkani-Hamed, Baumann, Lee and Pimentel 2018

Exchanges in dS



$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u, \bar{u}\right) + \cancel{\beta_{\pm\hat{\pm}}}\, \omega_N\left(u, \bar{u}\right)\right]$$



$$\sim \left(|\mathbf{k}_I|\right)^{-2(u+\bar{u})}$$

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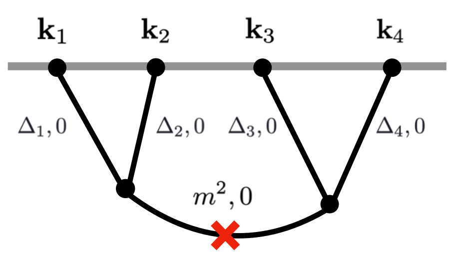
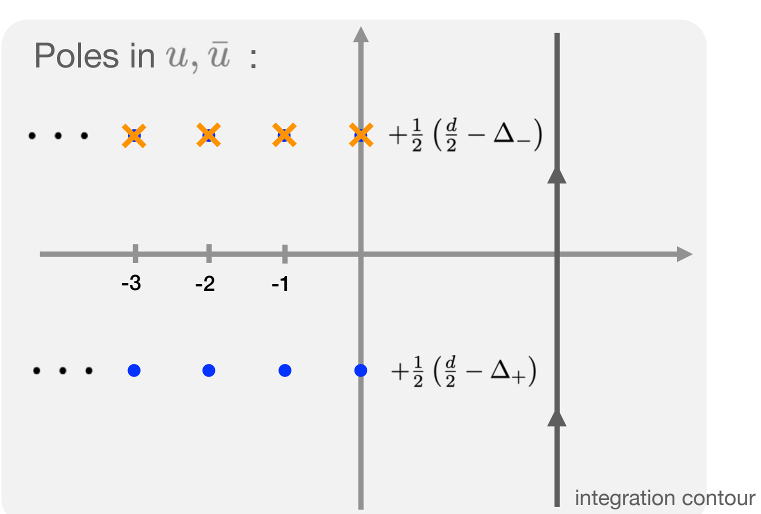


non-analytic

$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_+-\frac{d}{2}} \left[1+\mathcal{O}\left(k_I^2\right)\right]$$

$$+ \cancel{\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_--\frac{d}{2}} \left[1+\mathcal{O}\left(k_I^2\right)\right]} + \ldots$$



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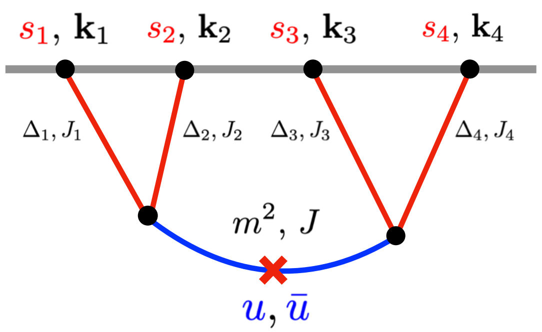
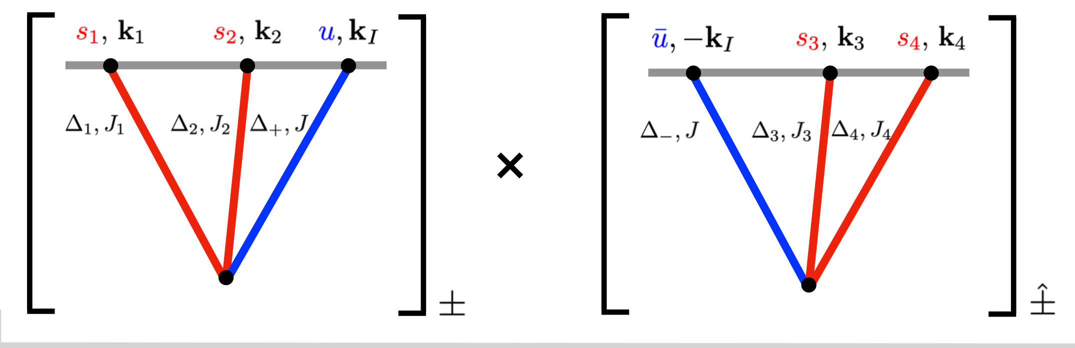
Exchanges in dS

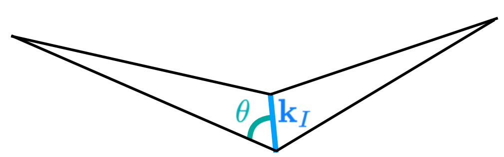


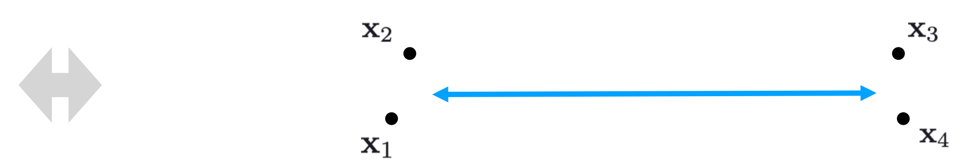
$$= \sum_{\pm\hat{\pm}} \left[\cancel{\alpha_{\pm\hat{\pm}}}\,\omega_D\,(u,\bar{u}) + \beta_{\pm\hat{\pm}}\,\omega_N\,(u,\bar{u})\right]$$



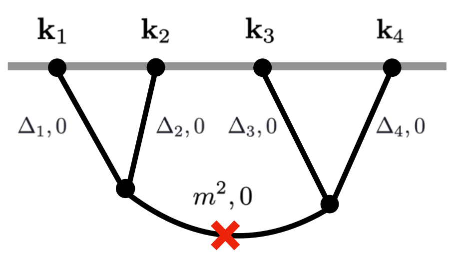
$$\sim (|\mathbf{k}_I|)^{-2(u+\bar{u})}$$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)





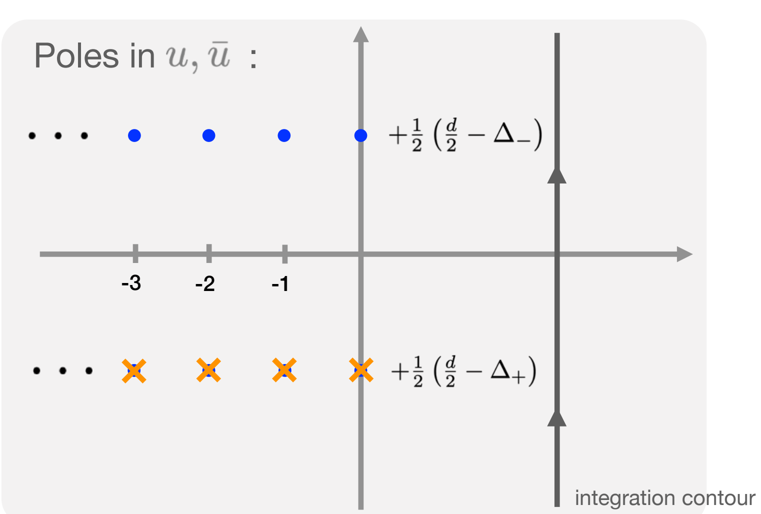
The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If all the fields are **scalars**:



non-analytic

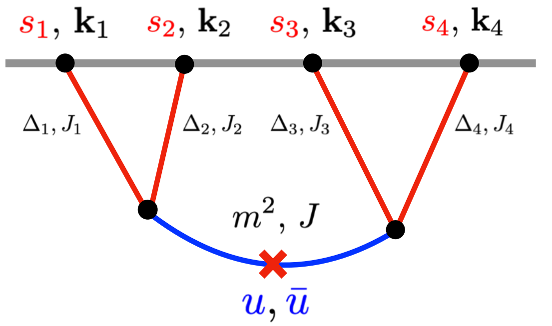
$$\overset{|\mathbf{k}_I|\ll|\mathbf{k}_i|}{\sim} \cancel{\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}\left[1+\mathcal{O}\left(k_I^2\right)\right]}$$

$$+ \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\left[1+\mathcal{O}\left(k_I^2\right)\right] + \ldots$$

