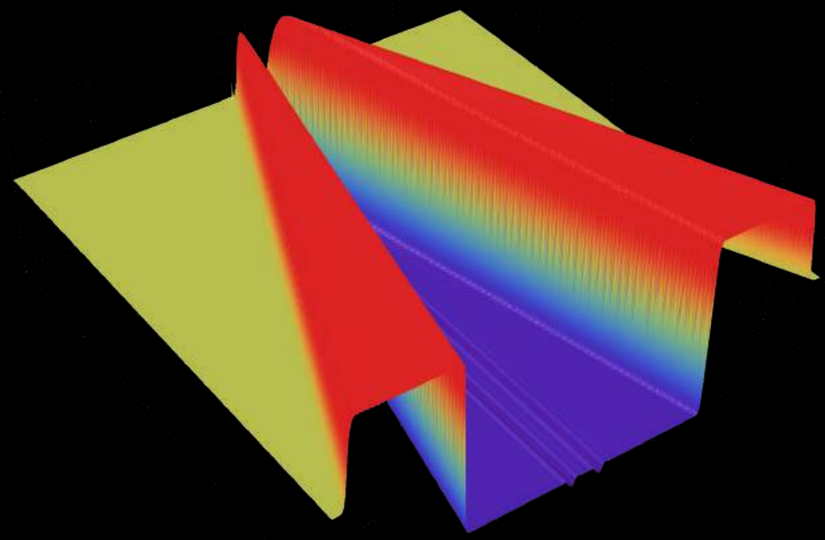
Bubble Dynamics from Holography



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JHEP 01 (2020) 106 [1905.12544 [hep-th]]
2103.xxxxx [hep-th]

Plan

- Motivation.
- Holographic model.
- Dynamics of phase separation.
- Bubble dynamics.
- Outlook.

Motivation

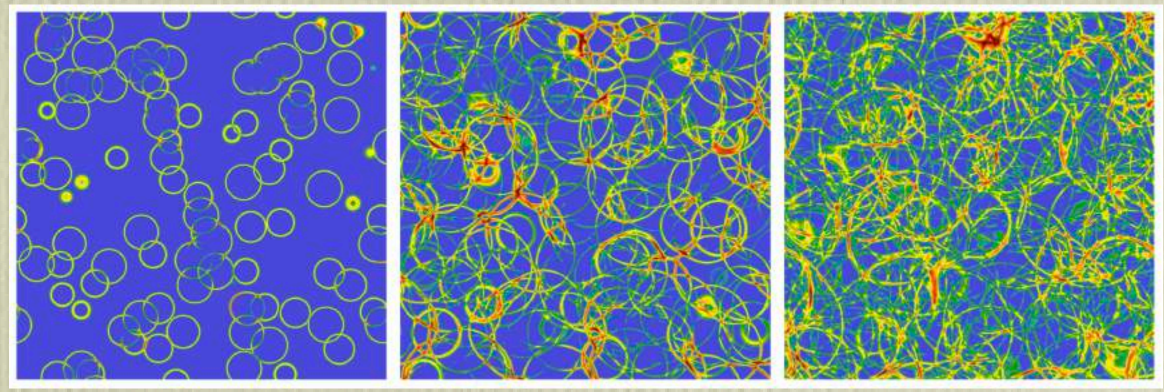
Motivation

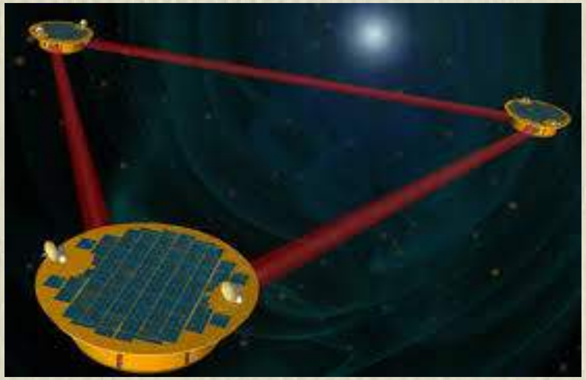
- First-order phase transitions are ubiquitous in Nature (e.g. boiling water).
- They proceed via the nucleation of bubbles.

Motivation

- You may wonder whether they occur in particle physics.
- This would be exciting because the Universe would have undergone this phase transition.
- The resulting bubbles could have produced GWs detectable by e.g. LISA.

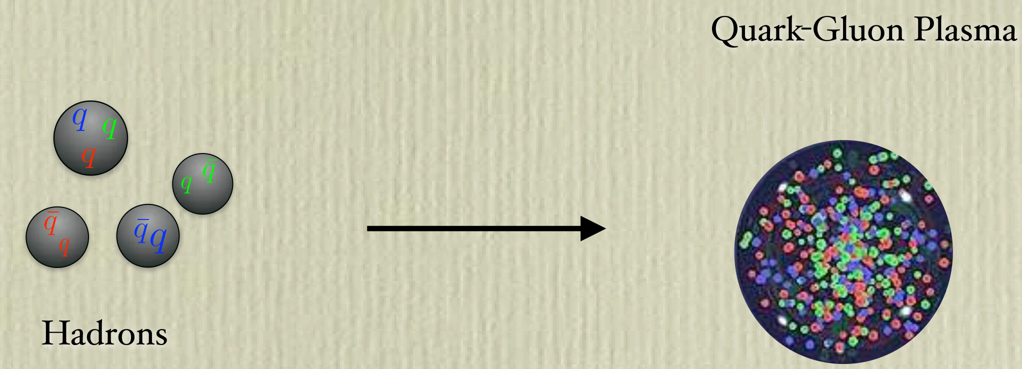
Picture from Hindmarsh, Huber, Rummukainen & Weir '15





Motivation

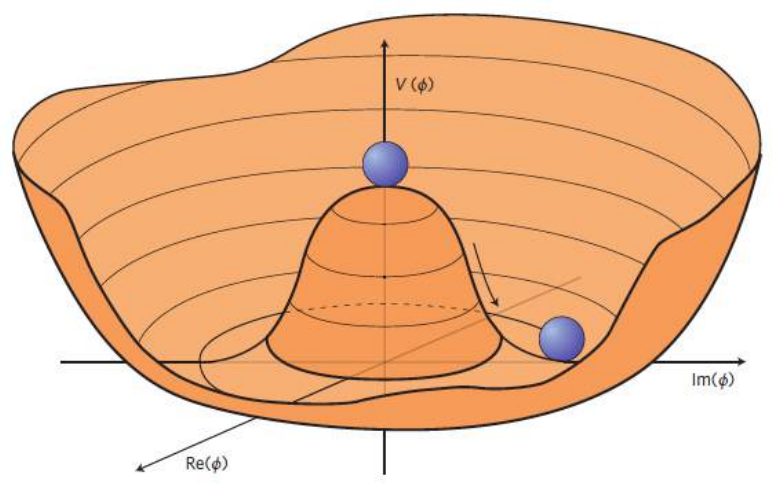
- Within the Standard Model you may first look at QCD: Witten ‘84



- Unfortunately this turns out to be a cross-over. Aoki, Endrodi, Fodor, Katz & Szabo ‘06

Motivation

- The next place is the Electro-Weak phase transition:



- But this is also believed to be a cross-over.

Kajantie, Laine, Rummukainen & Shaposhnikov '96
Laine & Rummukainen '98
Rummukainen, Tsypin, Kajantie, Laine & Shaposhnikov ' 98

Motivation

- However, the EW transition is 1-st order even in minimal extensions of the SM.

Carena, Quiros & Wagner '96
Delepine, Gerard, Felipe & Weyers'96
Laine & Rummukainen '98
Huber & Schmidt, '01
Grojean, Servant & Wells, '04
Huber, Konstandin, Prokopec & Schmidt '06
Profumo, Ramsey-Musolf & Shaughnessy '07
Barger, Langacker, McCaskey, Ramsey-Musolf & Shaughnessy '07
Laine, Nardini & Rummukainen '12
Dorsch, Huber & No '13
Damgaard, Haarr, O'Connell & Tranberg '15

Motivation

- Additional scenarios with 1-st order phase transitions include:

  - Grand Unified Theories at scale much higher than EW scale, which could have their own phase transitions.

    Georgi & Glashow '74
    Pati & Salam '74

    Guth & Weinberg '81
    Kuzmin, Shaposhnikov & Tkachev '82

  - Strongly interacting Dark Matter.

    Kribs & Neil '16
    Tulin & Yu '17
    Schwaller '15

Motivation

- Pessimist: This is disappointing.

- Optimist: This is great.

    - Discovery of GWs from a cosmological phase transition would be the discovery of physics BSM.

    - In some case this may be our only widow into such physics.

- Maximising the discovery potential requires accurate calculation of GW spectrum.

Motivation

- In turn, this requires calculation of several properties of the phase transition:
  - Equation of state (critical temperature).
  - Nucleation temperature.
  - Strength of the transition.
  - Transition rate.

  } Equilibrium

Recent holographic calculations:
Ares, Hindmarsh, Hoyos & Jokela '20
Bigazzi, Caddeo, Cotrone & Paredes '20

Motivation

- In turn, this requires calculation of several properties of the phase transition:
  - Equation of state (critical temperature).
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  - Bubble wall velocity.

Equilibrium

Recent holographic calculations:
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Motivation

- In turn, this requires calculation of several properties of the phase transition:

    - Equation of state (critical temperature).
    - Nucleation temperature.
    - Strength of the transition.
    - Transition rate.
    - Bubble wall velocity.

Equilibrium: Equation of state, Nucleation temperature, Strength of the transition, Transition rate.

Out-of-equilibrium: Bubble wall velocity.

Recent holographic calculations:
Ares, Hindmarsh, Hoyos & Jokela '20
Bigazzi, Caddeo, Cotrone & Paredes '20

- Most challenging, even at weak coupling.

- Also most pressing, since GW signal is most sensitive to v.

- In this talk: calculation at strong coupling using Holography.

Motivation

- Assume bubble has been nucleated and determine subsequent dynamics.
- Planar bubbles for most of the talk (spherical bubbles at the end):

  Invariant along transverse directions: x, y

  Expansion along longitudinal direction: z

- First work in long-term program.
- We do to know what lies BSM:

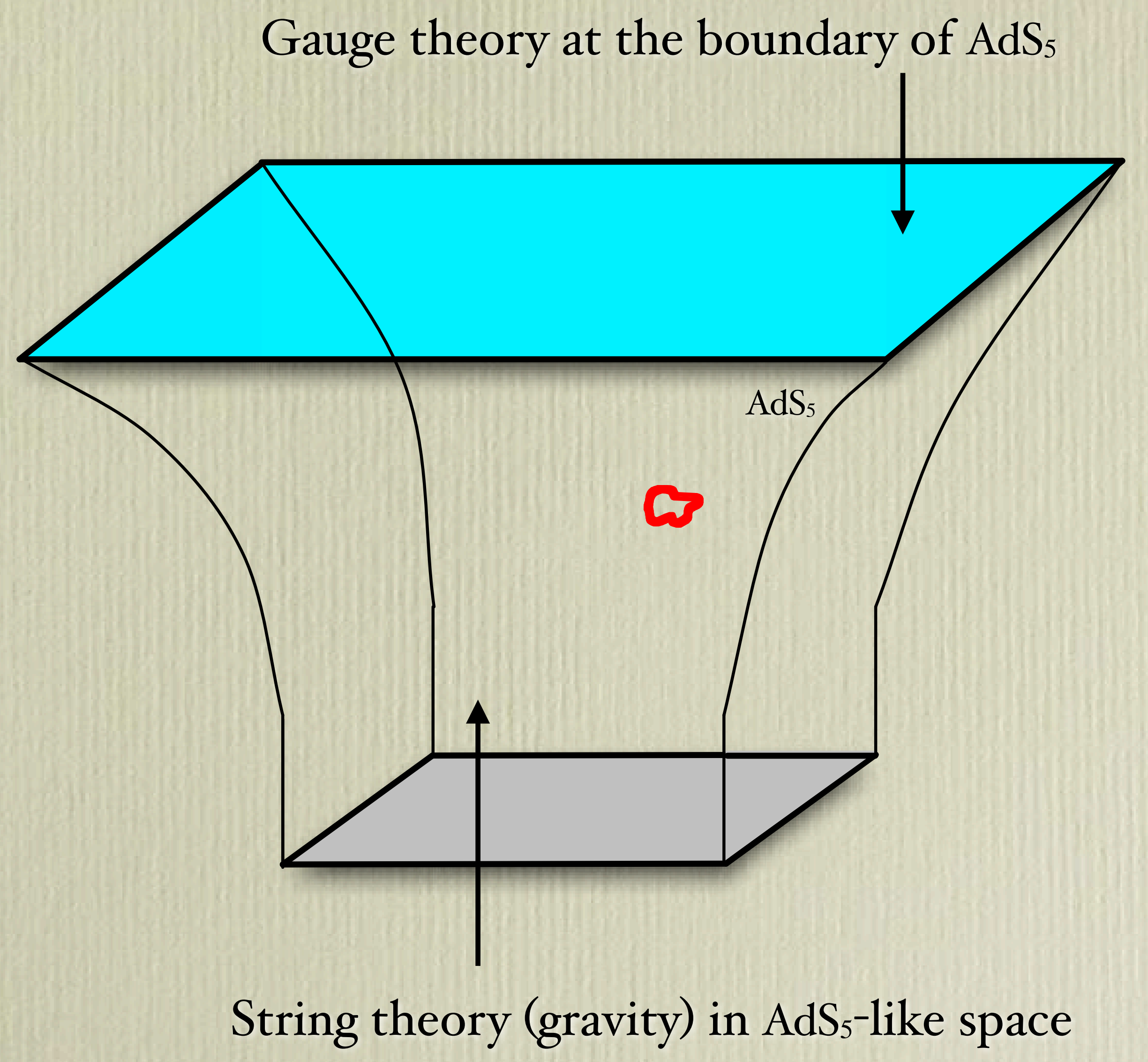
  → Choose simplest model

  → Focus on universal features

- Some plots are still preliminary but was too excited not to show you!
- All questions/feedback/criticism more than welcome.

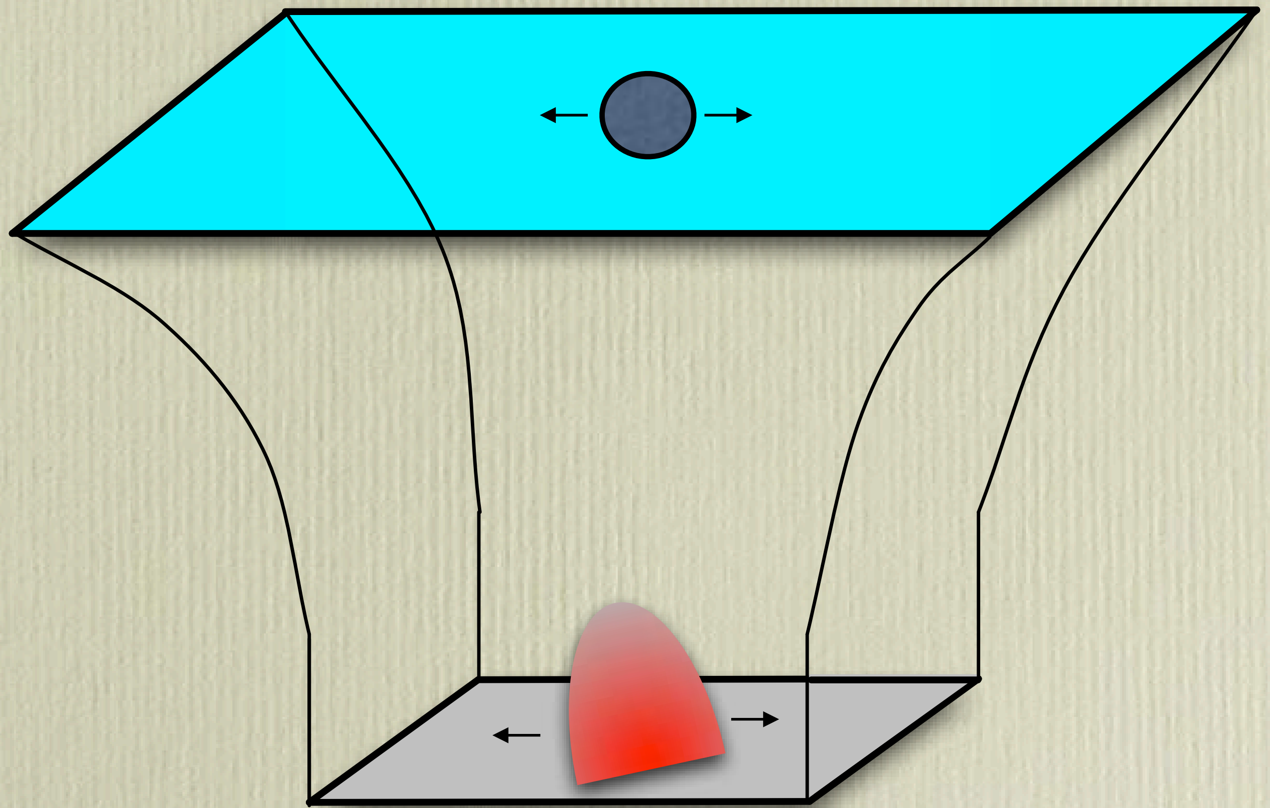
Holographic model

Holography


Gauge theory at the boundary of AdS5
AdS5
String theory (gravity) in AdS5-like space

