Causal Symmetry Breaking

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Holotube

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Based on

[2008.02271]	with: Alexander Altland (Cologne)	the spectrum
[2103.aabbc]	with: Alexander Altland, Pranjal Nayak Manuel Vielma (Geneva)	operators
[2012.07875]	with: Alexandre Belin (CERN) Jan de Boer (UvA) Pranjal Nayak (Geneva)	OPE

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The Mathematics of Physics
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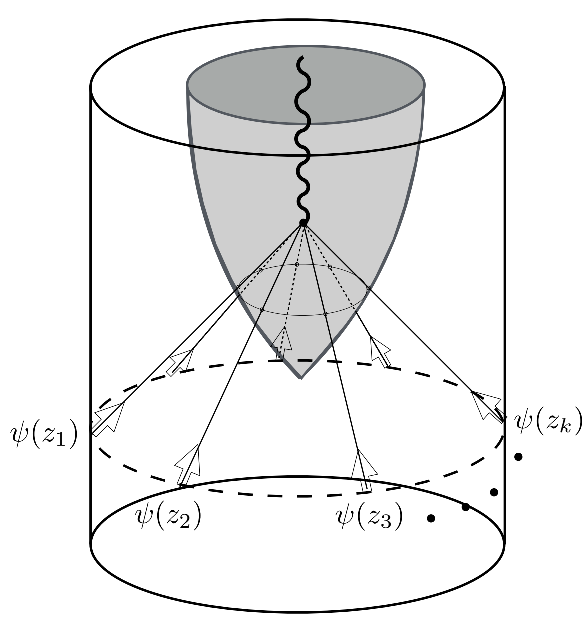
Introduction

Quantum mechanical unitarity and gravitational physics have long had a fraught relationship. Examples:

- Black hole information problem (e.g. the Page curve)
- Long-time behaviour of observables (e.g. 2-pt functions)

Such paradoxes arise when attempting to interpret black holes as thermodynamic entities [Bekenstein, Hawking,…]



Strategy: study quantum thermalisation at all relevant timescales

➢ Quantum chaos, quantum ergodicity

Gravity as an ensemble average

Both of our examples have recently enjoyed spectacular new progress, but also generated new kinds of fascinating questions

One of the most intriguing and important ones concerns the **role of the ensemble:**

Gravity contains contributions (wormholes) that strongly suggest an average over an ensemble of quantum systems

To my mind, we can have two attitudes:

1. The ensemble is fundamental: bulk theory ≅ boundary ensemble
2. The ensemble is emergent:disorder models, quantum chaos,…

Unus pro Omnibus, Omnes pro Uno

Here we develop a framework that allows us to understand the emergence of an ensemble using a universal EFT approach:

Causal Symmetry Breaking

The EFT of quantum chaos

Conceptual

Explains role of ensemble in individual theories

Wormholes in individual theories

Technical

Efficient tool

Makes specific quantitative predictions (RMT, ETH, OPE)

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context and motivation

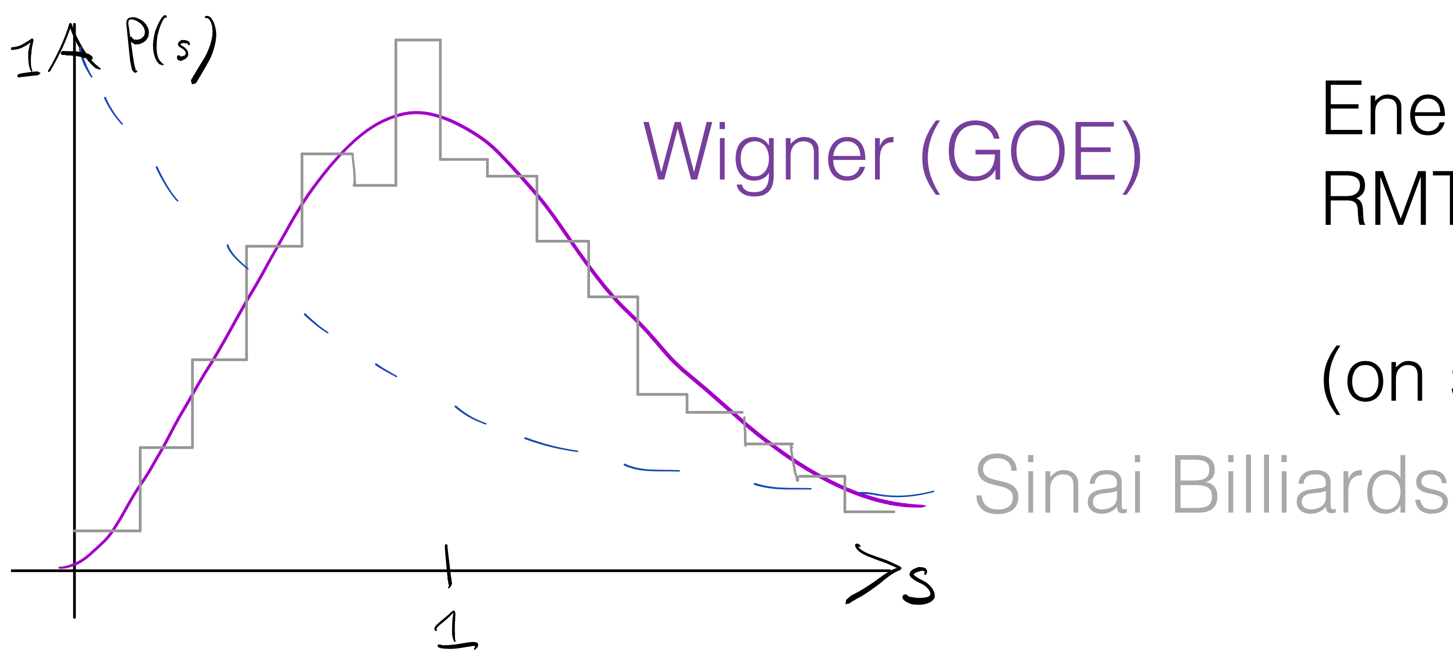
2. The effective field theory of quantum chaos