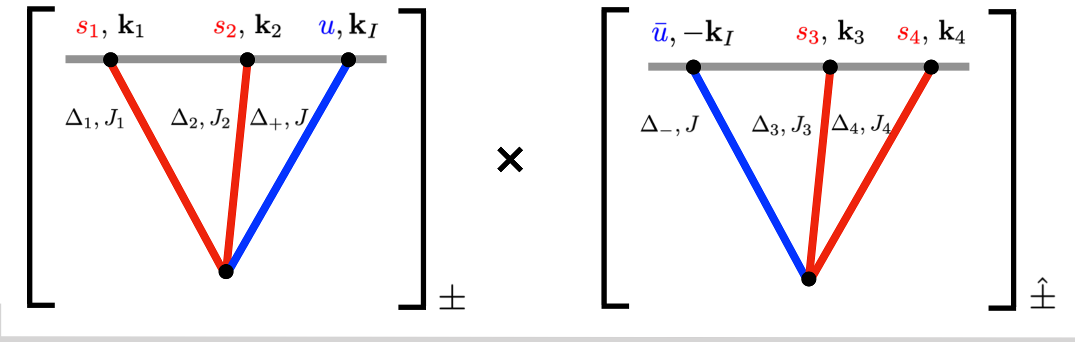


External conformally coupled/massless scalars: Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018

Exchanges in dS



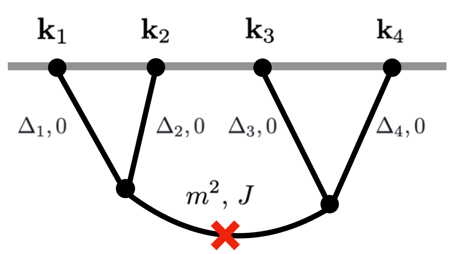
$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\,(u,\bar{u}) + \beta_{\pm\hat{\pm}}\, \omega_N\,(u,\bar{u})\right]$$



$$\sim (|\mathbf{k}_I|)^{-2(u+\bar{u})}$$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)

The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If the exchanged field has **spin J**:



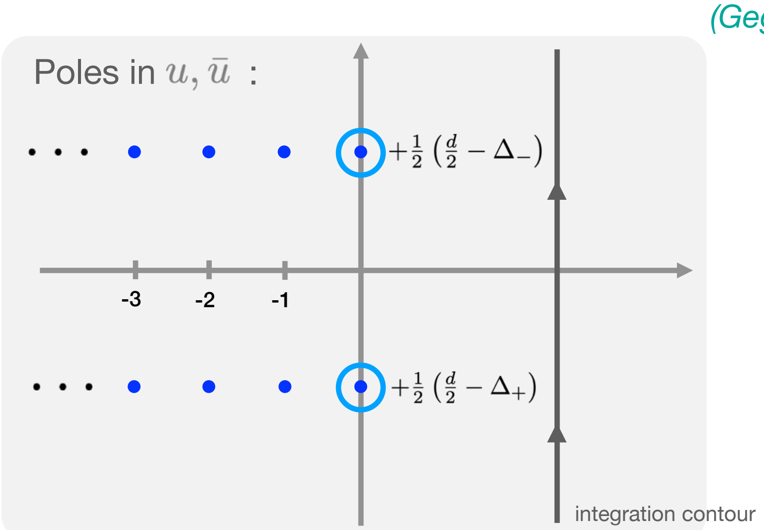
non-analytic

$$\overset{|\mathbf{k}_I|\ll|\mathbf{k}_i|}{\sim} C_J^{\left(\frac{d-2}{2}\right)}(\cos\theta)\Bigg[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right)(k_I^2)^{\Delta_+-\frac{d}{2}}$$

$$+\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right)(k_I^2)^{\Delta_--\frac{d}{2}}\Bigg]$$

angular dependence, (Gegenbauer polynomial)

$$+\dots$$



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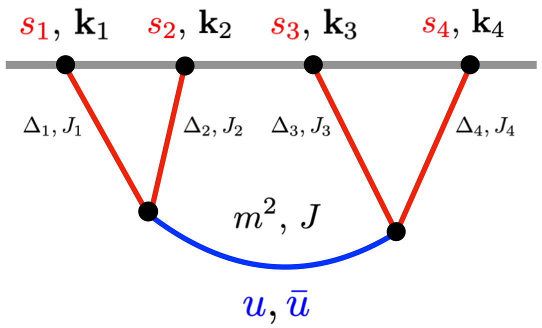
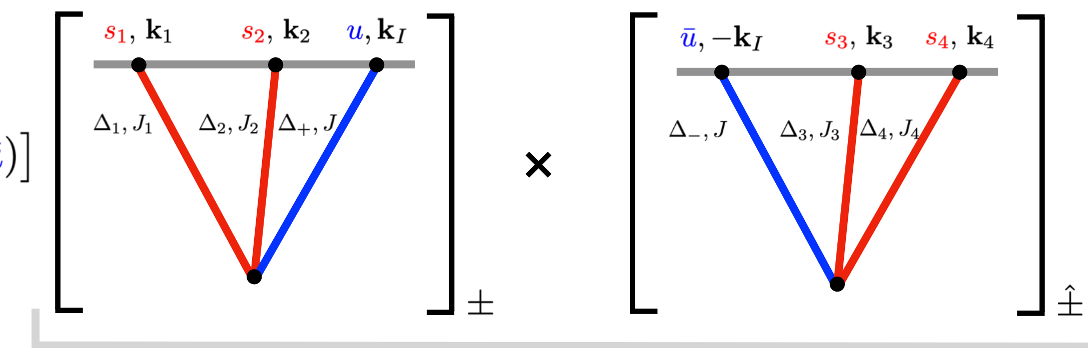
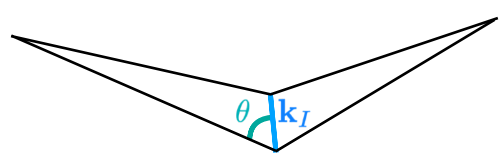
Exchanges in dS



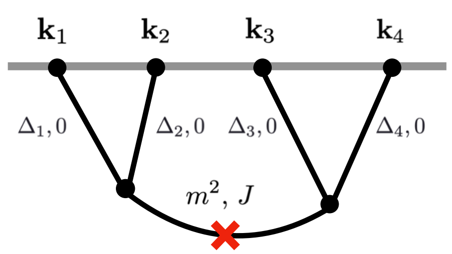
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$



The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)



The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If the exchanged field has **spin J**:

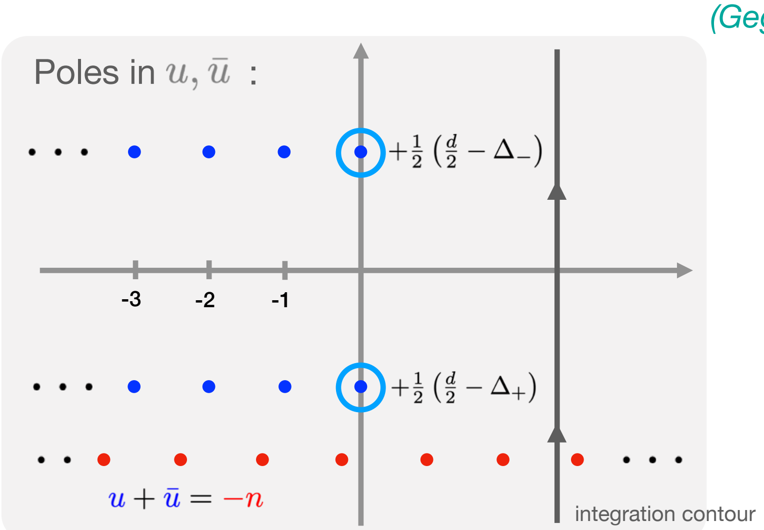


$$\underset{|\mathbf{k}_I|\ll|\mathbf{k}_i|}{\sim} C_J^{\left(\frac{d-2}{2}\right)}(\cos\theta)\Bigg[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}$$

