Stochastic gravity and turbulence

A. Yarom and S. Waeber

Turbulence

Recall:

$$\dot{\vec{v}} + \vec{v} \cdot \vec{\nabla} \vec{v} = - \vec{\nabla} p + \nu \nabla^2 \vec{v}$$

$$\vec{\nabla} \cdot \vec{v} = 0$$

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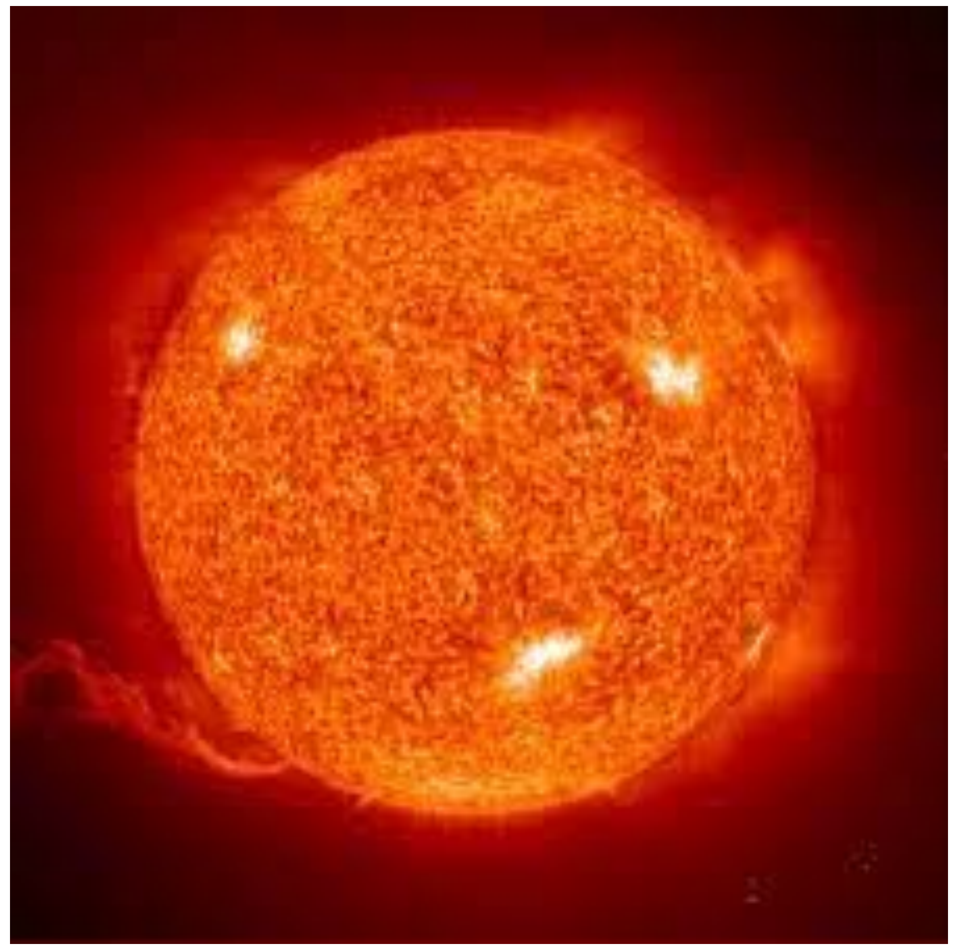
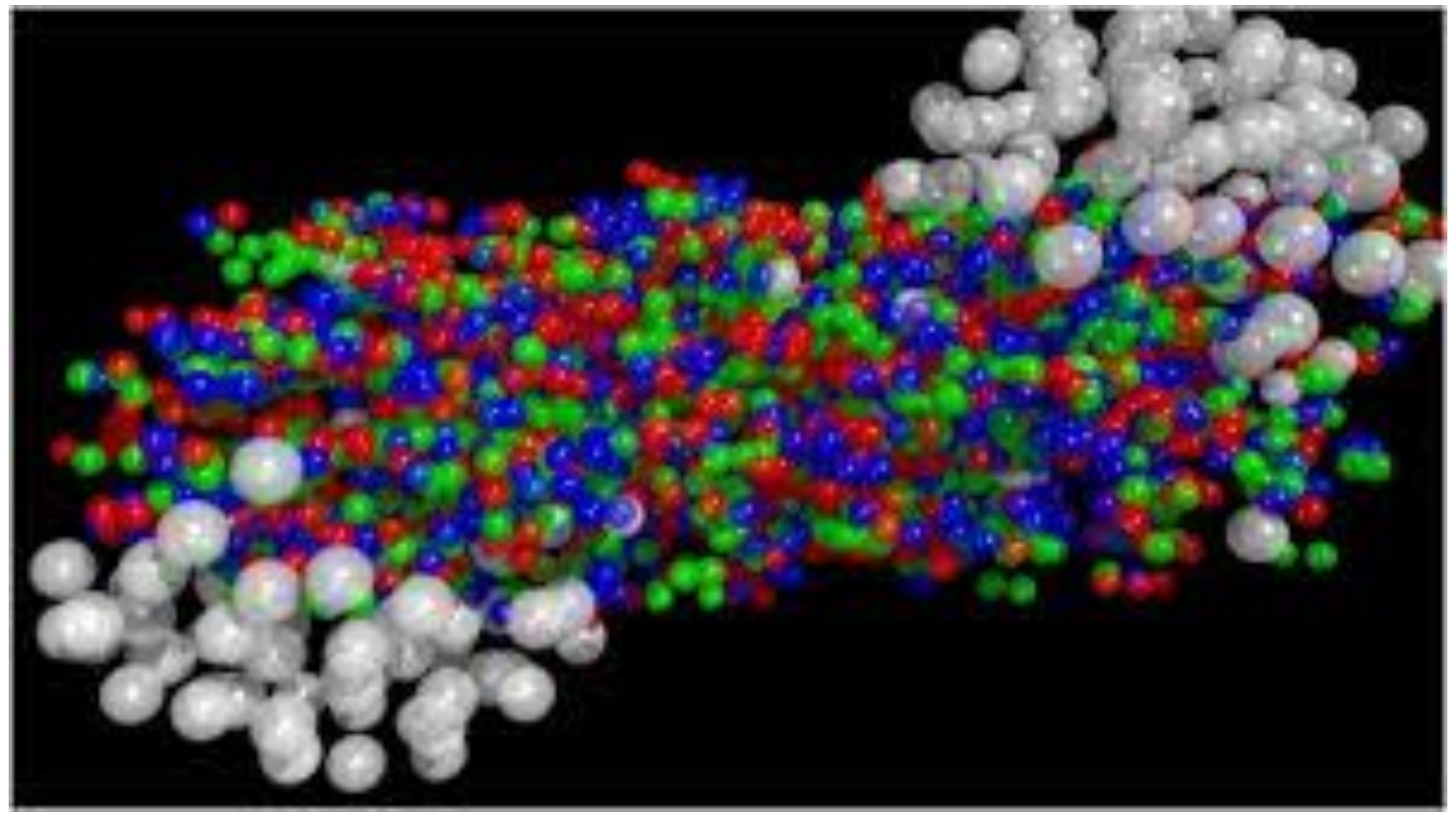
The Navier Stokes equations describe a multitude of phenomenon:





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Kolmogorov's theory suggests that turbulence will occur when

$$Re = \frac{|\overrightarrow{v}_0| L_0}{\nu} \gg 1$$

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$$\dot{\vec{v}} + \vec{v} \cdot \vec{\nabla} \vec{v} = - \vec{\nabla} p + \nu \nabla^2 \vec{v} + \vec{f}$$

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$$Re = \frac{|\overrightarrow{v}_0| L_0}{\nu} \gg 1$$

and that

$$\overline{\left((\overrightarrow{v}(\vec{r}) - \overrightarrow{v}(0)) \cdot \hat{r}\right)^n} \propto |r|^{\frac{n}{3}}$$

Turbulence

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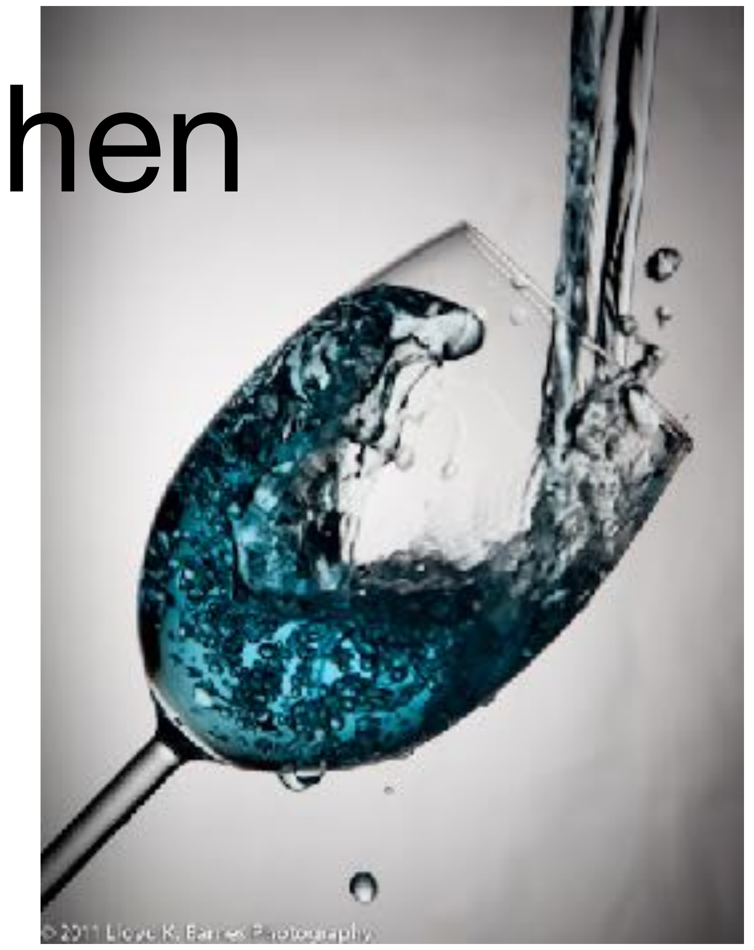

$$\overline{\left((\overrightarrow{v}(\vec{r}) - \overrightarrow{v}(0)) \cdot \hat{r}\right)^n} \propto |r|^{\frac{n}{3}}$$

Turbulence

Kolmogorov's theory suggests that turbulent flow satisfies

$$\overline{\left((\overrightarrow{v}(\vec{r}) - \overrightarrow{v}(0)) \cdot \hat{r}\right)^n} \propto |r|^{\frac{n}{3}}$$

For n=2 we get

$$\epsilon = \overline{\frac{1}{2}\rho|\overrightarrow{v}|^2} \propto |r|^{\frac{2}{3}}$$

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Usually written in Fourier space

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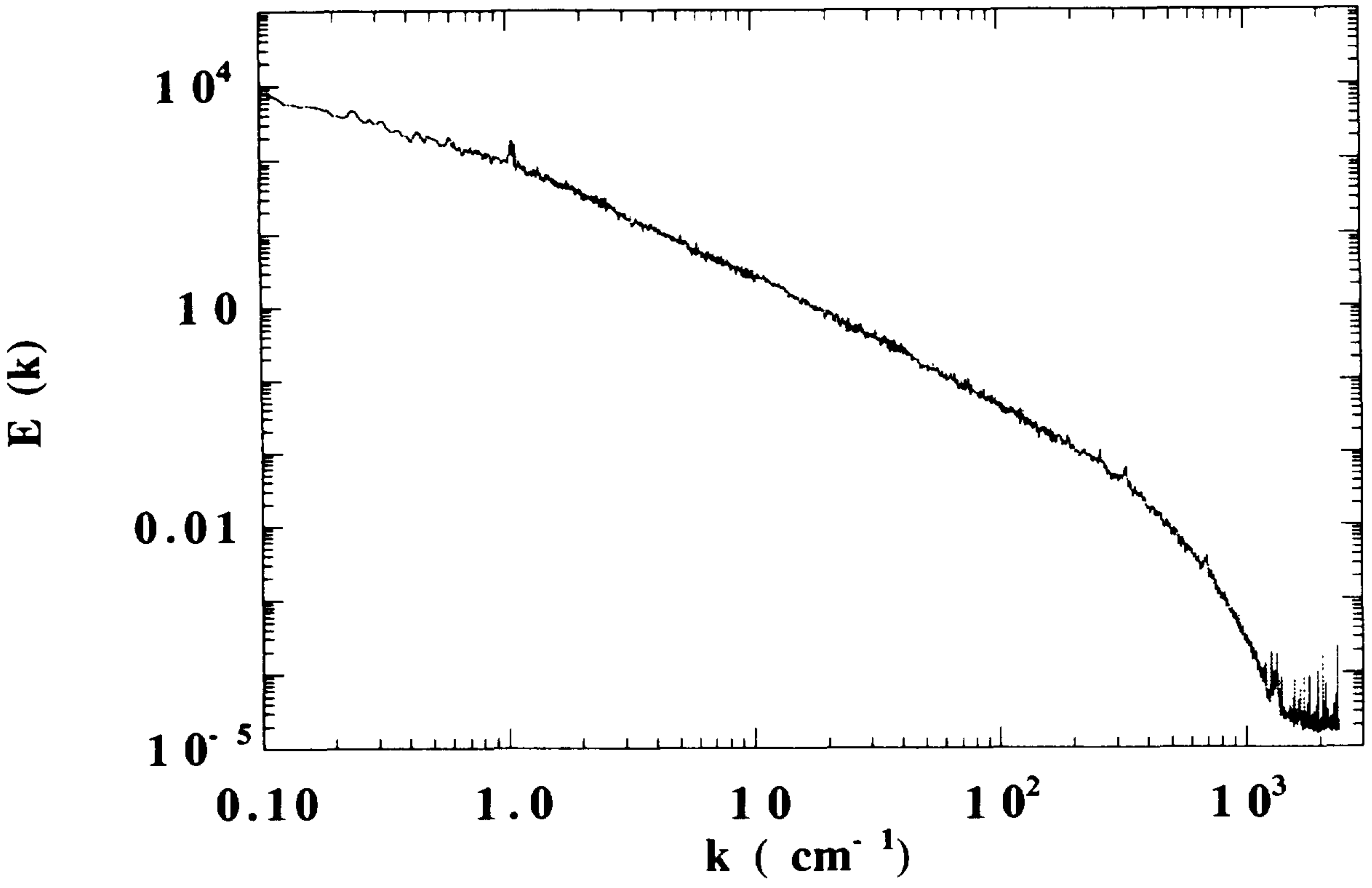

(Zocchi et. al. 1994)

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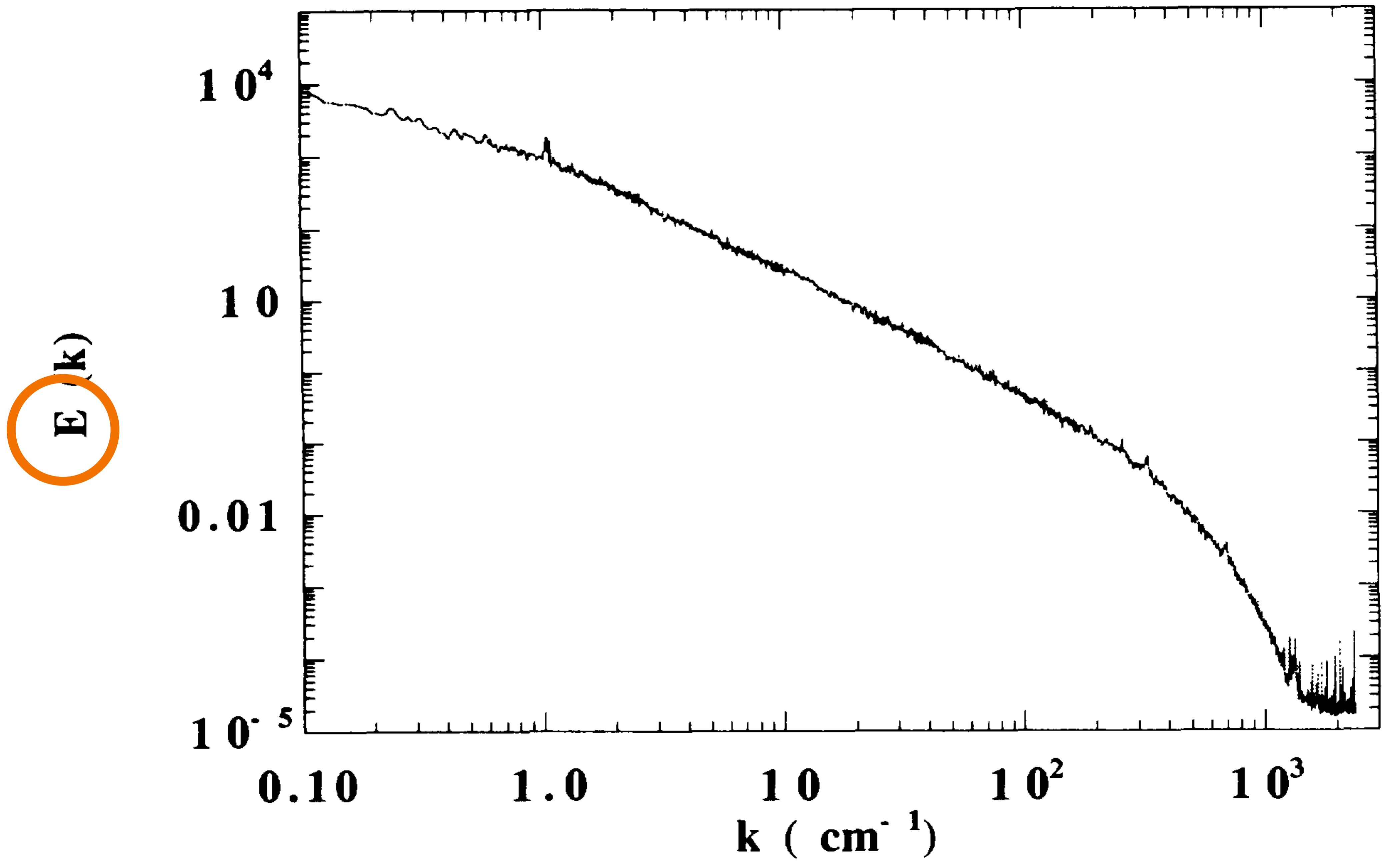

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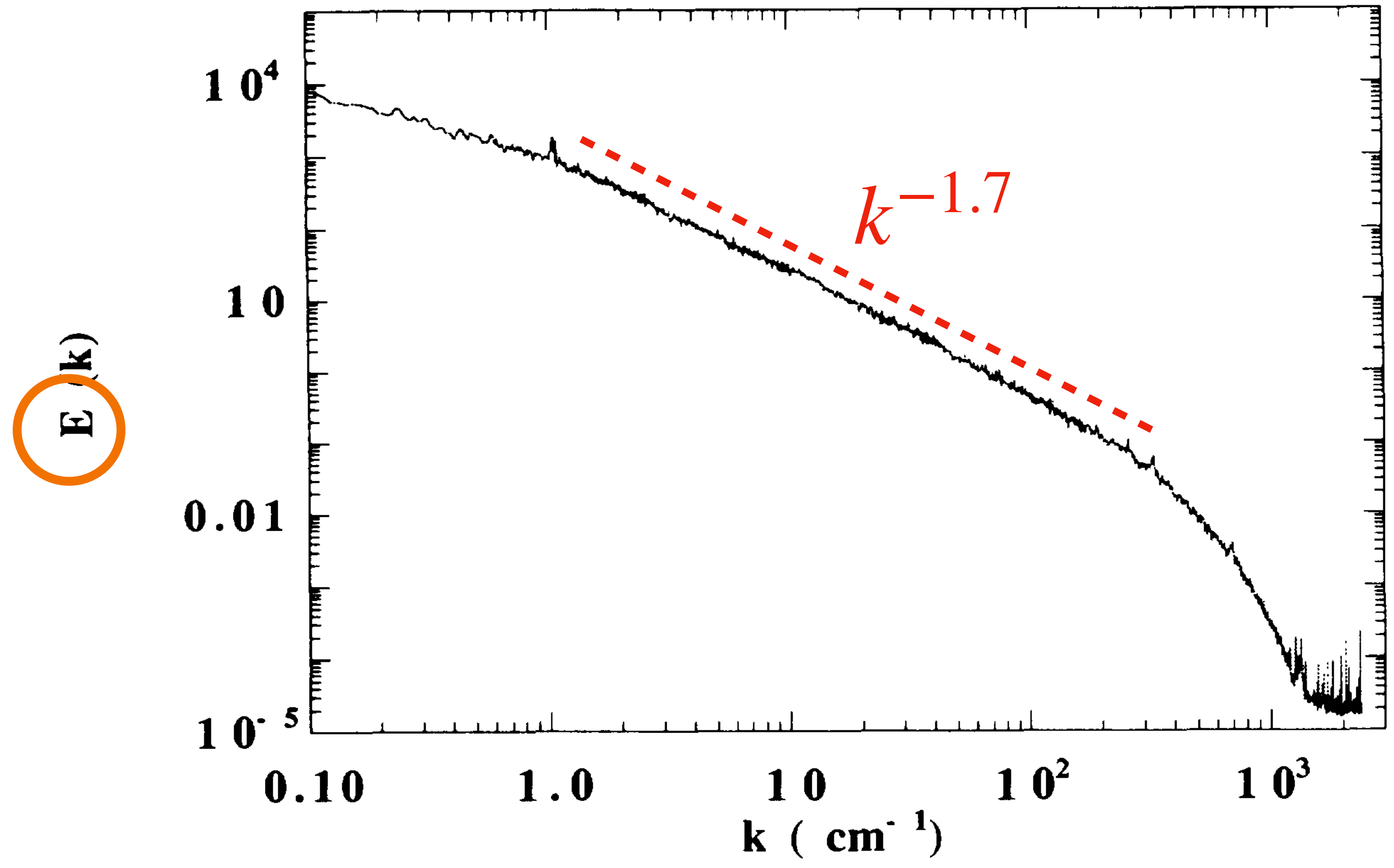

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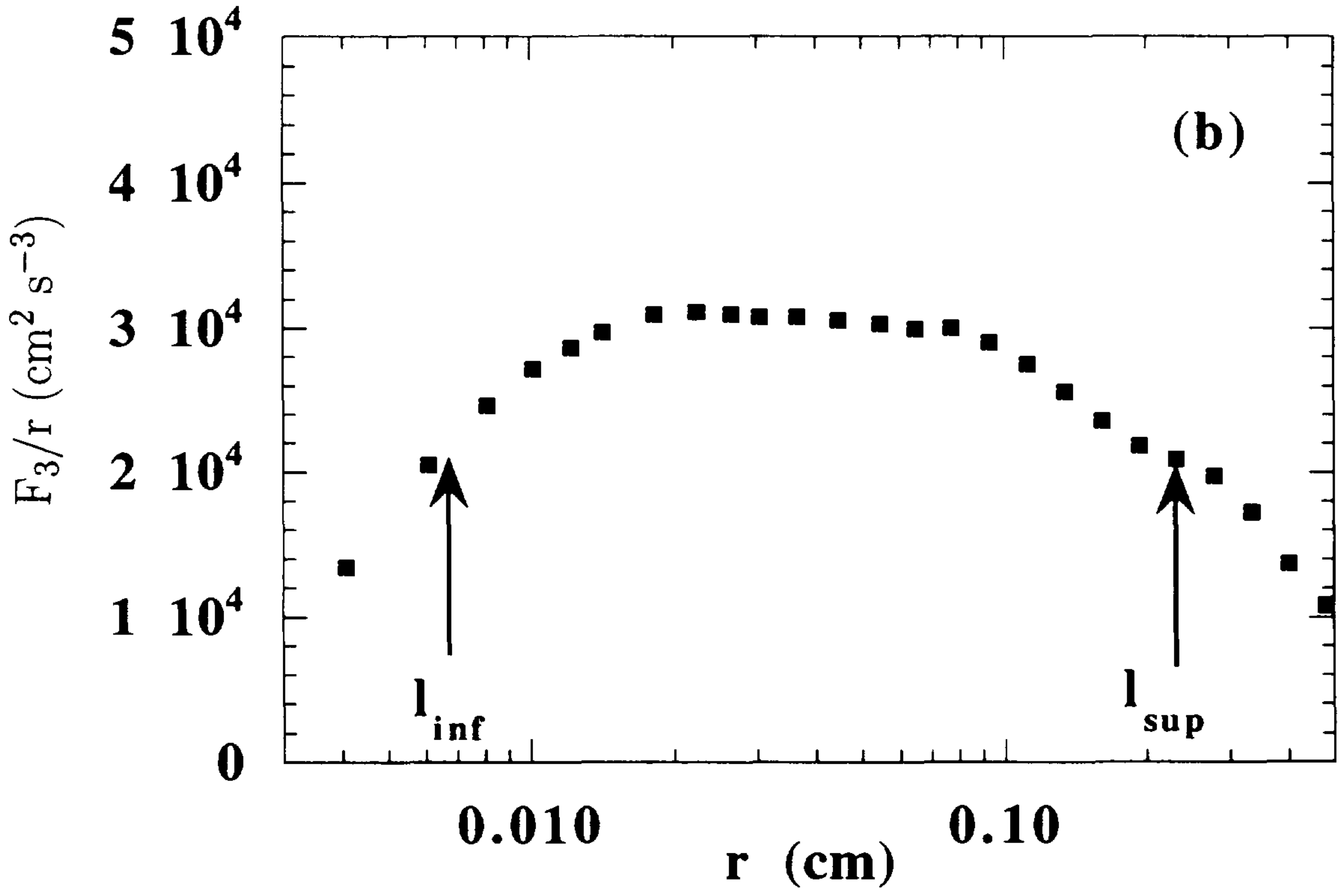

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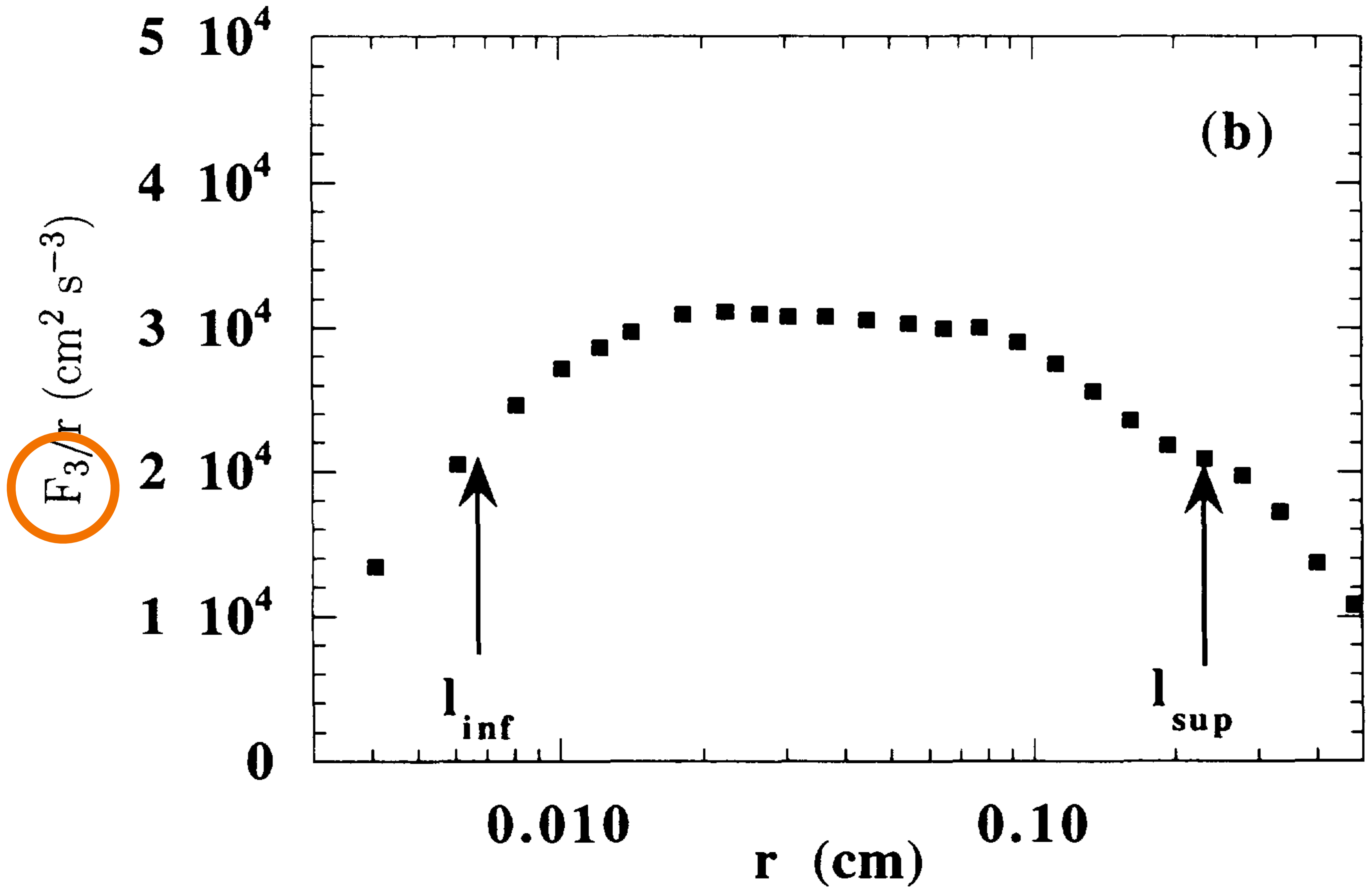

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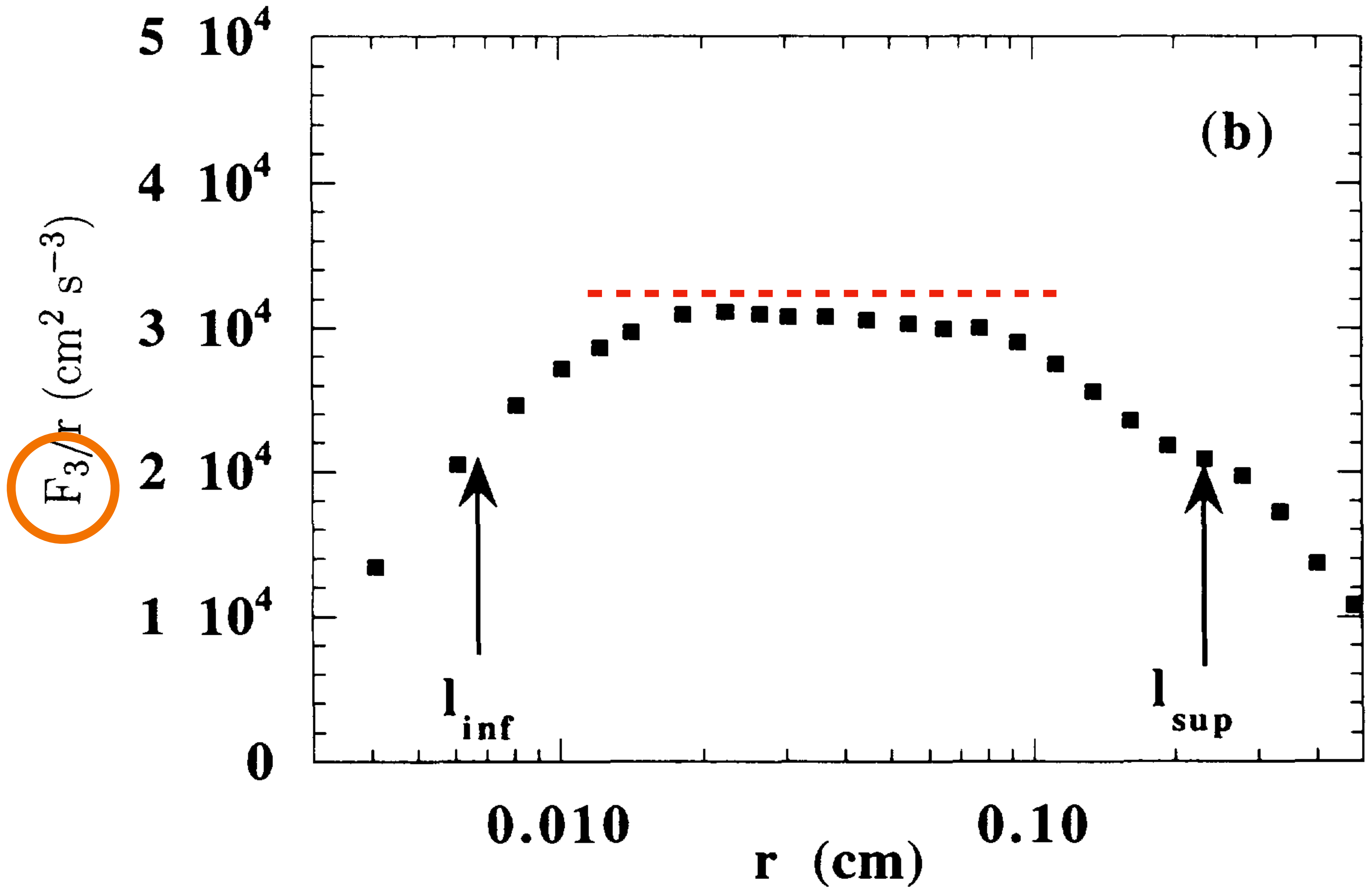

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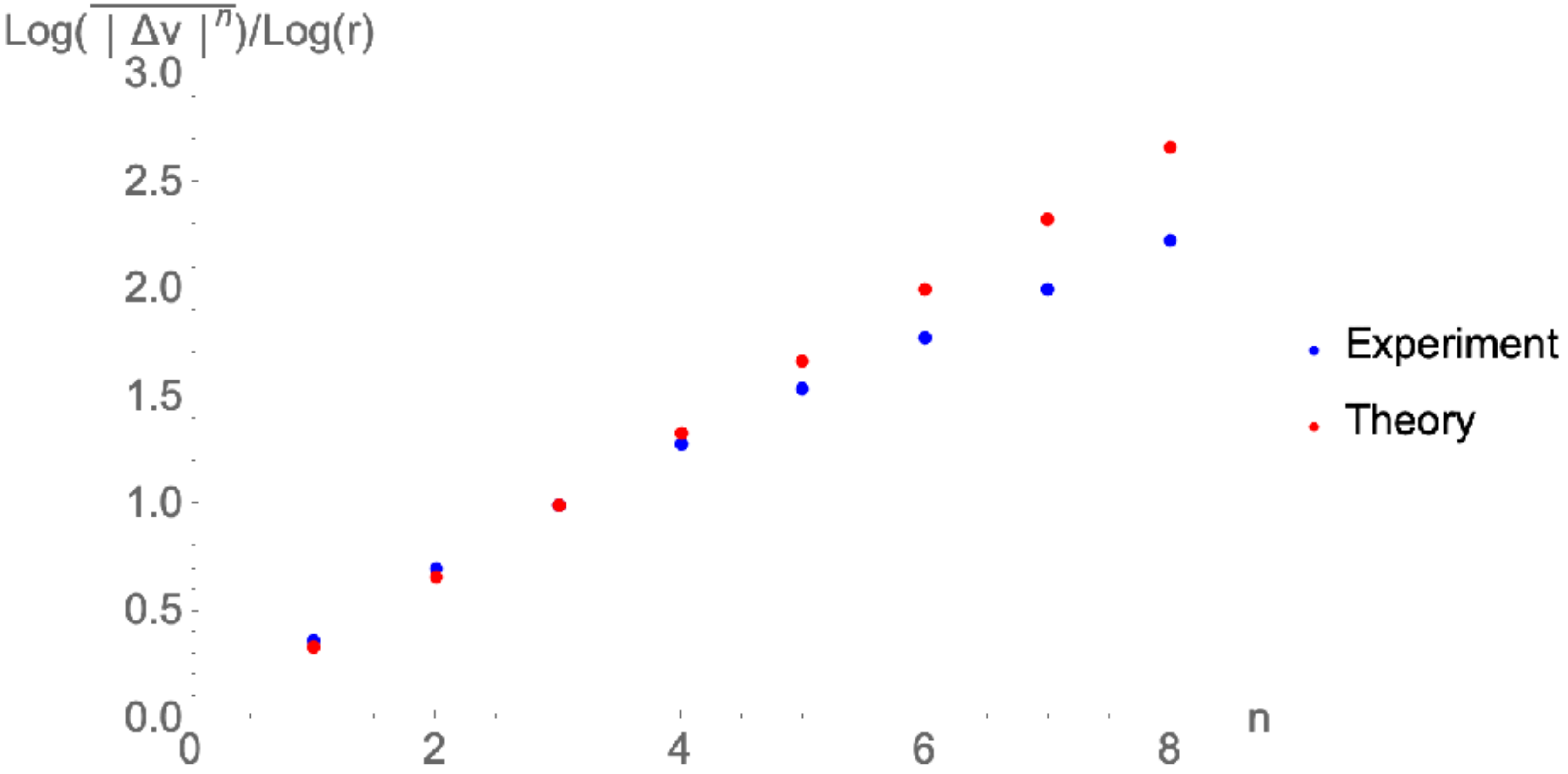

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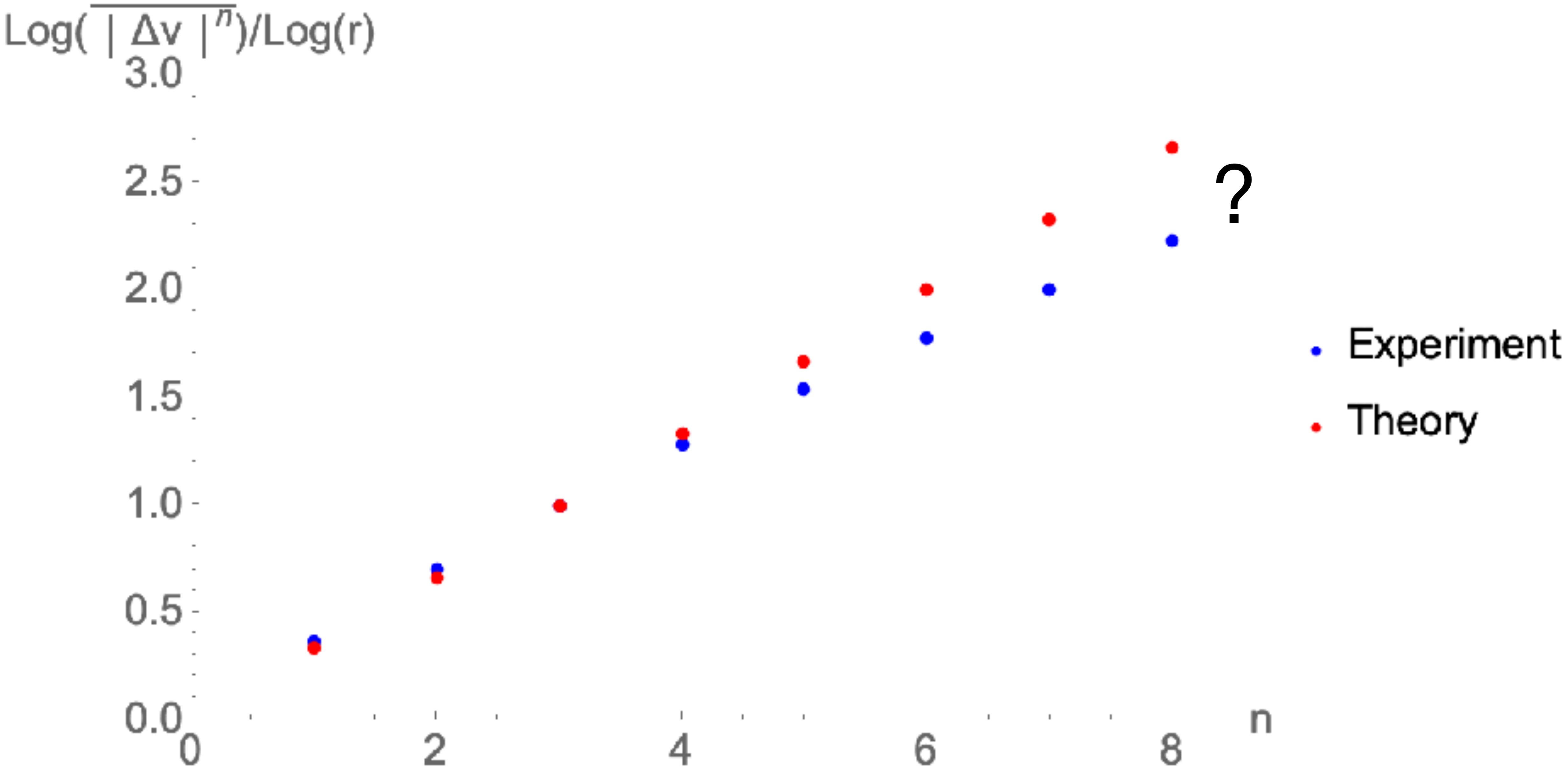

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Holographic turbulence

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Holographic turbulence

- Maldacena, 1997

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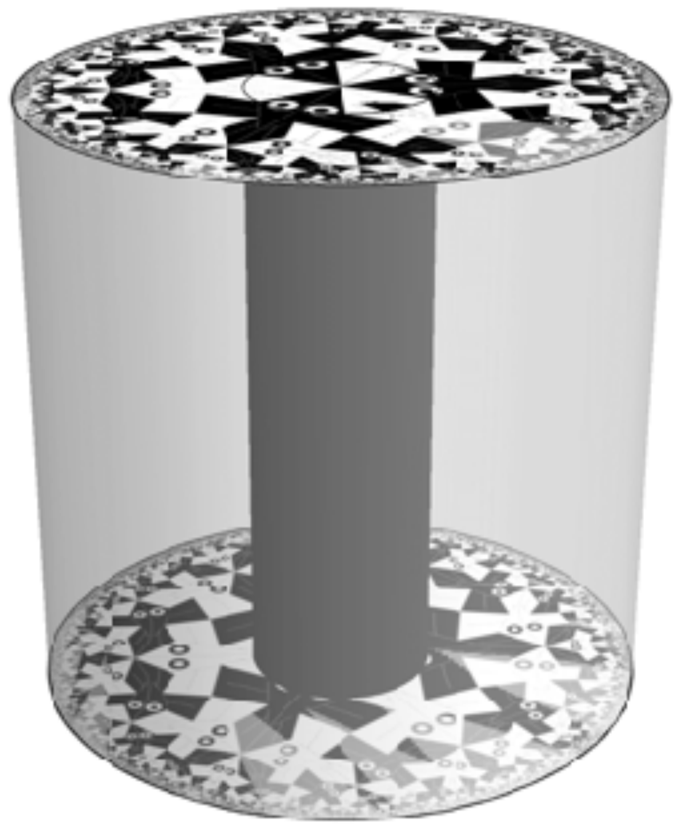

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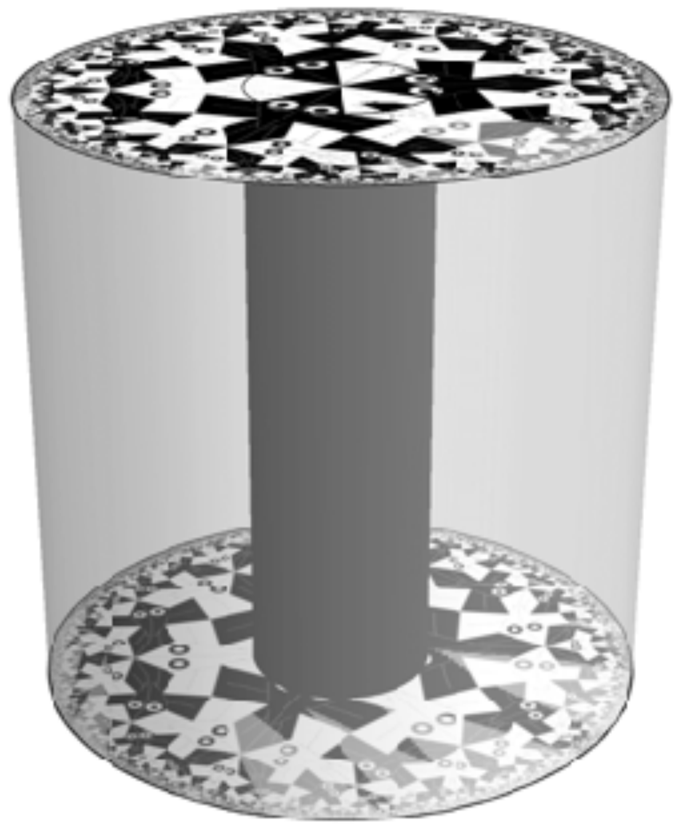

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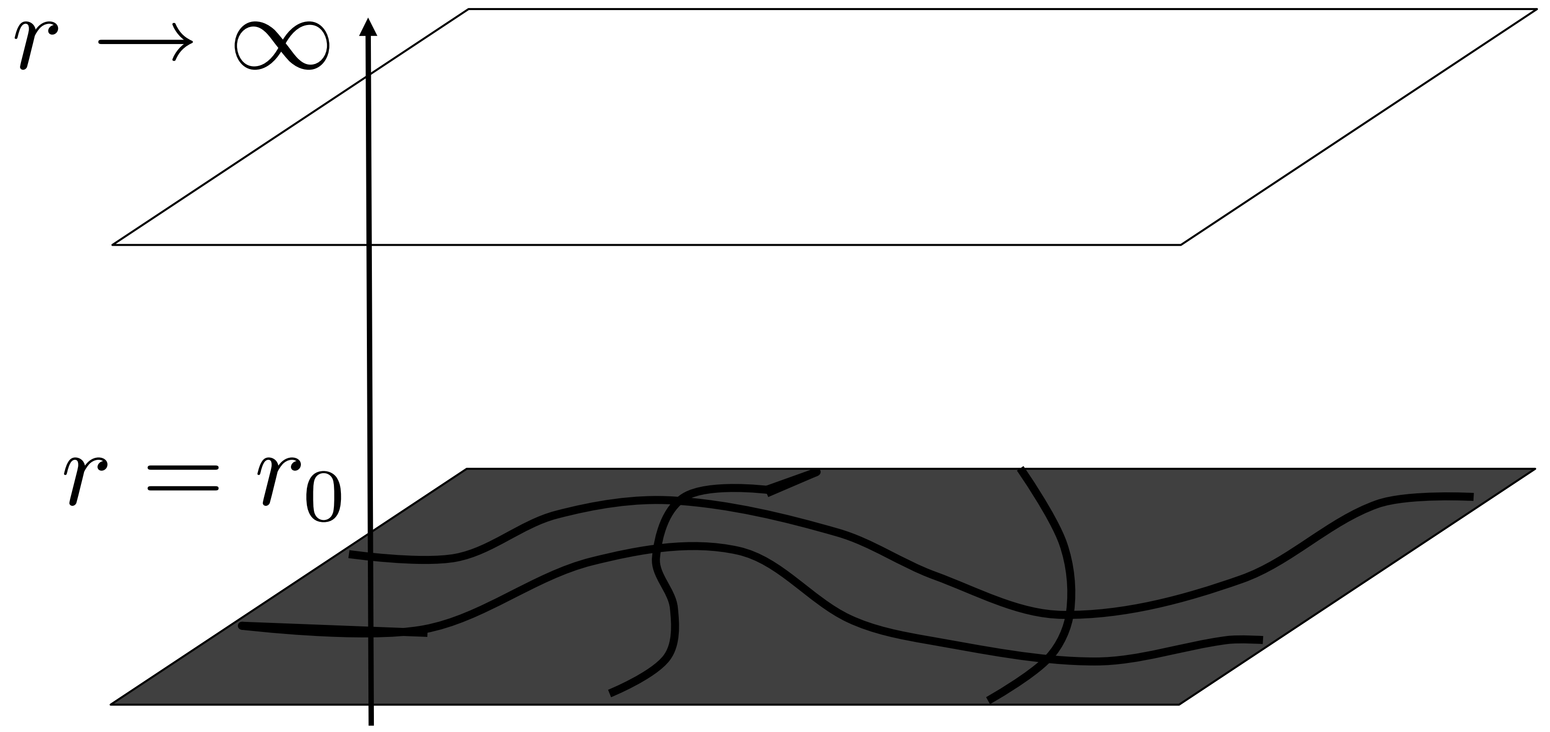
$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \ldots$$



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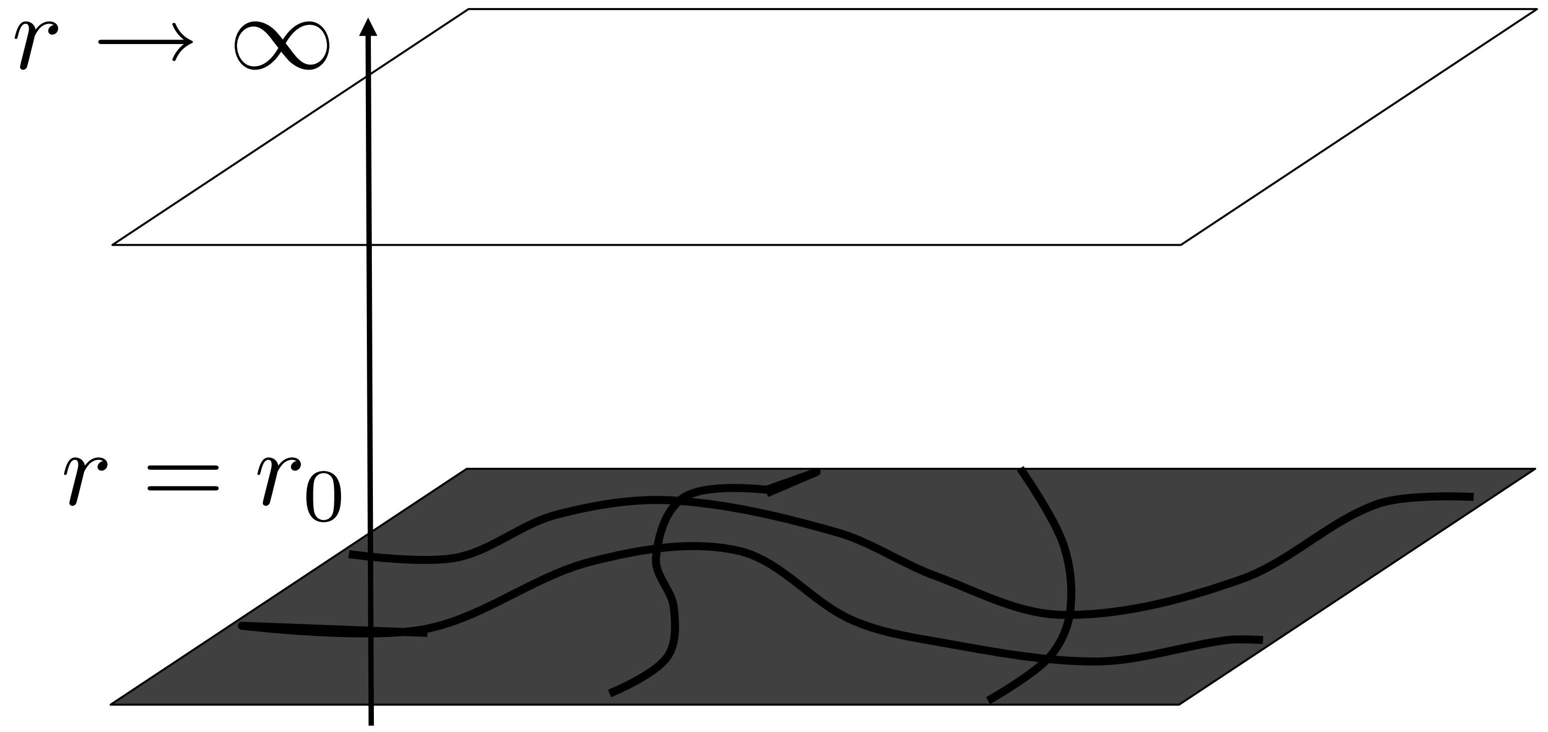

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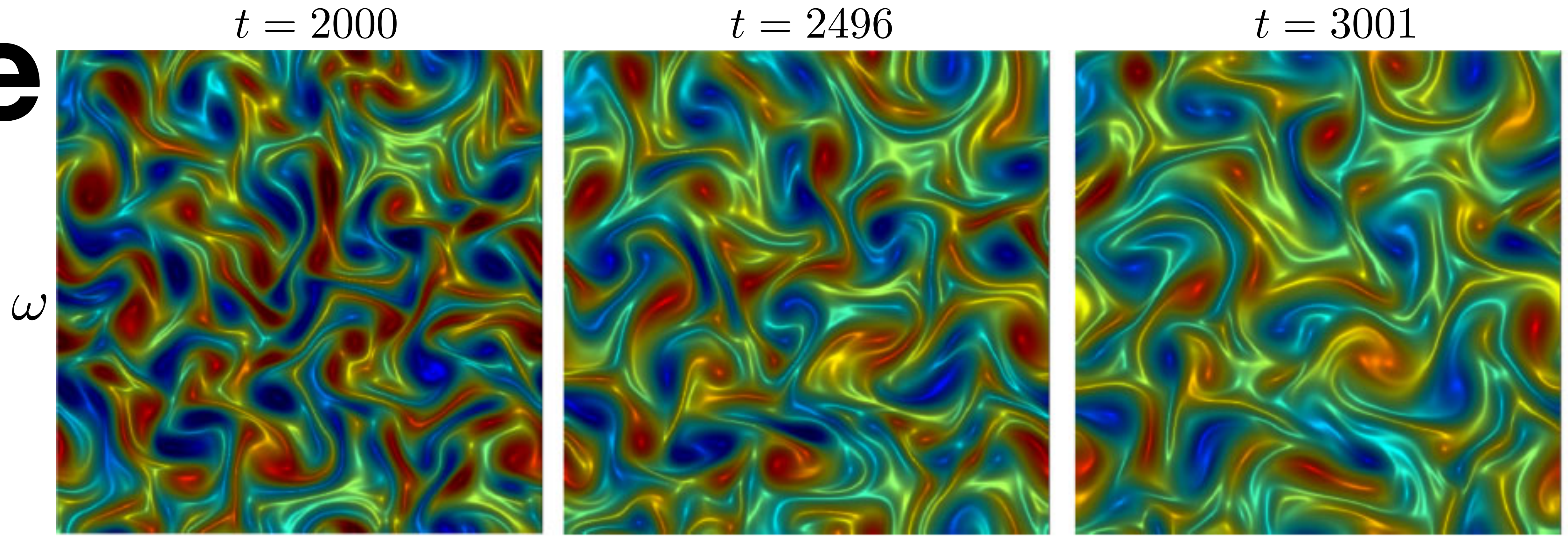

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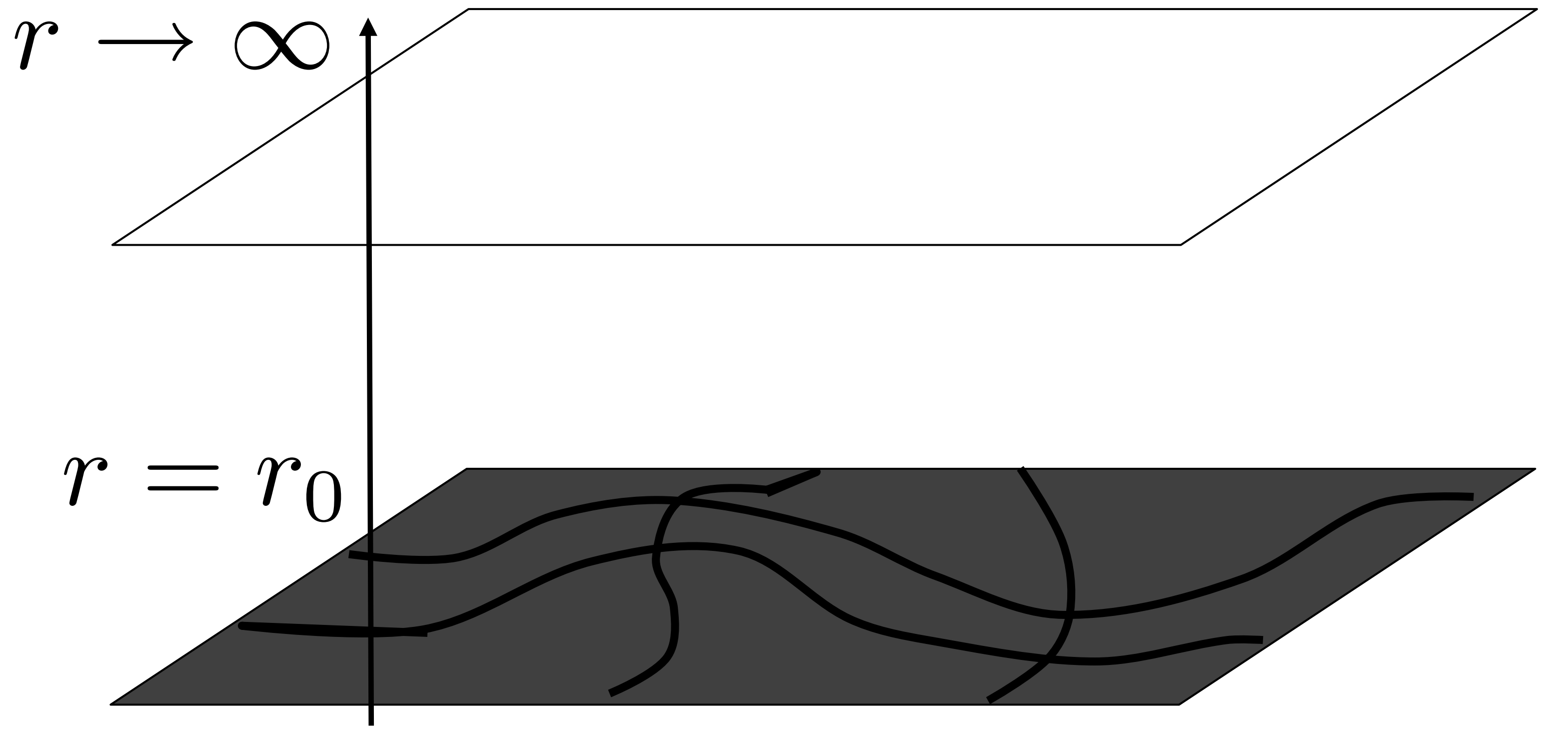

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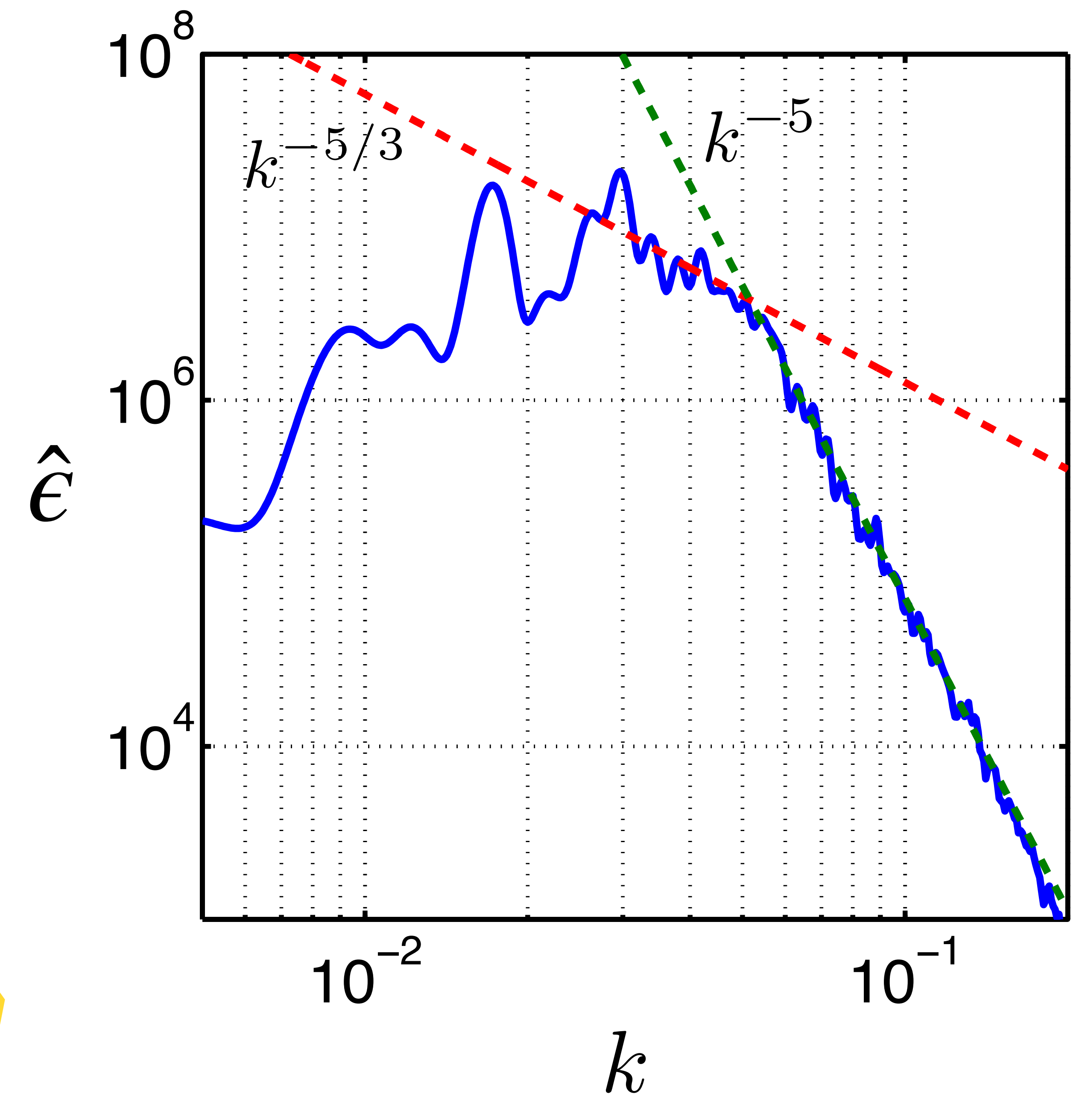

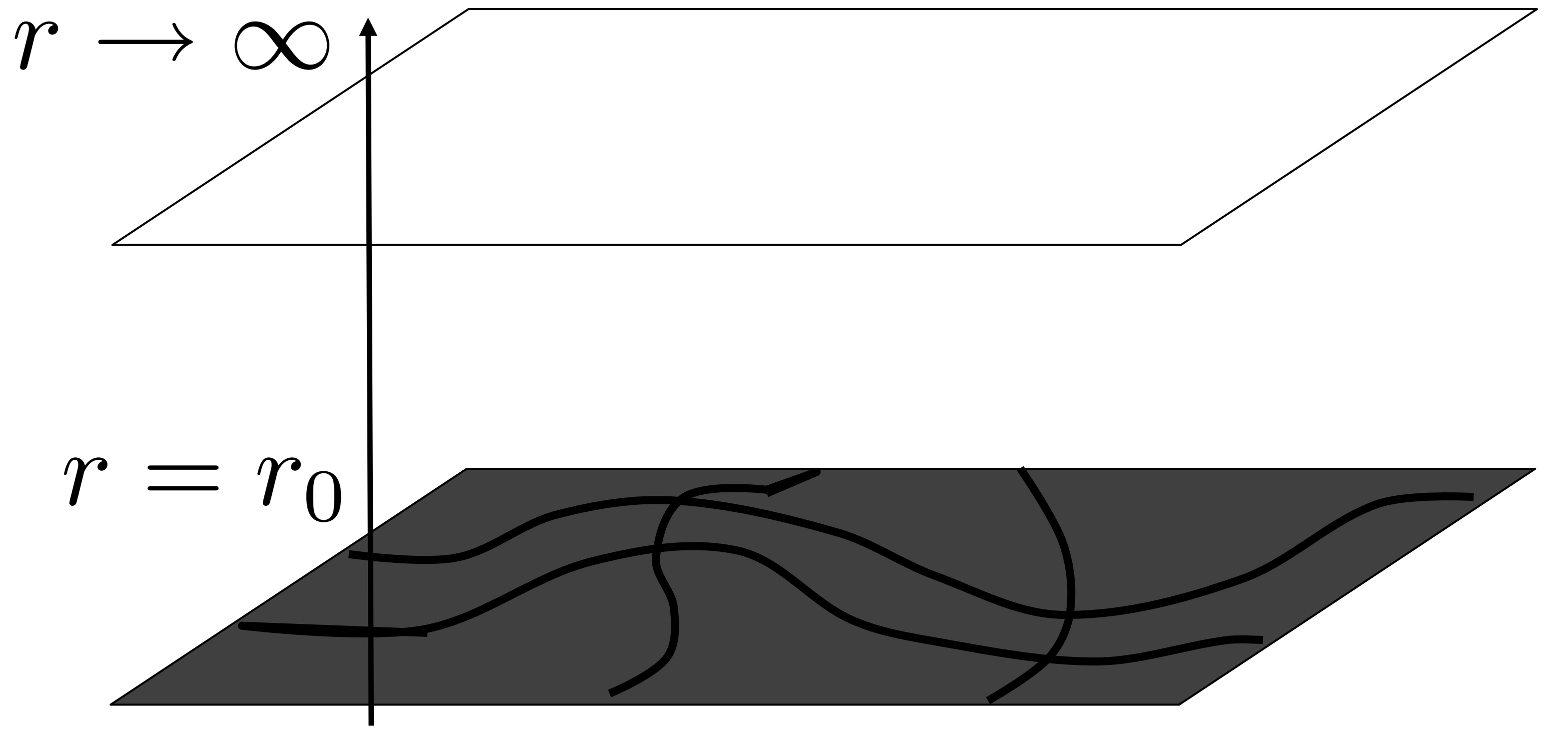

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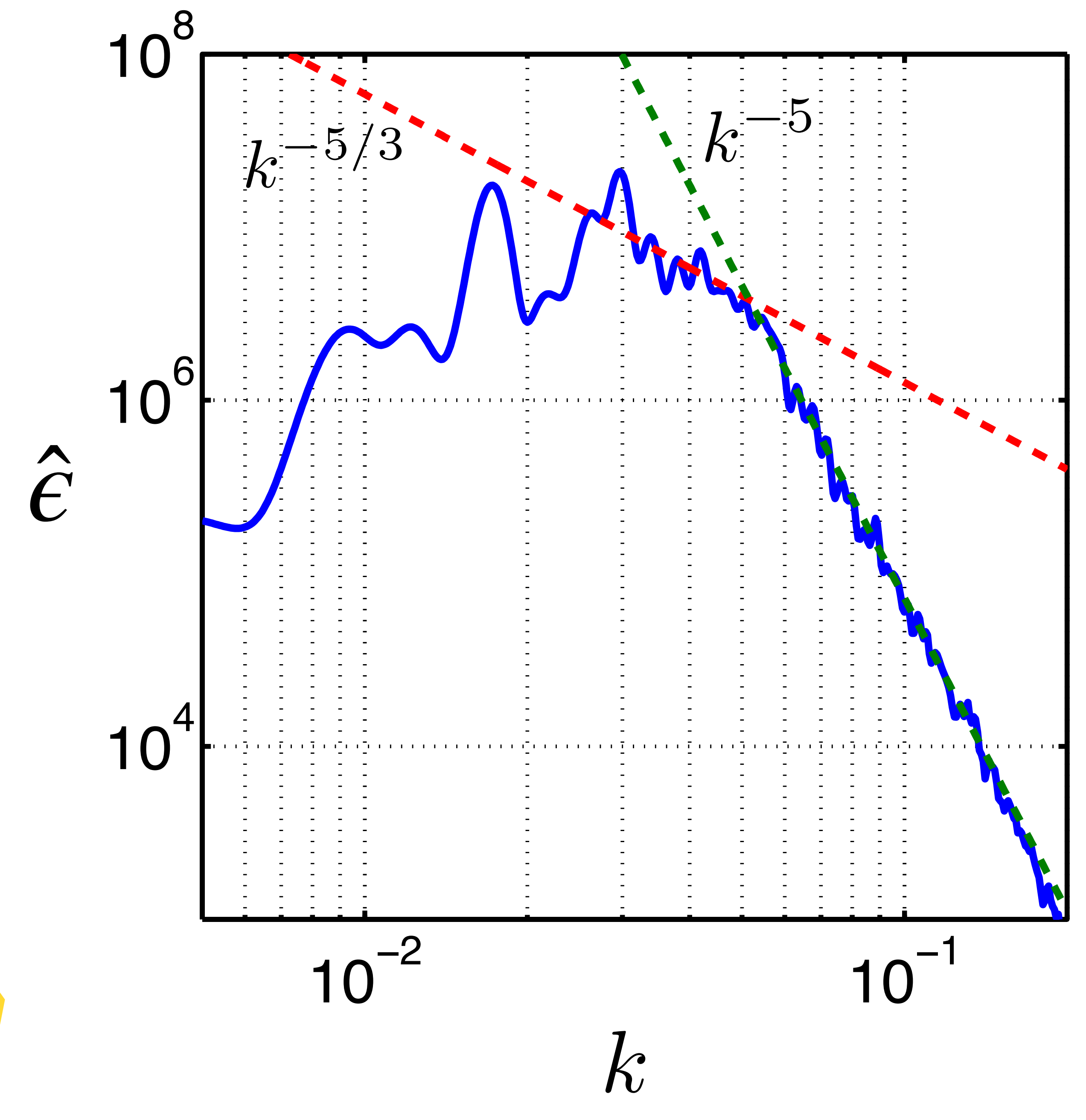
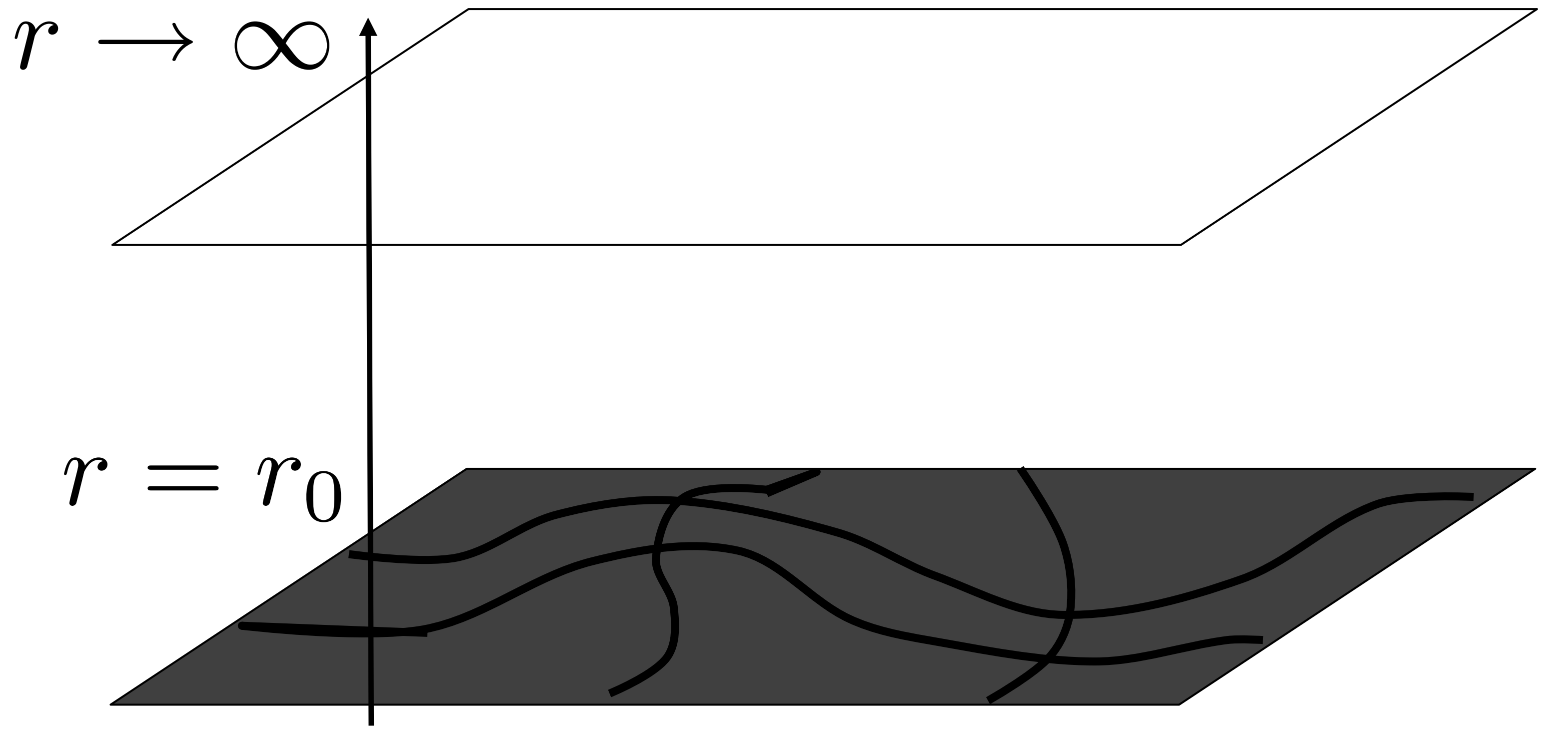
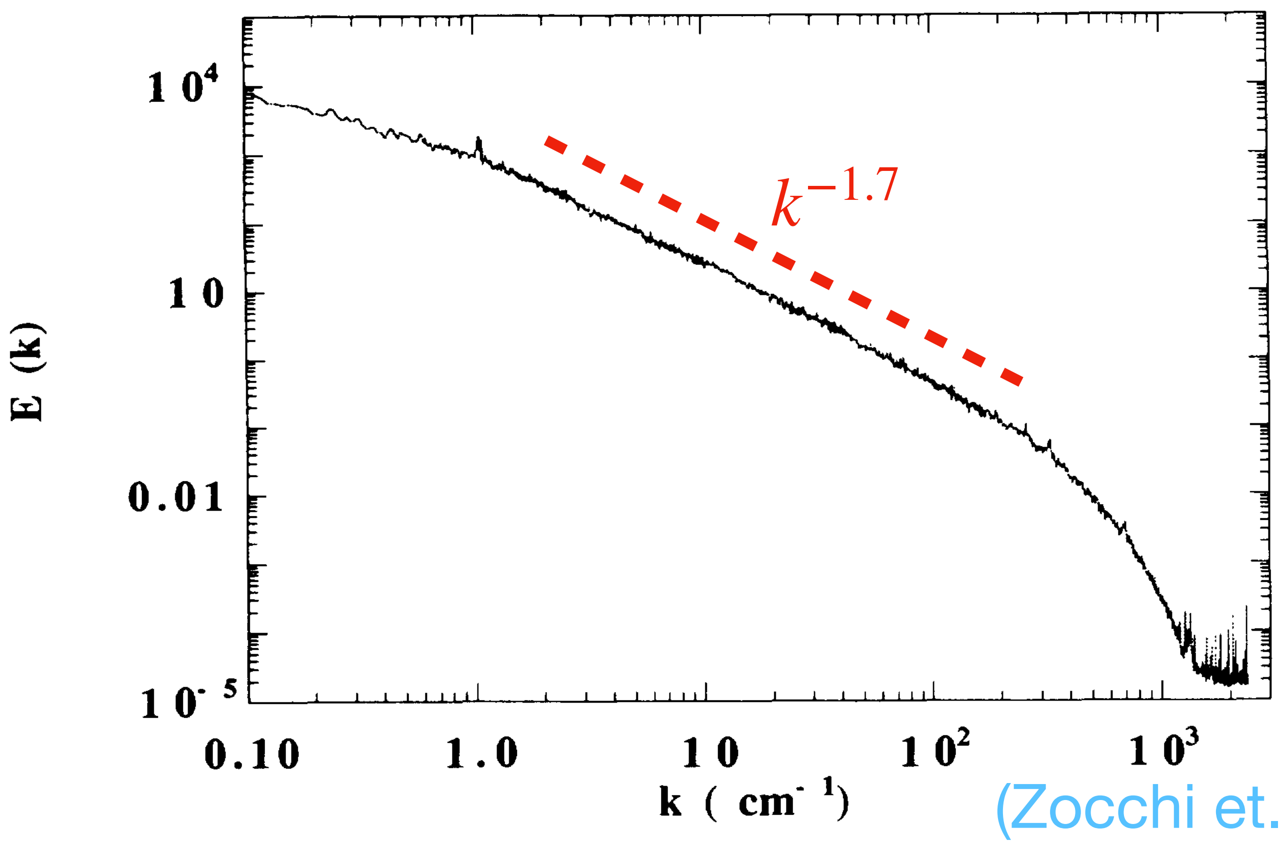




Recall:



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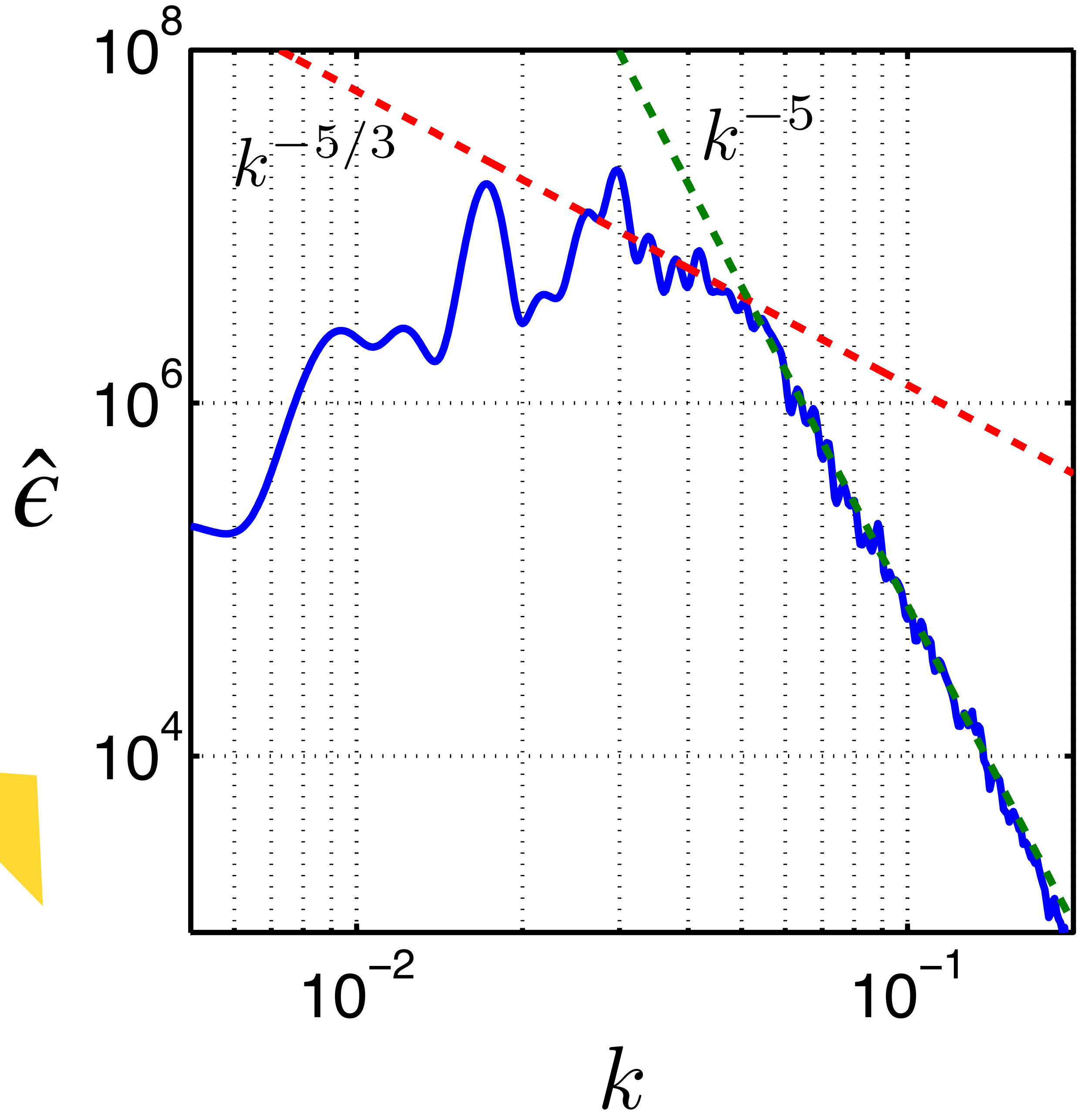
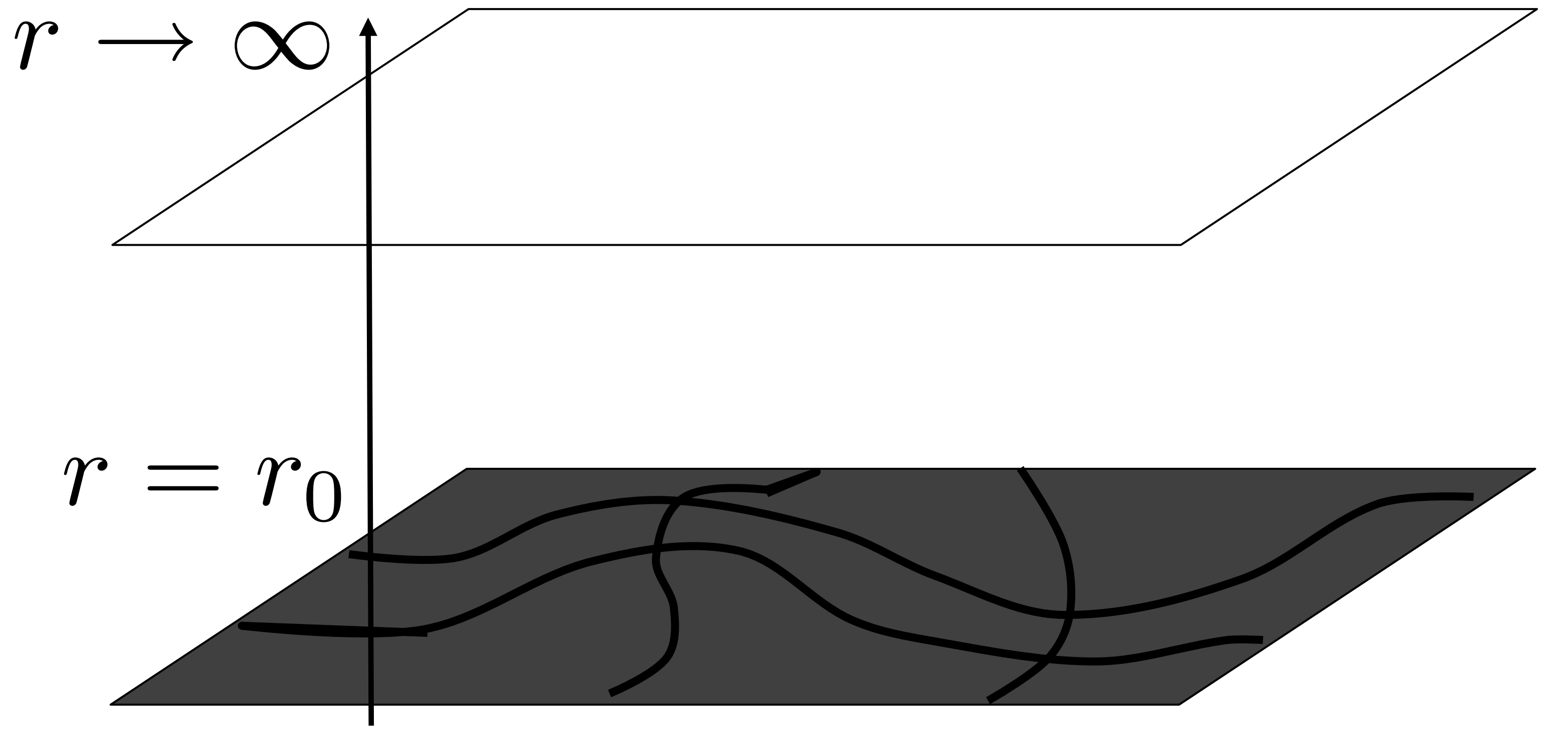
Holographic turbulence



$$\frac{|\vec{v}|}{c} \ll 1$$



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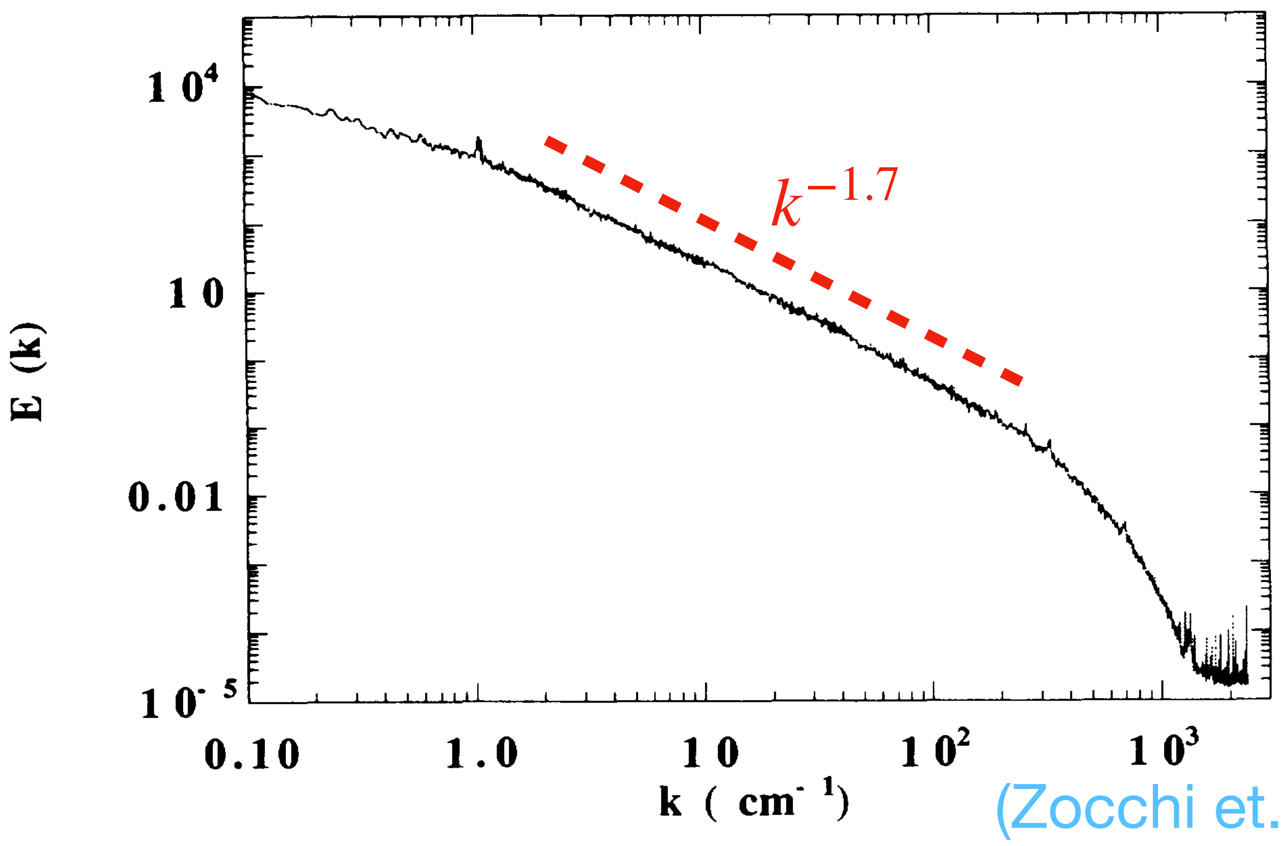

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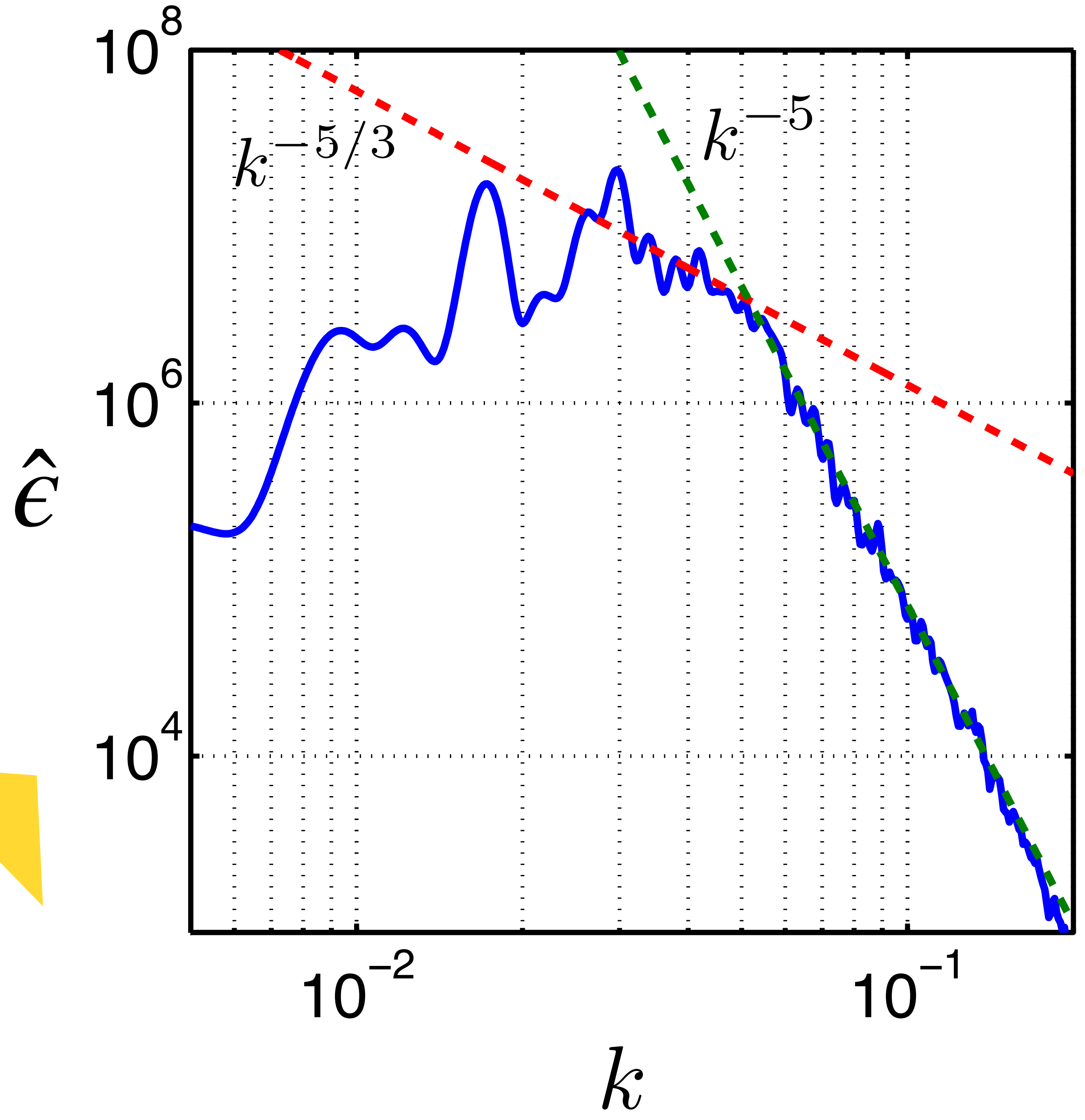

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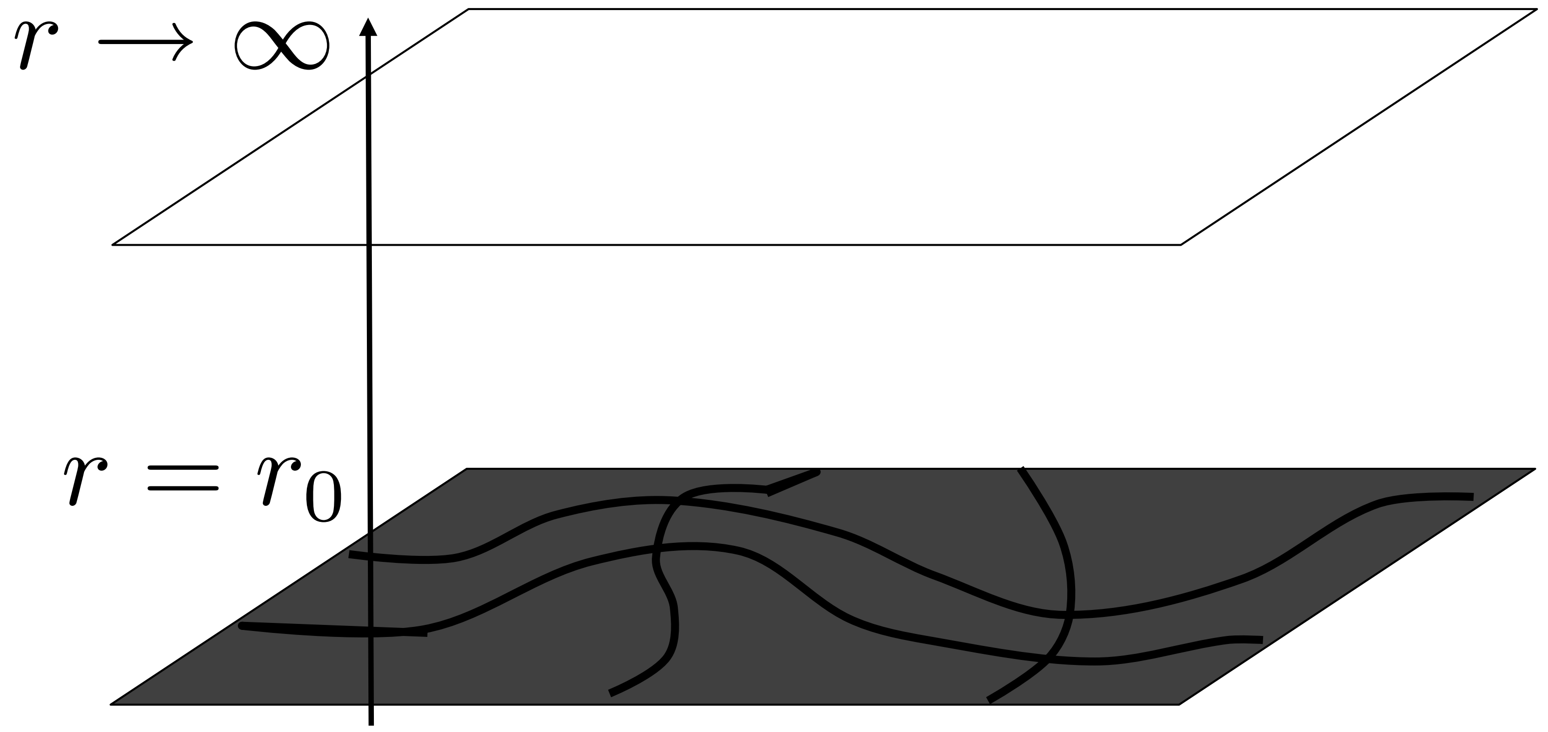

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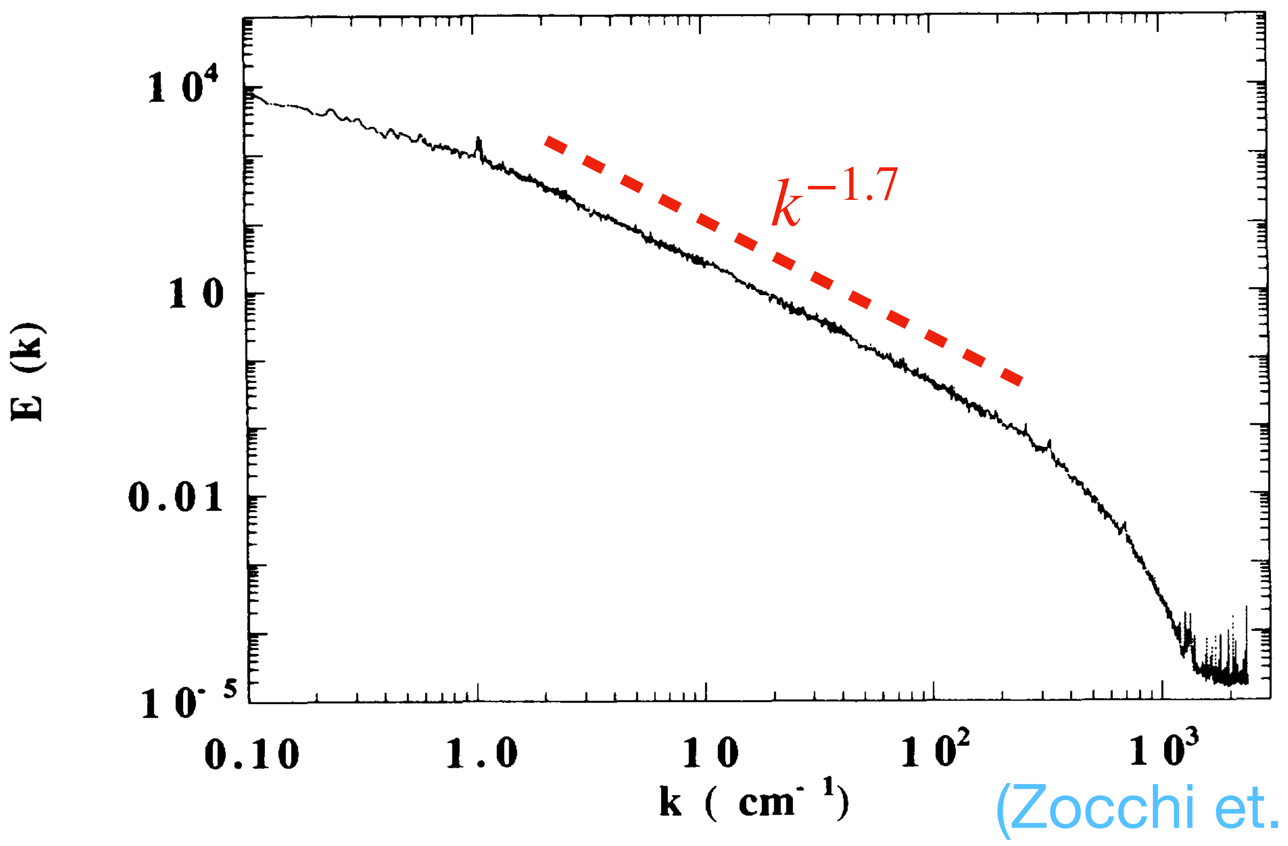

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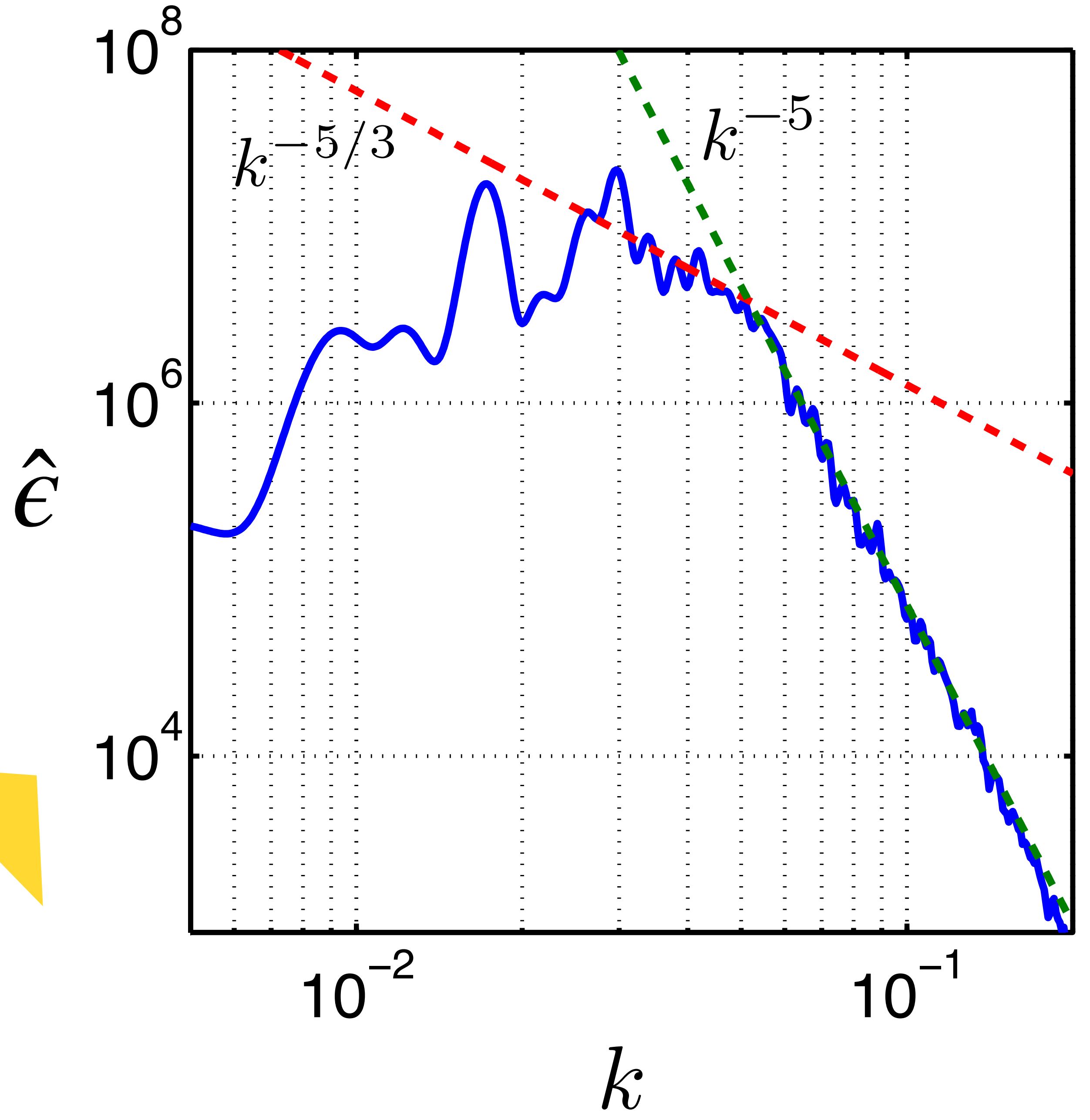

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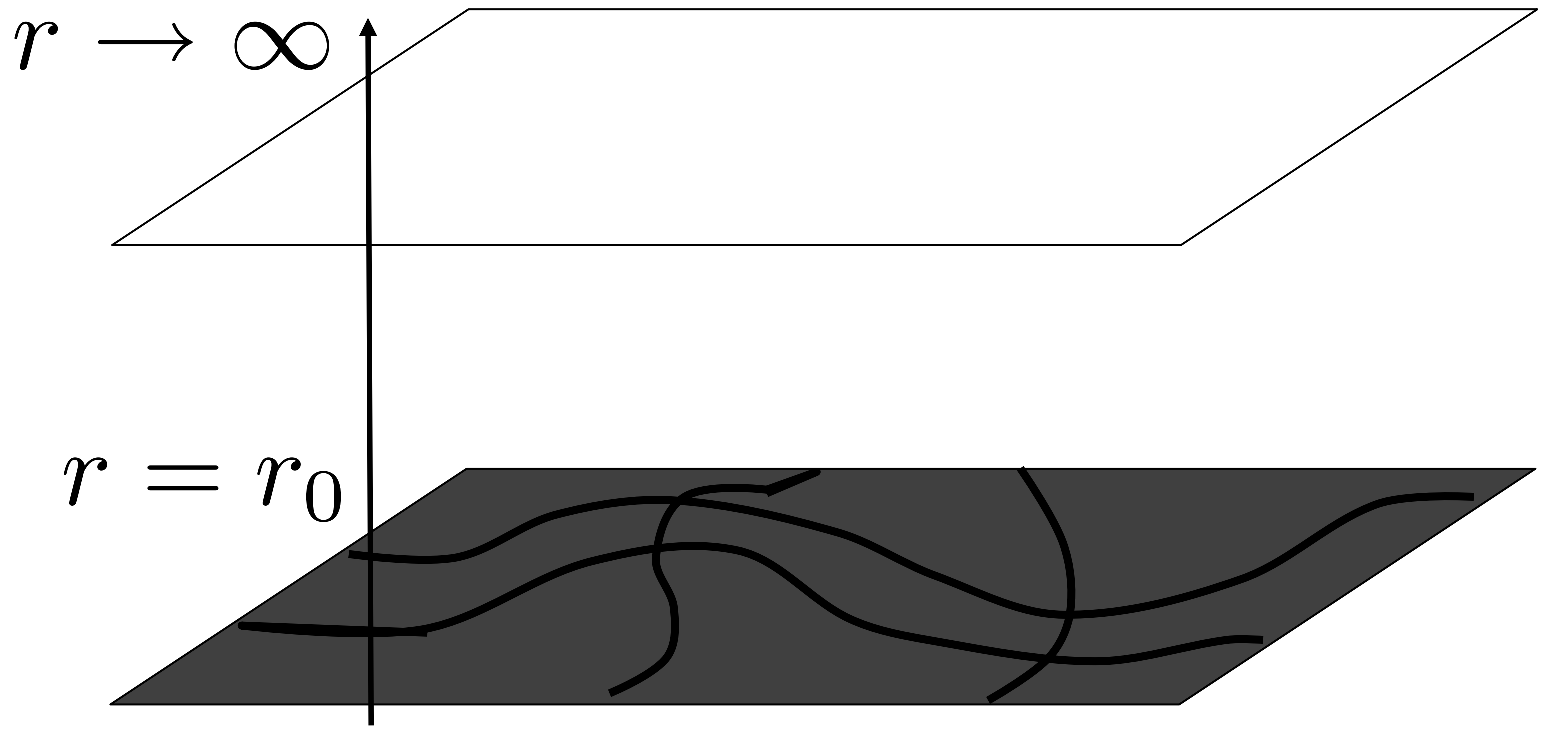

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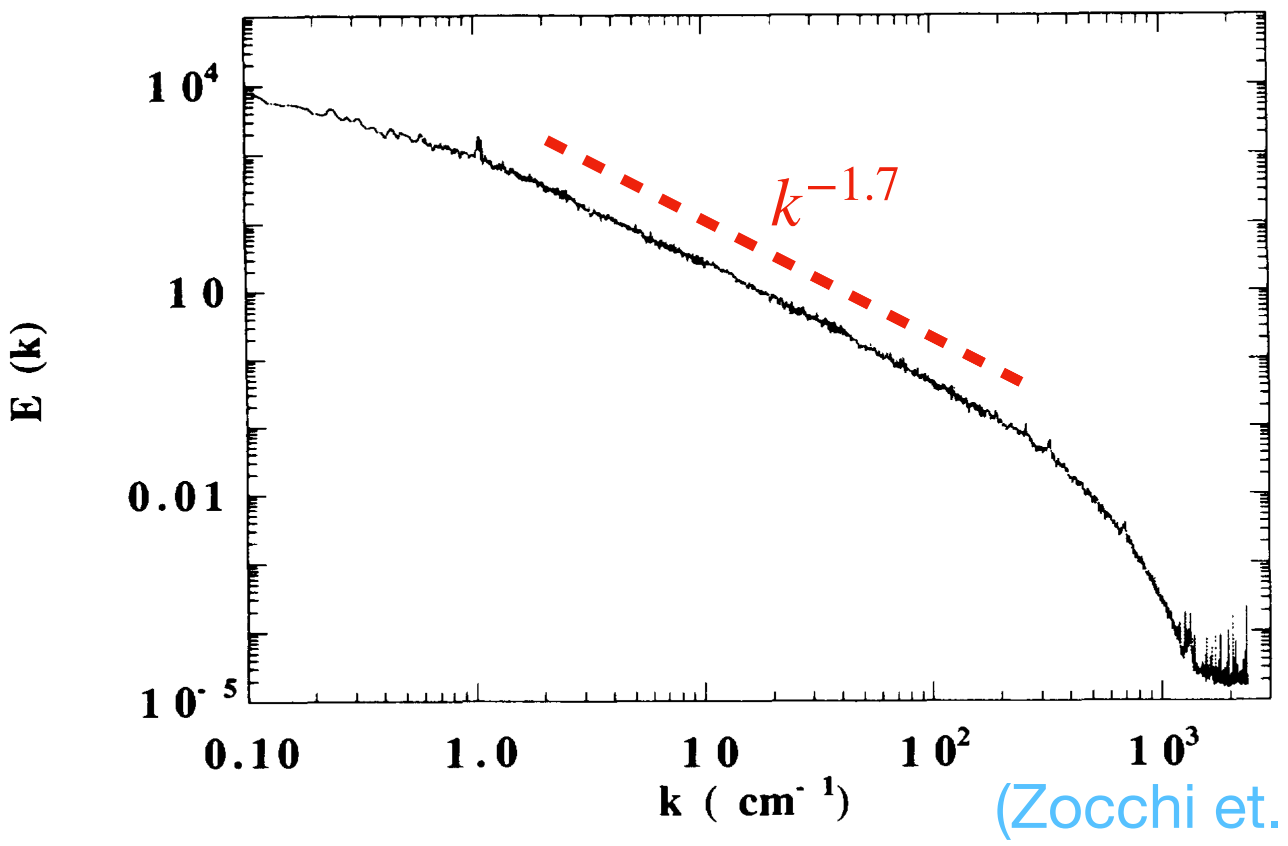

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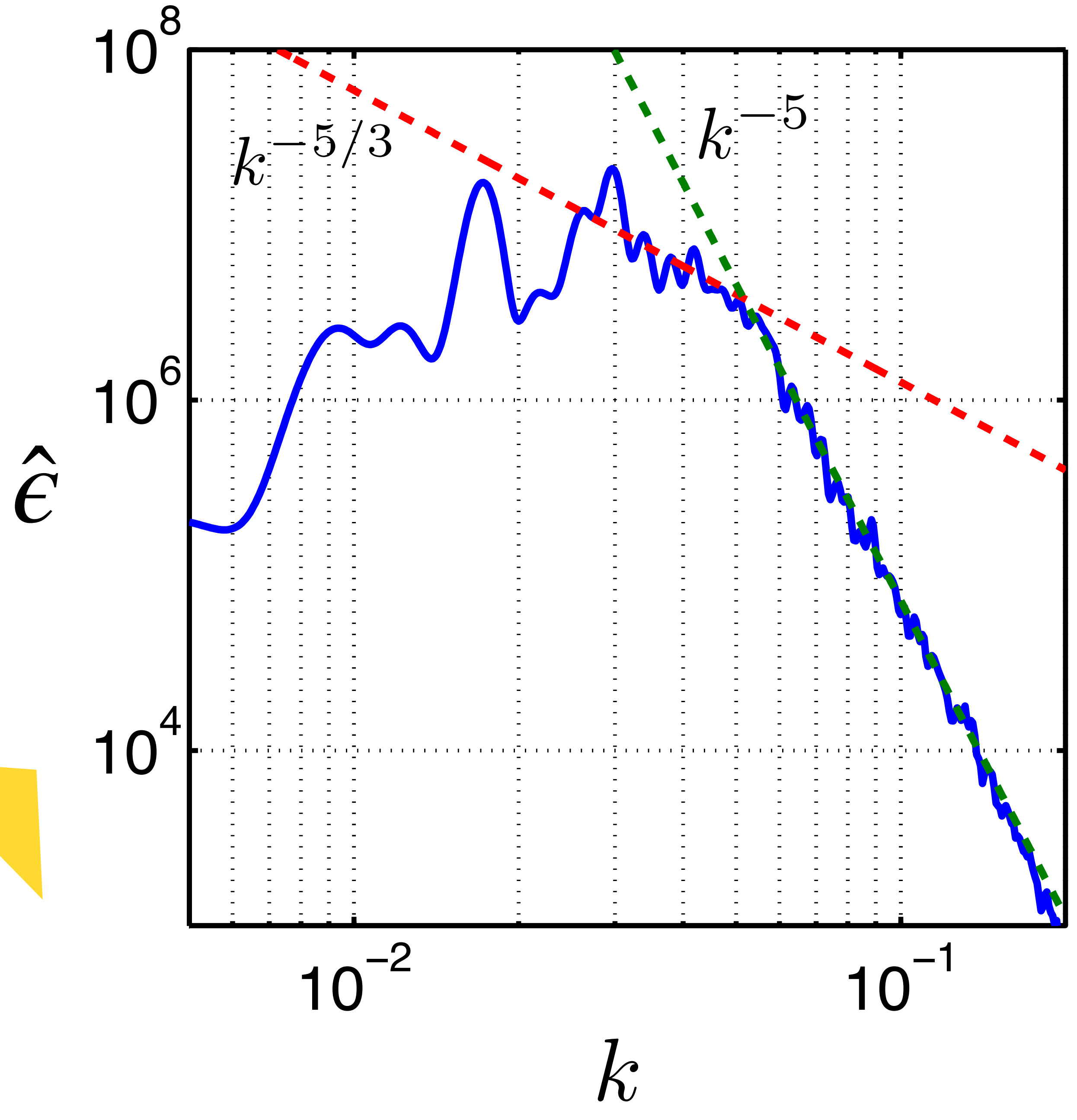

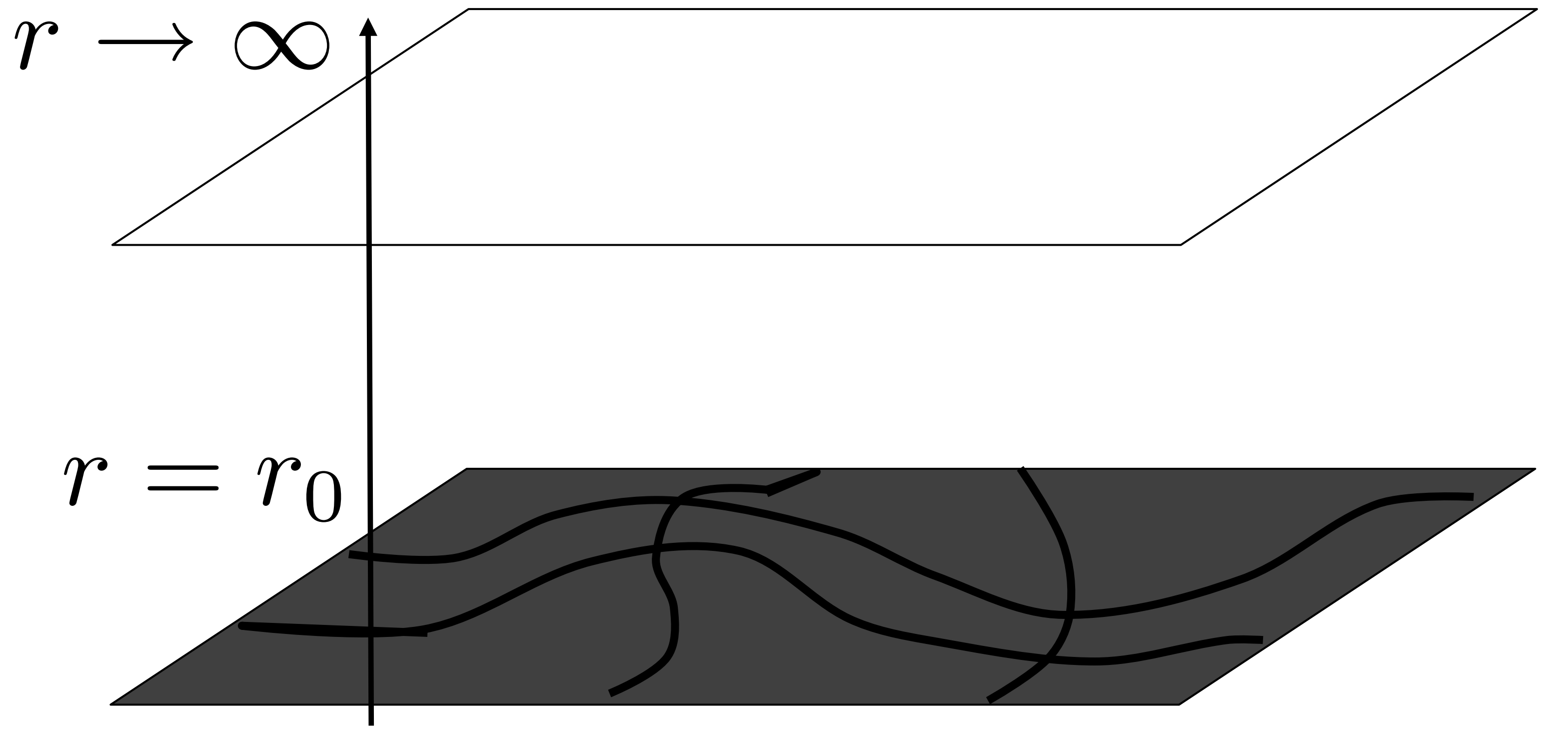

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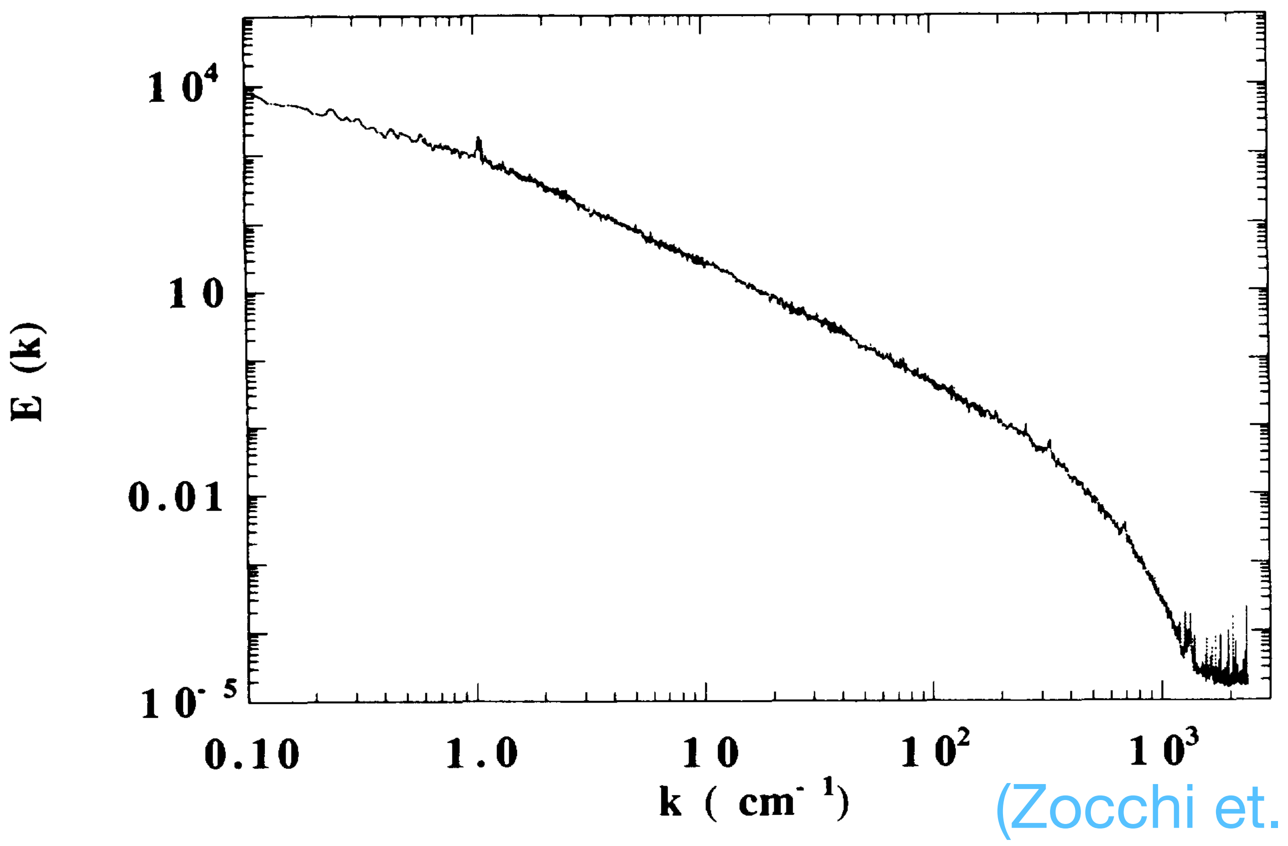

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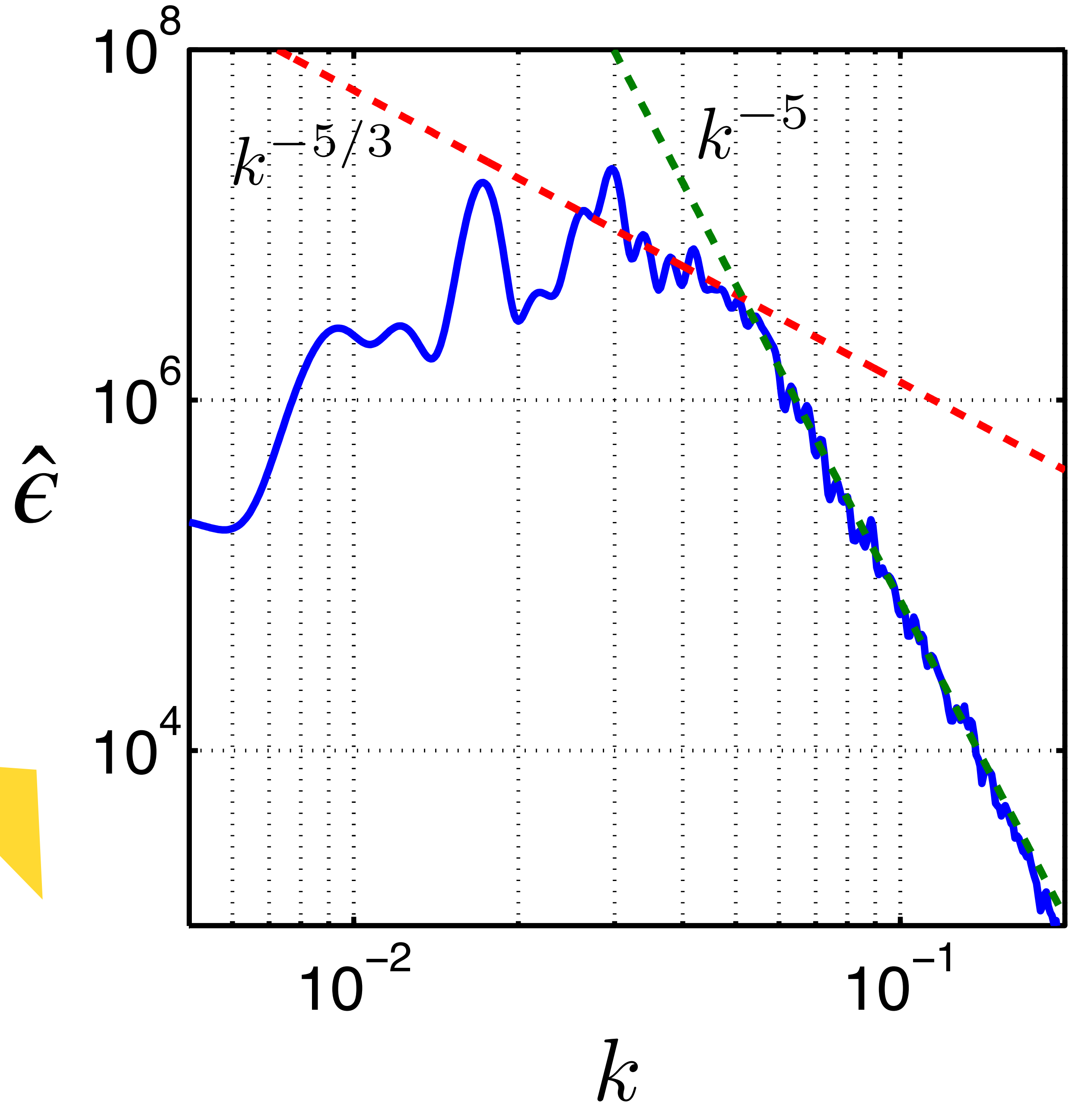

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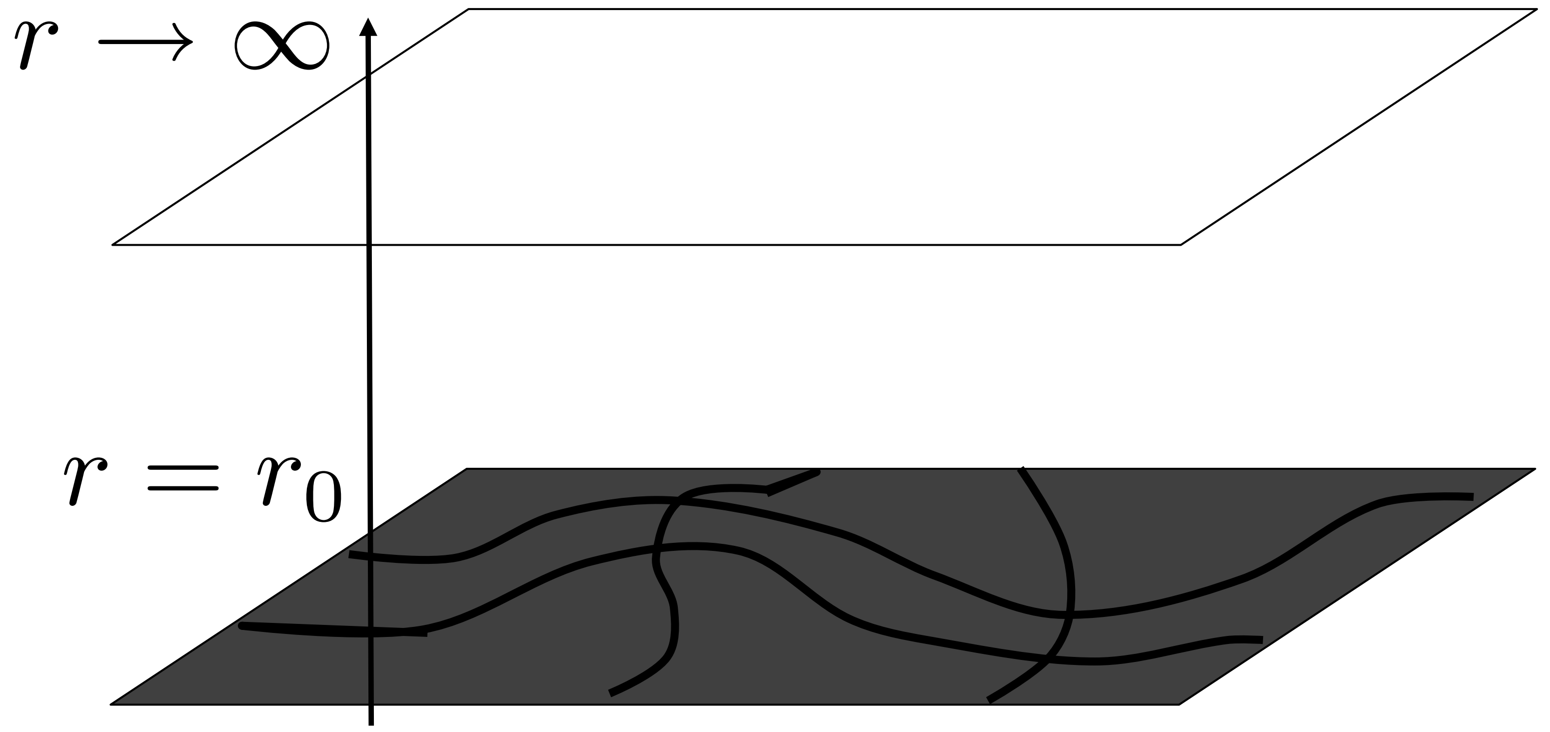

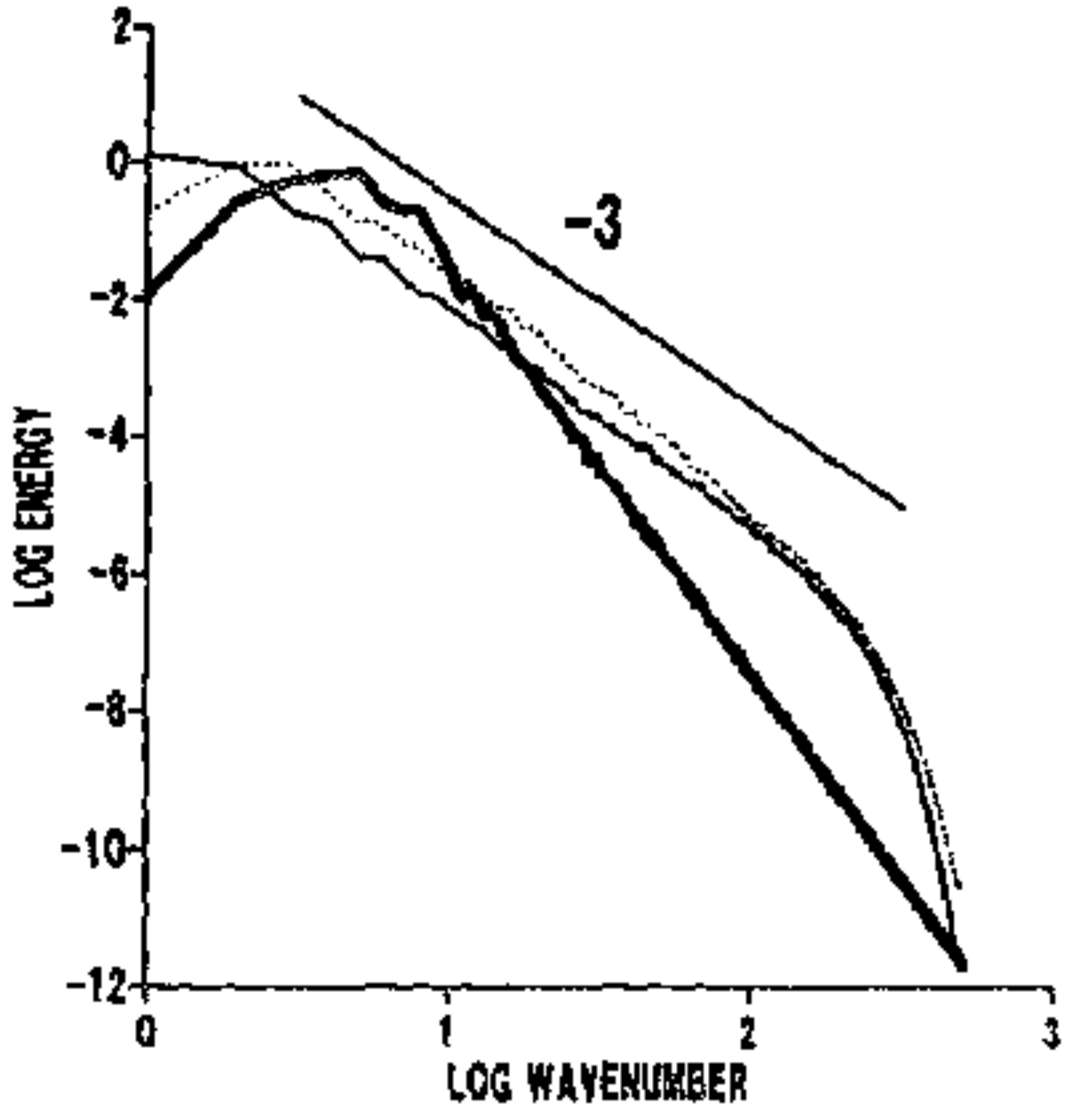

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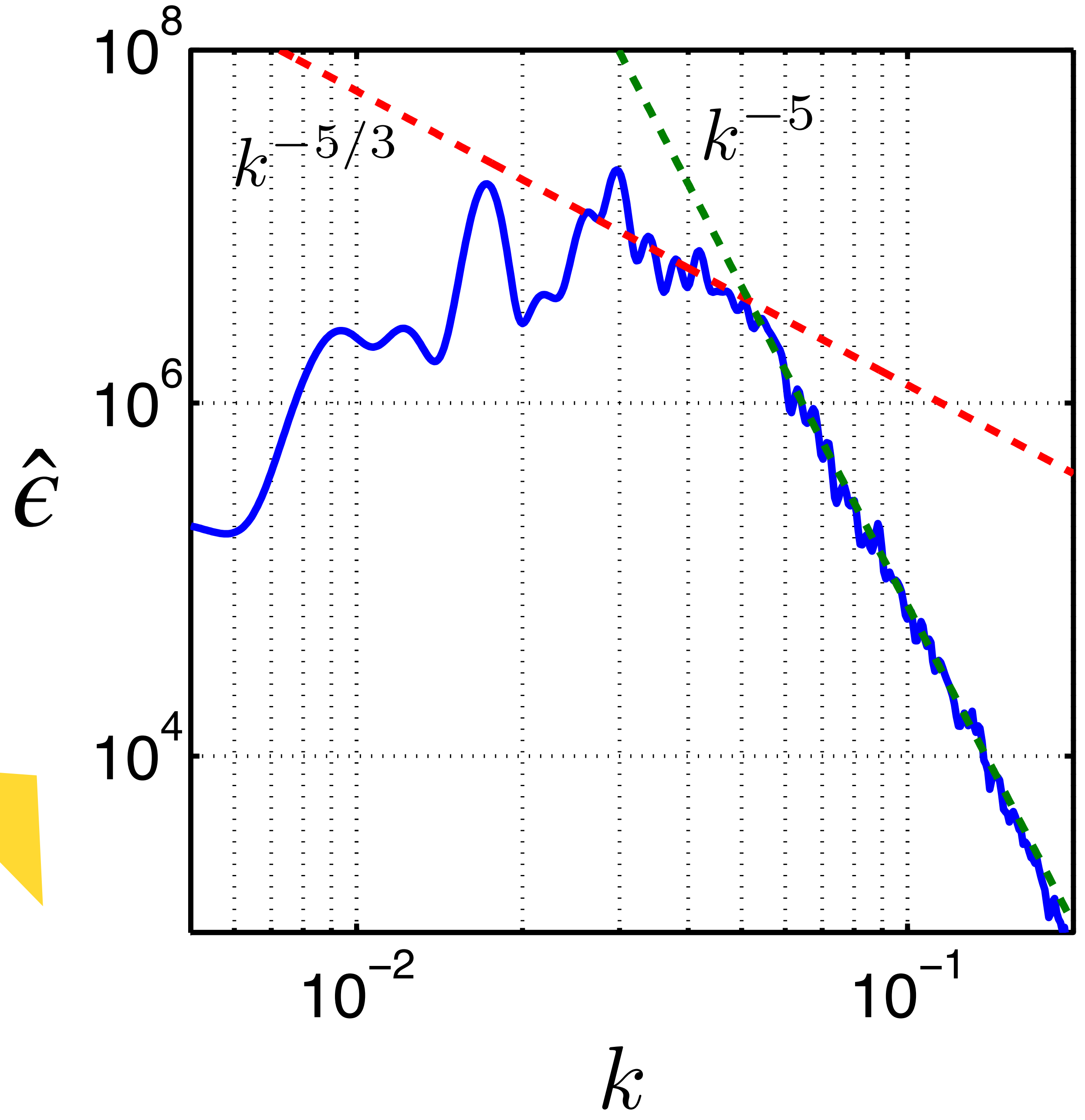

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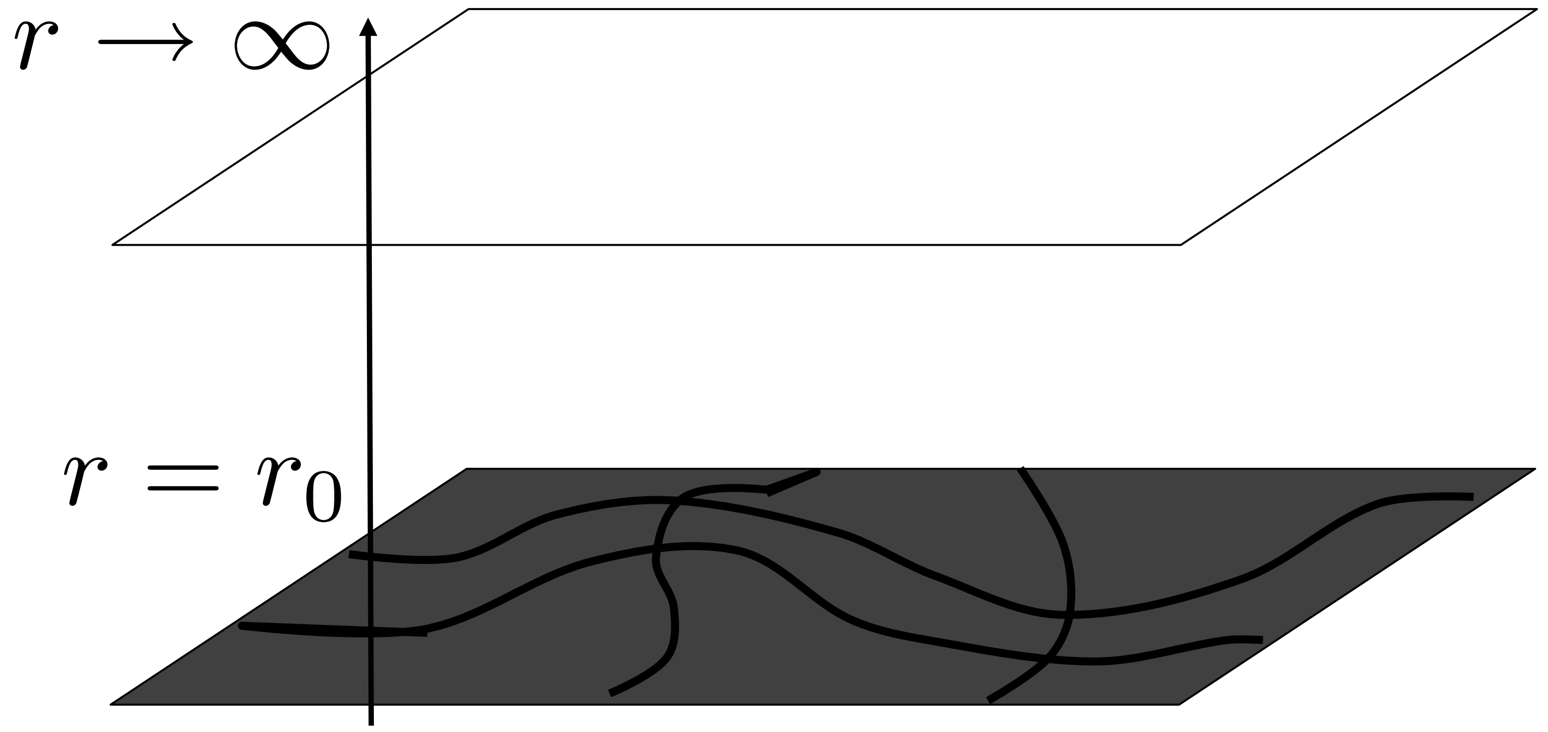

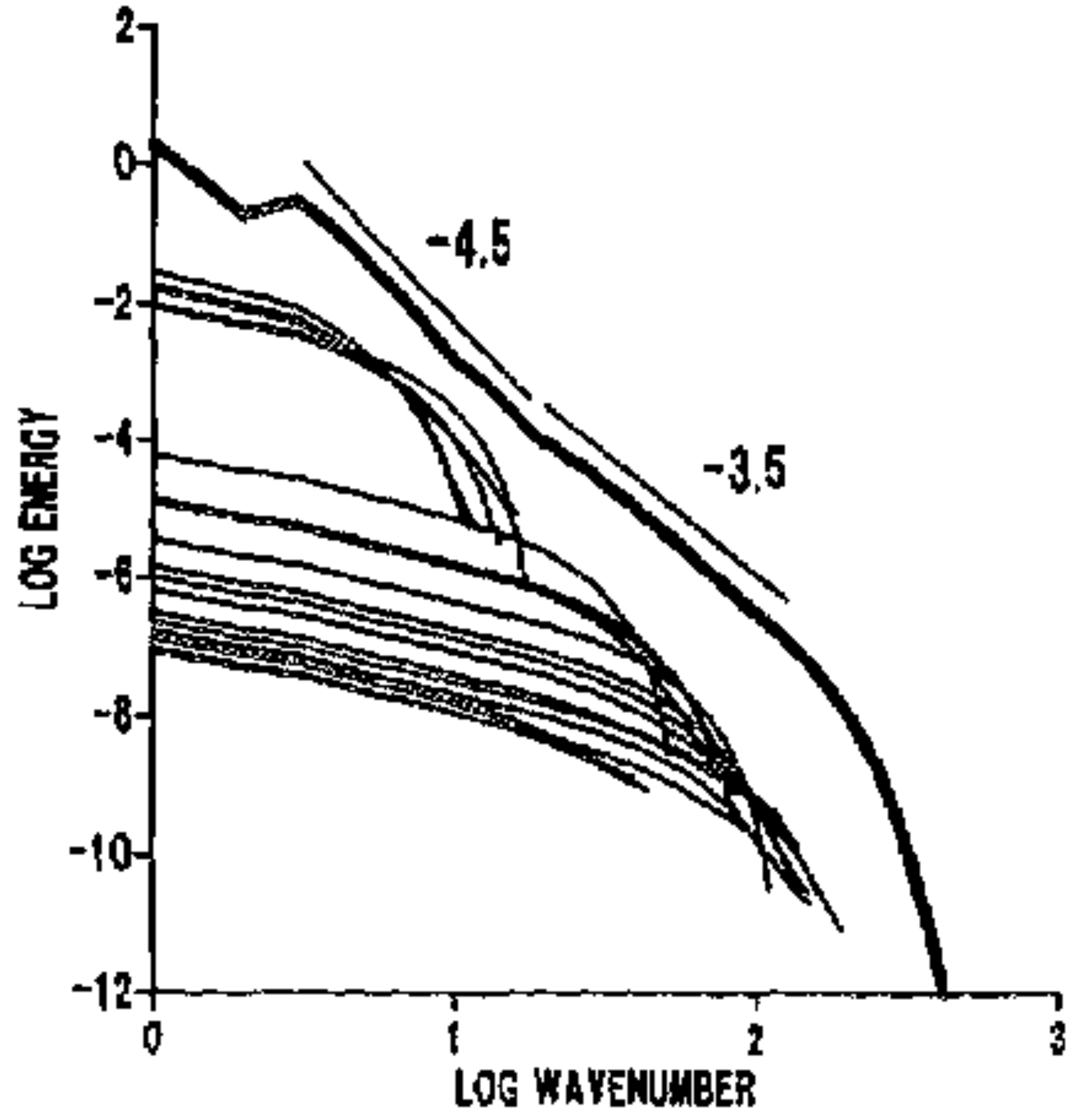

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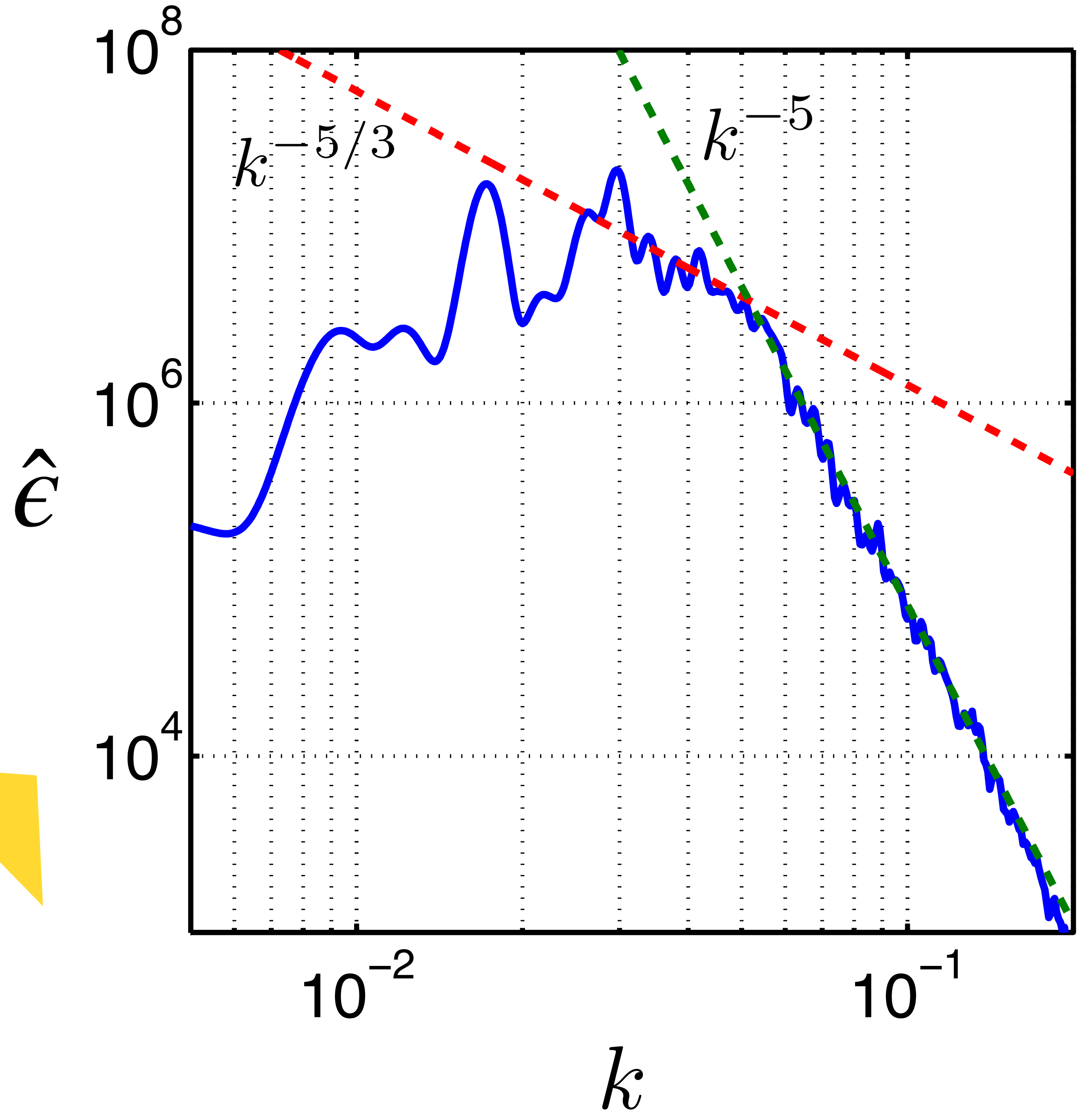

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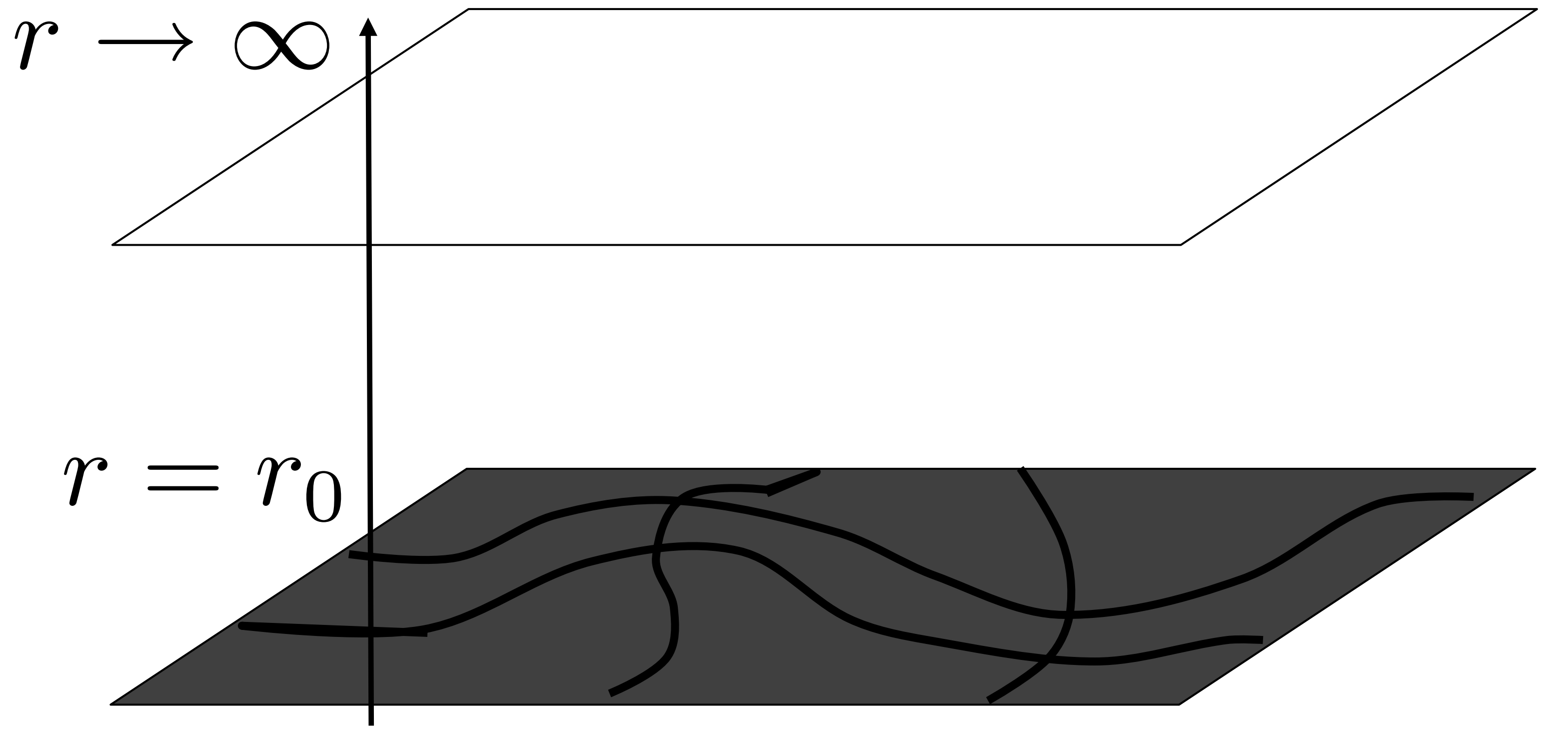

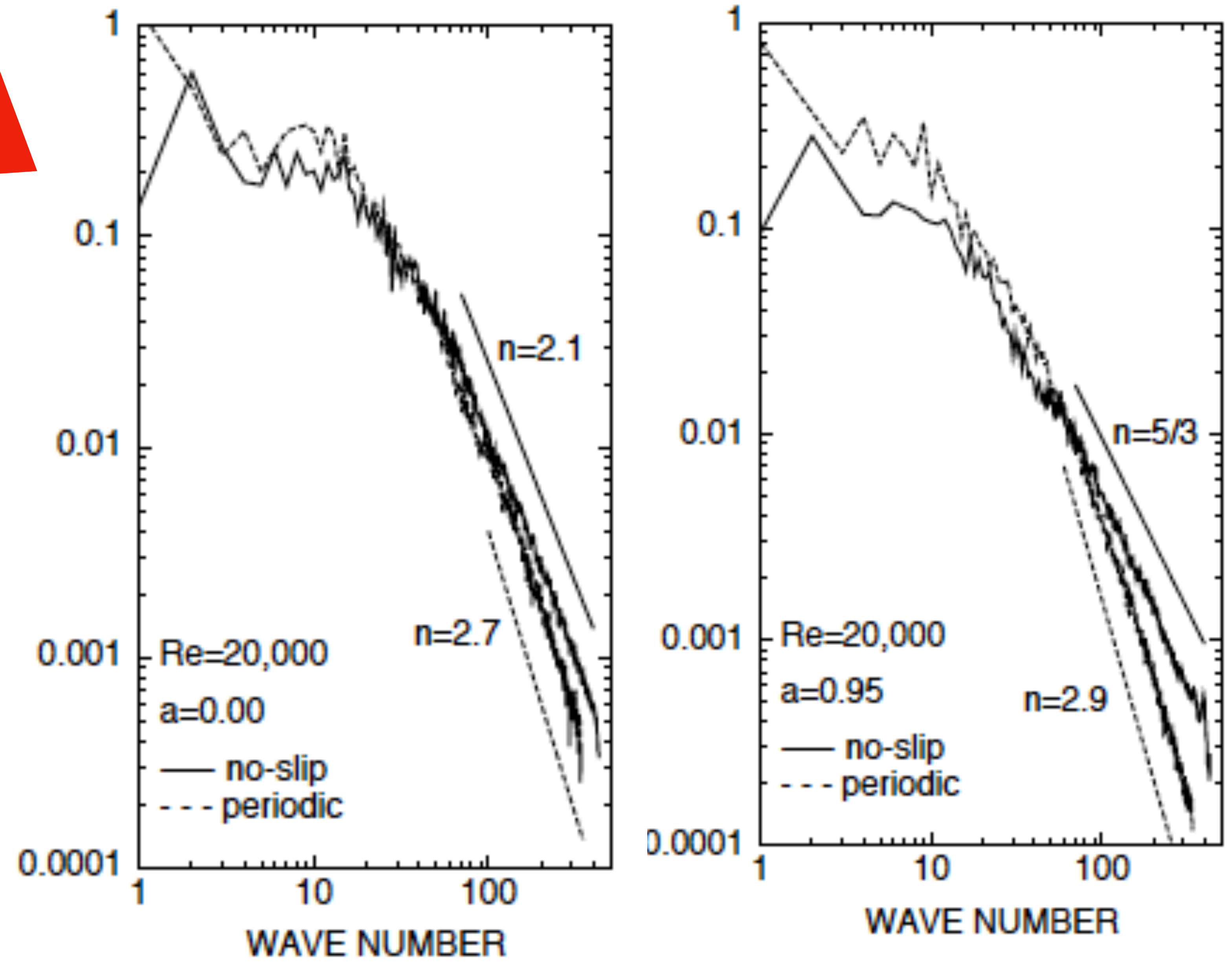

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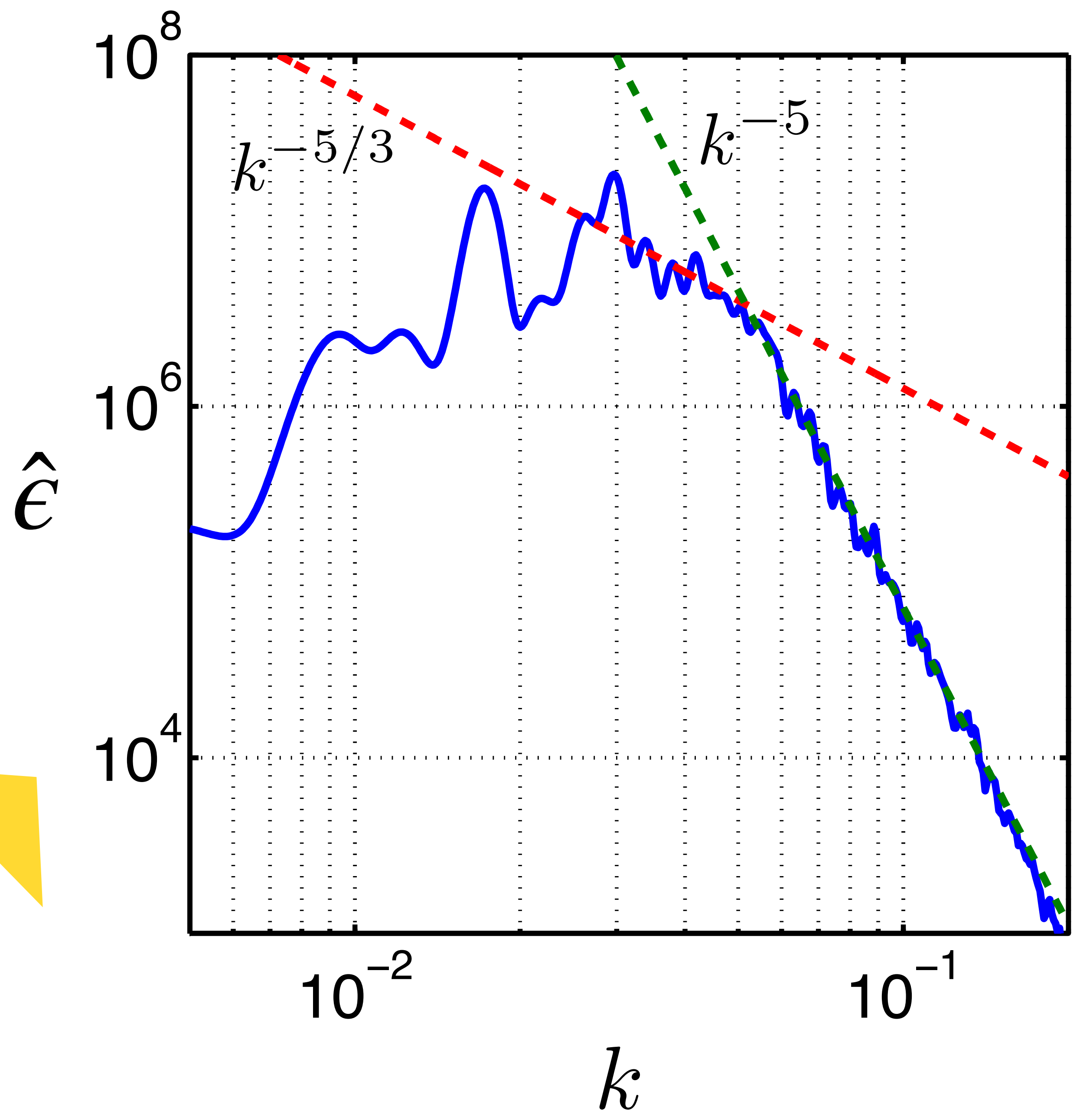