causal symmetry breaking and sigma model

3. Applications

(i) spectral correlations; (ii) wave-function statistics

Quantum Chaos

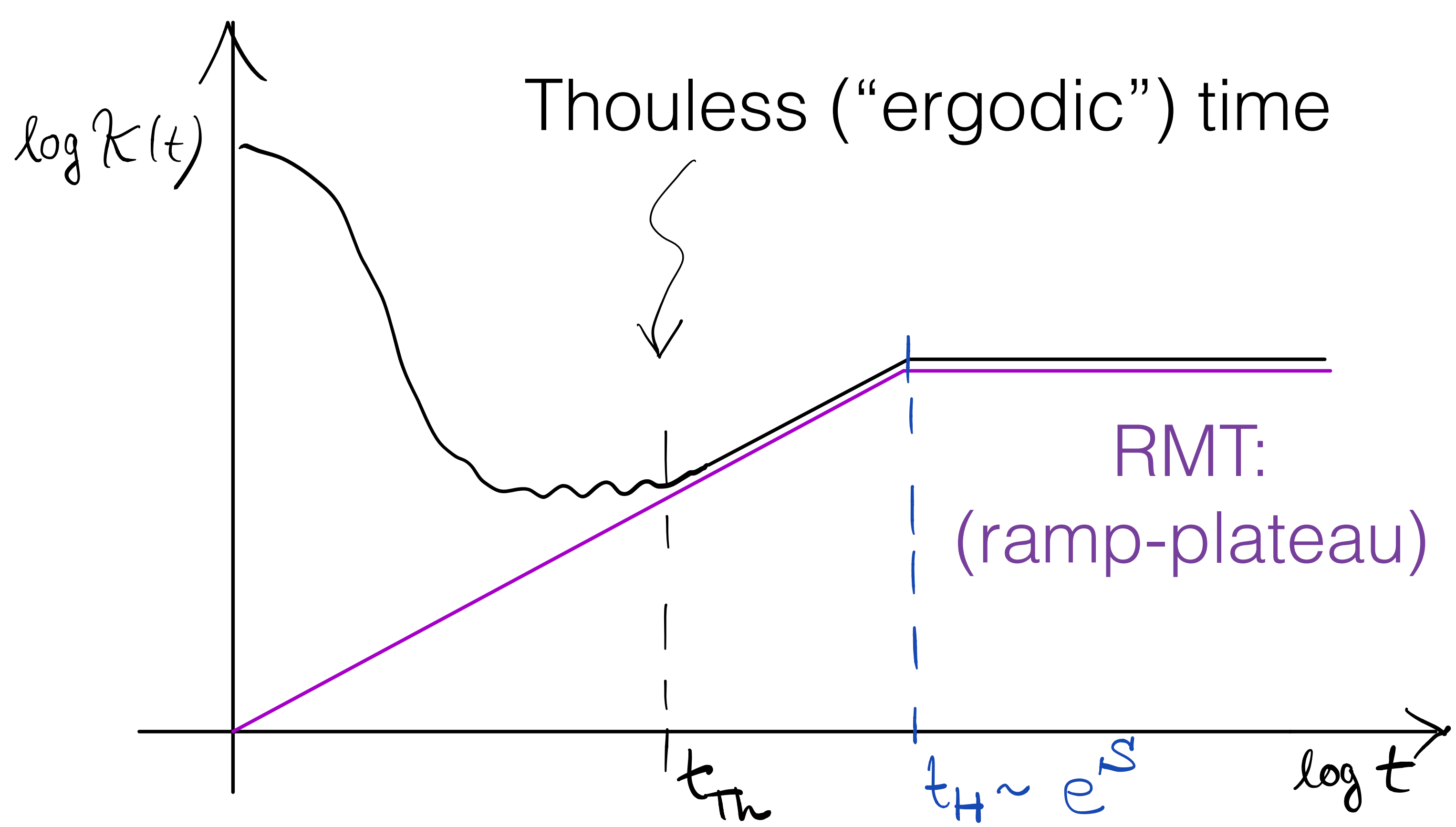


Energy levels follow RMT law

(on scale of a few Δ)

Related quantity:
spectral form factor

(= Fourier transform of two-level correlation $R_2(\omega)$)



Causal Symmetry and its breaking

Let us illustrate the main idea of causal symmetry breaking

$$\rho(E) = G^+(E) - G^-(E)$$

A non-zero spectral density along a finite interval (cut):

$$\rho(E) \neq 0 \quad \Leftrightarrow \quad G^+(E) \neq G^-(E)$$

Key idea: can understand this as a spontaneous symmetry breaking

$$\overline{G^+(E)} \neq \overline{G^-(E)}$$

$\overline{(\quad\cdot\quad)}$ Denotes average w.r.t.