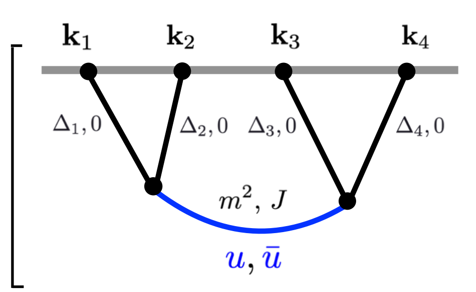
$$+\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\Bigg]$$

non-analytic

angular dependence, (Gegenbauer polynomial)



$$+\sum_{n=0}^{\infty} \mathrm{Res}_{u+\bar{u}=-n}$$



$+\dots$

analytic $\propto \left(k_I^2\right)^n$

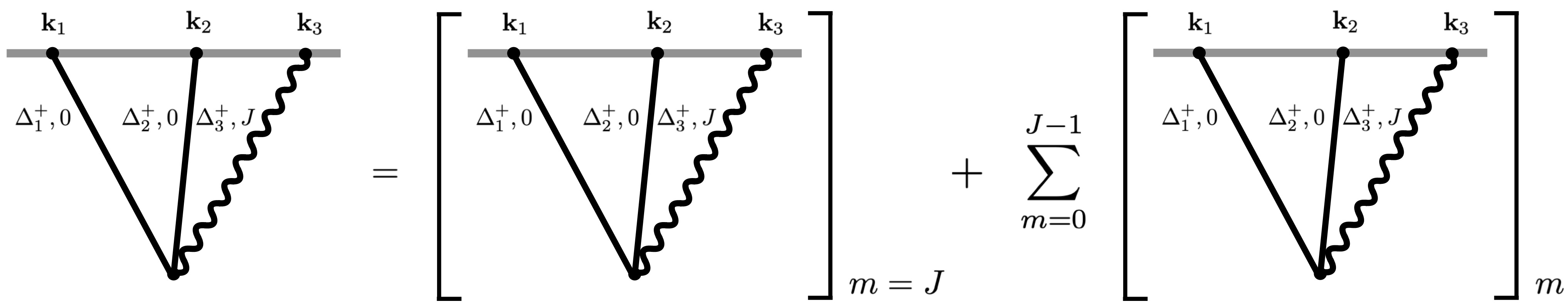
$\sim H^2/m^2$, EFT expansion

External conformally coupled/massless scalars: Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Decomposition into helicities m = 0, 1, …, J :



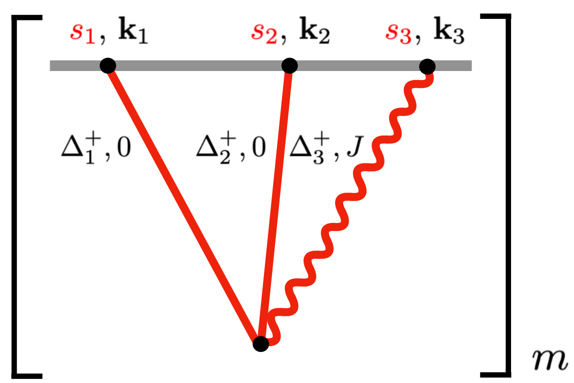
gauge invariance → these lower helicity components cannot contain bulk contact terms (they are at most boundary terms)

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Recall that bulk contact singularities are encoded in Dirac delta functions in the external Mellin variables.

For the helicity-m component we have:



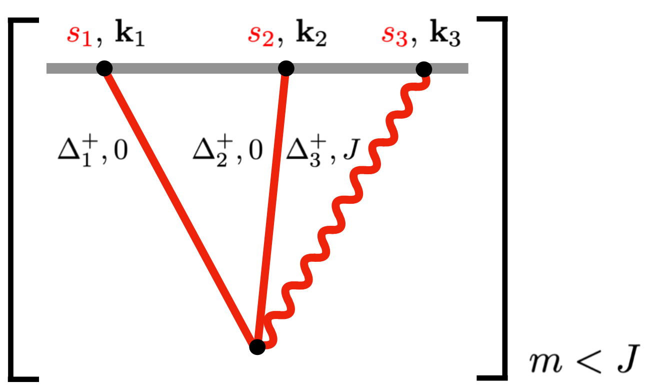
$$\propto \underbrace{\delta\left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)}_{\text{This must be here by the Dilatation Ward identities}}$$

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Recall that bulk contact singularities are encoded in Dirac delta functions in the external Mellin variables.

Gauge invariance requires that for the lower helicity components m < J we *must have*:



This factor cancels the bulk contact singularity encoded by the Dirac delta function

$$\underbrace{\propto \overbrace{\left(\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)} \delta\left(\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)}$$

$$\lim_{\eta_0 \to 0} \int_{-\infty}^{\eta_0} d\eta \, \partial_\eta \left(\eta^{\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3}\right) = \lim_{\eta_0 \to 0} \left(\eta_0^{\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3}\right)$$

This is a boundary term.

Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✔

Scalars of unequal mass ✘

(Consistent with Berends, Burgers and van Dam 1986)

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

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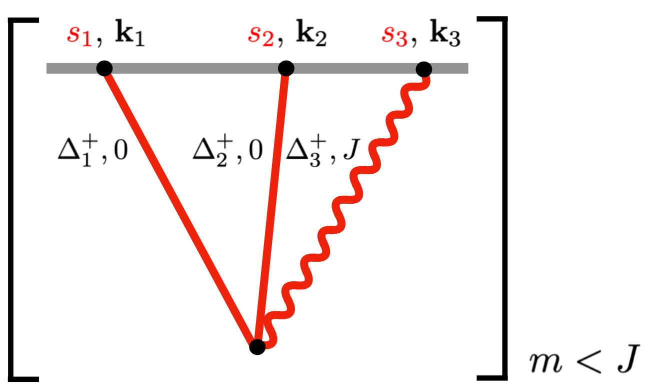

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This is a boundary term.

Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✓

A non-trivial Ward-Takahashi identity is generated by the finite number of poles that satisfy:

$$\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3 = 0$$

which are: $s_1 = \pm\frac{1}{2}\left(\Delta - \frac{d}{2}\right) - n_1, \qquad s_2 = \mp\frac{1}{2}\left(\Delta - \frac{d}{2}\right) - n_2, \qquad s_3 = \frac{1}{2}\left(\Delta_3^+ - \frac{d}{2}\right) - n_3, \qquad n_i \in \mathbb{N}$

with $\quad (n_1 + n_2 + n_3) = (J - 1 - m)$

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Recall that bulk contact singularities are encoded in Dirac delta functions in the external Mellin variables.