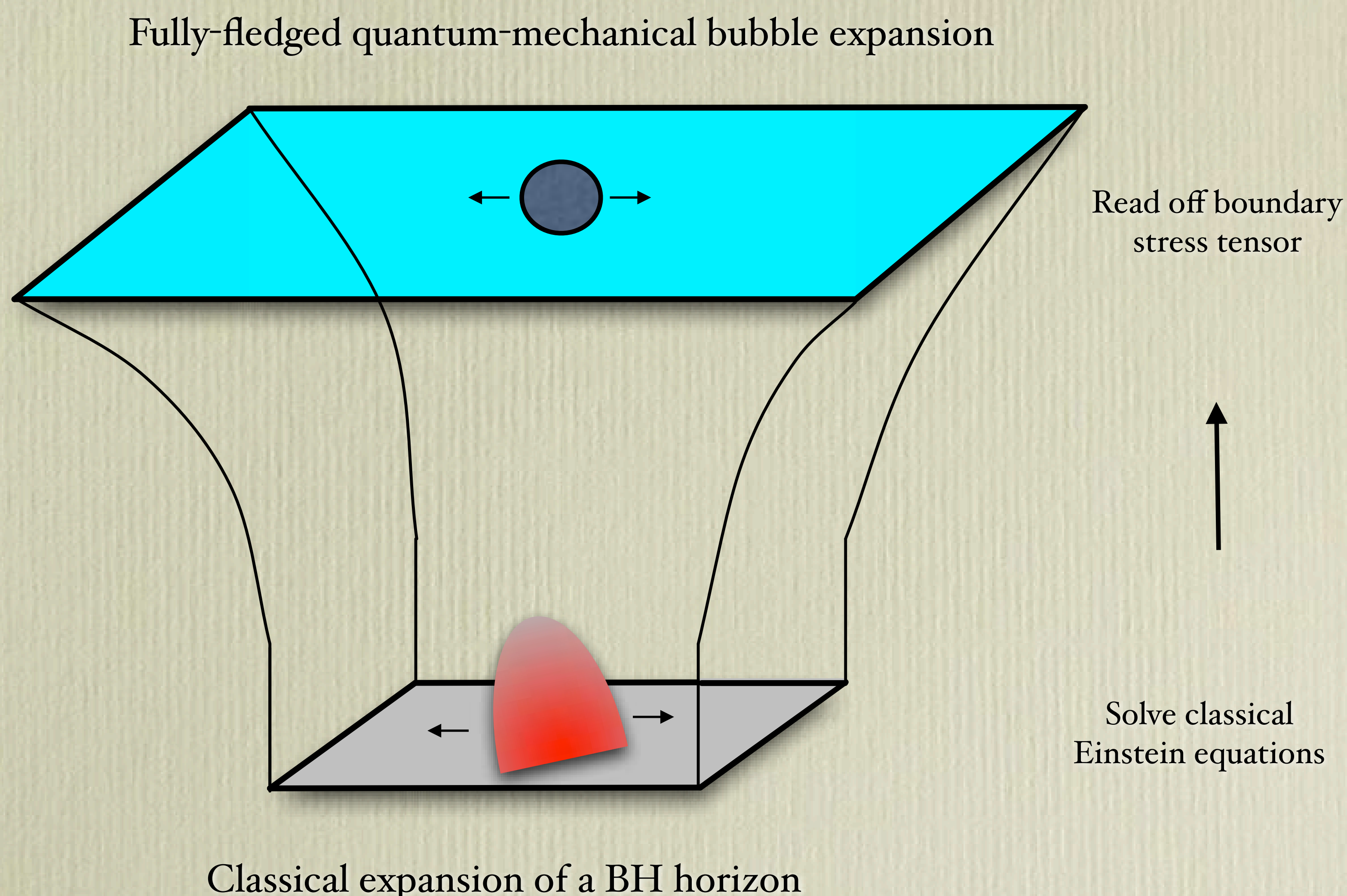
Holography

Fully-fledged quantum-mechanical bubble expansion



Classical expansion of a BH horizon

Strategy


Fully-fledged quantum-mechanical bubble expansion
Read off boundary stress tensor
Solve classical Einstein equations
Classical expansion of a BH horizon

Holographic model

Bea & Mateos '18

- Einstein-scalar action:

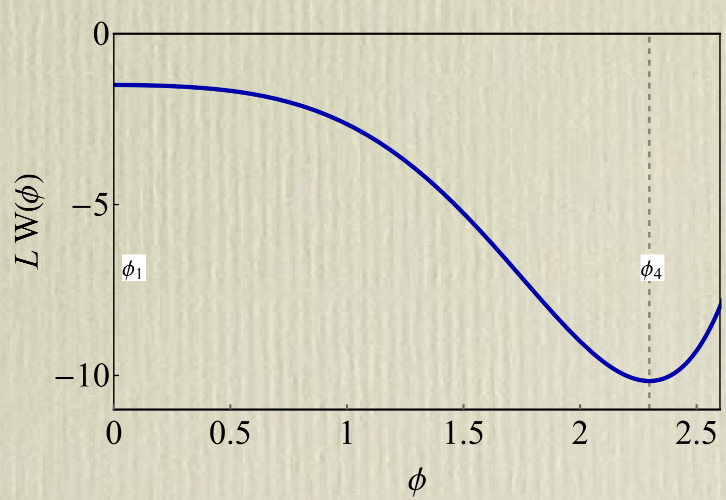
$$S = \frac{2}{\kappa_5^2} \int d^5x \sqrt{-g} \left[\frac{1}{4}\mathcal{R} - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right]$$

Encodes properties of the gauge theory

- Choose: $$V(\phi) = -\frac{4}{3} W(\phi)^2 + \frac{1}{2} W'(\phi)^2$$

$$LW(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{\phi^4}{4\phi_M^2} + \frac{\phi^6}{\phi_Q}$$

Parameters



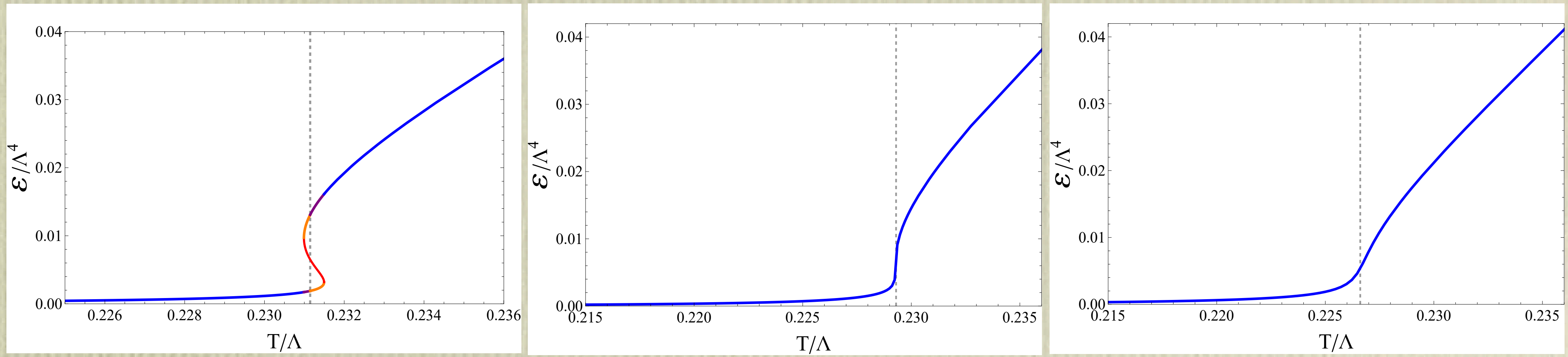
- Simplest non-conformal model with completely regular geometry even at *T=0*.

Gauge theory thermodynamics

Attems, Bea, Casalderrey, D.M., Triana & Zilhão '18

- Non-conformal: Has characteristic energy scale Λ.
- Transition depends on values of parameters:

Varying parameters →



1st order 2nd order Crossover

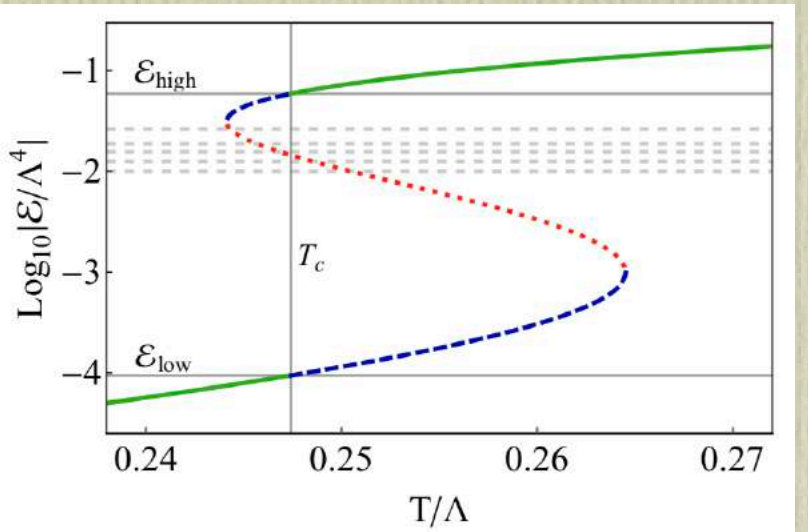
- In all cases: $T_c \sim \Lambda$

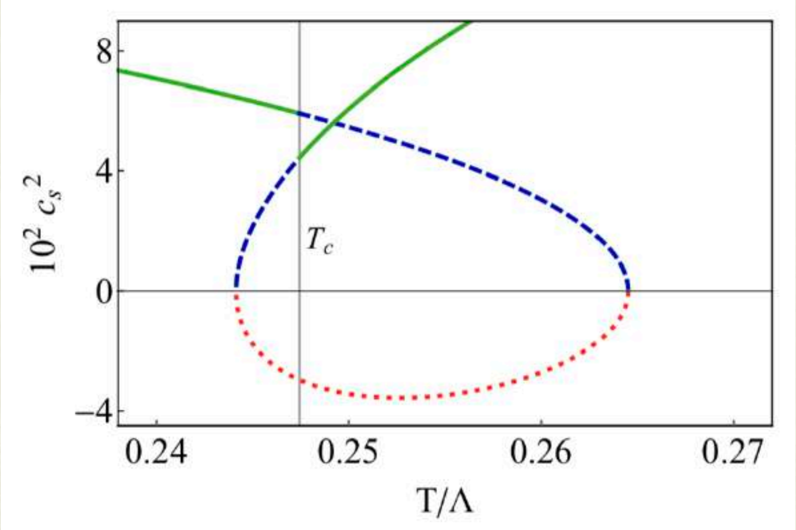
Dynamics of phase separation

1st-order phase transition: Spinodal instability

Attems, Bea, Casalderrey, D.M., Triana & Zilhão '17
Janik, Jankowski, Soltanpanahi '17
Attems, Bea, Casalderrey, D.M. & Zilhão '19
Bellantuono, Janik, Jankowski, Soltanpanahi '19

ϕ_M=2.3, ϕ_Q=∞



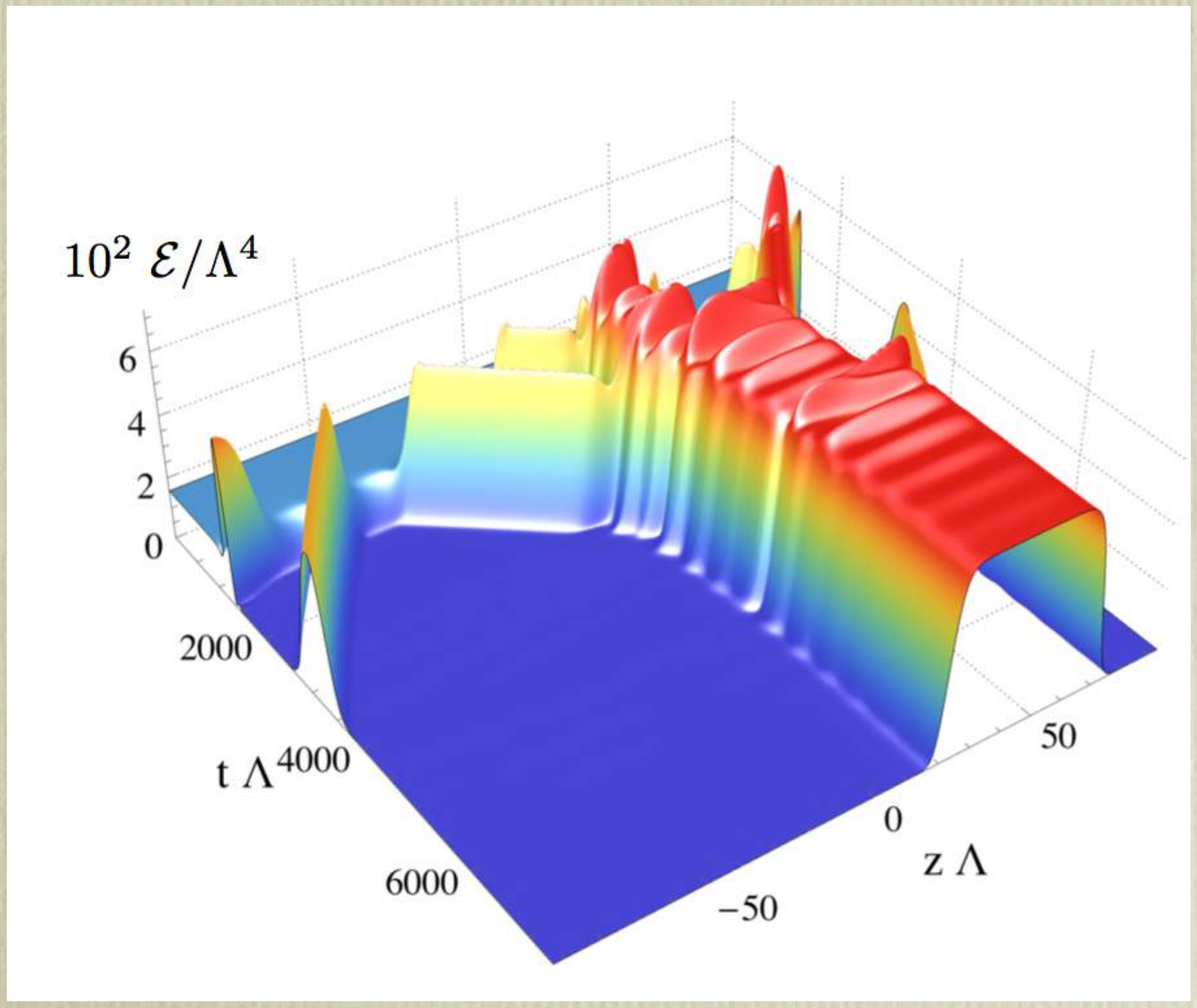


- Thermodynamic instability implies dynamical instability for red states.

1st-order phase transition: Phase separation

Attems, Bea, Casalderrey, D.M. & Zilhão '19

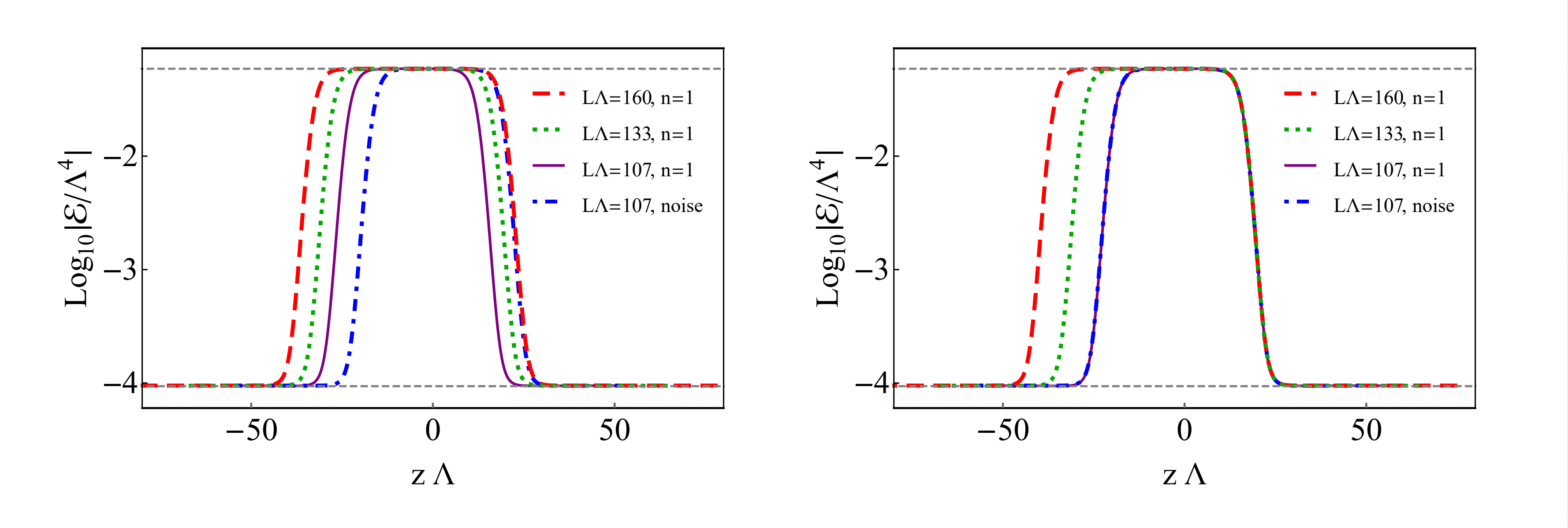
Perturbed homogeneous state evolves to phase-separated configuration:



1st-order phase transition: Phase separation

Attems, Bea, Casalderrey, D.M. & Zilhão '19

- Wall profile is universal (independent of initial conditions):



1st-order phase transition: Phase separation

Attems, Bea, Casalderrey, D.M. & Zilhao ’19

- Describing evolution in detail could fill an entire talk.

- Instead of that I will show you that entire evolution is well described by 2nd-order hydrodynamics.