- Eigenstate ensemble (c.f. Sinai)
- small set of parameters ('t Hooft coupling,SYK,…)
- coarse graining

The emergent ensemble

Point of departure: the generating functional of spectral resolvents

$$\mathcal{Z}^{(4)}(\hat{z}) = \frac{\det(z_1 - H)\det(z_2 - H)}{\det\left(z_3^+ - H\right)\det\left(z_4^- - H\right)} = \int d(\bar\psi, \psi) e^{i\bar\psi(\hat z - H^{\otimes 4})\psi}$$

"Weyl symmetry" under $z_1 \leftrightarrow z_2$

ψ is a (2L | 2L) supervector (L $= \dim(\mathcal{H})$)

This has an exact U(2|2) causal symmetry, weakly explicitly broken by energy differences $z_i - z_j \neq 0$

$$\rho(E) \neq 0 \;\Rightarrow\; U(2|2) \longrightarrow U(1|1) \times U(1|1)$$

Strong spontaneous breaking of causal symmetry by saddle point(s) (stabilised by L $\gg 1$)

[Wegner], [Efetov]

The geometry of the ensemble

➢ Goldstones of this symmetry breaking = EFT of quantum chaos

Reproduces physical content of RMT (i.e. an ensemble!)

The Goldstone physics = geometry of coset [CCWZ]

$$\int dQ\, e^{-S[Q;\omega]} \quad \text{where} \quad Q \in \frac{U(2|2)}{U(1|1) \times U(1|1)} := \mathcal{M}(Q)$$

Cf pion EFT within QCD:

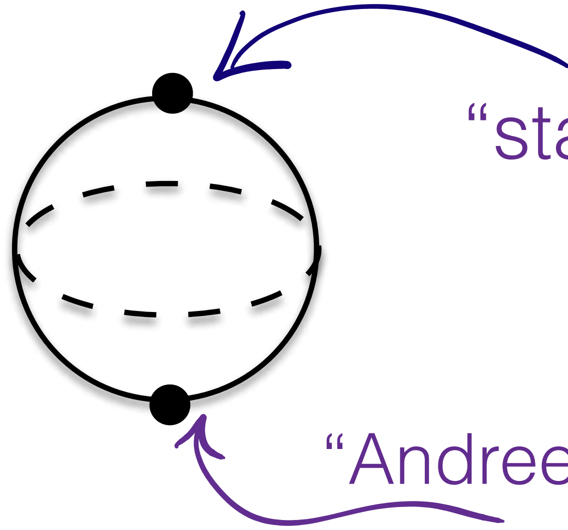
$$\text{chiral condensate } \langle \bar{q}_i q_i \rangle \leftrightarrow \text{causal condensate } \overline{\bar{\psi}_i \psi_i} = \overline{G}$$

$$\text{quark mass } m \;\leftrightarrow\; \text{energy difference } \Delta z = \omega \sim e^{-S}$$

A tale of two saddles

Key point: there are two symmetry breaking saddles

$$\mathcal{M} = H_2 \times S^2$$



"standard" saddle $\quad I_{\rm SS} = 0$

"Weyl symmetry"

"Andreev-Altshuler" saddle $\quad I_{\rm AA} \propto e^S$

Perturbative expansion of EFT in s^{-n} $\qquad \left(s := \dfrac{\omega}{\pi\Delta}\right)$