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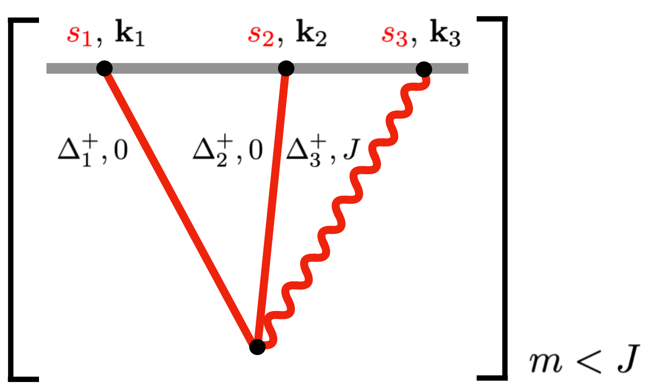

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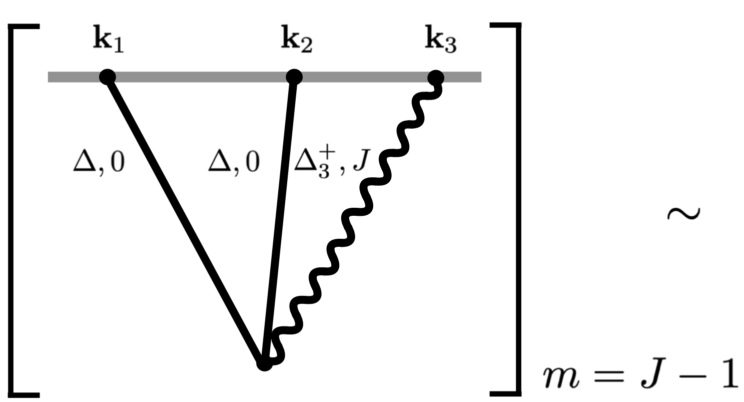
$$\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3 = 0$$

For example:

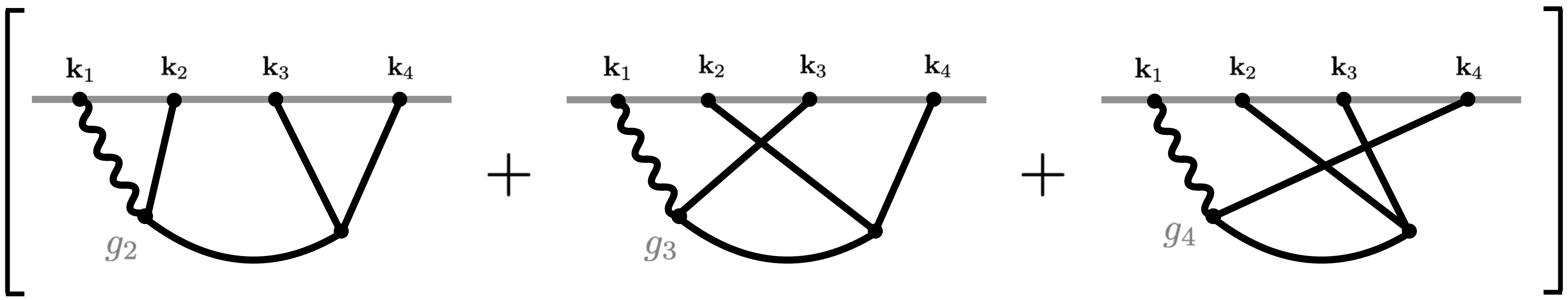


$$\sim \quad (\xi \cdot \mathbf{k}_{12})^{J-1} \langle \mathcal{O}_\Delta(\mathbf{k}_1) \mathcal{O}_\Delta(-\mathbf{k}_1) \rangle - (\xi \cdot \mathbf{k}_{12})^{J-1} \langle \mathcal{O}_\Delta(\mathbf{k}_2) \mathcal{O}_\Delta(-\mathbf{k}_2) \rangle$$

where $\xi \cdot \xi = 0, \quad \xi \cdot \mathbf{k}_3 = 0$

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

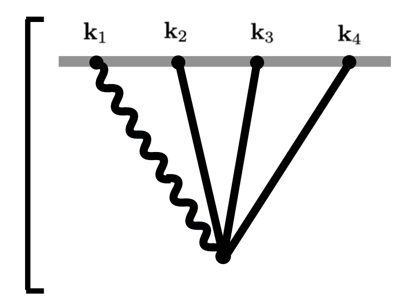


$m = J - 1$

~ [Ward-Takahashi identity] +] Bulk contact singularities $E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4| \to 0$]

Comes from the on-shell exchange, inherited from the gauge invariant 3pt functions

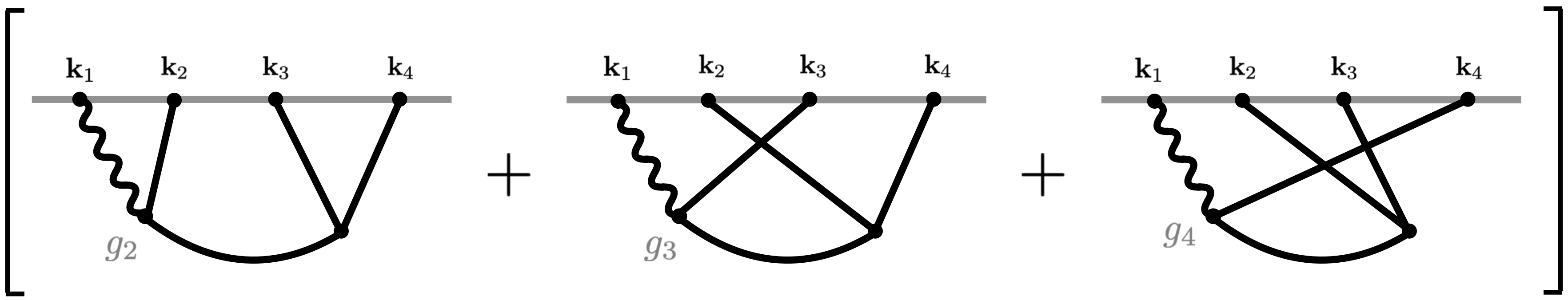
Violates gauge invariance. *Can they be compensated by:*



?

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.



$m = J-1$

~ [Ward-Takahashi identity] +] Bulk contact singularities $E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4| \to 0$]

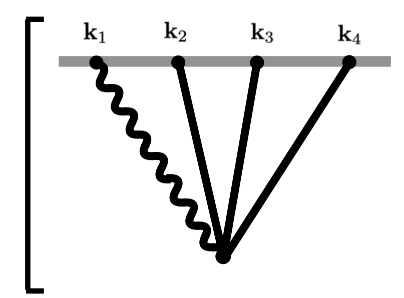
Comes from the on-shell exchange, inherited from the gauge invariant 3pt functions

Violates gauge invariance. *Can they be compensated by:*

Some singularities in E_T

cannot be cancelled!

(by local quartic vertices)



?

These singularities must therefore cancel by themselves → constrains g_i

The helicity-(J-1) component this gives the constraint:

$$\left[\sum_{i=2}^{4} g_i \left(\xi \cdot \mathbf{k}_i\right)^{J-1}\right] = 0$$

$J = 1: \; g_2 + g_3 + g_4 = 0,$ *charge conservation*

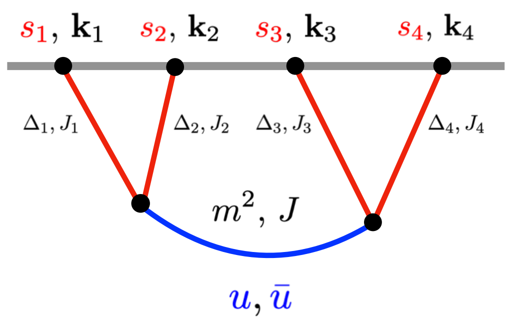
$J = 2: \; g_2 = g_3 = g_4,$ *equivalence principle*

See also Baumann et al. May 2020

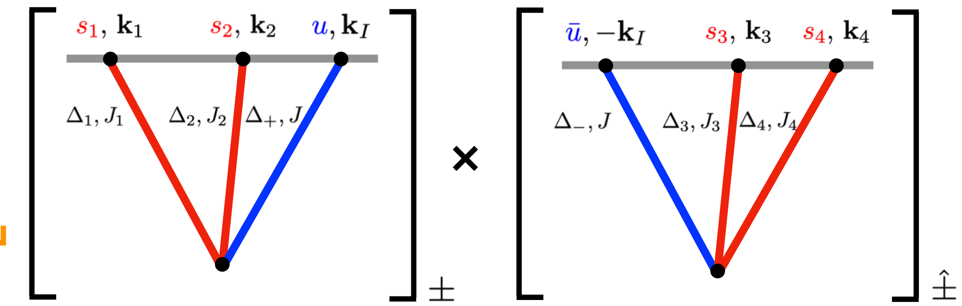
$J > 2: \; g_2 = g_3 = g_4 = 0,$ *no consistent coupling (in local theories)*

cf. Weinberg soft theorem in flat space!

