#### Applying gauge/gravity duality to neutron stars

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HoloTube 4th of May 2021

[arXiv:1809.07770 with N. Jokela, J. Remes; arXiv:1903.06169 with T. Ishii, G. Nijs; arXiv:1908.03213 with C. Ecker, G. Nijs, W. van der Schee; arXiv:2005.14205 with C. Hoyos, N. Jokela, J. Subils, J. Tarrio, A. Vuorinen; arXiv:2006.01141 with N. Jokela, G. Nijs, J. Remes; arXiv:2009.10731 with T. Demircik, C. Ecker]

# Outline

- 1. Introduction, motivation and overview
- 2. Dense holographic quark matter
- 3. Dense holographic nuclear matter and hybrid equations of state
- 4. Applications to neutron stars
  - Properties of isolated (static and spinning) neutron stars
  - Holographic neutron star mergers

# 1. Introduction

# The QCD phase diagram

- $\blacktriangleright$  Nuclear matter: dense liquid of protons and neutrons density  $\gtrsim$  density of atomic nuclei
- Quark matter: densely packed phase of free quarks and gluons



Laboratory experiments challenging ( $T_{QCD} \sim 10^{12}$  K), in particular at high density – lots of effort

Recent and future progress: LHC, RHIC, FAIR, NICA, ...



Lattice data only available at zero/small chemical potentials



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 Effective field theory works at small densities



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- Effective field theory works at small densities
- Perturbative QCD: only at high densities and temperatures



- Lattice data only available at zero/small chemical potentials
- Effective field theory works at small densities
- Perturbative QCD: only at high densities and temperatures
- Open questions at intermediate densities relevant for neutron stars



- 1. Improving theoretical predictions important!
- 2. Incoming experimental data from neutron star measurements!

#### Neutron stars

Neutron stars: extremely dense cold QCD matter

- Tolman-Oppenheimer-Volkoff (TOV) equations map equation of state (EoS) to mass-radius relation<sup>2p</sup>
- Uncertainty in EoS  $\Rightarrow$  inner core composition unknown
- EoS can be constrained by measuring masses and radii

Mass measurements: dozens of results using various methods

 Highest masses from Shapiro delay measurement of NS – white dwarf binaries J0348+0432 and J0740+6620:

[Antoniadis et al arXiv:1304.6875 Cromartie et al arXiv:1904.06759]

Radius measurements: more challenging, high uncertainties

 $M_{\rm max} \gtrsim 2 M_{\odot}$ 

► Cooling after X-ray bursts ⇒ radii around 10-15 km

More and better results expected in near future! E.g. NICER



Crust e Z n

Outer Core n-p Fermi liquid

Inner

Core

 $\sim 0.5\rho$ 

//~1-2km

-10km

#### Neutron star mergers

- Significant sources of gravitational radiation
- Microscopic properties of dense matter encoded in GW and EM signal
- Likely the origin of short gamma-ray-bursts and heaviest elements



# LIGO/Virgo constraints from GW170817

- ► The tidal deformability ∧ measures how strongly neutron stars deform in gravitational field
- Inspiral phase GW signal gives an upper bound Λ ≤ 580
- Implies a rough upper bound for neutron star radius:  $R \lesssim 13.5$  km





# Constraints on equation of state (EoS)

State of the art for QCD EoS at T = 0: interpolations between nuclear EoS and pQCD, constrained by

- 1. Mass bound  $M_{
  m max} > 2 M_{\odot}$  (excludes cyan area)
- 2. LIGO constraint from GW170817: (excludes red area)



Source of uncertainties: physics at strong coupling  $\Rightarrow$  Can holographic methods be used to reduce uncertainties further?