$$\Updownarrow$$

Non-perturbative expansion of microscopic theory in $\left(e^{-S}\right)^n$

Second saddle contributes $\quad e^{-ce^S}$

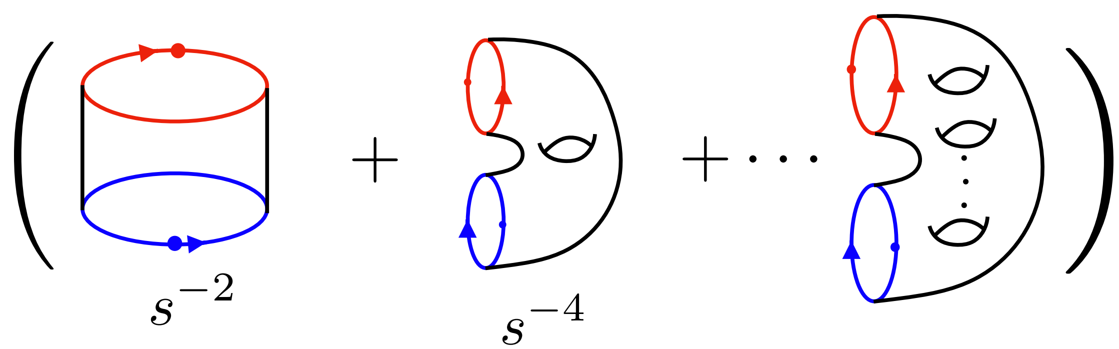
$$\Updownarrow$$

Doubly non-perturbative effects in microscopic theory

EFT expansion ↔ topological expansion

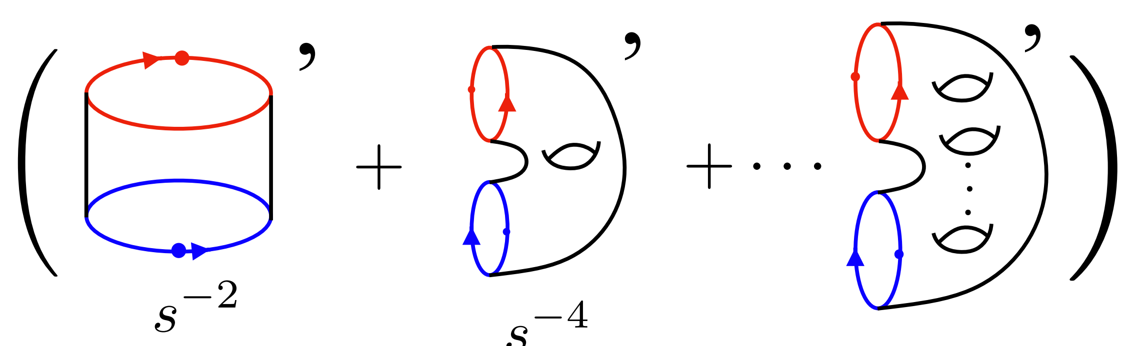
Perturbation theory in “pions” gives fat-graph expansion

$R_2(s) = e^{s\times 0}$



“standard” saddle

“Andreev-Altshuler” saddle $+e^{2is}$



[A. Altland, JS]

Each topology predicts coefficient of leading singular term
(e.g. as computed from JT)

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Applications, a summary

(I) Spectral Correlations

- Ramp-plateau
(and comparison to leading singularities from wormholes)

- Comments on SYK

[Altland, Bagrets ('17), Altland, JS ('20)] [Altland, Nayak, Vielma, JS ('20)]

(II) Wave-function statistics

- Operators correlations and ETH

- OPE statistics and genus-2 wormhole

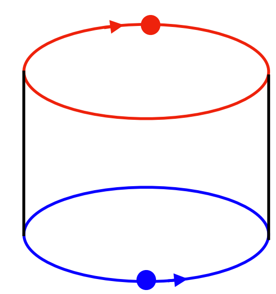
[Altland, Nayak, Vielma, JS ('20)] [Belin, de Boer, Nayak, JS ('20)]

Spectral correlations: the leading singularity

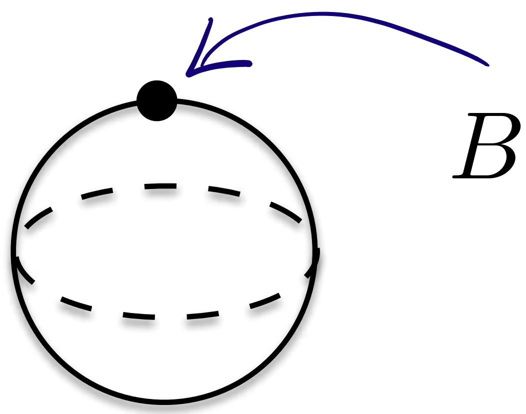
Recall: $\mathcal{Z}^{(4)}(\hat{z})$ is generator of spectral resolvent, R(E), correlations

Let's evaluate $\overline{\rho(E)\rho(E')}$ correlations with the EFT (leading diagram)

$$\left\langle \mathrm{str}(B\tilde{B}P^{\mathrm{f}})\mathrm{str}(\tilde{B}BP^{\mathrm{f}})\right\rangle =$$



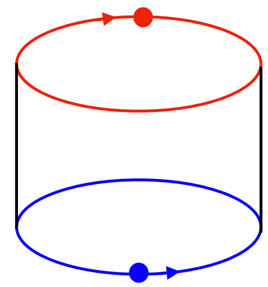
$$= -\frac{1}{2s^2}$$



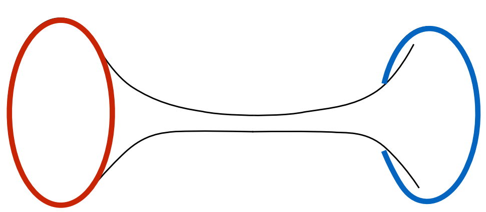
$B, \tilde{B}$ Are causal pions around the north pole (SS) with propagator $B\tilde{B} = s^{-1}$

In real time this gives the linear ramp $K(t) = \begin{cases} \tau & \text{unitary} \\ 2\tau & \text{orthogonal} \end{cases}$

Spectral correlations: bulk manifestations

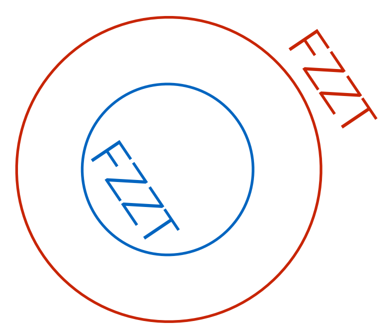


$$= \quad -\frac{1}{2s^2}$$



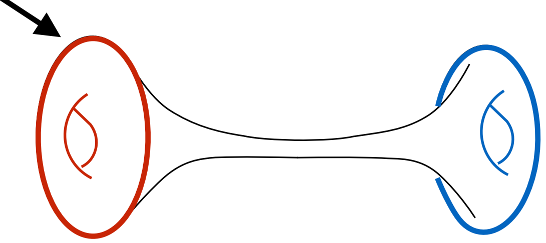
JT double trumpet
[SSS ('19)]

$$\sim -\frac{1}{2s^2} + \mathrm{reg}$$



FZZT annulus
[Martinec ('03)]