Summary



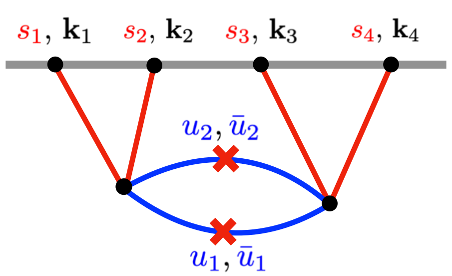
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{EFT}} \underbrace{\sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



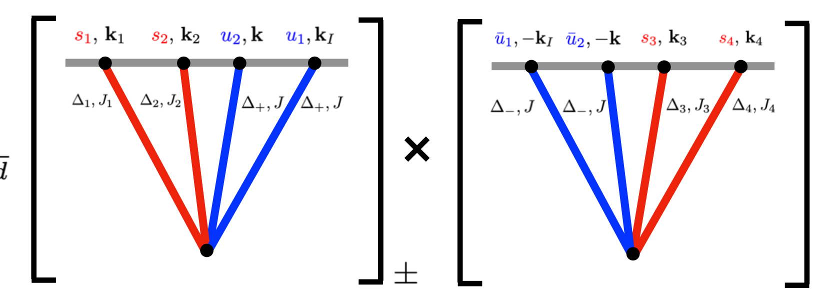
Factorisation + Conformal Symmetry

Plenty of diverse directions for the future!

- Higher points and Loops. Nice parallel with generalised unitarity methods/Cutkosky rules:



$$= \sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u_1,\bar{u}_1\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u_1,\bar{u}_1\right)\right]\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u_2,\bar{u}_2\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u_2,\bar{u}_2\right)\right]\int\frac{d^d\mathbf{k}}{(2\pi)^d}$$



- Bootstrap of Euclidean CFTs dual to dS physics?

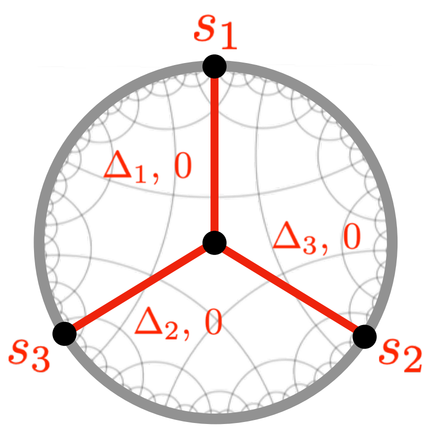
- Celestial Amplitudes?

$\vdots$

Back up slides

Contact Amplitudes

AdS 3pt contact amplitude of generic scalars



$$\propto \delta\left(\tfrac{d}{4} - (s_1 + s_2 + s_3)\right) \prod_{j=1}^{3} \Gamma\left(s_j + \tfrac{i\nu_j}{2}\right)\Gamma\left(s_j - \tfrac{i\nu_j}{2}\right)\left(\frac{k_j}{2}\right)^{-2s_j + i\nu_j}$$

bulk integration

external legs

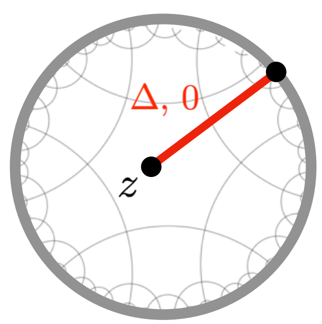
$$m_j^2 R^2 = \Delta_j\left(\Delta_j - d\right)$$

$$\Delta_j = \frac{d}{2} + i\nu_j$$

The connection to the bulk is manifest!

Bulk-boundary propagator in Poincaré coordinates:

$$\mathrm{d}s^2 = R^2 \frac{\mathrm{d}z^2 + \mathrm{d}\mathbf{y}^2}{z^2}$$



$$= z^{\frac{d}{2}} \left(\frac{k}{2}\right)^{i\nu} K_{i\nu}\left(zk\right) = \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s}{2\pi i} z^{\frac{d}{2} - 2s}\, \Gamma\left(s + \tfrac{i\nu}{2}\right)\Gamma\left(s - \tfrac{i\nu}{2}\right)\left(\frac{k}{2}\right)^{-2s + i\nu}$$

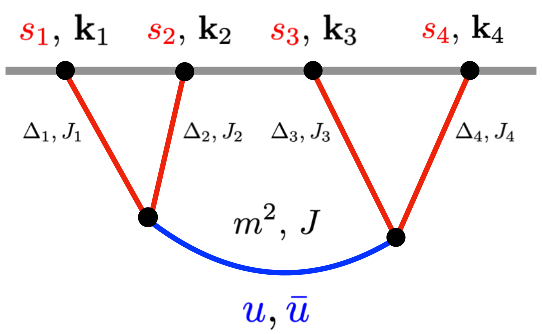
Modified Bessel function of the second kind

Bulk integration is encoded in a **Dirac delta function:**

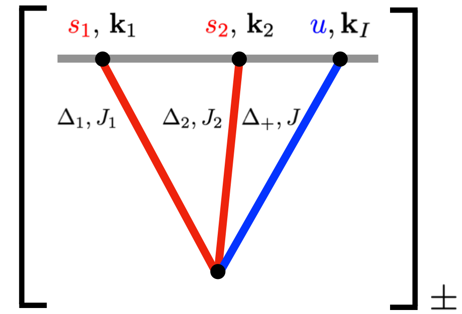
$$\delta\left(\tfrac{d}{2} - 2\left(s_1 + s_2 + s_3\right)\right) = \lim_{z_0 \to 0} \int_{z_0}^{\infty} \frac{dz}{z^{d+1}} z^{\sum_{j=1}^{3}\left(\frac{d}{2} - 2s_j\right)}$$

The sum of the Mellin variables is thus conserved: $s_1 + s_2 + s_3 = \dfrac{d}{4}$

Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right)\sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$



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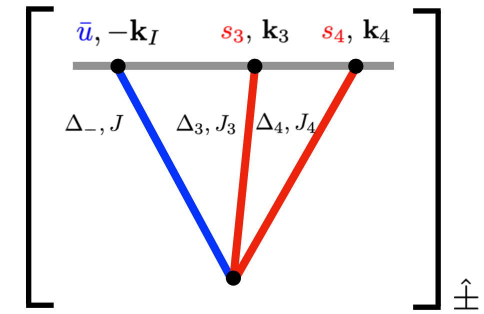


The Mellin-Barnes representation above captures the full analytic structure of the boundary correlators.