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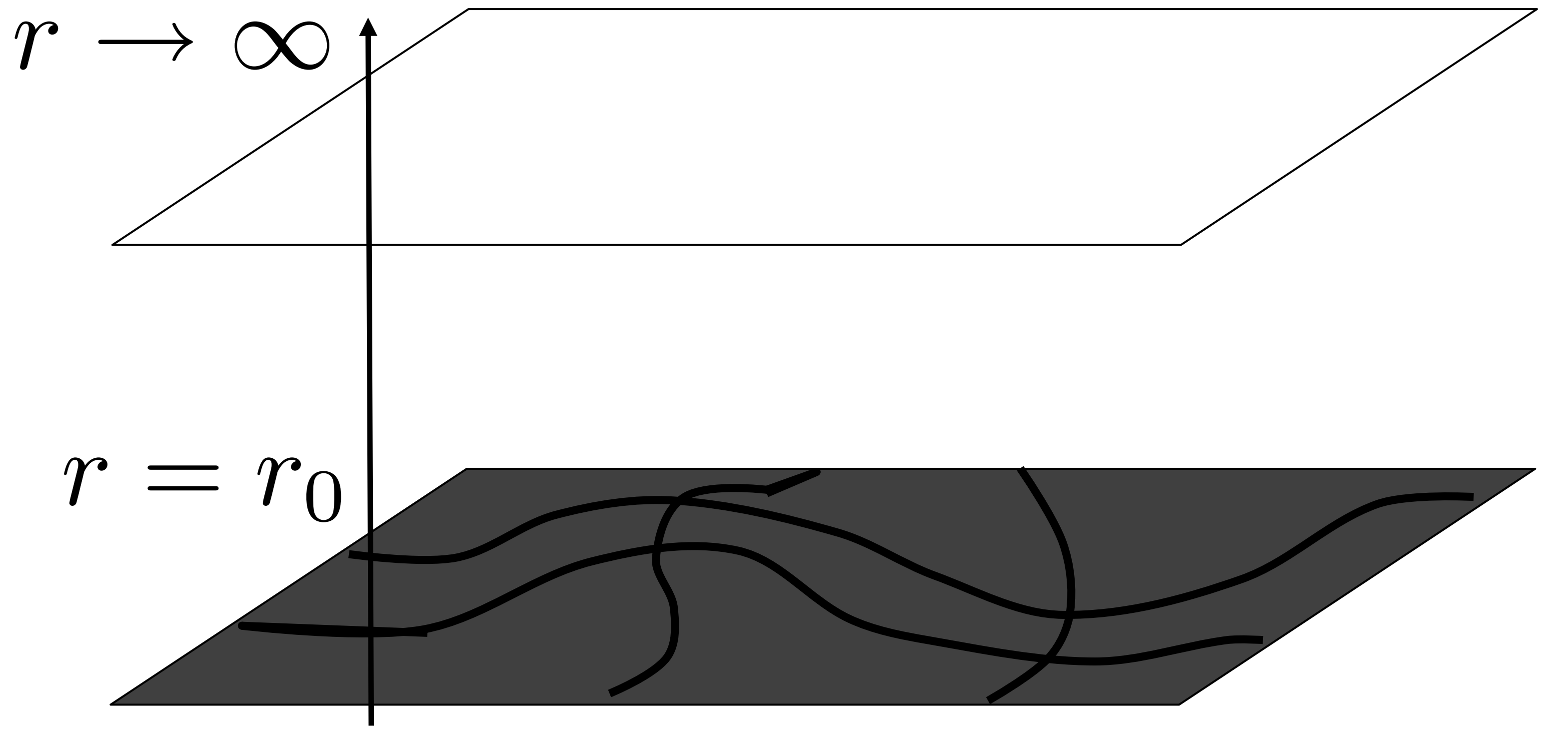
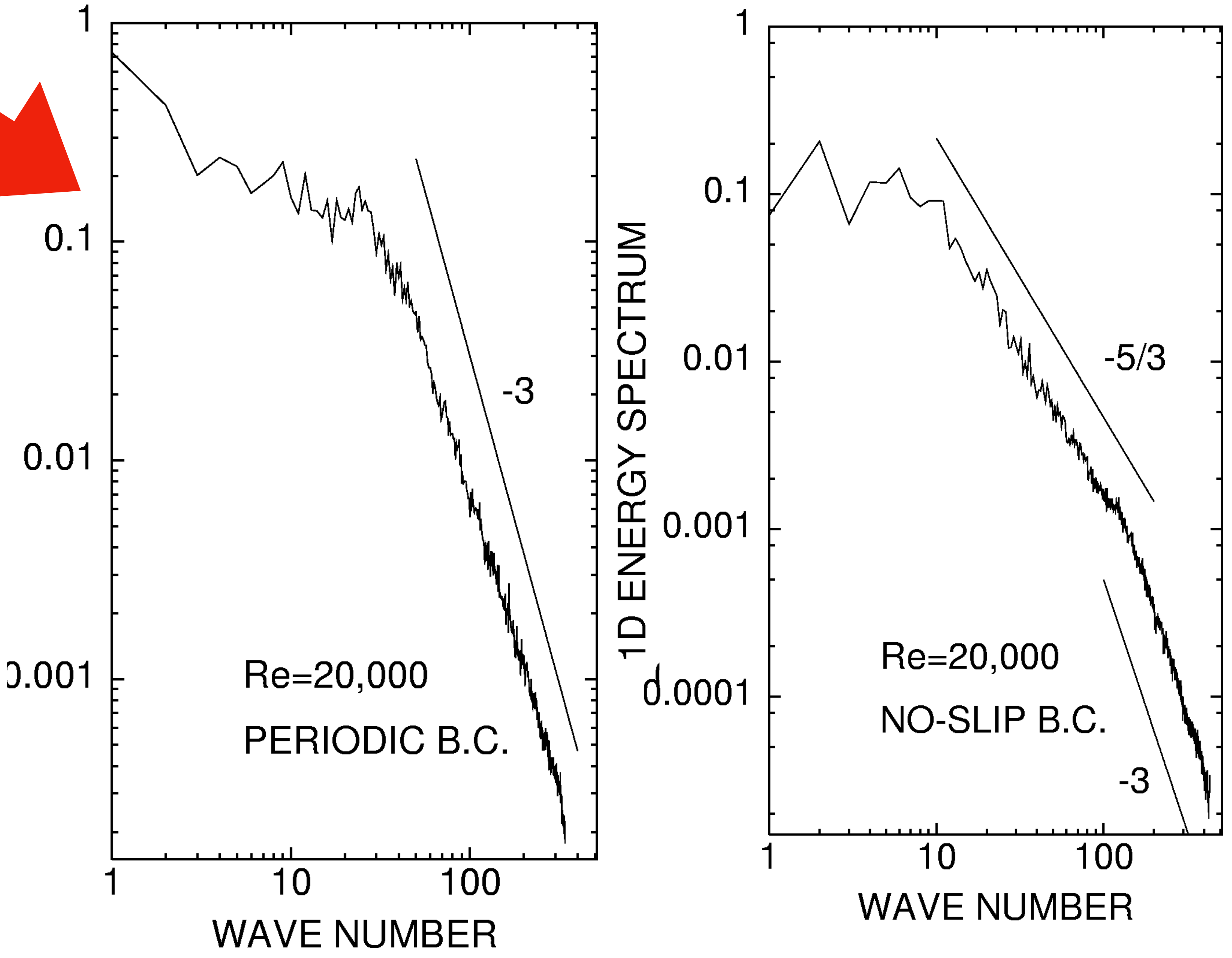
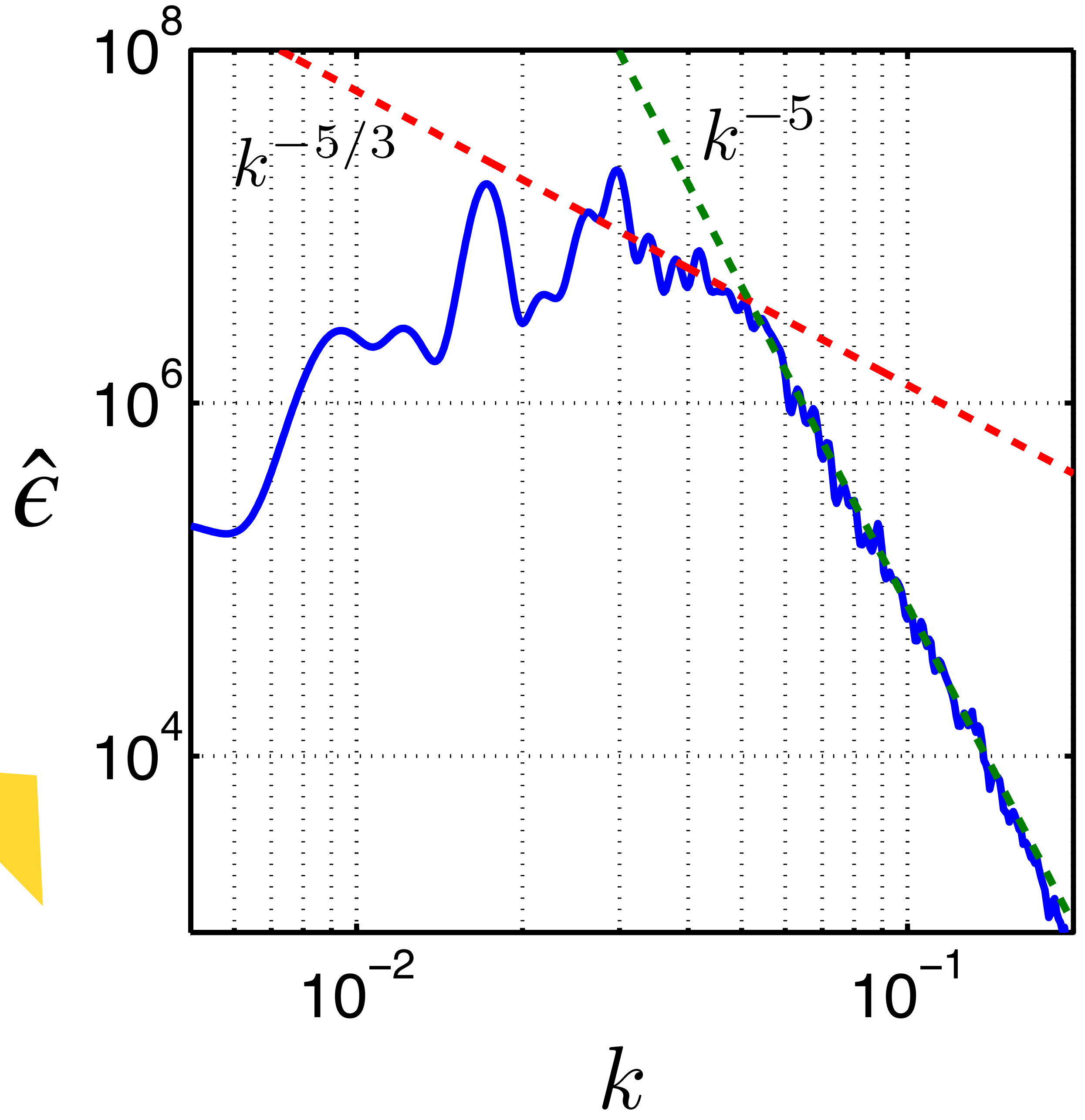


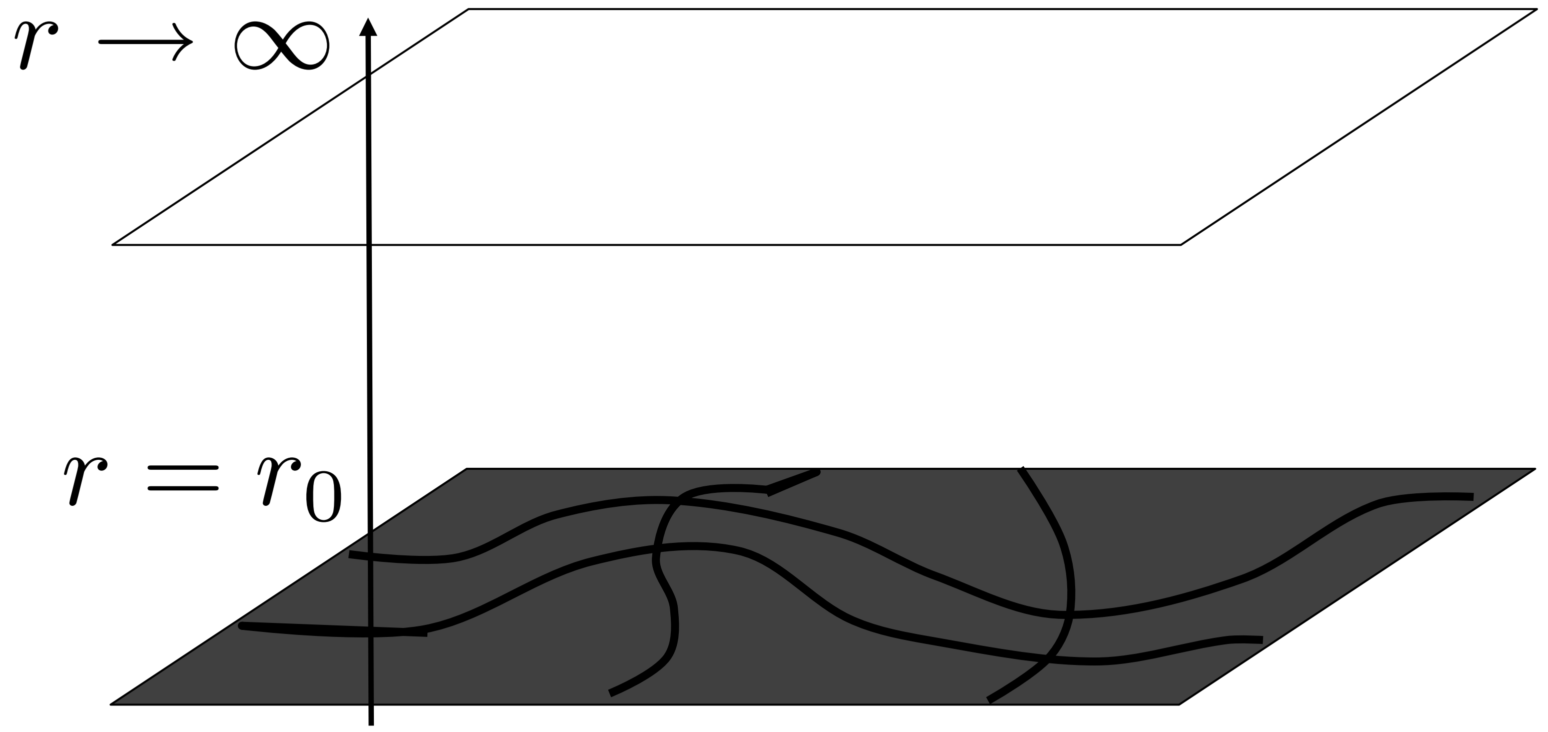


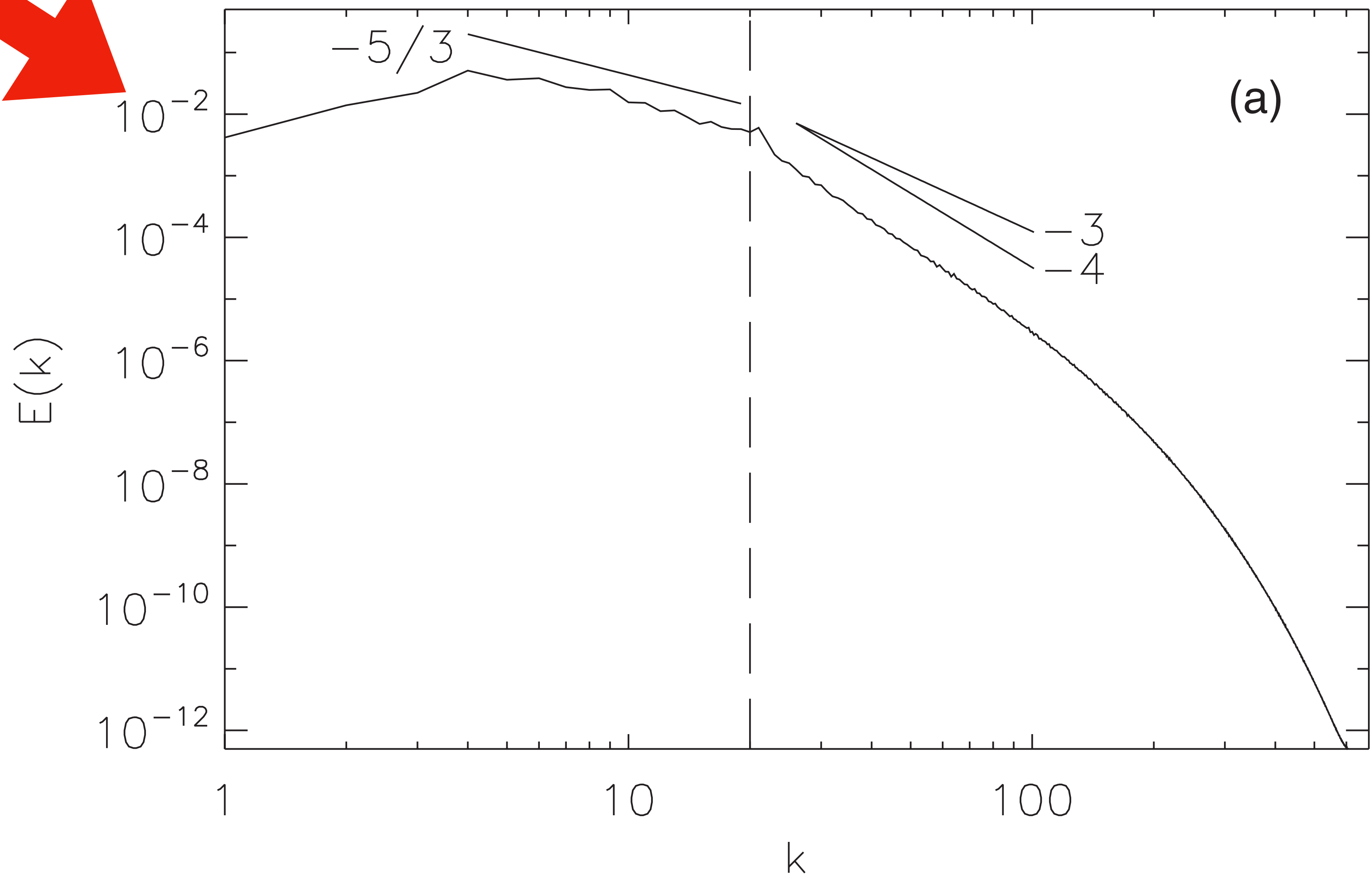
Holographic turbulence



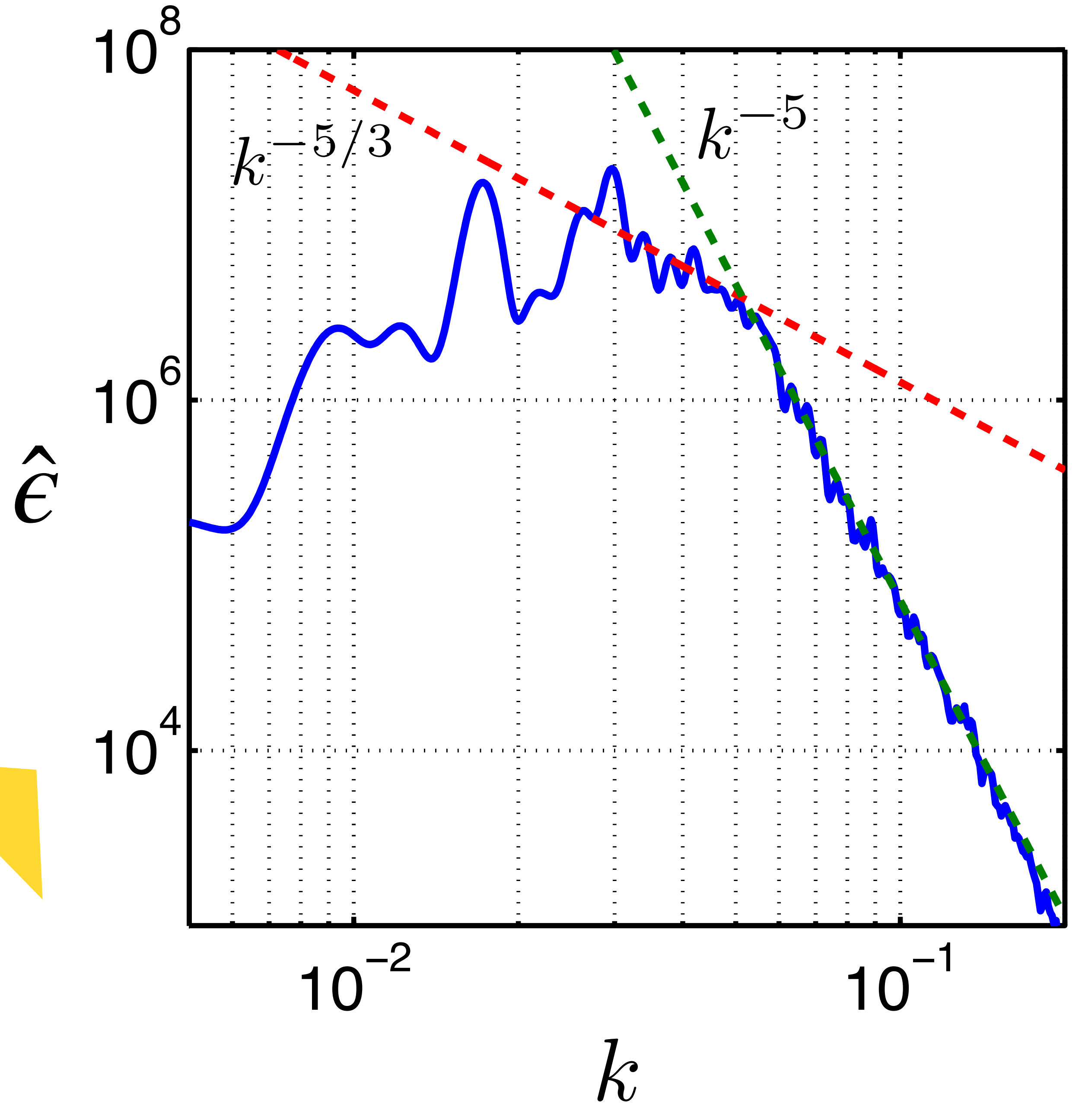
$$\frac{|\vec{v}|}{c} \ll 1 \qquad \vec{f} = 0$$

$$\dot{\vec{v}} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} p + \nu \nabla^2 \vec{v} + \vec{f}$$





Holographic turbulence



$$\frac{|\vec{v}|}{c} \ll 1 \qquad \vec{f} \neq 0$$

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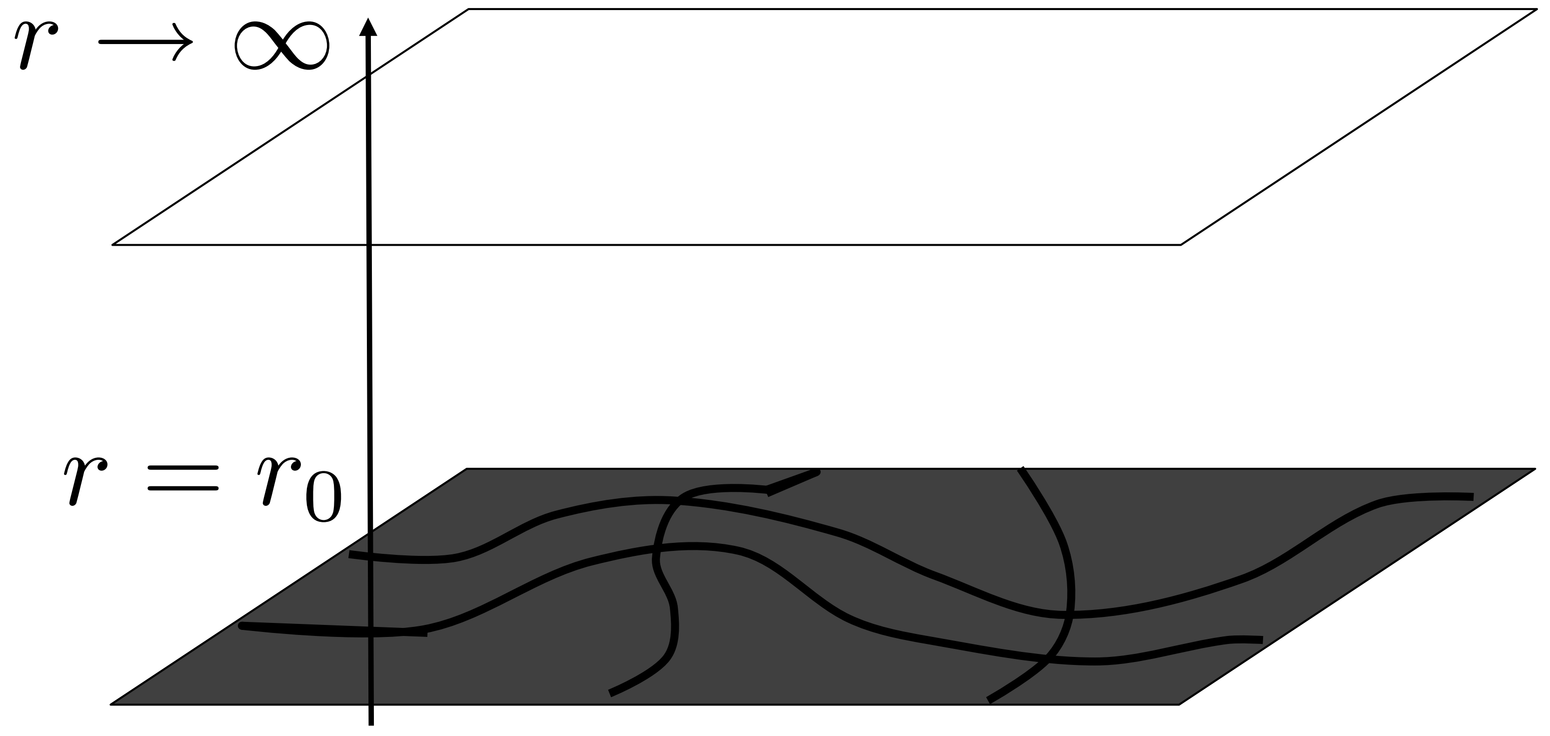
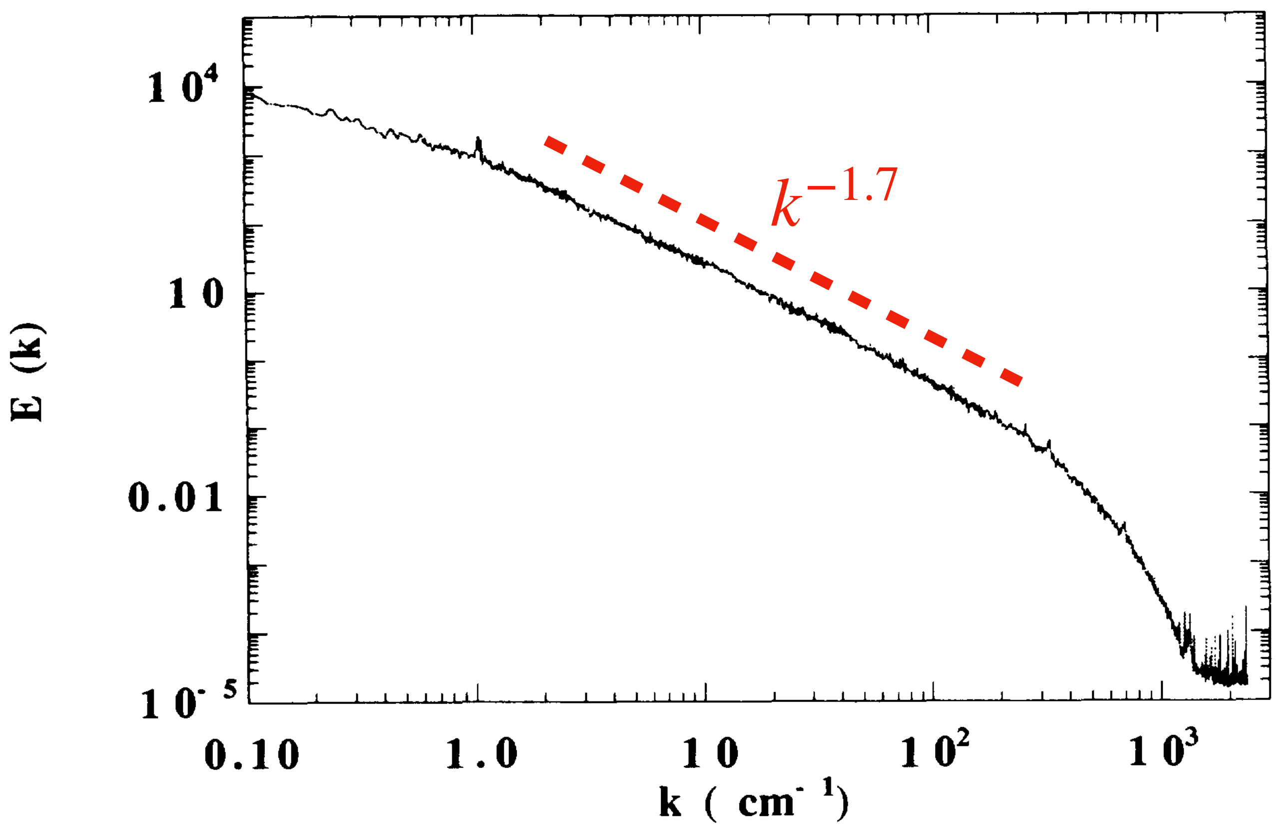
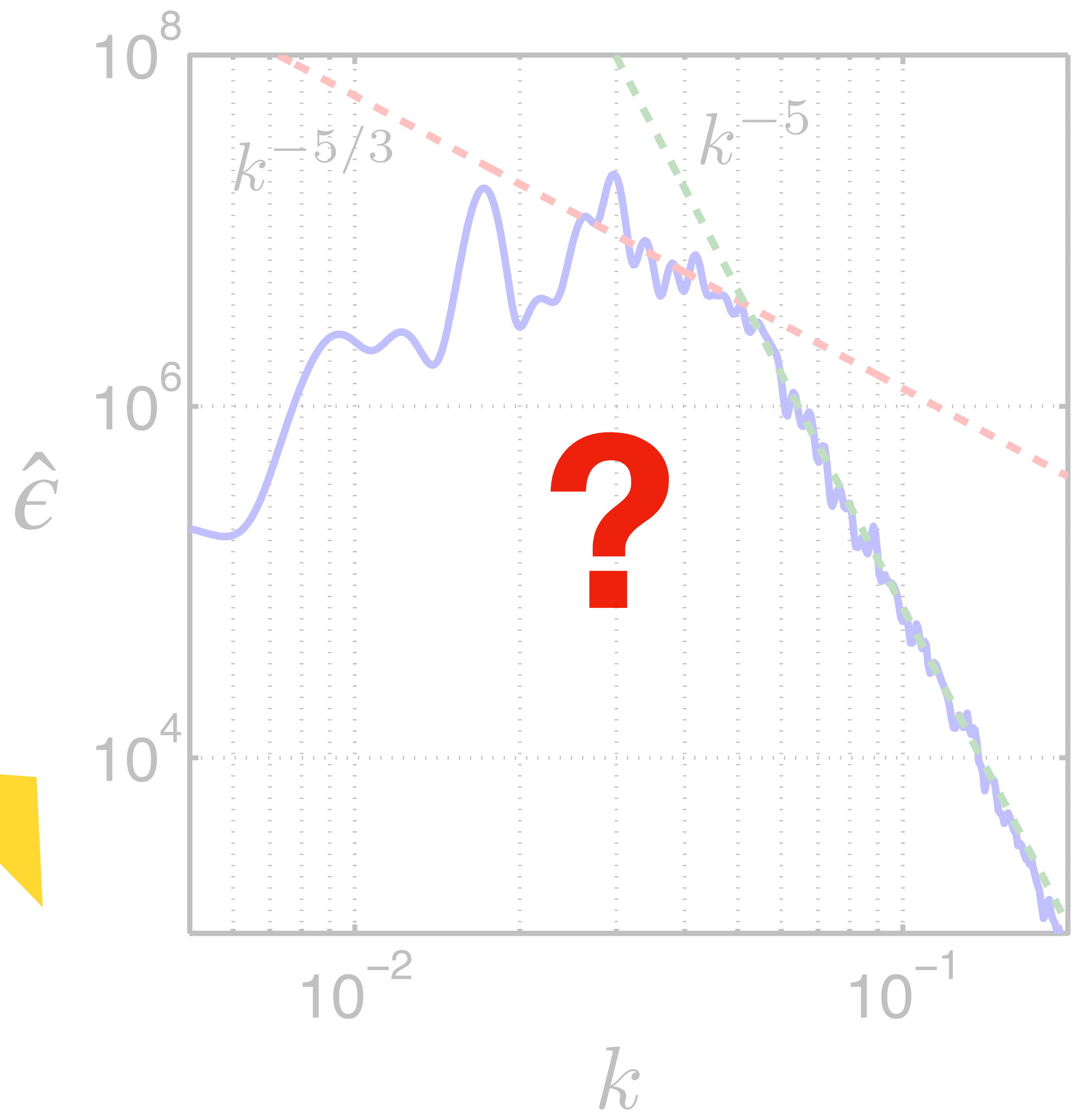




Holographic turbulence



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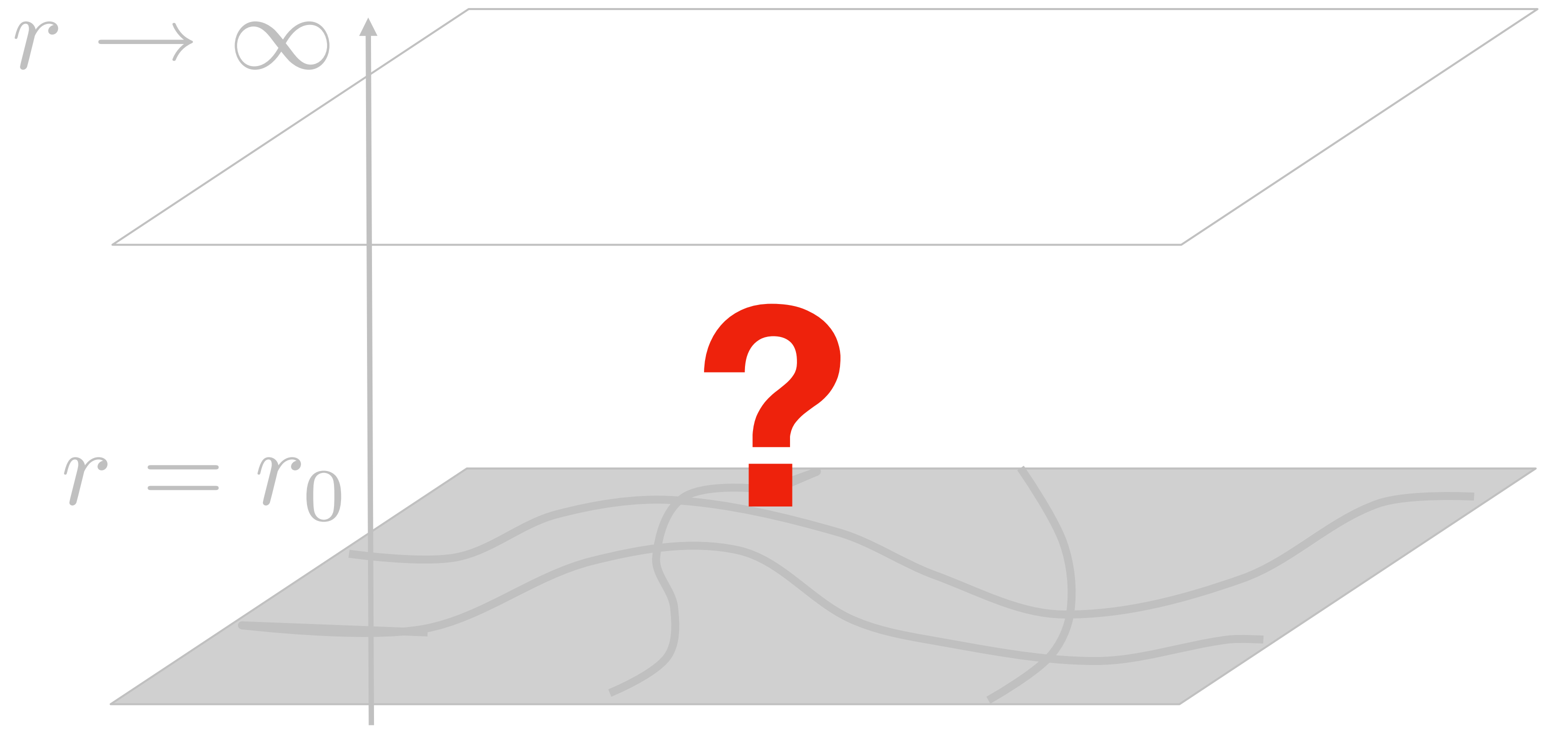
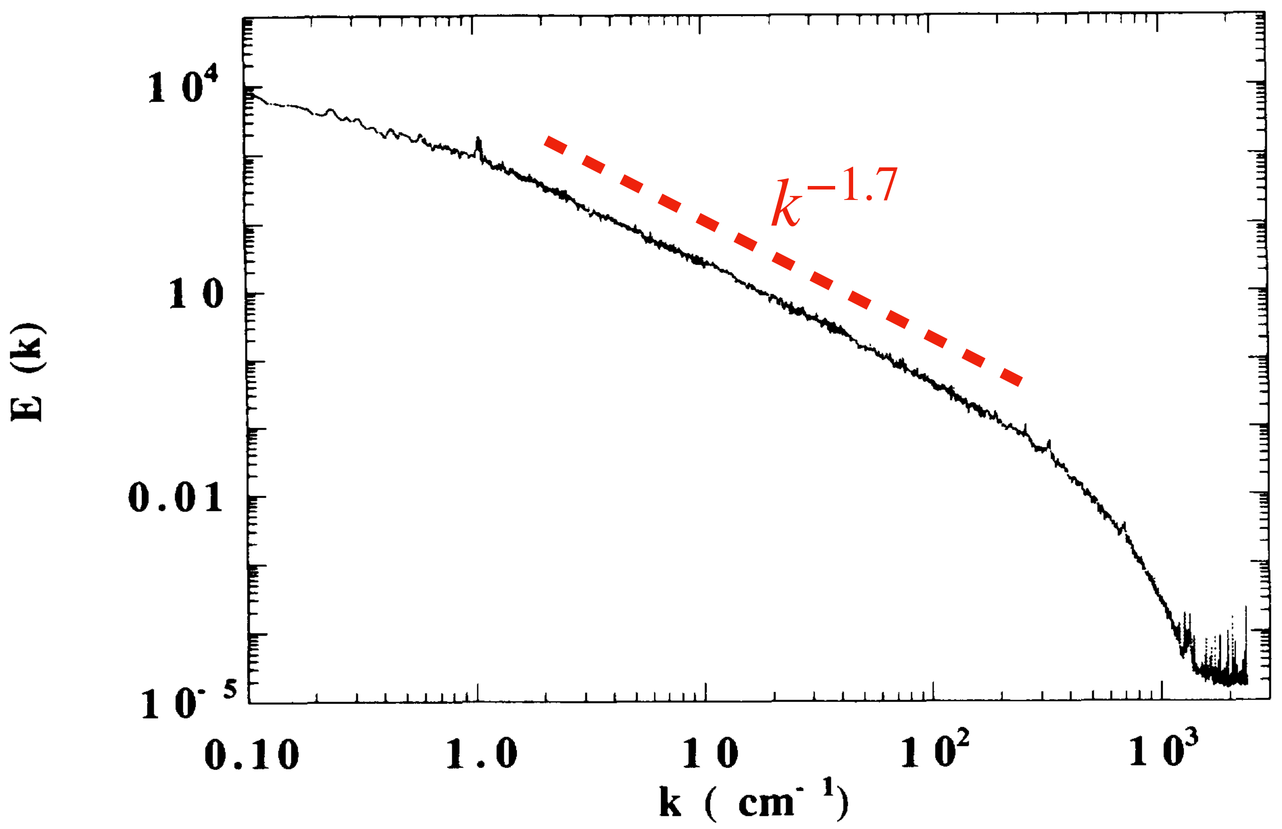
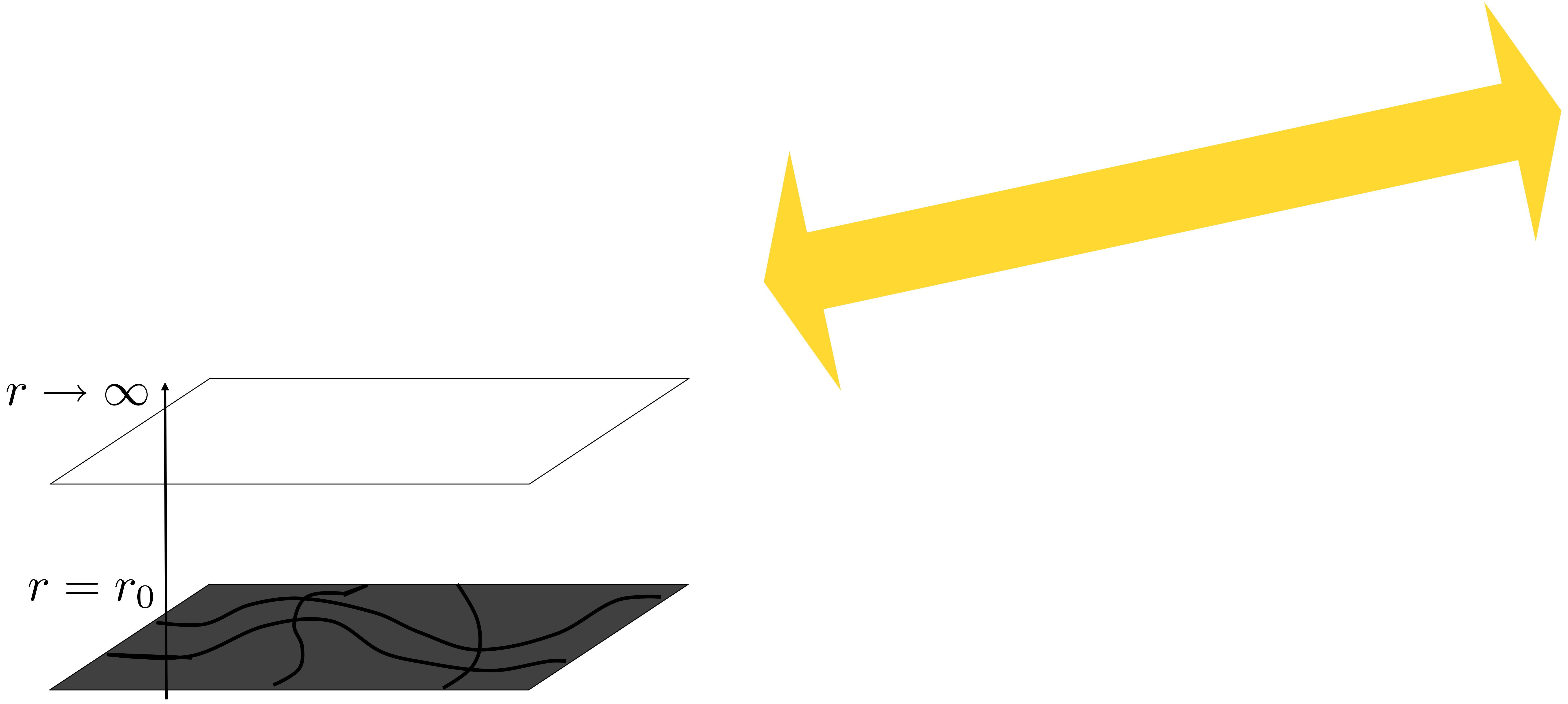




Holographic turbulence

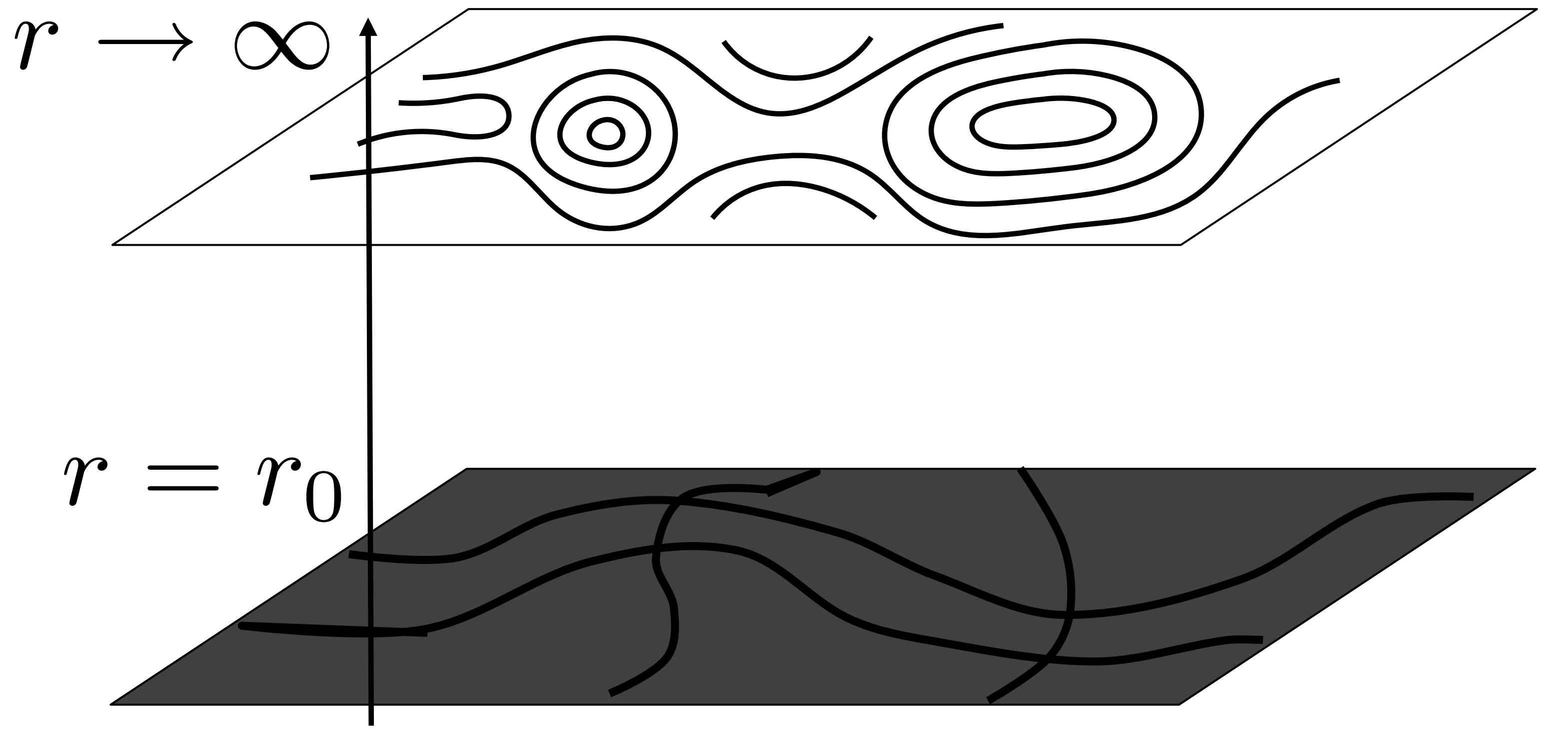
$$\nabla_\mu T^{\mu\nu} = 0$$



Holographic turbulence

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu}$$



Holographic turbulence

$$\nabla_\mu T^{\mu\nu} = 0$$

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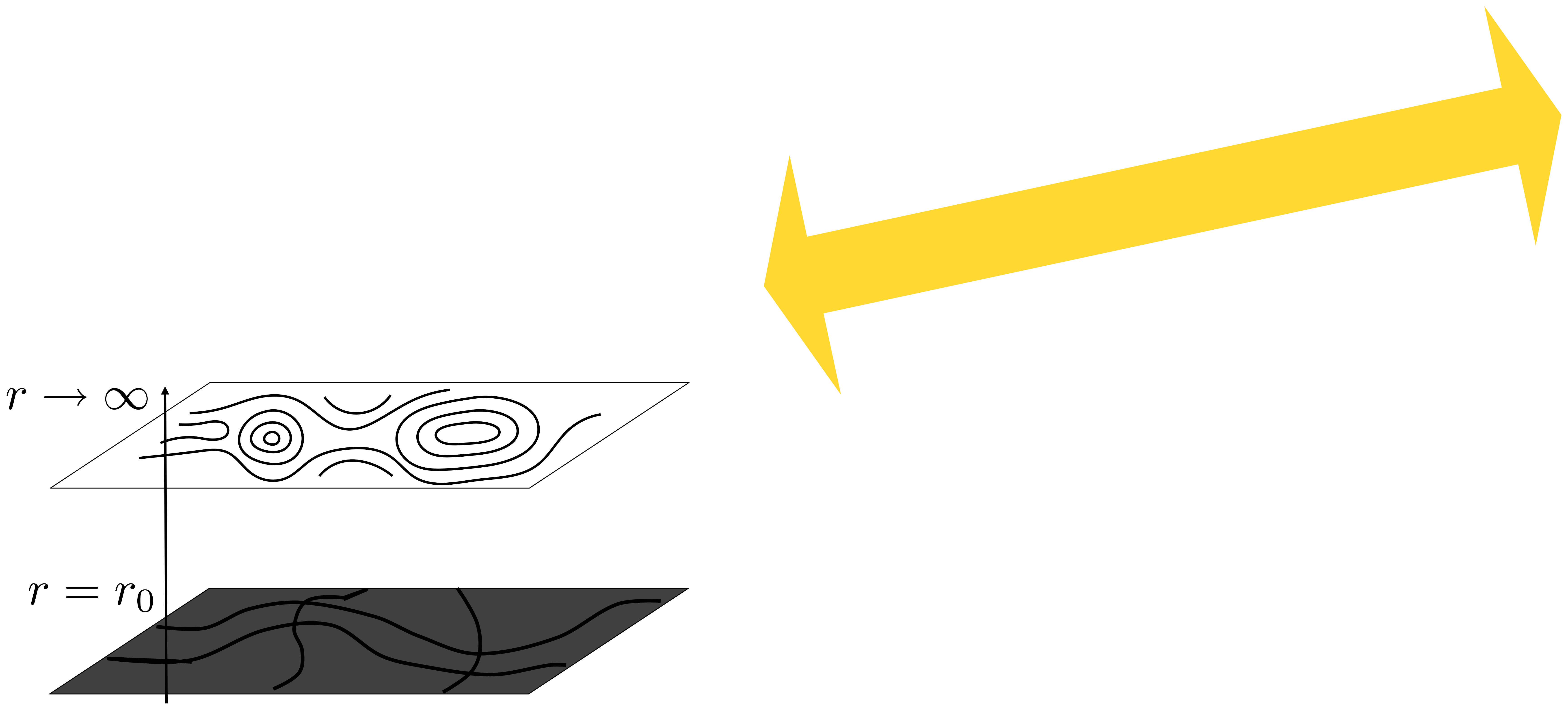
Write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Holographic turbulence



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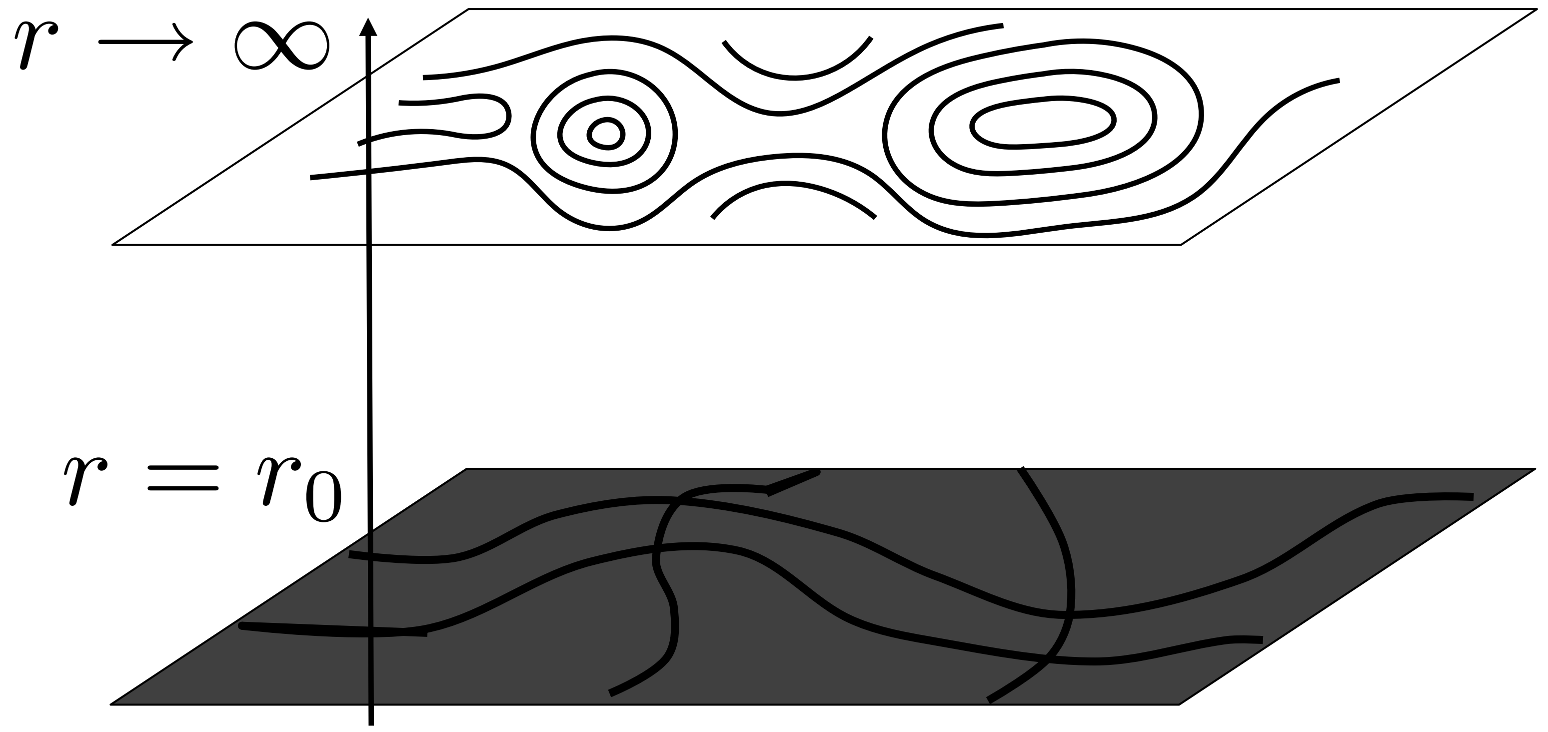
$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \tilde{T}^{\mu\nu}$$

Holographic turbulence



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Write

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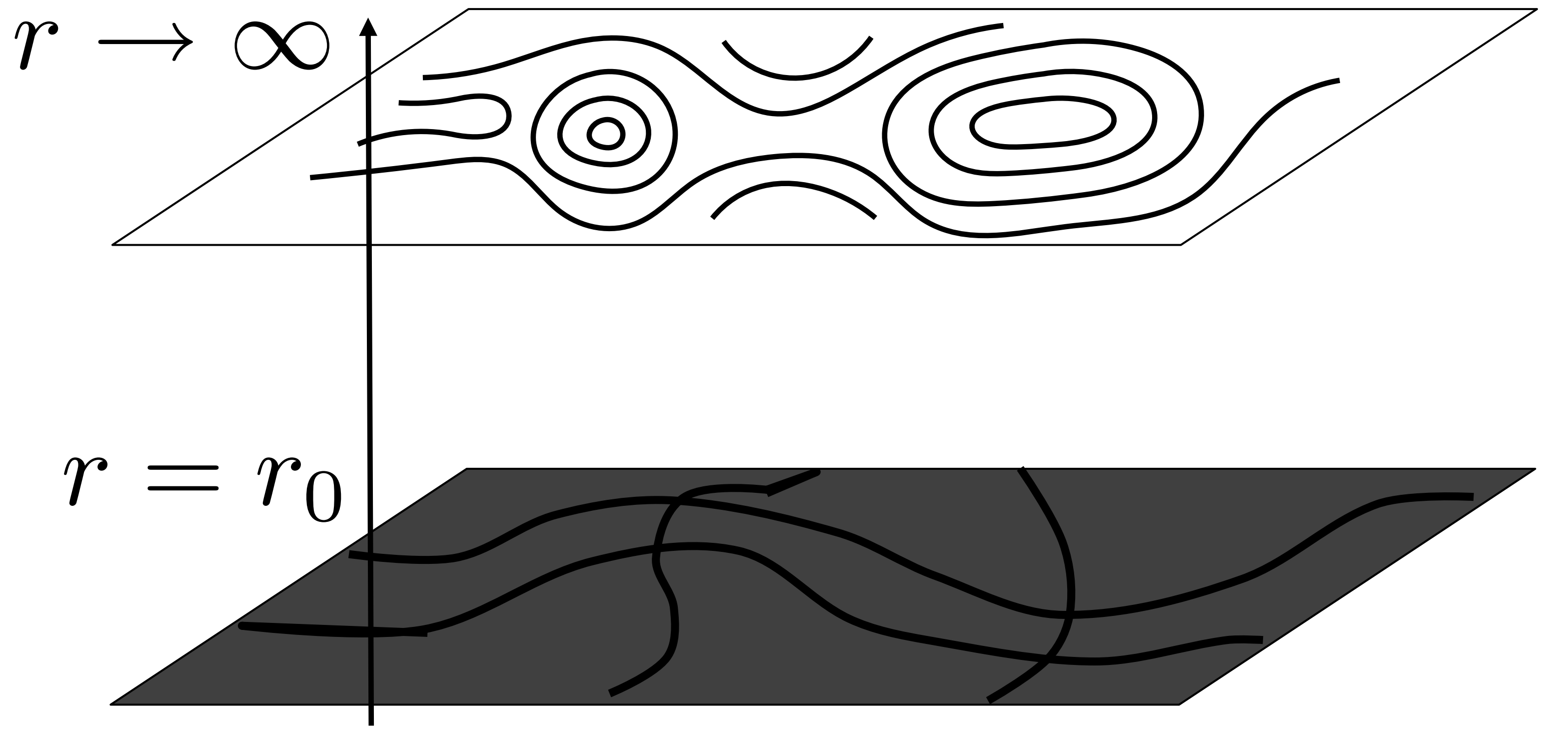
$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \tilde{T}^{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = \nabla^{(0)}_\mu T^{\mu\nu}_{(0)} + F^\nu$$

Holographic turbulence

$$\nabla_\mu^{(0)} T_{(0)}^{\mu\nu} = -F^\nu$$

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu}$$



Holographic turbulence

- Bhattacharyya et. al. 2007

$$\nabla_\mu^{(0)} T_{(0)}^{\mu\nu} = -F^\nu$$

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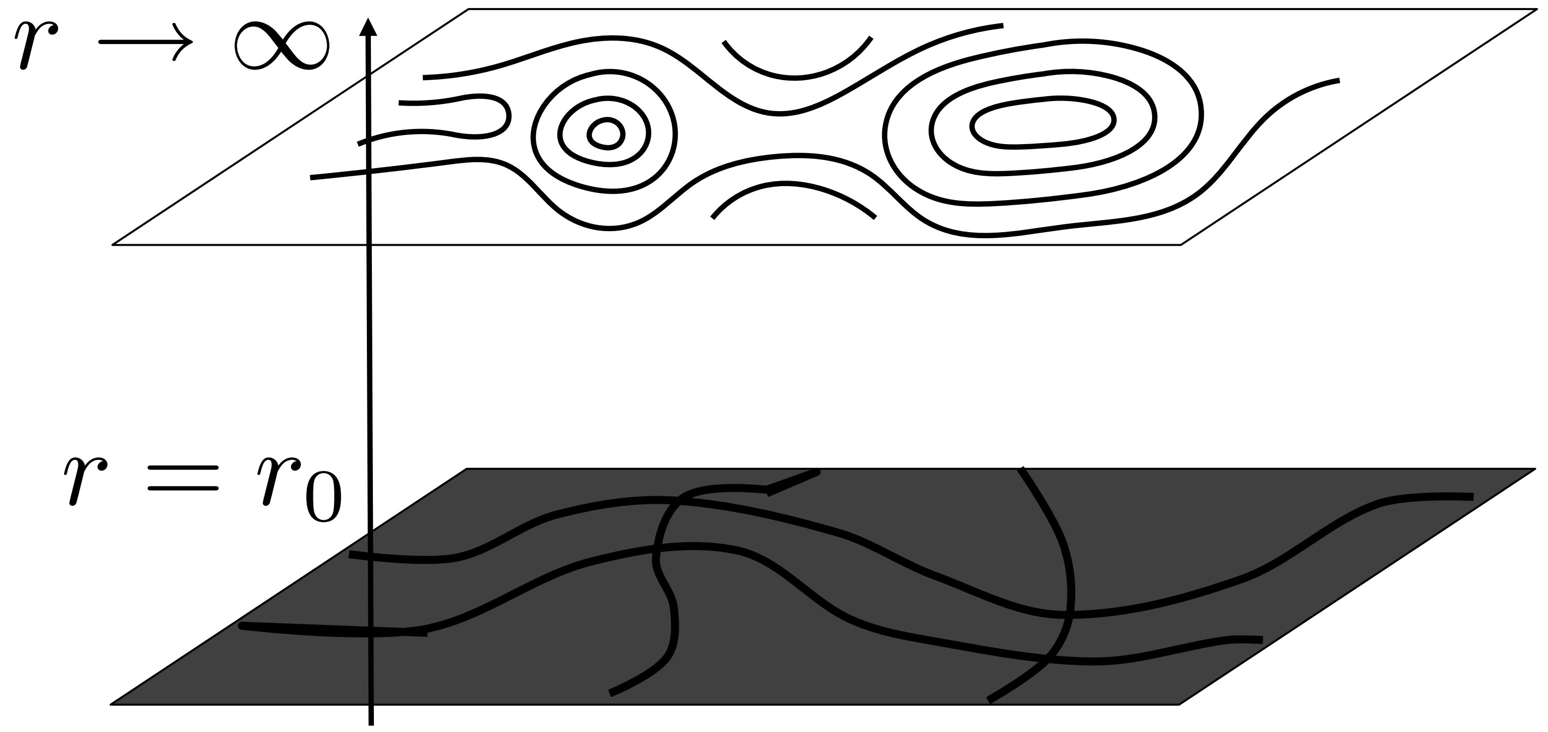

Holographic turbulence

- Bhattacharyya et. al. 2007
- Andrade et. al. 2019

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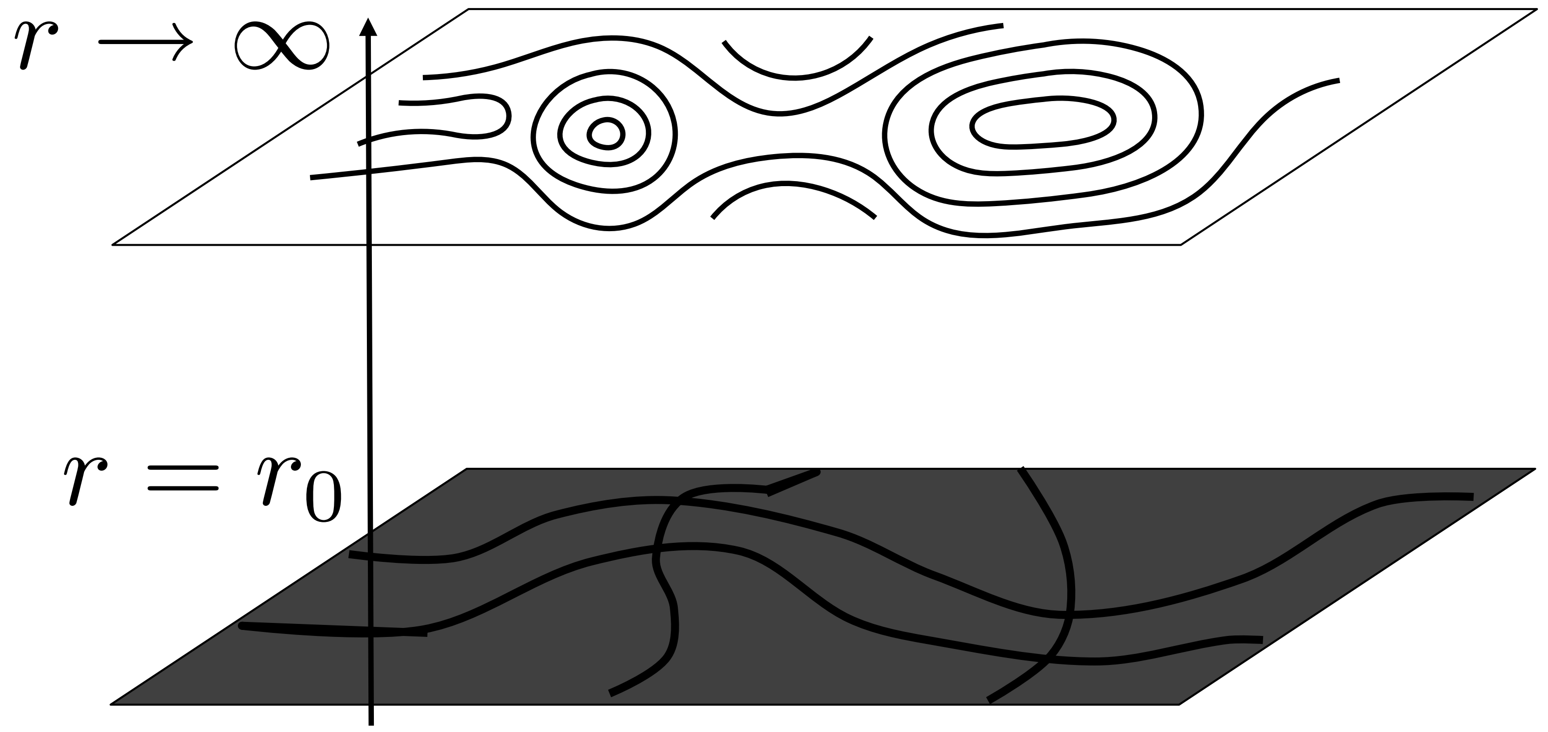

Holographic turbulence

Stochastic differential equation

$$\nabla_\mu^{(0)} T_{(0)}^{\mu\nu} = -F^\nu$$

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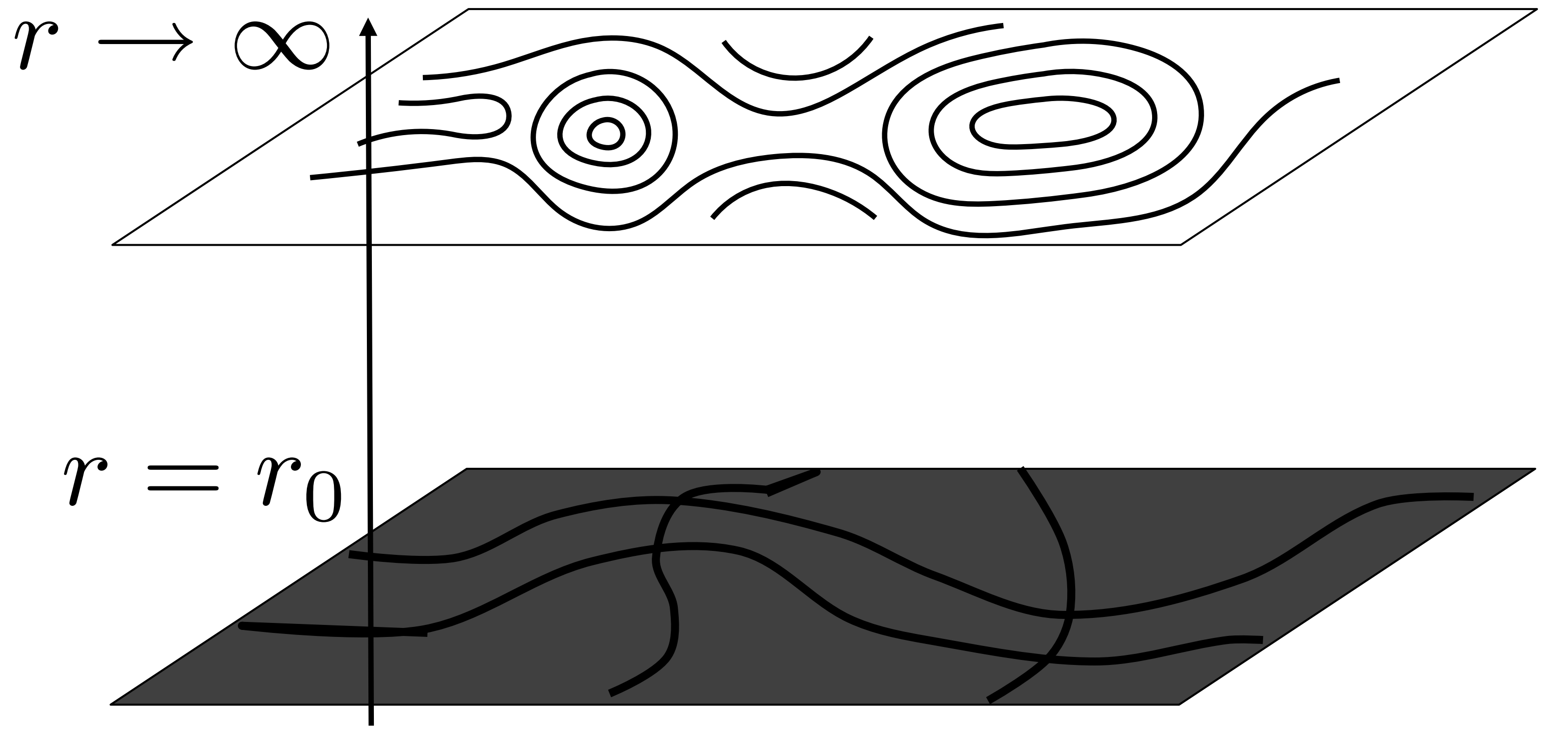

Stochastic differential equations

Consider

$$\dot{Q}(t) = g(Q(t)) + h(Q(t))\xi(t)$$

where

$$\overline{\xi(t)} = 0 \qquad \overline{\xi(t)\xi(t')} = D\delta(t - t')$$

Stochastic differential equations

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In integral form

$$Q(t) = Q(0) + \int_0^t g(Q(t'))dt' + \int_0^t h(Q(t'))\xi(t')dt'$$

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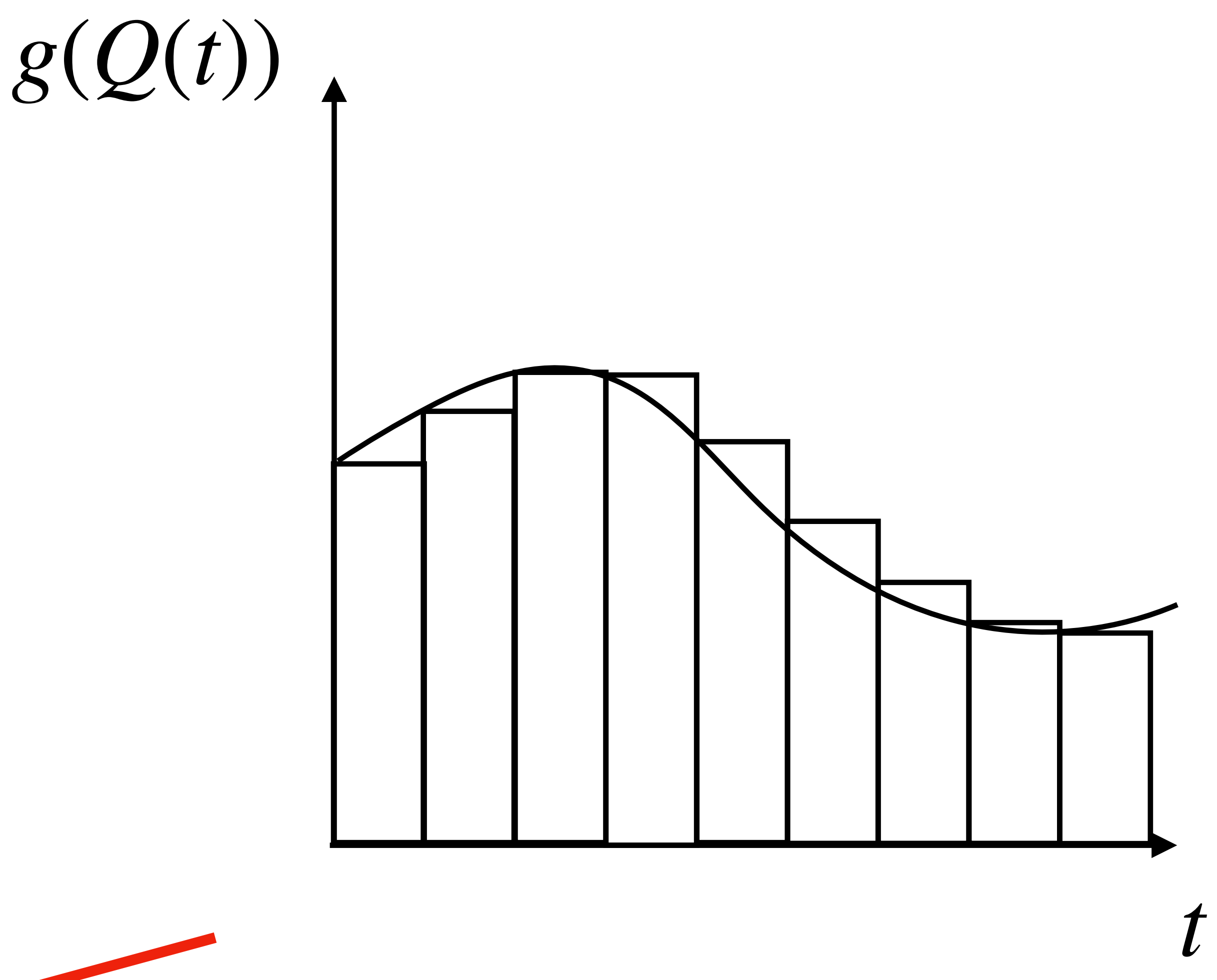

$$\int_0^t g(Q(t'))dt' = \lim_{\Delta t \to 0} \sum_{n=0}^{N-1} g(Q(n\Delta t))\Delta t$$