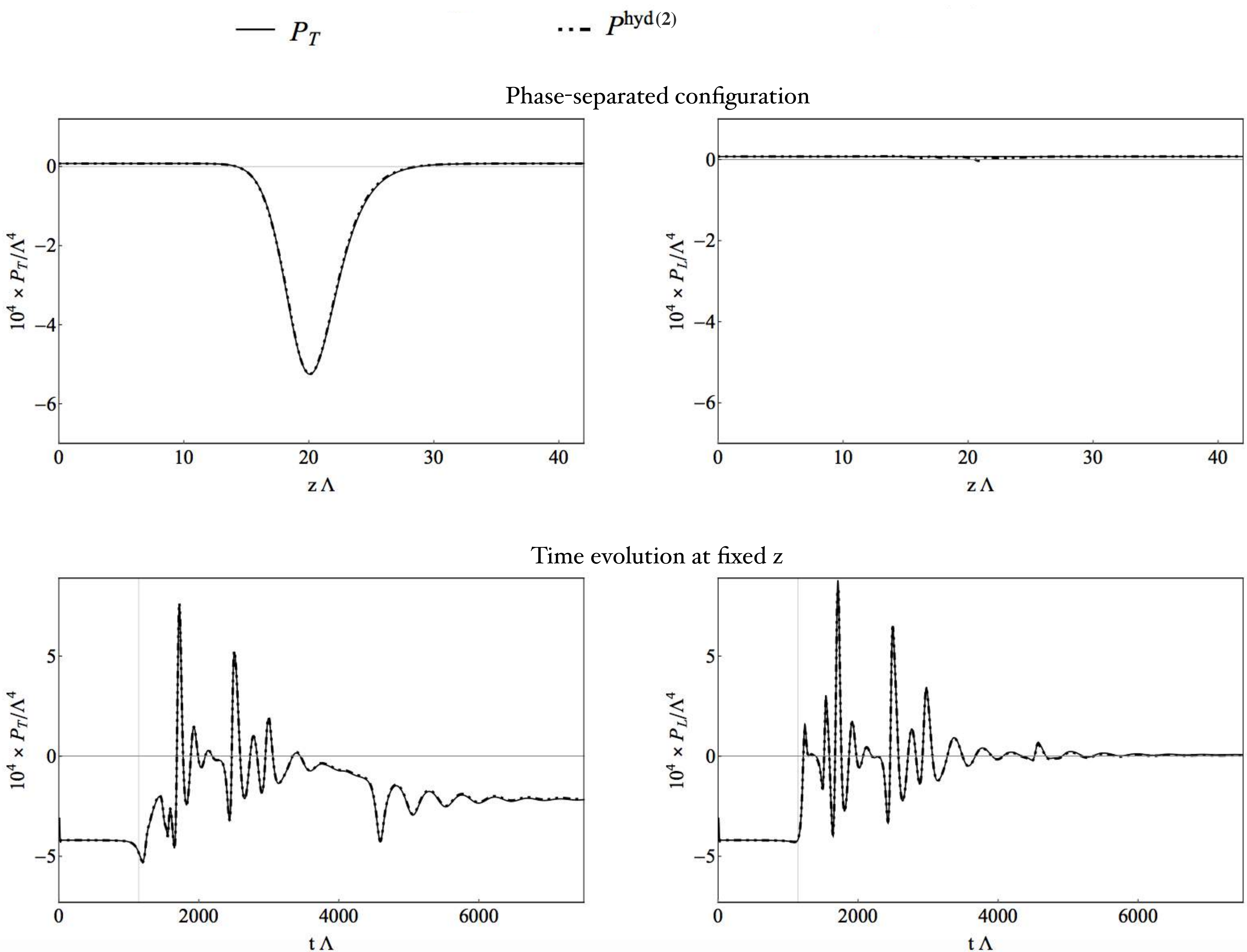
Evolution described by 2nd-order hydrodynamics

Attems, Bea, Casalderrey, D.M., Triana & Zilhão '17
Attems, Bea, Casalderrey, D.M. & Zilhão '19

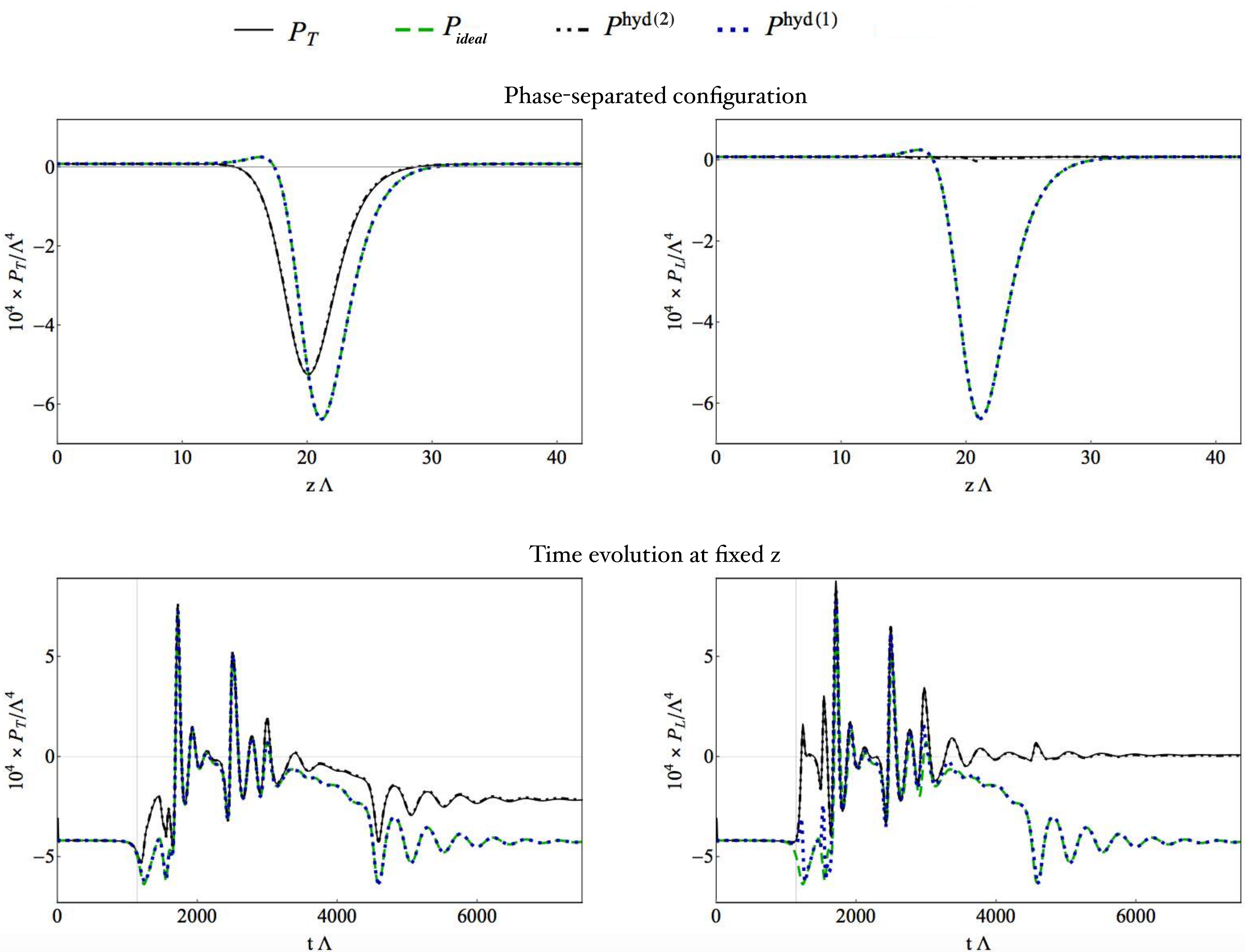
bulk & shear viscosities

↓

$$T_{\mu\nu} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial_{\text{spatial}}^2$$



P_T
$P^{\mathrm{hyd}(2)}$
Phase-separated configuration
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
z Λ
Time evolution at fixed z
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
t Λ
t Λ



P_T
P_{ideal}
$P^{\text{hyd}(2)}$
$P^{\text{hyd}(1)}$
Phase-separated configuration
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
Time evolution at fixed z
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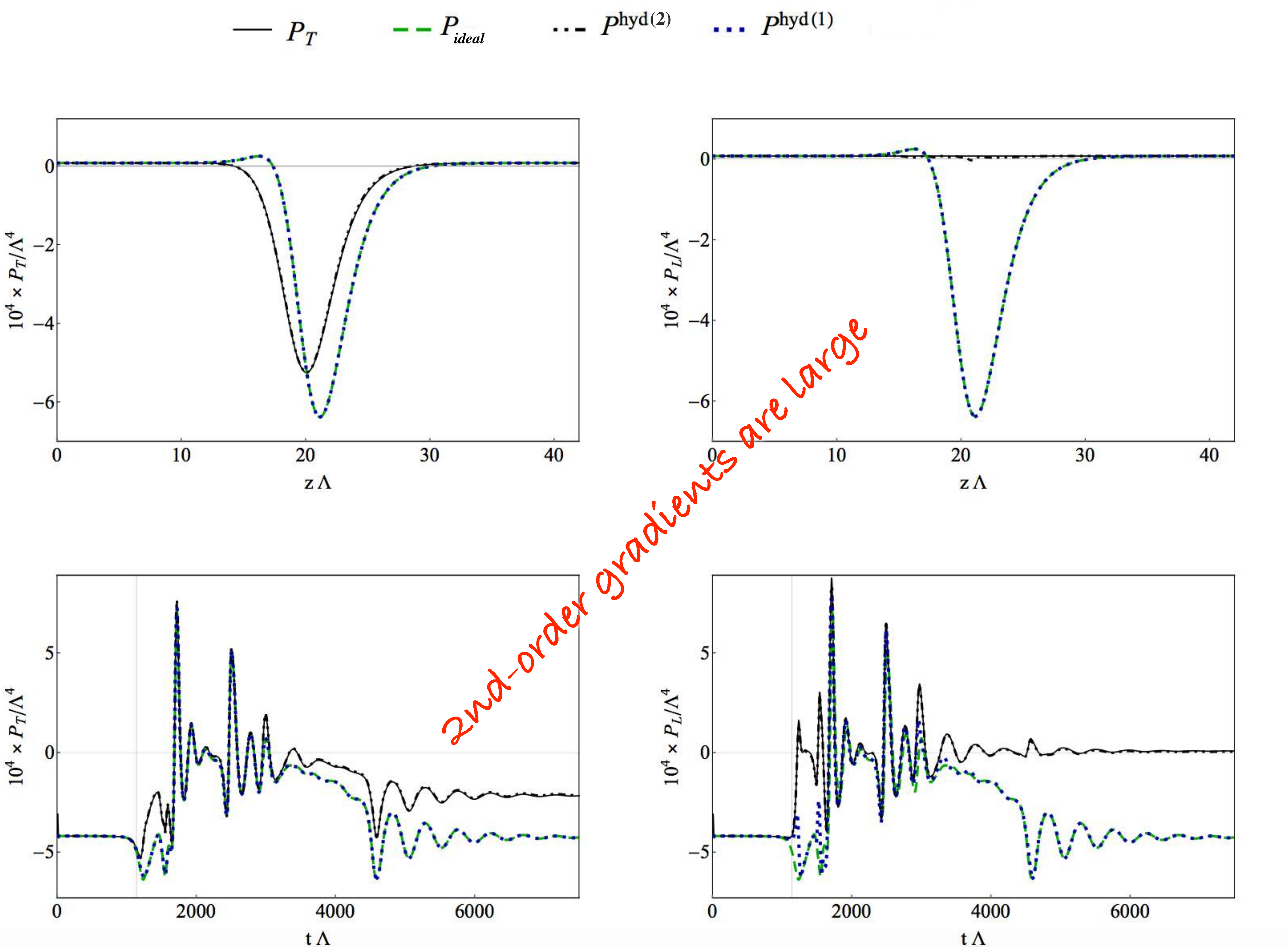


P_T
P_{ideal}
$P^{\text{hyd}(2)}$
$P^{\text{hyd}(1)}$
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
t Λ
2nd-order gradients are large

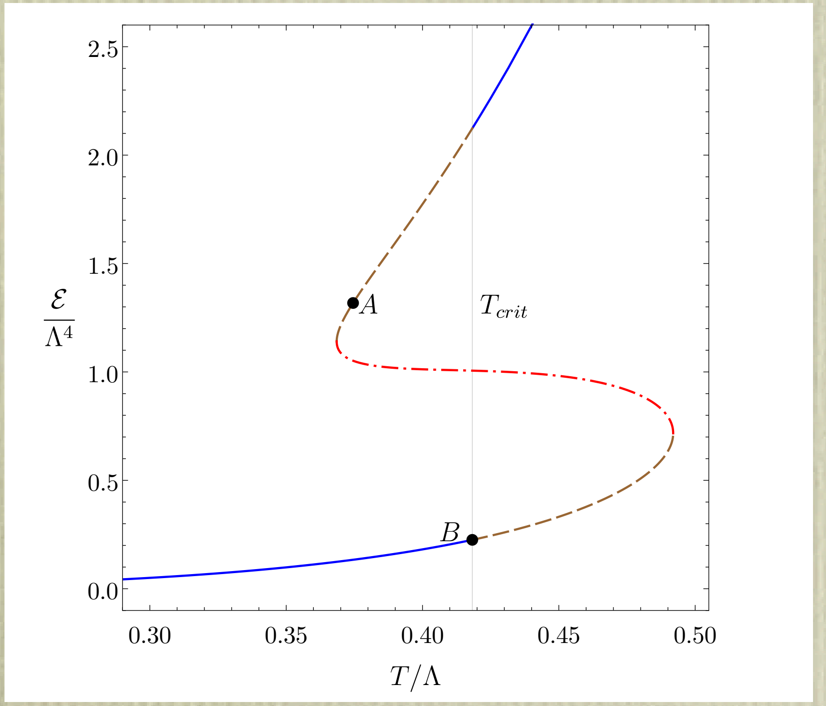
Bubble dynamics

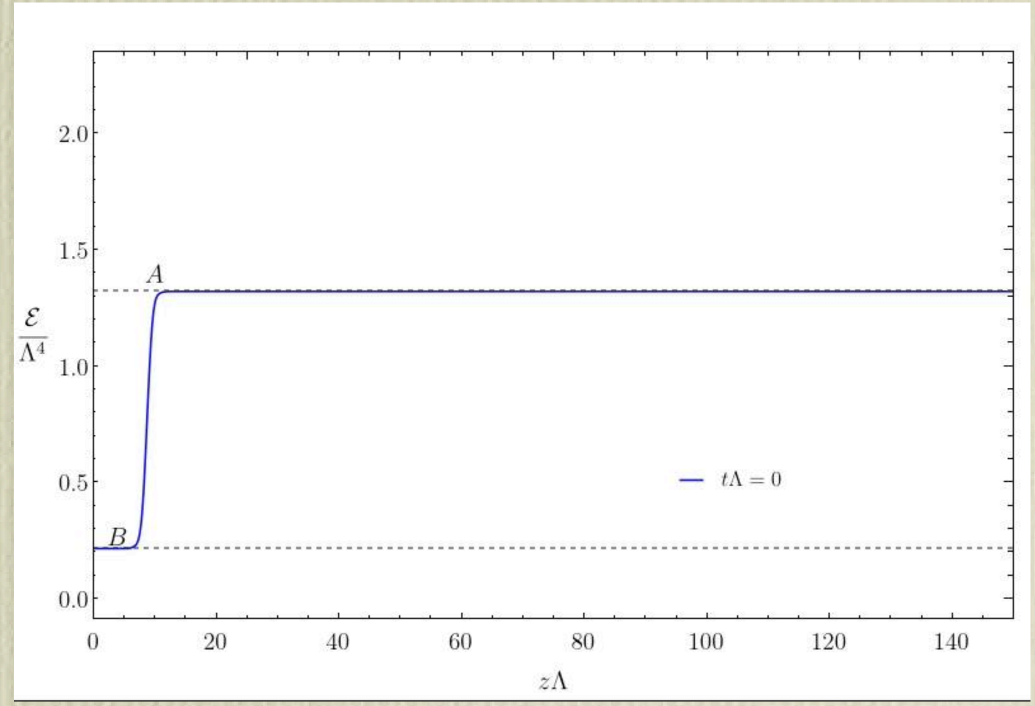
Bubble dynamics

ϕ_M=0.85, ϕ_Q=10

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

- Assume bubble has been nucleated at $T_N = T_A$ and "let it go".
- B does not have to be at same temperature!