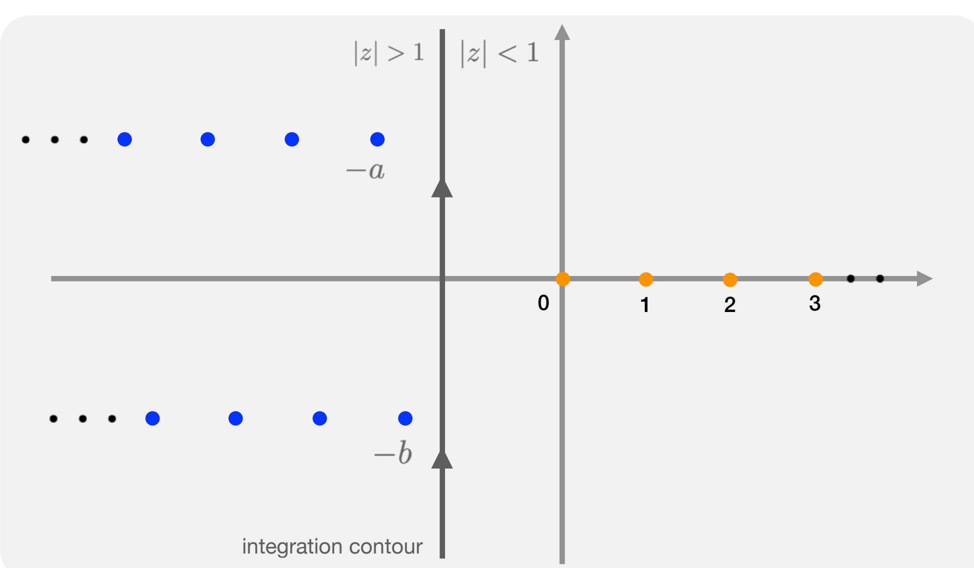
cf. Mellin-Barnes representation of the Gauss Hypergeometric function

$${}_2F_1\left(a,b;c;z\right)=\frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\,\frac{\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(-s\right)}{\Gamma\left(c+s\right)}\left(-z\right)^s$$

residues of the ●

$$=\sum_{n=0}^{\infty}\frac{\left(a\right)_n\left(b\right)_n}{\left(c\right)_n}\frac{z^n}{n!},\qquad |z|<1$$

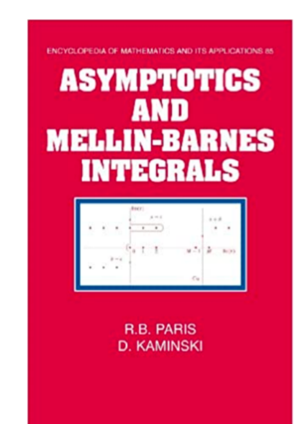
Poles in s :



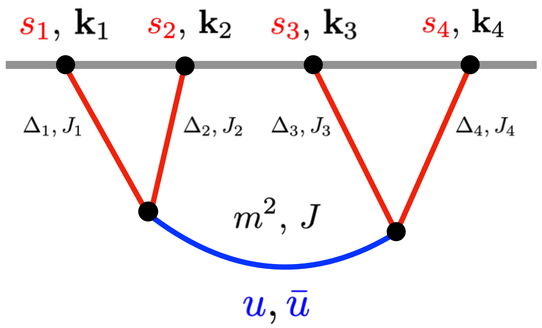
Useful to establish:

- Identities and Transformation formulas
- Asymptotic Expansions
- Resolution of Singularities

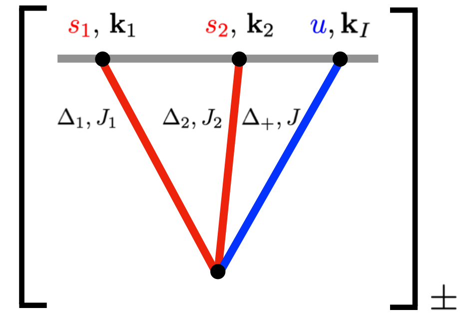
⋮



Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right) \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$



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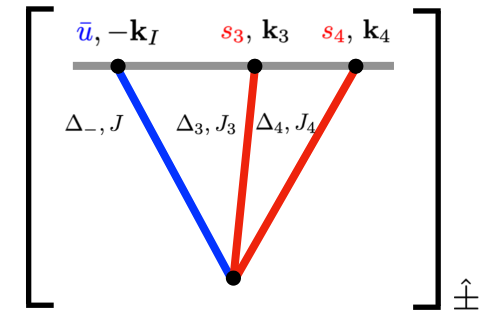

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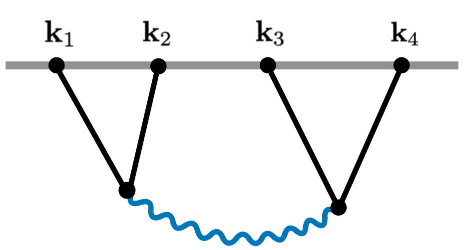
cf.

For some special representations [e.g. (partially)-massless fields] the Mellin-Barnes integrals can be lifted.

e.g. massless spin-J exchange between conformally coupled scalars:



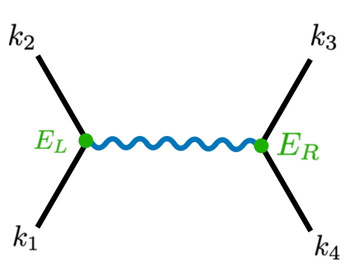
$$\propto \left(2\pi\right)^3 \delta^{(3)}\Big(\sum_{i=1}^{4} \mathbf{k}_i\Big) \frac{1}{\left(E_L\right)^J \left(E_R\right)^J \left(E_T\right)^{2J-1}} \left[\sum_{n=0}^{J-1} c_n \left(\left(|\mathbf{k}_1|+|\mathbf{k}_2|\right)\left(|\mathbf{k}_3|+|\mathbf{k}_4|\right)+|\mathbf{k}_I|^2\right)^{J-1-n} \left(|\mathbf{k}_I| E_T\right)^n\right] \Xi_J + \ldots$$

helicity-J

Simple integer coefficients.

Lower helicity, contact terms

cf. corresponding amplitude in (d+1)-dimensional flat space:



$$= \left(2\pi\right) \delta\left(E_T\right) \left(2\pi\right)^3 \delta^{(3)}\Big(\sum_{i=1}^{4} \mathbf{k}_i\Big) \frac{1}{\mathsf{s}} \Xi_J$$

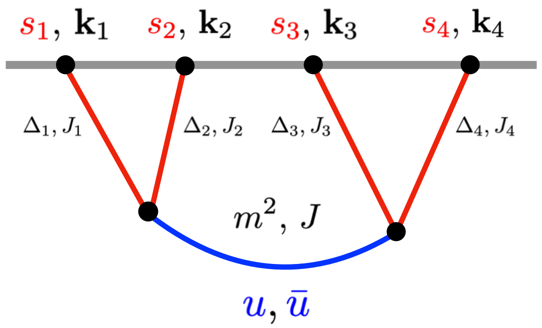
(d+1)-dim. Mandelstam variable

(d+1)-dim. null momentum: $k_i = \left(|\mathbf{k}_i|, \mathbf{k}_i\right)$

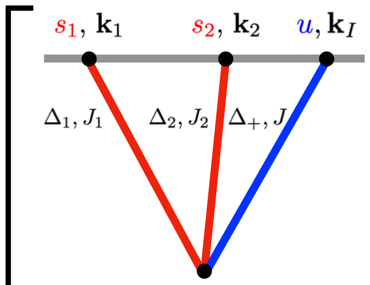
$$E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4|$$