$$\sim -\frac{1}{2s^2} + \mathrm{reg}$$



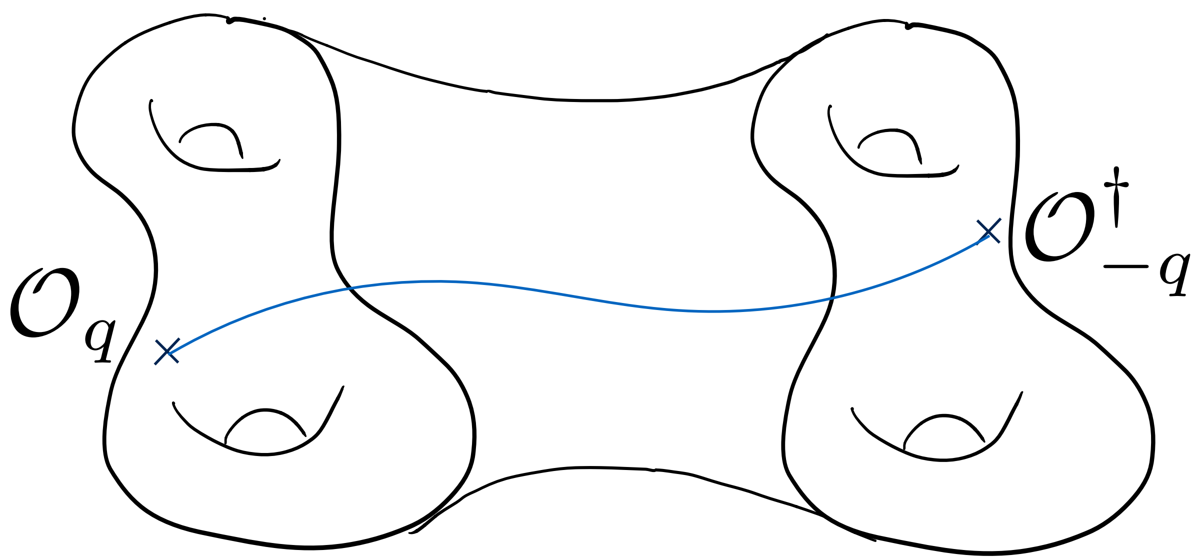
3D torus wormhole
[Cotler, Jensen ('20)]

$$\sim -\frac{1}{2s^2} + \mathrm{reg}$$

Leading EFT diagram is dual to bulk wormhole

Interlude: some implications of bulk wormholes

Example: global charge and ETH [Belin, de Boer, Nayak, JS]



"genus-two wormhole" (more later)

$$\langle \mathcal{O}_q \rangle \equiv 0$$

Non-vanishing correlation function → non-vanishing variance of $\langle \mathcal{O}_q \rangle$

Bulk symmetry is **gauged**: wormhole correlation = 0

Bulk symmetry is **global**: wormhole correlation ≠ 0

Absence of bulk global symmetries [Belin, de Boer, Nayak, JS; Harlow, Ooguri]

Microscopic origins: SYK

SYK: a rare example where we can explicitly derive the chaos EFT (via random coupling average)