Bubble dynamics

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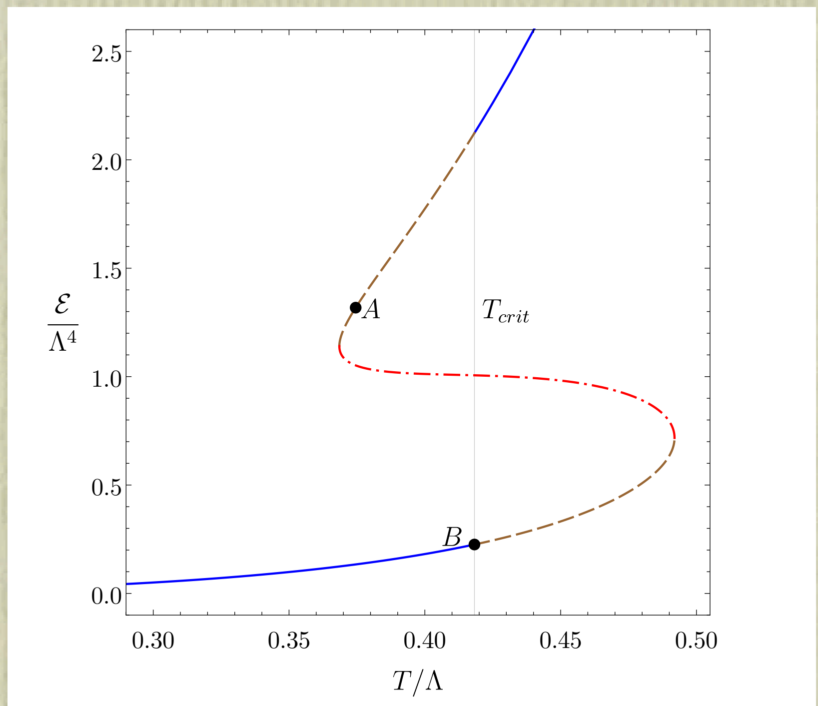
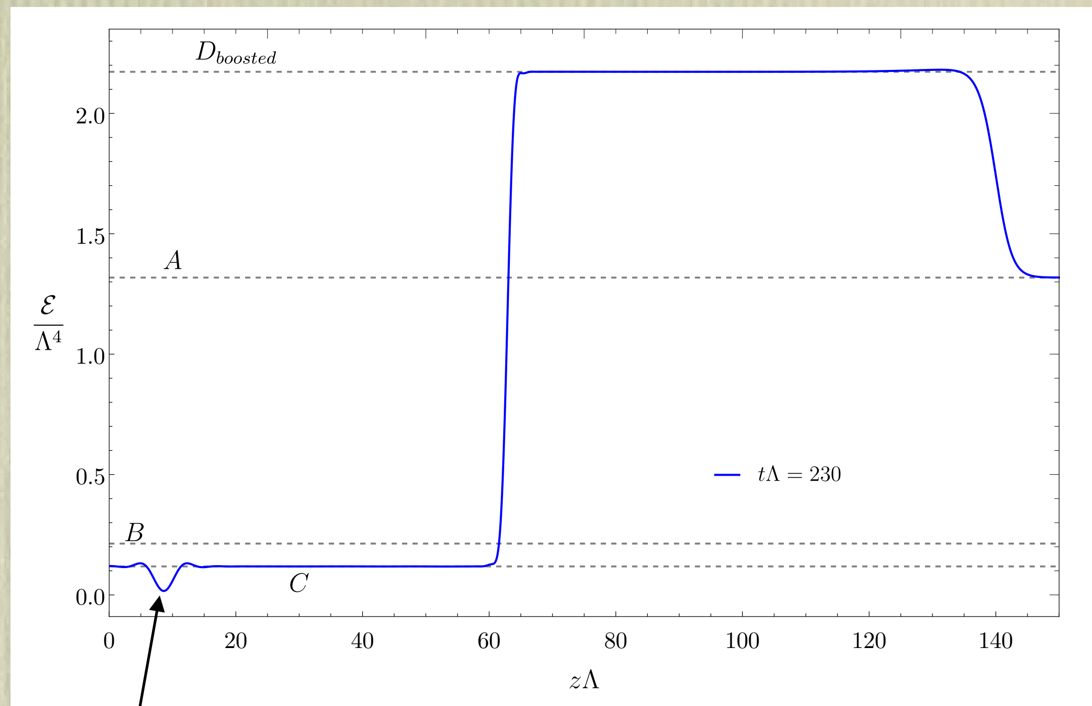
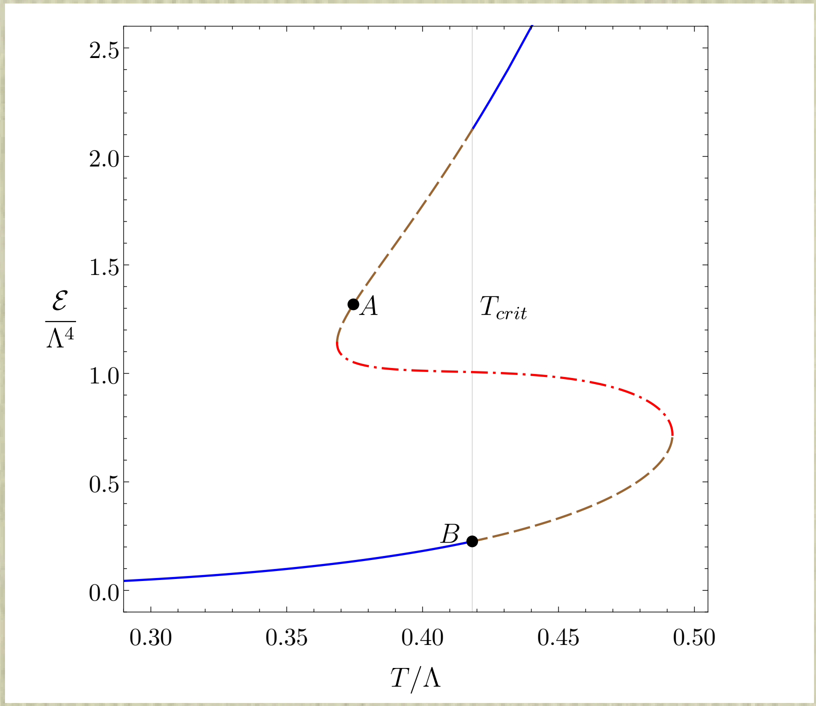


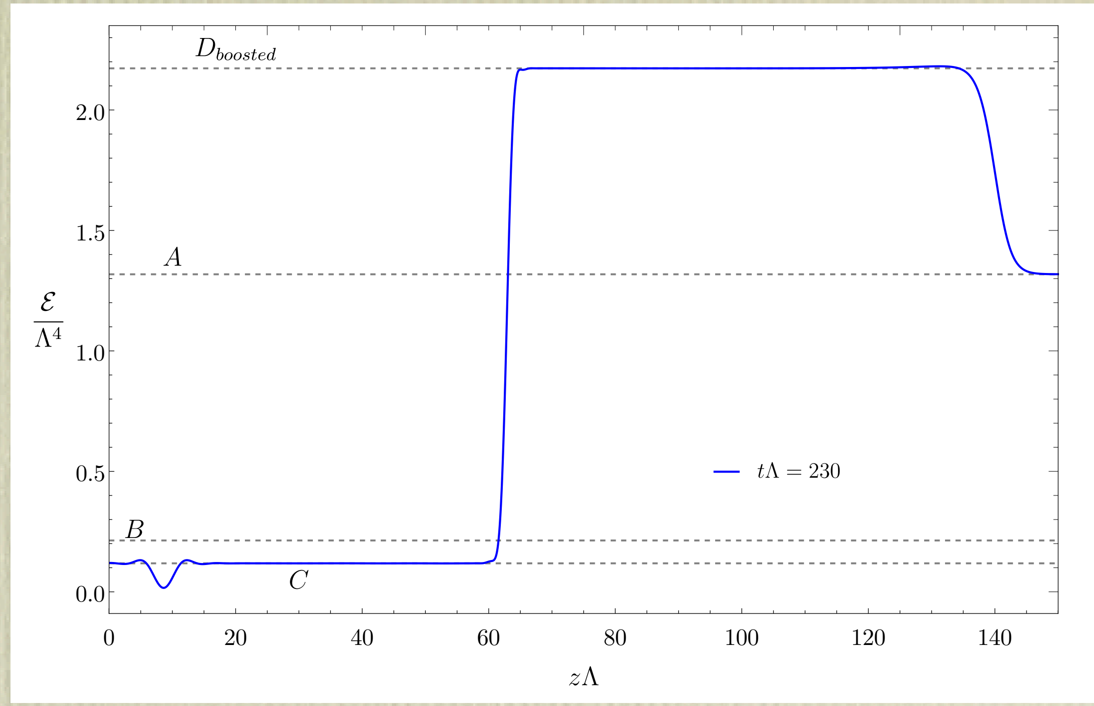


Most likely numerical artifact, please ignore

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

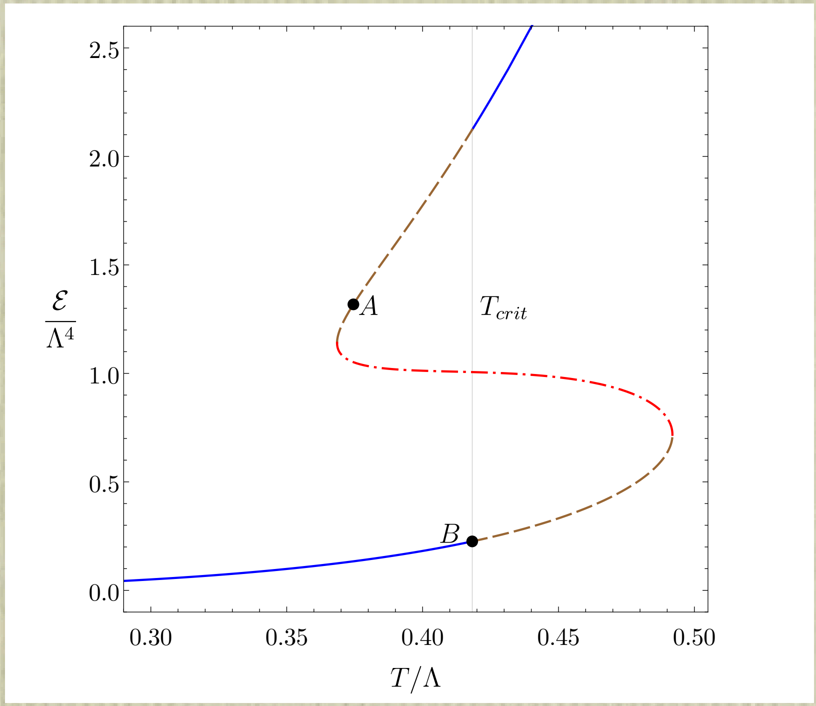


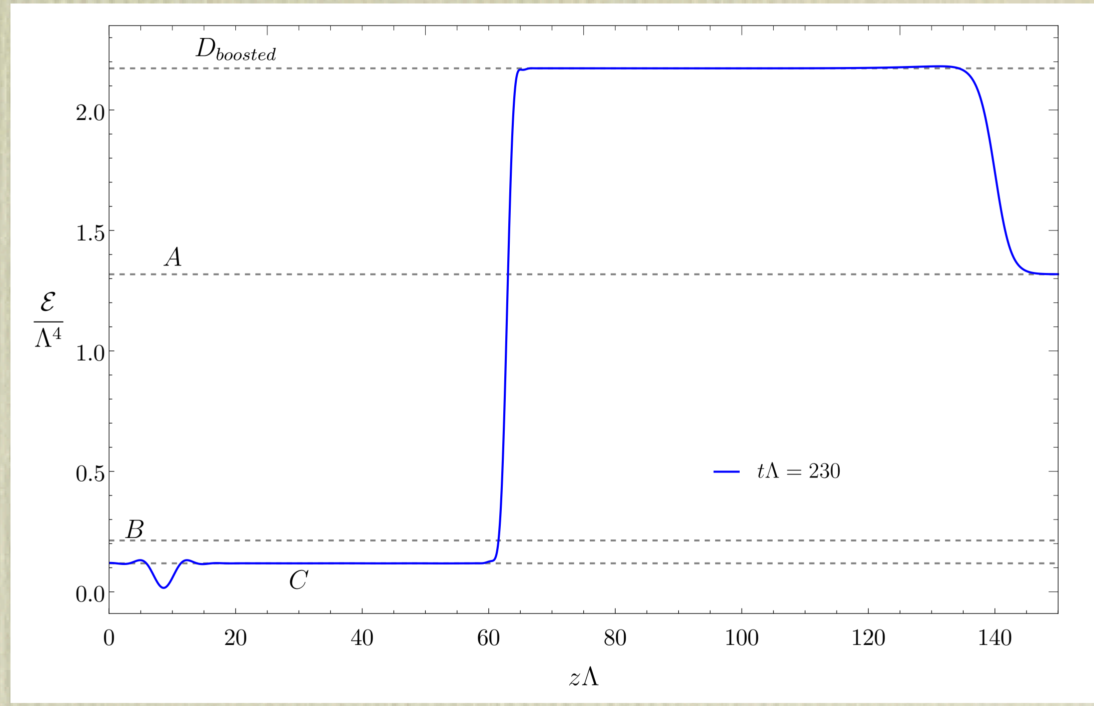


- Wall moves at constant $v_{wall} = 0.236$
- Fluid in $D_{boosted}$ moves at $v = 0.219$

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)



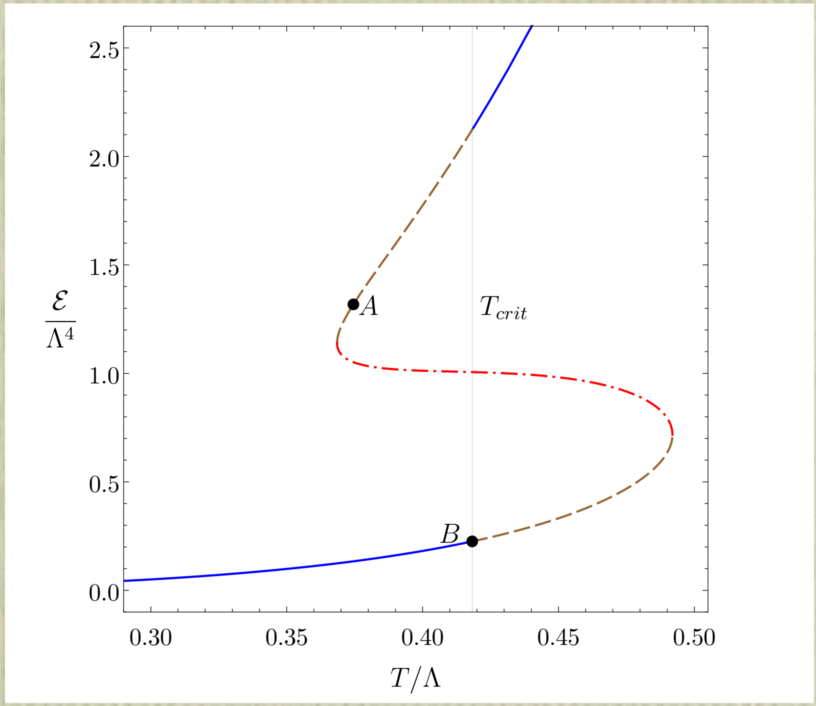


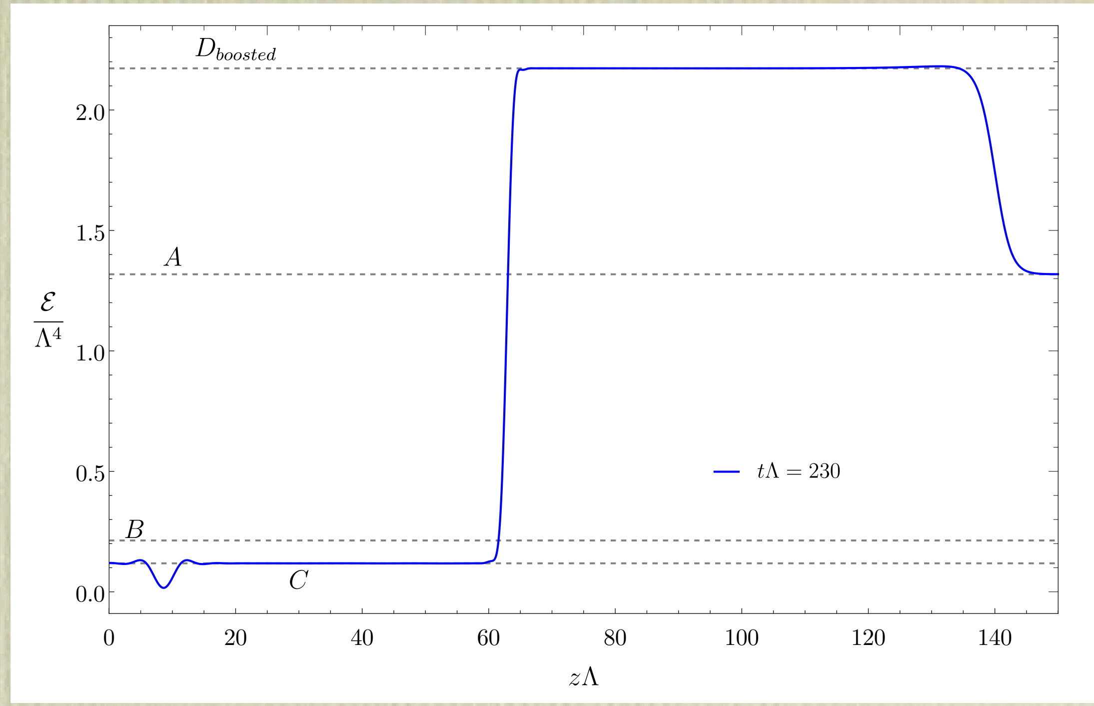
- Wall moves at constant $v_{wall} = 0.236$
- Fluid in $D_{boosted}$ moves at $v = 0.219$
- States C and D dynamically determined in terms of T_A
- Size of C and $D_{boosted}$ grow linearly with *t*, but C-$D_{boosted}$ interfase grows more slowly