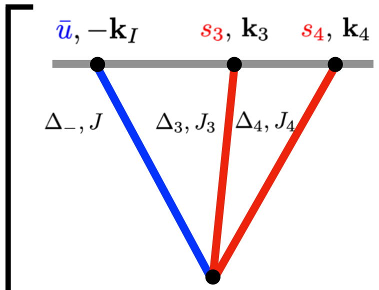
$$E_L = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_I|, \quad E_R = |\mathbf{k}_I| + |\mathbf{k}_3| + |\mathbf{k}_4|$$

Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right)\sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]\Big[\ \cdot\ \Big]_{\pm}\times\Big[\ \cdot\ \Big]_{\hat{\pm}}$$





The Mellin-Barnes representation above captures the full analytic structure of the boundary correlators.

cf. Mellin-Barnes representation of the Gauss Hypergeometric function

$$ {}_2F_1\left(a,b;c;z\right)=\frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\frac{\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(-s\right)}{\Gamma\left(c+s\right)}\left(-z\right)^s \overset{c=b}{=} \left(1-z\right)^{-a}$$

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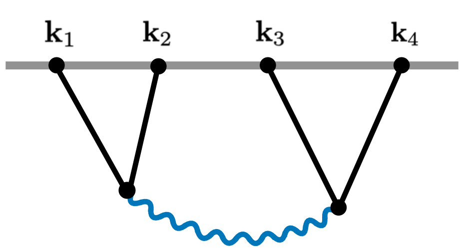
For some special representations [e.g. (partially)-massless fields] the Mellin-Barnes integrals can be lifted.

e.g. massless spin-J exchange between conformally coupled scalars:



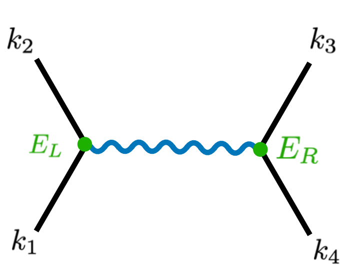
$$\propto (2\pi)^3\,\delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big)\frac{1}{\left(E_L\right)^J\left(E_R\right)^J\left(E_T\right)^{2J-1}}\left[\sum_{n=0}^{J-1}c_n\left(\left(|\mathbf{k}_1|+|\mathbf{k}_2|\right)\left(|\mathbf{k}_3|+|\mathbf{k}_4|\right)+|\mathbf{k}_I|^2\right)^{J-1-n}\left(|\mathbf{k}_I|E_T\right)^n\right]\Xi_J+\ldots$$

helicity-J

Lower helicity, contact terms

Maldacena, Pimentel & Raju 2011-2012

$E_T\to 0$



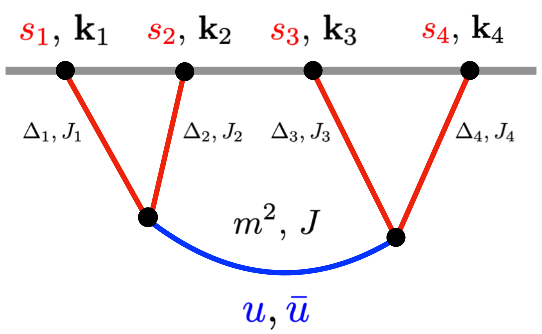
$$\times\ \frac{1}{\left(E_T\right)^{\text{power}}}$$

(d+1)-dim. null momentum: $k_i=\left(|\mathbf{k}_i|,\mathbf{k}_i\right)$

$E_T=|\mathbf{k}_1|+|\mathbf{k}_2|+|\mathbf{k}_3|+|\mathbf{k}_4|$

$E_L=|\mathbf{k}_1|+|\mathbf{k}_2|+|\mathbf{k}_I|,\quad E_R=|\mathbf{k}_I|+|\mathbf{k}_3|+|\mathbf{k}_4|$

Exchanges in dS



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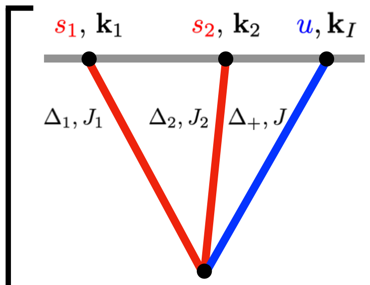

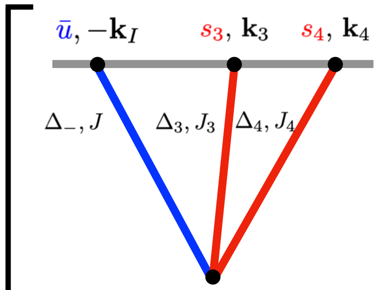

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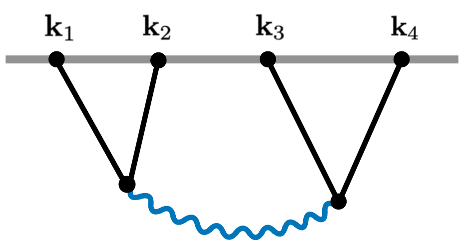
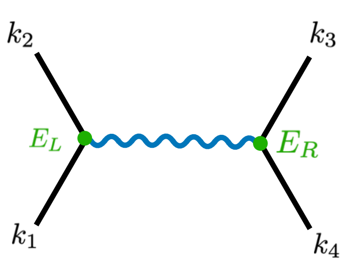


$$\propto (2\pi)^3\,\delta^{(3)}\Big(\sum_{i=1}^4 \mathbf{k}_i\Big)\frac{1}{\left(E_L\right)^J\left(E_R\right)^J\left(E_T\right)^{2J-1}}\underbrace{\left[\sum_{n=0}^{J-1} c_n\left(\left(|\mathbf{k}_1|+|\mathbf{k}_2|\right)\left(|\mathbf{k}_3|+|\mathbf{k}_4|\right)+|\mathbf{k}_I|^2\right)^{J-1-n}\left(|\mathbf{k}_I|E_T\right)^n\right]}_{E_T\to 0:\;\left(E_L E_R\right)^{J-1}=\,\mathsf{s}^{J-1}}\Xi_J+\dots$$

helicity-J

Lower helicity, contact terms

Flat limit $\quad E_T\to 0$



$$\times\;\frac{1}{\left(E_T\right)^{\text{power}}}=(2\pi)^3\,\delta^{(3)}\Big(\sum_{i=1}^4\mathbf{k}_i\Big)\frac{\Xi_J}{\mathsf{s}}\times\frac{1}{\left(E_T\right)^{2J-1}}$$

(d+1)-dim. null momentum: $k_i=\left(|\mathbf{k}_i|,\mathbf{k}_i\right)$

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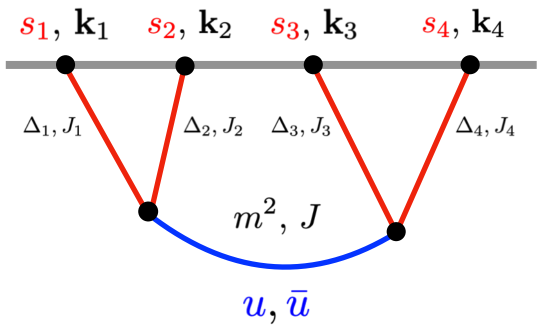
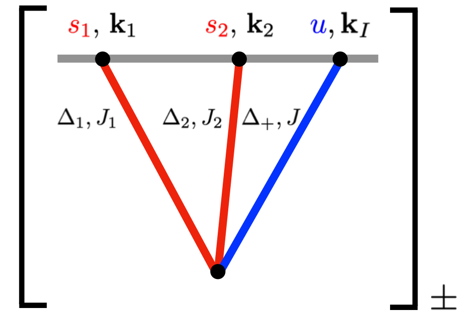
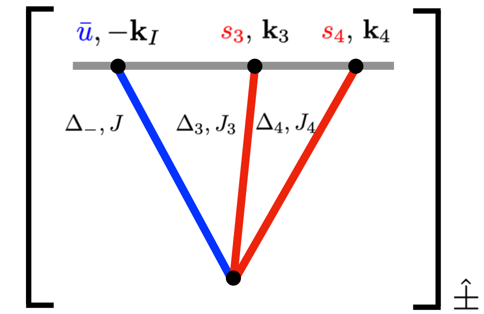
Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right) \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$

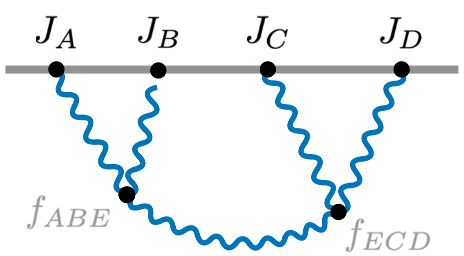


$\times$



For some special representations [e.g. (partially)-massless fields] the Mellin-Barnes integrals can be lifted.