In integral form

$$Q(t) = Q(0) + \int_0^t g(Q(t'))dt' + \int_0^t h(Q(t'))\xi(t')dt'$$

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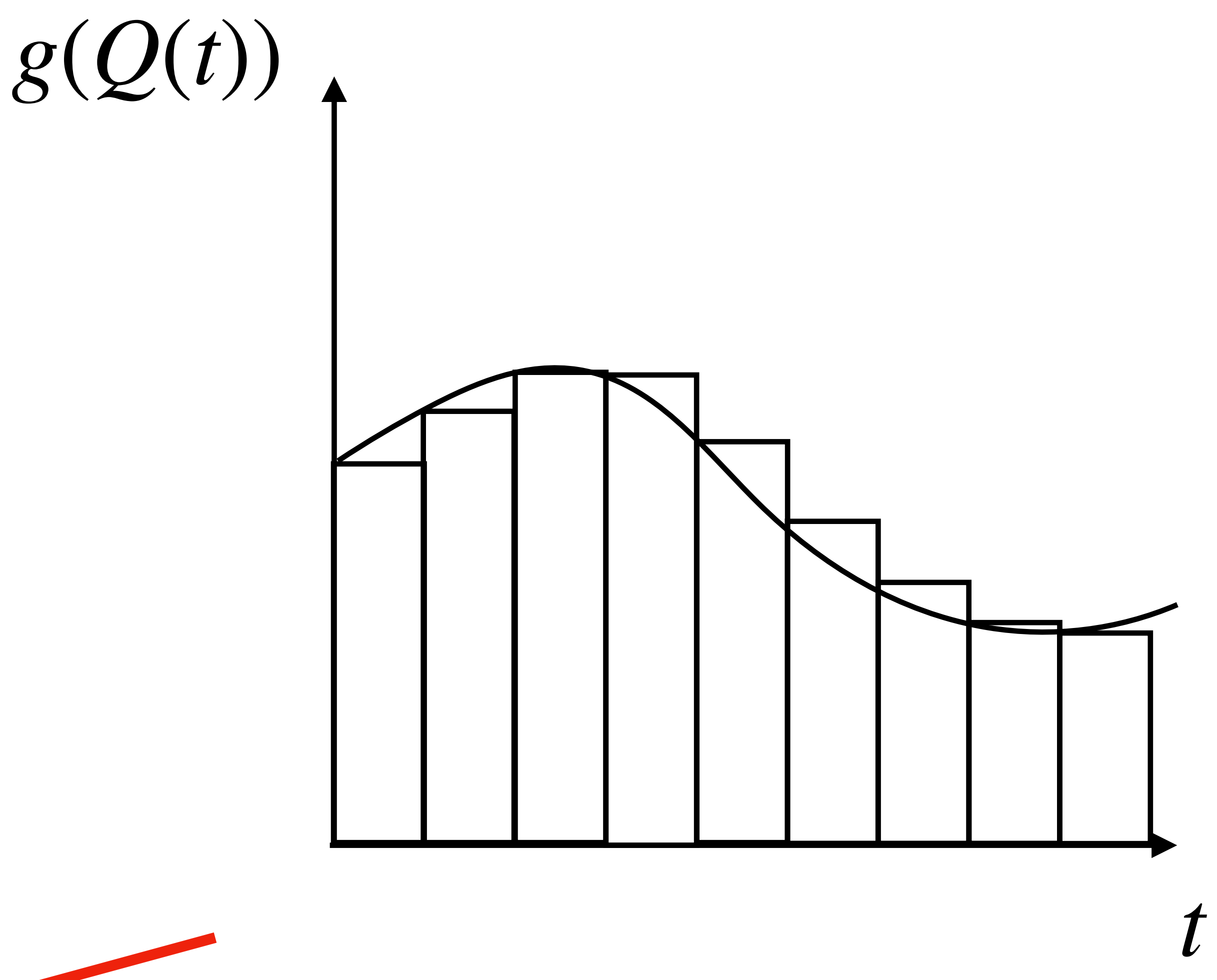
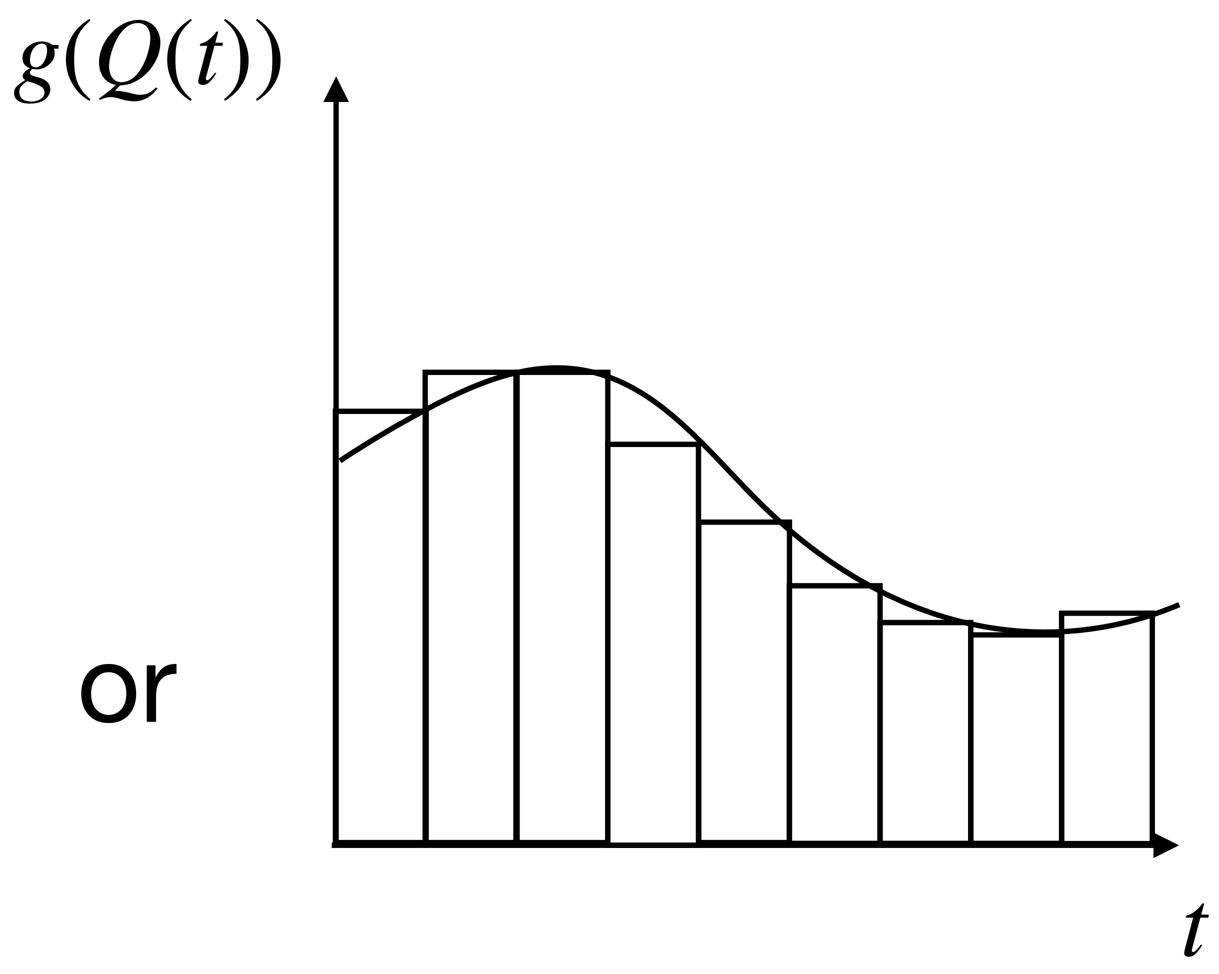


or



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In integral form

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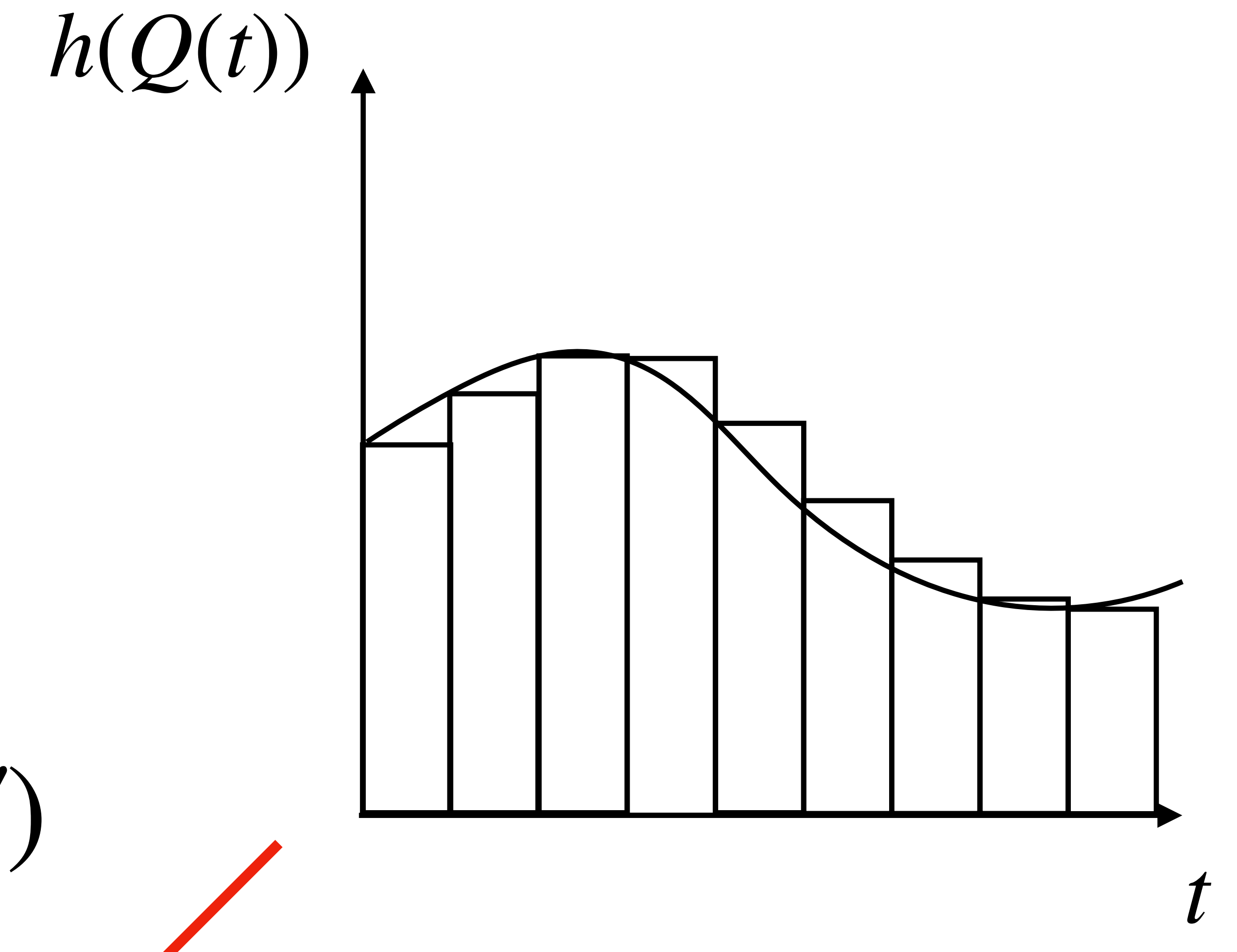

$$\int_0^t h(Q(t'))\xi(t')dt' = \lim_{\Delta t \to 0} \sum_{n=0}^{N-1} h(Q(n\Delta t)) \int_{n\Delta t}^{(n+1)\Delta t} \xi(t')dt'$$

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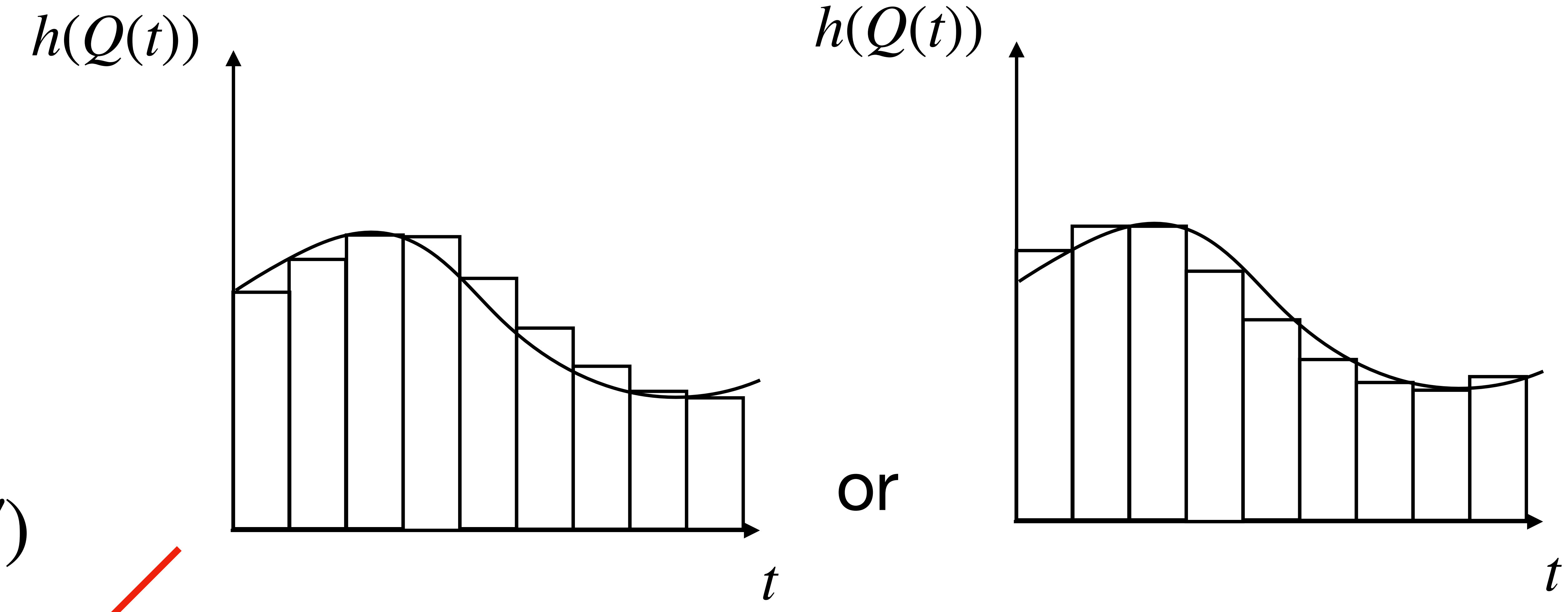

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$$Q(t) = Q(0) + \int_0^t g(Q(t'))dt' + \int_0^t h(Q(t'))\xi(t')dt'$$

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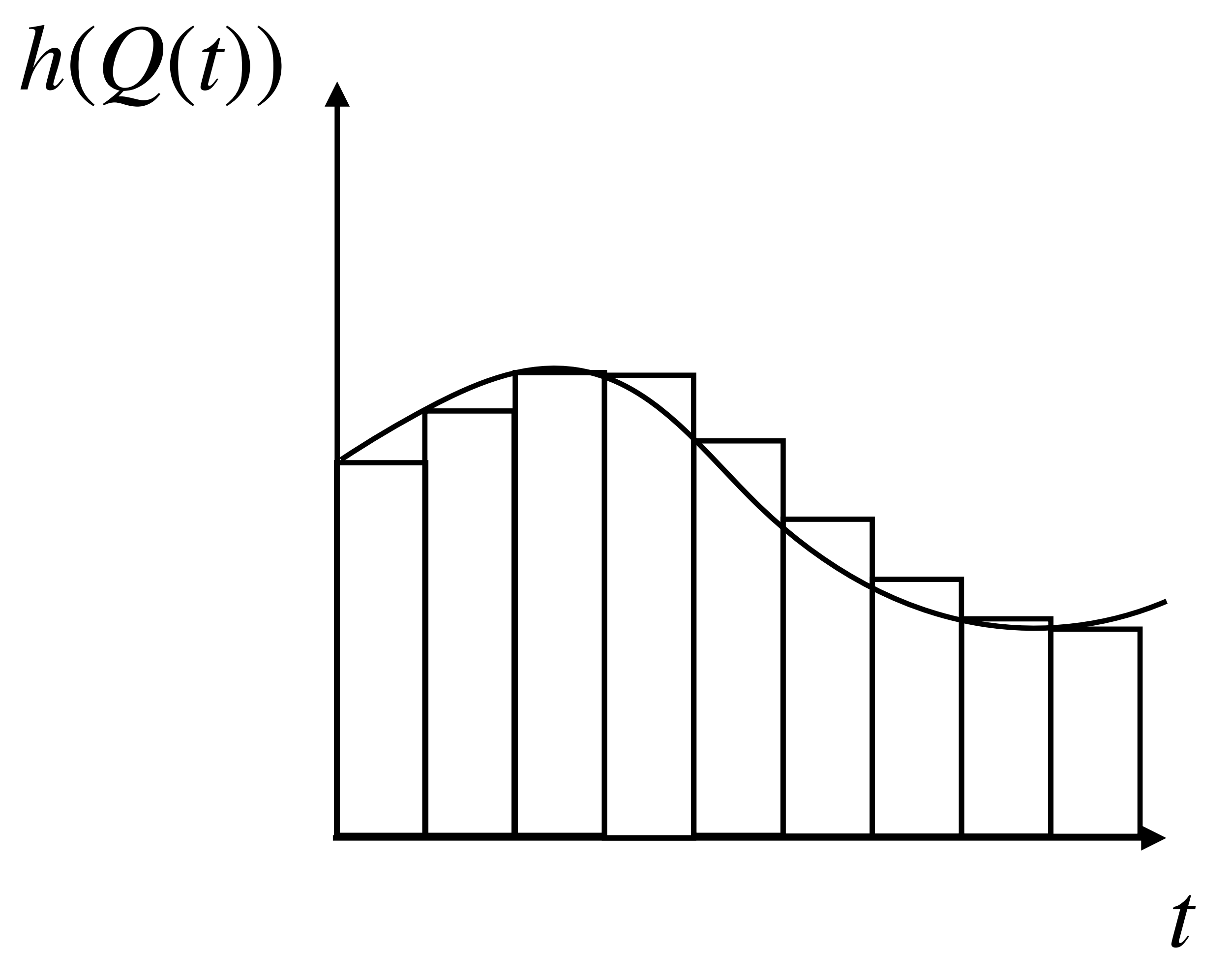
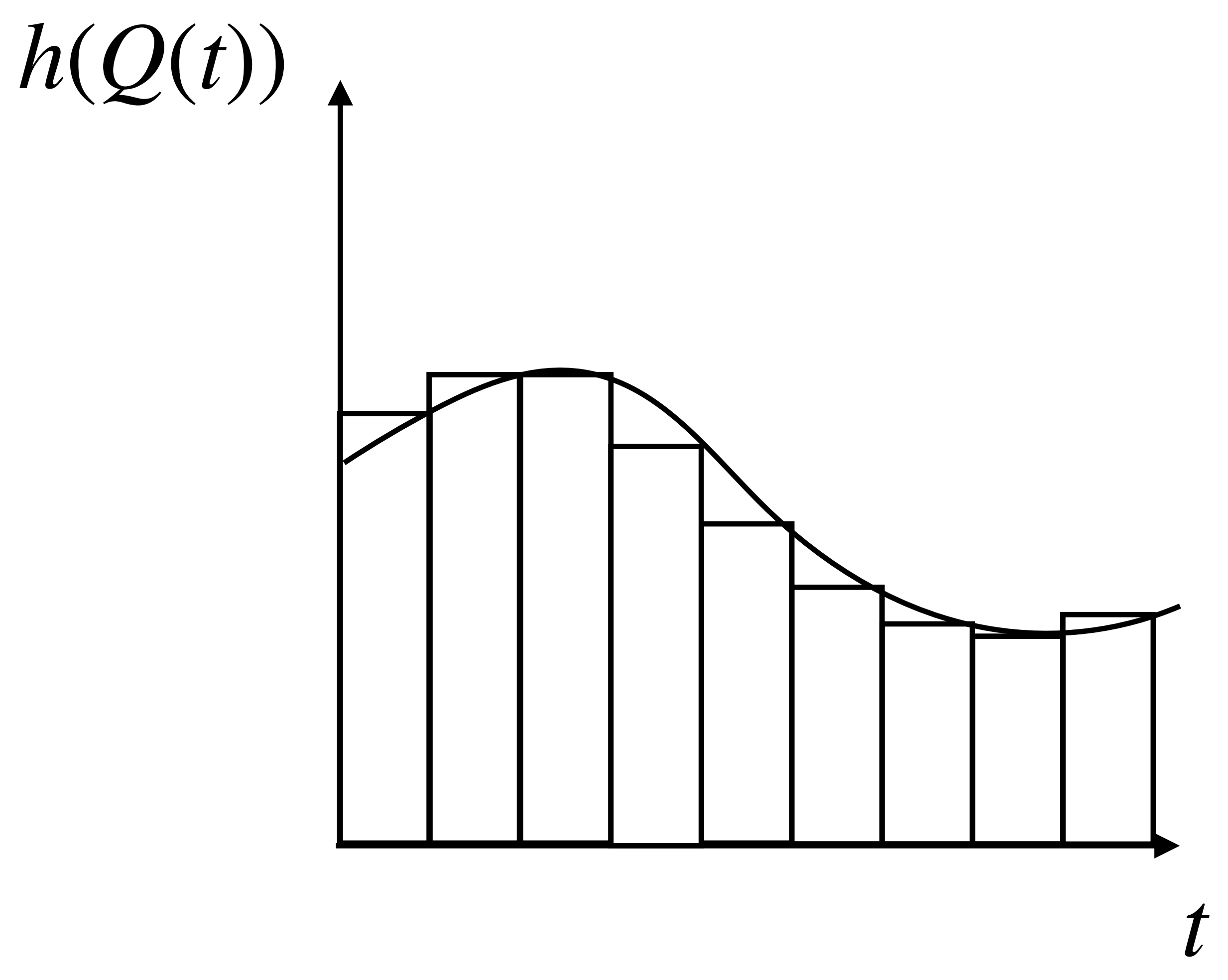
Stochastic differential equations

$$\dot{Q}(t) = g(Q(t)) + h(Q(t))\xi(t)$$





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Stochastic differential equations

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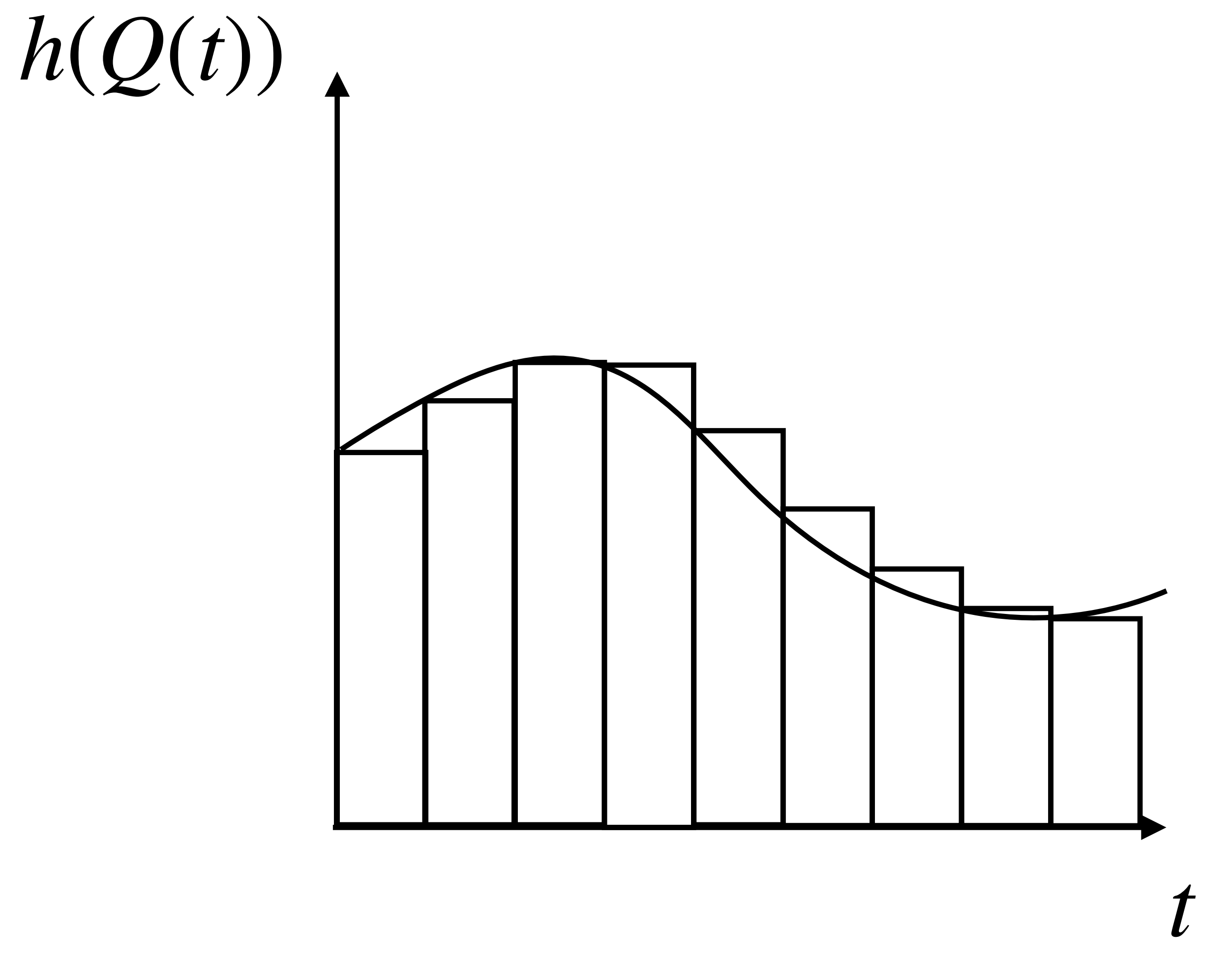

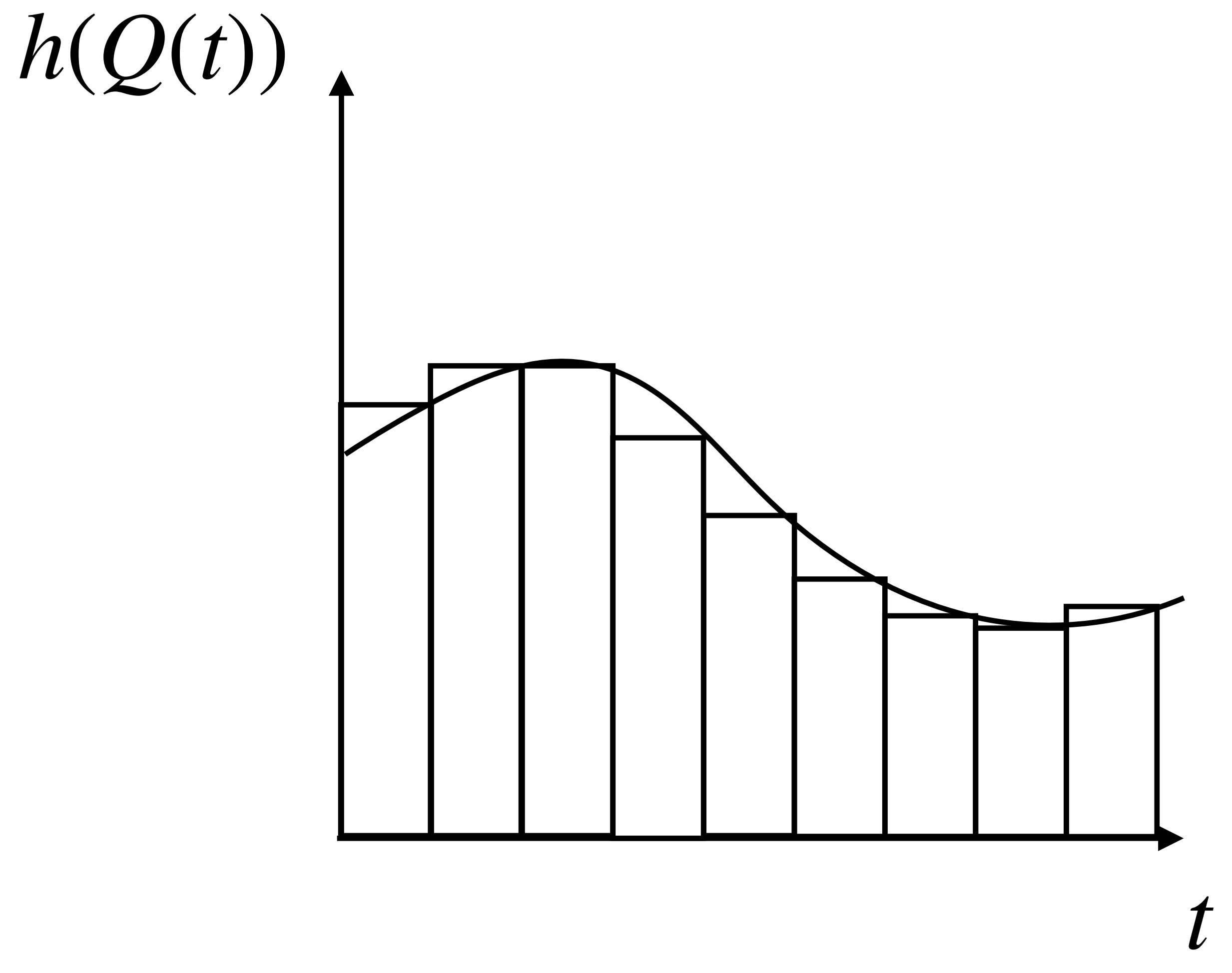

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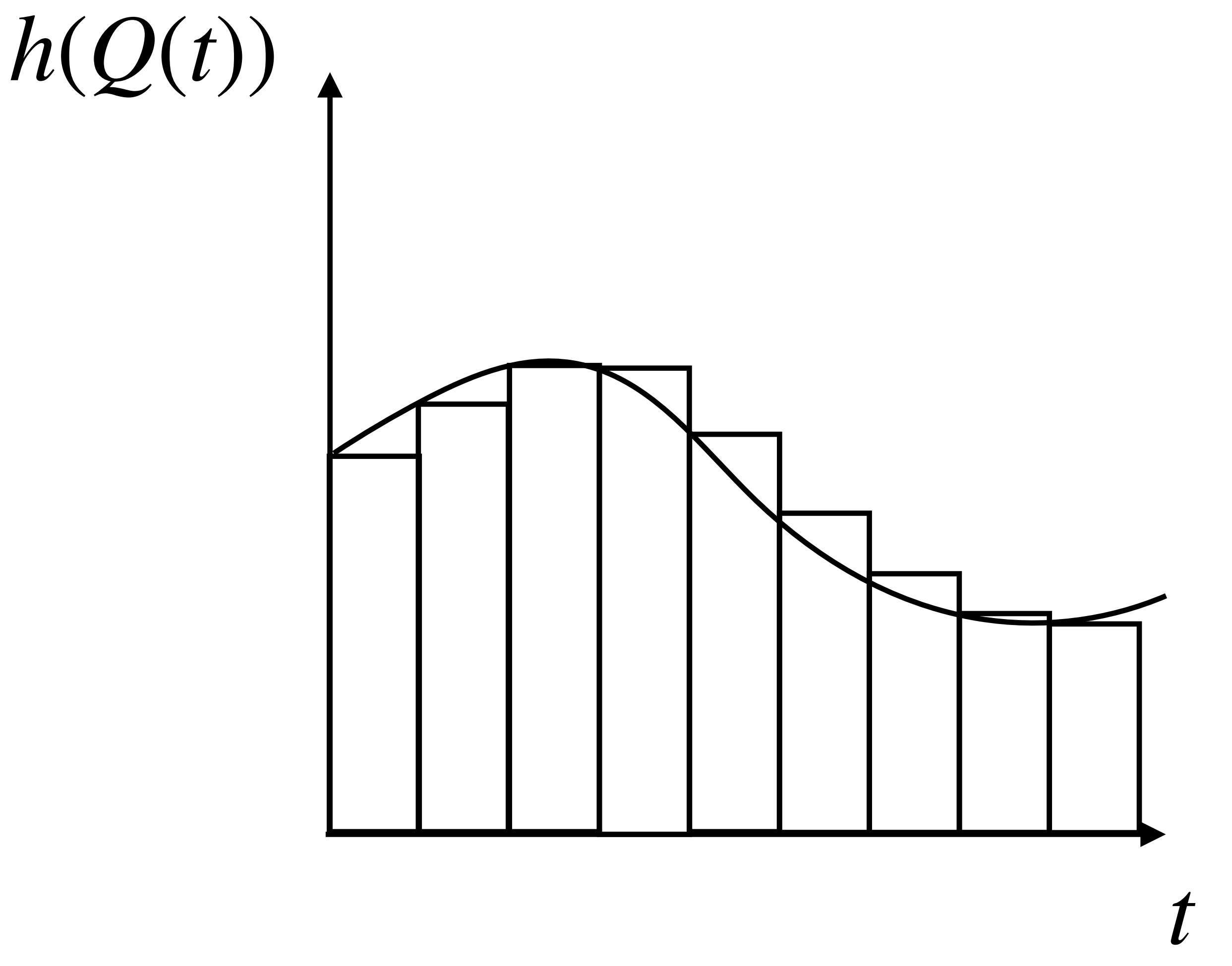

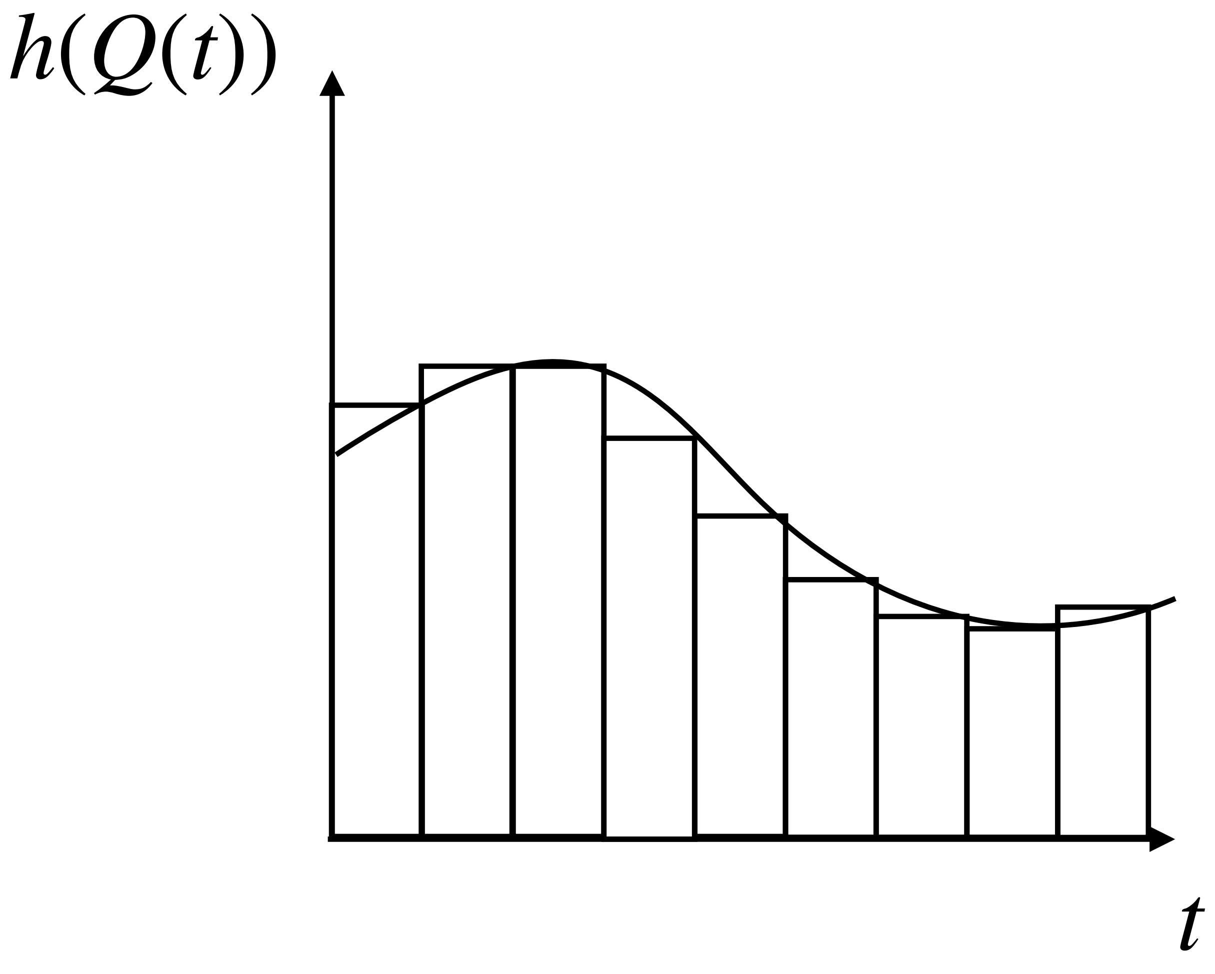

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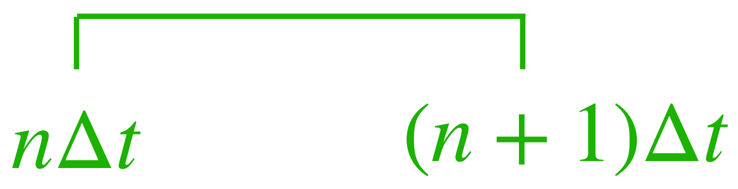

Stochastic differential equations

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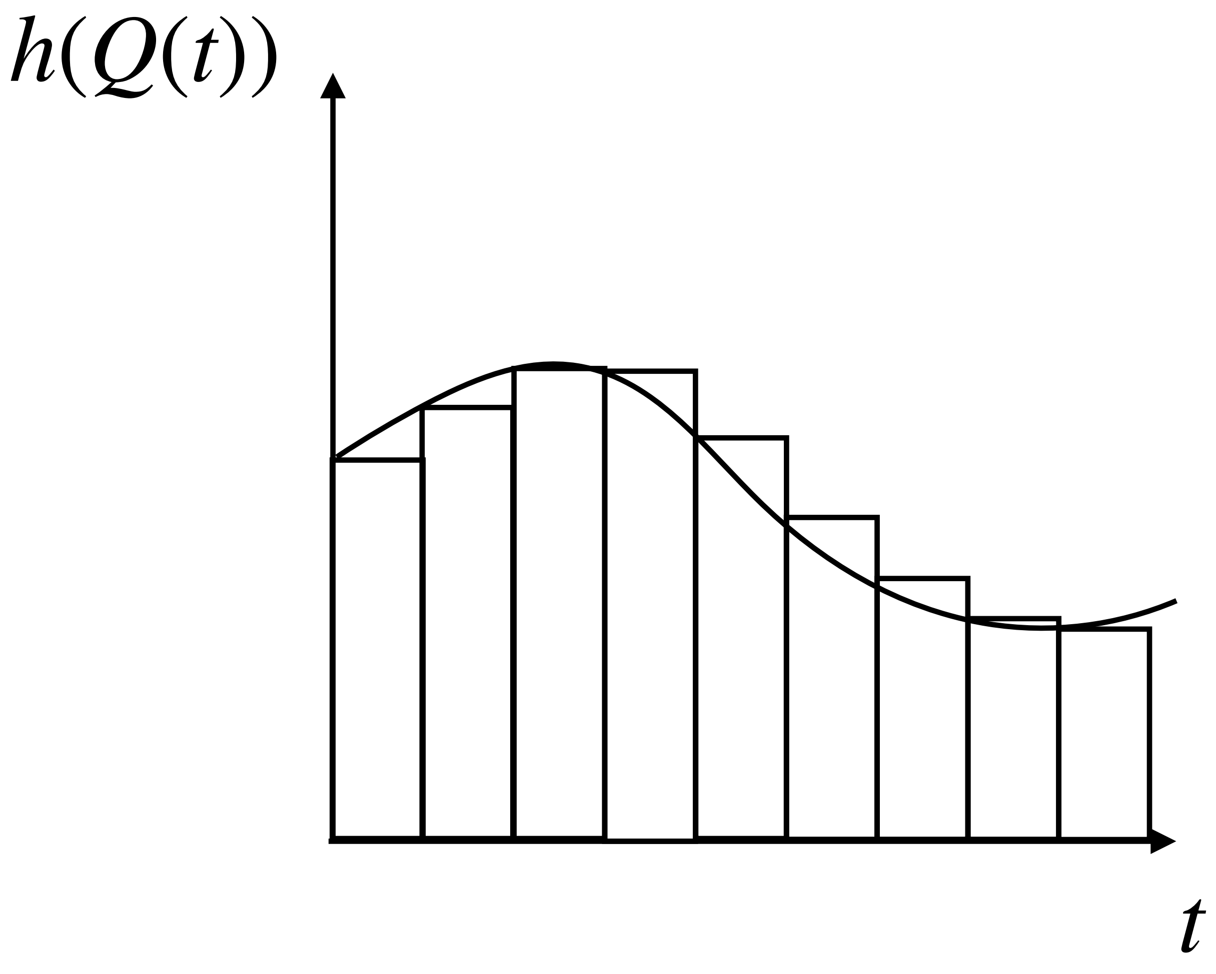

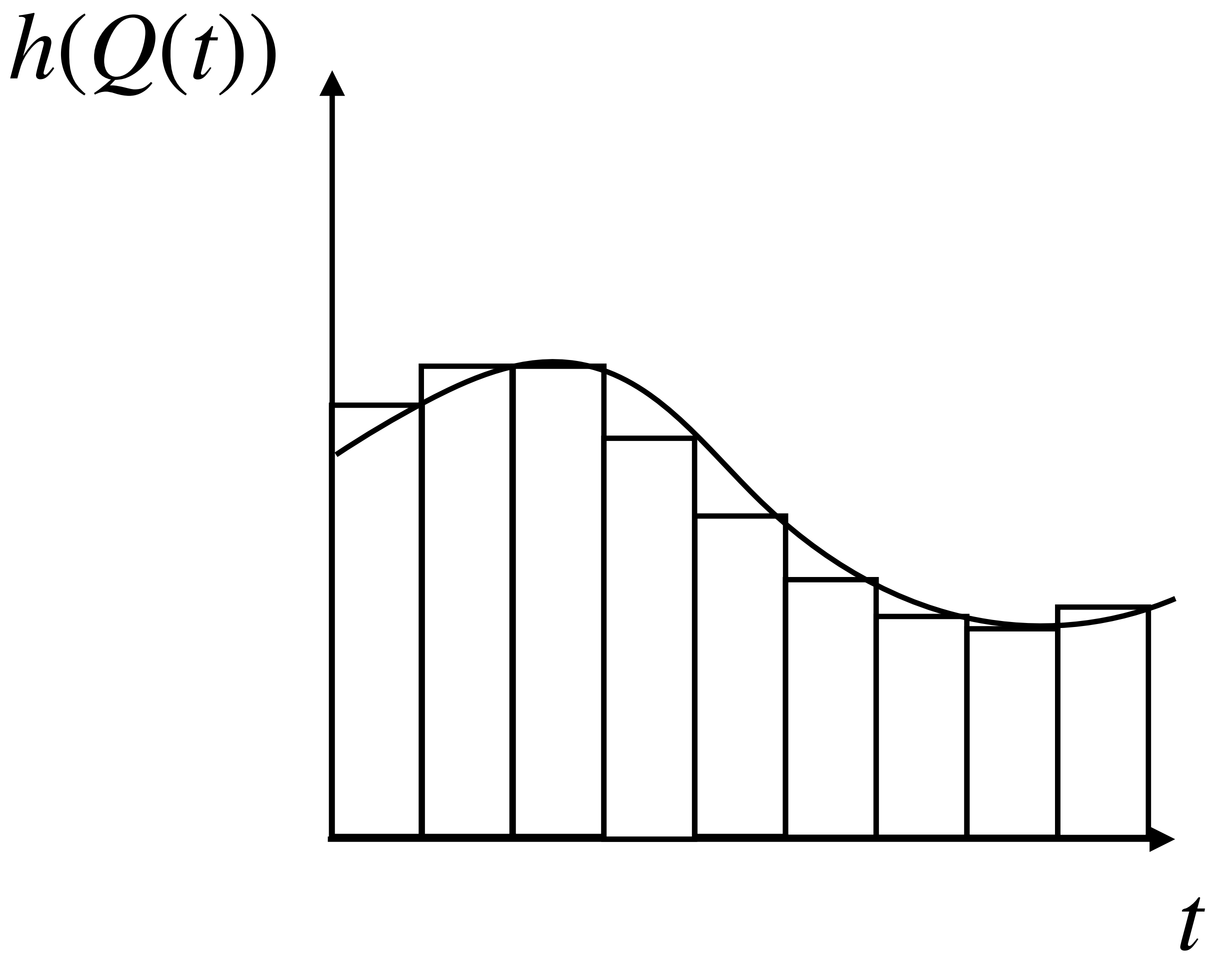

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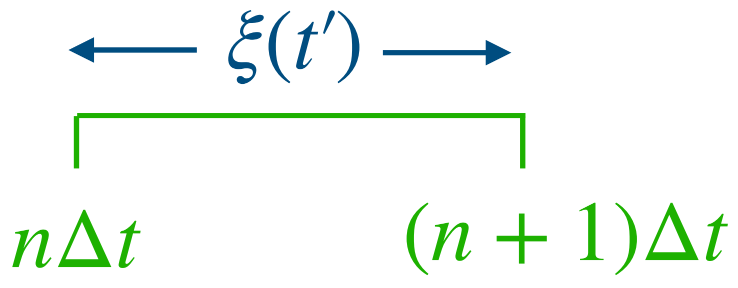

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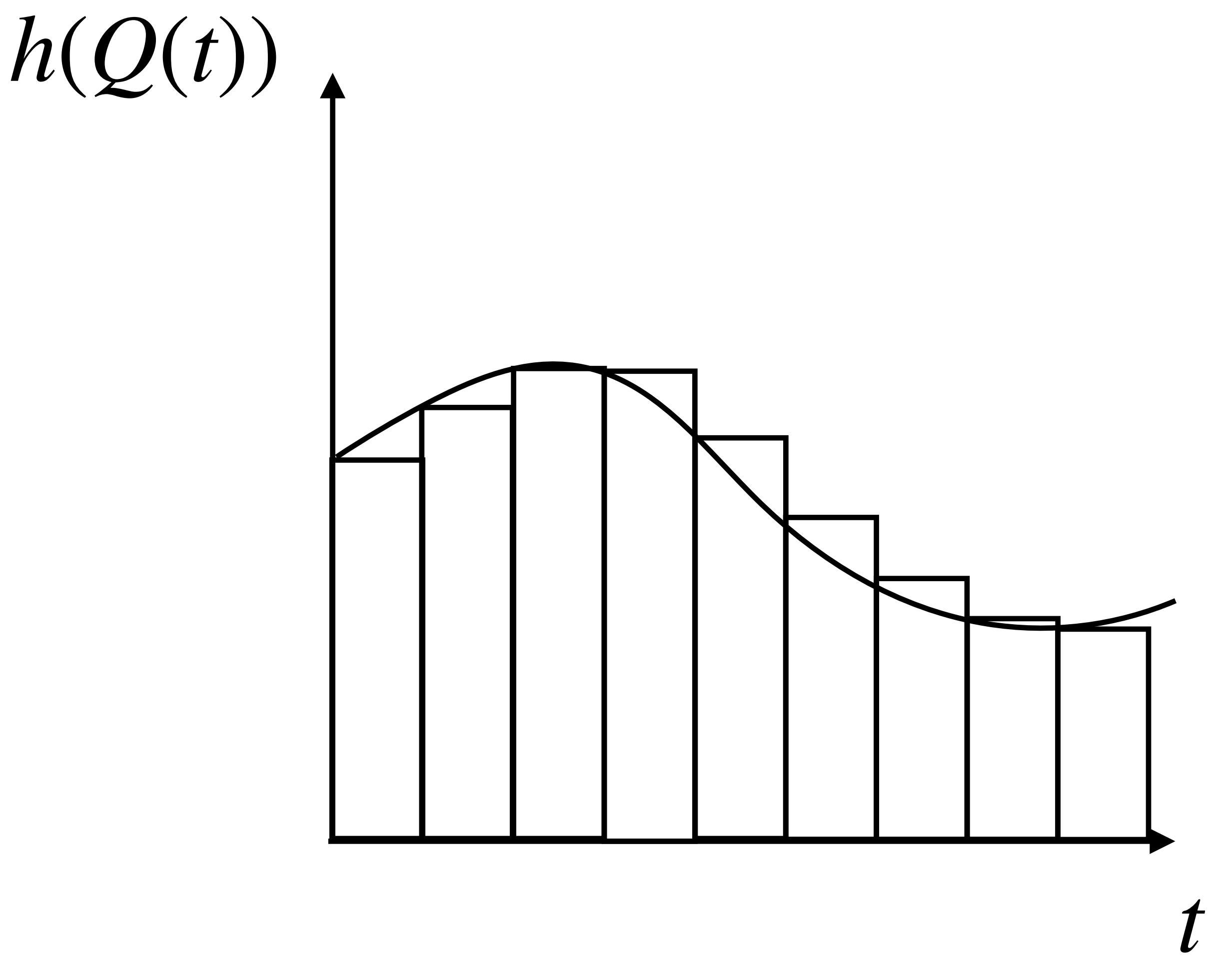

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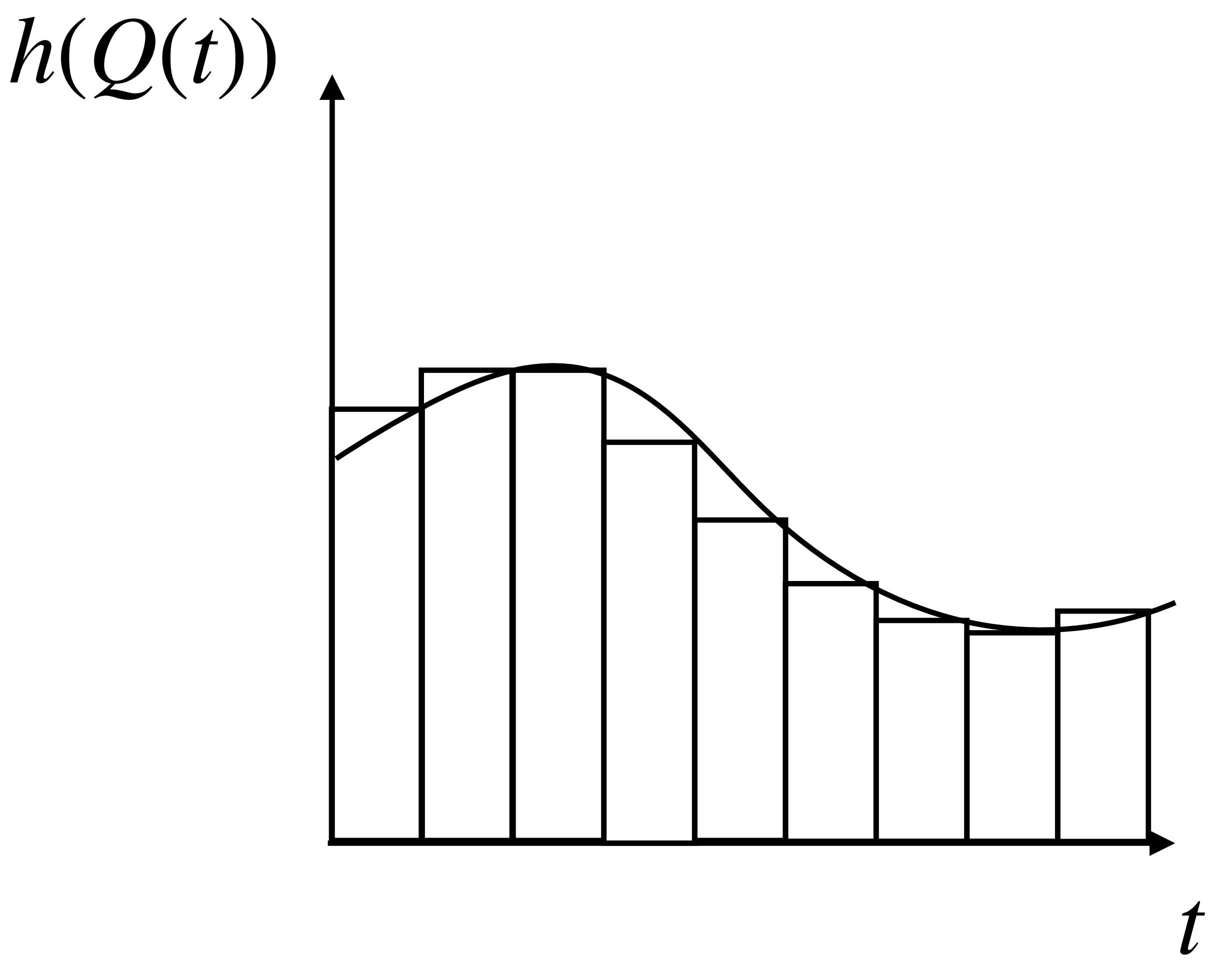

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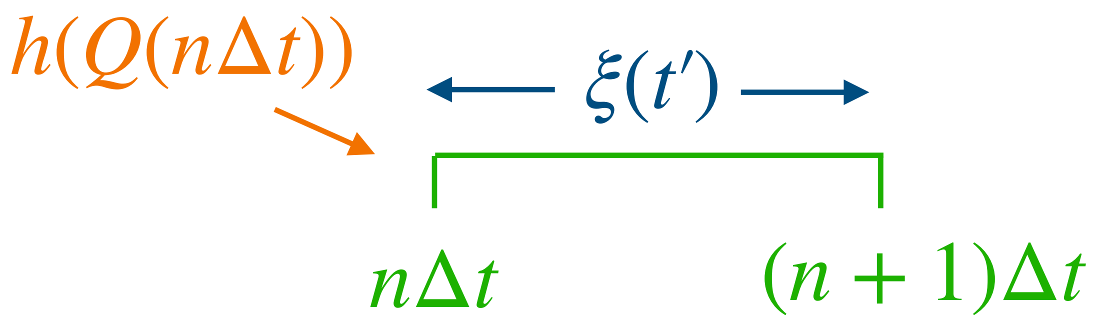

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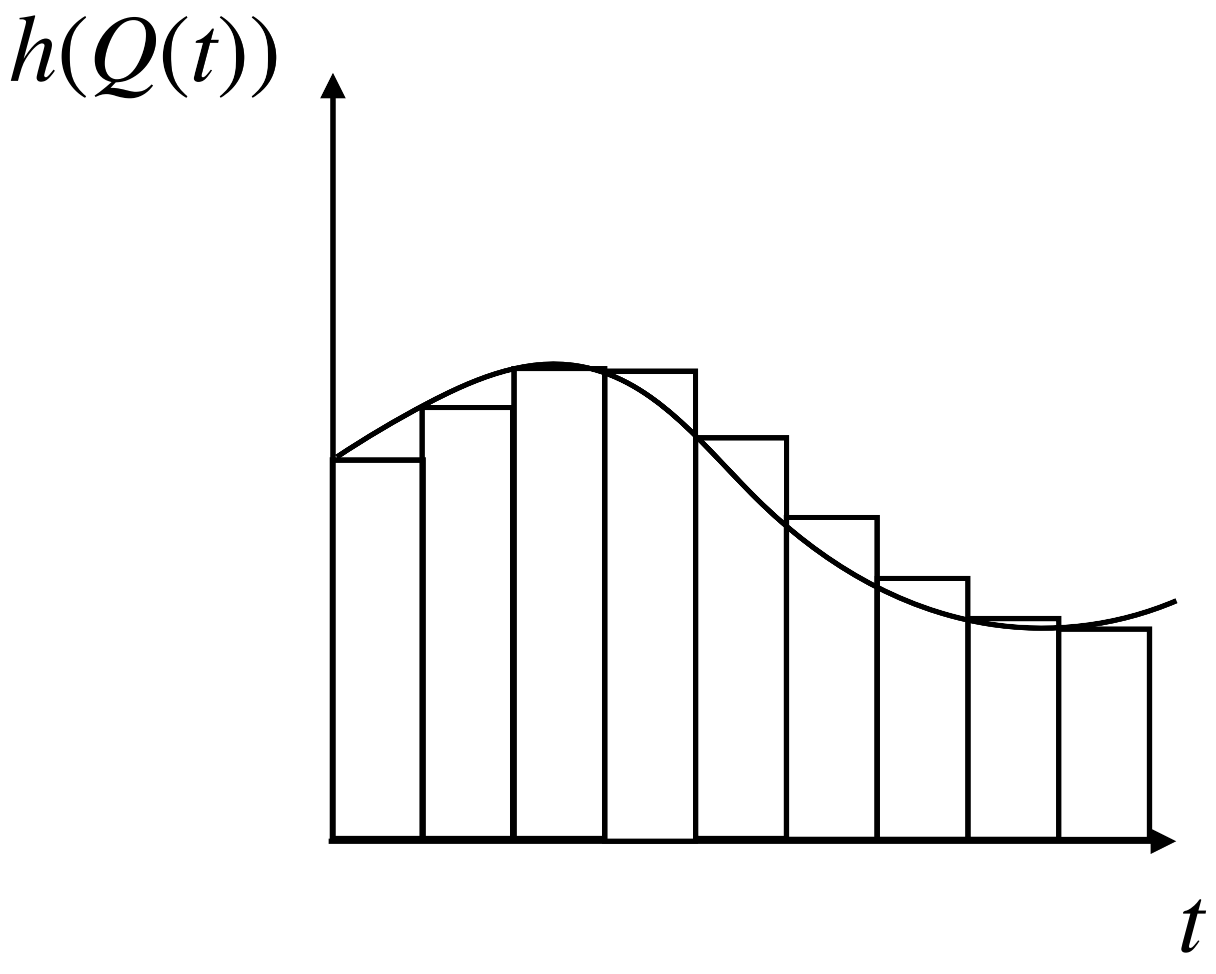

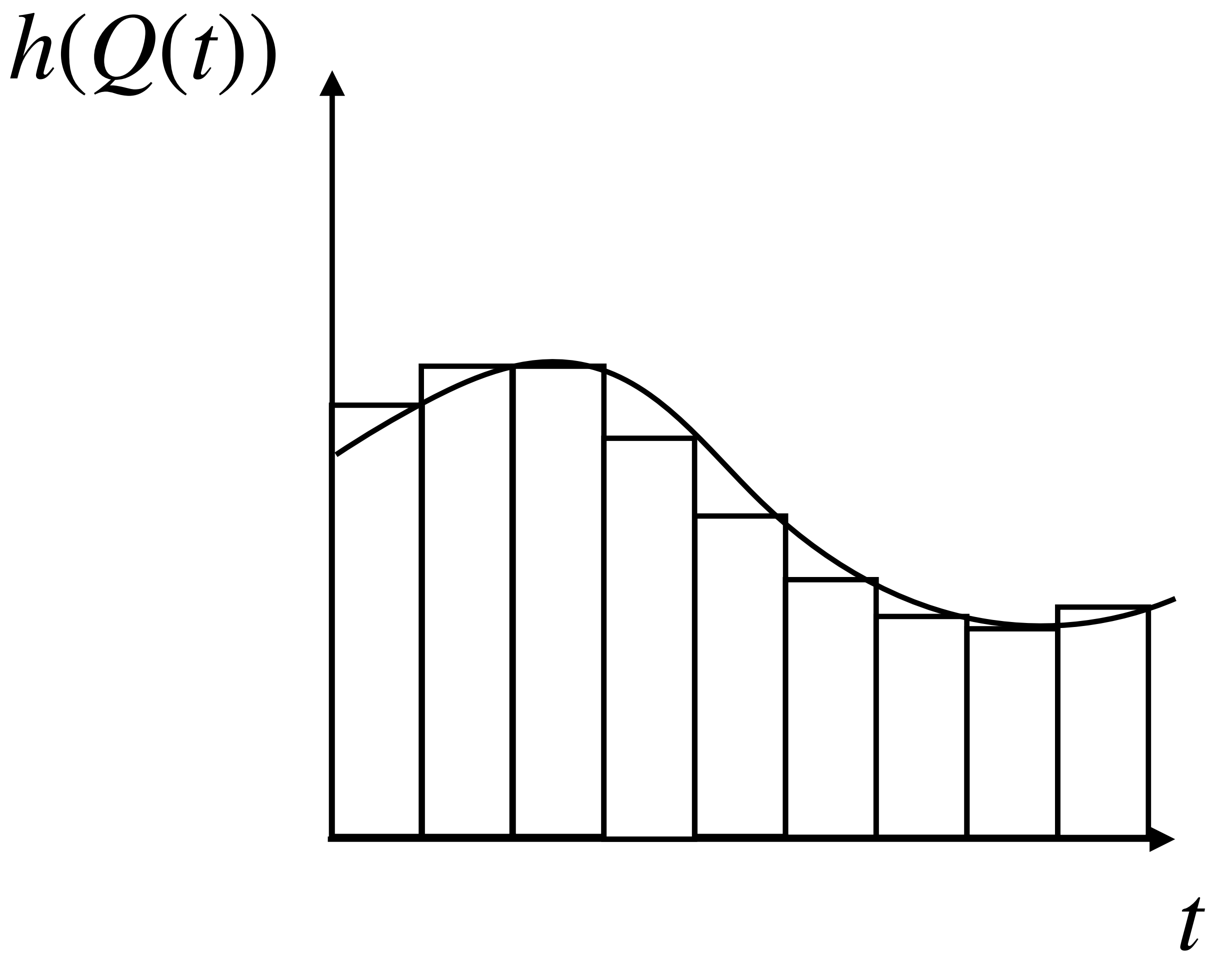

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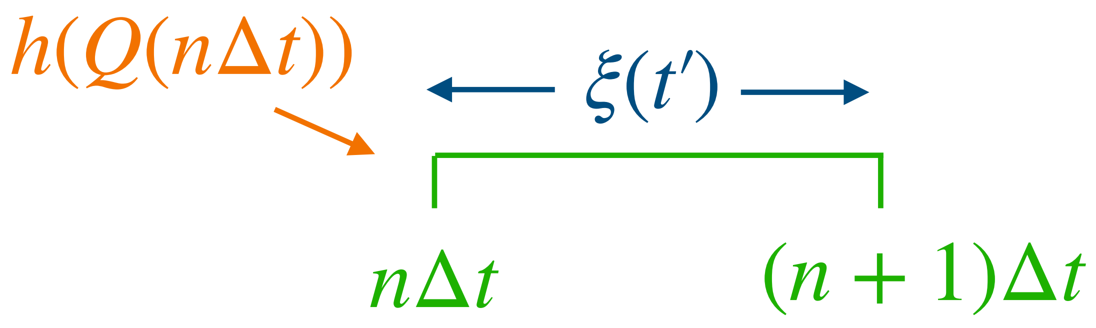
$$= 0$$



Stochastic differential equations

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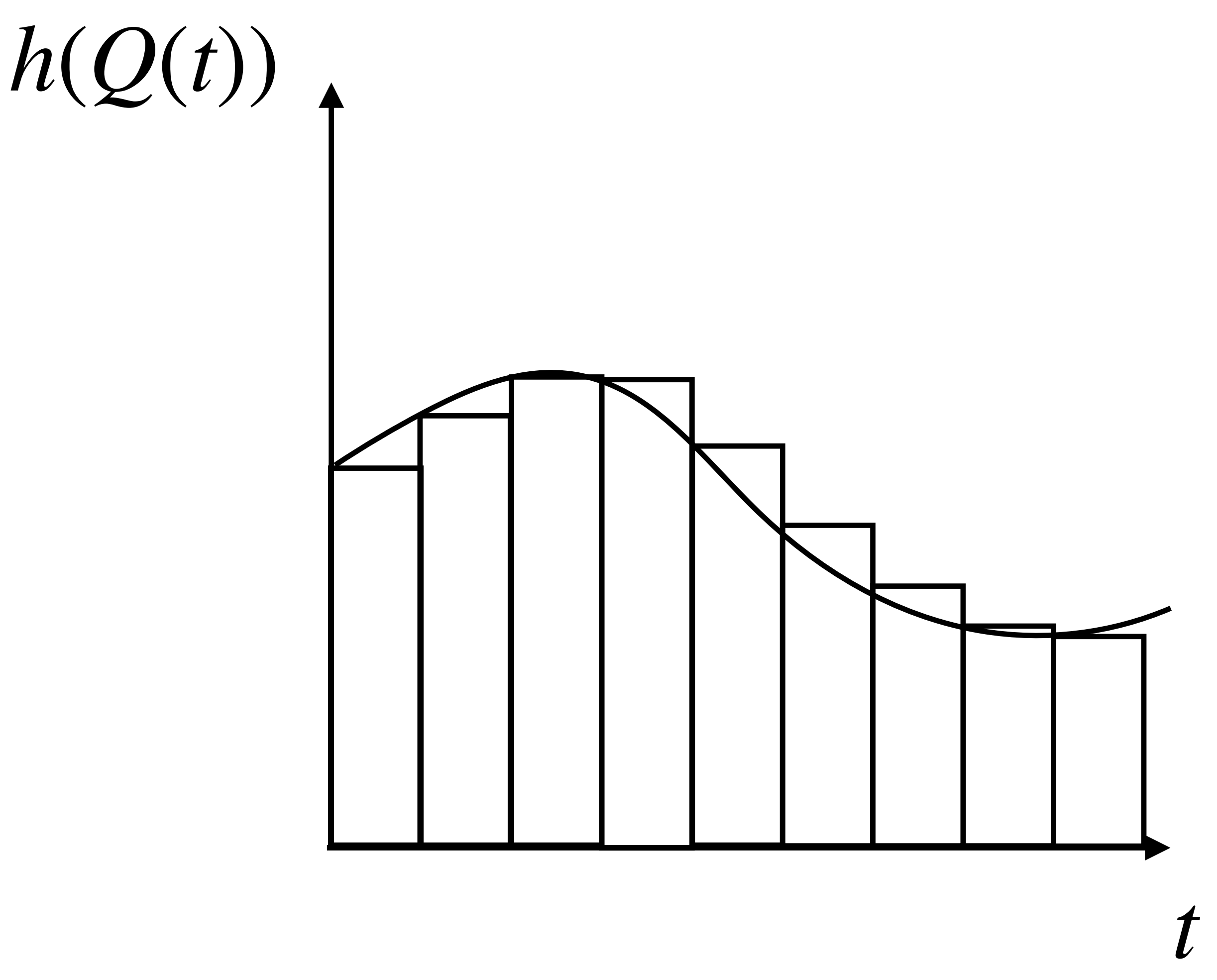

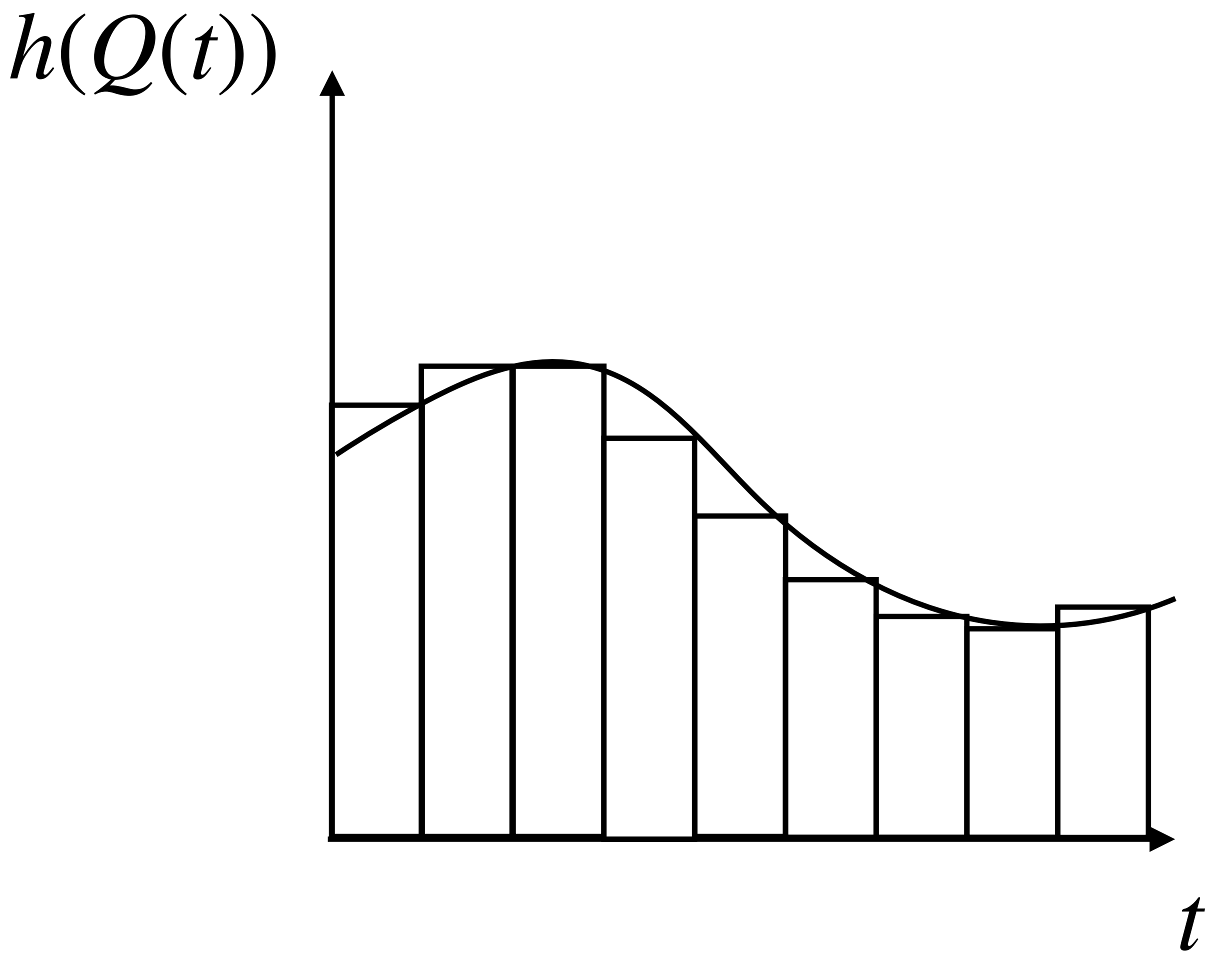

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Stochastic differential equations

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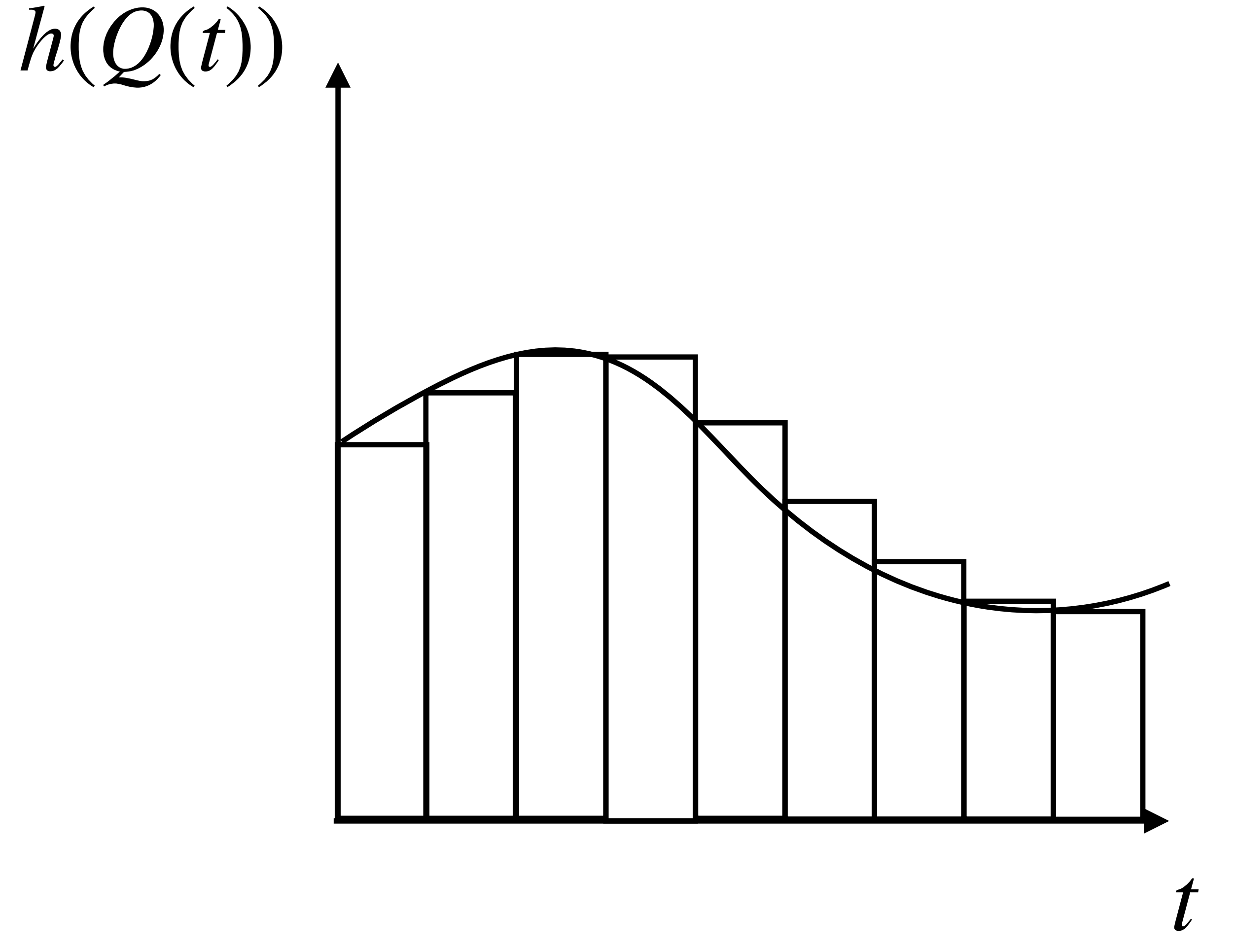

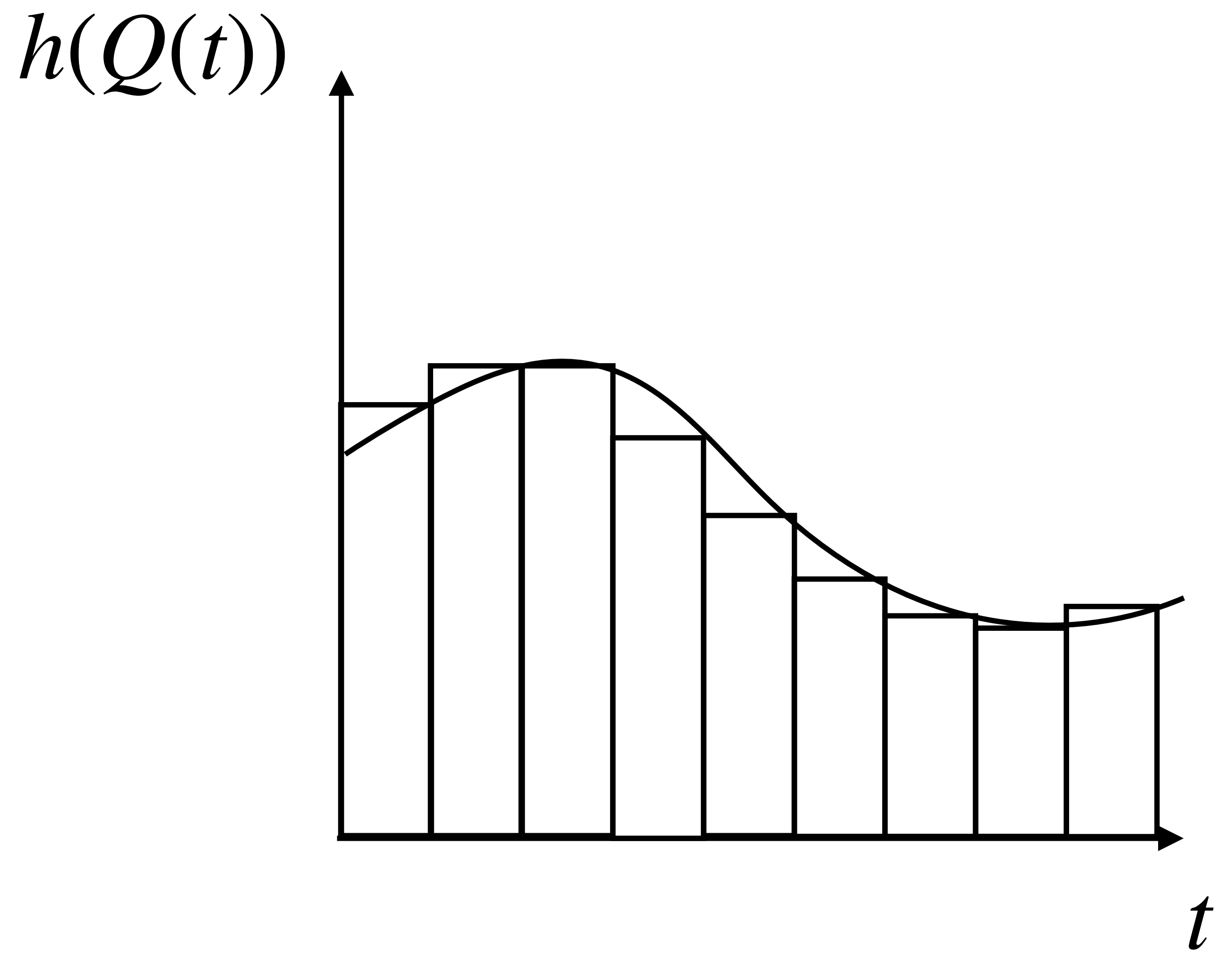

$$\int_0^t h(Q(t'))\xi(t')dt' = \lim_{\Delta t \to 0} \sum_{n=0}^{N-1} h(Q(n\Delta t)) \int_{n\Delta t}^{(n+1)\Delta t} \xi(t')dt'$$

$$\overline{\int_0^t h(Q(t'))\xi(t')dt'} = \lim_{\Delta t \to 0} \sum_{n=0}^{N-1} \int_{n\Delta t}^{(n+1)\Delta t} \overline{h(Q(n\Delta t))\xi(t')}dt'$$

$$= 0$$

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Stochastic differential equations