→ Strictly speaking no *z/t* scaling at late times

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

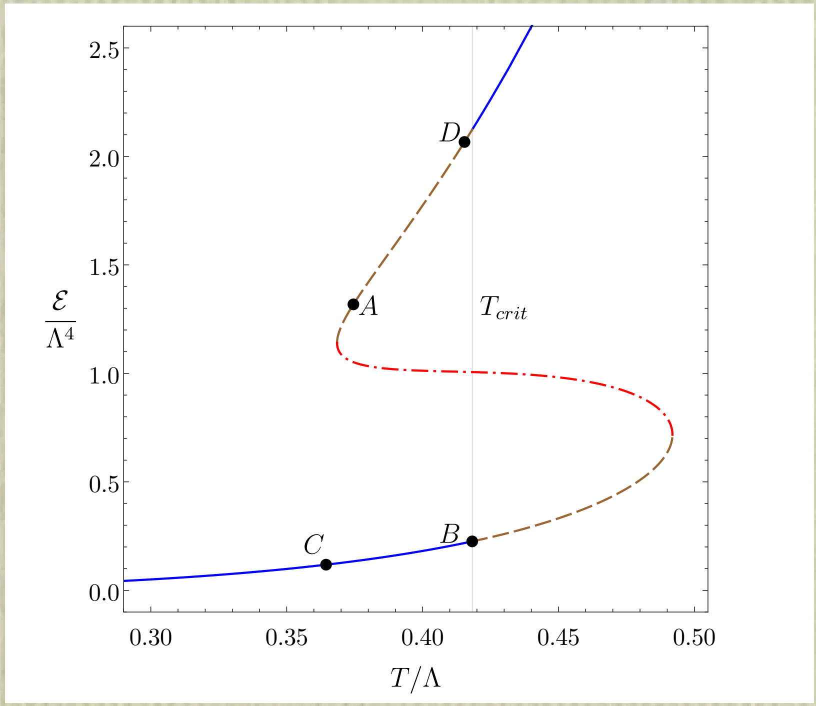


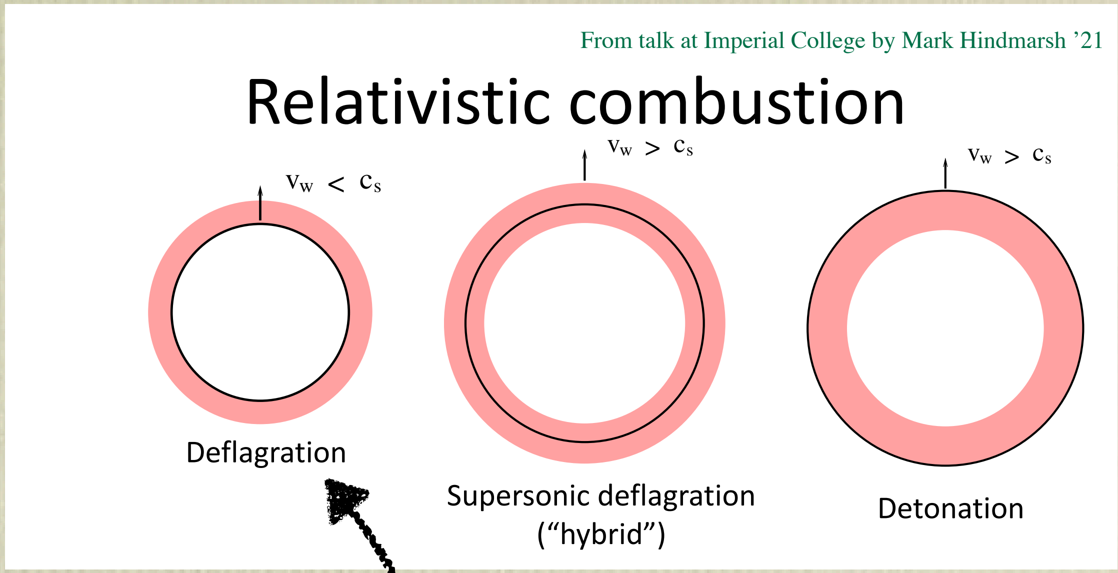


- Wall moves at constant $v_{wall} = 0.236$
- Fluid in $D_{boosted}$ moves at $v = 0.219$
- States C and D dynamically determined in terms of T_A
- Speed of sound: $c_{s,A} = 0.402$, $c_{s,D} = 0.507$

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)





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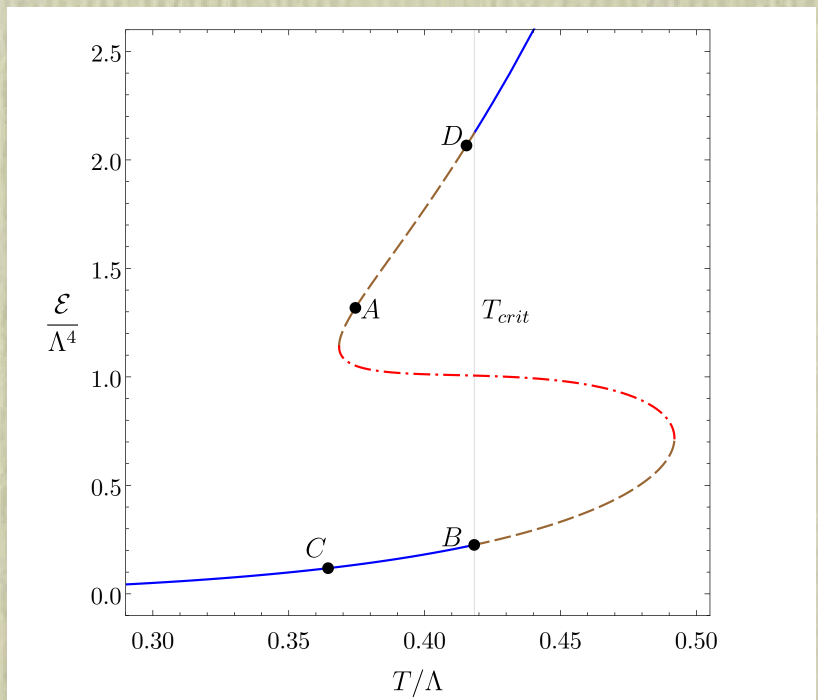
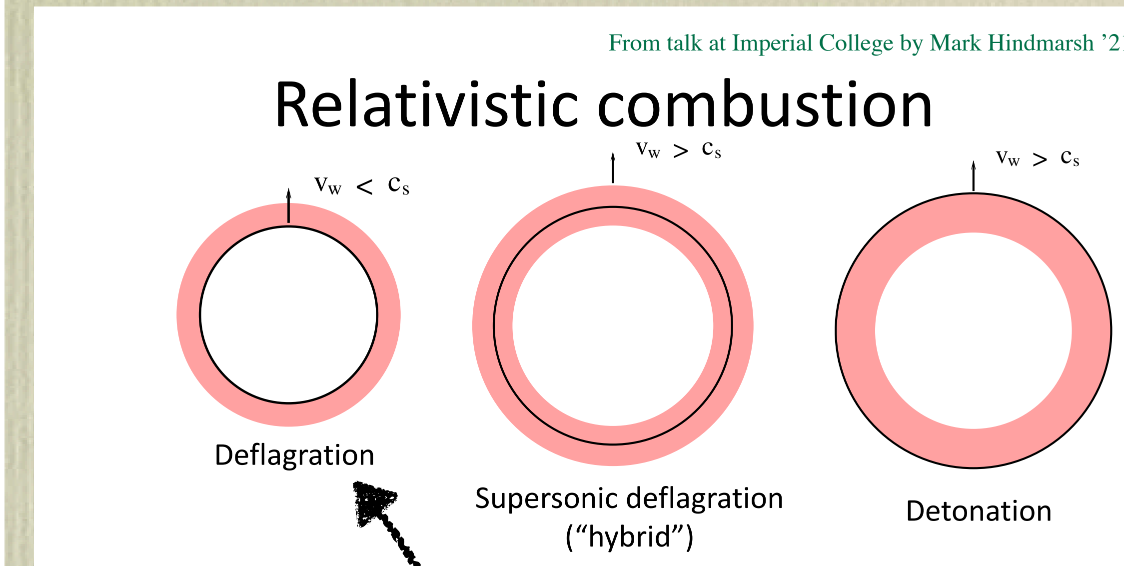
- In this case wall is subsonic w.r.t. both A and D

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)





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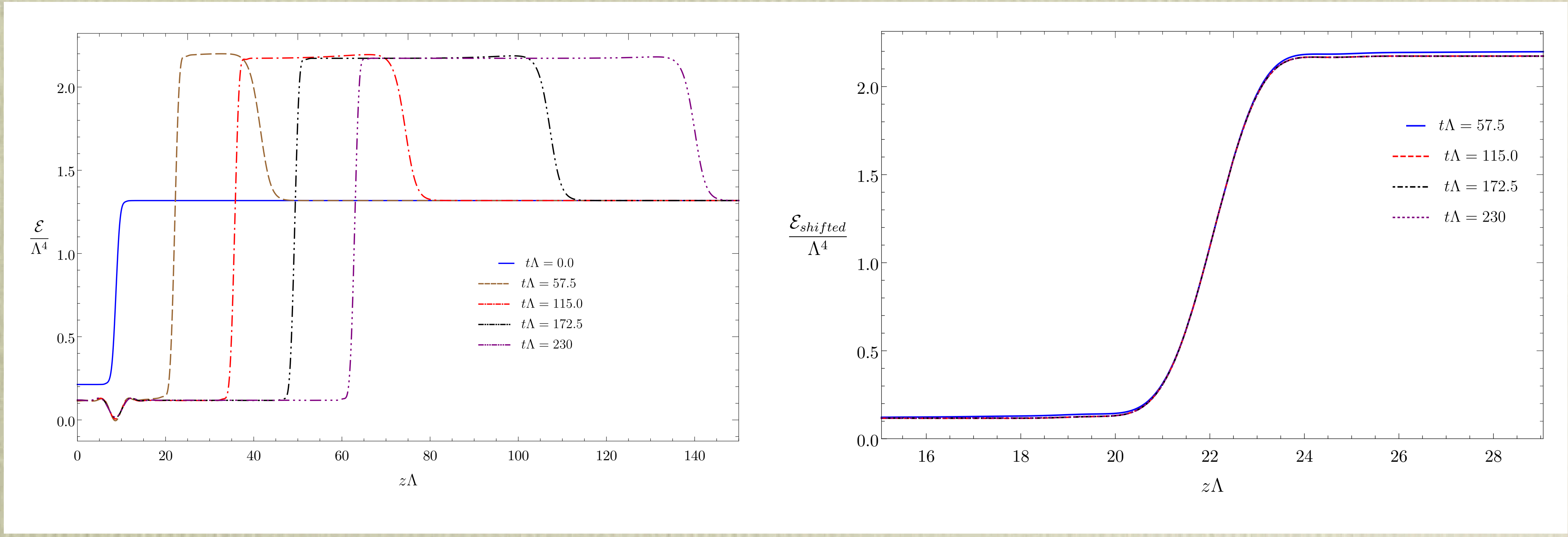
- In this case wall is subsonic w.r.t. both A and D
- Just one example, will come back to this

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

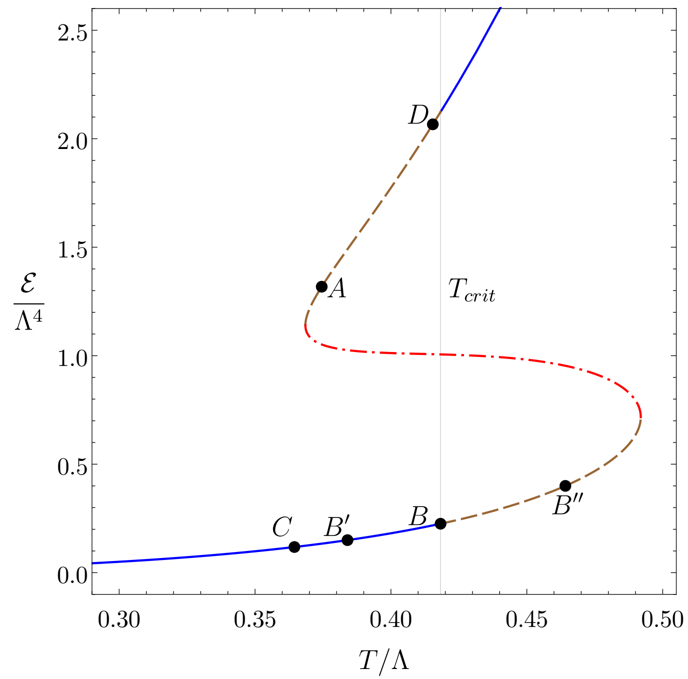
- Wall profile is constant in time:

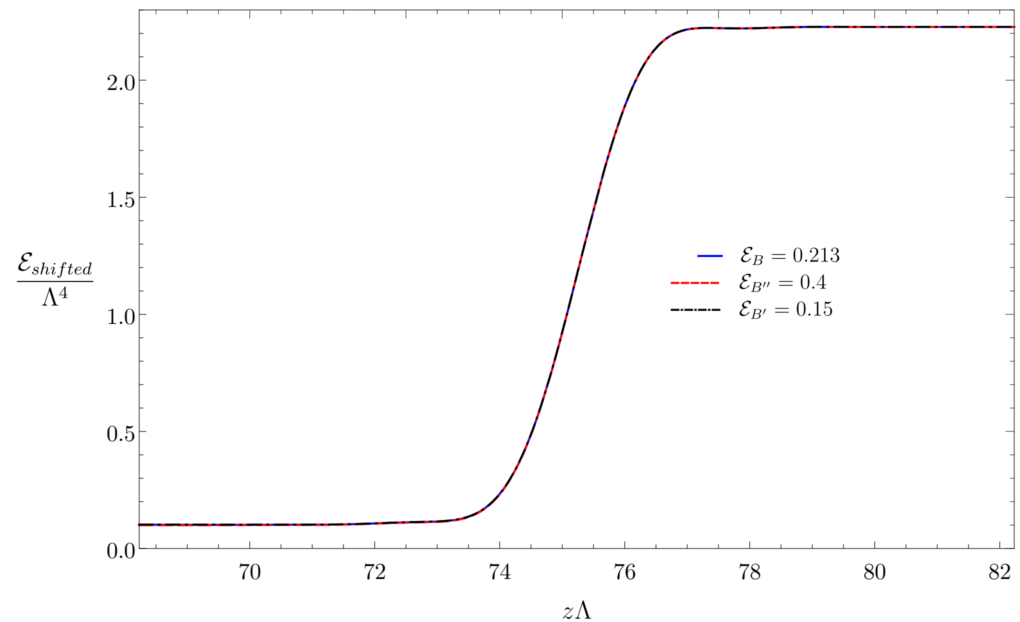


Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

- Steady state (v_{wall}, C, D and wall profile) is independent of initial conditions.
- Will illustrate it with wall profile.
- For example, changing *B*:

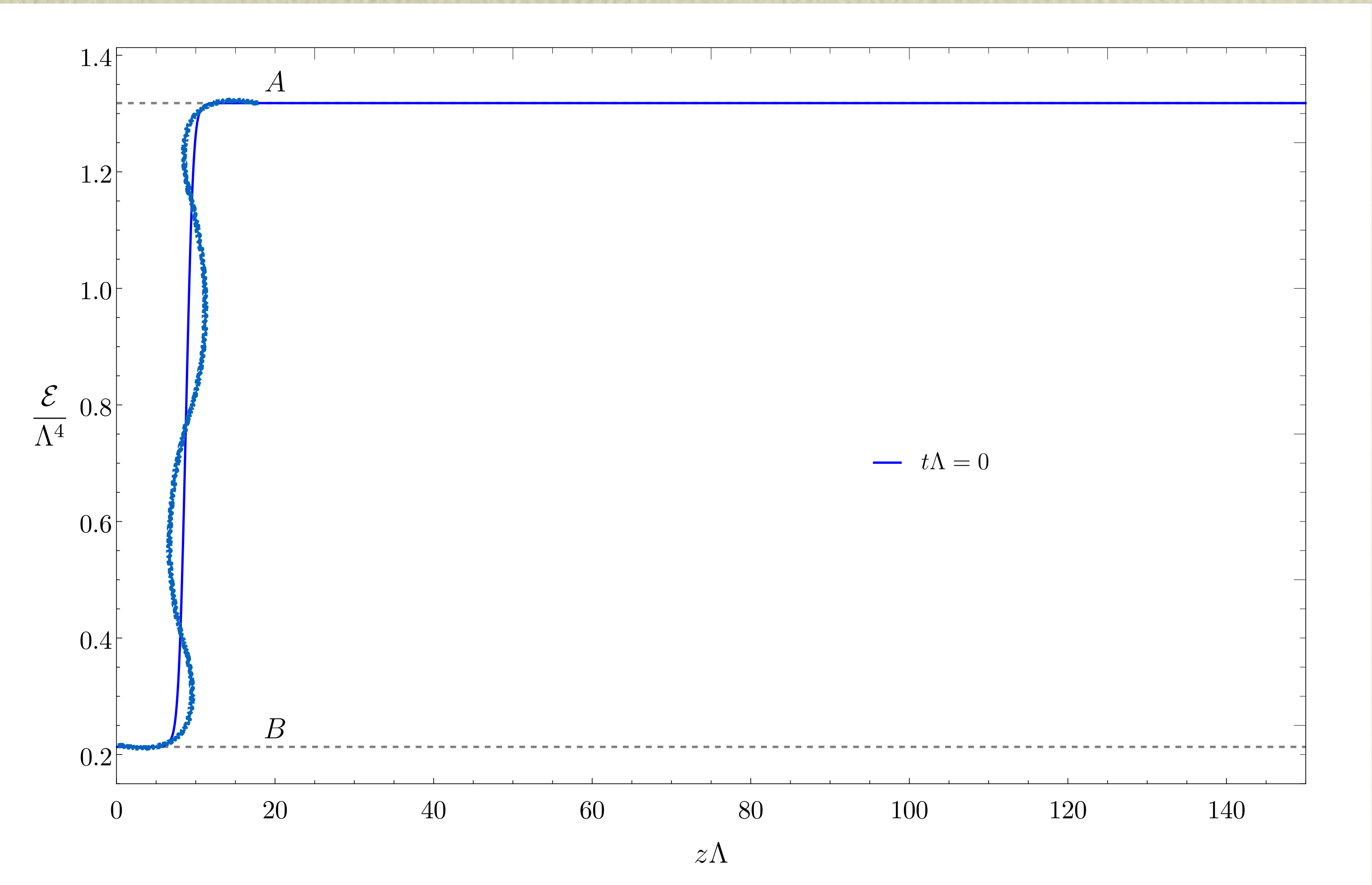




Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

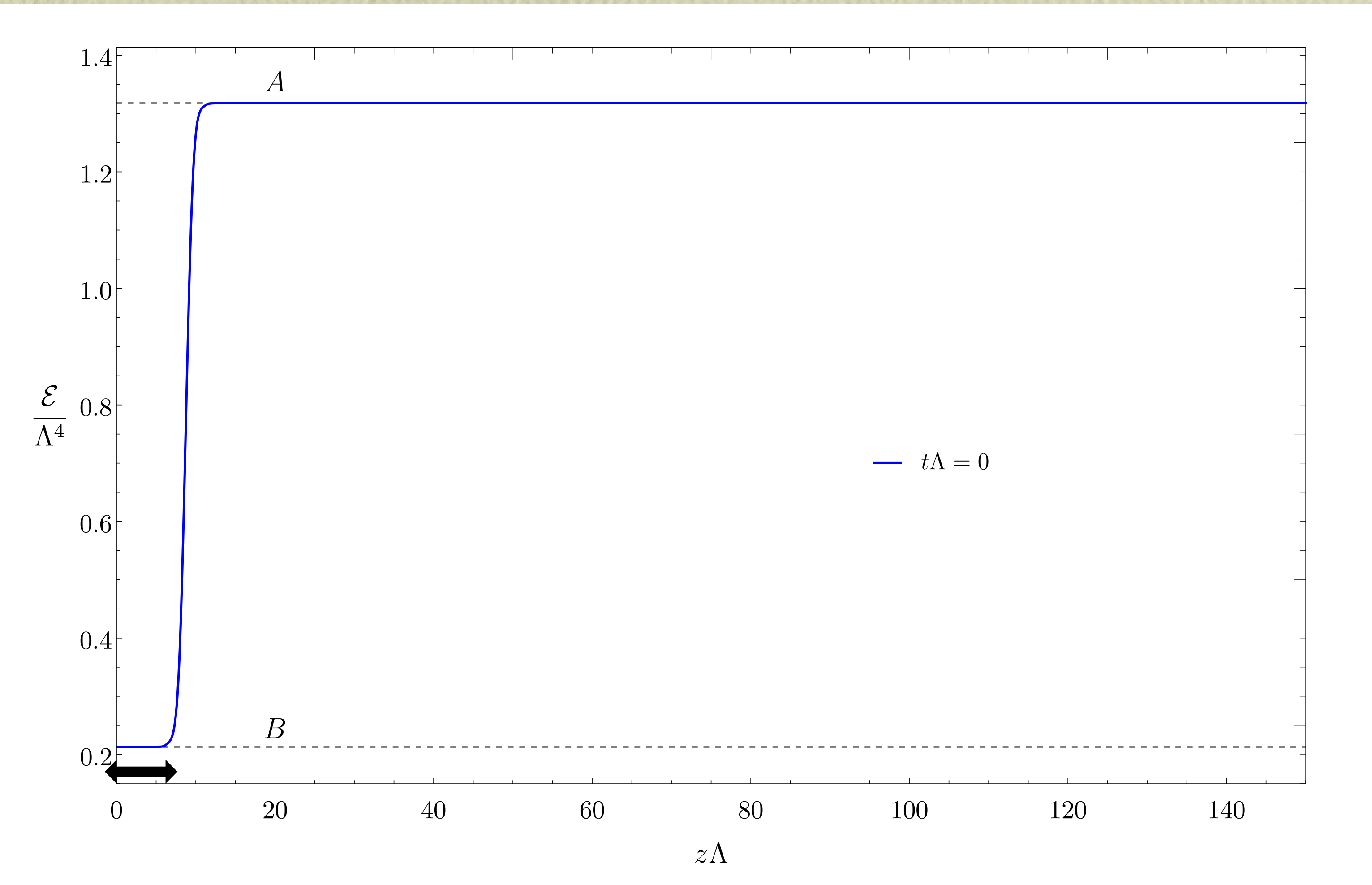
- Analogous result under changes in:
  - Initial wall profile.



Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

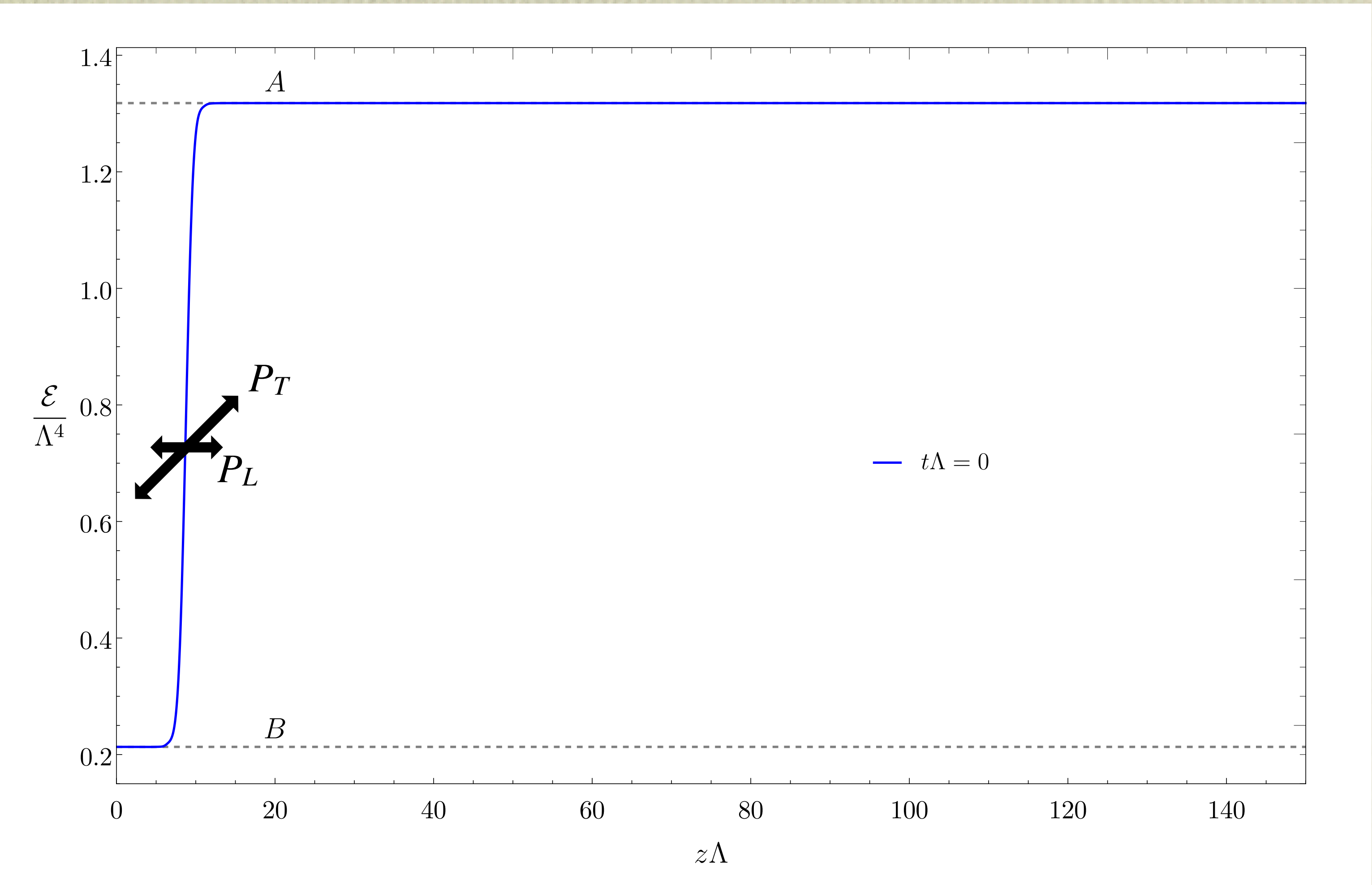
- Analogous result under changes in:
  - Initial wall profile.
  - Initial bubble size.



Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

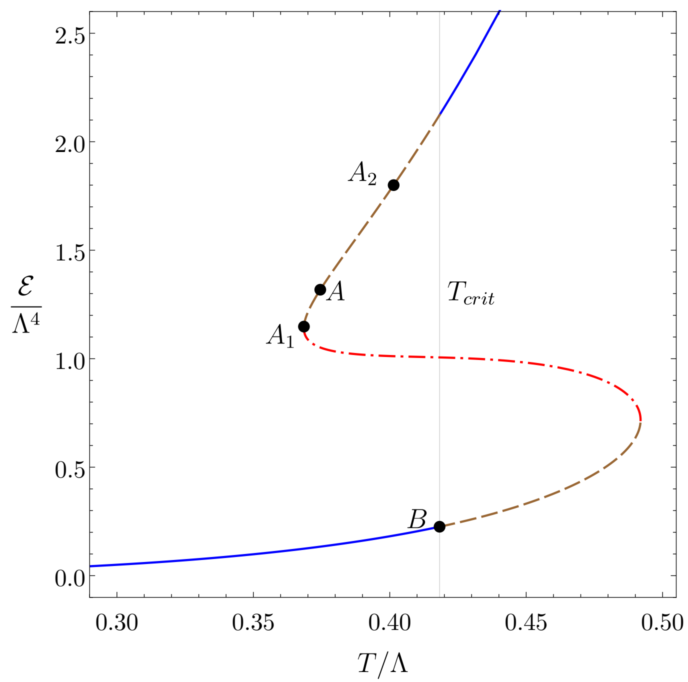
- Analogous result under changes in:
    - Initial wall profile.
    - Initial bubble size.
    - Initial pressure anisotropy.



Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

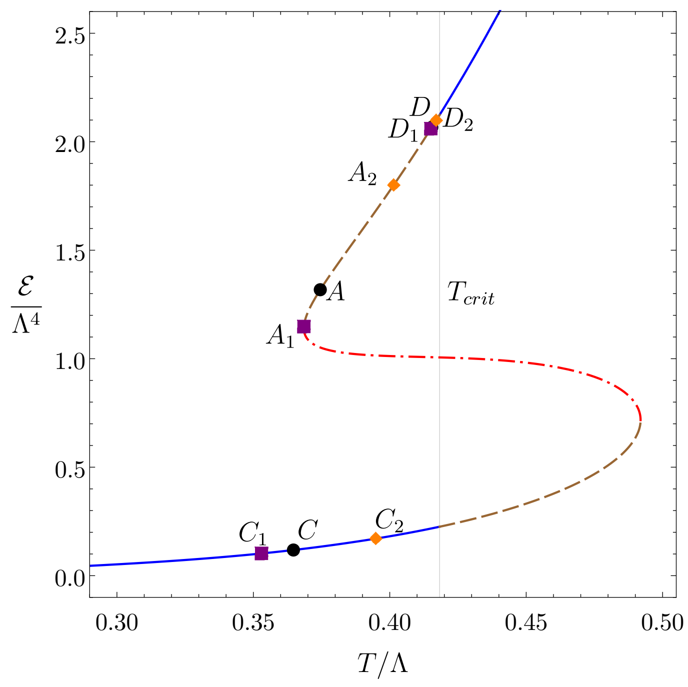
- We now turn to dependence on the nucleation temperature T_A .



Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

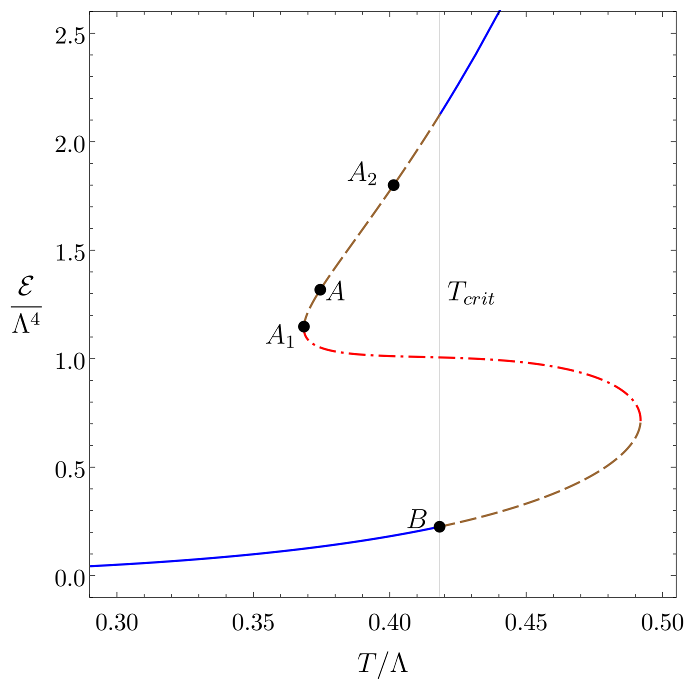
- We now turn to dependence on the nucleation temperature T_A .
- C and D seem to depende weakly on A.
- In contrast, v_{wall} is very sensitive to A, as expected.



Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

- As A approaches the turning point:
    - Pressure difference increases.
    - Energy density in A decreases.
    - $c_{s,A}$ approaches zero.
- Therefore v_{wall} grows and becomes supersonic w.r.t. A (but not w.r.t. to D):
    - For A_1 we have: $v_{wall} = 0.29$, $c_{s,A} = 0.12$, $c_{s,D} = 0.51$



Bubble dynamics

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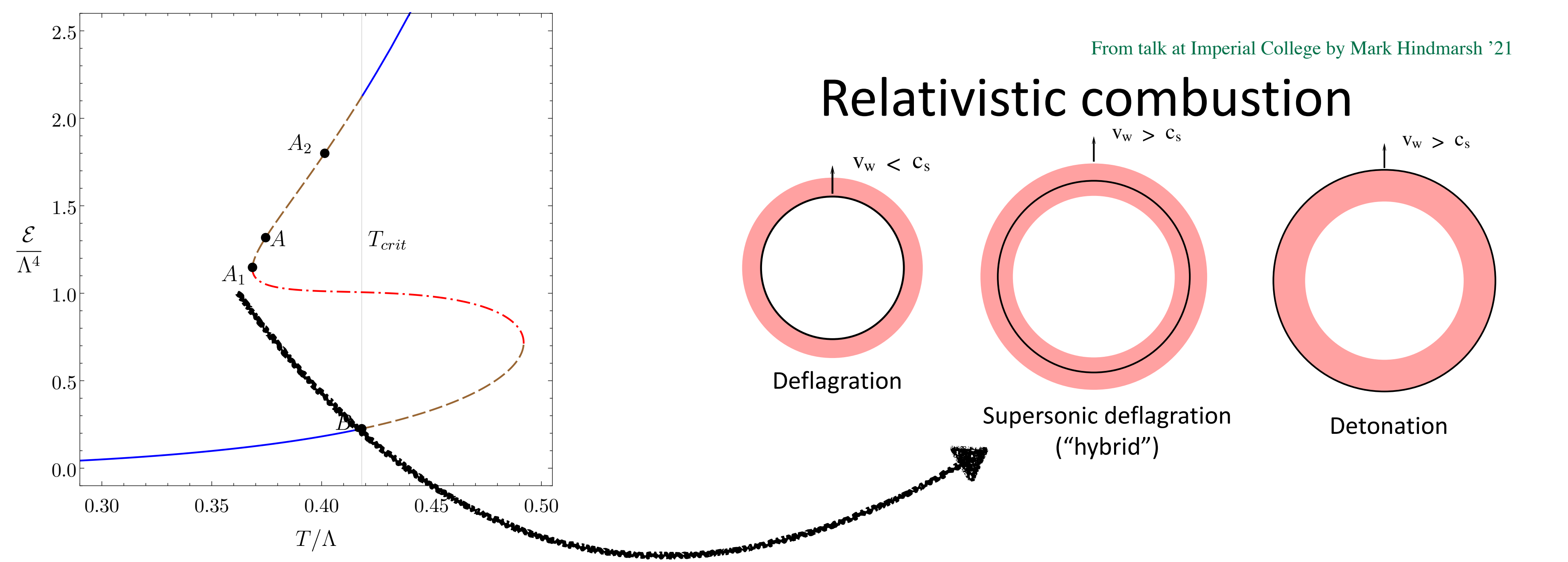
- Therefore v_{wall} grows and becomes supersonic w.r.t. A (but not w.r.t. to D):
  - For A_1 we have: $v_{wall} = 0.29$, $c_{s,A} = 0.12$, $c_{s,D} = 0.51$

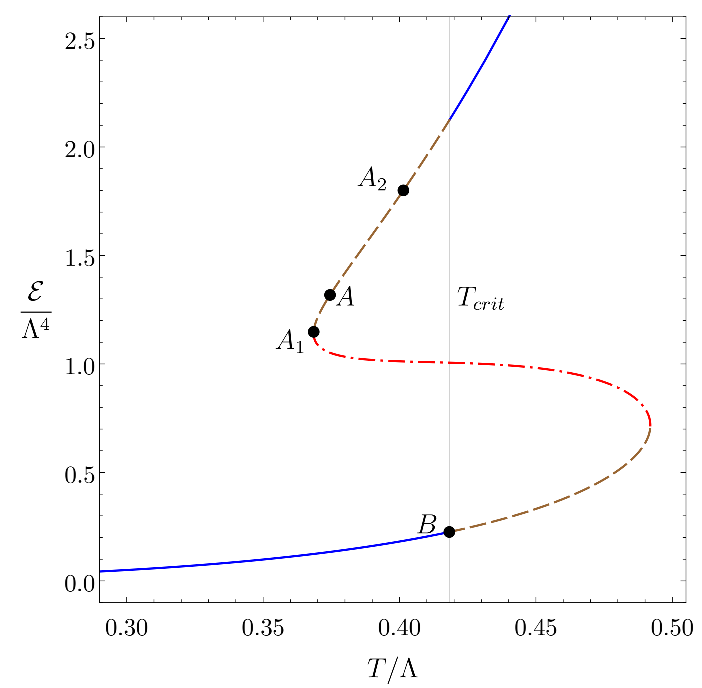


From talk at Imperial College by Mark Hindmarsh '21

Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

- In contrast, as A approaches T_C:
  - Pressure difference approaches zero.
  - Energy density in A increases.
  - $c_{s,A}$ approaches a finite value.
- Therefore v_{wall} approaches zero:
  - For A_2 we have: $v_{wall} = 0.085$, $c_{s,A} = 0.493$, $c_{s,D} = 0.508$