Generically have massive contributions in addition to Goldstones

$$S^{\sigma}_{\text{SYK}} = S_{\text{hom.}} + S_{\text{massive}}$$

RMT physics → $S_{\text{hom.}}$

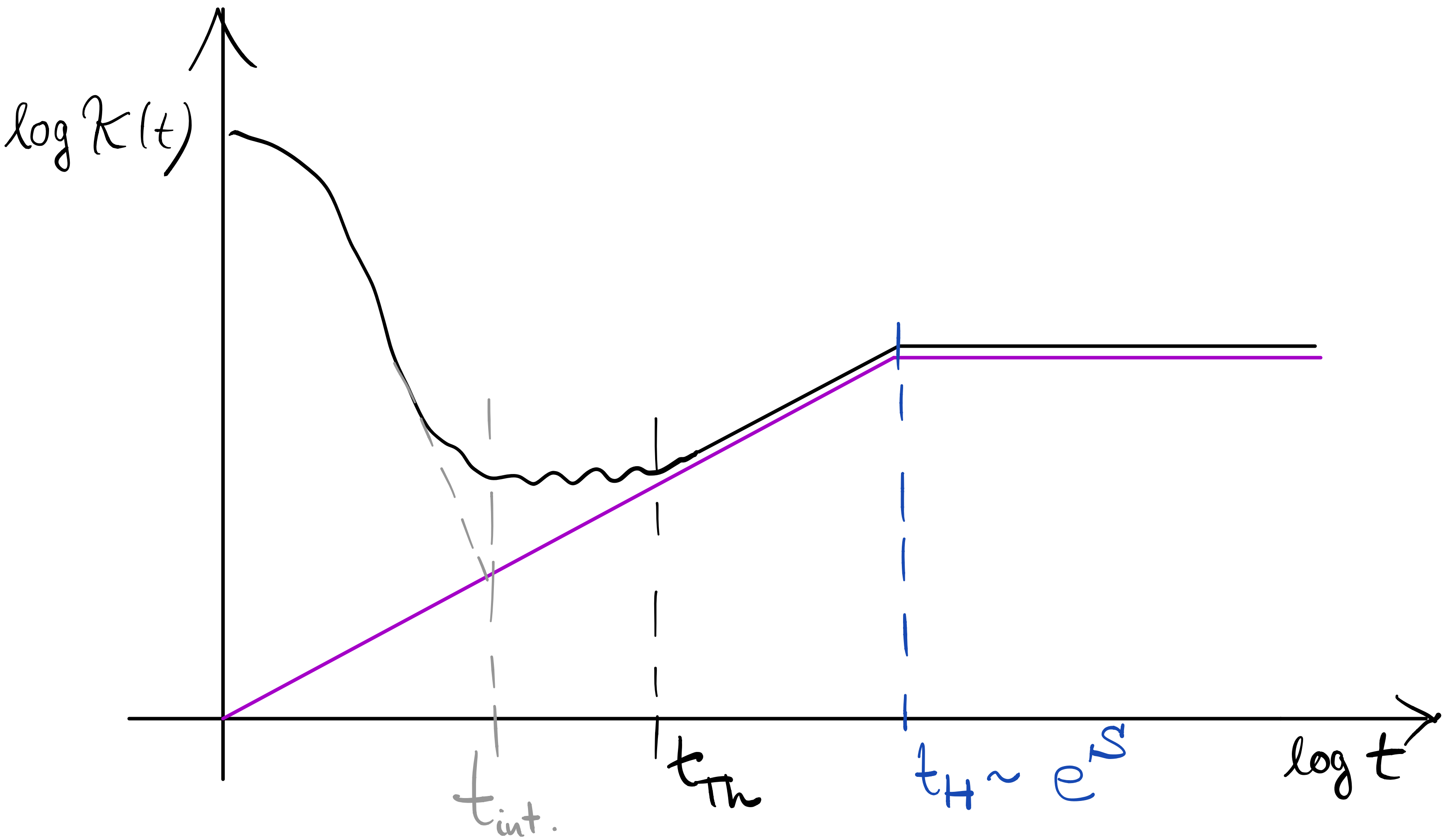
Early-time corrections → S_{massive}

$$\left.\begin{matrix} R^{(2)}(\omega) \\ R^{(2)}_{\mathcal{O}}(\omega) \end{matrix}\right\} = \left\{\begin{matrix} R^{(2)}_{\text{RMT}}(\omega) \\ R^{(2)}_{\mathcal{O},\text{RMT}}(\omega) \end{matrix}\right\} + \frac{1}{2}\left(\frac{\Delta}{\pi}\right)^2 \sum_{k\neq 0} \text{Re}\frac{1}{(i\omega + m(k))^2}$$

RMT dominates if $\omega \sim \Delta N^2 \sim N^{3/2} e^{-N}$ (q=4 SYK)

SYK Thouless physics

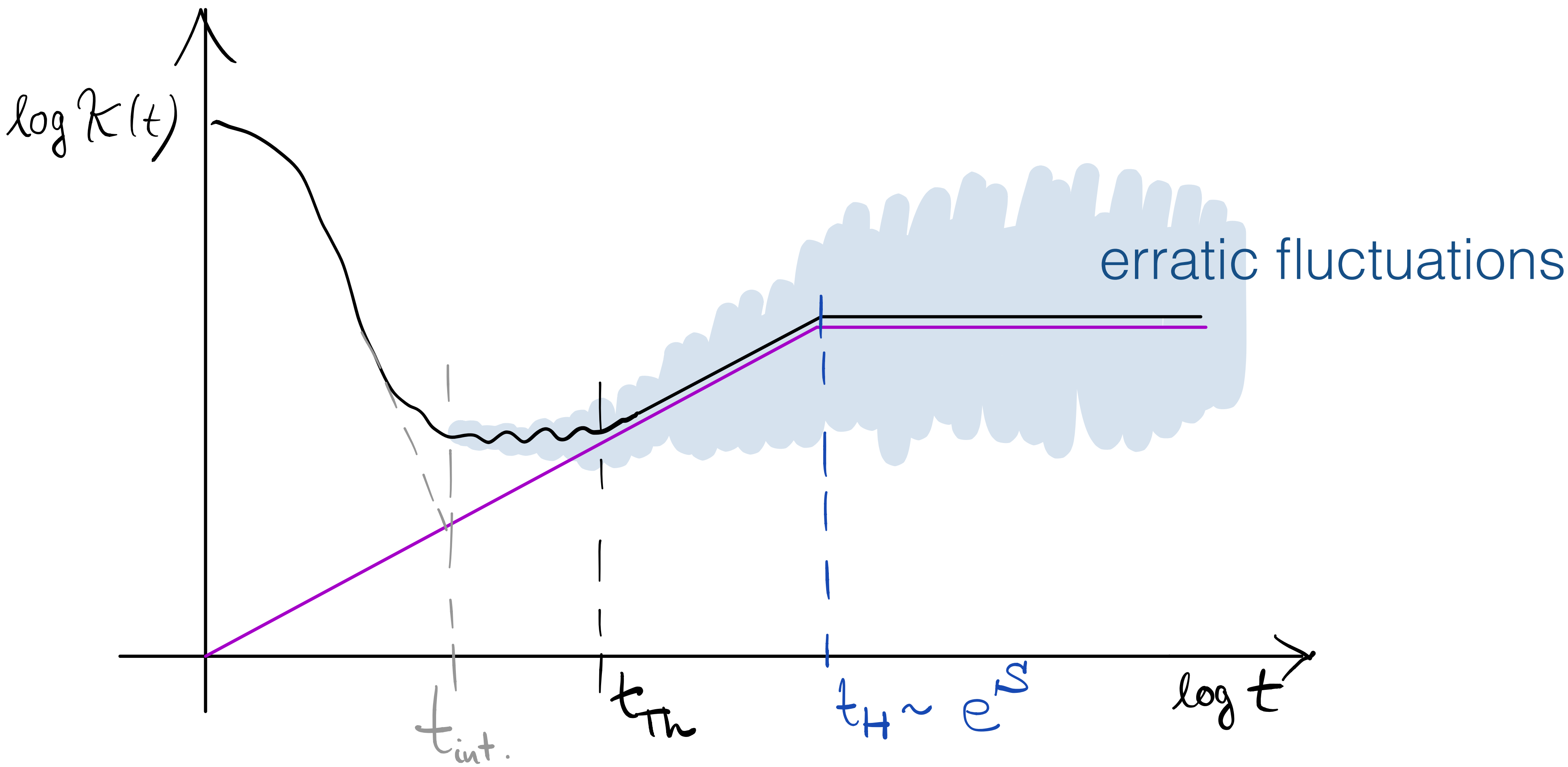
This results in a Thouless time $t_{\mathrm{Th}} \sim N^{1/2} \log N$