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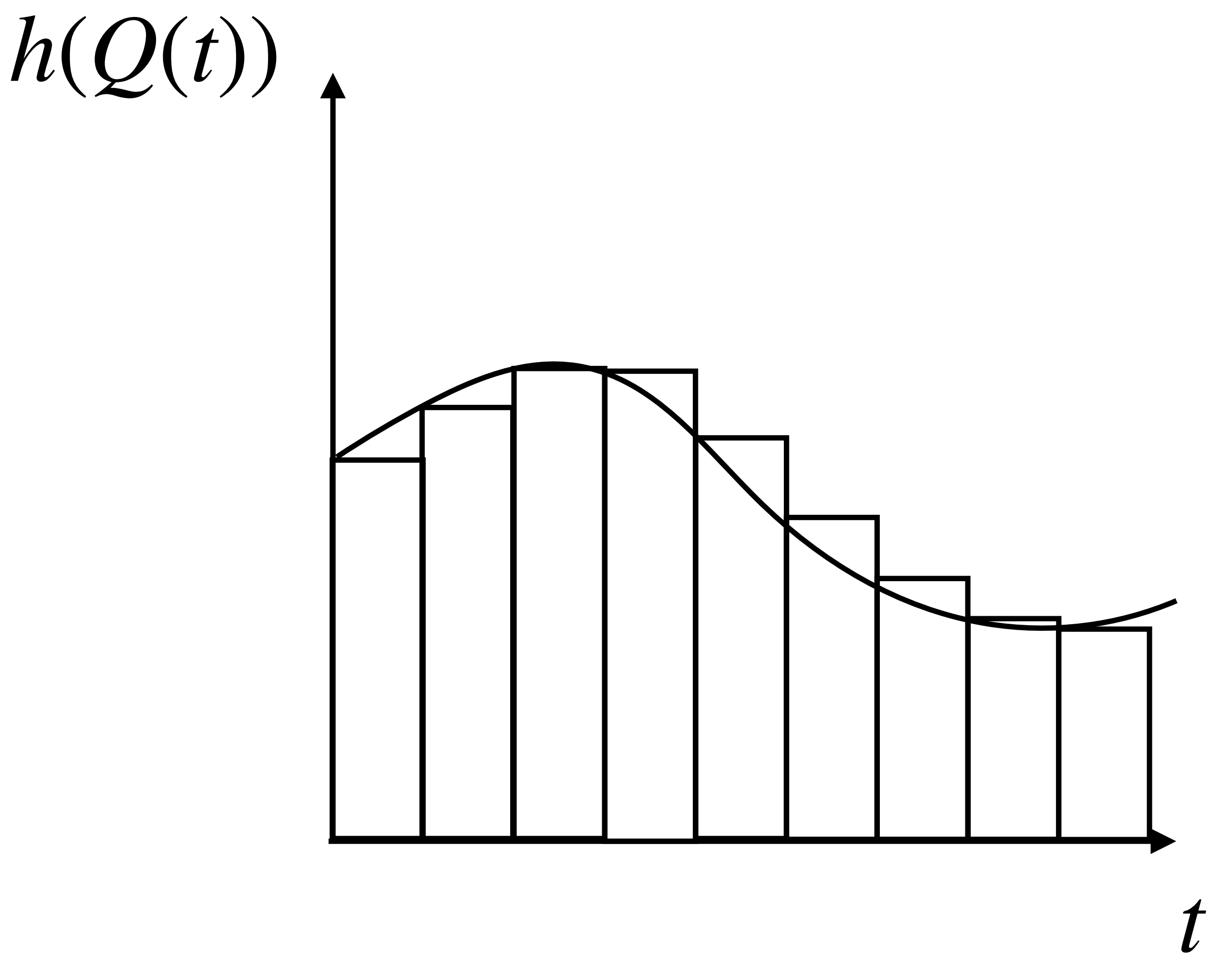

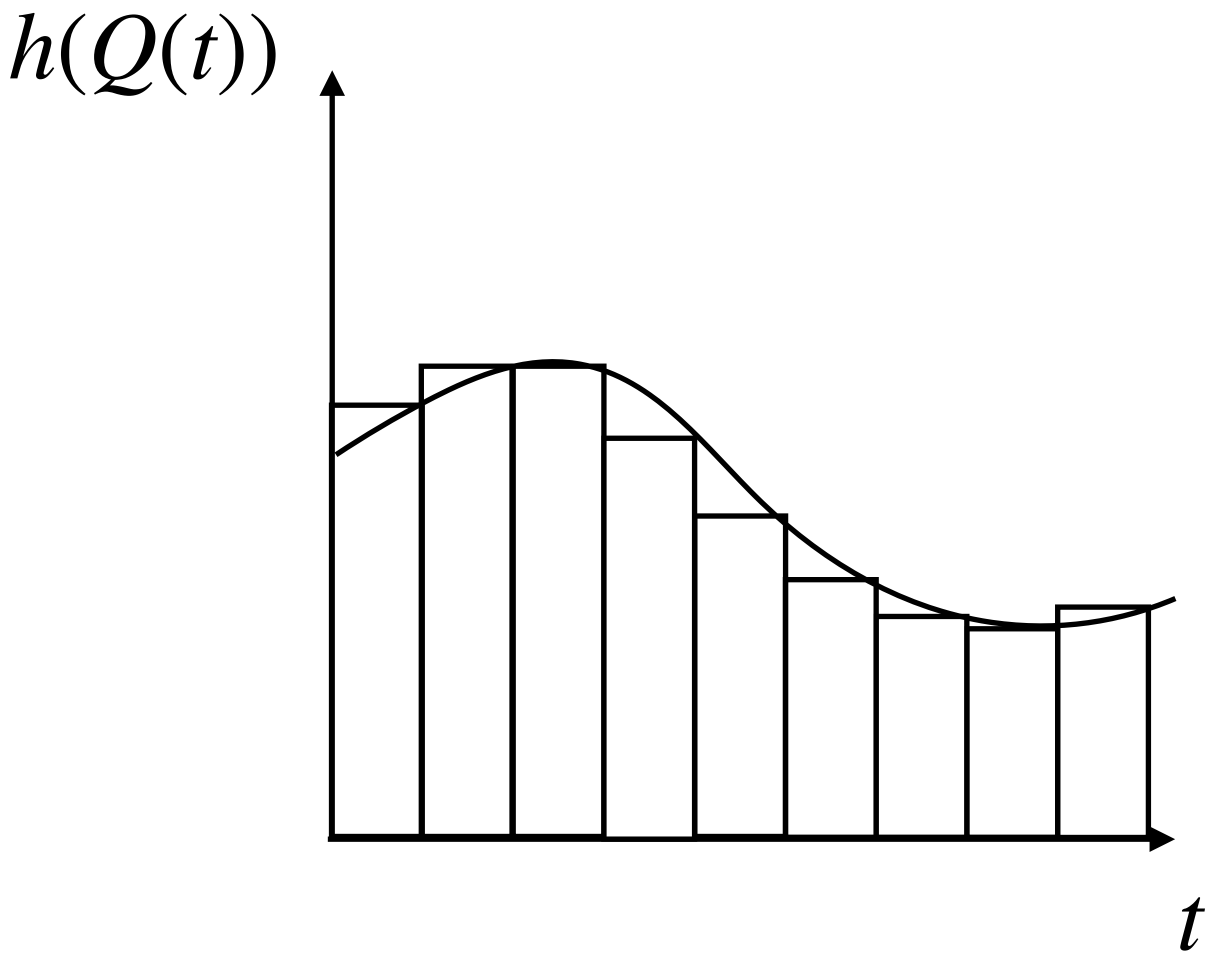

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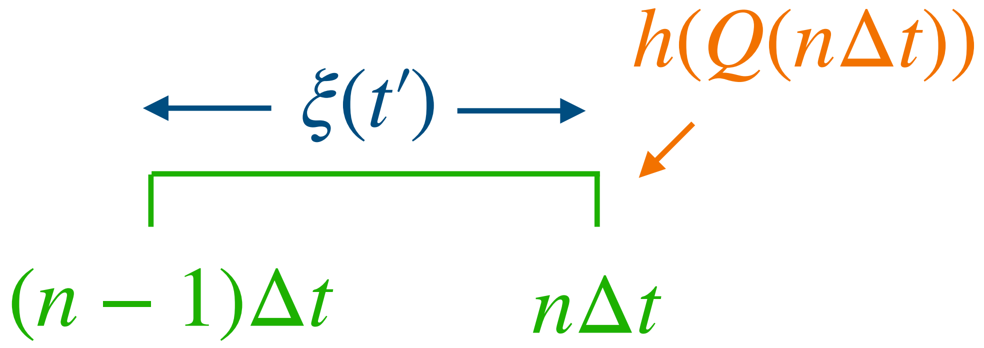

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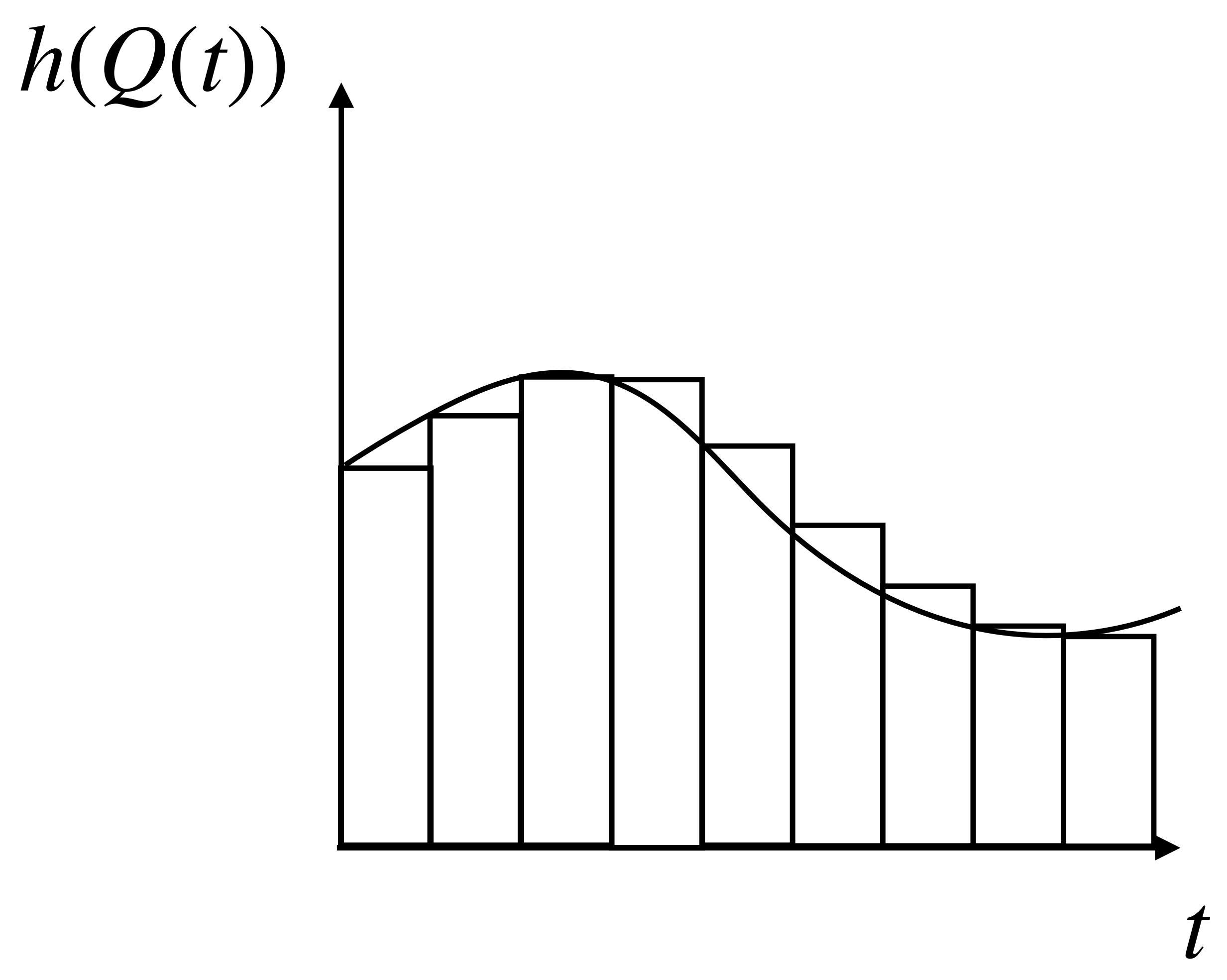

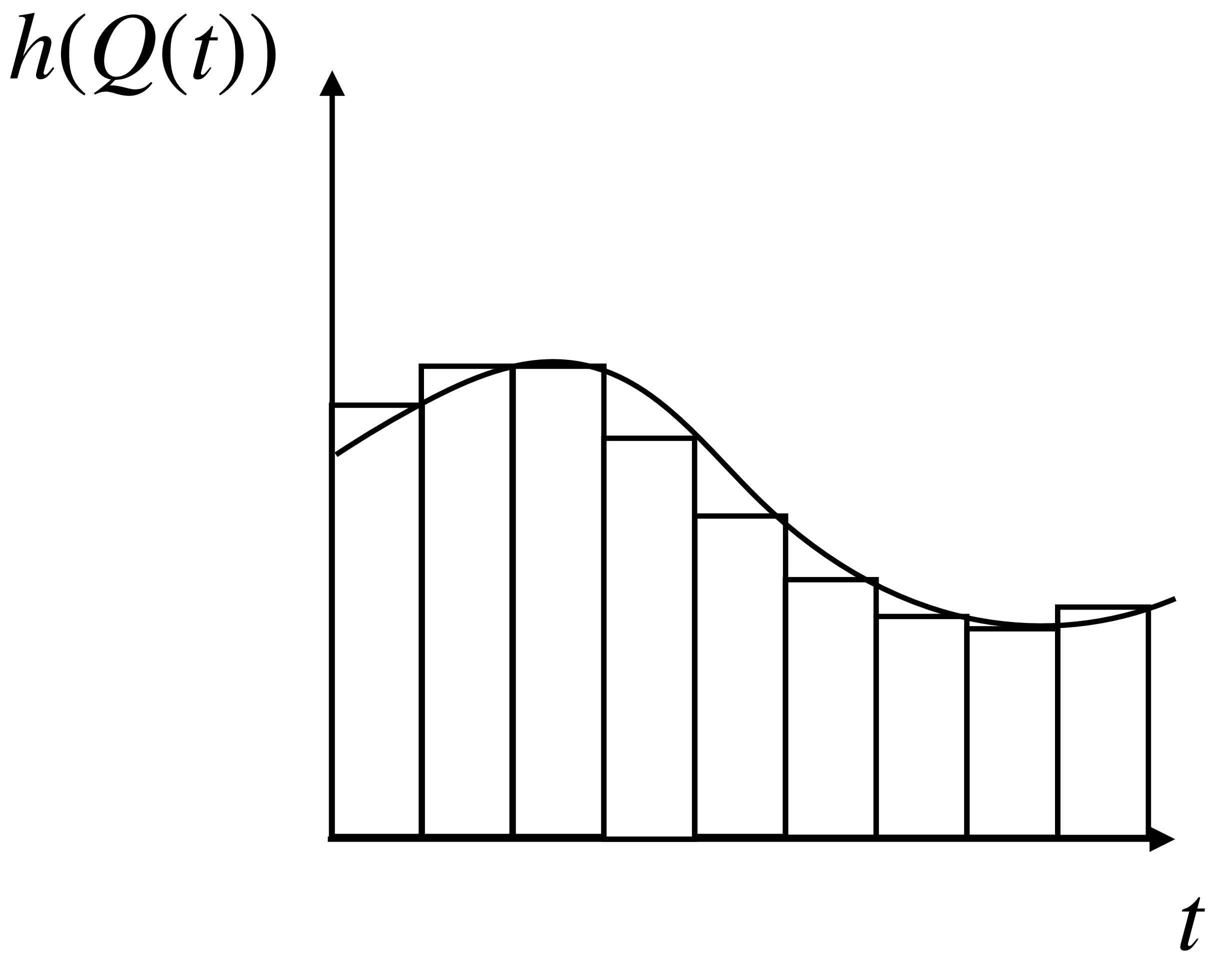

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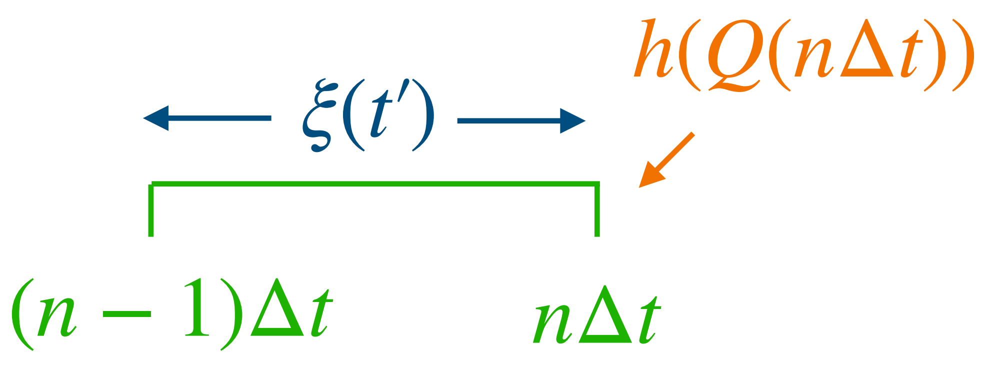

Stochastic differential equations

Consider

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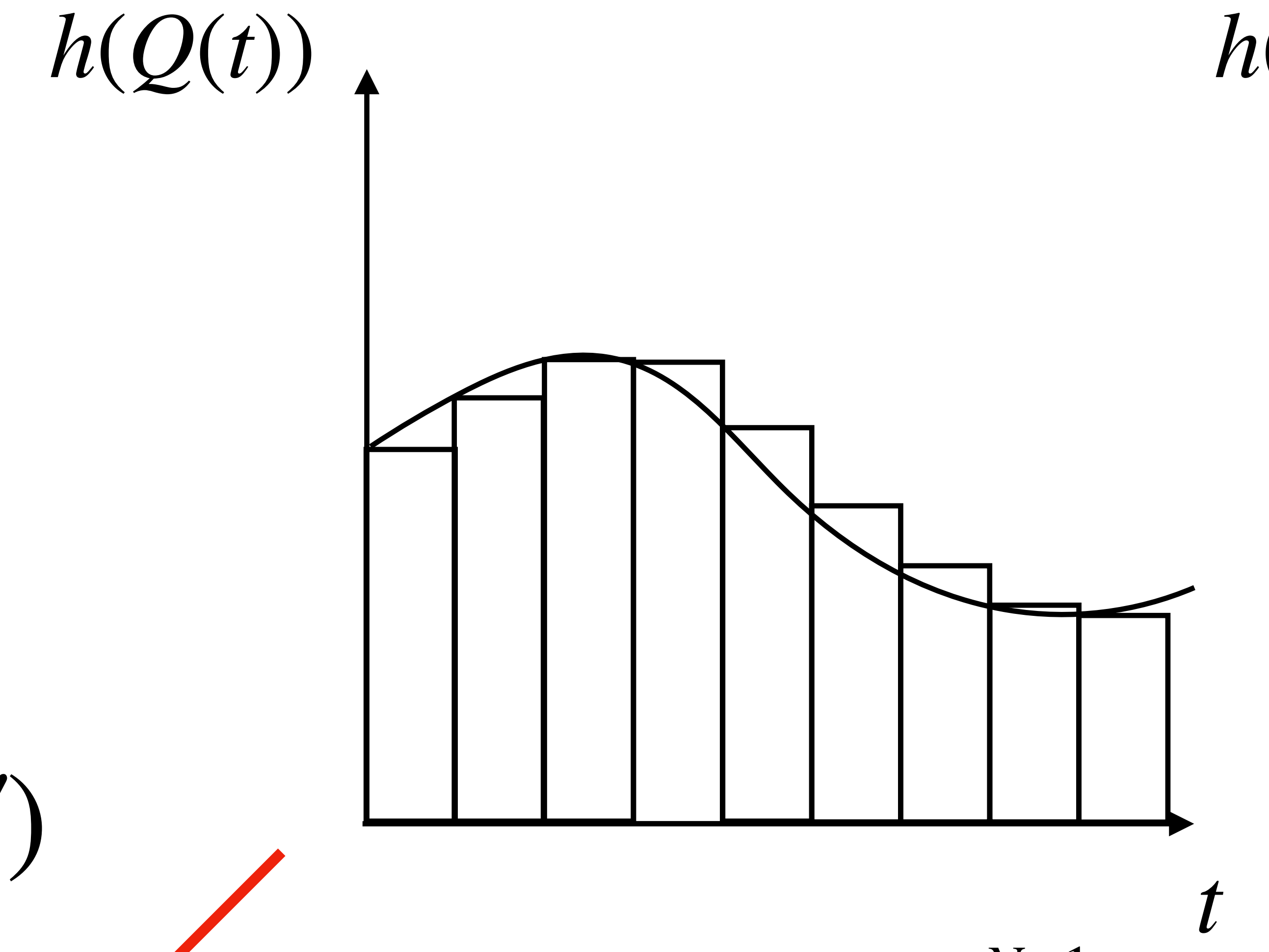
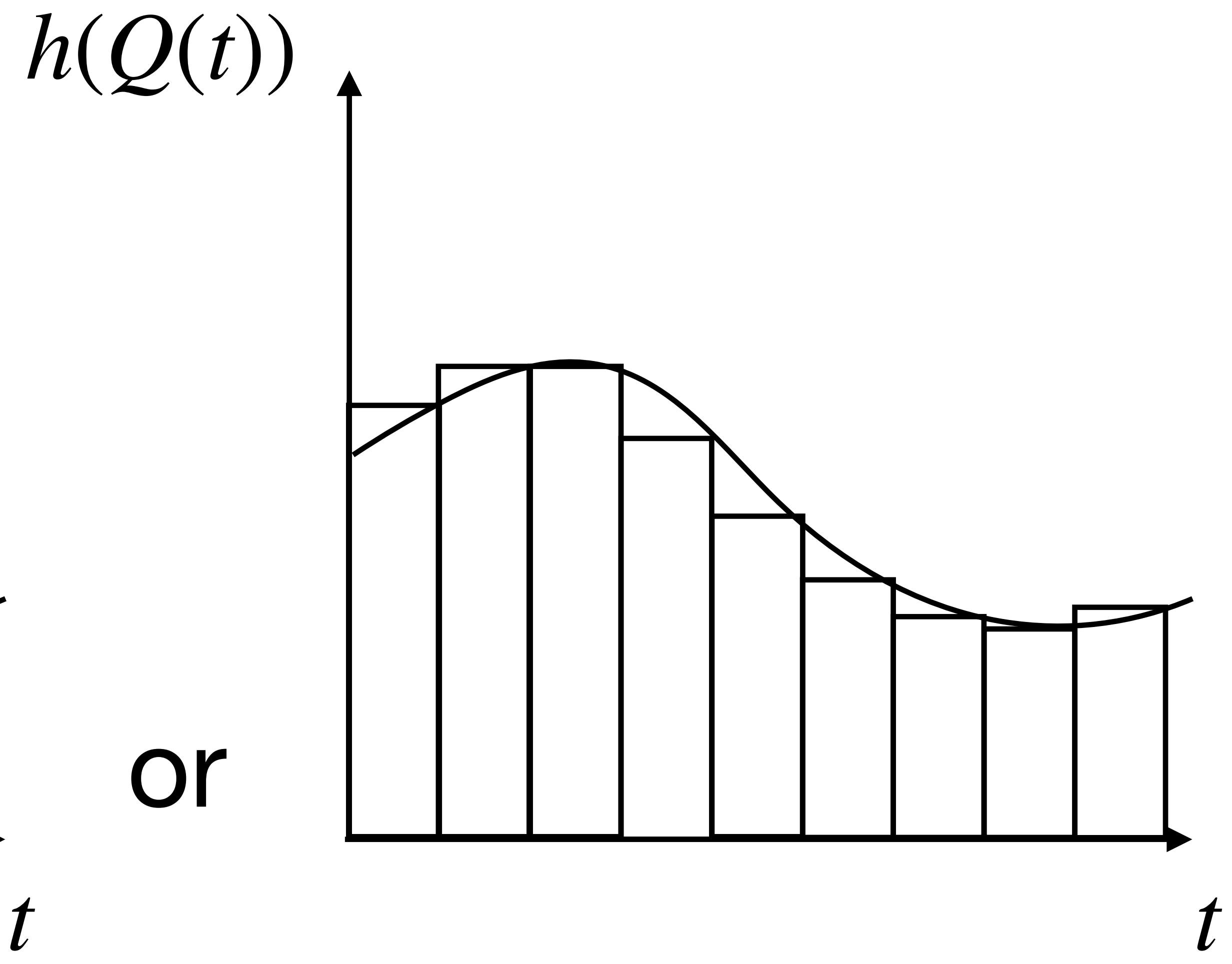
where

$$\overline{\xi(t)} = 0 \qquad \overline{\xi(t)\xi(t')} = D\delta(t - t')$$



or



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In integral form

$$Q(t) = Q(0) + \int_0^t g(Q(t'))dt' + \int_0^t h(Q(t'))\xi(t')dt'$$

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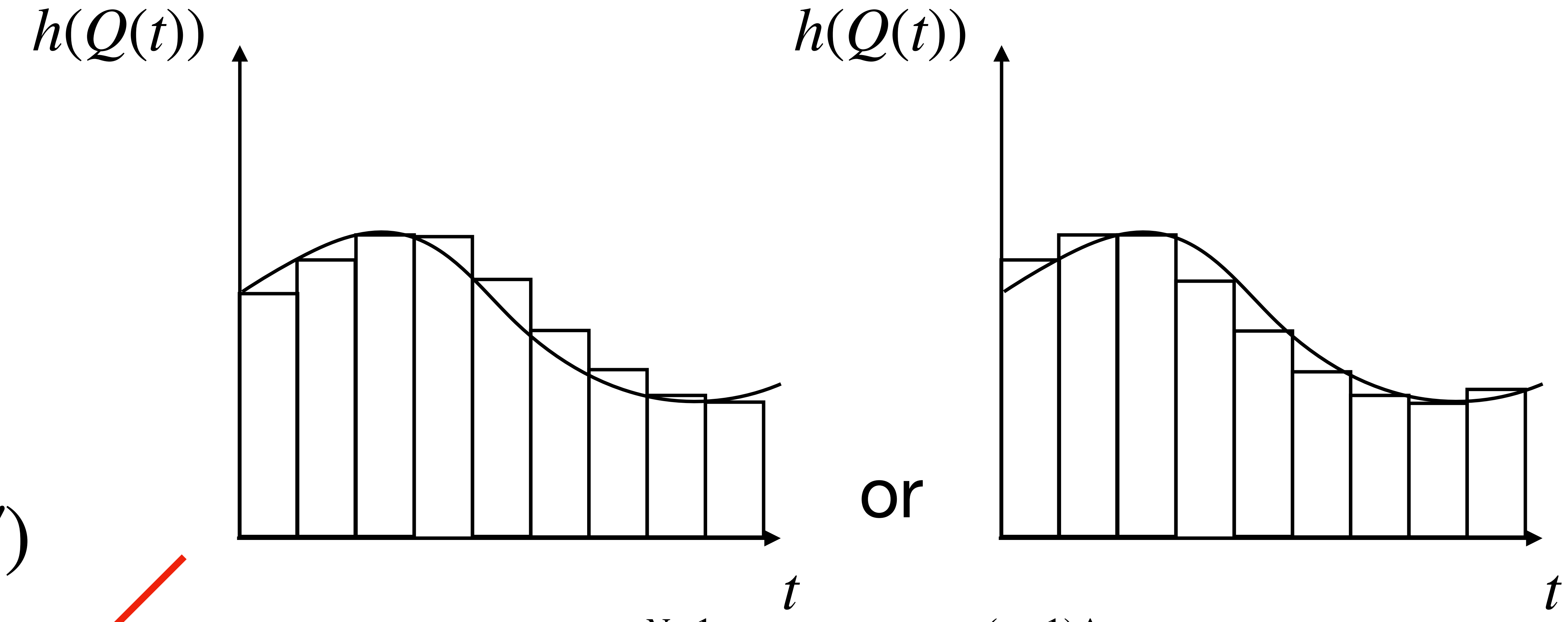

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The SDE must be supplemented by an integration scheme.

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Ito

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Stratonovich

$$\int_0^t h(Q(t'))\xi(t')dt' = \lim_{\Delta t \to 0} \sum_{n=0}^{N-1} h\left(\frac{Q((n+1)\Delta t) + Q(n\Delta t)}{2}\right) \int_{n\Delta t}^{(n+1)\Delta t} \xi(t')dt'$$

Stochastic differential equations

Consider the Stratonovich SDE

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In some instances, one can solve this analytically, e.g., the solution to

$$\dot{Q}(t) = -\frac{Q(t)}{\tau} + \frac{\xi(t)}{\tau}$$

is

$$Q(t) = Q_0 e^{-\frac{t}{\tau}} + \frac{1}{\tau}\int_0^t e^{-\frac{t-s}{\tau}}\xi(s)ds$$

Stochastic differential equations

Consider the Stratonovich SDE

$$\dot{Q}(t) = g(Q(t)) + h(Q(t))\xi(t)$$

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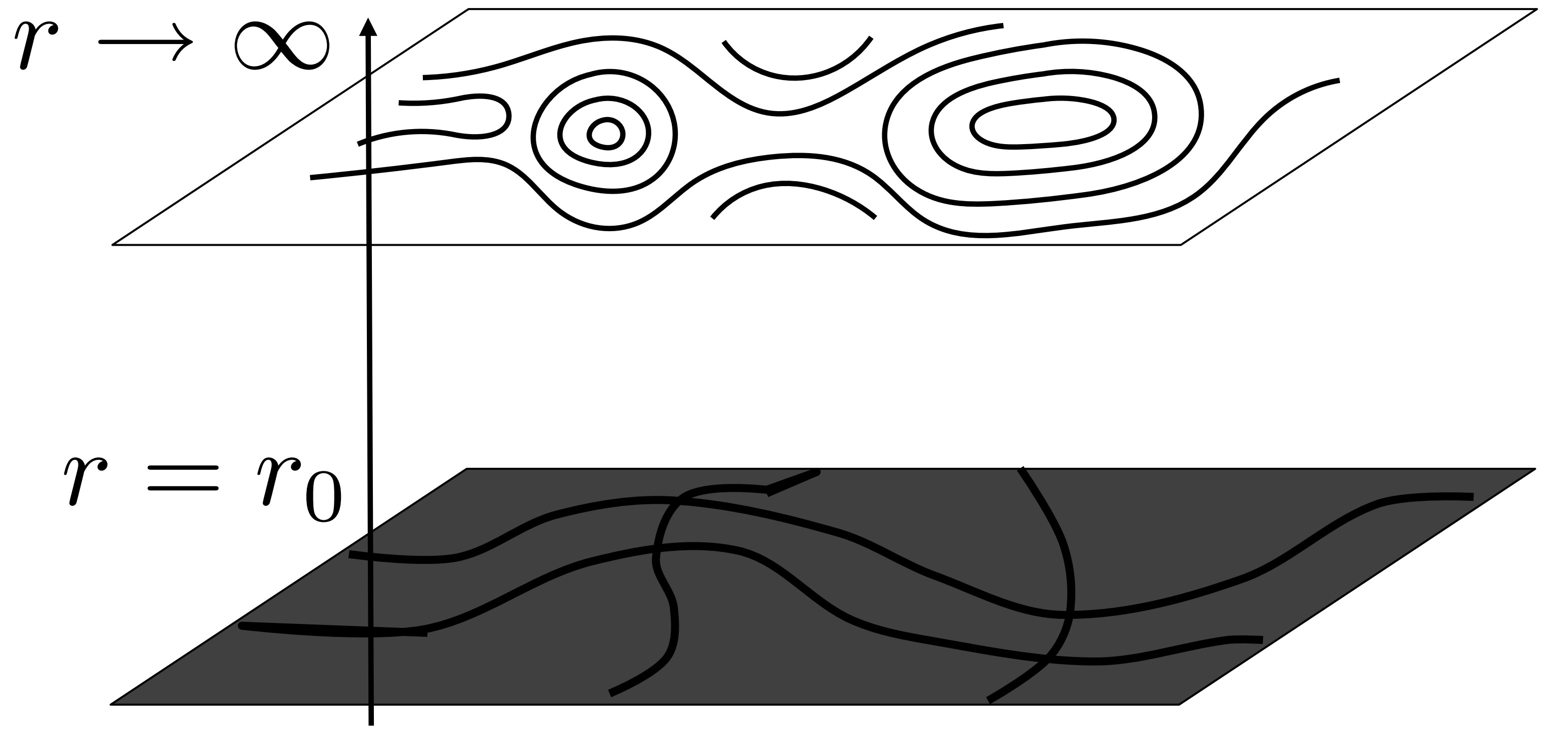
More generally, one has to resort to numerics, e.g.,

$$Q(t + \Delta t) = Q(t) + h(Q(t))\sqrt{\Delta t D}\phi + \left(g(Q(t)) + \frac{D}{2}h(Q(t))h'(Q(t))\right)\Delta t + \mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)$$

Holographic turbulence

$$\nabla_\mu^{(0)} T_{(0)}^{\mu\nu} = -F^\nu$$

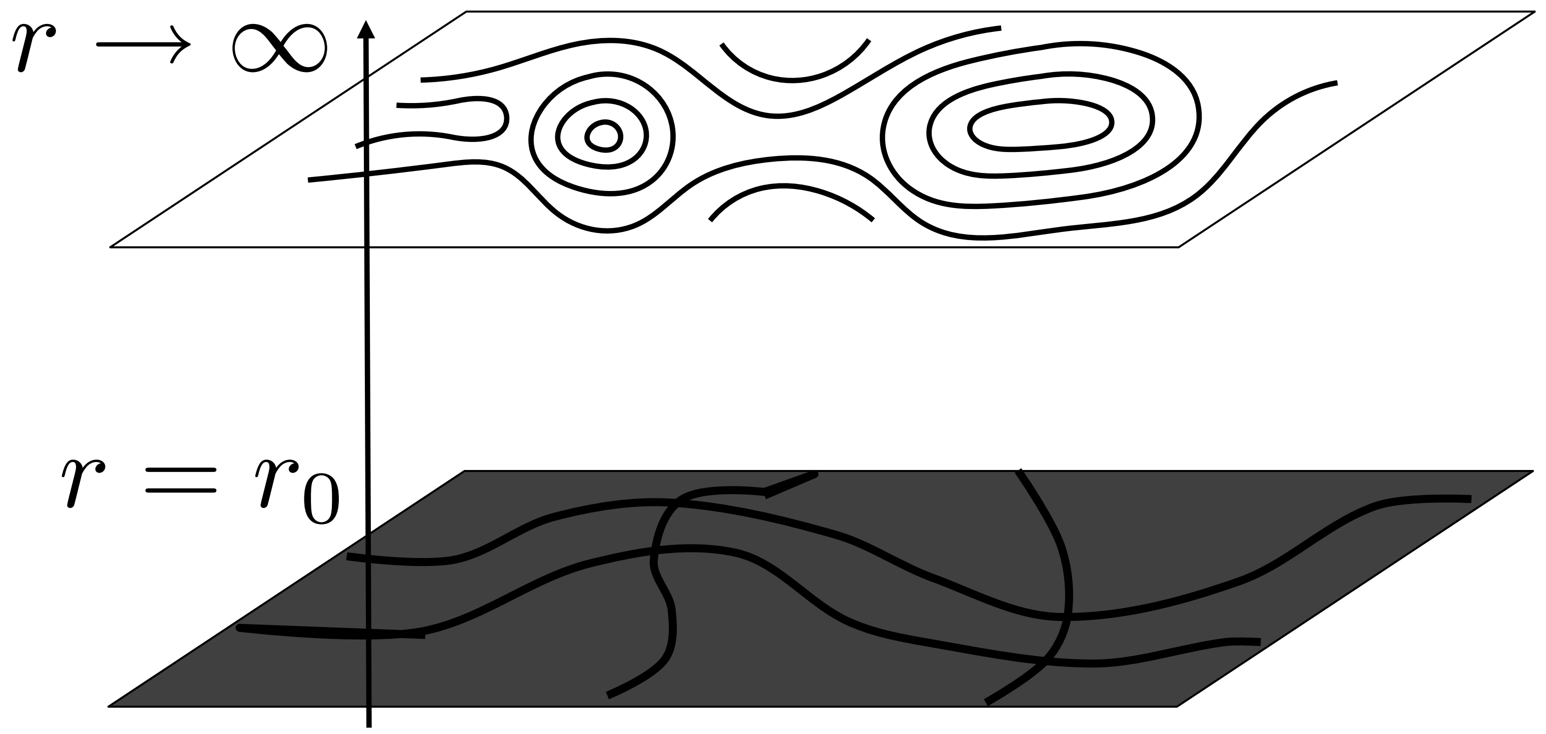
$$\bar{g}_{\mu\nu} = \eta_{\mu\nu}$$



Holographic turbulence

We wish to solve

$$R_{mn} - \frac{1}{2}Rg_{mn} - \frac{12}{\ell^2}g_{mn} = 0$$



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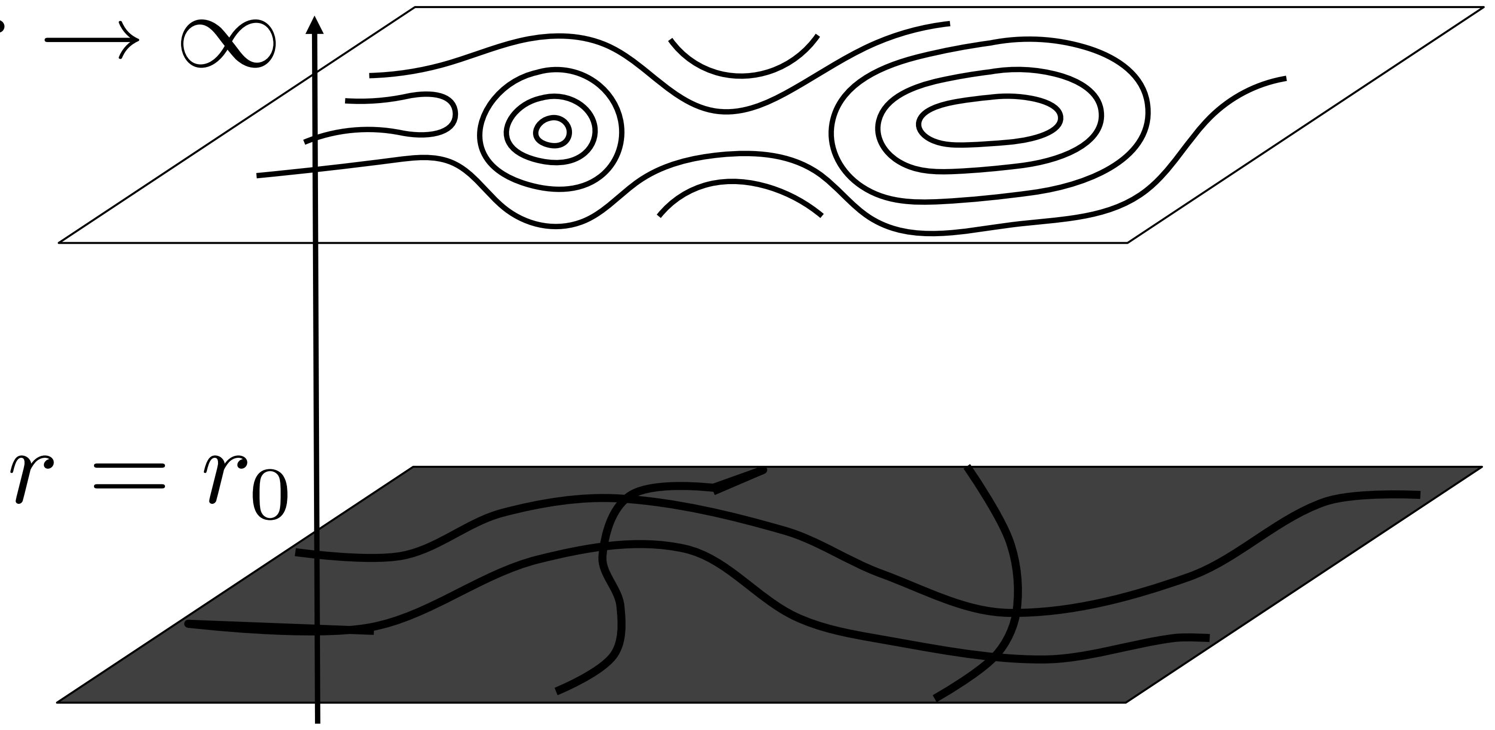

$$R_{mn} - \frac{1}{2} R g_{mn} - \frac{12}{\ell^2} g_{mn} = 0$$

such that at t<0 we have

$$ds^2 = r^2(-f(r)dt^2 + (dx^1)^2 + (dx^2)^2) + \frac{dr^2}{r^2 f(r)}$$

$$f(r) = \left(1 - \left(\frac{r_0}{r}\right)^3\right)$$

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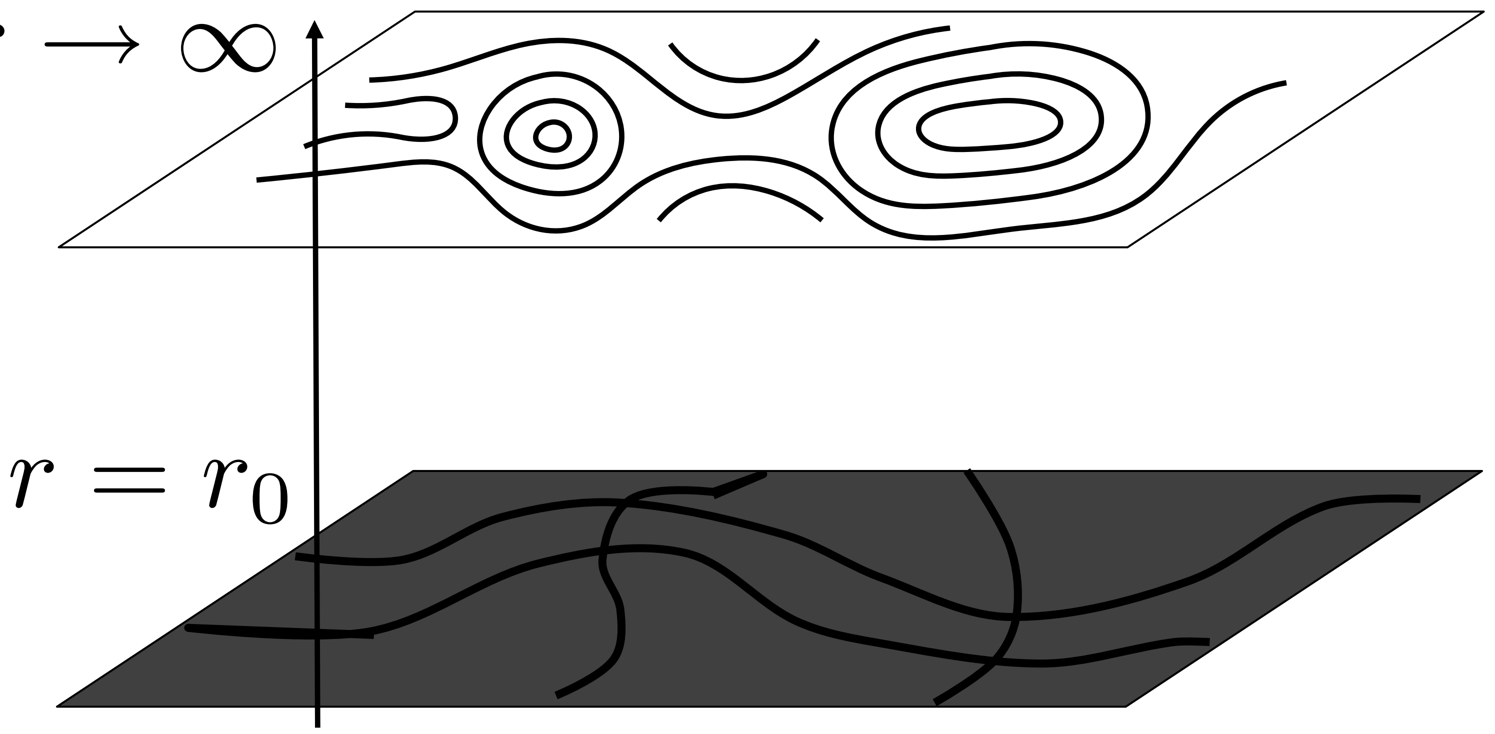

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At t>0 we could have

$$g_{mn} \xrightarrow[r\to\infty]{} g^{(0)}_{\mu\nu} \qquad g^{(0)}_{\mu\nu} dx^\mu dx^\nu = (1+\xi)\big((dx^1)^2 + (dx^2)^2\big) - dt^2$$

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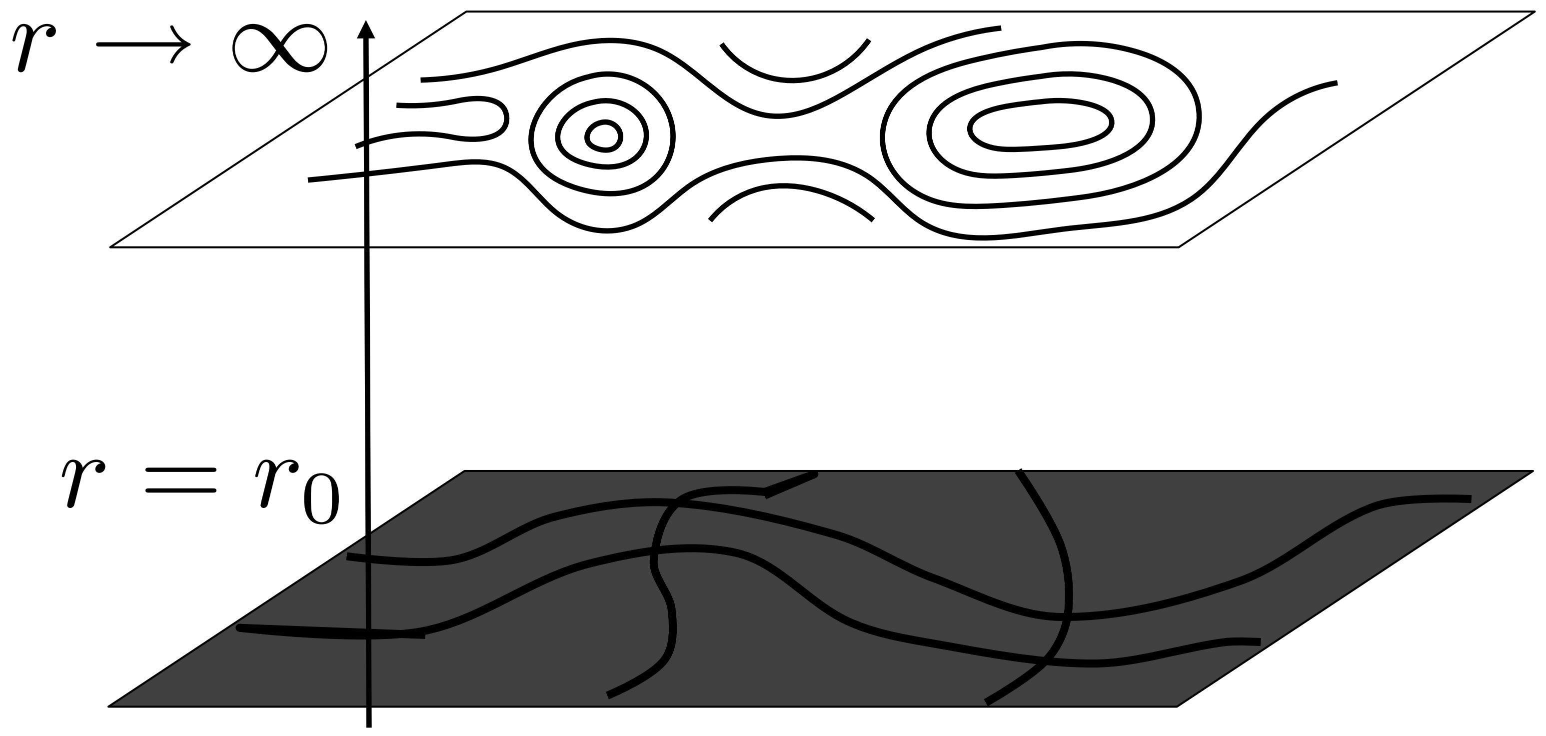

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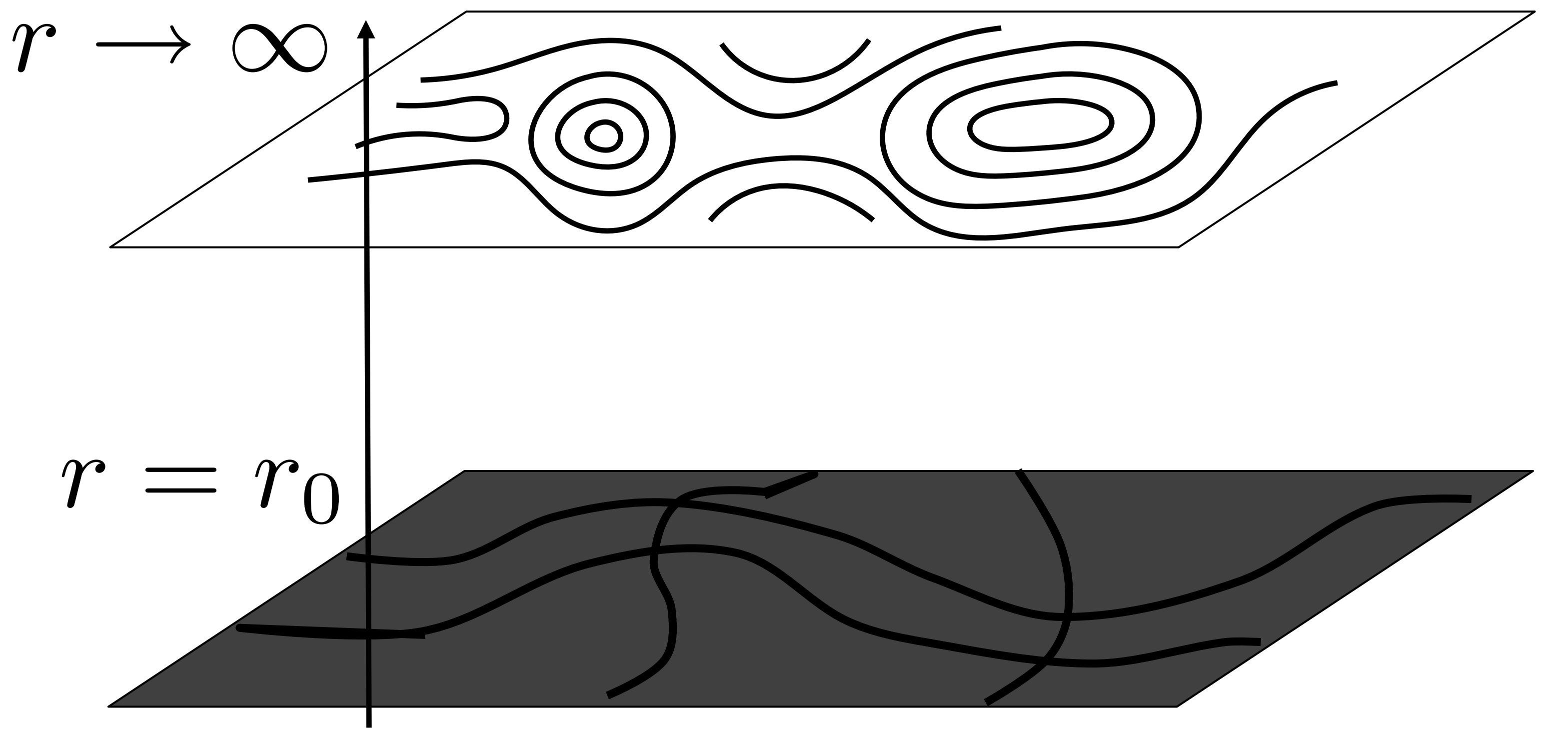

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It is straightforward to show that this results in SDE's which are polynomial in ξ.

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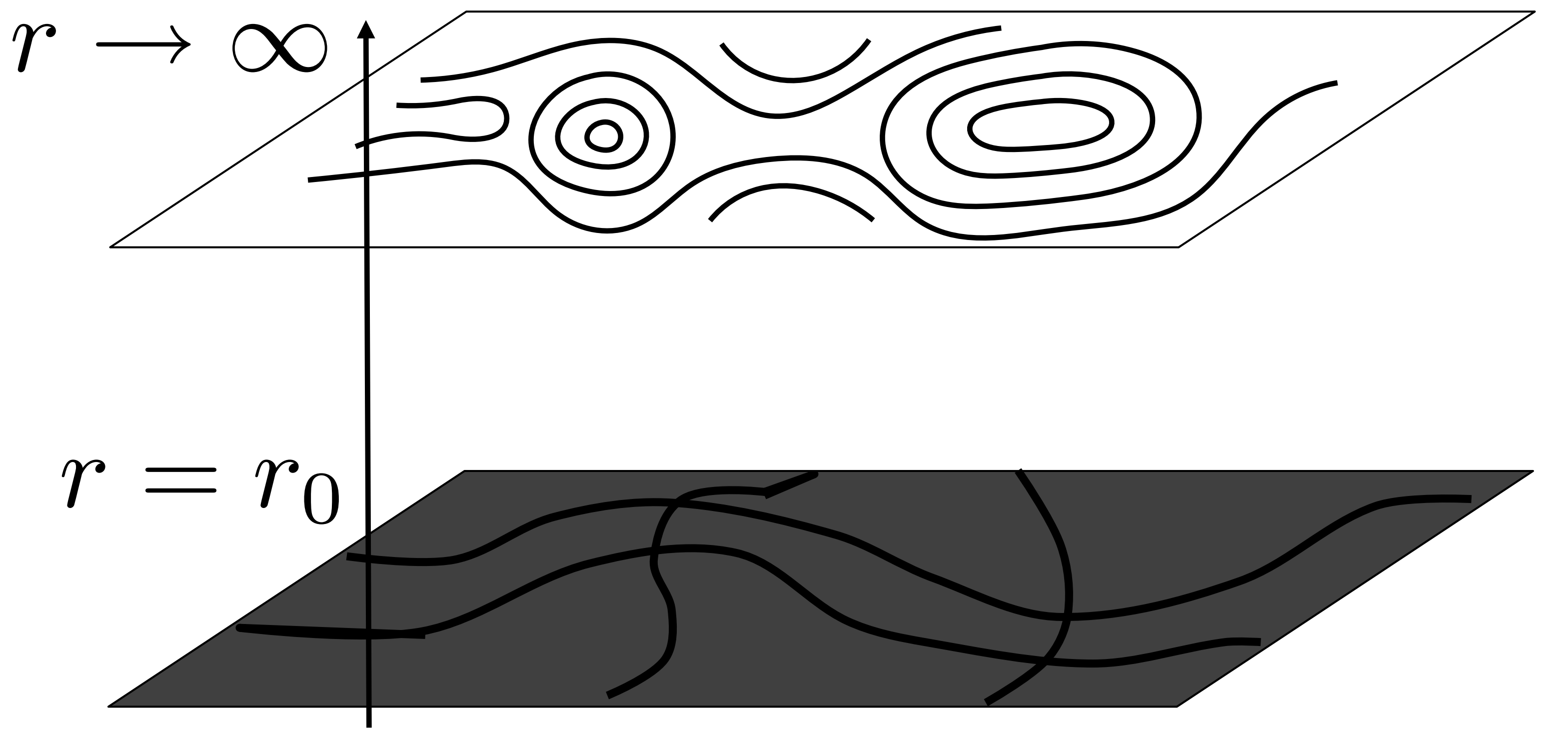

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But then,

$$\overline{\xi(t, \vec{x})^2} = D(0)\delta(0)$$

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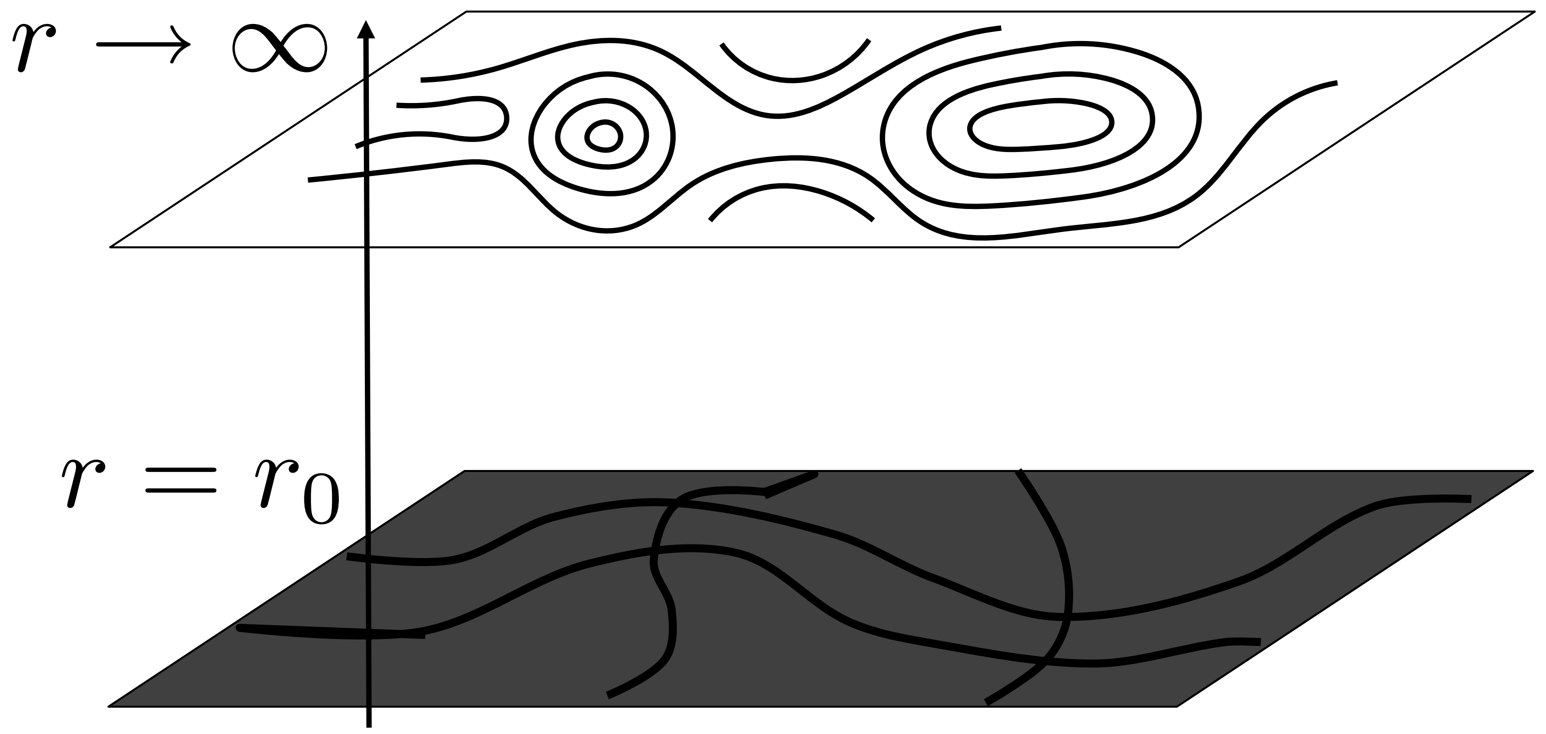

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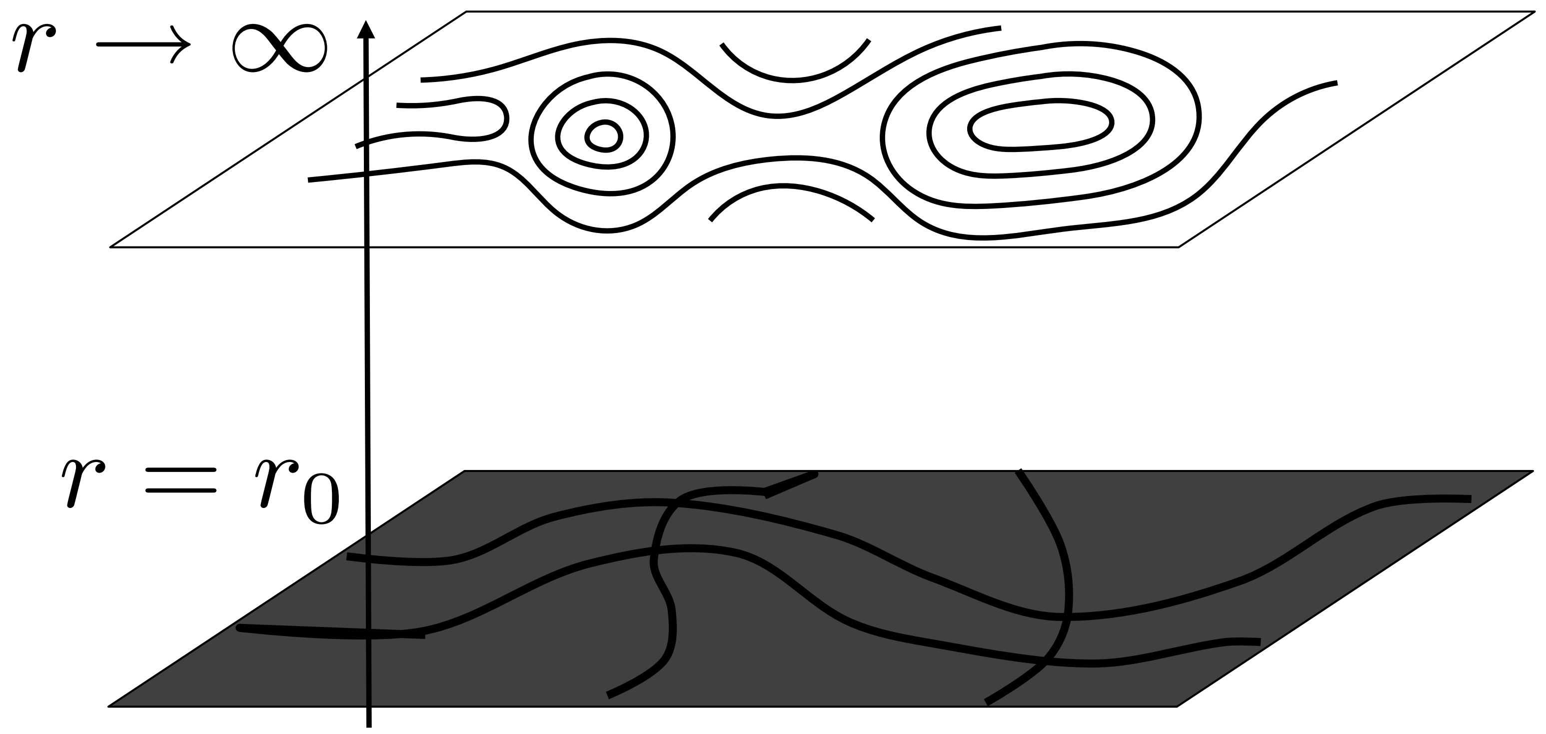

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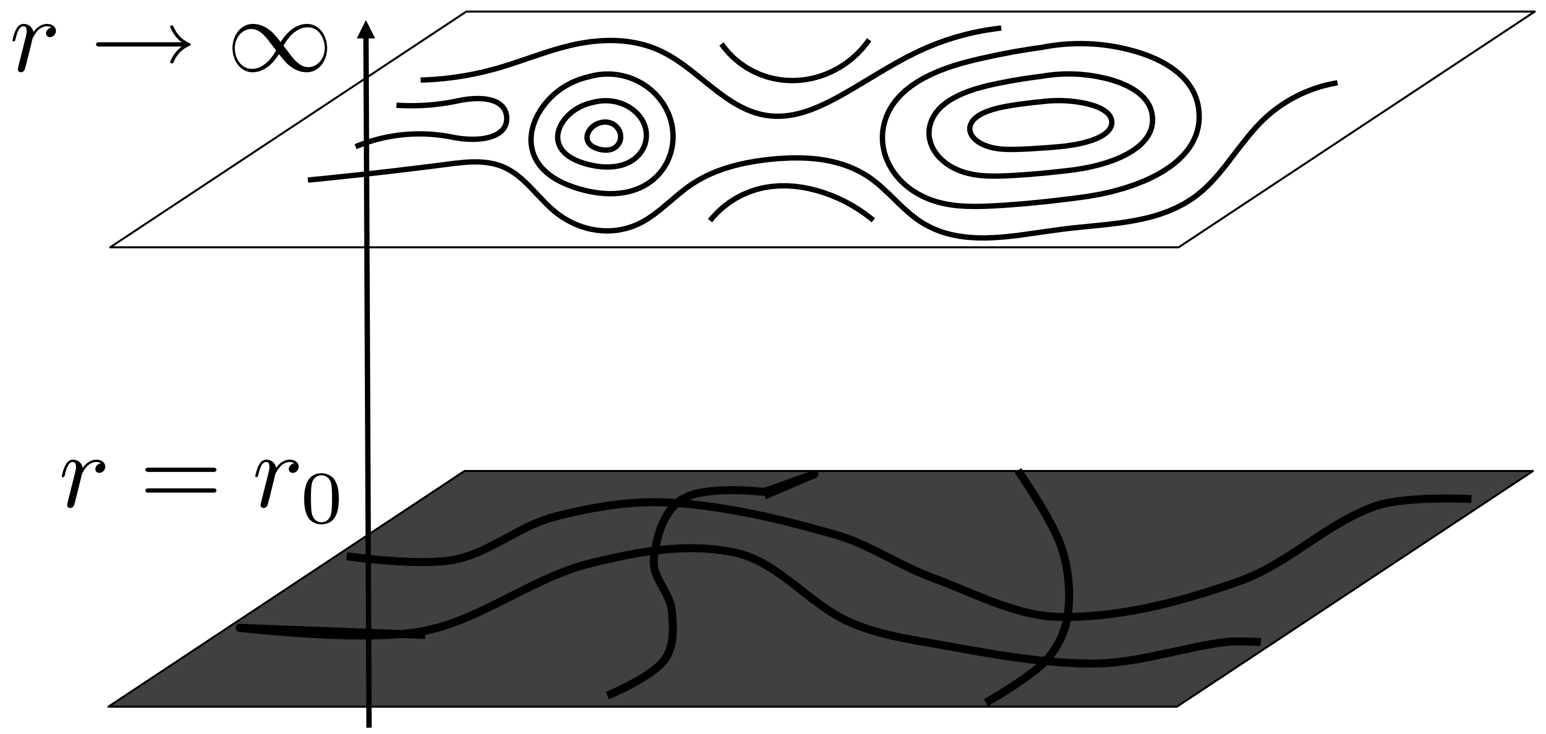

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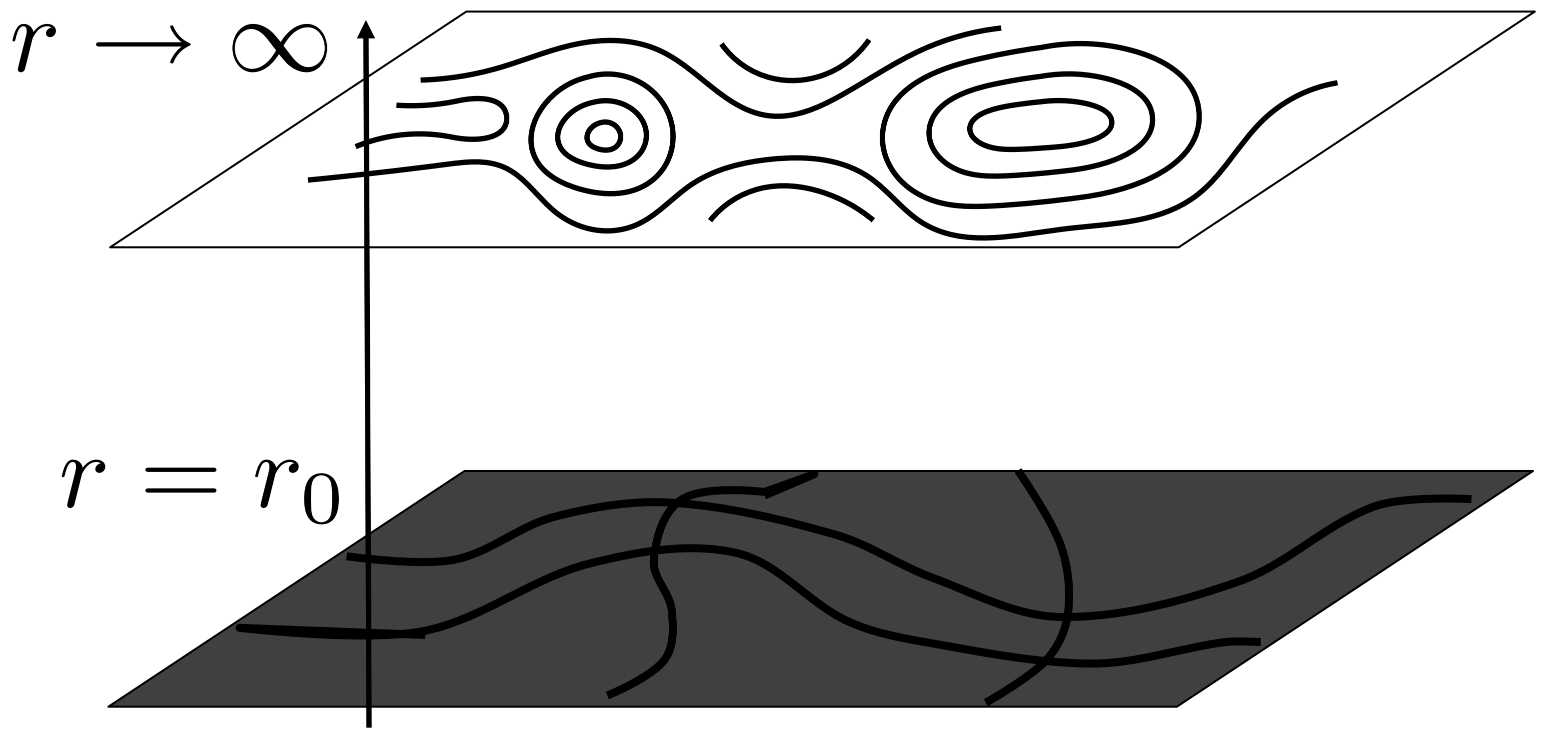

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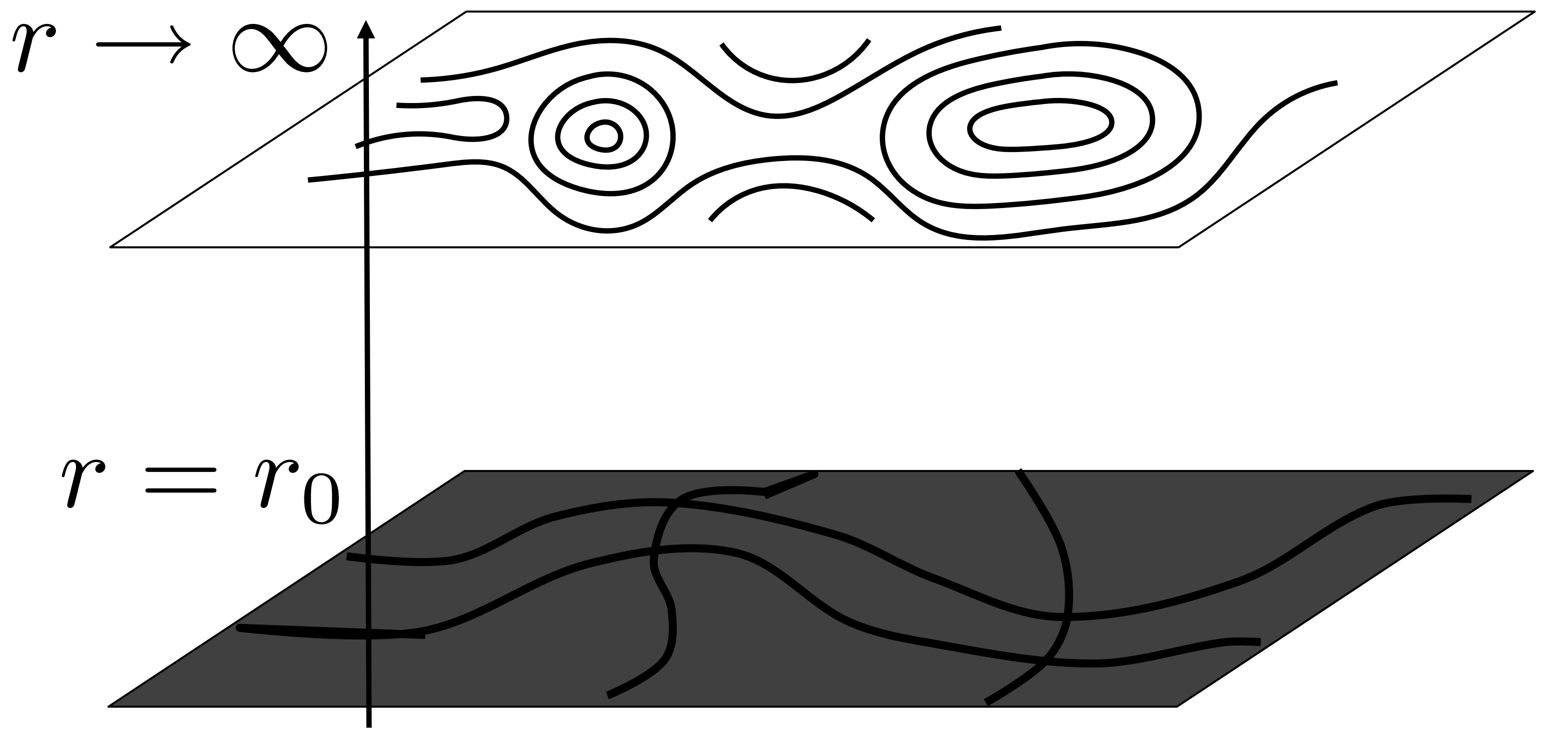