Bubble dynamics

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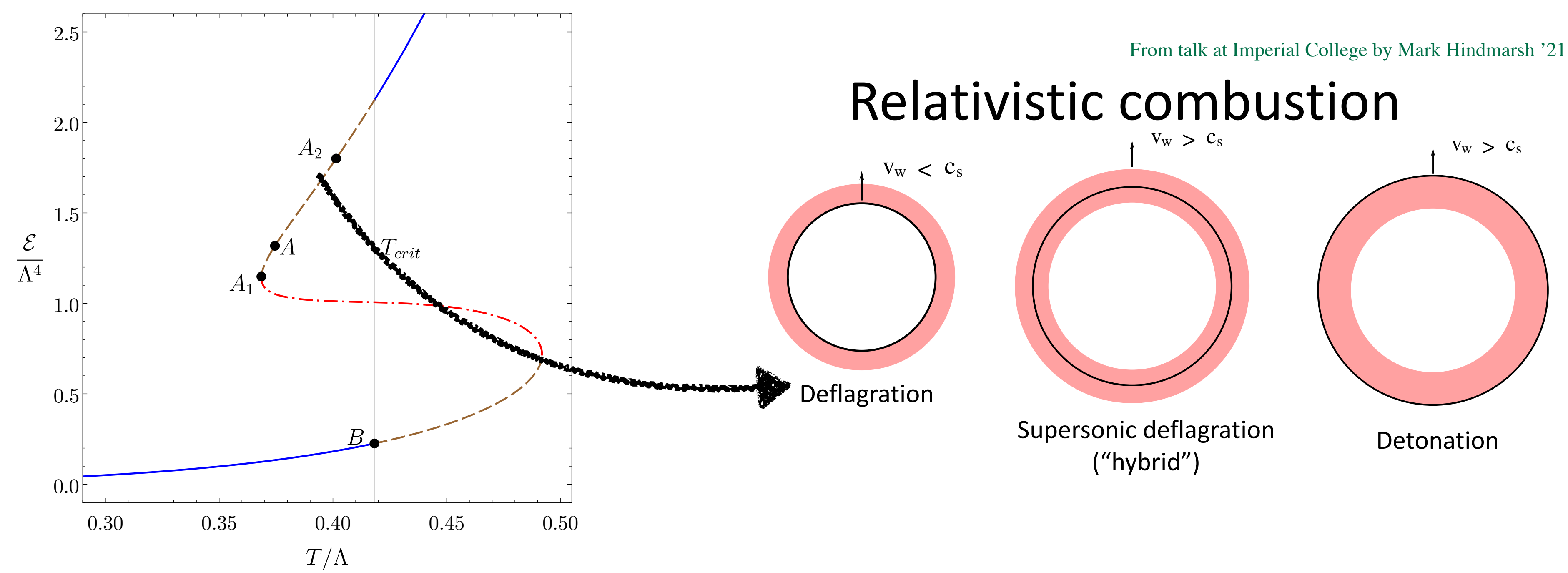
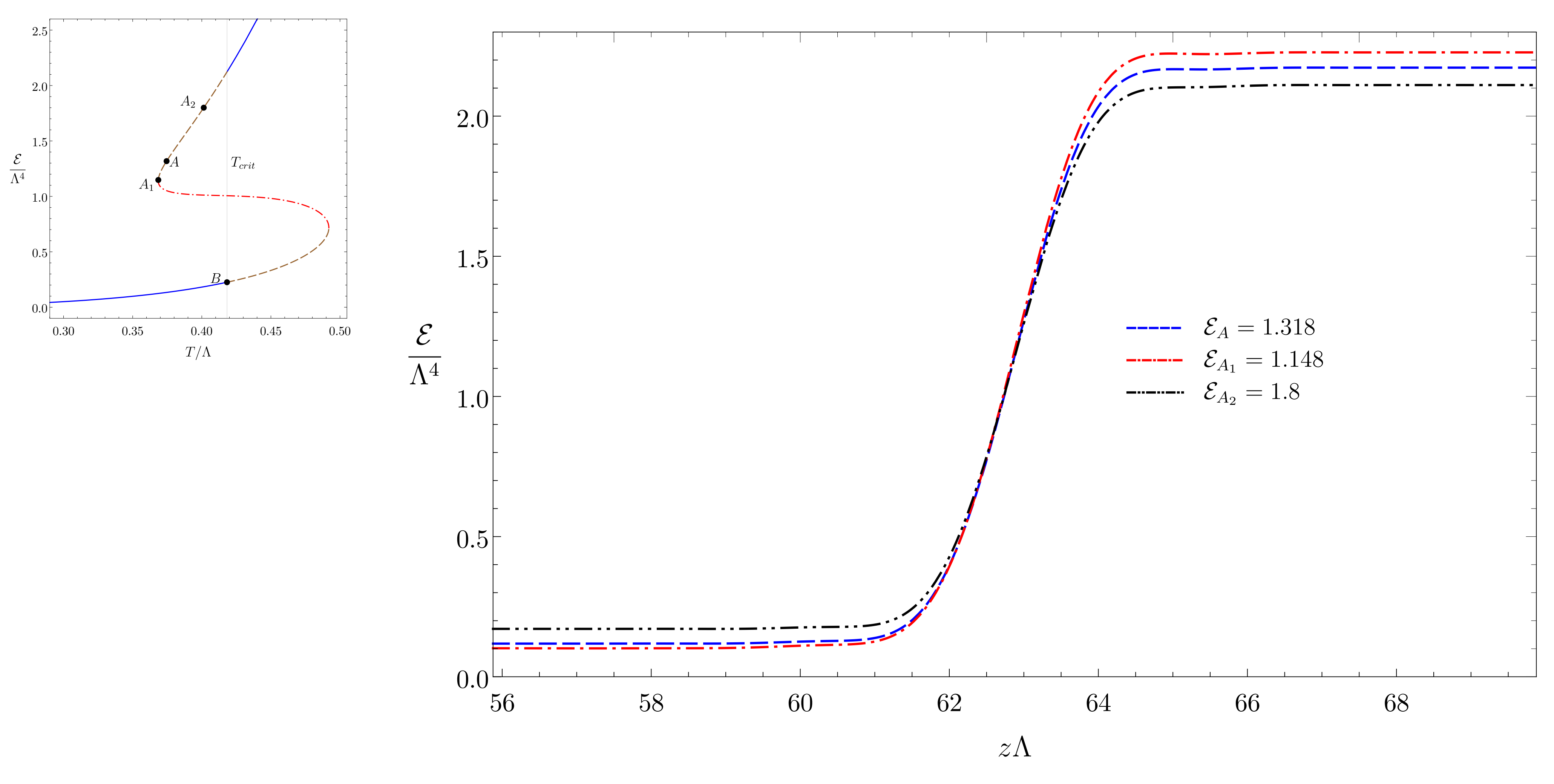


Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

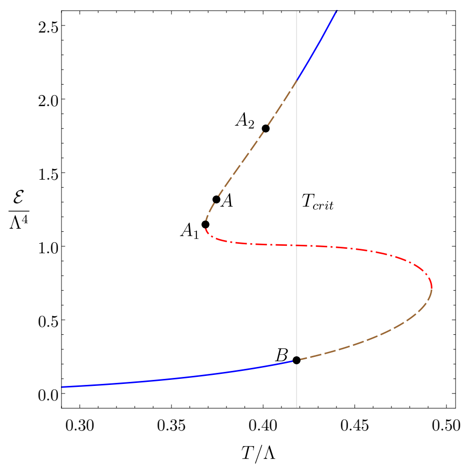
- What about the wall profile?

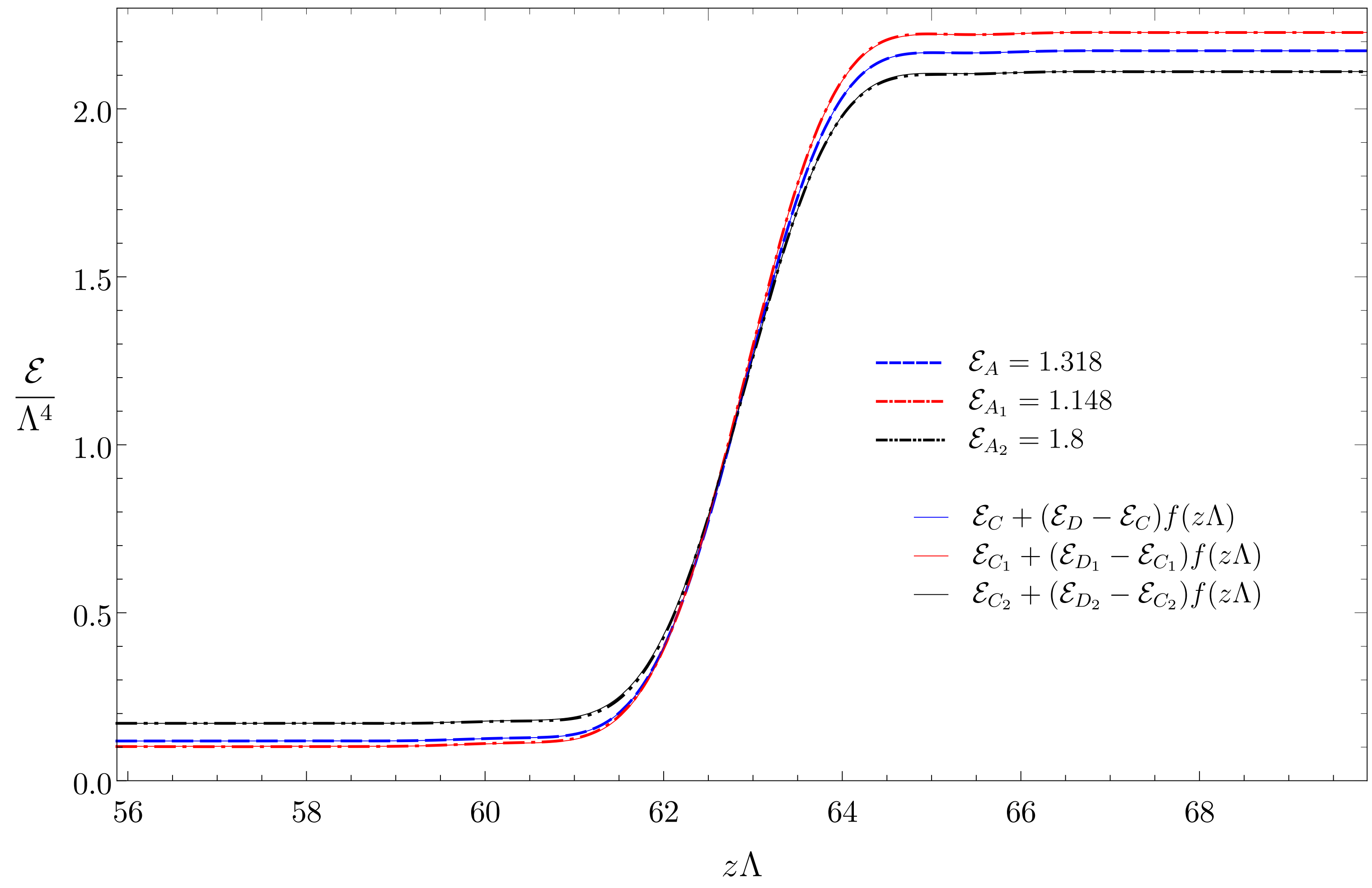


Bubble dynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

- What about the wall profile?
- It is universal (up to rescalings):

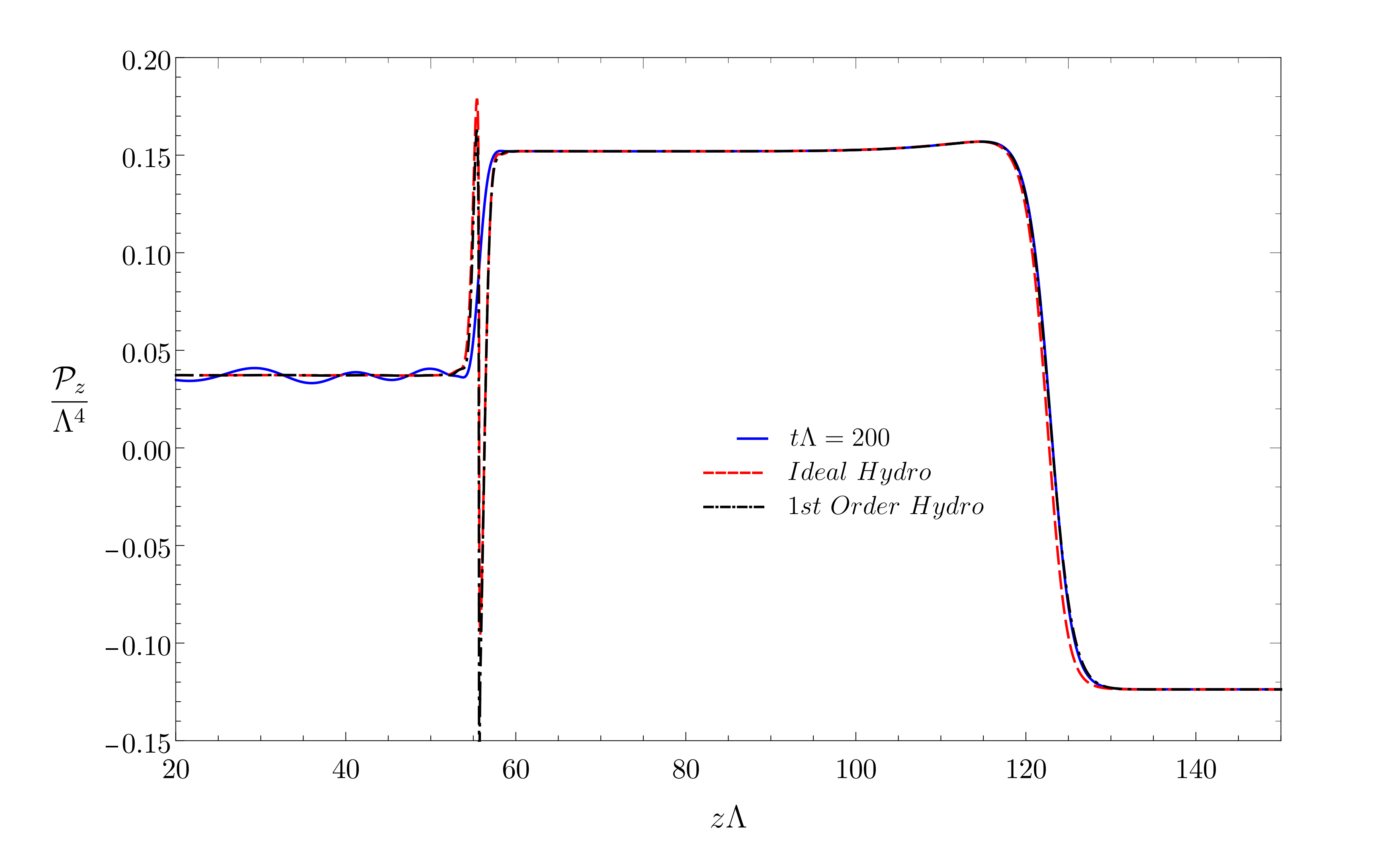




Hydrodynamics

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (to appear)

- As expected, everything but the wall is well described by ideal hydro.
- First-order hydro improves the description away from the wall.
- Second-order hydro in ~ 2 weeks.