Intersection time: $t_{\mathrm{int.}} \sim \log N$

SYK Thouless physics

This results in a Thouless time $t_{\mathrm{Th}} \sim N^{-3/2} e^{N}$



Intersection time: $t_{\mathrm{int.}} \sim \log N$

Wave function statistics

Another class of observables of interest are matrix elements

$$\mathcal{O}_{ij} = \langle \psi_i | \mathcal{O} | \psi_j \rangle$$

The EFT implies not only E_i statistics but also that of the associated $|\psi_i\rangle$
These induce O_{ij} correlations

$$R_{\mathcal{O}}(\omega) = \sum_{i,j} \left| \langle \psi_i | \mathcal{O} | \psi_j \rangle \right|^2 \delta \left(E_i - E_j - \omega \right)$$

"operator resolvent"

Computable by adding sources to the EFT, again governed by causal symmetry

Universal operator correlations

An analysis that parallels that of spectral correlations results in

$$R_{\mathcal{O}}(s) = \delta(s)\left[\mathrm{tr}\mathcal{O}\mathcal{O}^\dagger - \mathrm{tr}\mathcal{O}\,\mathrm{tr}\mathcal{O}^\dagger\right] - \left(1 - \frac{\sin^2 s}{s^2}\right)\mathrm{tr}\mathcal{O}\mathcal{O}^\dagger$$

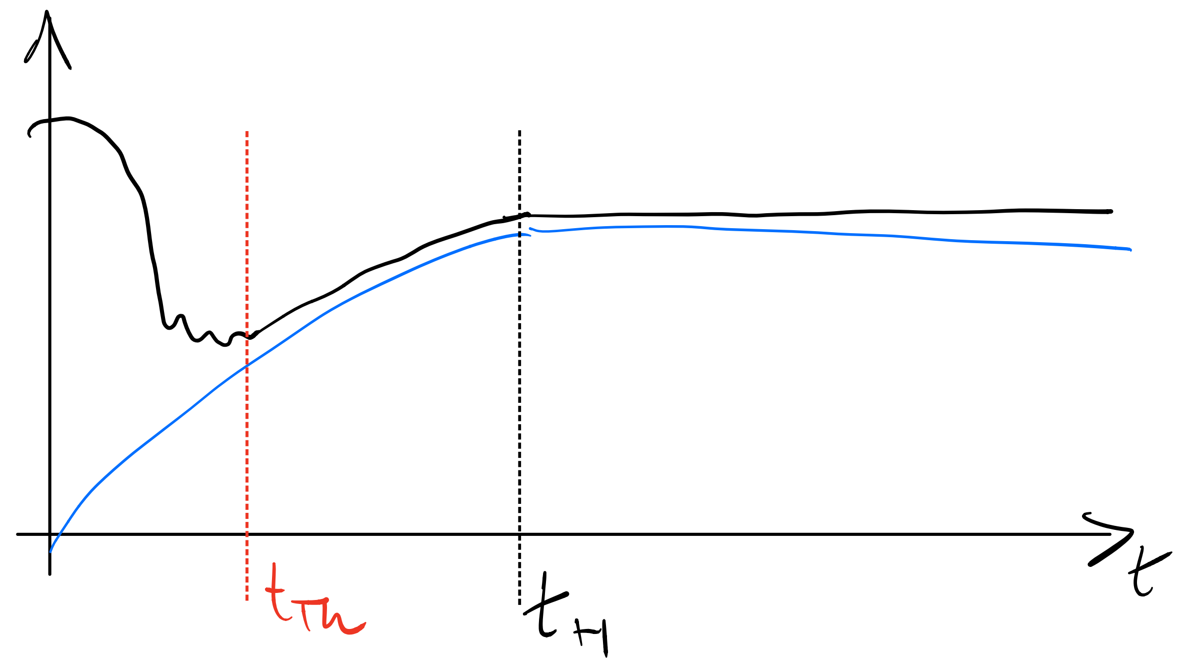
"operator sine kernel" (OSK) [Altland, Nayak, JS, Vielma ('20)]

$$\text{"tr"} = \begin{cases} \text{- canonical ensemble} \\ \text{- any micro canonical window} \\ \text{- eigenstate projector} \end{cases}$$

Operator ramp-plateau

Fourier transforming into the time domain we get

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle =$$



Recall ETH ansatz for operator statistics

$$\langle \psi_i|\mathcal{O}|\psi_j\rangle = \overline{\mathcal{O}(E)} + f(E_i, E_j)e^{-S(E)/2}R_{ij}$$