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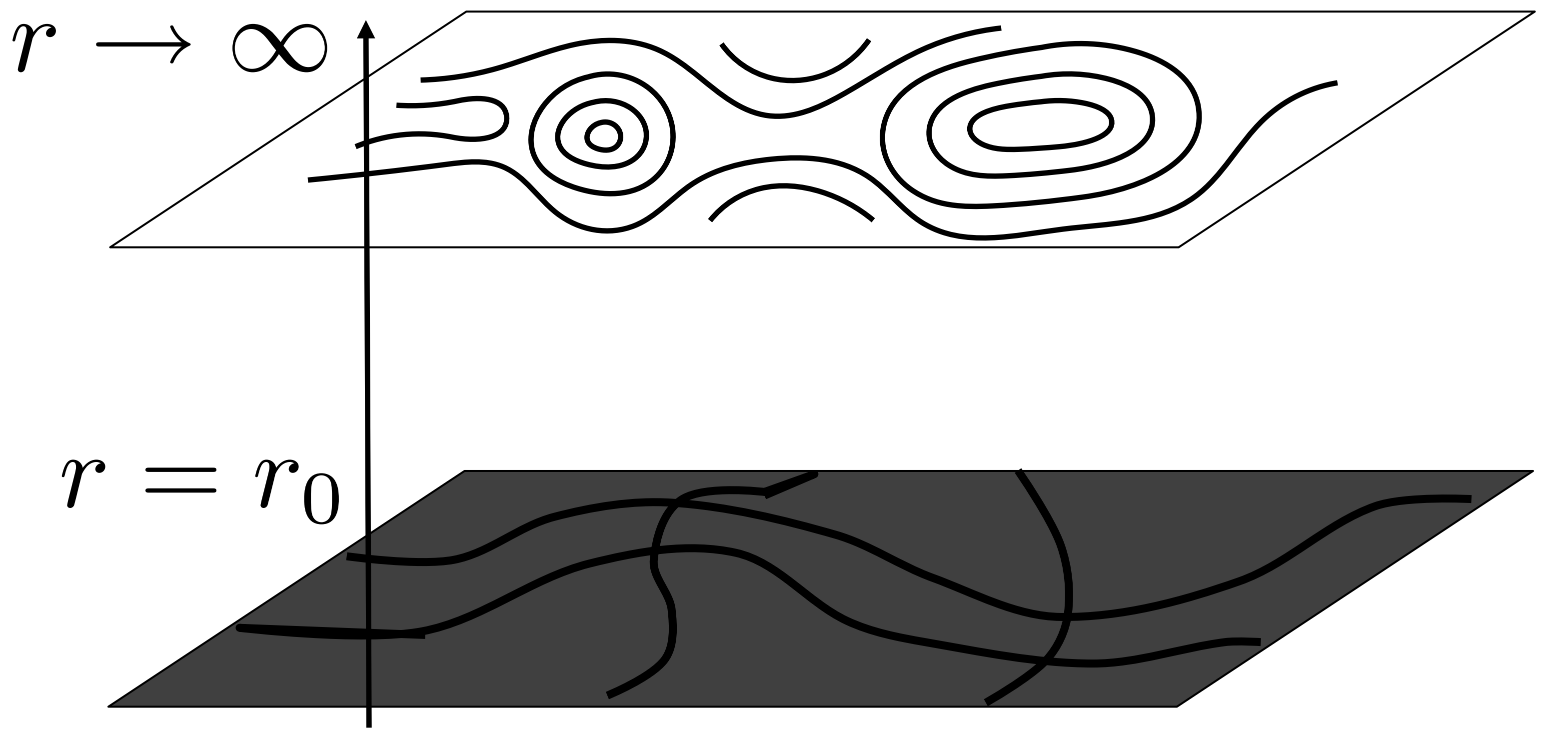

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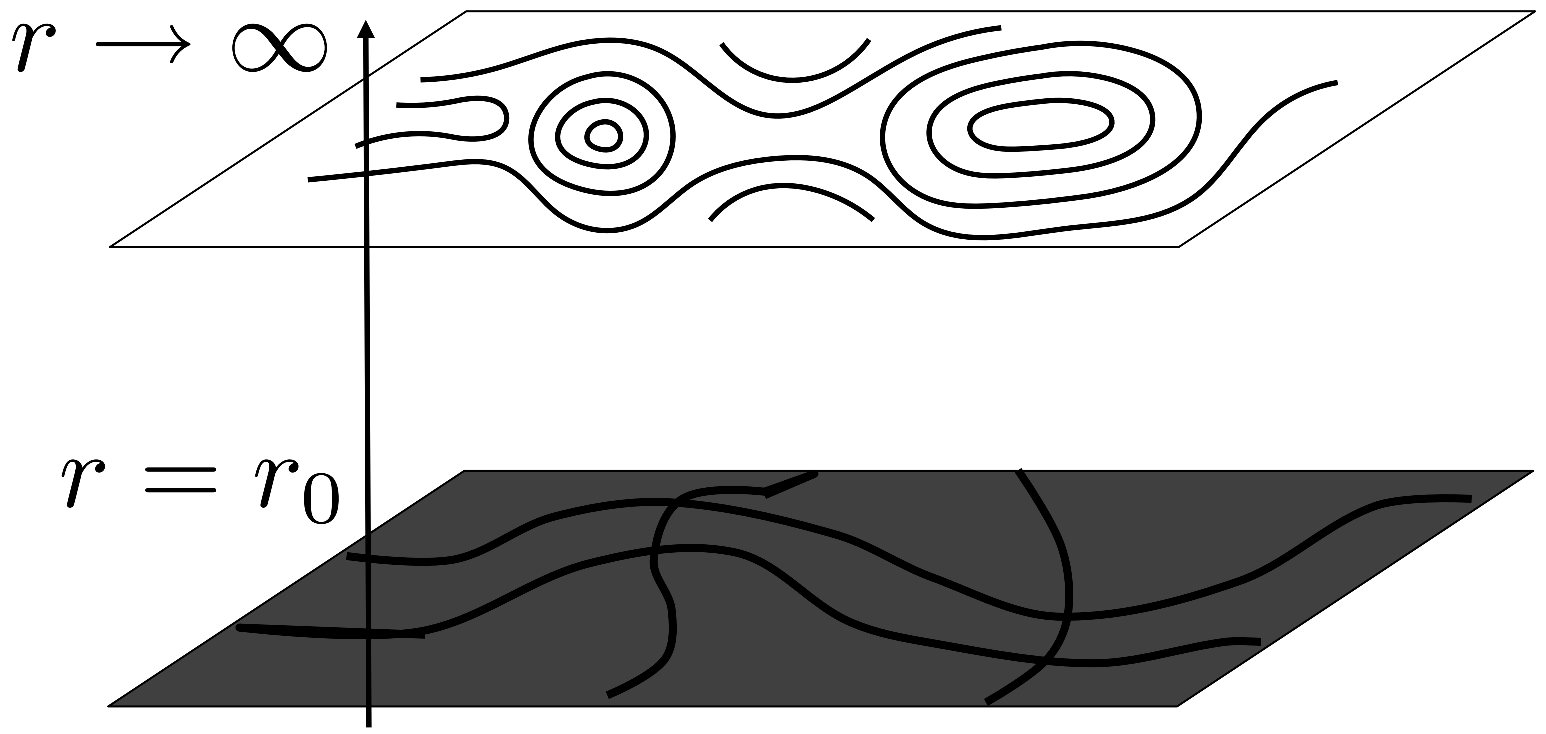

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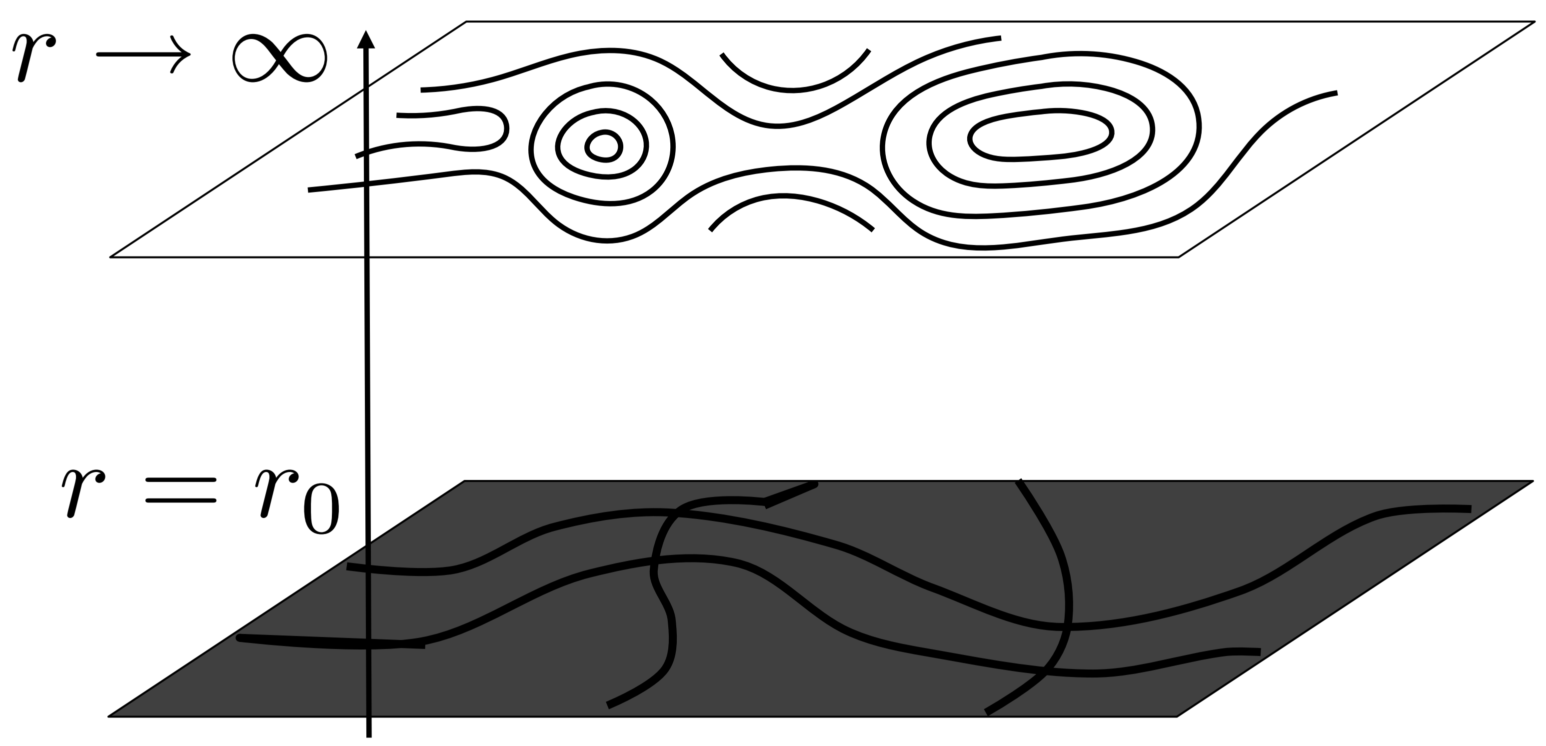

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The dual energy momentum tensor can be read off of the metric

$$ds^2 = \frac{\ell^2}{\zeta^2}\left(\sum_{k=0}^{\infty} g^{(k)}_{\mu\nu} \zeta^k dx^\mu dx^\nu + d\zeta^2\right) \qquad T_{\mu\nu} = g^{(3)}_{\mu\nu}$$

Holographic turbulence

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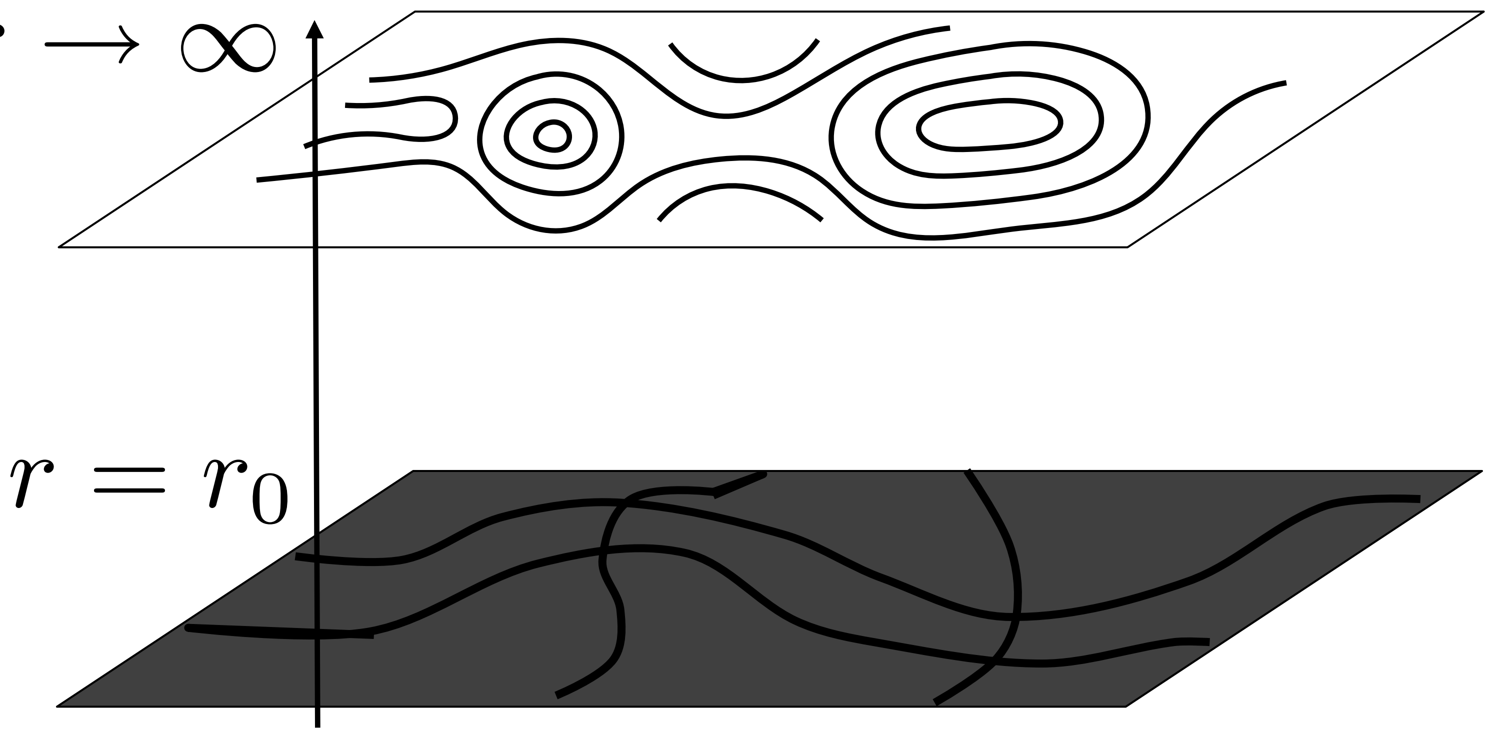

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$$ds^2 = \frac{\ell^2}{\zeta^2}\left(\sum_{k=0}^{\infty} g^{(k)}_{\mu\nu} \zeta^k dx^\mu dx^\nu + d\zeta^2\right) \qquad T_{\mu\nu} = g^{(3)}_{\mu\nu}$$

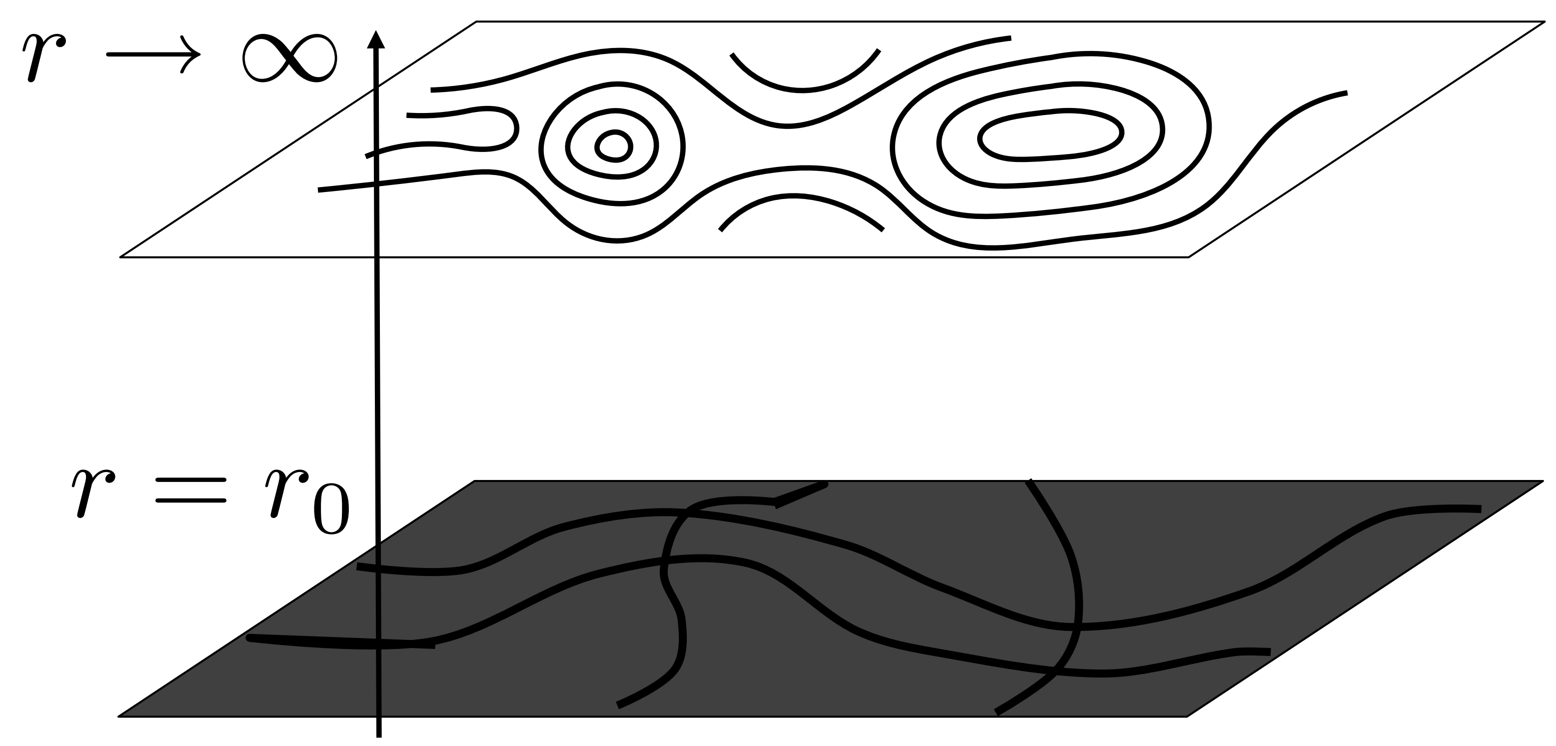
Then

$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$

Holographic turbulence

We wish to solve

$$R_{mn} - \frac{1}{2}Rg_{mn} - \frac{12}{\ell^2}g_{mn} = 0$$



At t>0 we use

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In practice, we need to solve numerically.

Holographic turbulence

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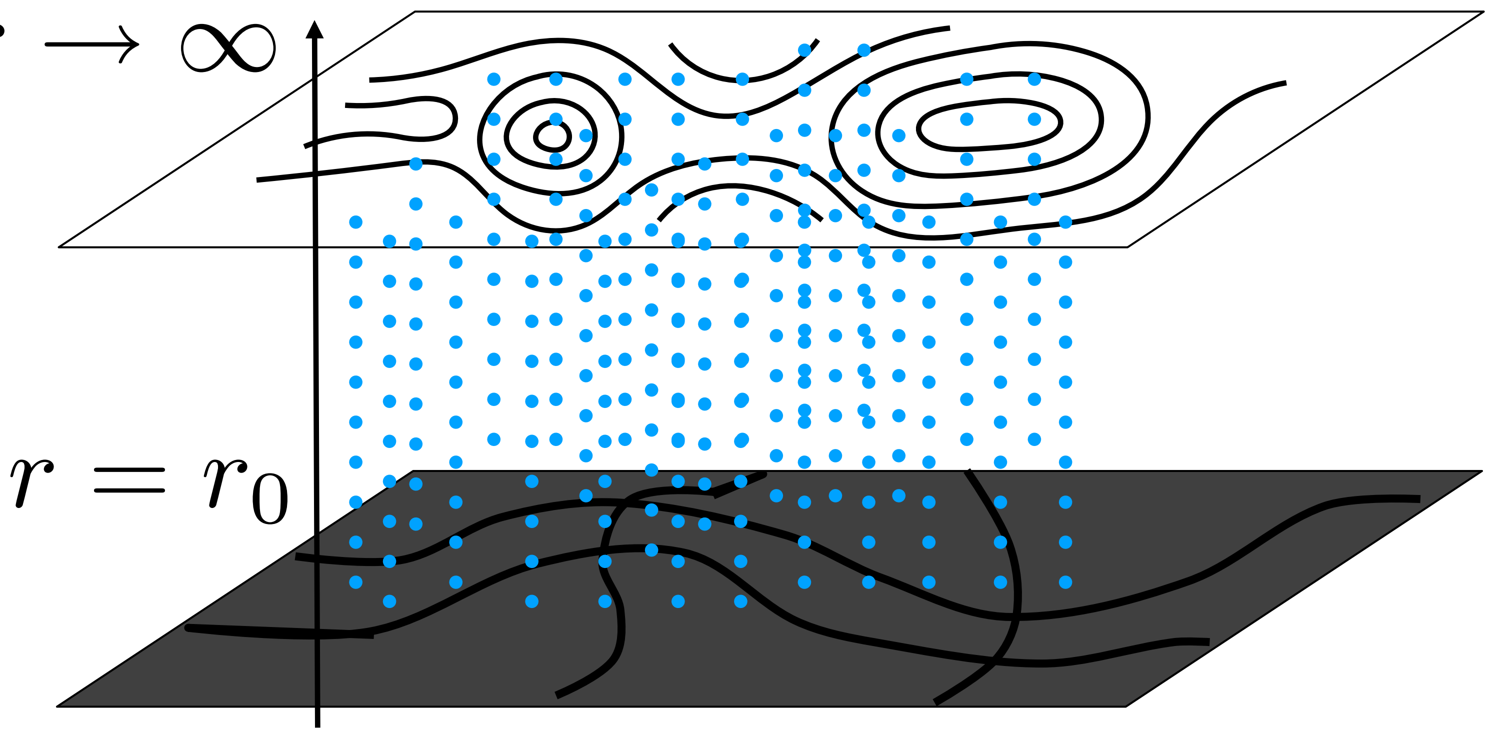

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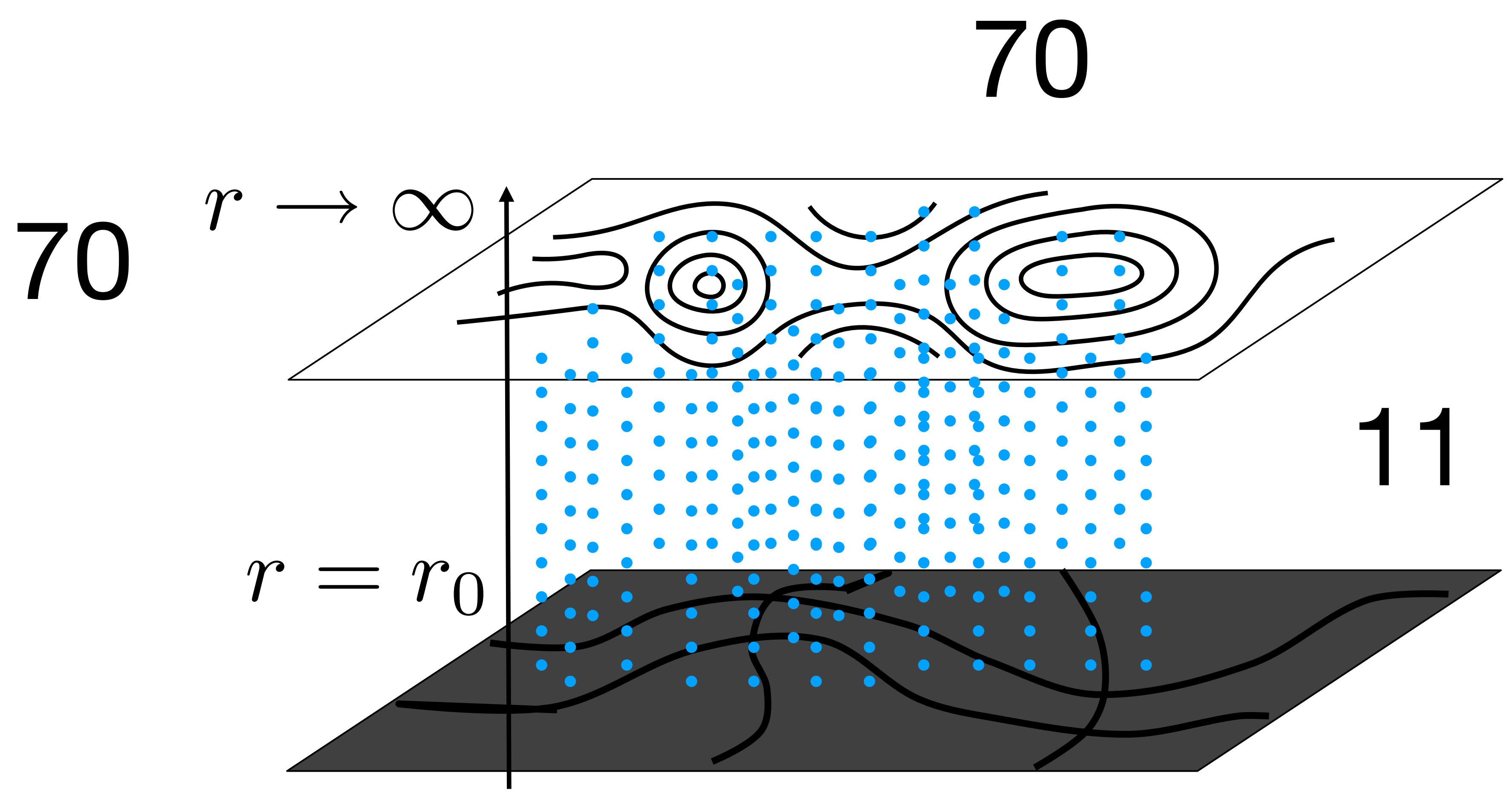

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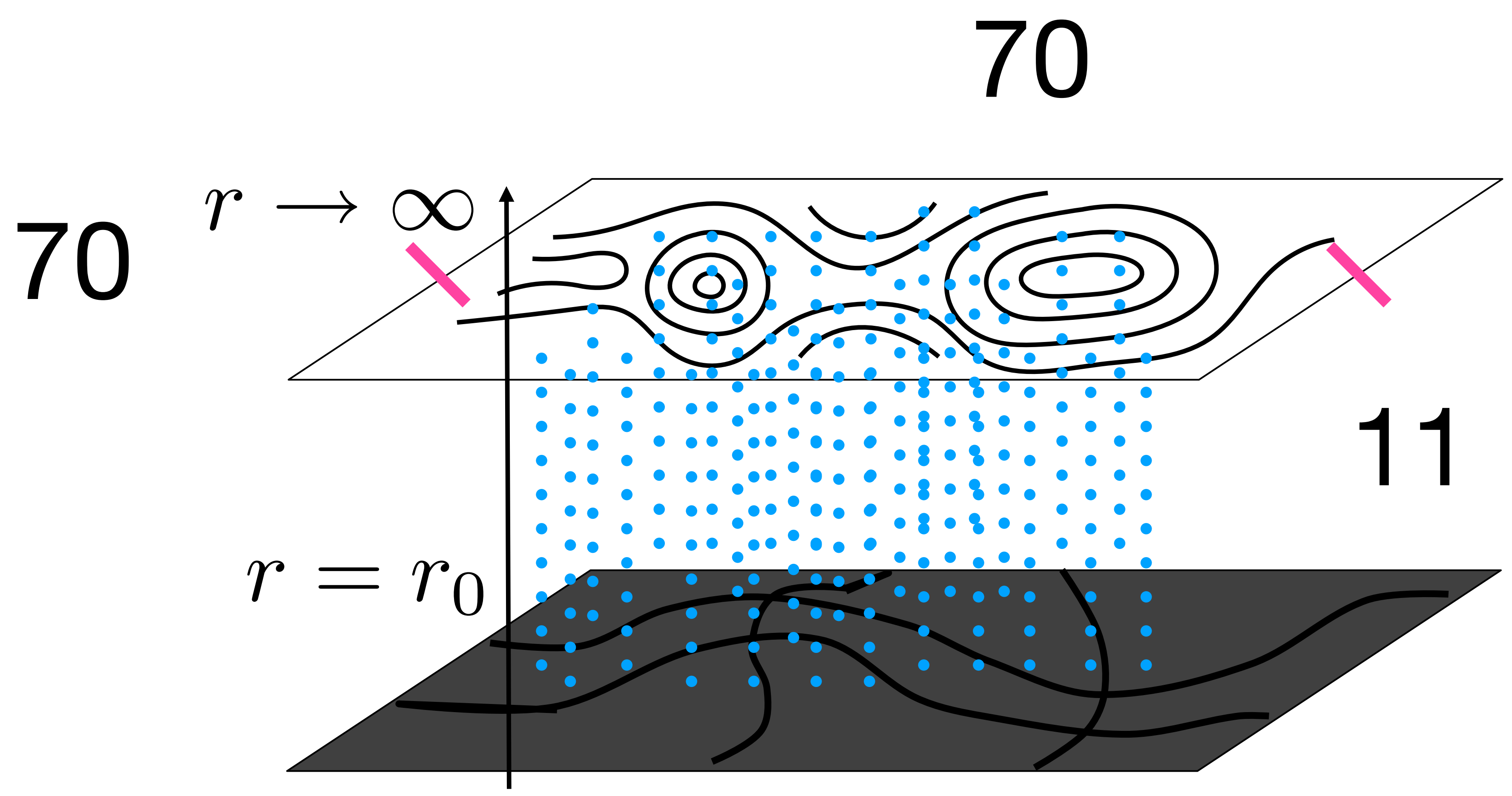

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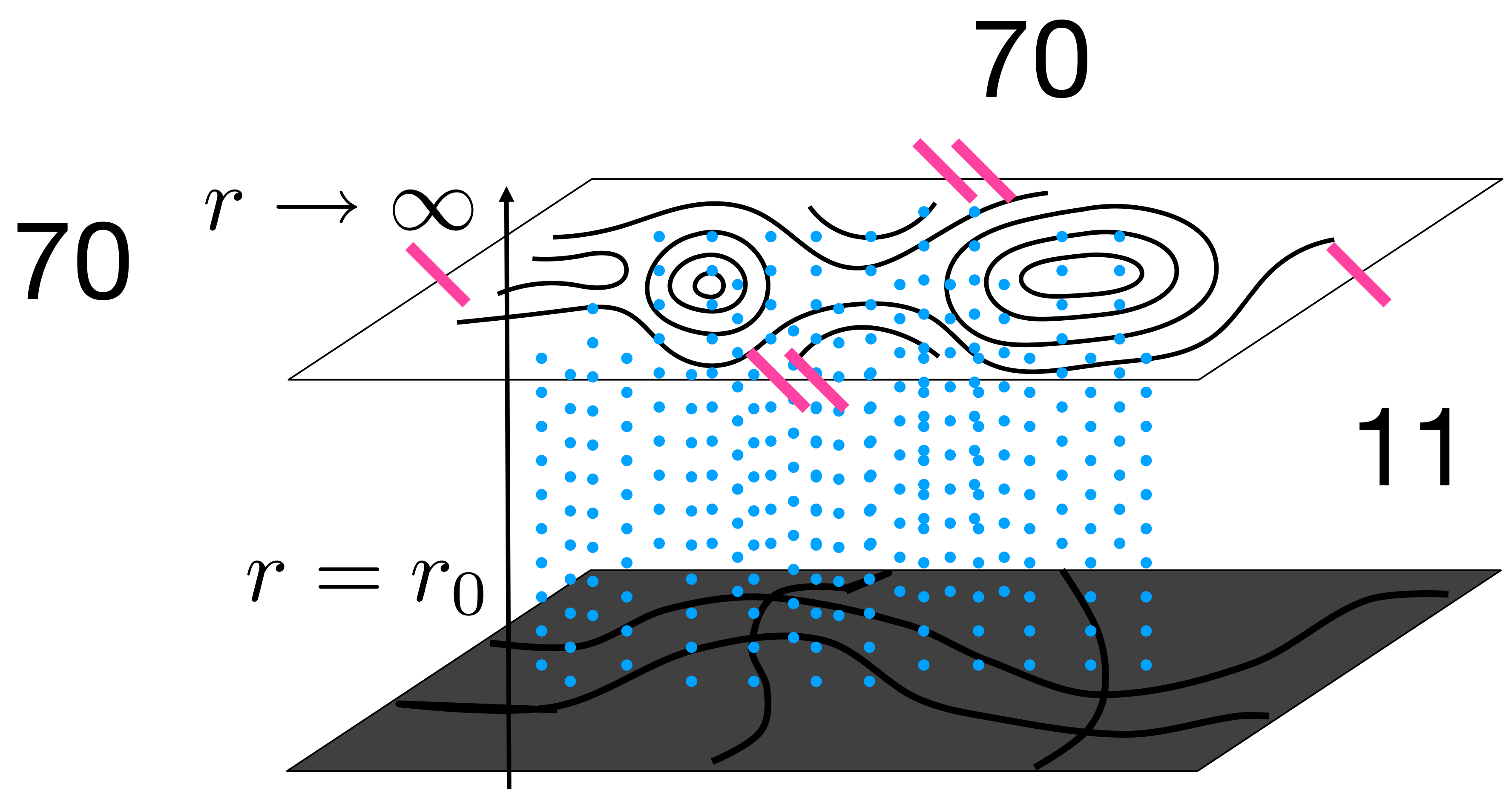

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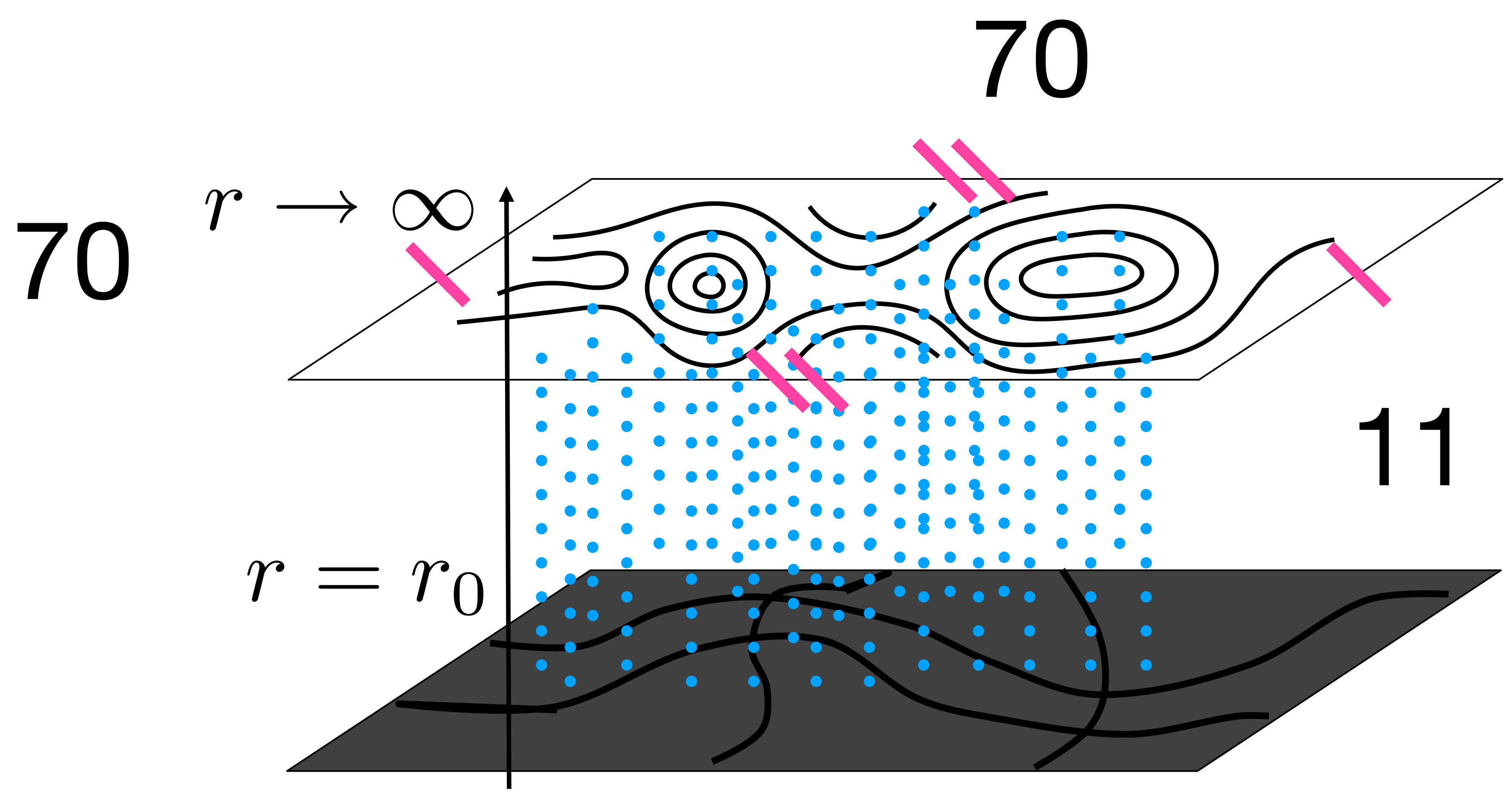

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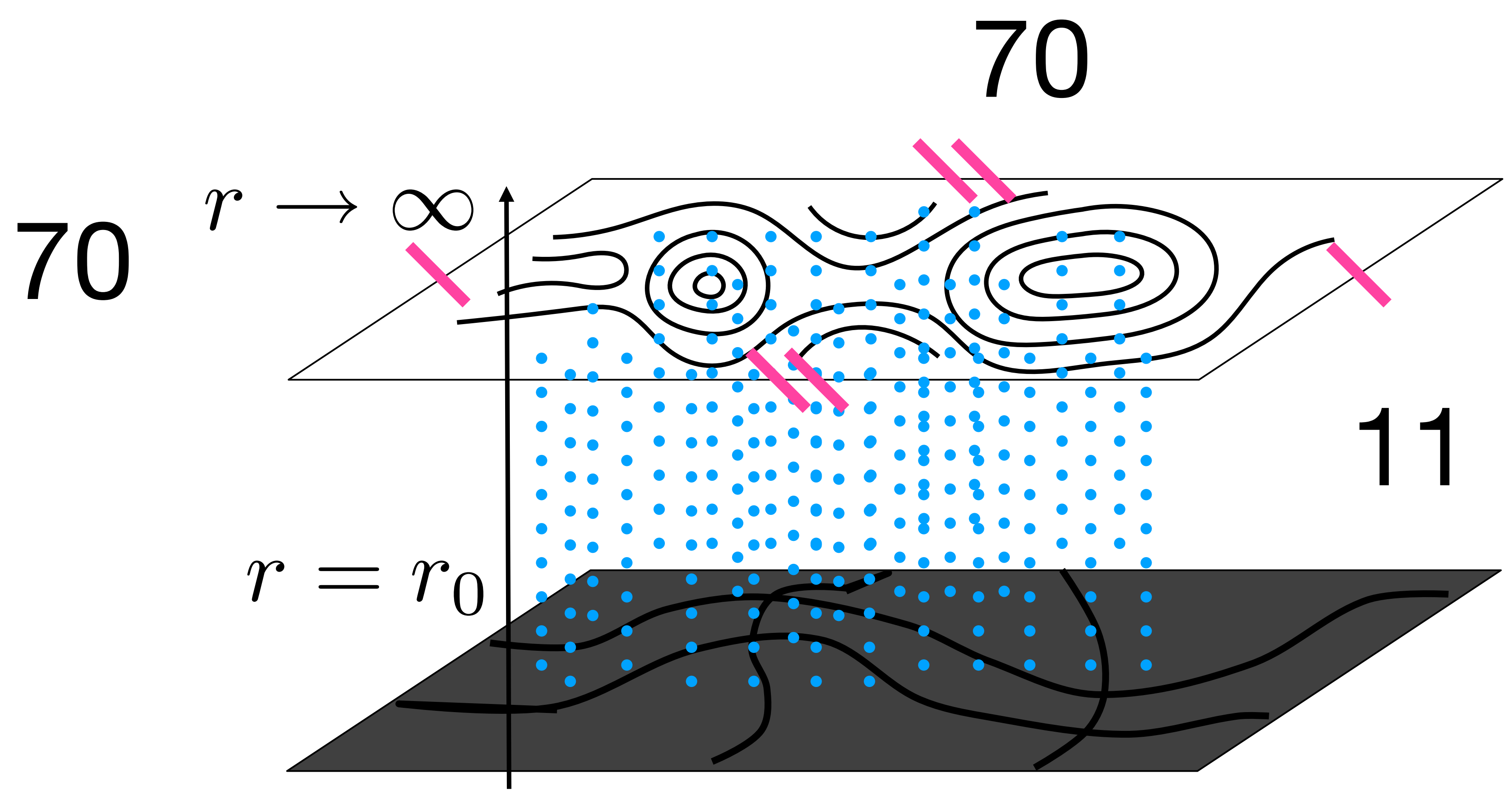
In practice, we need to solve numerically.

Solving in the right order allows us to rewrite the Einstein equations as a set of ordinary stochastic differential equations. (Chesler, Yaffe, 2013)

Holographic turbulence

We solved for the metric

$$ds^2 = \frac{\ell^2}{\zeta^2}\left(\sum_{k=0}^{\infty} g_{\mu\nu}^{(k)}\zeta^k dx^\mu dx^\nu + d\zeta^2\right)$$

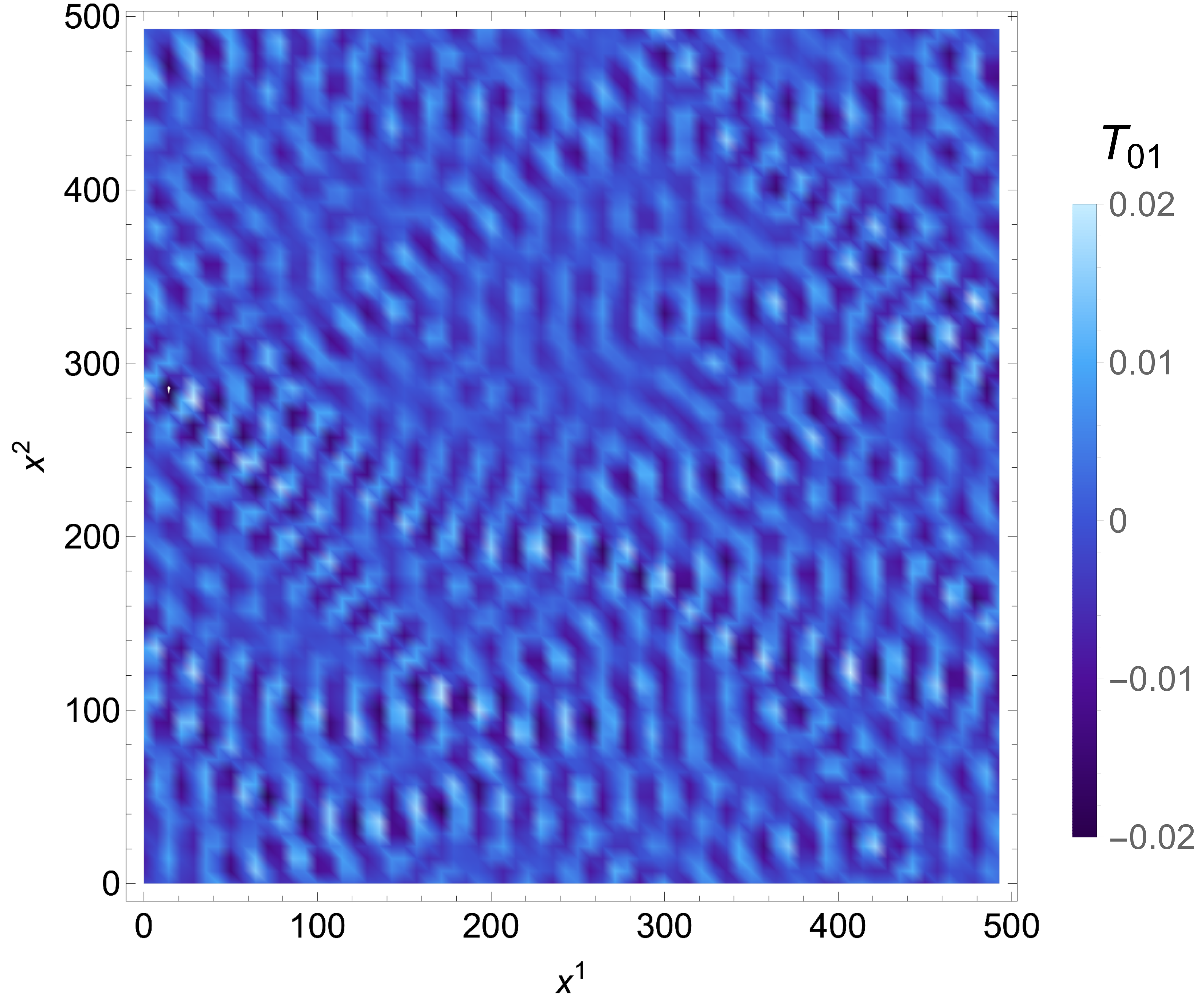


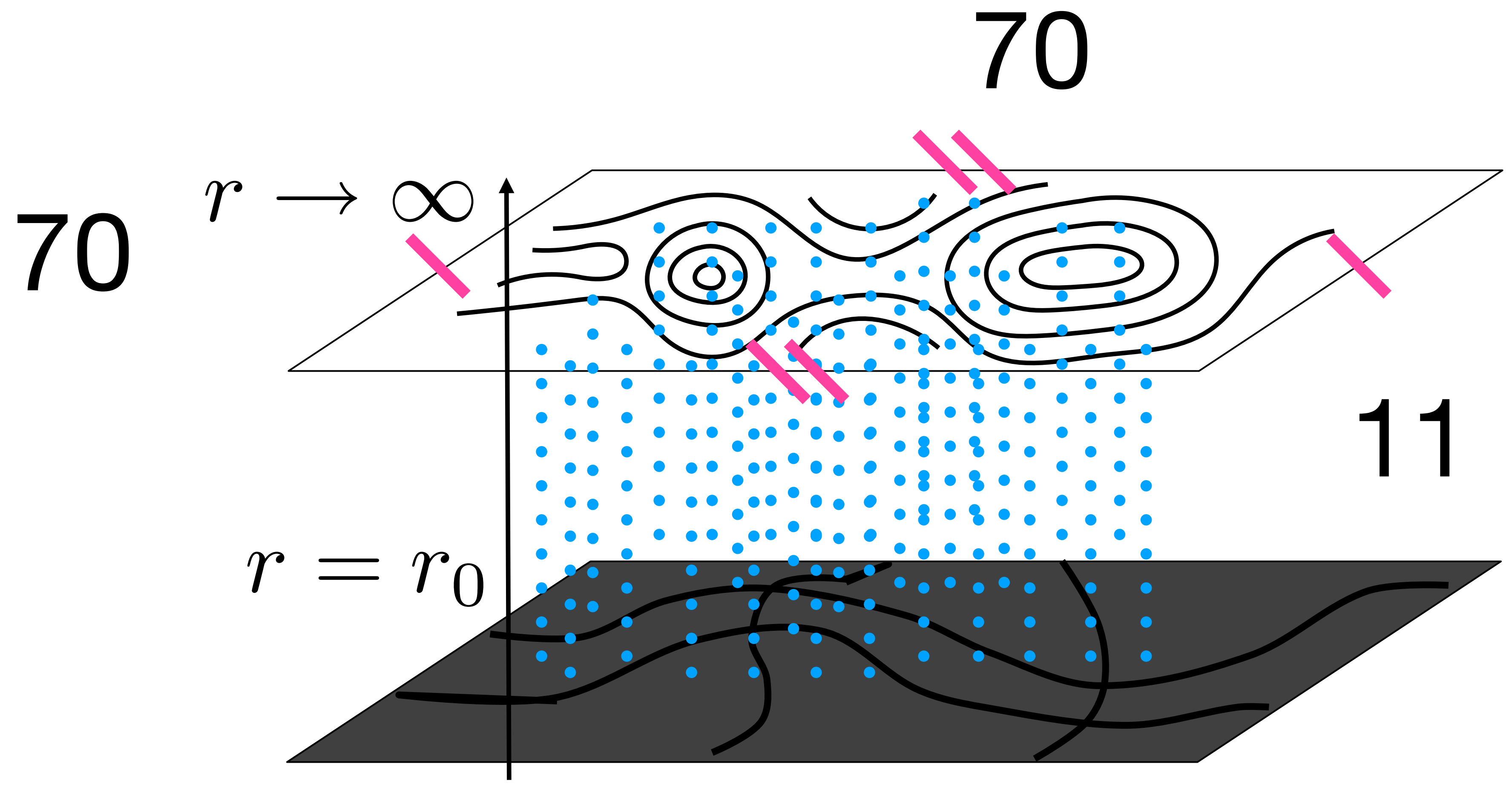
Did this many times, and then computed the average:

$$\overline{T_{\mu\nu}} = \overline{g_{\mu\nu}^{(3)}}$$

Holographic turbulence

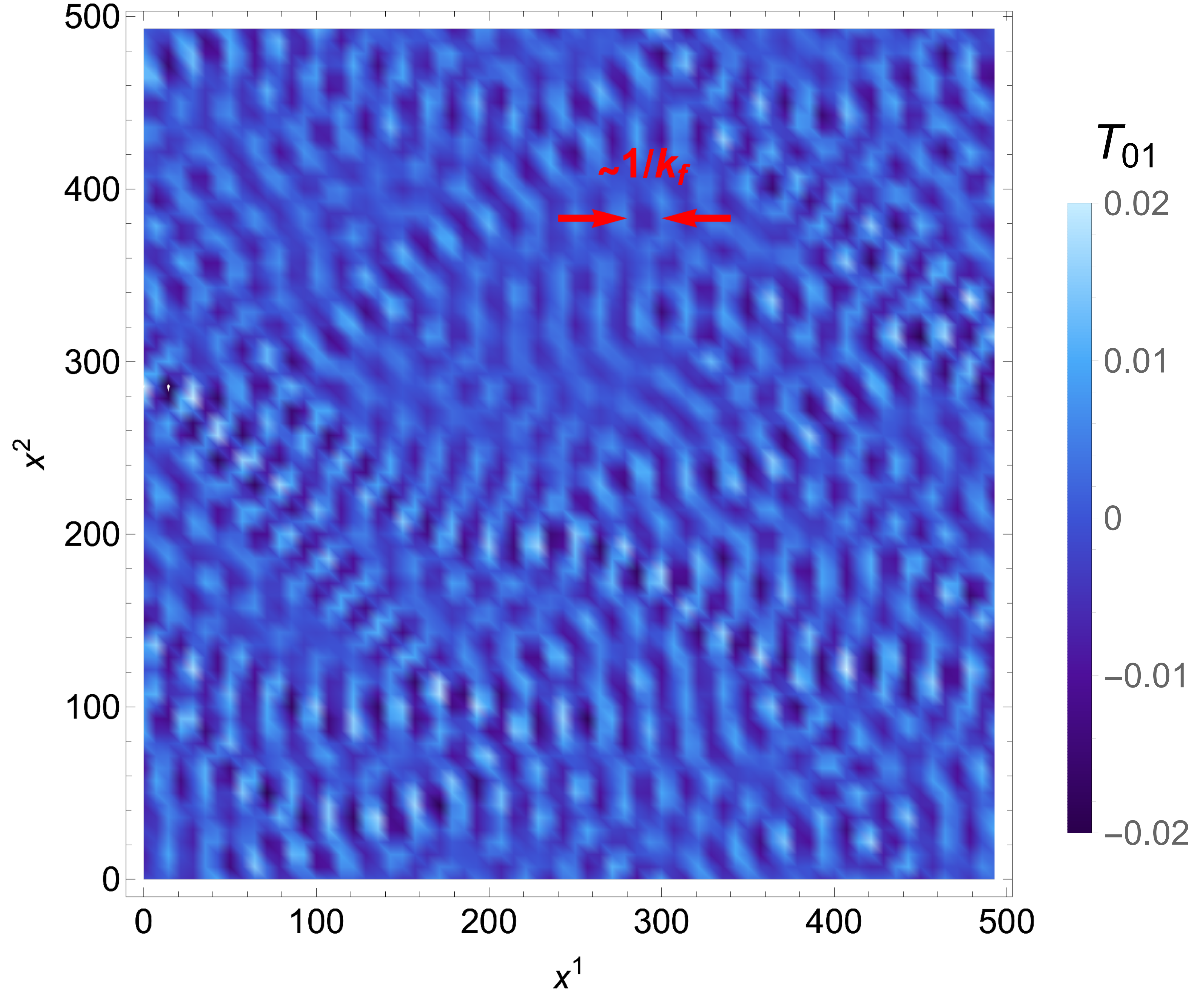
$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$

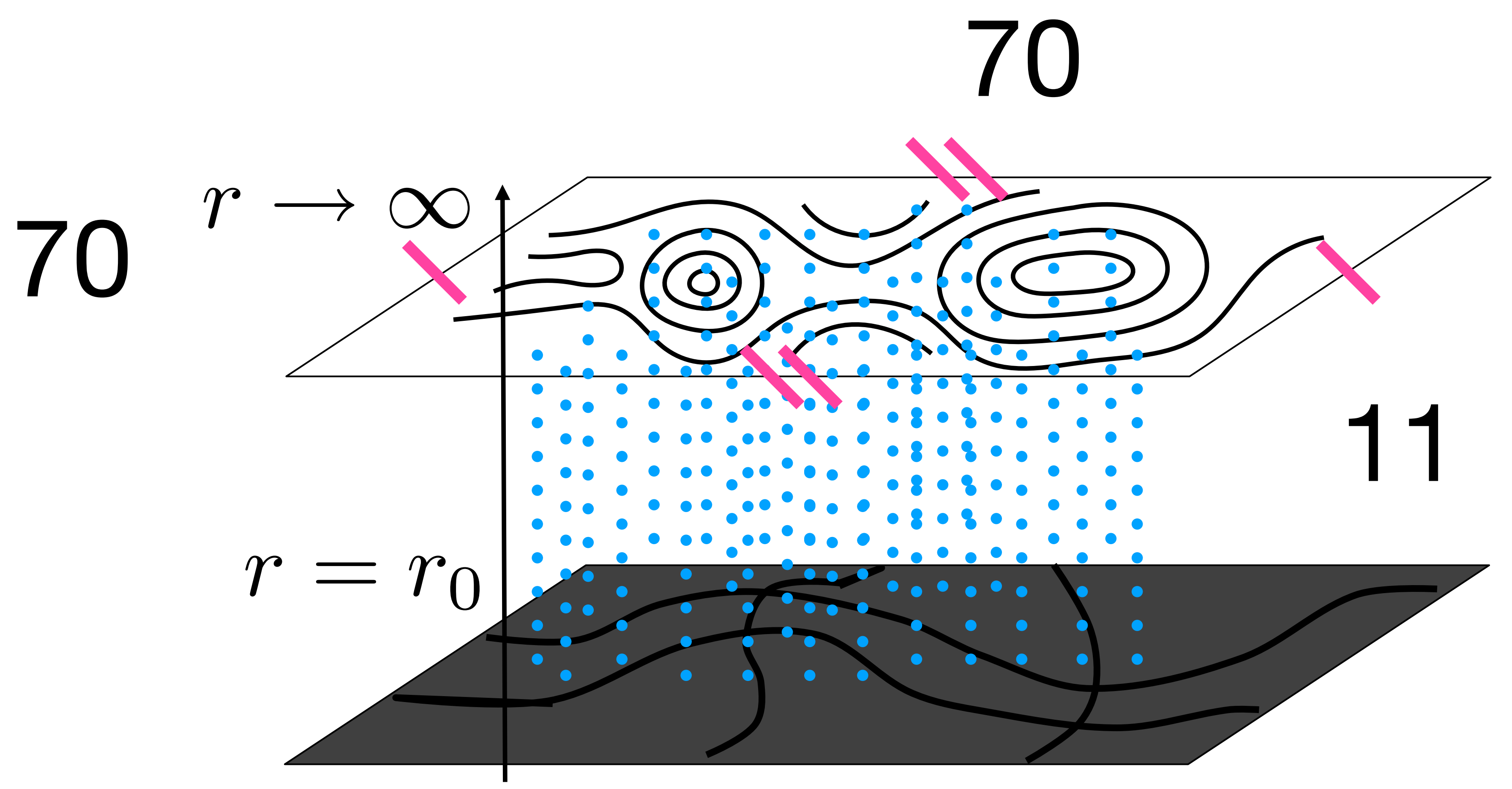




Holographic turbulence

$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$





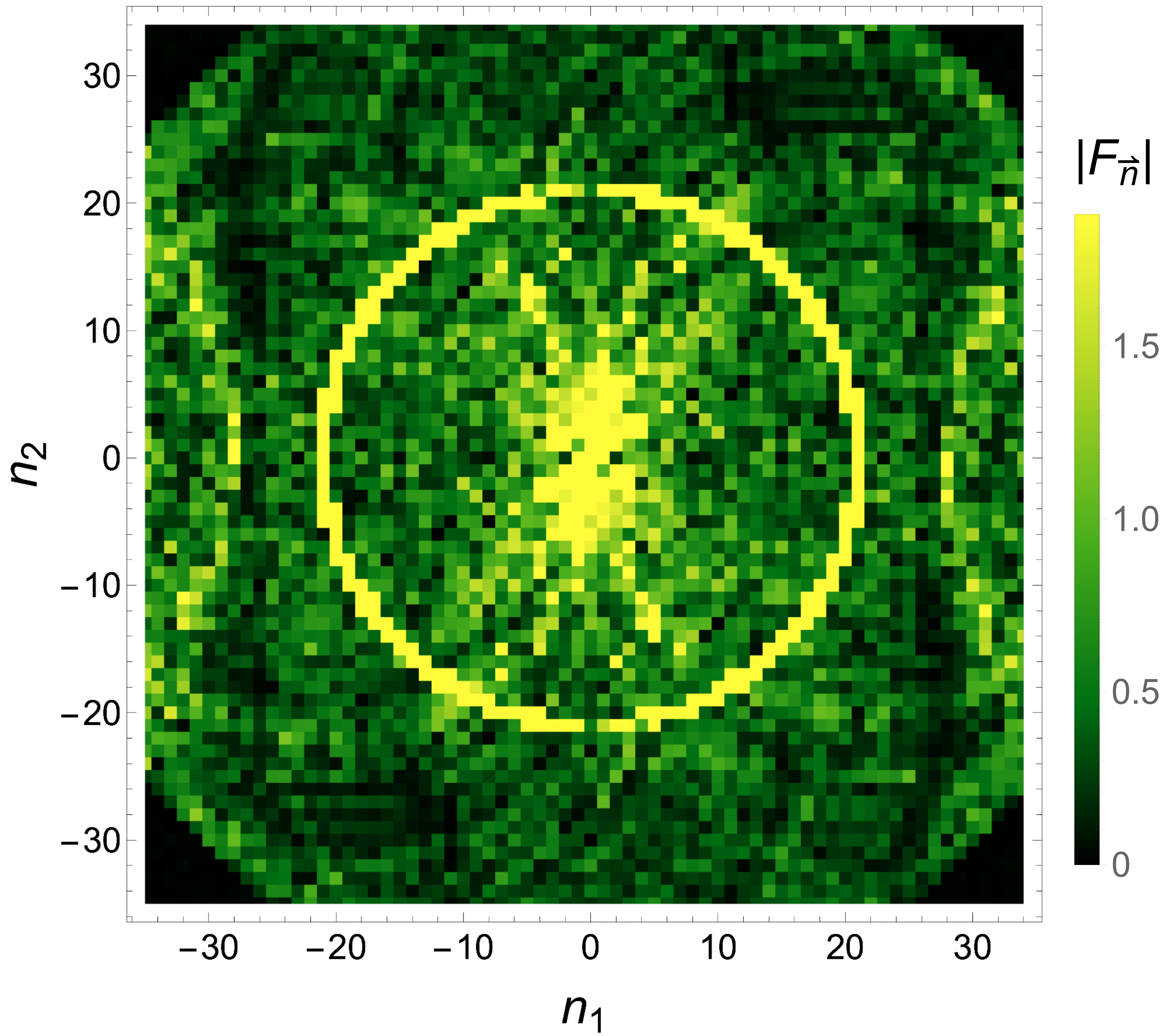
$$\overline{\xi(t, \vec{x})} = 0$$

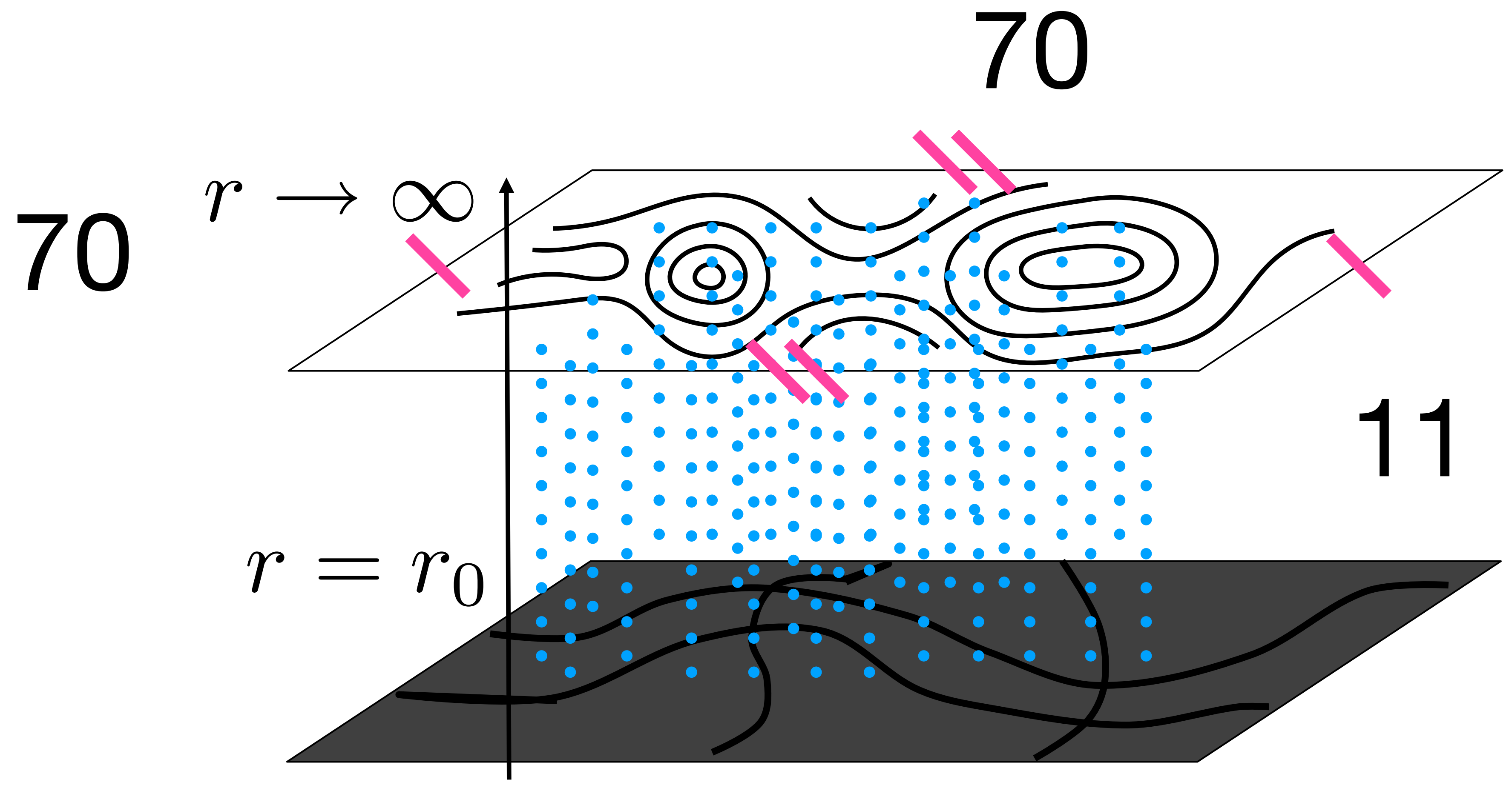
$$\overline{\xi(t, \vec{x})\xi(t', \vec{x}')} = D(\vec{x} - \vec{x}')\delta(t - t')$$

$$\hat{D}(\vec{k}) = \delta(|\vec{k}| - k_f)$$

Holographic turbulence

$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$

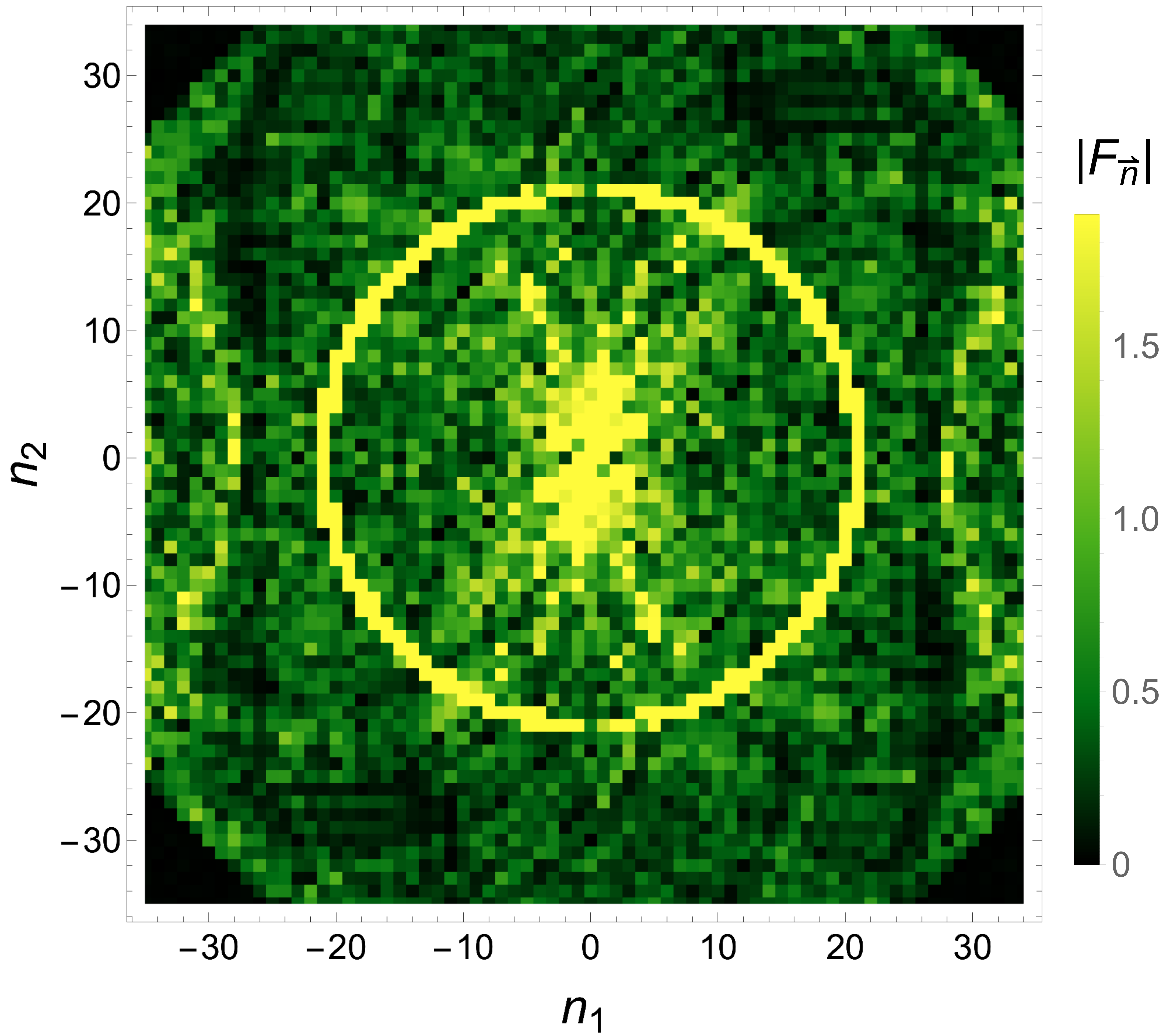


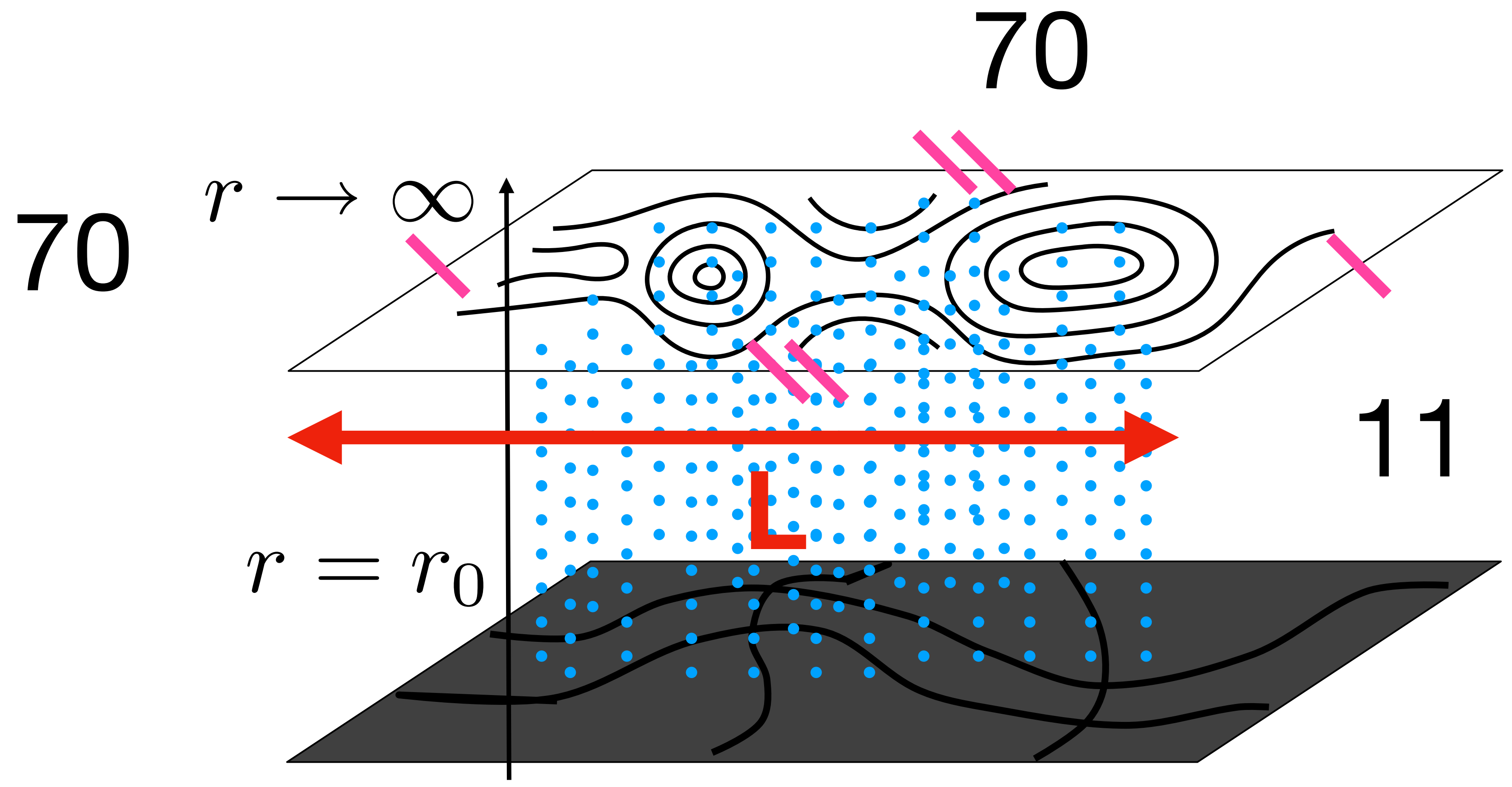


$$T_{01} = \frac{1}{L} \sum_{\vec{n}} F_{\vec{n}} e^{i \frac{2\pi \vec{n}}{L} \vec{x}}$$

Holographic turbulence

$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$





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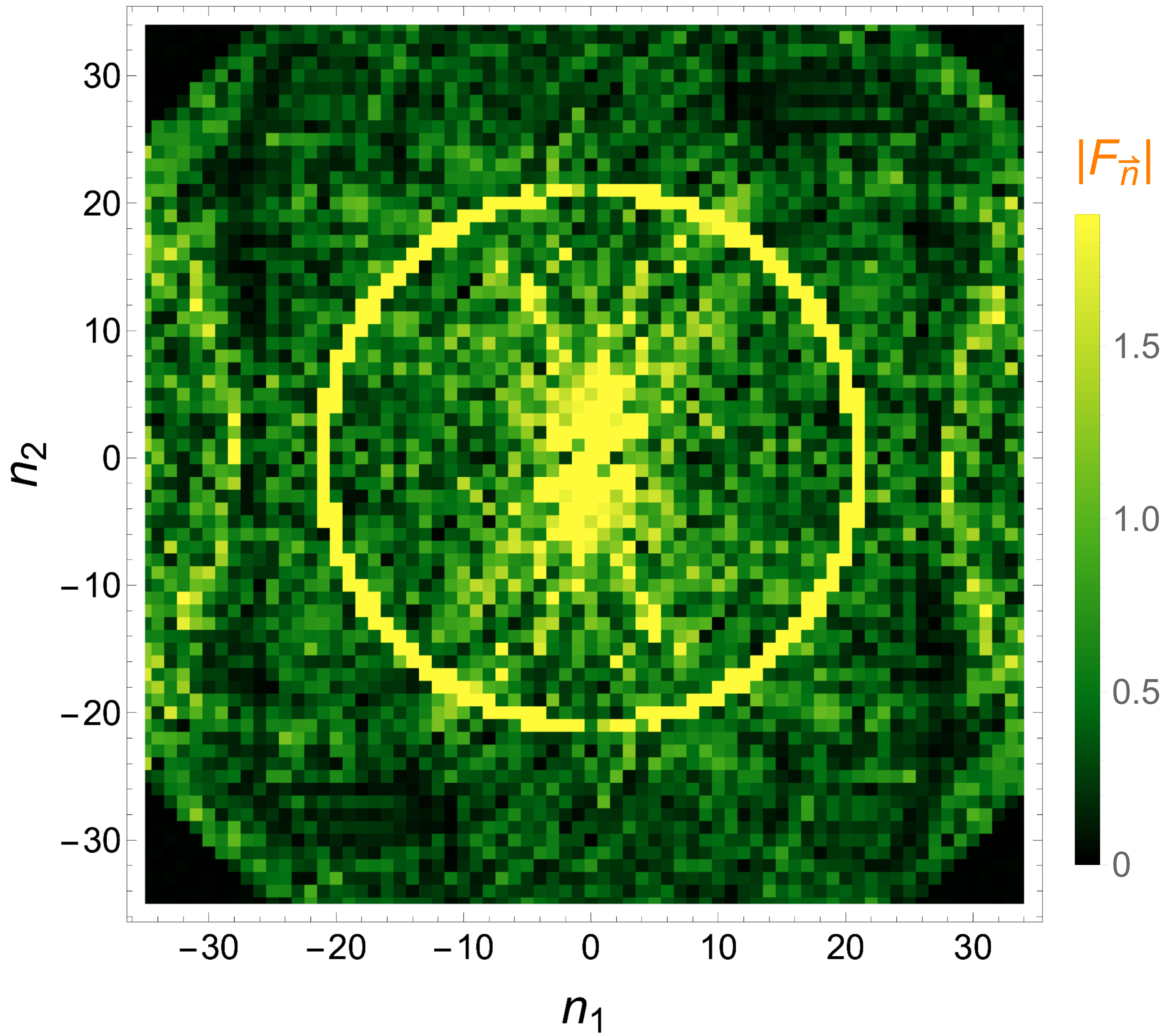