# 2. Dense holographic quark matter

# Recent progress on dense holographic QCD

For quark matter, use D3-D7 top down model:  $\epsilon = 3p + \frac{\sqrt{3}m^2}{2\pi}\sqrt{p}$ [Karch, O'Bannon, arXiv:0709.0570]

▶ N = 4 SYM +  $N_f = 3$  probe hypermultiplets in the fundamental representation

For nuclear matter use with stiff, intermediate, and soft





Strong first order nuclear to quark matter transitions

 Neutron stars with "holographic" quark matter core (black curves) are unstable

[Hoyos, Rodriguez, Jokela, Vuorinen arXiv:1603.02943]11/32

Varying the quark mass *m* one can get quark stars and hybrid stars [Annala, Ecker, Hoyos, Jokela, Rodriguez-Fernandez, Vuorinen arXiv:1711.06244]

Sizeable deviations from universal I-Love-Q relations [Yagi, Yunes, arXiv:1303.1528]

Including running of the quark mass + color superconductivity [Bitaghsir Fadafan, Cruz Rojas, Evans, arXiv:1911.12705; 2009.14079]

- Possibility of an intermediate  $\chi$ SB deconfined phase
- Stiffer holographic equations of state (high speed of sound)
- Quark matter cores

Also studying Witten-Sakai-Sugimoto model suggests a weaker transition/crossover possible [Bitaghsir Fadafan, Kazemian, Schmitt, arXiv:1811.08698]

Using Einstein-Maxwell-dilaton for quark matter [Mamani, Flores, Zanchin, arXiv:2006.09401]

(Largish) quark stars also studied in Witten-Sakai-Sugimoto and in D4-D6 models [Burikham, Hirunsirisawat, Pinkanjanarod, arXiv:1003.5470 Kim, Shin, Lee, Wan, arXiv:1404.3474] This talk: towards more realistic model of quark matter?

# Holographic V-QCD

A holographic bottom-up model for QCD in the Veneziano limit (large  $N_f$ ,  $N_c$  with  $x = N_f/N_c$  fixed): V-QCD

- Bottom-up, but trying to follow principles from string theory as closely as possible
- Many parameters: effective description of QCD
- Comparison with QCD data essential
- Works surprisingly well! (I will show examples)

The model is obtained through a fusion of two building blocks:

[MJ, Kiritsis arXiv:1112.1261]

1. IHQCD: model for glue inspired by string theory (dilaton gravity)

[Gürsoy, Kiritsis, Nitti; Gubser, Nellore]

 Adding flavor and chiral symmetry breaking via tachyon brane actions [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,latrakis,Kiritsis,Paredes]

Full backreaction between the two sectors in the Veneziano limit!

# V-QCD at finite T and $\mu$ (without nucleons)

Two bulk scalars:  $\lambda \leftrightarrow {
m Tr} F^2$  ( $\lambda \approx g^2 N_c$  near boundary),  $au \leftrightarrow ar q q$ 

$$S_{V-QCD} = N_c^2 M^3 \int d^5 x \sqrt{g} \left[ R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$-N_f N_c M^3 \int d^5 x V_{f0}(\lambda) e^{-\tau^2}$$
$$\times \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a \tau \partial_b \tau + \mathbf{w}(\lambda)F_{ab})}$$

$$F_{rt} = \Phi'(r) \qquad \Phi(0) = \mu$$

[Alho, Kajantie, Kiritsis, MJ, Tuominen arXiv:1210.4516; Alho, Kajantie, Kiritsis, MJ, Rosen, Tuominen arXiv:1312.5199]

Effective model: choose potentials by comparing to QCD data

- Many potentials V<sub>g</sub>, V<sub>f0</sub>, w, κ however need to be "simple" functions – still lot of predictivity!
- Constrained asymptotically by properties of QCD
- Remaining degrees of freedom fitted to lattice data

# Constraining the model at $\mu \approx 0$

Stiff fit to lattice data near  $\mu = 0$  (many parameters, but results insensitive to them) [Gürsoy, Kiritsis, Mazzanti, Nitti arXiv:0903.2859; MJ, Jokela, Remes, arXiv:1809.07770]

- Many parameters already fixed by requiring qualitative agreement with QCD
- Good description of lattice data nontrivial result!