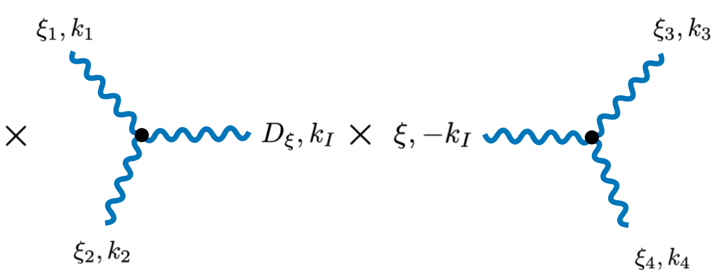
e.g. 4pt exchange in Yang-Mills theory



$$\propto\ f_{ABE}f_{ECD}\left(2\pi\right)^3 \delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big) \times \frac{1}{|\mathbf{k}_I| E_L E_R E_T} \times \left(|\mathbf{k}_I| + E_T\right) \times \underbrace{\mathsf{p}_+^{\mathrm{YM}}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_I; \xi_1, \xi_2, D_\xi\right) \mathsf{p}_-^{\mathrm{YM}}\left(-\mathbf{k}_I, \mathbf{k}_3, \mathbf{k}_4; \xi, \xi_3, \xi_4\right)}_{\text{contraction of the 3pt tensorial structures}}$$

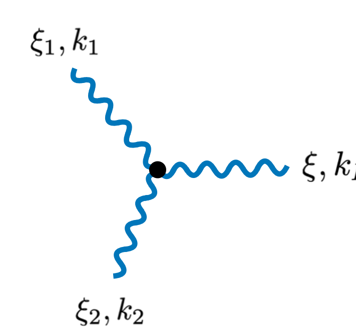
Flat limit $\quad E_T \to 0$

$$f_{ABE}f_{ECD}\left(2\pi\right)^3 \delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big)\frac{1}{\mathsf{s}} \times$$

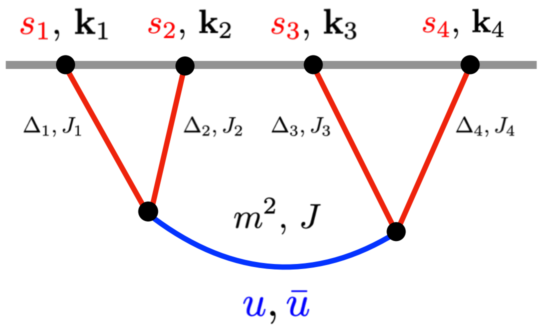


The highest helicity component is simple and given by the 3pt Yang-Mills amplitude in flat space:

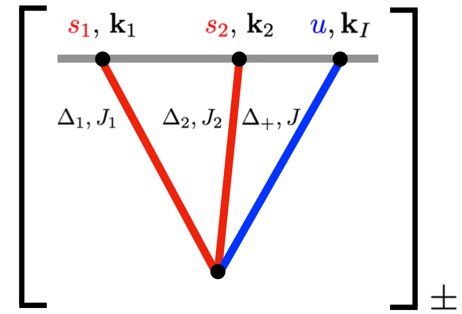
$$\mathsf{p}_\pm^{\mathrm{YM}}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_I; \xi_1, \xi_2, \xi\right)\Big|_{\lambda=\pm 1} = -i\left[\left(\xi_1\cdot\mathbf{k}_2\right)\left(\xi_2\cdot\xi\right) + \left(\xi_2\cdot\mathbf{k}_I\right)\left(\xi\cdot\xi_1\right) + \left(\xi\cdot\mathbf{k}_1\right)\left(\xi_1\cdot\xi_2\right)\right] \quad =$$



Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right) \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$



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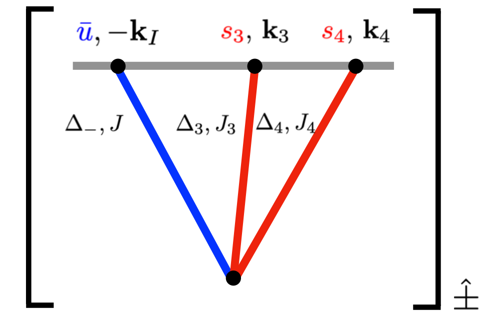

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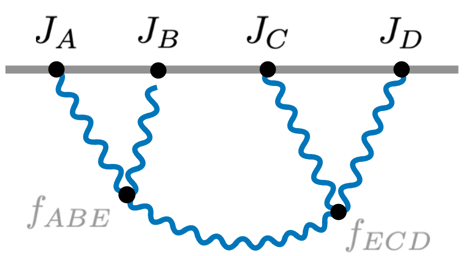
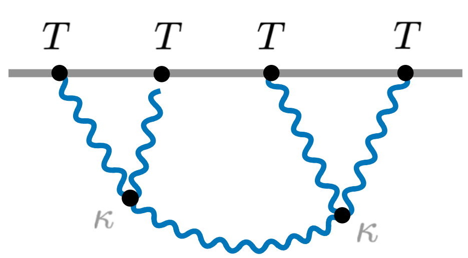


$$\propto f_{ABE} f_{ECD}\, (2\pi)^3\, \delta^{(3)}\Big(\sum_{i=1}^{4} \mathbf{k}_i\Big) \times \frac{1}{|\mathbf{k}_I| E_L E_R E_T} \times \left(|\mathbf{k}_I| + E_T\right) \times \underbrace{\mathsf{p}_+^{\mathrm{YM}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,D_\xi\right) \mathsf{p}_-^{\mathrm{YM}}\left(-\mathbf{k}_I,\mathbf{k}_3,\mathbf{k}_4;\xi,\xi_3,\xi_4\right)}_{\text{contraction of the 3pt tensorial structures}}$$

e.g. 4pt exchange in Gravity



$$\propto \kappa^2\, (2\pi)^3\, \delta^{(3)}\Big(\sum_{i=1}^{4} \mathbf{k}_i\Big) \times \frac{\mathrm{Poly}\left(|\mathbf{k}_i|, |\mathbf{k}_I|\right)}{|\mathbf{k}_I|^3 \left(E_L\right)^2 \left(E_R\right)^2 \left(E_T\right)^3} \times \mathsf{p}_+^{\mathrm{GR}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,D_\xi\right) \mathsf{p}_-^{\mathrm{GR}}\left(-\mathbf{k}_I,\mathbf{k}_3,\mathbf{k}_4;\xi,\xi_3,\xi_4\right)$$

The double-copy structure of flat scattering amplitudes is encoded in dS correlators:

$$\mathsf{p}_\pm^{\mathrm{GR}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,\xi\right)\Big|_{\lambda=\pm 2} = \left(\mathsf{p}_\pm^{\mathrm{YM}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,\xi\right)\Big|_{\lambda=\pm 1}\right)^2$$