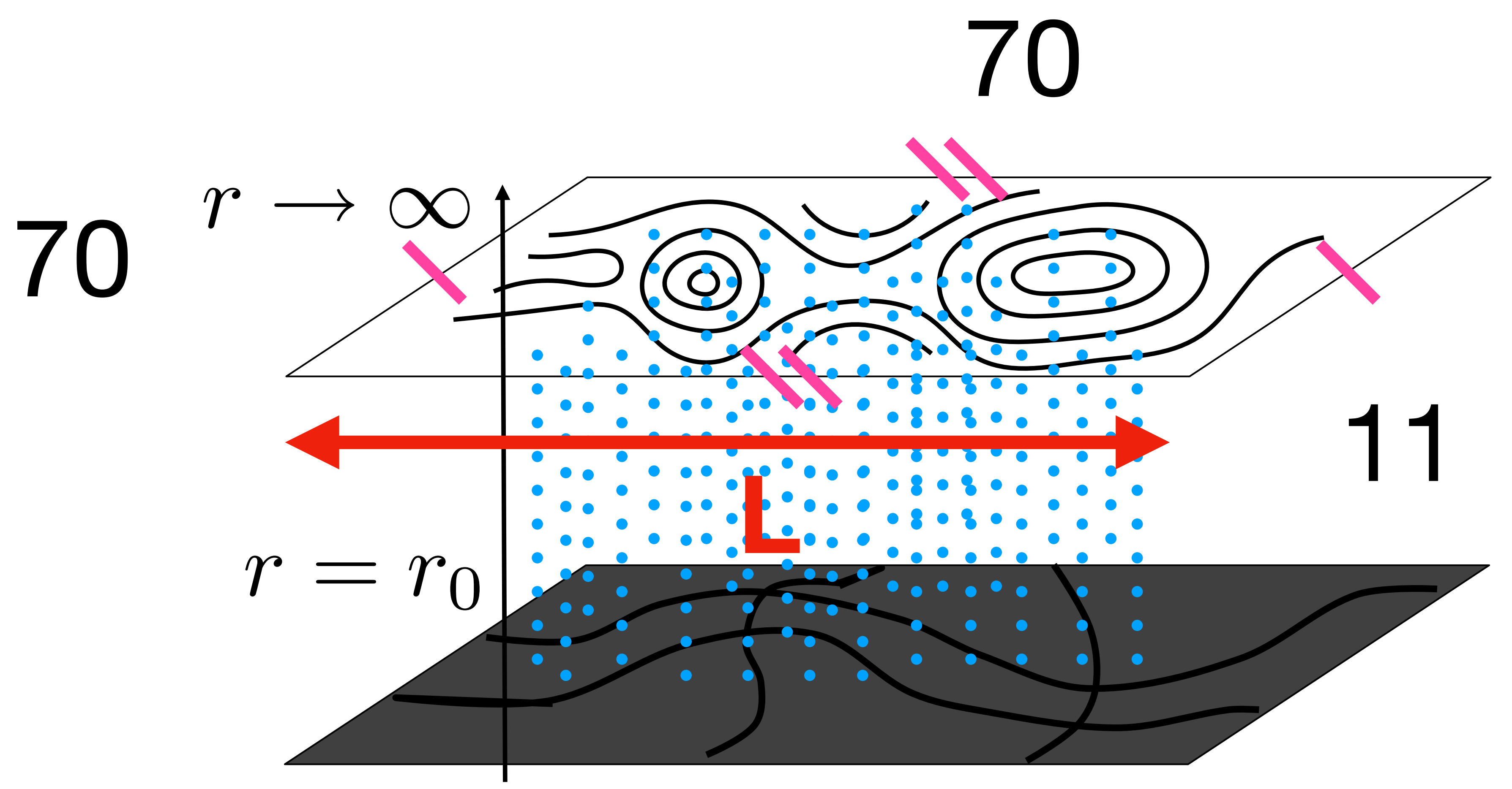

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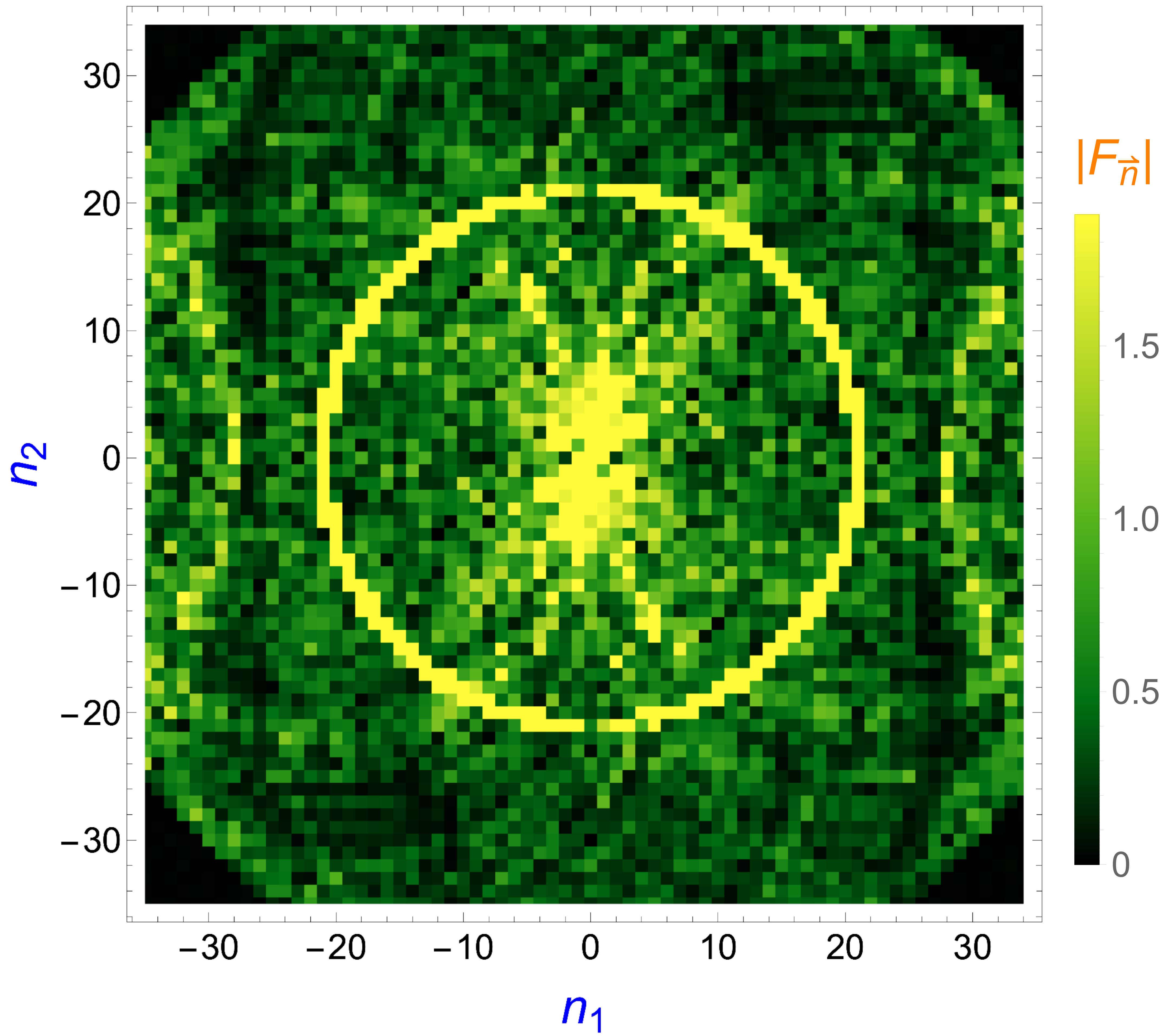

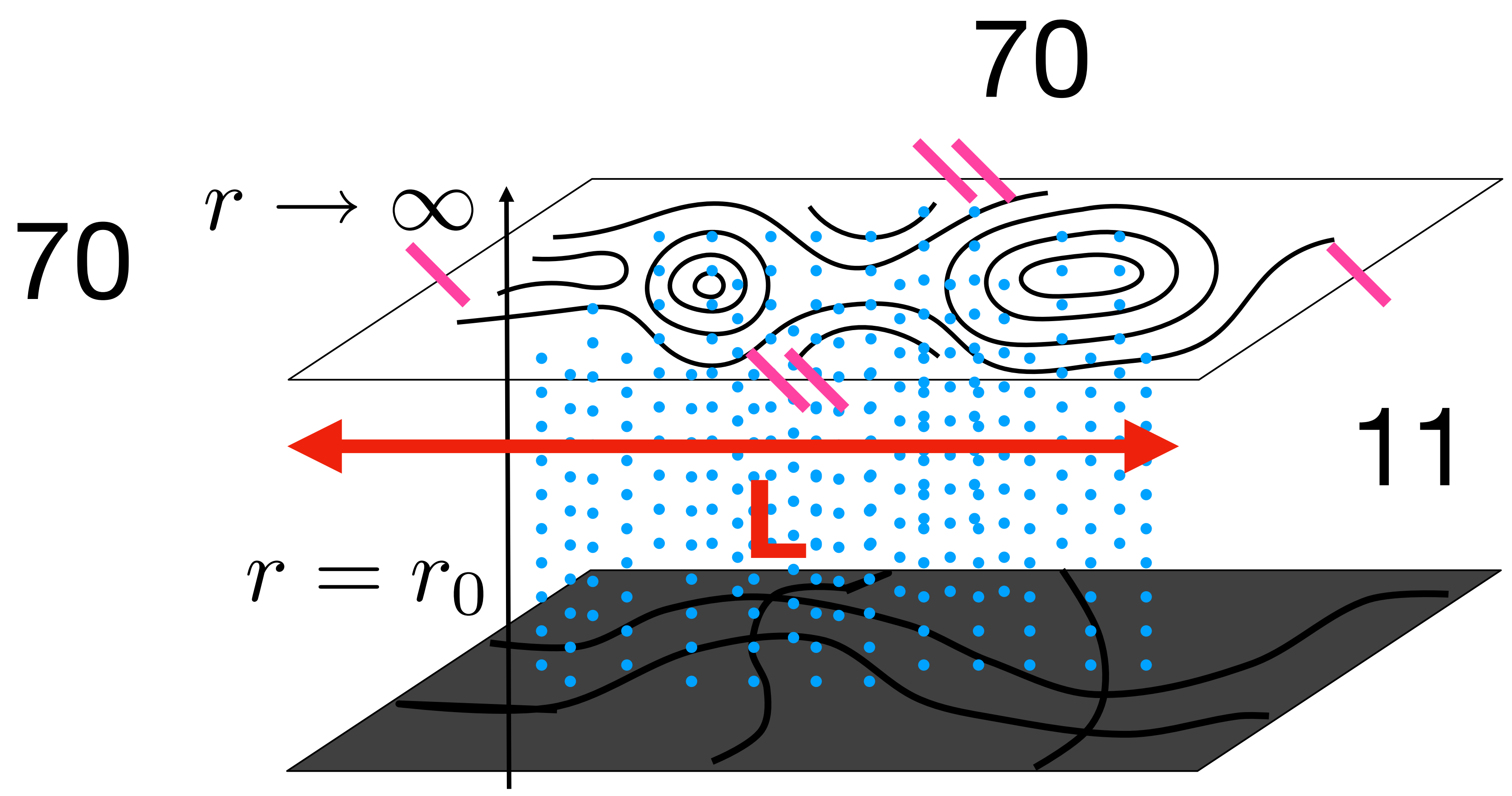

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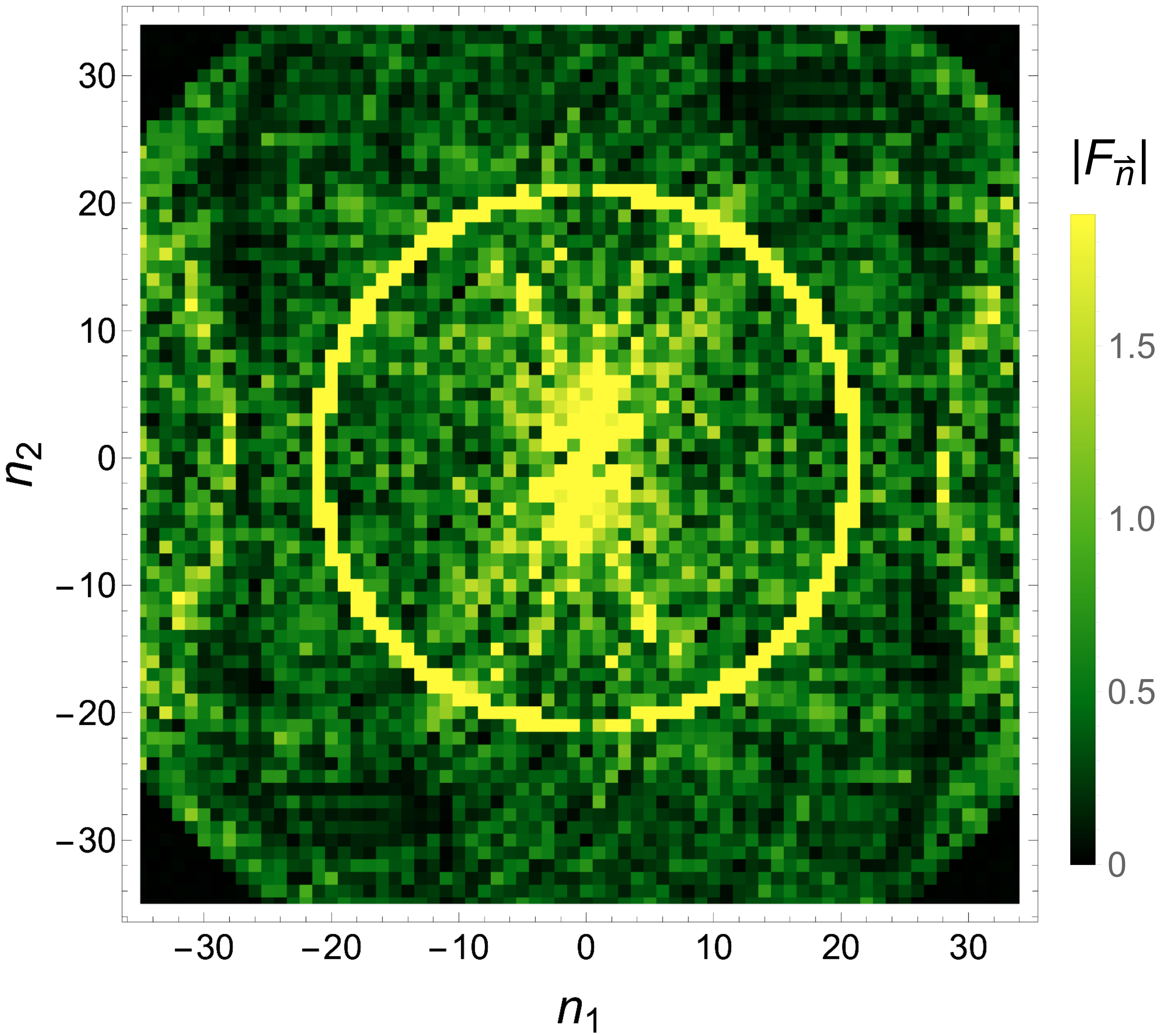

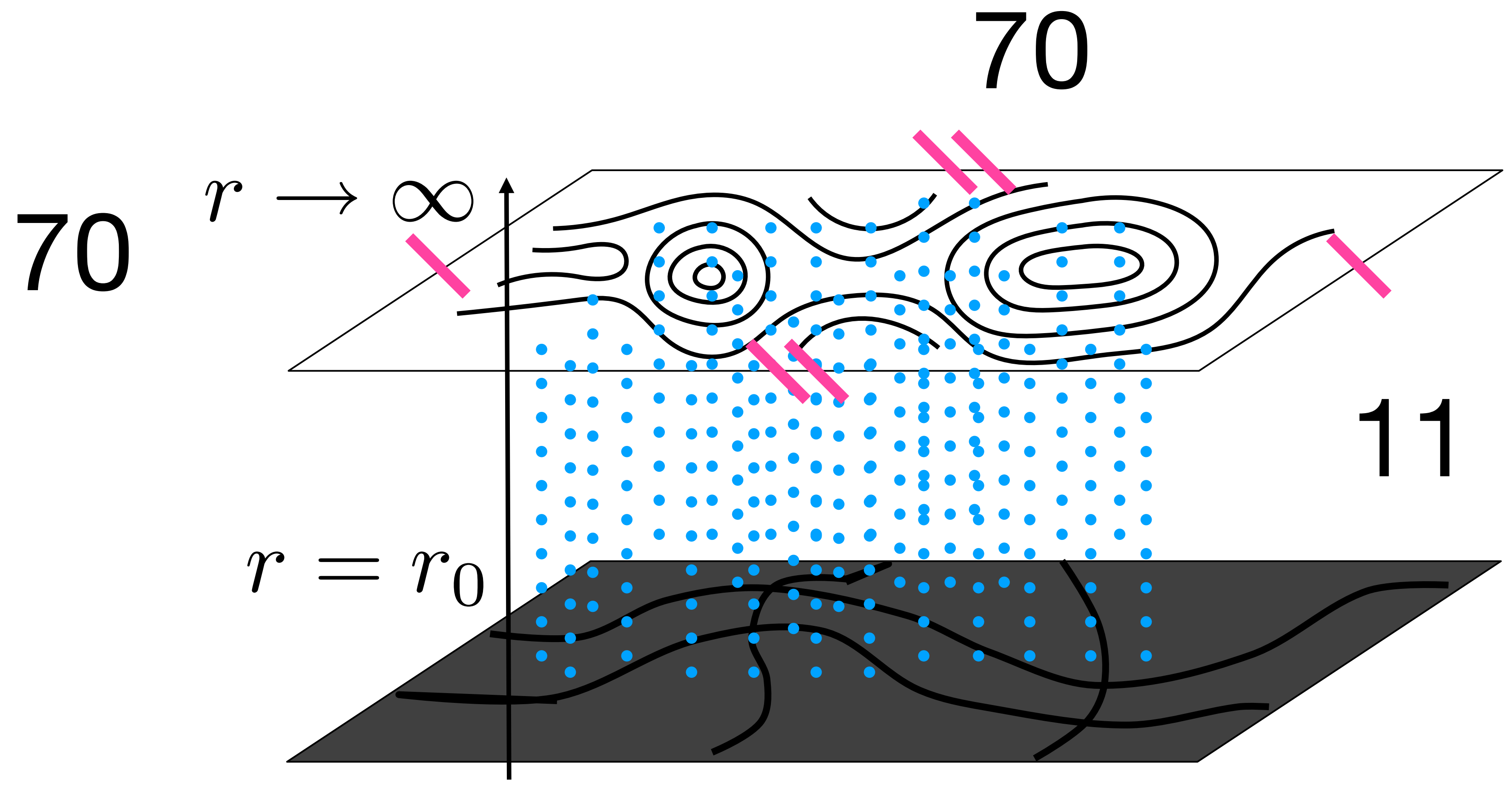

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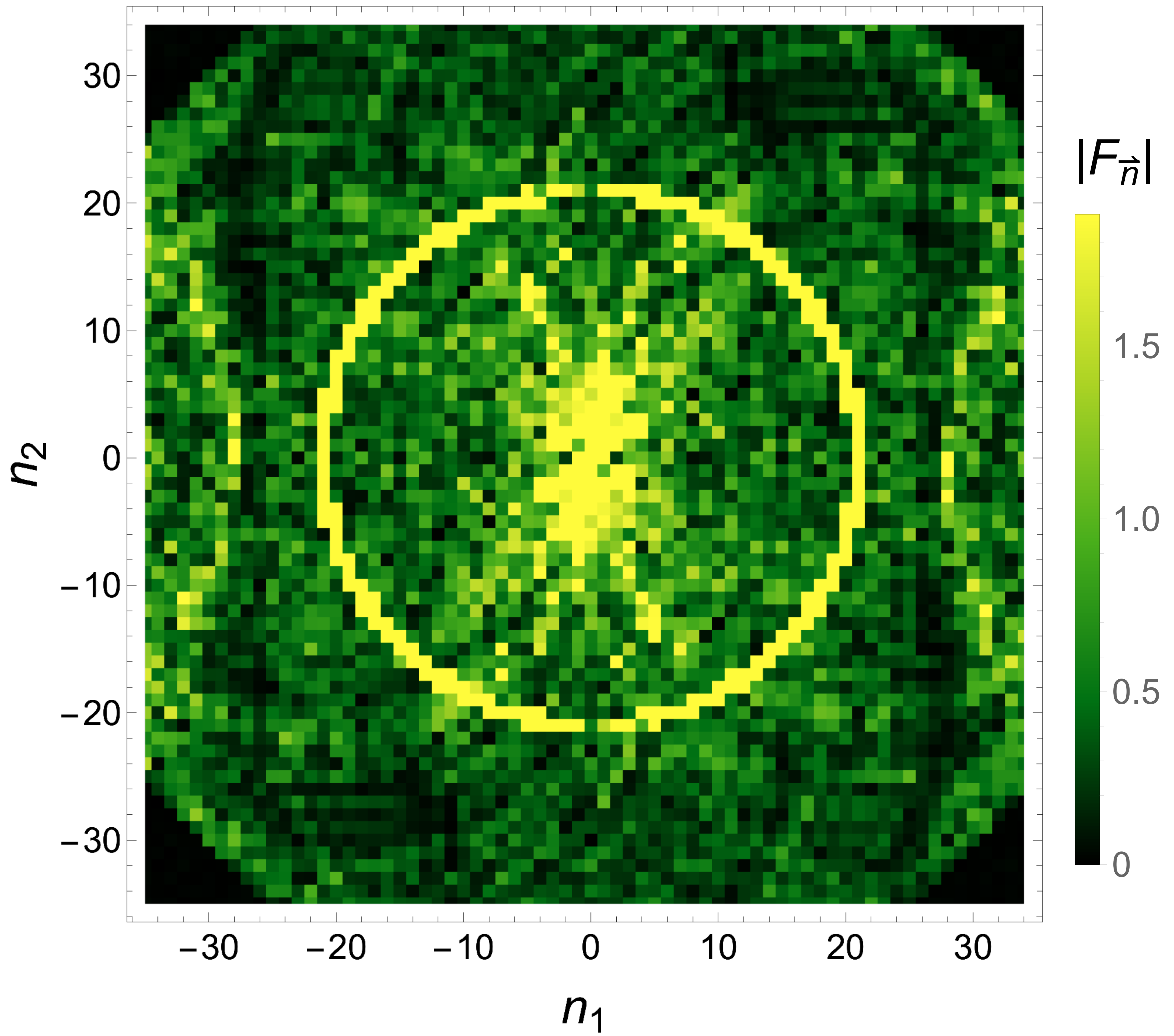

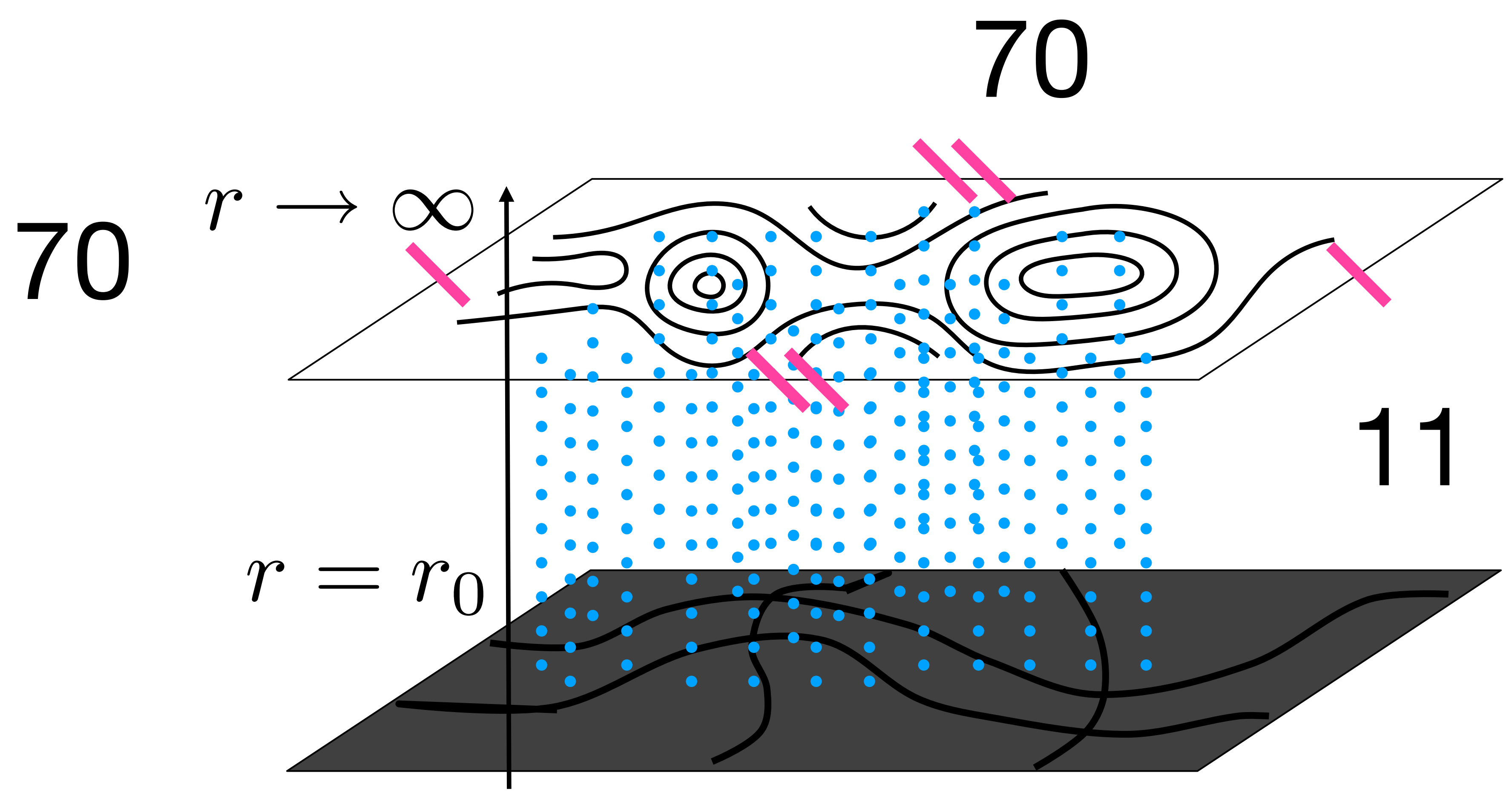

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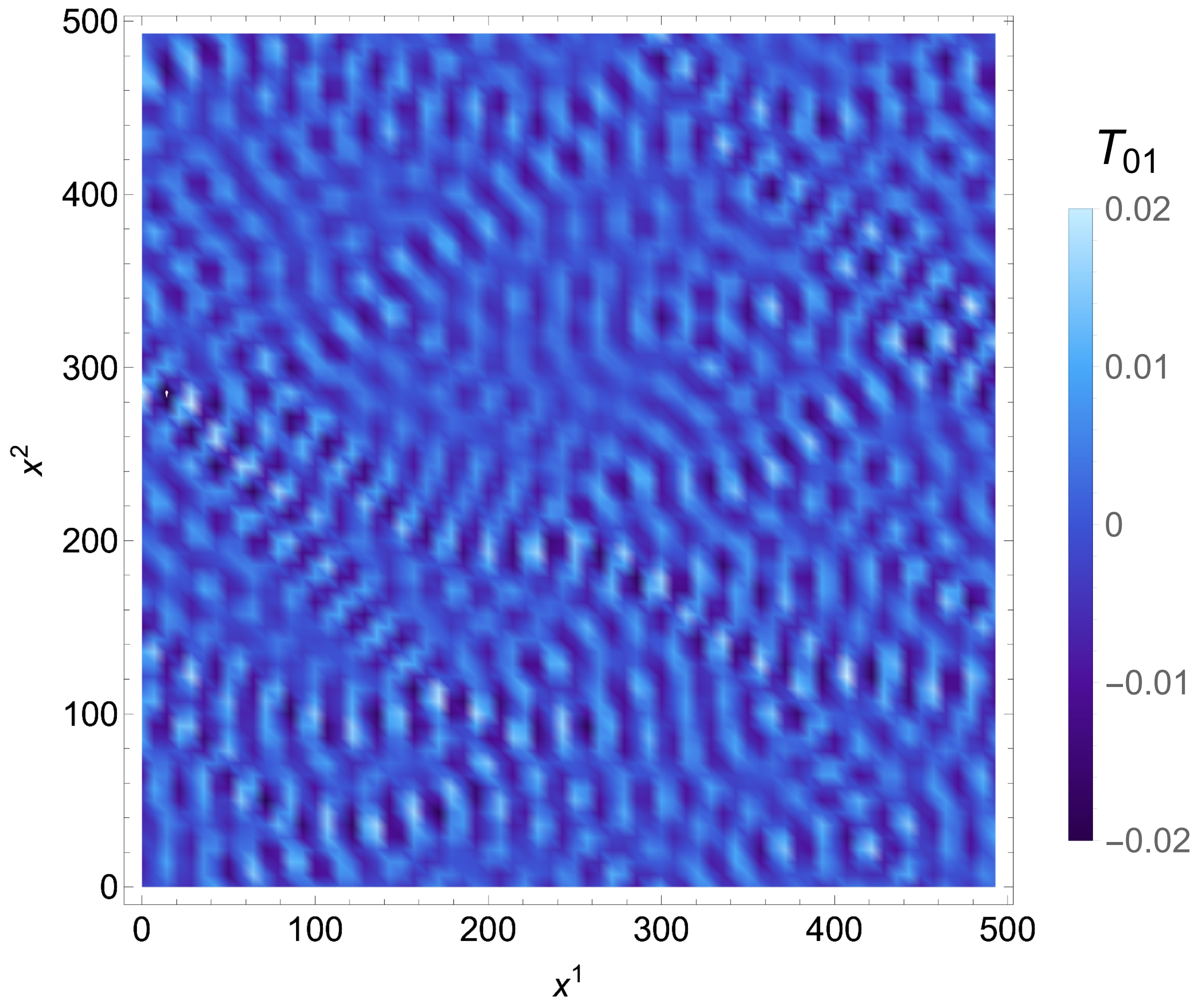
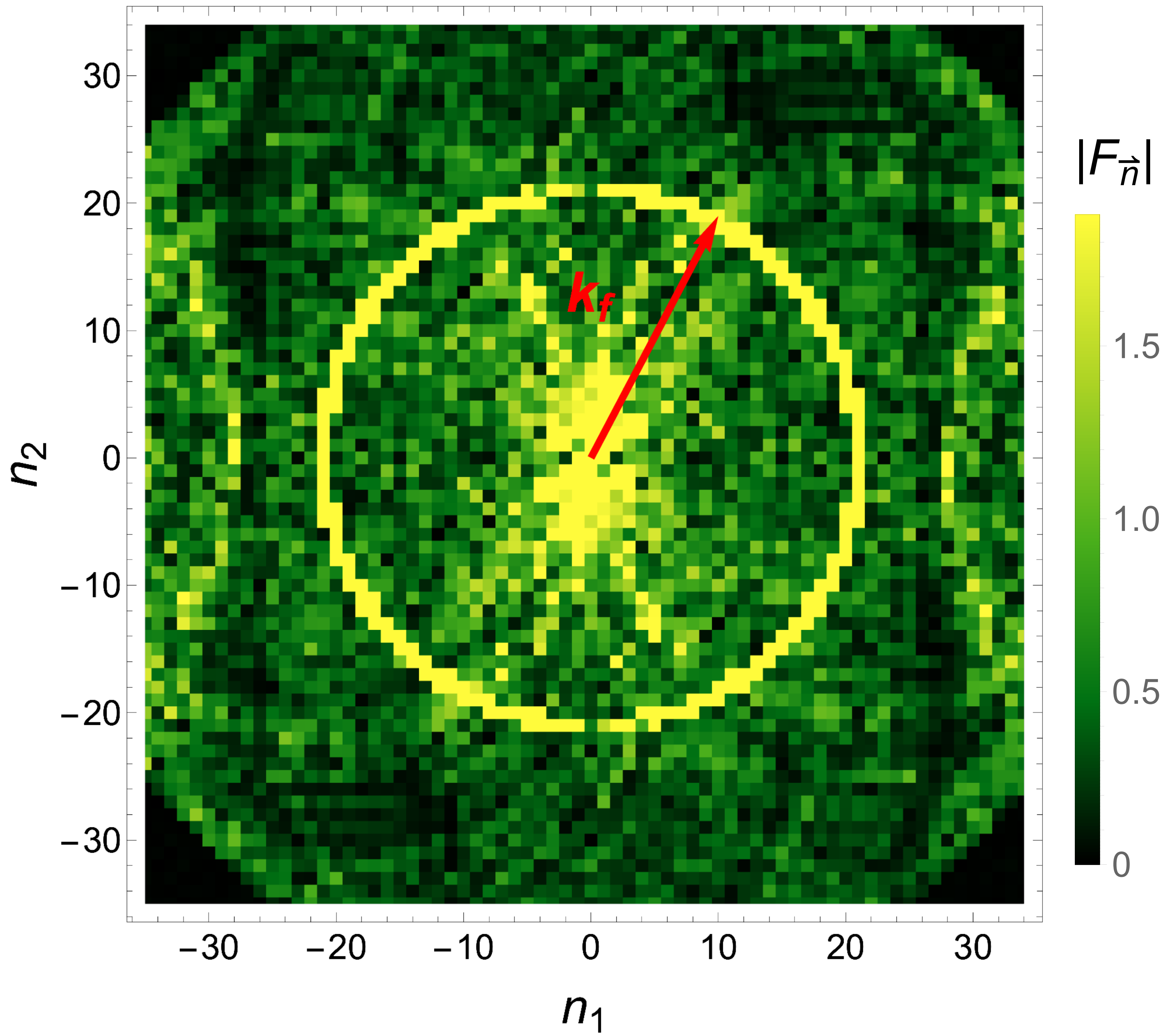
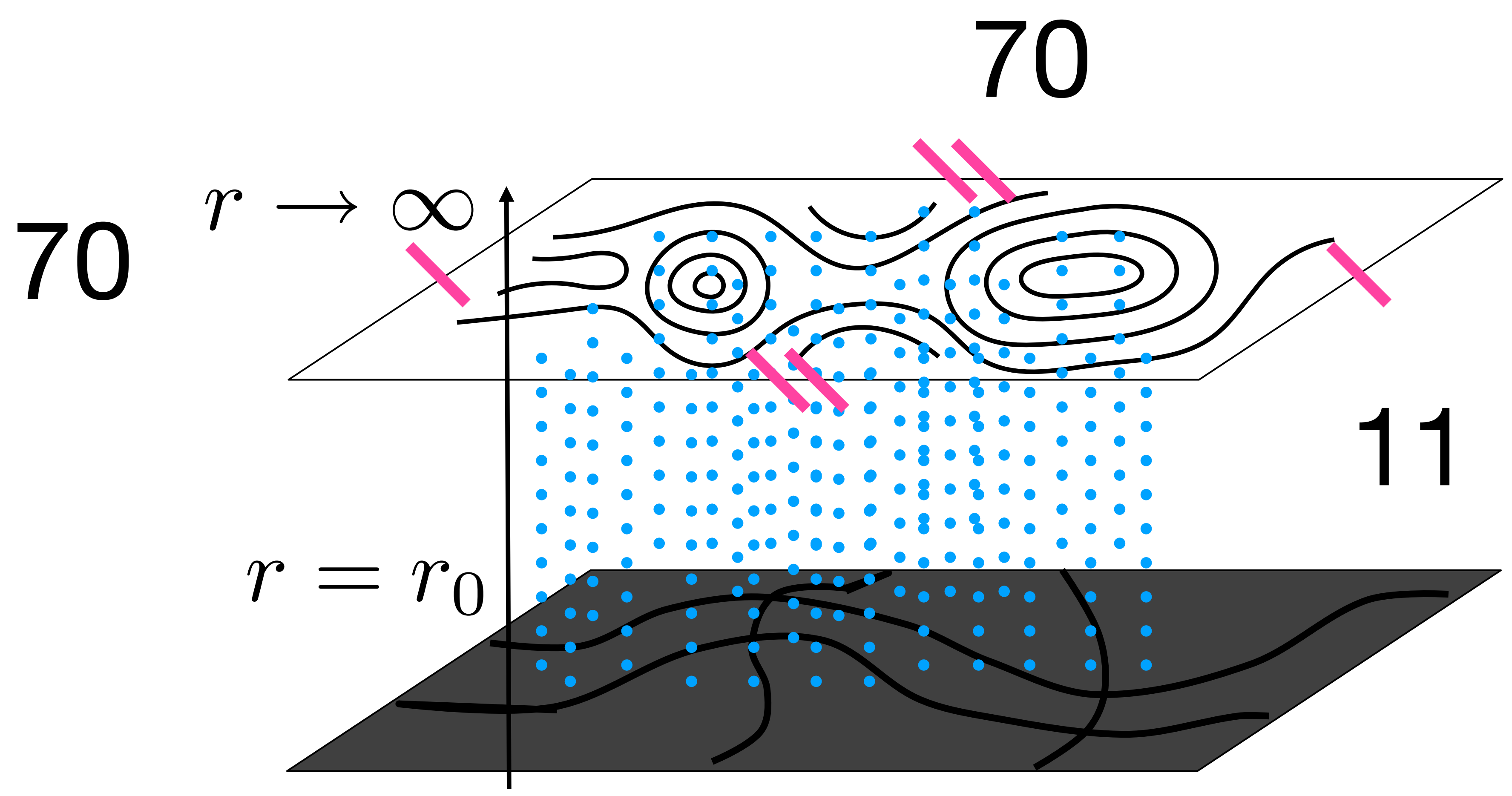


Holographic turbulence

$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$





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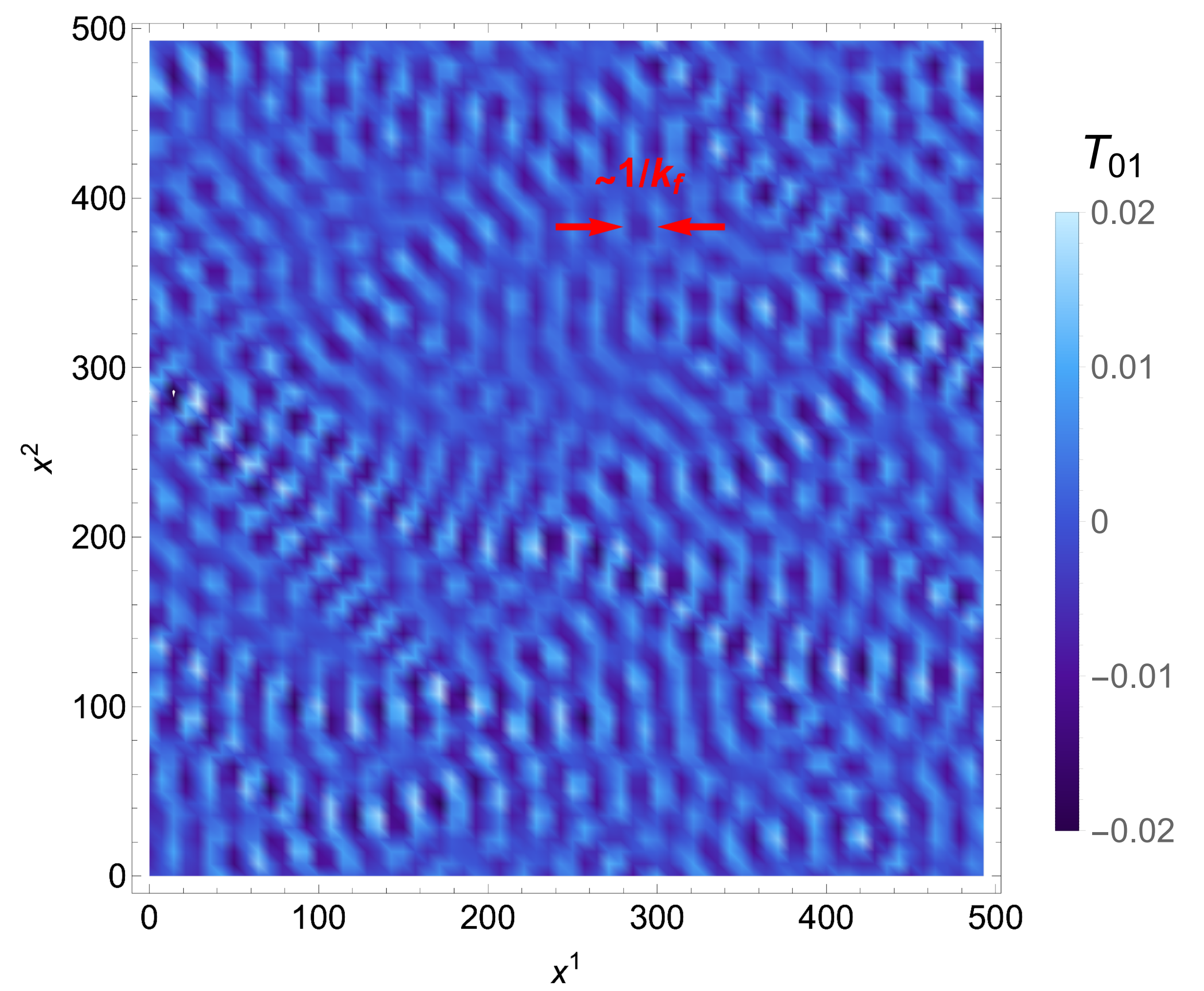

Holographic turbulence

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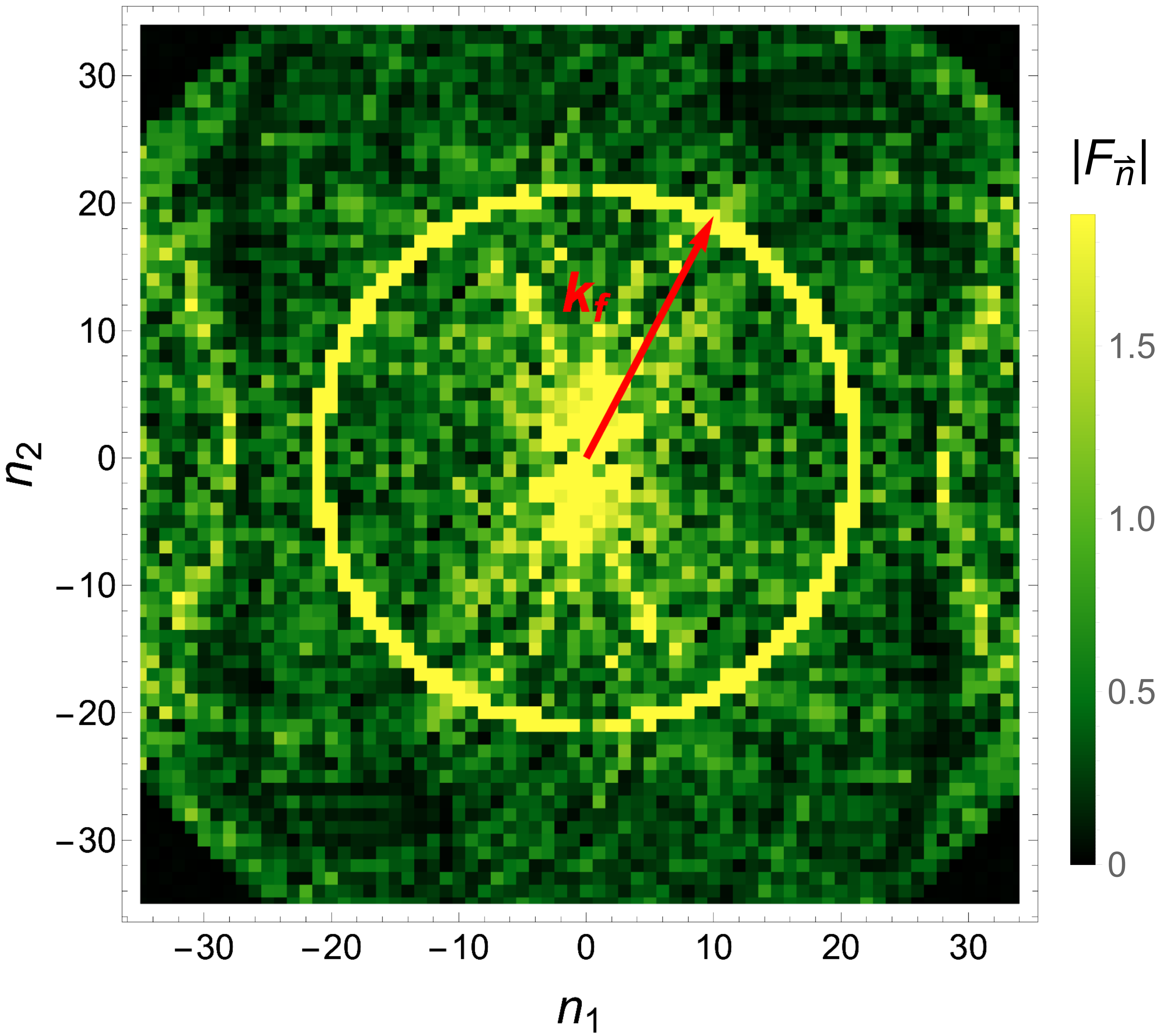

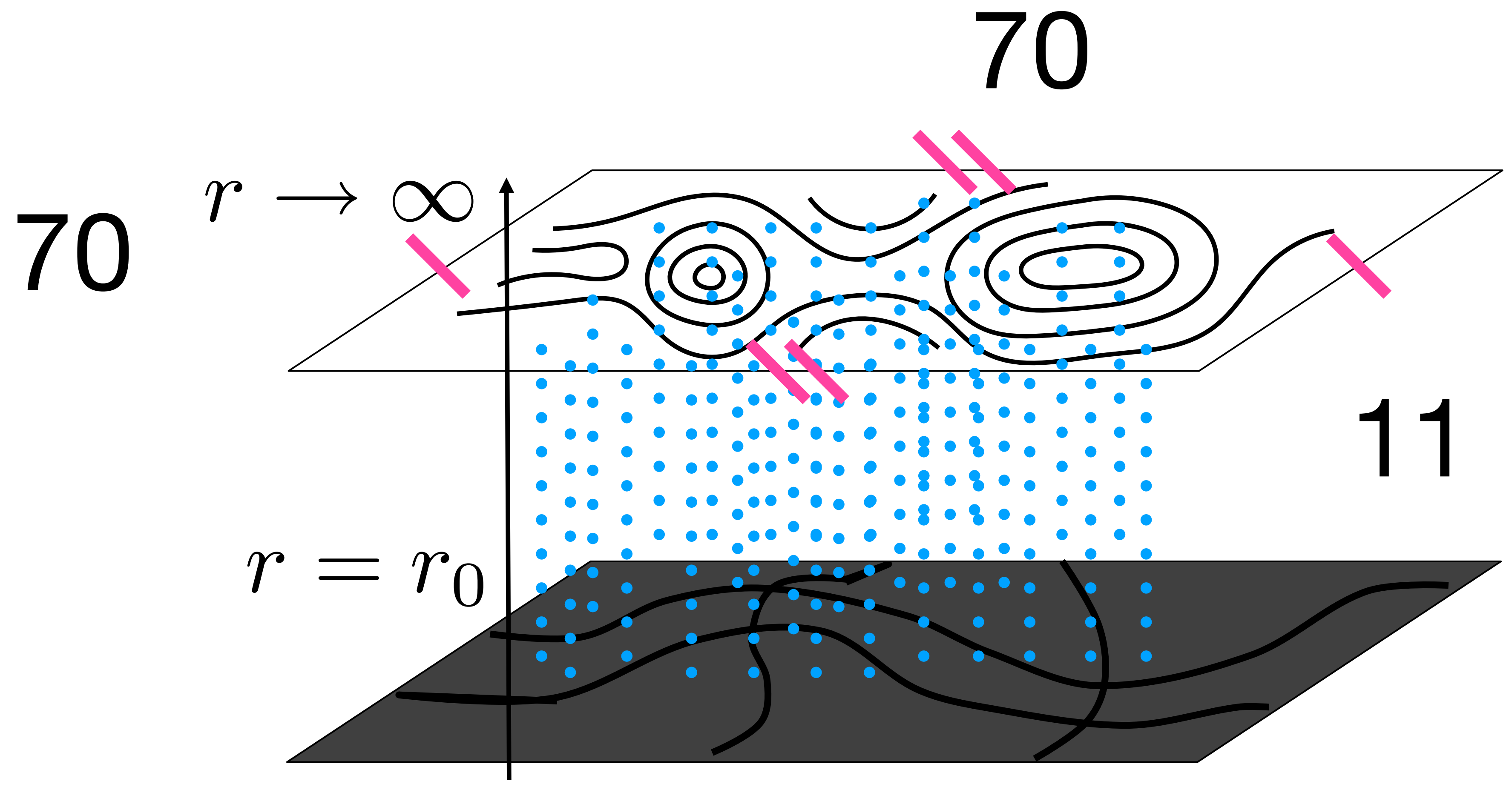

$$\hat{F}(|\vec{n}|) = \int F_{\vec{n}} \vec{n}\, d\theta$$

Holographic turbulence

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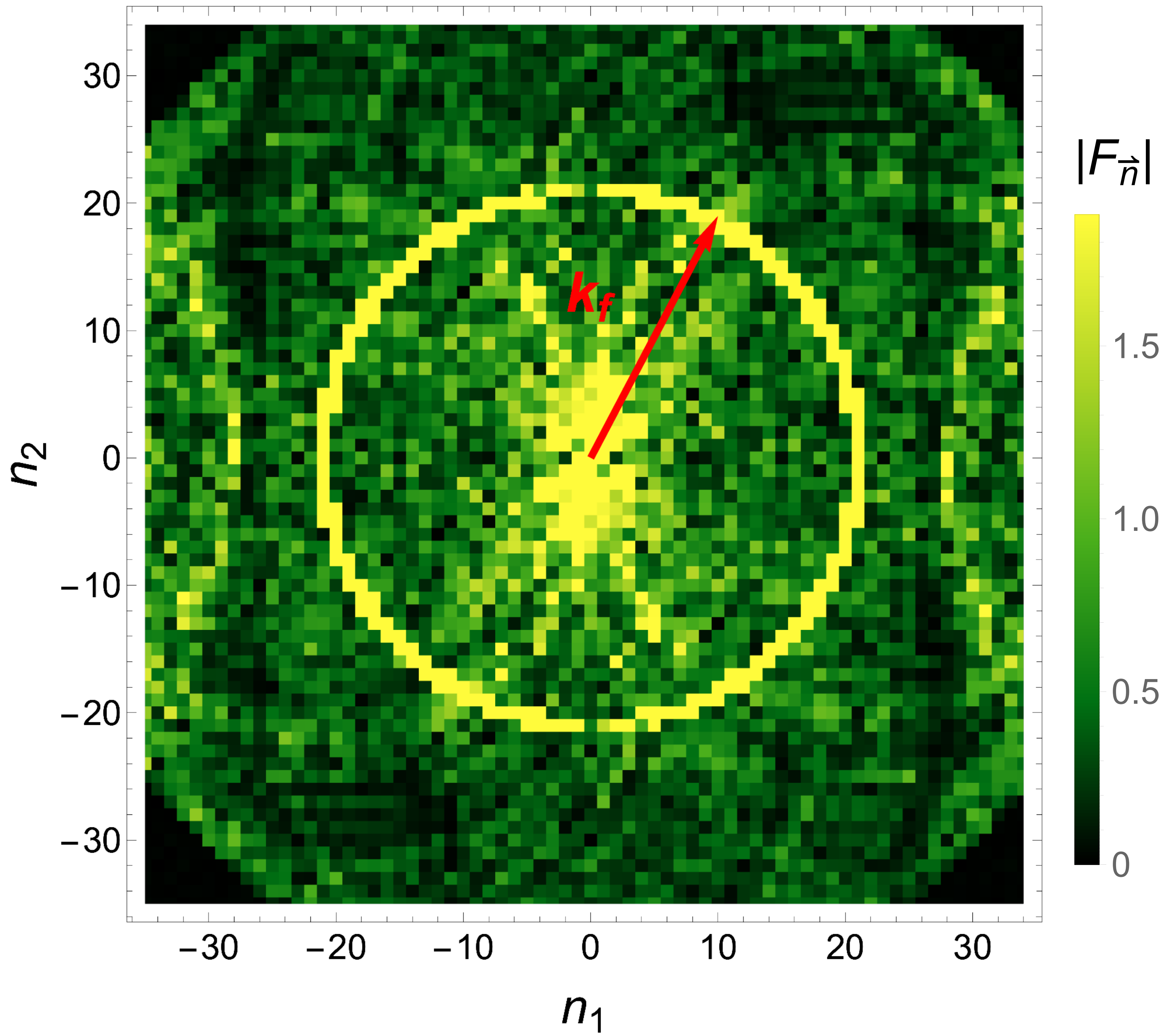

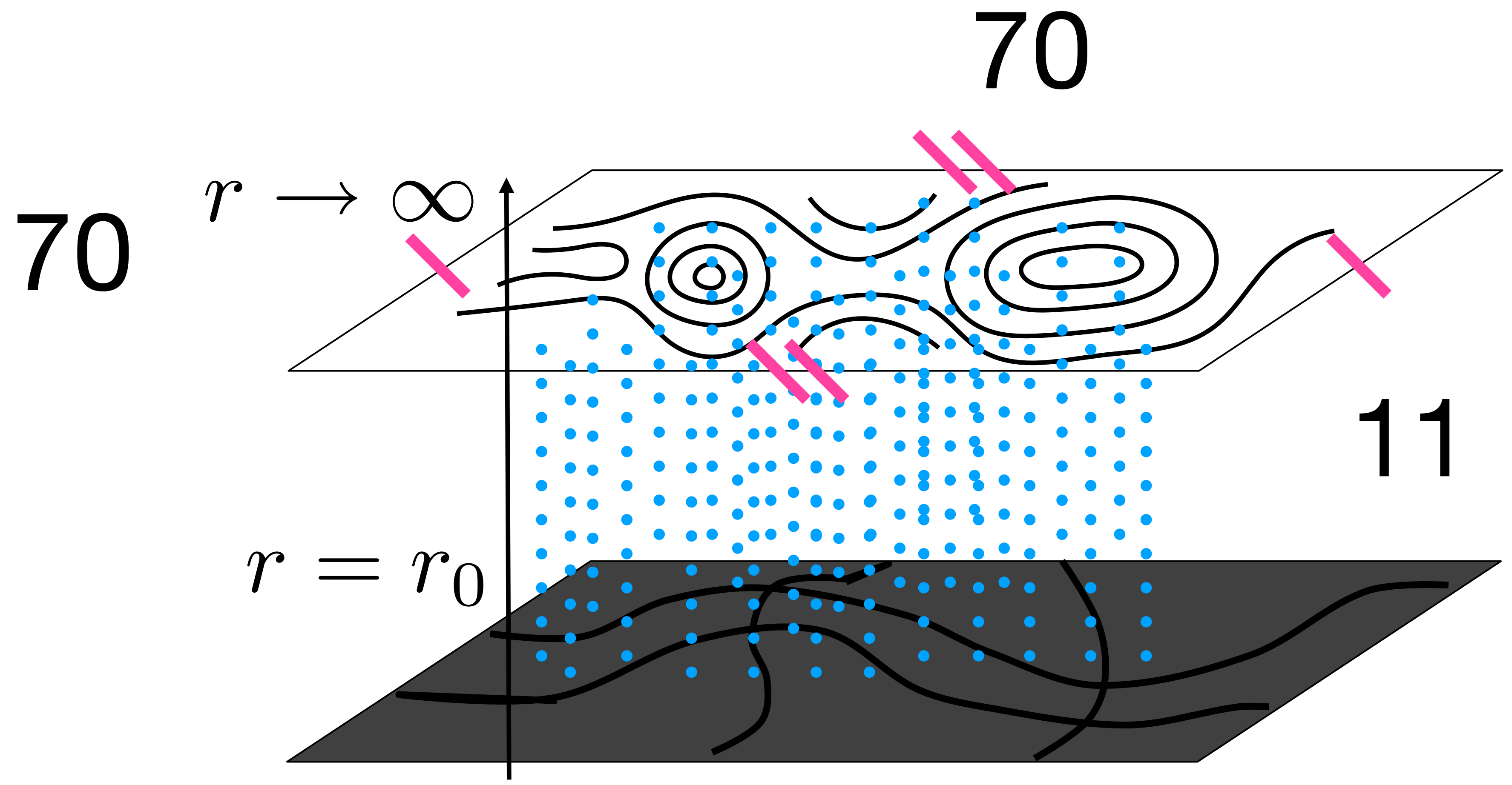

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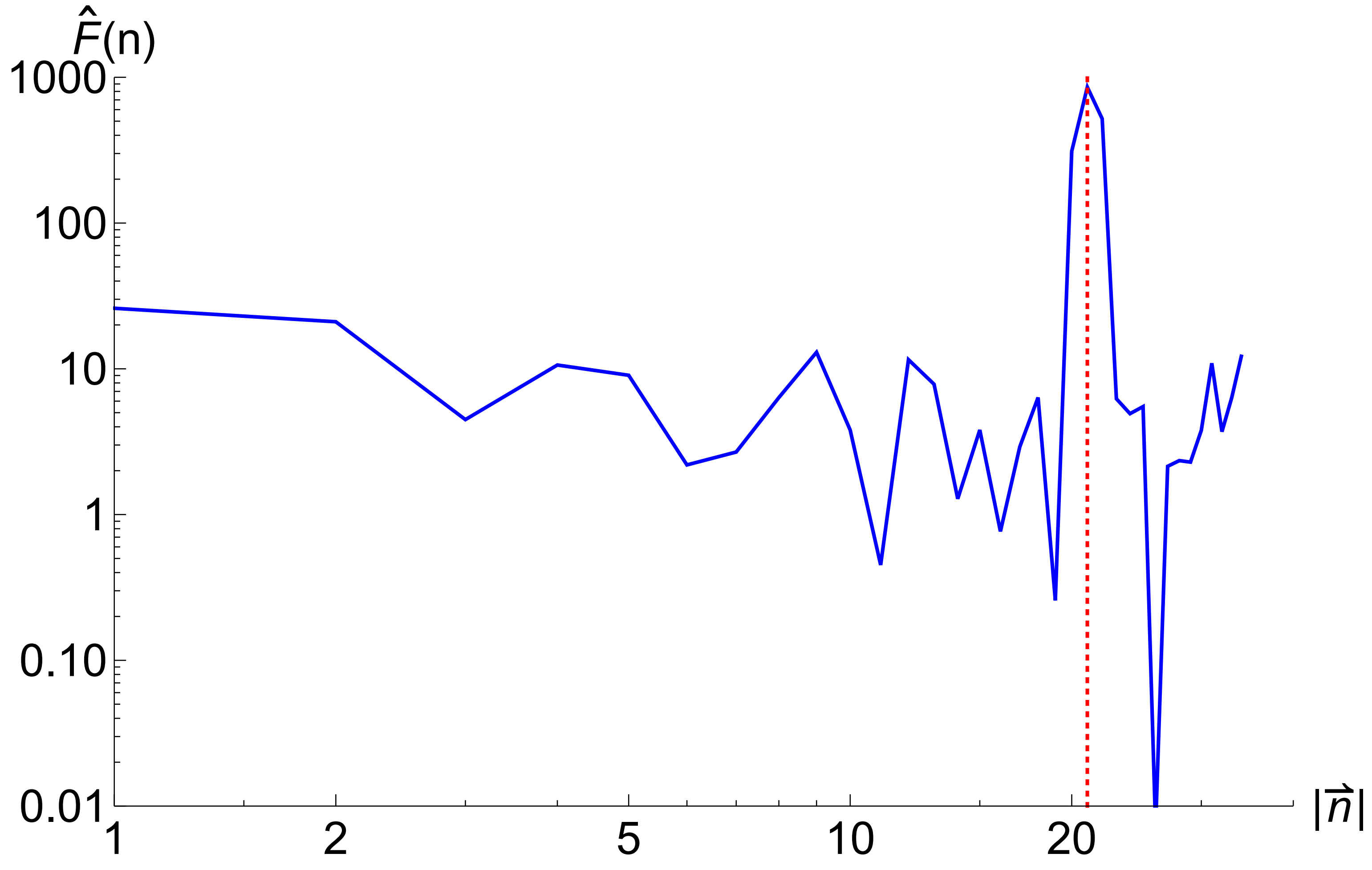

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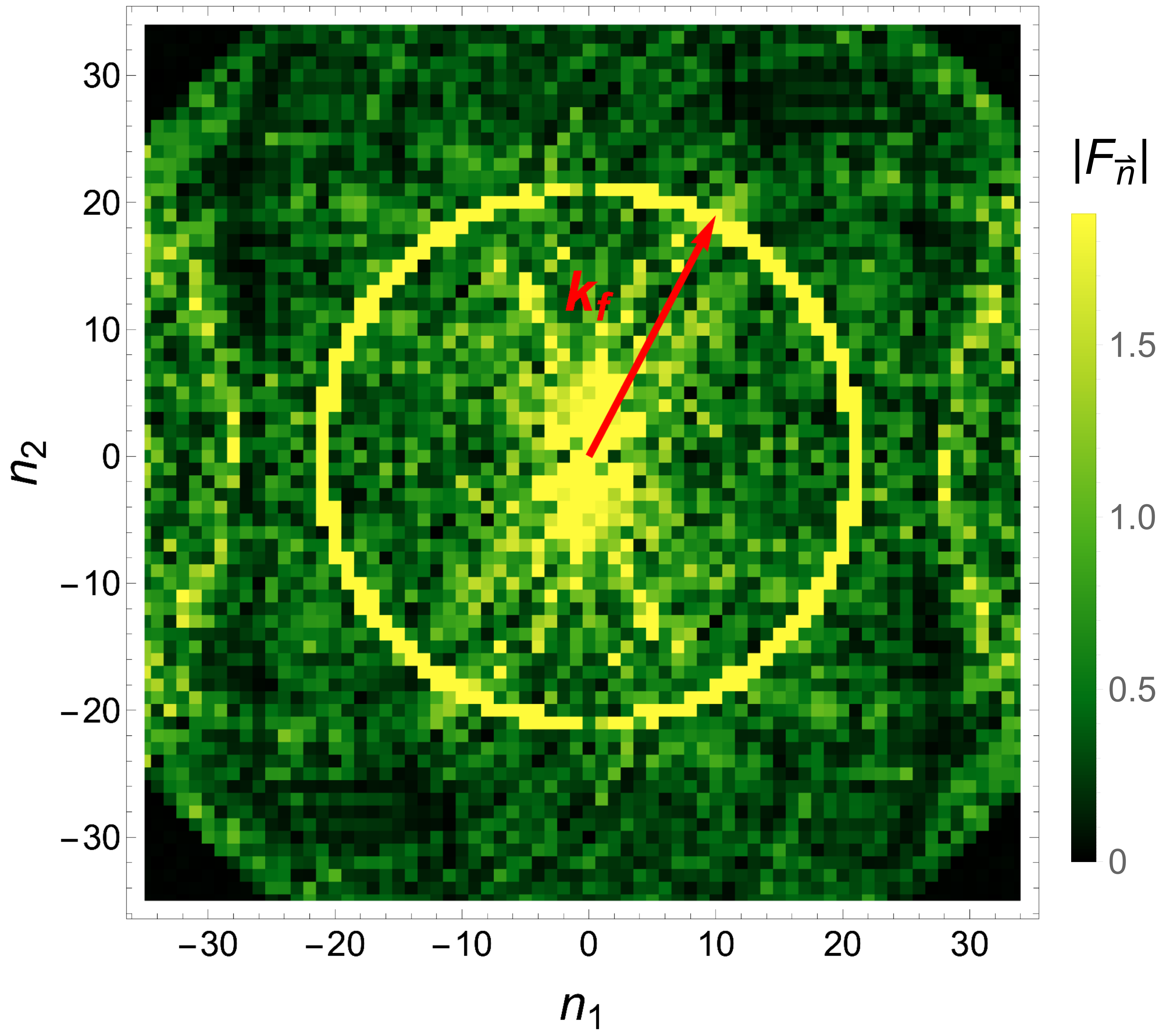

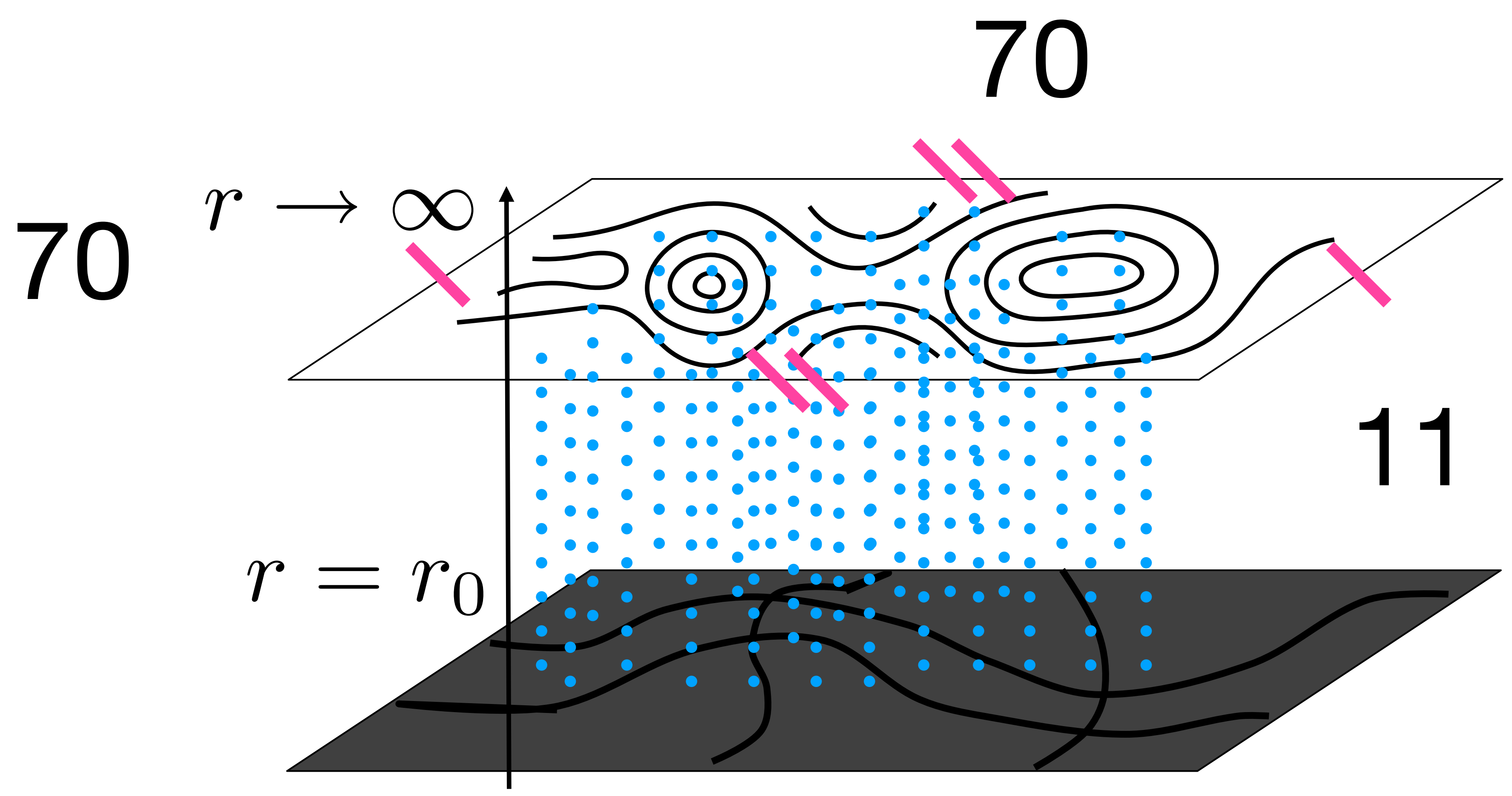

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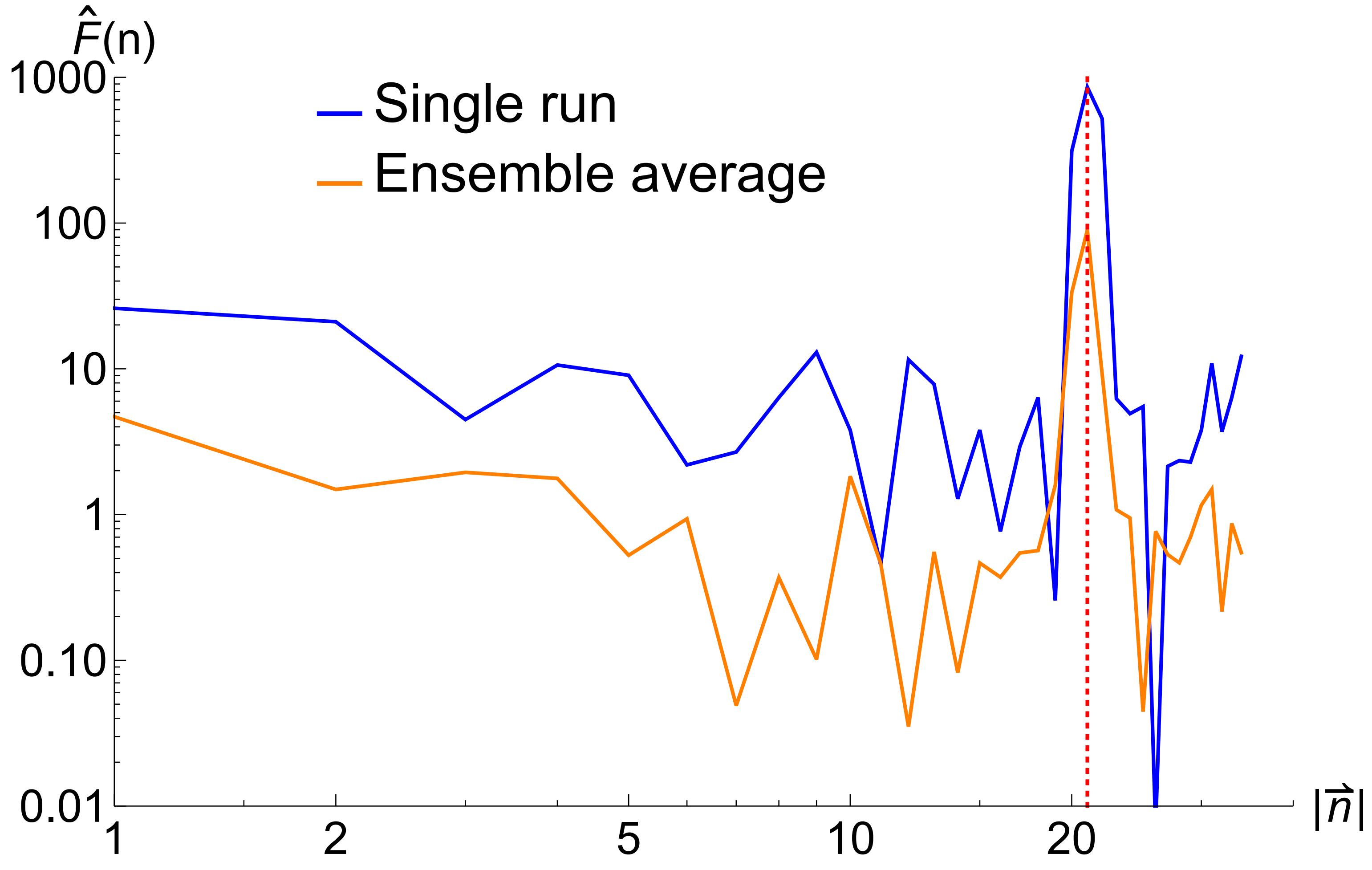

Holographic turbulence

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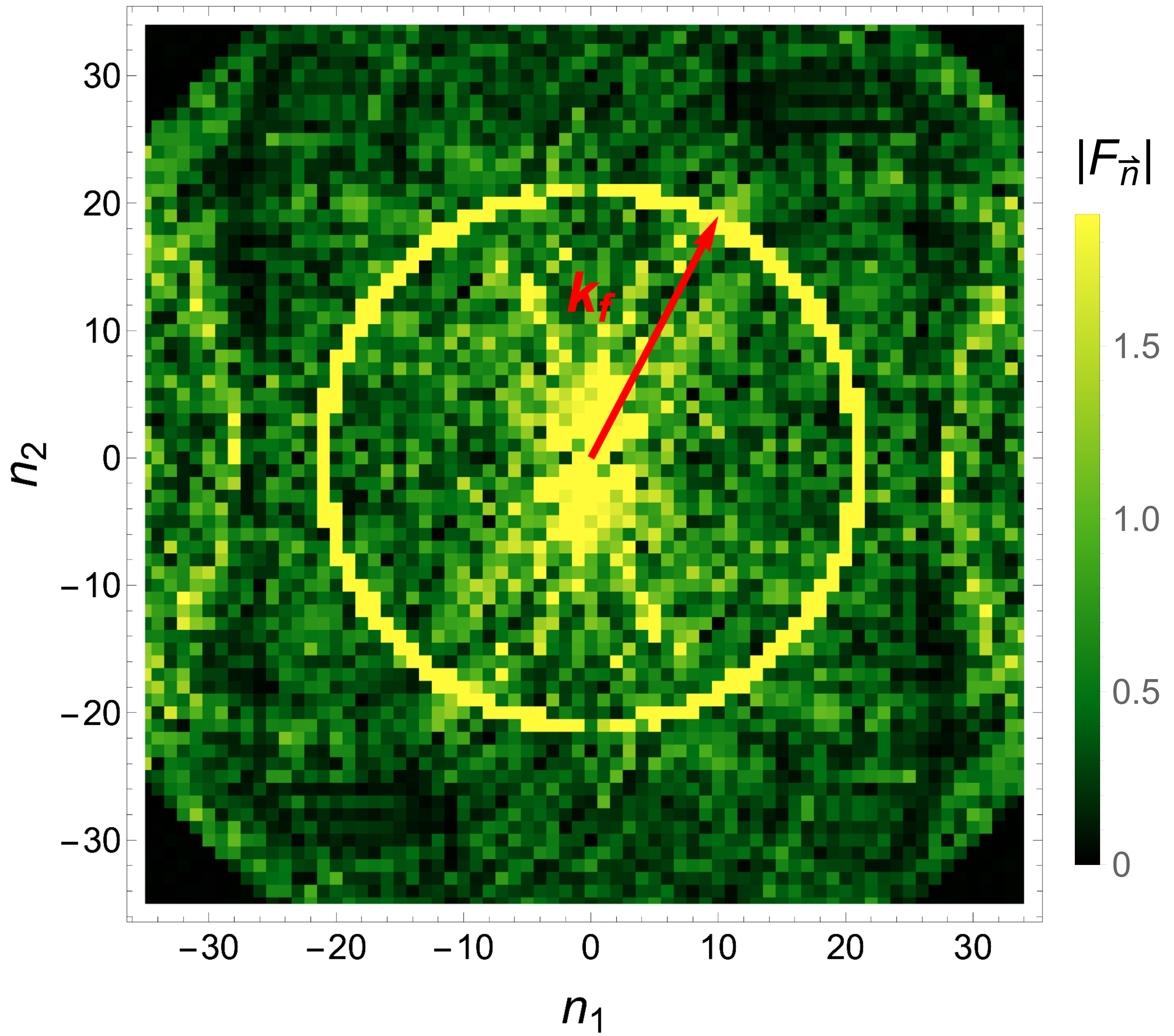