Interaction measure, 2+1 flavors

Lattice data: Borsanyi et al. arXiv:1309.5258

Baryon number susceptibility

Lattice data: Borsanyi et al. arXiv:1112.4416



# Extrapolated EoSs of cold QCD

The V-QCD quark matter result compares nicely to



Approach similar in spirit to studies of the QCD critical point [DeWolfe,Gubser,Rosen 1012.1864; Knaute,Yaresko,Kämpfer 1702.06731; Critelli, Noronha, Noronha-Hostler, Portillo, Ratti, Rougemont, arXiv:1706.00455] Holographic curves do not intersect nuclear models on  $(\epsilon, p)$  plane  $\Rightarrow$  Strongly first order transitions Similar to the simplest D3-D7 model but in some contrast

with Witten-Sakai-Sugimoto model where the transition appears smoother

#### Transport of cool quark matter

Beyond the EoS: transport properties

- (Bulk) viscosity relevant for neutron star merger dynamics?
- ▶ Viscosities ↔ instabilities (*r*-modes) in spinning NSs
- Conductivities relevant for NS cooling and equilibration after NS merger [Review: Schmitt, Shternin, arXiv:1711.06520]

However transport is challenging to analyze...

- While the EoS of dense and cold QCD matter has large uncertainties, even less is known about transport
- Only available first-principles result for quark matter: leading order pQCD analysis in the unpaired phase

[Heiselberg, Pethick, PRD 48(1993)2916]

 We carry out the strong coupling analysis in both D3-D7 and V-QCD models

[Hoyos, Jokela, MJ, Subils, Tarrio, Vuorinen, arXiv:2005.14205]

#### Transport of cool quark matter



- ▶ Predictions for viscosities for unpaired quark matter (dashed  $\mu = 450$  MeV, solid  $\mu = 600$  MeV)
- Large deviation from perturbative results
- Notice that our (small) results assume "idealized" case: only QCD contributions, no weak interactions or electrons

# 3. Holographic nuclear matter and hybrid EoSs

### Nuclear matter in holographic models

Each baryon maps to a solitonic "instanton" configuration of gauge fields in the bulk [Witten: Gross, Ooguri; ...]

Studied a lot in Witten-Sakai-Sugimoto model
 [Sakai, Sugimoto; Kim, Sin, Zahed; Hong, Rho, Yee, Yi;
 Hata, Sakai, Sugimoto, Yamato; Hashimoto, Sakai, Sugimoto;...]
 Rough model for physics of QCD at finite density

[Bergman, Lifschytz, Lippert; Rozali, Shieh, Van Raamsdonk,Wu; Kim, Sin, Zahed; Preis, Schmitt;...]

Baryonic phase has soliton crystals with nontrivial transitions
 [Kaplunovsky, Melnikov, Sonnenschein; ...]

 [MJ, Kaplunovsky, Sonnenschein]

Quarkyonic phase possible

[de Boer, Chowdhury, Heller, Jankowski; Kovensky, Schmitt]

Solution constructed in "hard-wall" models

[Pomarol, Wulzer]

And will also be constructed in V-QCD

[MJ, Kiritsis, Nitti, Préau, in progress...]

# Simple approach: homogeneous bulk field

Extending the instanton approach reliably to neutron star core densities is hard

- At low densities, one can work with instantons + two-body interactions
- At higher densities, many body interactions important, instantons overlap
- Phase is crystal instead of fluid at large  $N_c$

►  $\Rightarrow$  Set  $N_f = 2$  and try first a simple approximation scheme (homogeneous), perhaps more reasonable at high densities? [Rozali, Shieh, Van Raamsdonk, Wu arXiv:0708.1322]

$$A^i = h(r)\sigma^i$$

[Li,Schmitt,Wang arXiv:1505.04886; Elliot-Ripley,Sutcliffe,Zamaklar arXiv:1607.04832]

Our approach: add homogeneous nuclear matter in a rich bottom-up model (V-QCD)  $\Rightarrow$  realistic results? [Ishii, MJ, Nijs, arXiv:1903.06169]

### Discontinuity and smeared instantons

With the homogeneous Ansatz  $A_i^a(r) = h(r)\delta_i^a$  baryon number vanishes for any smooth h(r):

$$N_b \propto \int dr \frac{d}{dr} \left[ \text{CS} - \text{term} \right] = 0$$

How can this issue be avoided?

Smearing the BPST soliton in singular Landau gauge:

$$\langle A_i^a \rangle \sim \int \frac{d^3 \times \eta_{i4}^a \, \delta r}{(\delta r^2 + x^2 + \rho^2)(\delta r^2 + x^2)} \\ \sim -\frac{\delta_i^a \, \delta r}{\sqrt{\delta r^2 + \rho^2} + |\delta r|}$$

- This suggests a solution: introduce a discontinuity in h(r) at r = r<sub>c</sub>
- The discontinuity sources nonzero baryon charge!