OSK gives a universal contribution to $f(E_i, E_j)$

→ Invites comparison of our EFT to ETH correlations as wormholes

[Blommaert,Mertens, Turiaci, Verlinde] [Pollack, Rozali, Sully, Wakeham ('20)] [Saad ('19)]

OPE correlations

In a CFT we have the OPE coefficients

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \langle \psi_i | \mathcal{O}_j | \psi_k \rangle = C_{ijk}$$

Thus, wave-function statistics ⇒ C_{ijk} randomness

Such an OPE randomness conjecture is implied in gravity [Belin, de Boer ('20)]

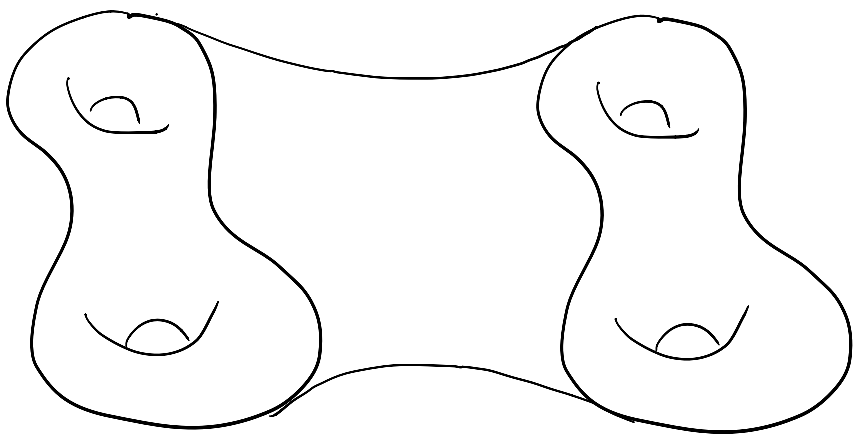
The motivation comes from the genus-two wormhole
[see also: Cardy, Maloney, Maxfield ('17),…]

$$[Z_{g=2\times g=2}]_{\text{gravity}} \neq [Z_{g=2}]^2_{\text{gravity}}$$

$$Z_{g=2} = \sum_{i,j,k} C_{ijk} C^*_{ijk} q_1^{\Delta_1} q_2^{\Delta_2} q_3^{\Delta_3}$$

Genus-two non-factorization

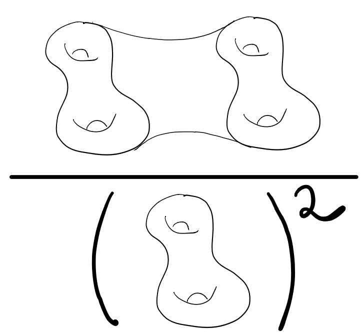
Non-factorisation on the gravity side: genus-2 wormhole:



3D gravity solution:

$$ds^2 = \ell_{\mathrm{AdS}}^2 \left(d\tau^2 + \cosh^2 \tau d\Sigma_{g=2}^2\right)$$

This causes non-factorisation of the bulk answer



$$\sim 1 + c_{\mathrm{grav}} e^{-3S}$$

If OPE coefficients are random, this comes from a variance

$$\overline{\left|C_{ijk}^* C_{lmn}\right|^2} \neq 0$$

Universal OPE statistics

We can make this precise using wave-function statistics [Belin, de Boer, Nayak, JS]

$$\mathbb{O} = C^*_{ijk} C_{lmn} \left(|\psi_i\rangle |\psi_j\rangle |\psi_k\rangle \right) \left(\langle\psi_l| \langle\psi_m| \langle\psi_n| \right)$$

We build chaos EFT with sources for $\mathbb{O} \in \mathcal{H}^{\otimes 3}$. This computes

$$\overline{\mathrm{Tr}\mathbb{O}\mathrm{Tr}\mathbb{O}^\dagger} = \partial_h^2 \mathcal{Z}^{(4)} \left(\hat{z}, h \right) \Big|_{h=0}$$

Using our well-established sigma-model expansion we find indeed

$$\frac{\overline{\mathrm{Tr}\mathbb{O}\mathrm{Tr}\mathbb{O}^\dagger}}{\overline{\mathrm{Tr}\mathbb{O}}\,\overline{\mathrm{Tr}\mathbb{O}^\dagger}} = 1 + c_{\mathrm{EFT}} e^{-3S}$$

I.e. a non-zero variance as required by the gravity “prediction”

Summary: chaos EFT

- Causal symmetry breaking $\Longrightarrow$ EFT of quantum chaos

  ↙ Ensemble av. from chaotic dynamics

  ↘ Powerful calculational framework

- Fully non-perturbative framework allows to control ramp and plateau analytically

- Simple geometric principle: coset-space sigma model; allowed cosets classified by Cartan (= Altland Zirnbauer)

Showcased applications to: SYK, eigenvalue statistics, wave function statistics; many more?

Some open issues

- Can we add erratic fluctuations in a controlled way, i.e. is there some degree of universality to these?

- What is the bulk picture of causal symmetry and its breaking?

  Some level of understanding in minimal strings [Altland, Sonner ('20)] and in JT gravity [Saad, Shenker, Stanford ('19)]

- Fat-graph expansion of chaos EFT works in higher-dimensional boundary theories. How to capture leading singular diagrams in the bulk?

10^6 € question: what is the AA saddle in higher-dimensional bulk spacetimes?

thank you very much
for your attention