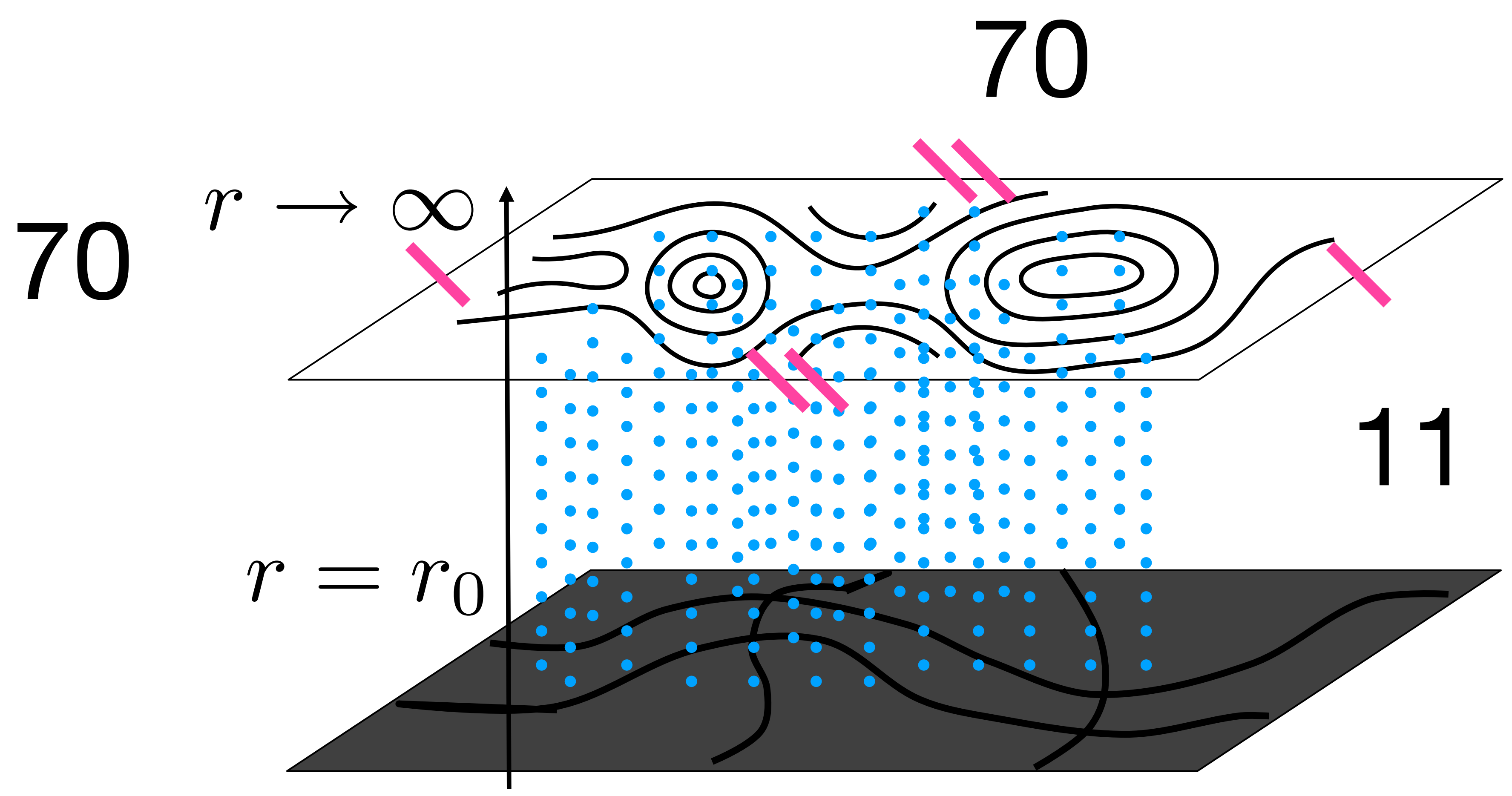

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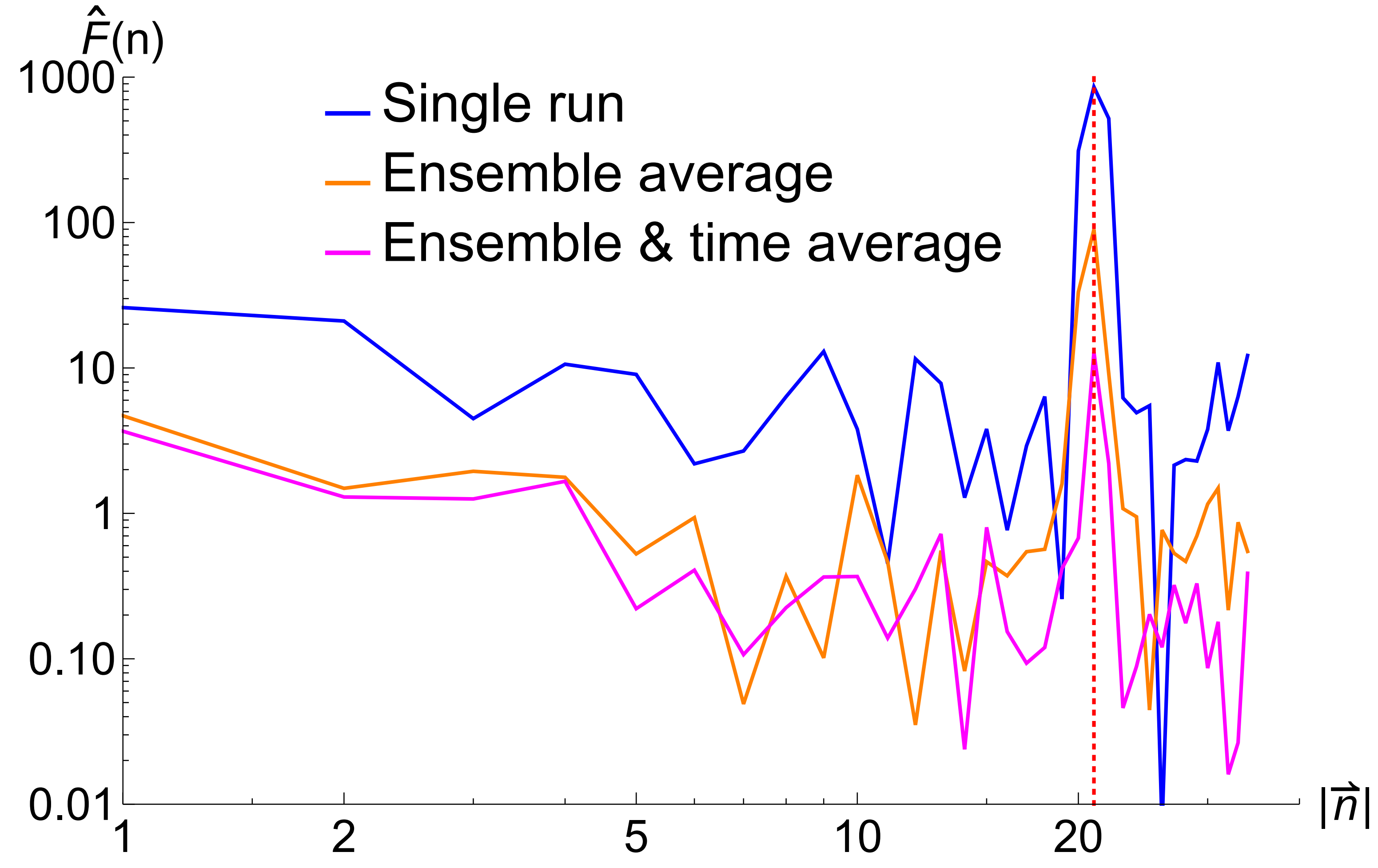
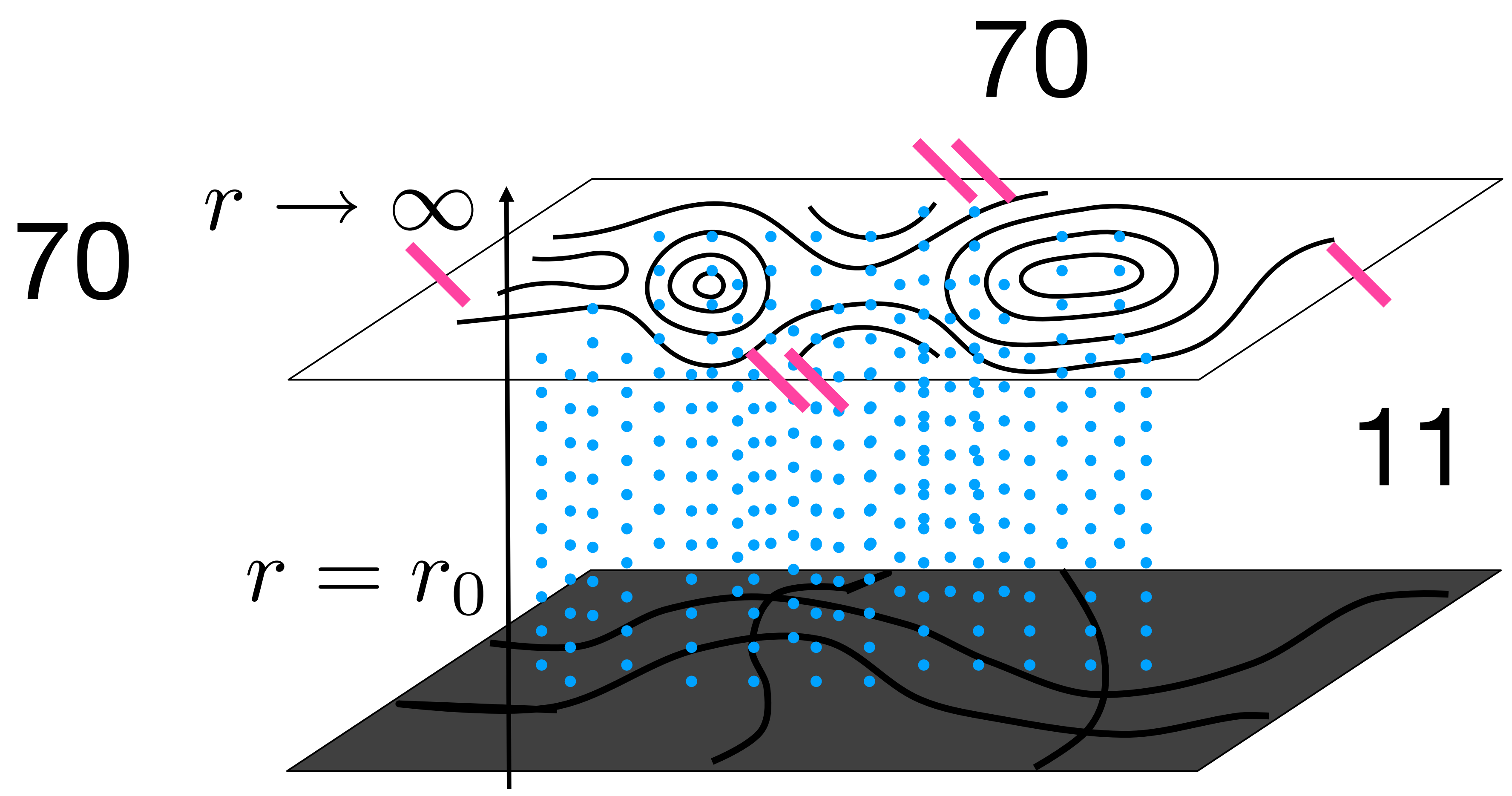


Holographic turbulence

$$\overline{T_{\mu\nu}} = \overline{g^{(3)}_{\mu\nu}}$$

Recall:

$$\hat{\epsilon} = \int \overline{\frac{1}{2}\rho\,|\,\hat{v}\,|^2 k d\theta_k} \propto k^{-\frac{5}{3}}$$



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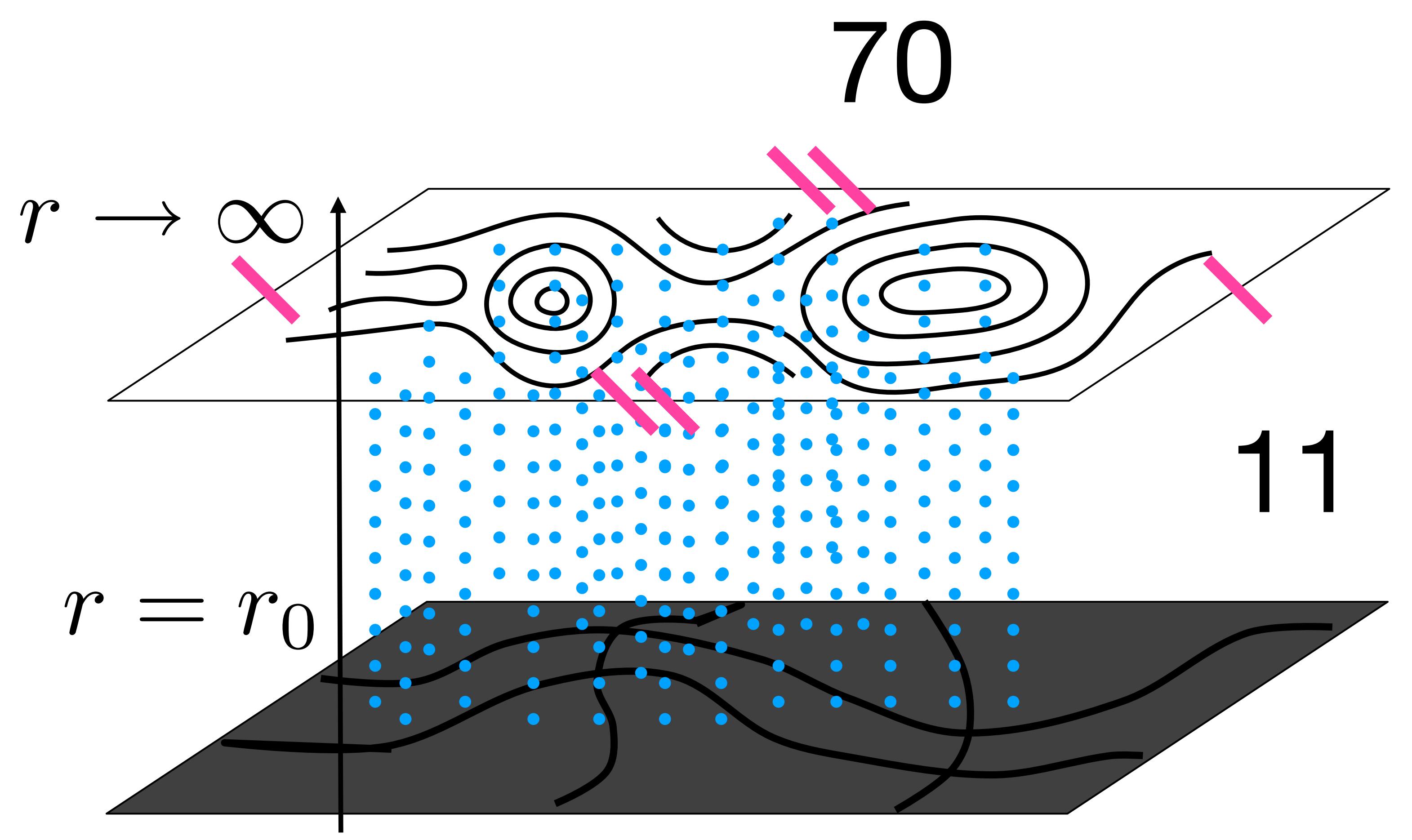
Define:

$$T^{\mu}{}_{\nu}u^{\nu} = -\,\epsilon u^{\nu}$$

70



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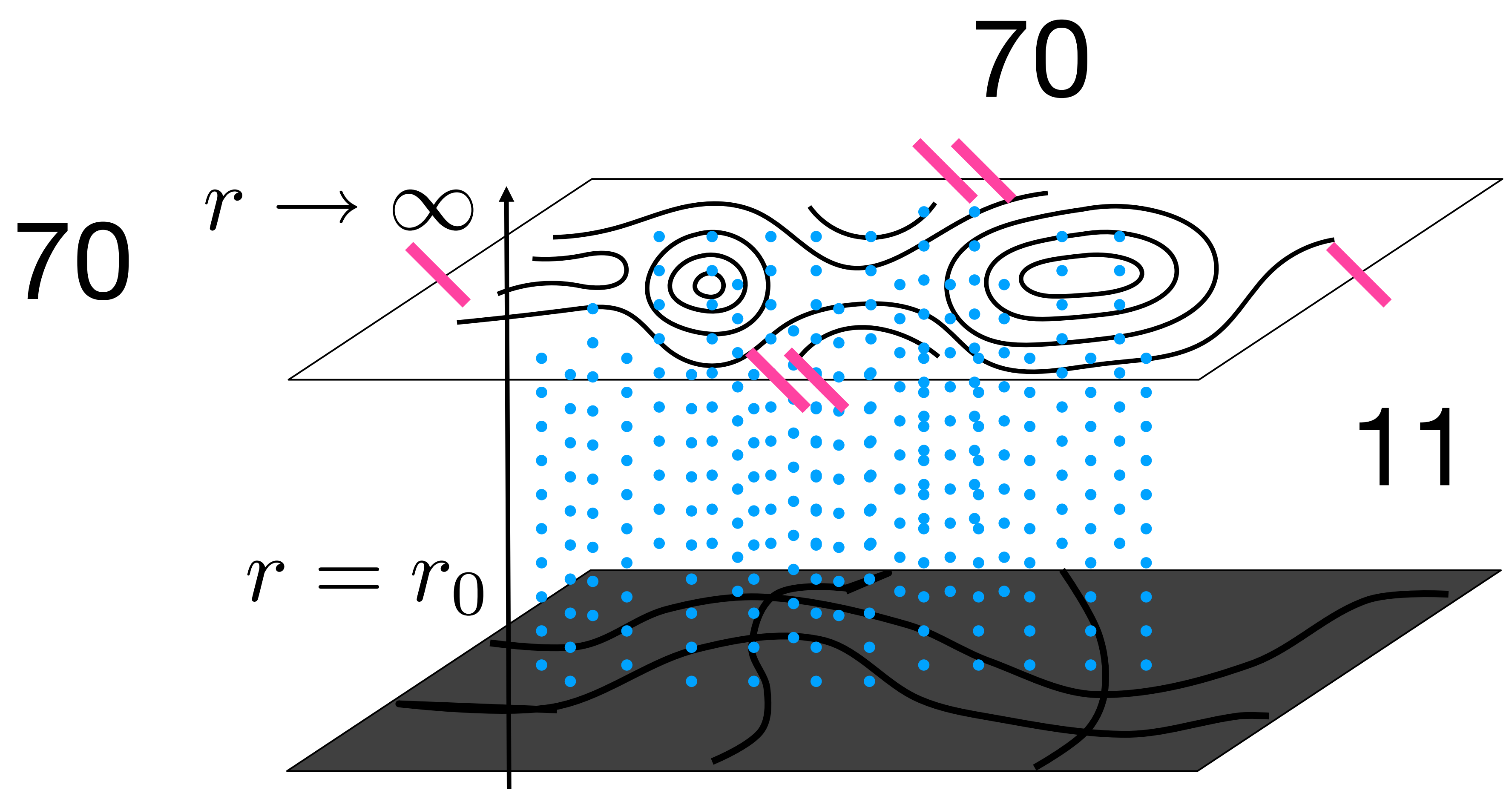
$$u^{\mu} = \gamma\left(1, \overrightarrow{v}\right)$$



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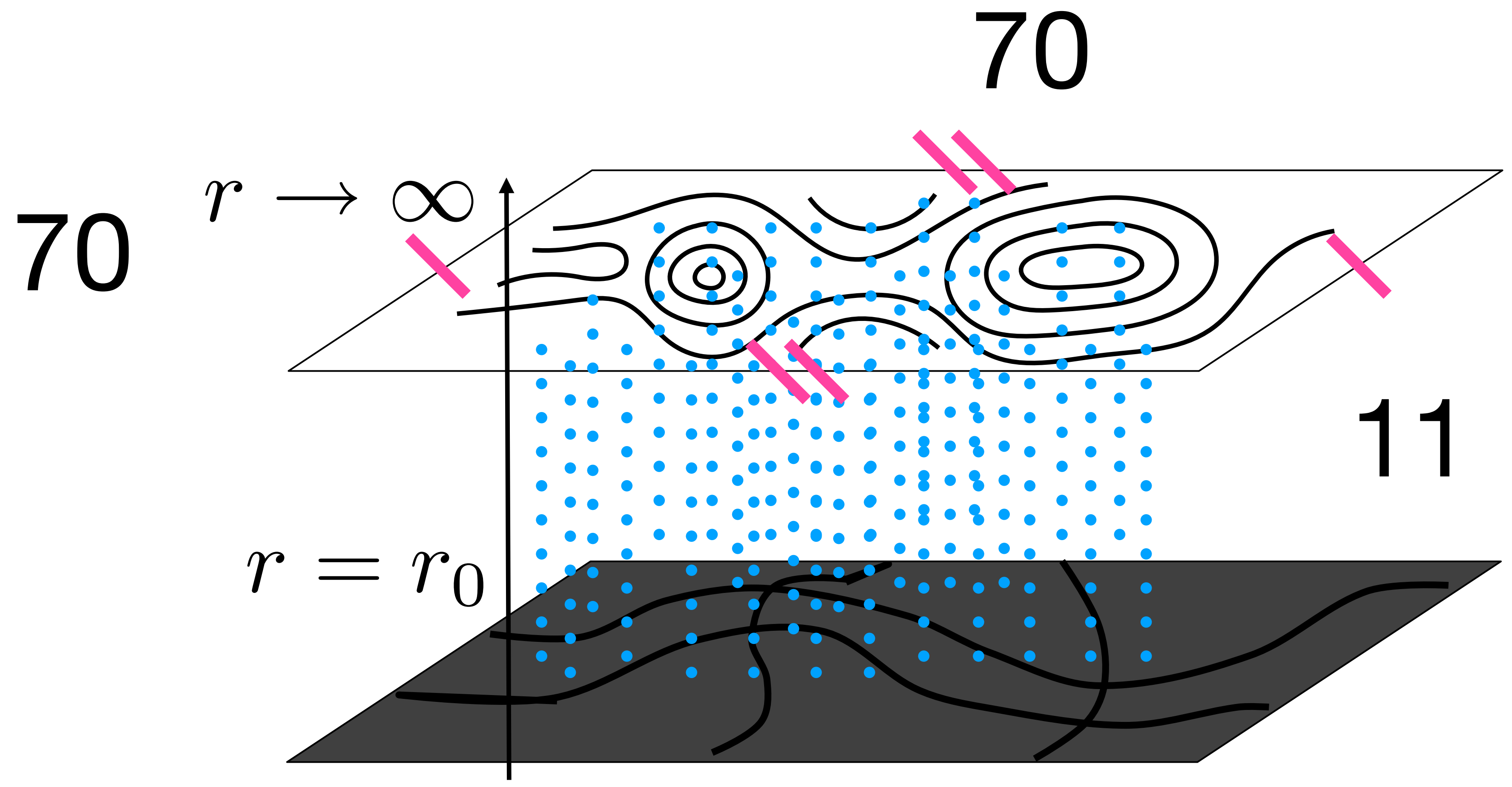

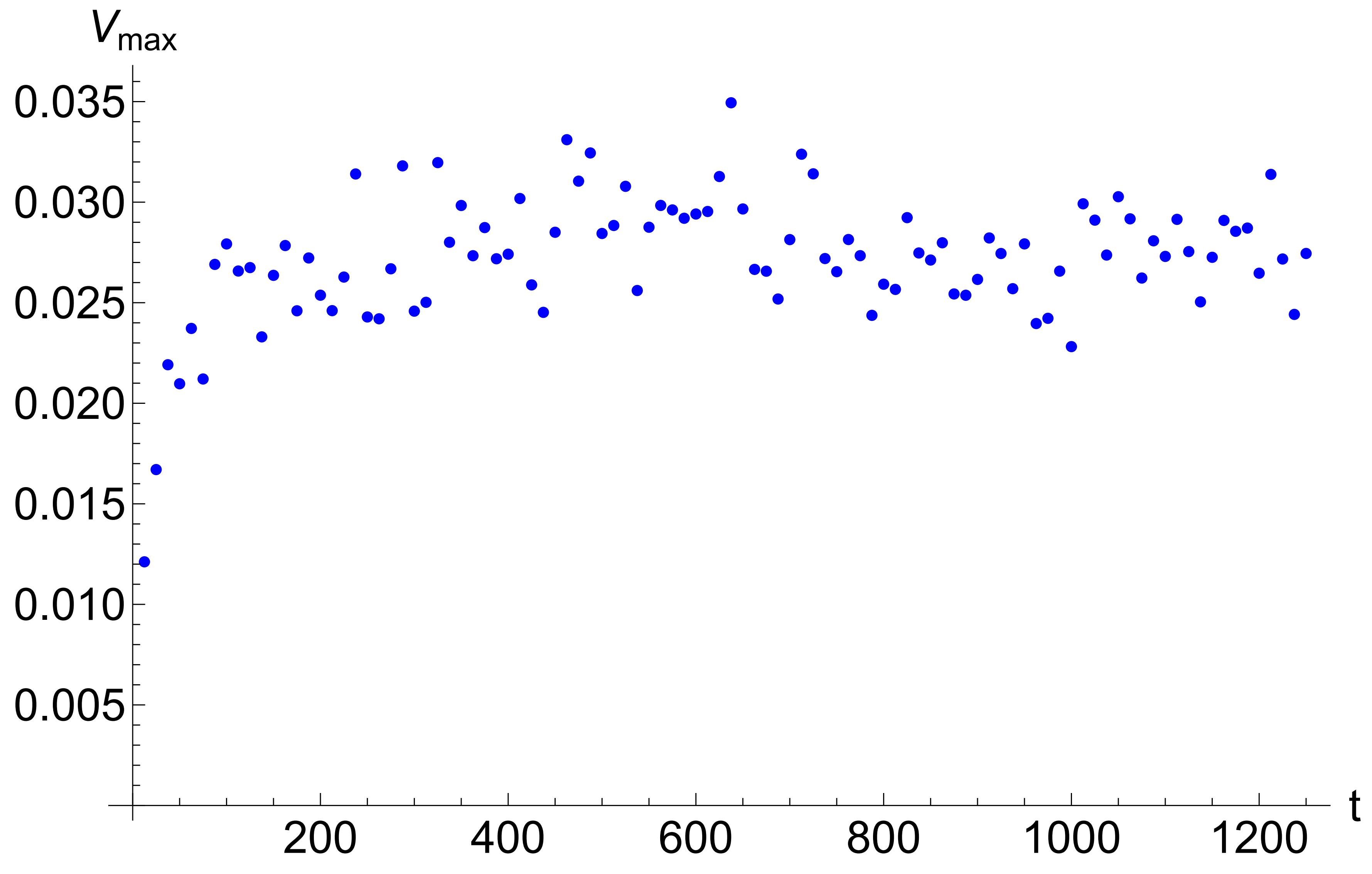

Holographic turbulence

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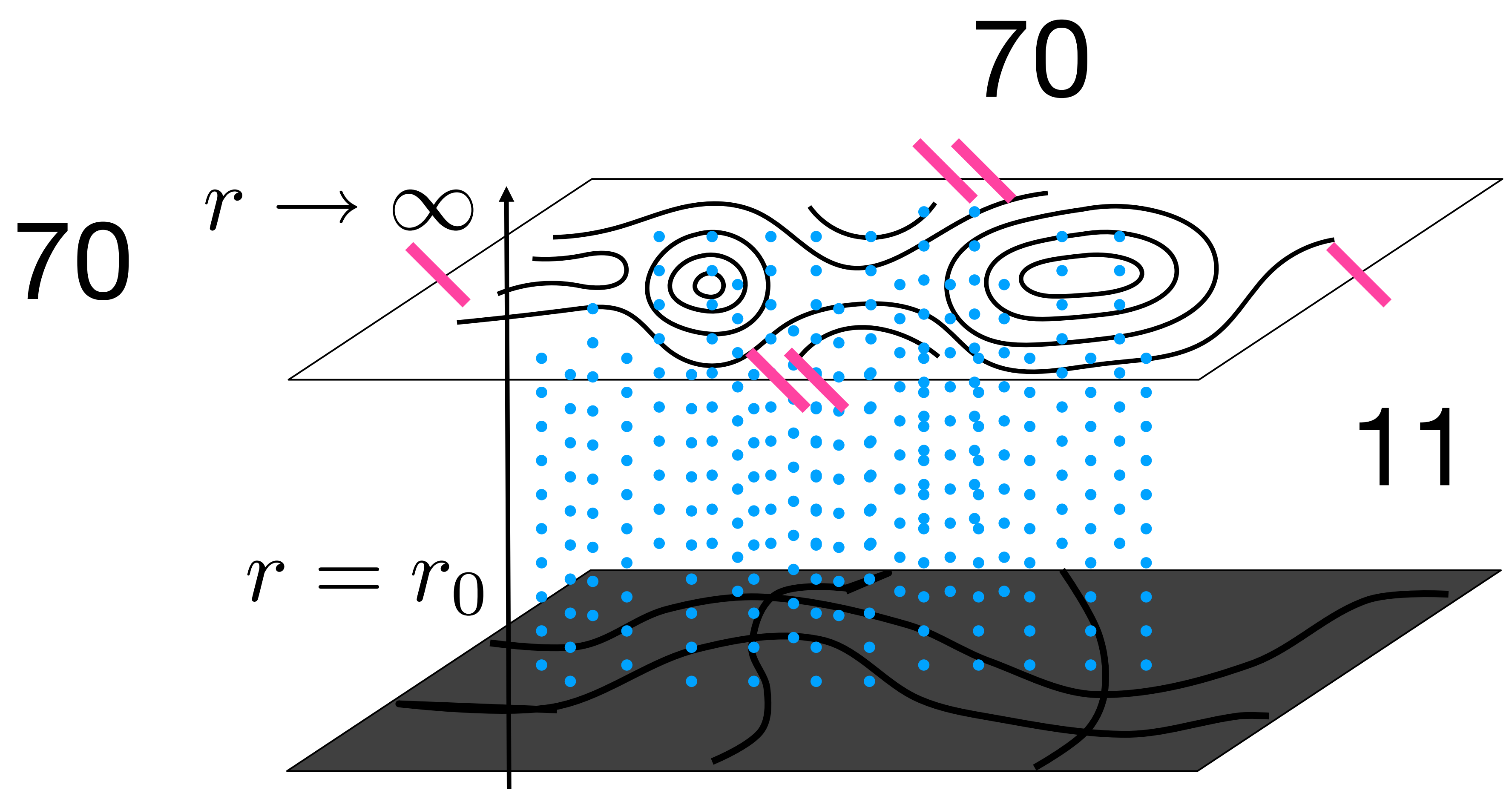
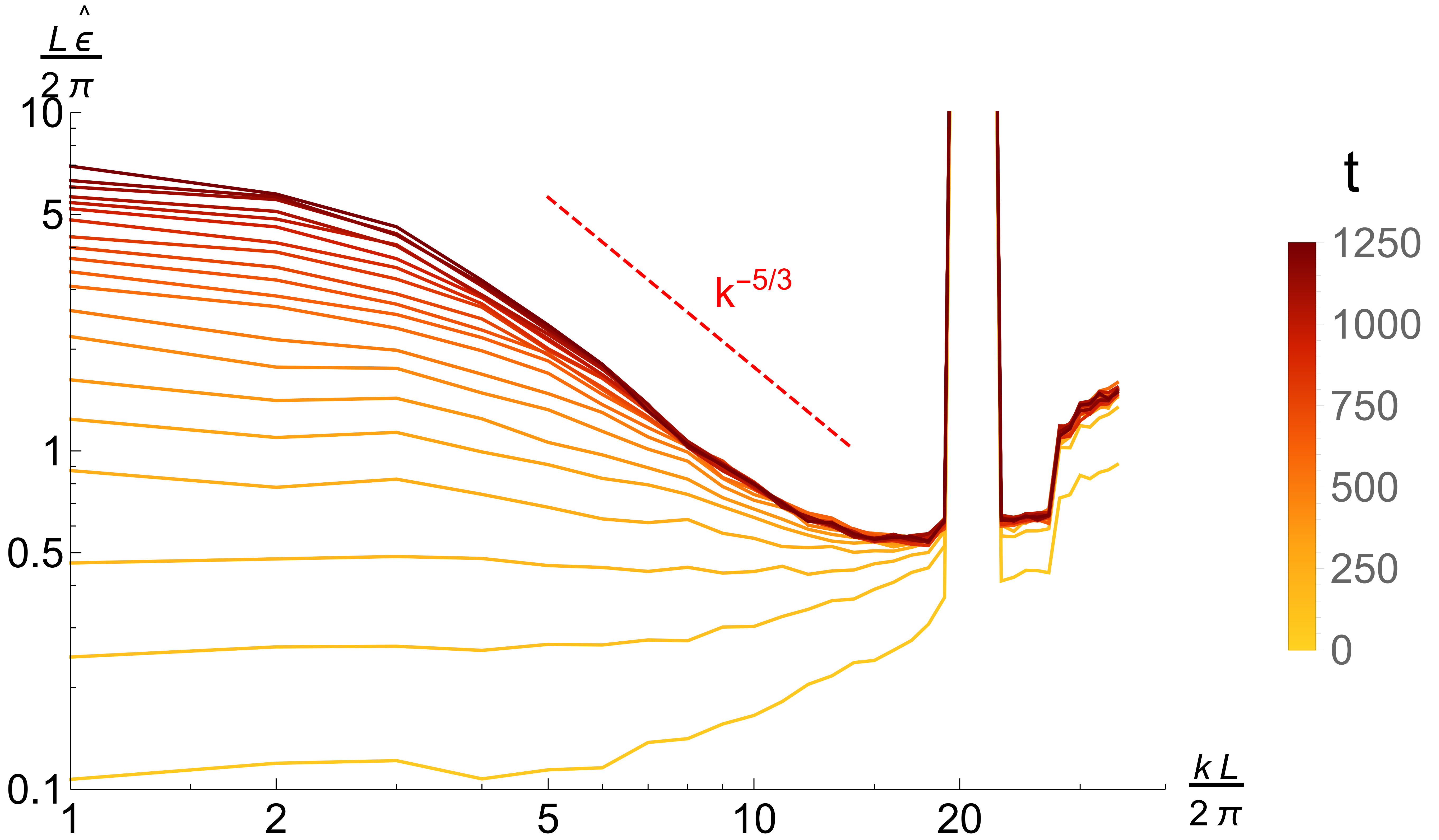
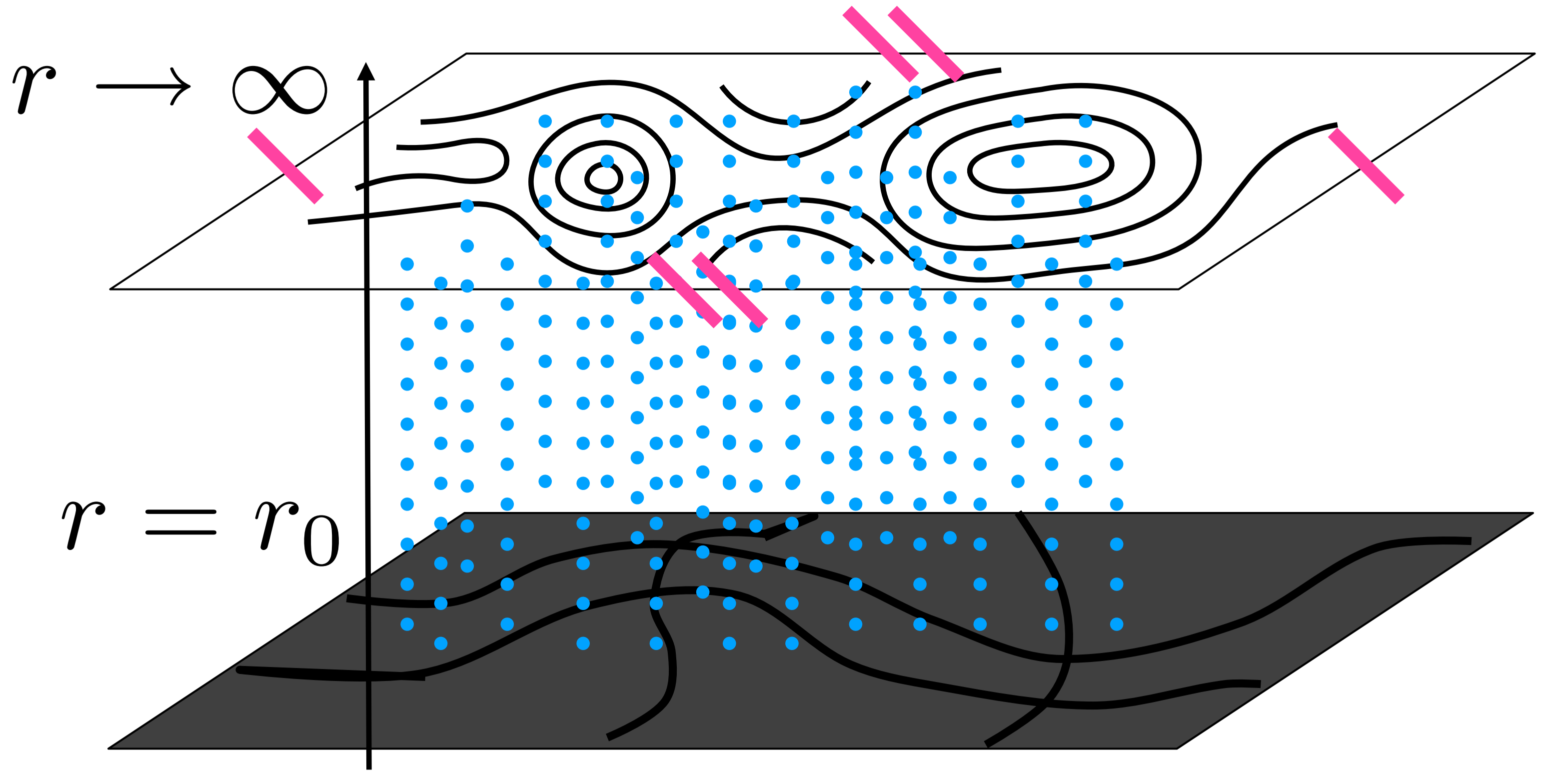


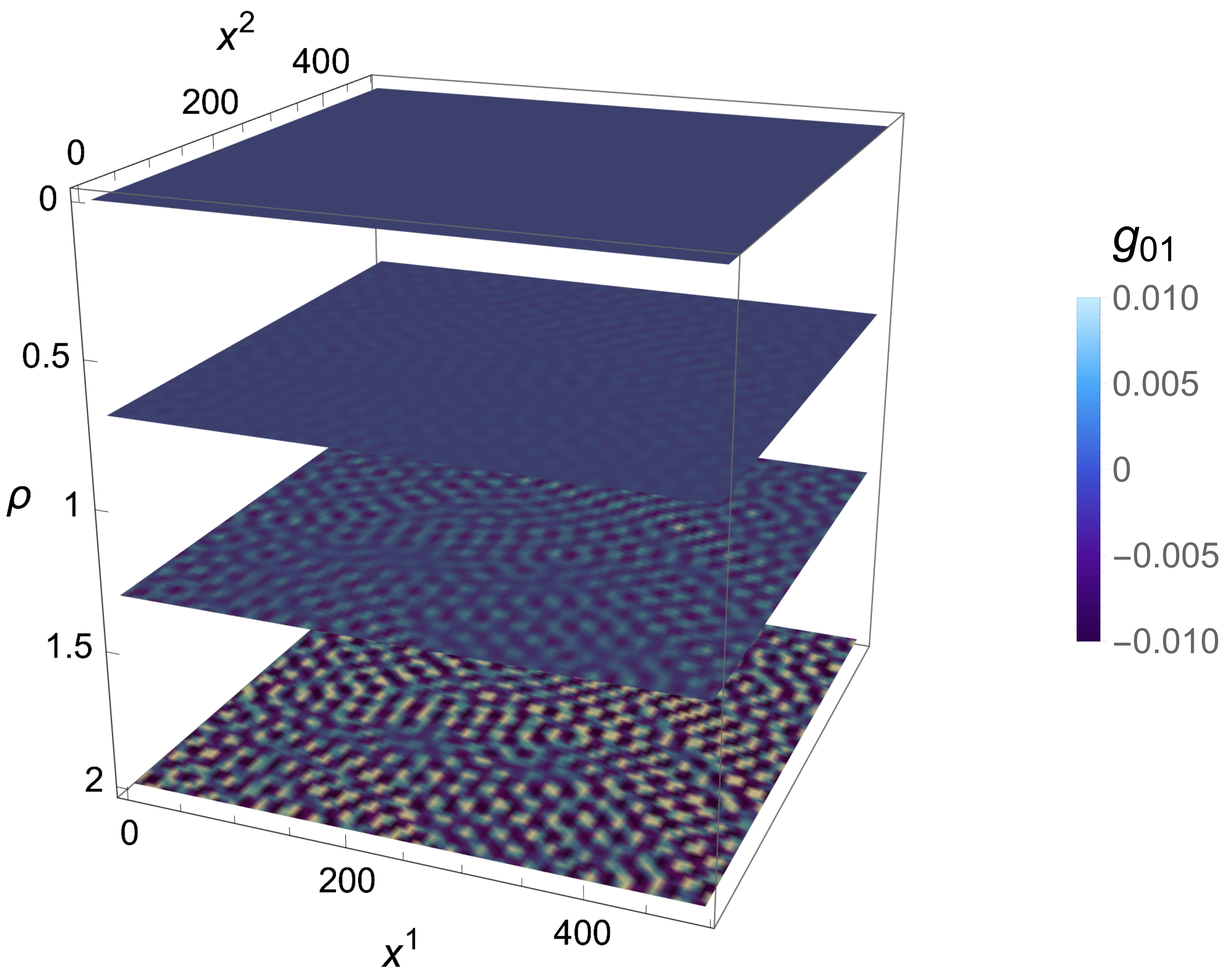
We find:



Holographic turbulence



Holographic turbulence



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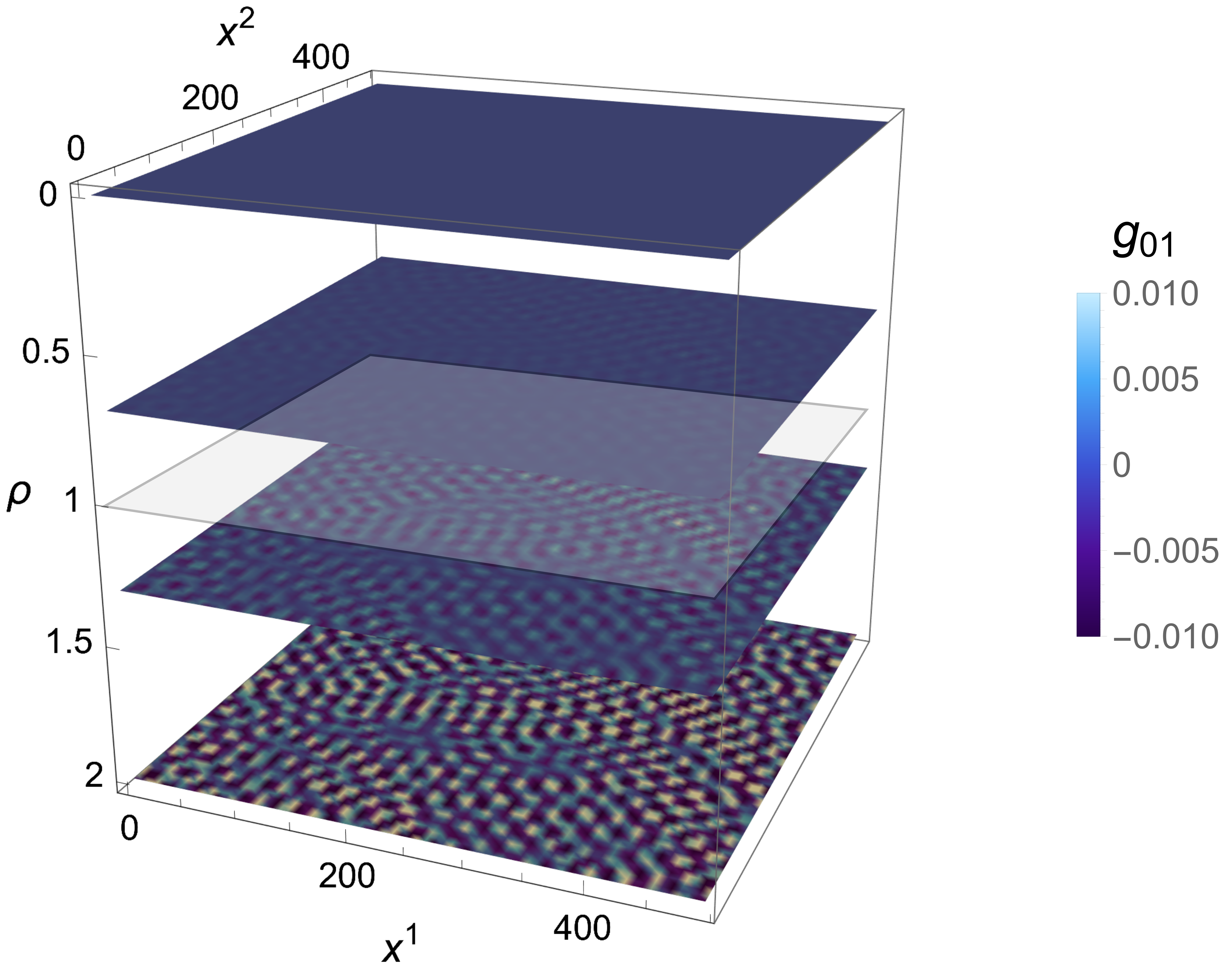
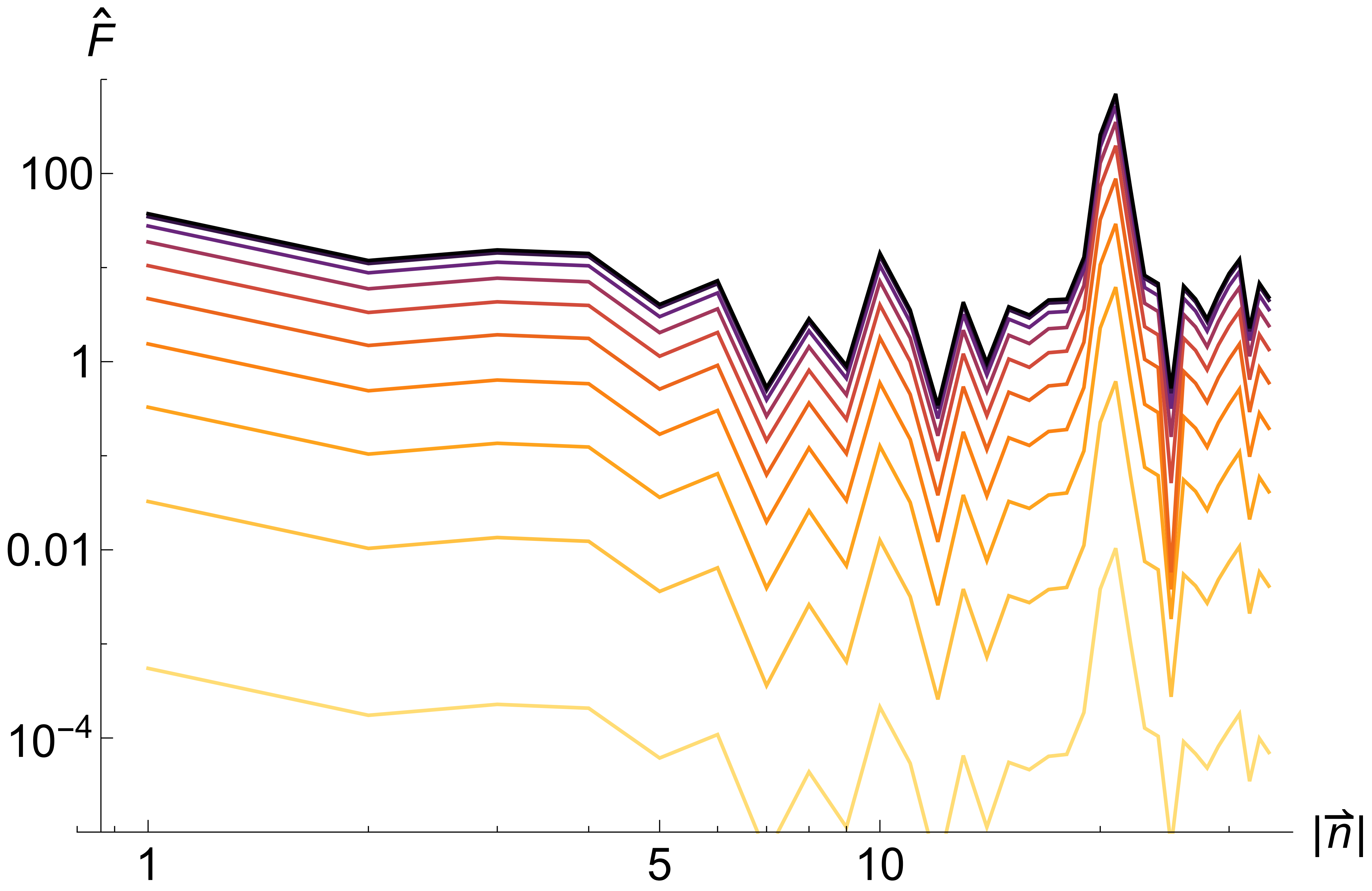
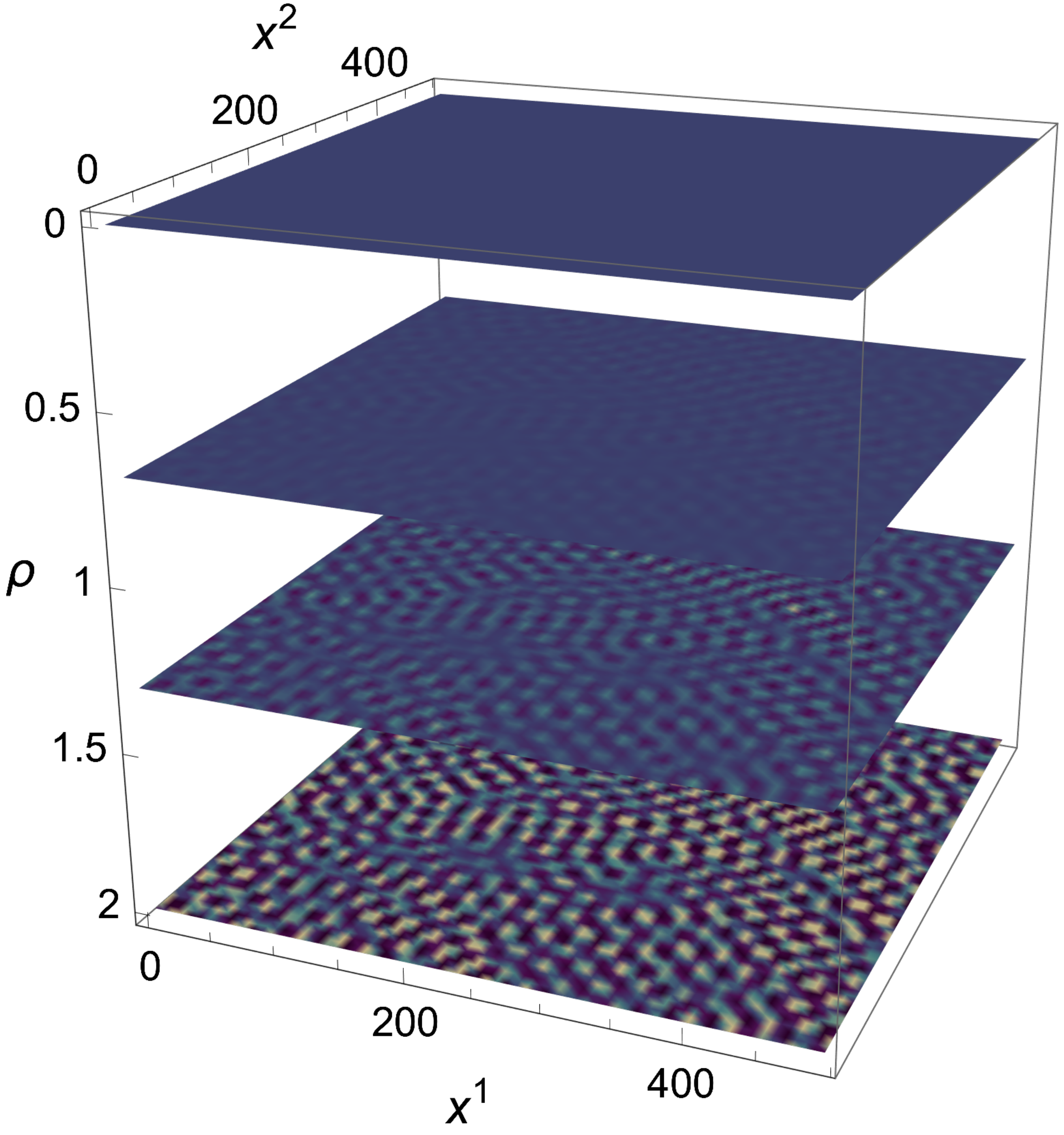


Holographic turbulence

$$\hat{F}(|\vec{n}|) = \int T^{01}_{\vec{n}} |\vec{n}| d\theta$$

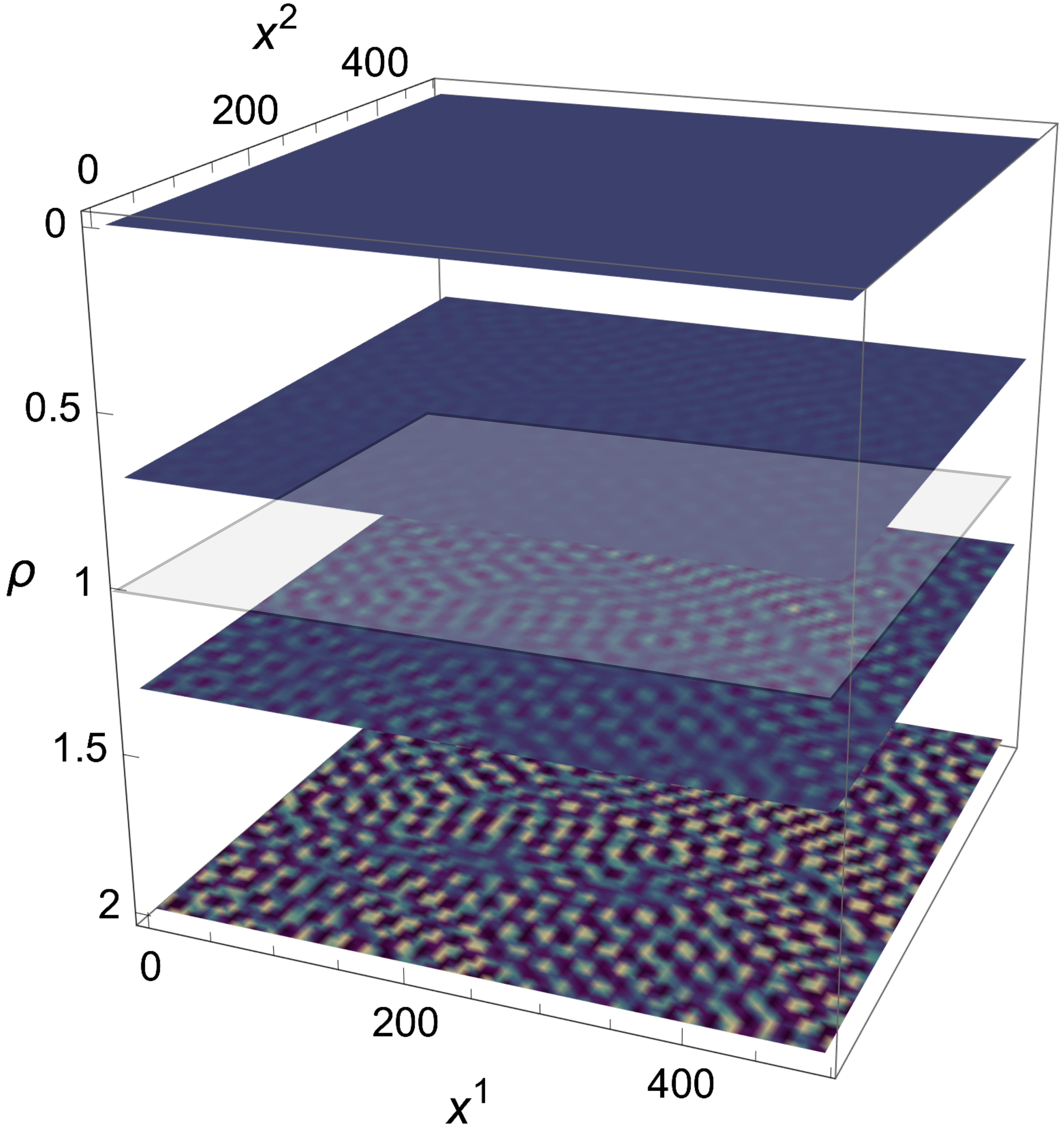




Holographic turbulence

There's an apparent horizon at

$$0.9 \leq \rho = \rho_h(t, x_1, x_2) \leq 1.1$$



Holographic turbulence

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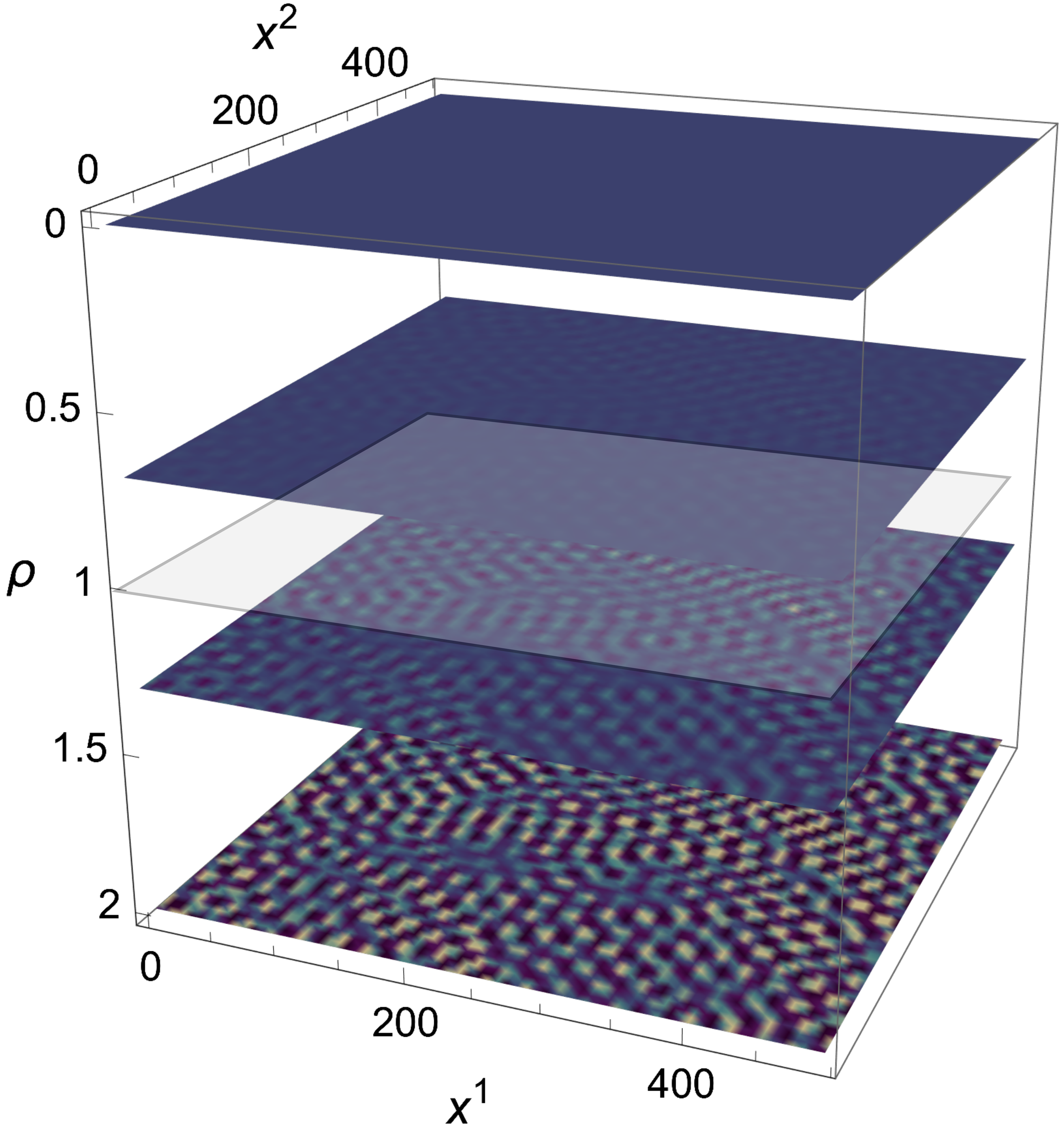
The area element of the apparent horizon is given by an expression of the form

$$\mathscr{A}(t, \overrightarrow{x}) = \Sigma(t, \overrightarrow{x})^2$$



Holographic turbulence

There's an apparent horizon at

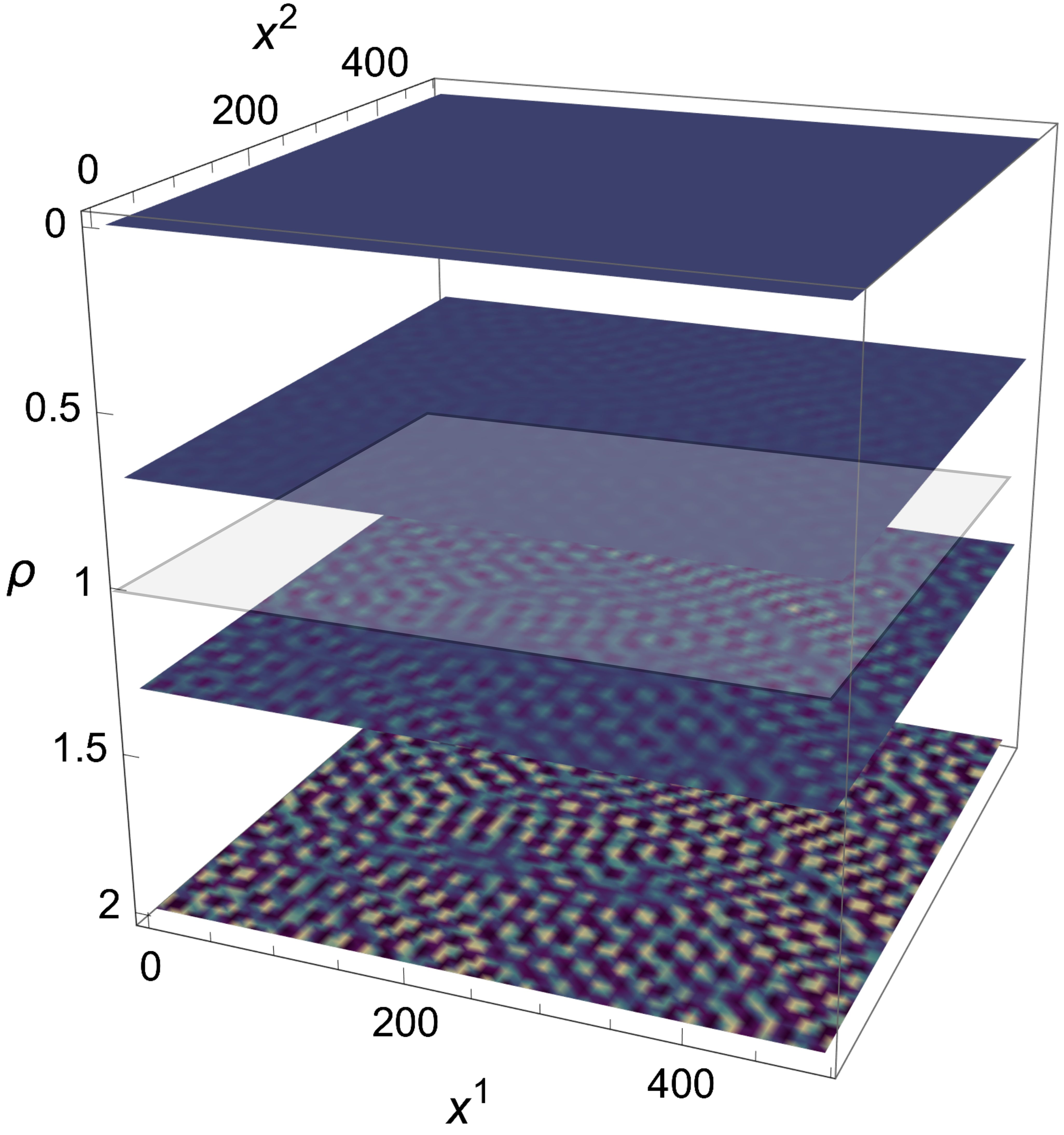
$$0.9 \leq \rho = \rho_h(t, x_1, x_2) \leq 1.1$$

The area element of the apparent horizon is given by an expression of the form

$$\mathscr{A}(t, \overrightarrow{x}) = \Sigma(t, \overrightarrow{x})^2$$

We define the area element power spectrum

$$\hat{a}(|\overrightarrow{n}|) = \frac{1}{(2\pi)^2} \int |\Sigma_{\overrightarrow{n}}|^2 |\overrightarrow{n}| \, d\theta$$



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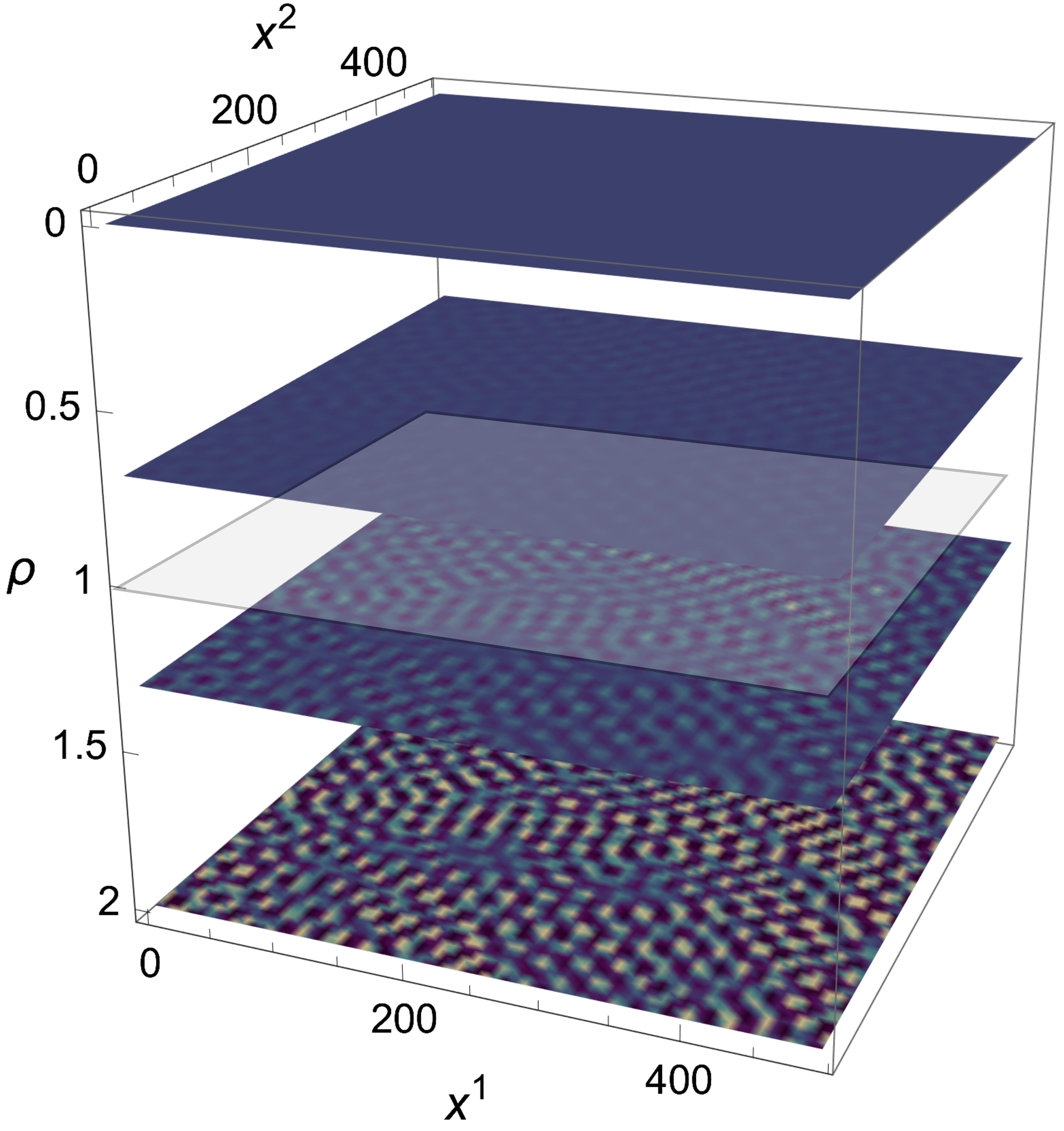

Holographic turbulence

The area element of the apparent horizon is given by an expression of the form

$$\mathscr{A}(t, \overrightarrow{x}) = \Sigma(t, \overrightarrow{x})^2$$

Compare with

$$\epsilon = |\overrightarrow{v}|^2$$

We define the area element power spectrum

$$\hat{a}(|\overrightarrow{n}|) = \frac{1}{(2\pi)^2} \int |\Sigma_{\overrightarrow{n}}|^2 |\overrightarrow{n}| d\theta$$

$$\hat{\epsilon} = \frac{1}{(2\pi)^2} \int |\hat{v}|^2 k d\theta$$

such that

$$A = \int \mathscr{A} d^2x = \int \hat{a} dn$$

$$E = \int \epsilon d^2x = \int \hat{\epsilon} dk$$

Holographic turbulence

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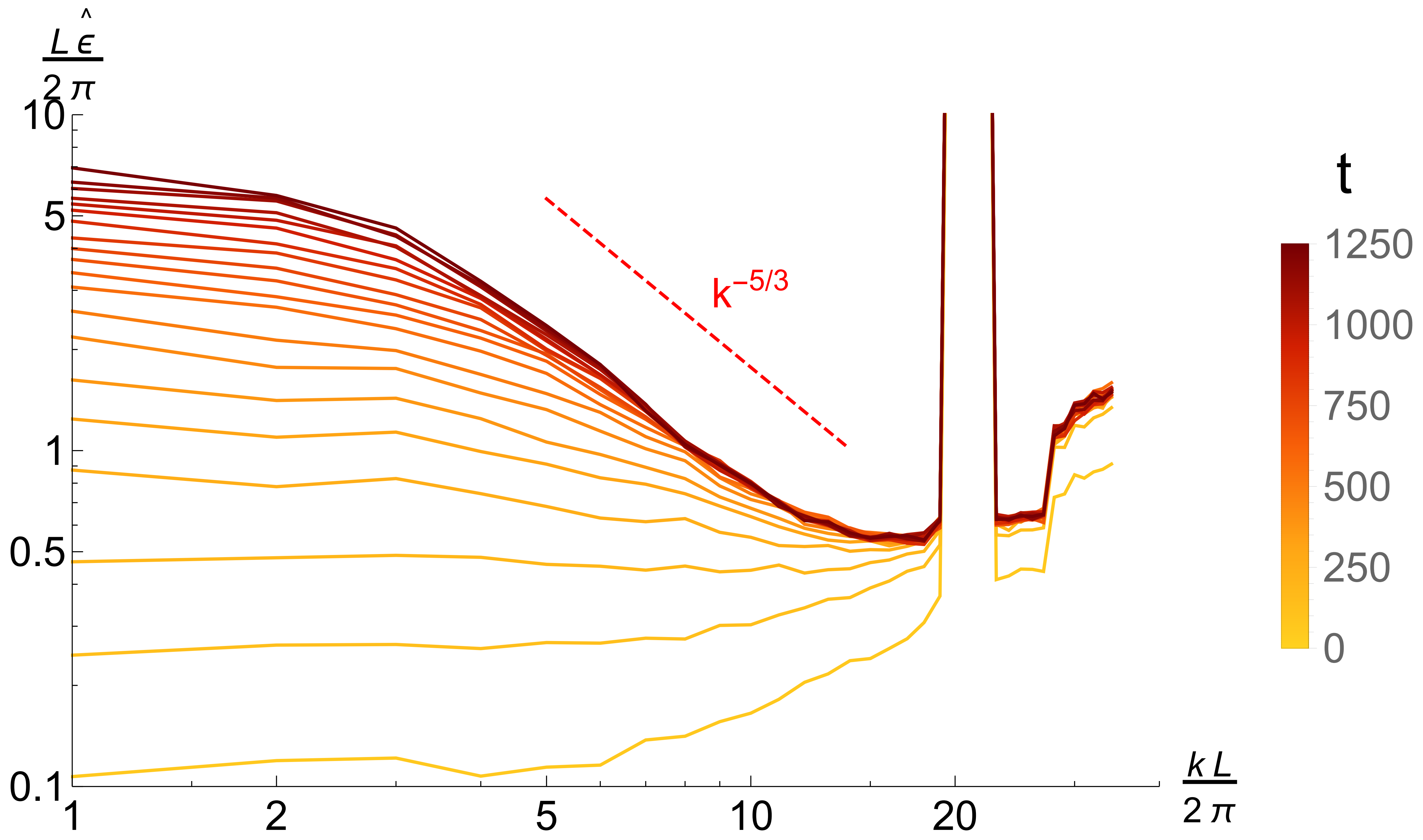
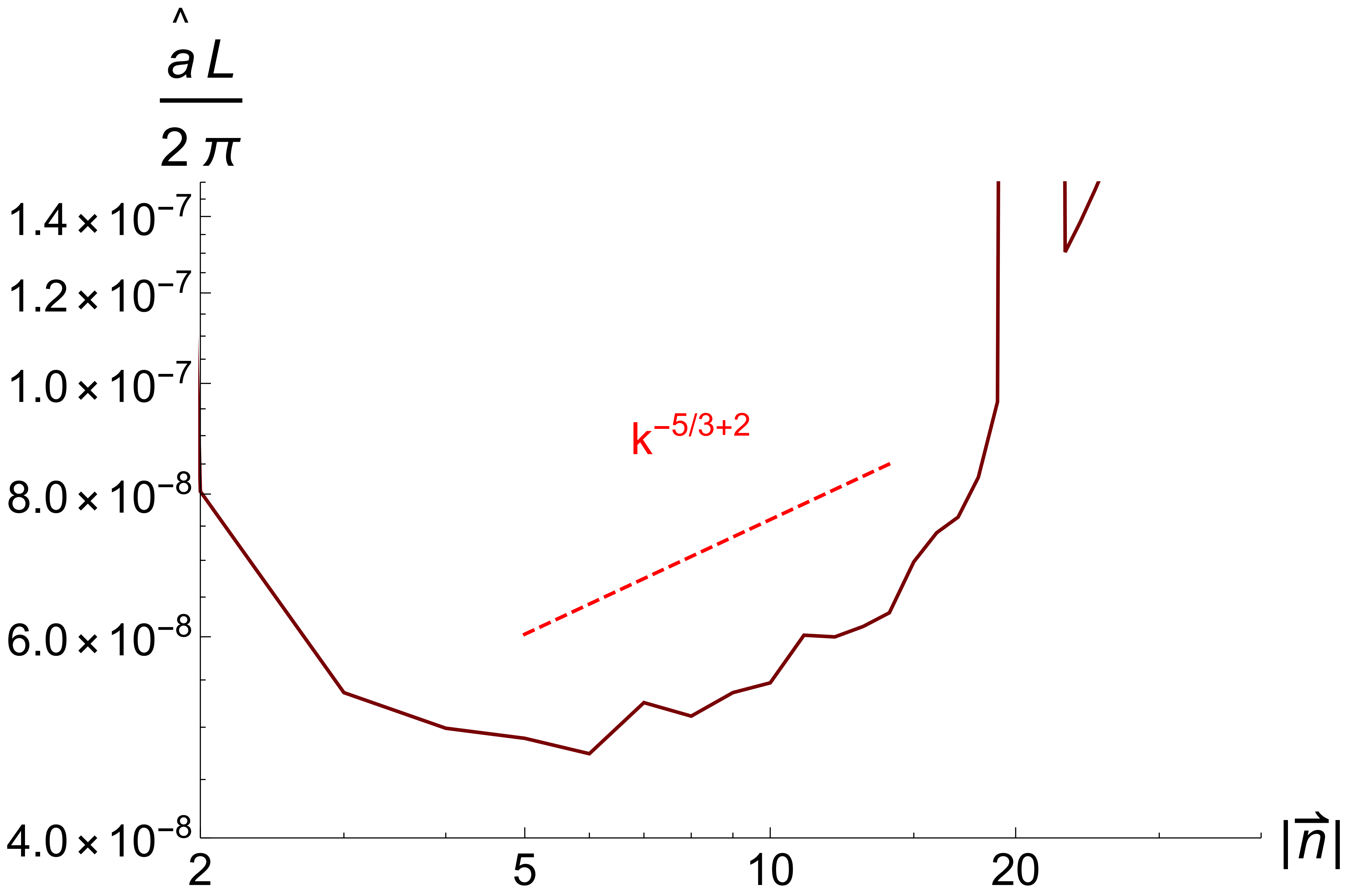
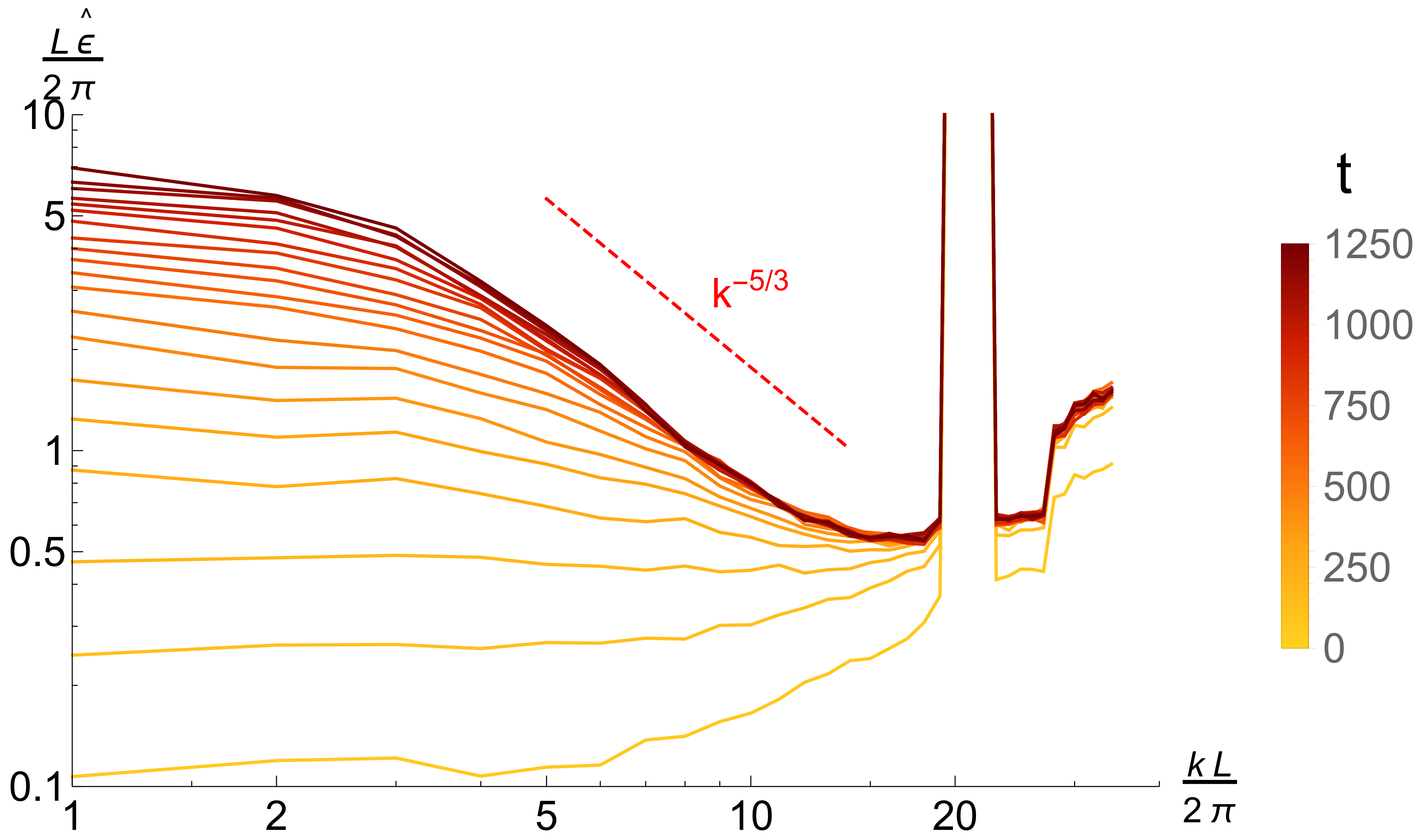


Holographic turbulence





Results

- Driven turbulent behavior can be achieved by placing the theory on a random metric
- The area power spectrum seems to follow the energy power spectrum in the non relativistic limit

Results

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Outlook

- Can we increase the inertial range?
- Is there relativistic turbulence?
- Does the power spectrum relate to a fractal horizon?
- If so, is there a way to predict it?