#### Phase diagram at zero quark mass



, [Ishii, MJ, Nijs, arXiv:1903.06169]

# Hybrid Equations of State

V-QCD nuclear matter description not reliable at low densities

- $\Rightarrow$  use traditional models (effective field theory) instead
  - Match nuclear models (low densities) with V-QCD (high densities)
  - Variations in model parameters give rise to the band
  - Same (holographic) model for nuclear and quark matter



#### Speed of sound and comparison to FRG



Similar predictions as with the functional renormalization group method!



[Drews, Weise arXiv:1610.07568; Otto, Oertel, Schaefer arXiv:1910.11929] $_{25/32}$ 

# 4. Applications to neutron stars

# Predictions for (non-rotating) neutron stars

Plug EoSs in the TOV equations  $\Rightarrow$  Mass-Radius relations

- 1. without holography
- 2. with holography (hybrid EoSs)

[Jokela, MJ, Nijs, Remes arXiv:2006:01141]



- Strong first order nuclear to quark matter phase transitions: quark cores unstable
- Large radii of neutron stars preferred

#### Rapidly spinning holographic neutron stars

GW190814: LIGO/Virgo observed a merger of a  $23M_{\odot}$  black hole with a  $2.6M_{\odot}$  compact object

[arXiv:2006.12611]

▶  $2.6M_{\odot}$  falls in the "gap": a black hole or a neutron star?

- Holographic EoSs easily compatible with the neutron star interpretation
- ► However requires fast rotation, f ≥ 1 kHz



[Demircik, Ecker, MJ, arXiv:2009.10731]

# Neutron star mergers with holographic EoS



Simulations: LORENE+EinsteinToolkit+WhiskyTHC on Cartesius [Ecker, MJ, Nijs, van der Schee arXiv:1908.03213] [We are grateful for support by Helvi Witek and Elias Most!]

### Power Spectral Density

Post-merger power spectral density (PSD) has typical three peak structure

$$ilde{h}(f)\equiv \sqrt{rac{| ilde{h}_+(f)|^2+| ilde{h}_ imes(f)|^2}{2}}\,,\quad ilde{h}_{+, imes}(f)\equiv \int h_{+, imes}(t)e^{-i2\pi ft}dt\,.$$

Characteristic frequencies  $f_1$ ,  $f_2$ ,  $f_3$  encode information about EoS



Holographic EoSs predict low frequencies f<sub>2</sub> of the strongest peak
 Signal will be visible at advanced LIGO for nearby mergers

# Ongoing work: temperature dependence



- Add realistic temperature dependence for nuclear matter [See also: Chesler, Jokela, Loeb, Vuorinen, arXiv:1906.08440]
- Required for state-of-the-art merger simulations
- Temperature dependence for nuclear matter is tricky using holography
- ► ⇒ Use simple van der Waals/mean field model adjusted to match with holography
  [Demircik, Ecker, MJ, in progress]
- Nuclear to quark matter transition ends at a critical point!

# Conclusions

- Gauge/gravity duality (combined with other approaches) is useful to study dense QCD
- Using V-QCD with simple approximations, many details work really well:
  - $\checkmark$  Precise fit of lattice thermodynamics at  $\mu \approx 0$
  - $\checkmark$  Extrapolated EoS for cold quark matter reasonable
  - $\checkmark$  Simultaneous model for nuclear and quark matter
  - ✓ Stiff EoS for nuclear matter
- Predictions for

. . .

- equation of state of cold matter
- transport in quark matter phase
- properties of static and spinning neutron stars
- gravitational wave spectrum in neutron star mergers

# Backup slides

# Ansatz for potentials, (x = 1)

$$\begin{split} V_g(\lambda) &= 12 \left[ 1 + V_1 \lambda + \frac{V_2 \lambda^2}{1 + \lambda/\lambda_0} + V_{\rm IR} e^{-\lambda_0/\lambda} (\lambda/\lambda_0)^{4/3} \sqrt{\log(1 + \lambda/\lambda_0)} \right] \\ V_{f0}(\lambda) &= W_0 + W_1 \lambda + \frac{W_2 \lambda^2}{1 + \lambda/\lambda_0} + W_{\rm IR} e^{-\lambda_0/\lambda} (\lambda/\lambda_0)^2 \\ \frac{1}{w(\lambda)} &= w_0 \left[ 1 + \frac{w_1 \lambda/\lambda_0}{1 + \lambda/\lambda_0} + \bar{w}_0 e^{-\lambda_0/\lambda_{W_s}} \frac{(w_s \lambda/\lambda_0)^{4/3}}{\log(1 + w_s \lambda/\lambda_0)} \right] \\ V_1 &= \frac{11}{27\pi^2} , \quad V_2 = \frac{4619}{46656\pi^4} \\ W_1 &= \frac{8 + 3W_0}{9\pi^2} ; \quad W_2 = \frac{6488 + 999W_0}{15552\pi^4} \end{split}$$

Fixed UV/IR asymptotics  $\Rightarrow$  fit parameters only affect details in the middle

# Constraining the potentials

In the UV (  $\lambda \rightarrow 0$ ):

► UV expansions of potentials matched with perturbative QCD beta functions ⇒ asymptotic freedom and logarithmic flow of the coupling and quark mass, as in QCD [Gürsoy, Kiritsis arXiv:0707.1324; MJ, Kiritsis arXiv:1112.1261]

In the IR  $(\lambda \to \infty)$ : various qualitative constraints

- Linear confinement, discrete glueball & meson spectrum, linear radial trajectories
- Existence of a "good" IR singularity
- Correct behavior at large quark masses
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

[Gürsoy, Kiritsis, Nitti arXiv:0707.1349; MJ, Kiritsis arXiv:1112.1261; Arean, latrakis,

MJ, Kiritsis arXiv:1309.2286, arXiv:1609.08922; MJ arXiv:1501.07272]

Final task: determine the potentials in the middle,  $\lambda = \mathcal{O}(1)$ 

Qualitative comparison to lattice/experimental data

# Transport from gauge/gravity duality

Transport: deviation from equilibrium  $\leftrightarrow$  fluctuations of the 5D metric

Leading order deviation characterized in terms of transport coefficients:

- Shear viscosity  $\eta$  "standard" viscosity
- Bulk viscosity  $\zeta$  viscosity in compression/expansion
- Electric conductivity  $\sigma$  defined by  $\vec{J} = \sigma \vec{E}$
- Thermal conductivity  $\kappa$  defined by  $\vec{Q} = -\kappa \nabla T$

Coefficients can be computed from correlators via using Kubo formulae + standard dictionary

► E.g. 
$$\eta = -\frac{1}{\omega} \operatorname{Im} \left\langle T_{xy}(\omega, \vec{k_1}) T_{xy}(\omega, \vec{k_2}) \right\rangle \bigg|_{\omega \to 0, k_i \to 0}$$

Famous result:  $\eta = \frac{s}{4\pi}$  ("universal", holds also in our models)

# Transport from gauge/gravity duality

The result for the bulk viscosity boils down to the Eling-Oz formula: [Eling, Oz arXiv:1103.1657]

$$\frac{\zeta}{\eta} = \left(s\frac{\partial\phi_H}{\partial s} + \rho\frac{\partial\phi_H}{\partial\rho}\right)^2$$

Expressed in terms of horizon data with  $\lambda = e^{\phi}$ ,  $\rho =$  charge density

For relativistic fluid at rest (no convection), charge and thermal conductivities are related:

$$\kappa = \frac{\mu s}{\rho} \sigma$$

 $\Rightarrow$  both can be expressed in terms of horizon data

#### Transport of cool quark matter



- ▶ Predictions for electric and thermal conductivities (dashed  $\mu = 450$  MeV, solid  $\mu = 600$  MeV)
- V-QCD deviates from D3-D7 and perturbative results due to the backreaction of quarks to the gluon background

EoSs of hot and dense QCD

EoS of V-QCD quark matter (with various potential choices) combined with DD2, SFHx, and IUF nuclear matter EoSs

[Chesler, Jokela, Loeb, Vuorinen, arXiv:1906.08440]



- Reasonable EoS at all  $\mu$  and T
- Sizable latent heat, decreases with T

#### Homogeneous nuclear matter in V-QCD

Nuclear matter in the probe limit: consider full brane action  $S = S_{\text{DBI}} + S_{\text{CS}}$  where [Bigazzi, Casero, Cotrone, Kiritsis, Paredes; Casero, Kiritsis, Paredes]  $S_{\text{DBI}} = -\frac{1}{2}M^3 N_c \operatorname{\mathbb{T}} r \int d^5 x \, V_{f0}(\lambda) e^{-\tau^2} \left( \sqrt{-\det A^{(L)}} + \sqrt{-\det A^{(R)}} \right) \\ A_{MN}^{(L/R)} = g_{MN} + \delta_M^r \delta_N^r \kappa(\lambda) \tau'(r)^2 + \delta_{MN}^{rt} w(\lambda) \Phi'(r) + w(\lambda) F_{MN}^{(L/R)}$ 

gives the dynamics of the solitons (will be expanded in  $F^{(L/R)}$ ) and

$$S_{\rm CS} = \frac{N_c}{8\pi^2} \int \Phi(r) e^{-b\tau^2} dt \wedge \left(F^{(L)} \wedge F^{(L)} - F^{(R)} \wedge F^{(R)} + \cdots\right)$$

sources the baryon number for the solitons

 $\blacktriangleright$  Extra parameter, b > 1, to ensure regularity of solutions Set  $N_f = 2$  and consider the homogeneous SU(2) Ansatz [Rozali, Shieh, Van Raamsdonk, Wu arXiv:0708.1322]

$$A_L^i = -A_R^i = h(r)\sigma^i$$

[Ishii, MJ, Nijs, arXiv:1903.06169]

# Hybrid Equations of State (T = 0)

V-QCD nuclear matter description not reliable at low densities  $\Rightarrow$  use traditional models (effective field theory) instead

- Match nuclear models (low densities) with V-QCD (high densities)
- Easy to pass astrophysical constraints thanks to stiffness of V-QCD EoS
- Same (holographic) model for nuclear and quark matter phases!

[Ecker,MJ,Nijs,van der Schee arXiv:1908.03213; Jokela, MJ, Nijs, Remes arXiv:2006.01141]

Construct "all possible" hybrid EoSs, depending on

- 1. The nuclear model EoS: different models with varying stiffness
- 2. Parameters of the holographic model still free after lattice fit
- 3. The matching density between nuclear matter/holography

#### No quark matter cores?!

1.0

0.8  $M\left[M_{\odot}\right]$  of QM core

0.6

0.4 0.2

0

A recent model independent study claims that most massive neutron stars have quark matter cores

> [Annala, Gorda, Kurkela, Nättilä, Vuorinen arXiv:1903.09121, Nature Phys.]



Seems to contradict our results, what's going on?

(Simplified) answer: our model predicts lower adiabatic index  $\gamma = d \log p / d \log \epsilon$  for nuclear matter than what they expect



R [km] of QM core

 $\max(c_*^2) \le$ 

1.0

0.8

1/3

6

#### Mechanical Toy Model



[Takami, Rezzolla, Baiotti arXiv:1412.3240] 43/32

#### EoS dependence of the Power Spectral Density



$M[M_{\odot}]$	EoS	b	<i>f</i> <sub>1</sub> [kHz]	<code>f₂[kHz]</code>	<i>f</i> ₃[kHz]
1.30	SLyVQCD105	10.5	1.93	2.53	3.77
1.30	SLyVQCD106	10.6	2.15	2.80	3.70 (4.06)
1.30	SLy	-	2.21	3.19	4.24

#### Density distributions



Evolution of density in the merger: no quark matter produced (outside possible horizon): transition at  $\sim 17 \times 10^{14} g/cm^3$ 

#### Frequencies and universal relations

Estimated characteristic frequencies of the gravitational wave signal may also be estimated by using (quasi) universal relations [Takami, Rezzolla, Baiotti arXiv:1412.3240; Tsang, Dietrich, Van Den Broeck arXiv:1907.02424; Breschi et al. arXiv:1908.11418]

 Functions of neutron star masses and tidal deformabilities fitted to simulation data



Again, holographic hybrid EoSs favor low frequencies
 Our simulation results fall within the band, and compare rather well with the predictions from the relations