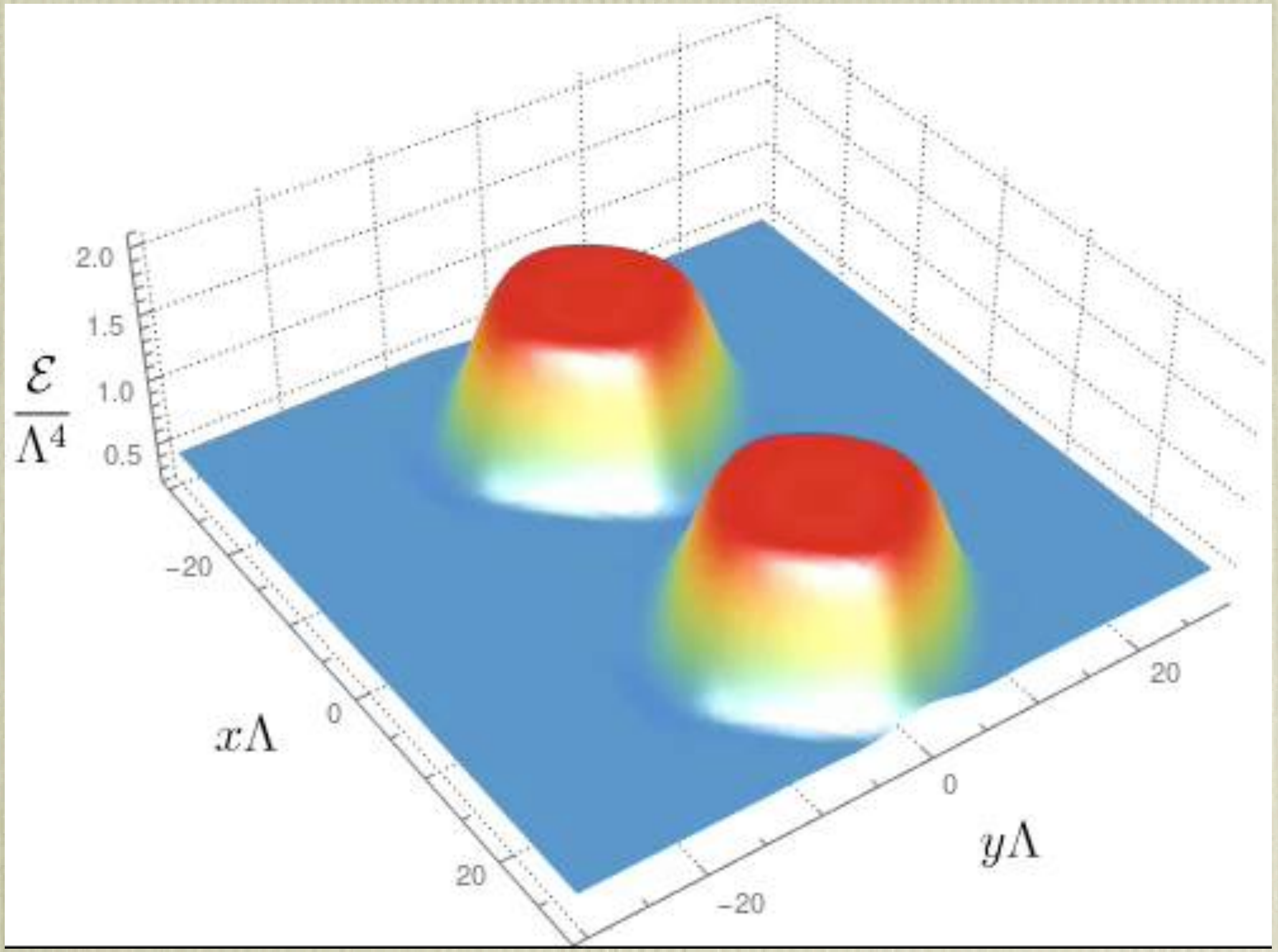
Spherical bubbles

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (in progress)

Spherical bubbles

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (in progress)

- More precisely, circular domains in too-small a box:



Outlook

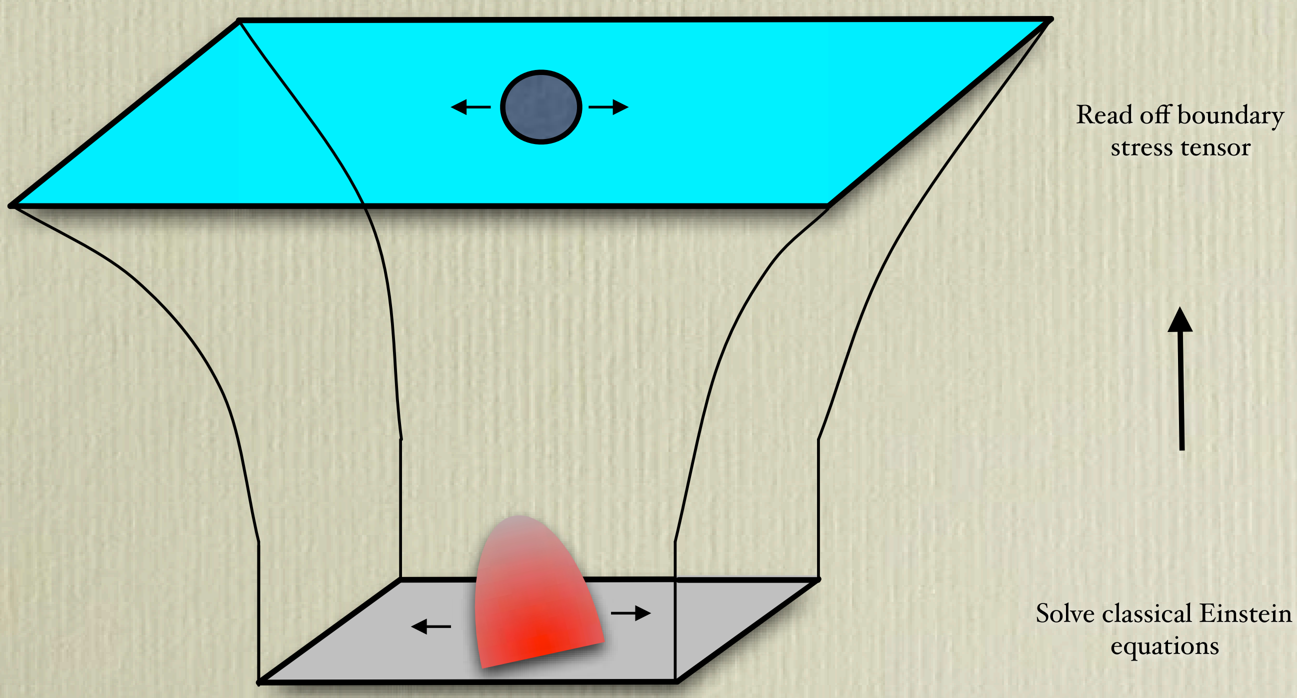
Outlook

Bea, Casalderrey, D.M., Giannakopoulos, Sanchez-Garitaonandia & Zilhao (in progress)

- In the near future holography will allow a direct calculation of the GW spectrum in strongly coupled theories *with a gravity dual.*

- All post-nucleation dynamics are included:

    - Bubble expansion.
    - Bubble collisions.
    - Sound modes.
    - Turbulence.
    - Etc.



Thank you

Evolution described by 2nd-order hydrodynamics

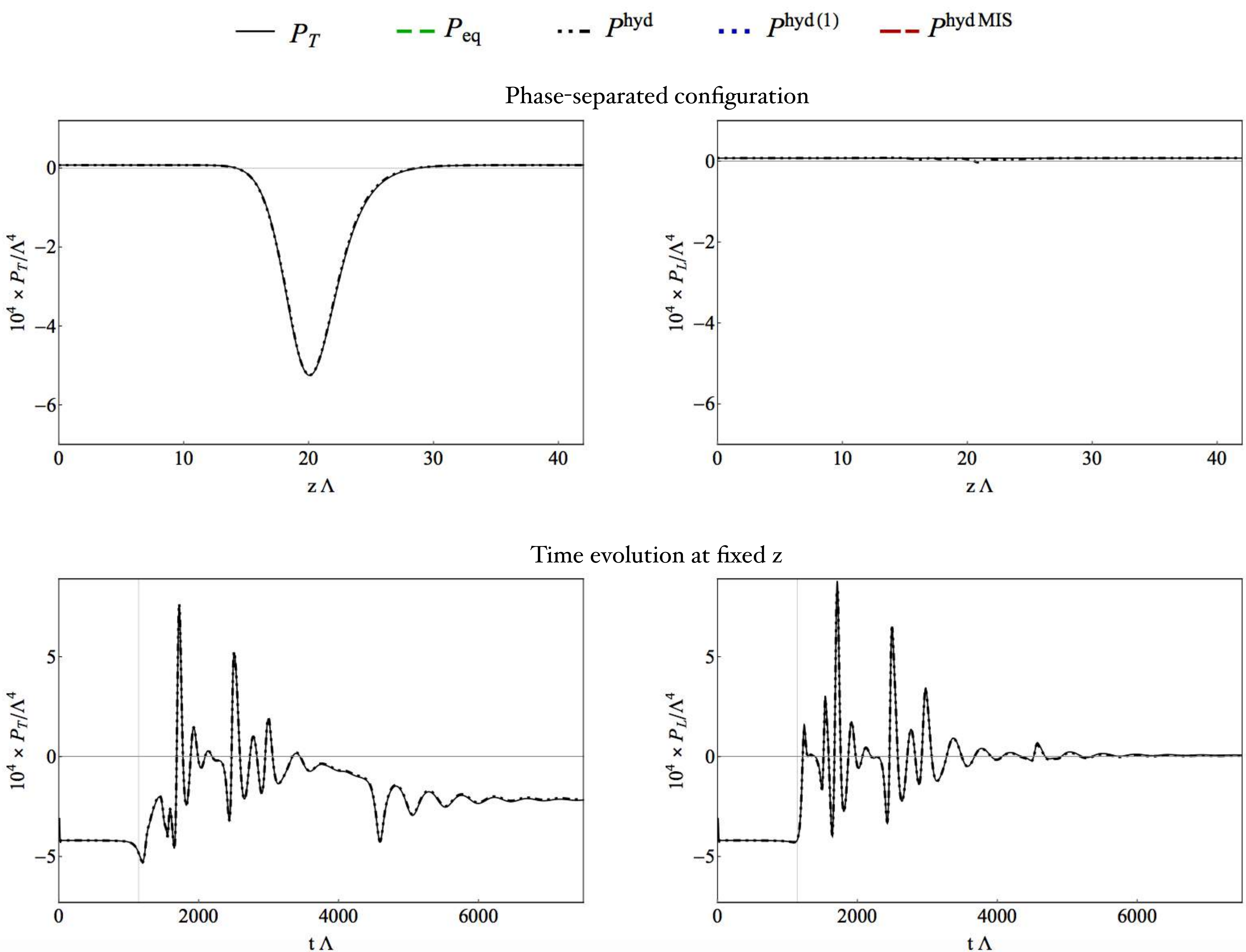
Attems, Bea, Casalderrey, D.M., Triana & Zilhao ’17
Attems, Bea, Casalderrey, D.M. & Zilhao ’19

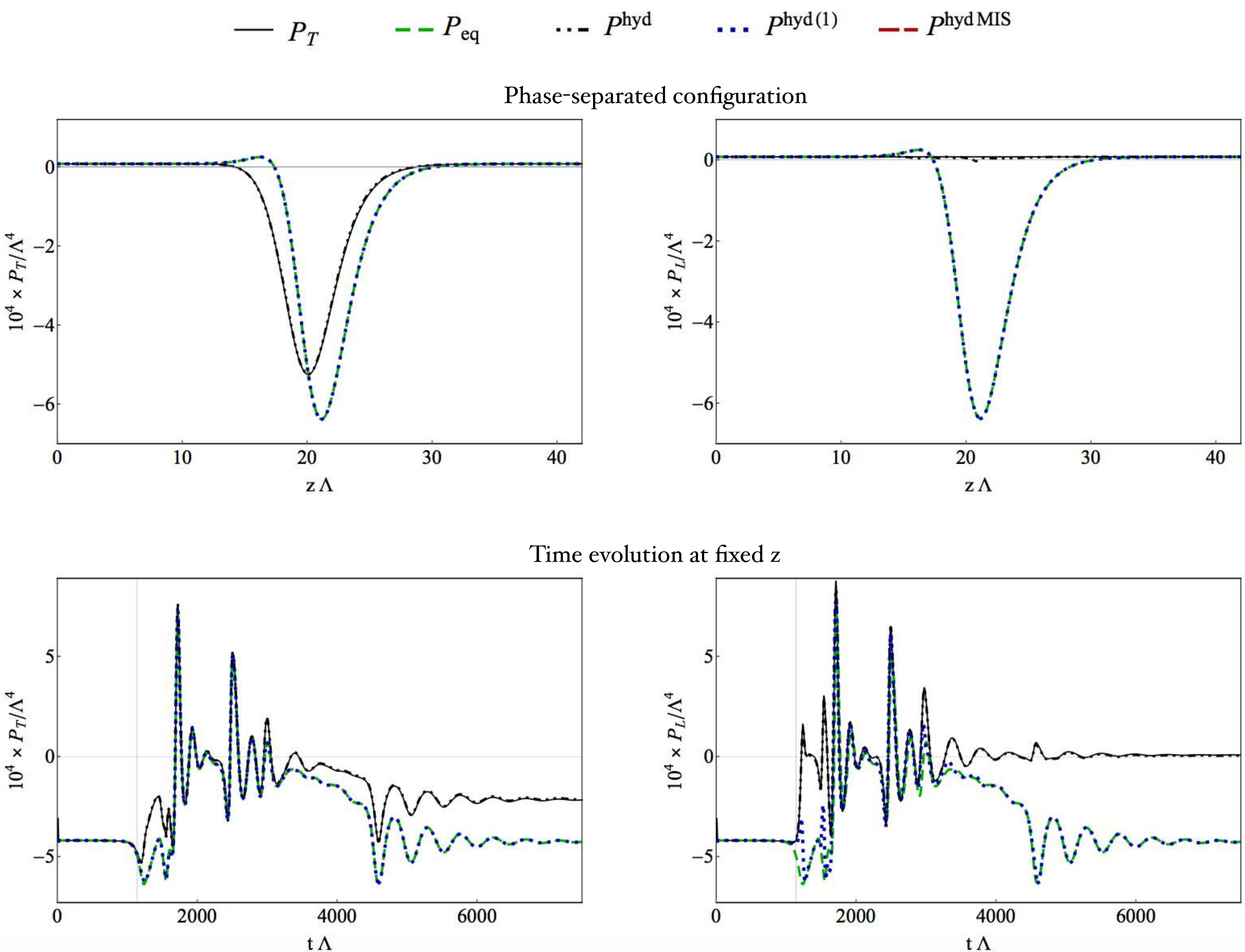
bulk & shear viscosities

↓

$$T_{\mu\nu} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial^2_{\text{spatial}}$$

“Purely spatial formulation”


P_T
P_{eq}
P^{hyd}
$P^{hyd\,(1)}$
$P^{hyd\,MIS}$
Phase-separated configuration
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
z Λ
Time evolution at fixed z
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
t Λ
t Λ



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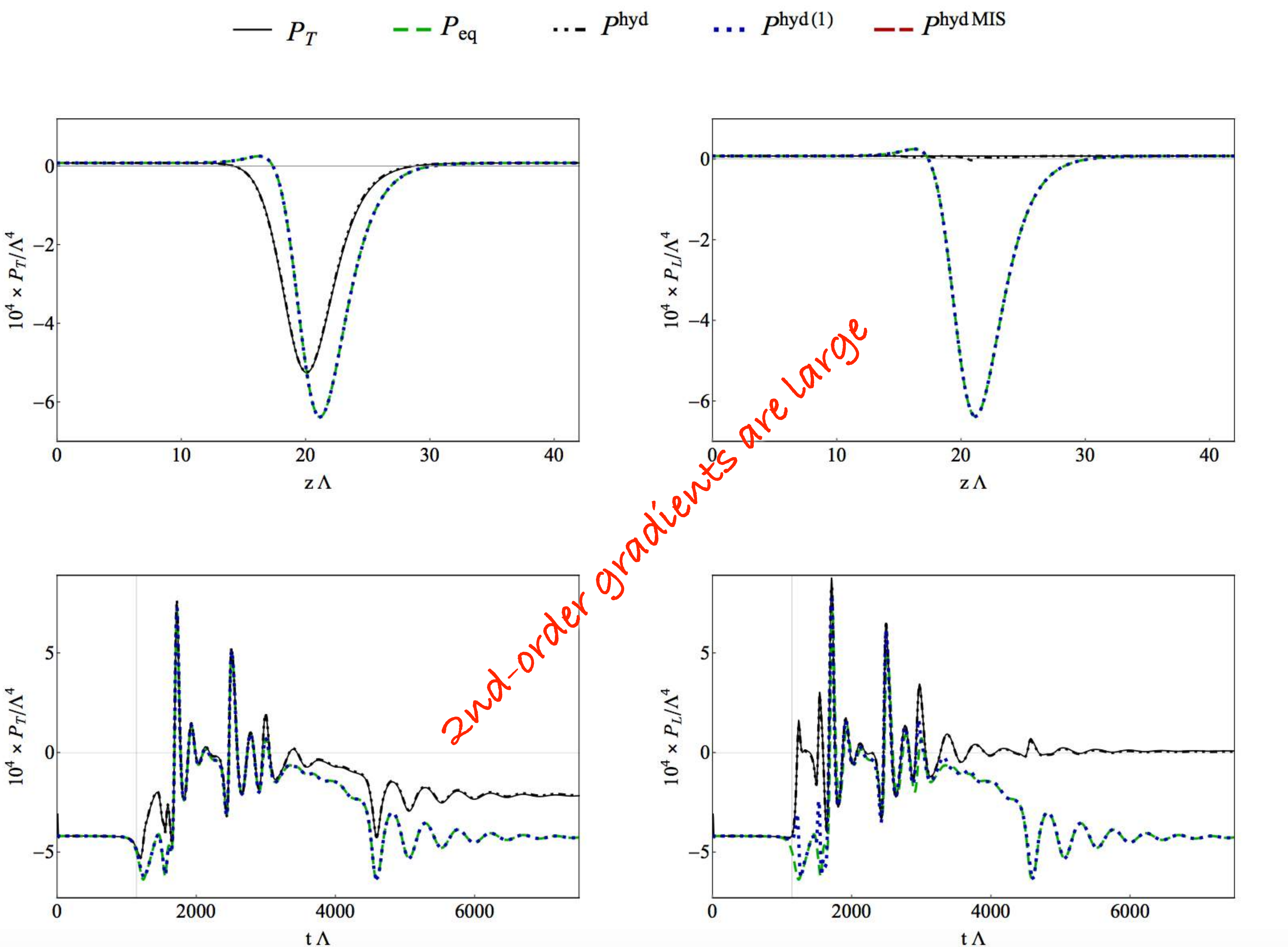

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2nd-order gradients are large

Evolution described by 2nd-order hydrodynamics

Attems, Bea, Casalderrey, D.M., Triana & Zilhao '17
Attems, Bea, Casalderrey, D.M. & Zilhao '19

- We are not doing time evolution, just checking constitutive relations.

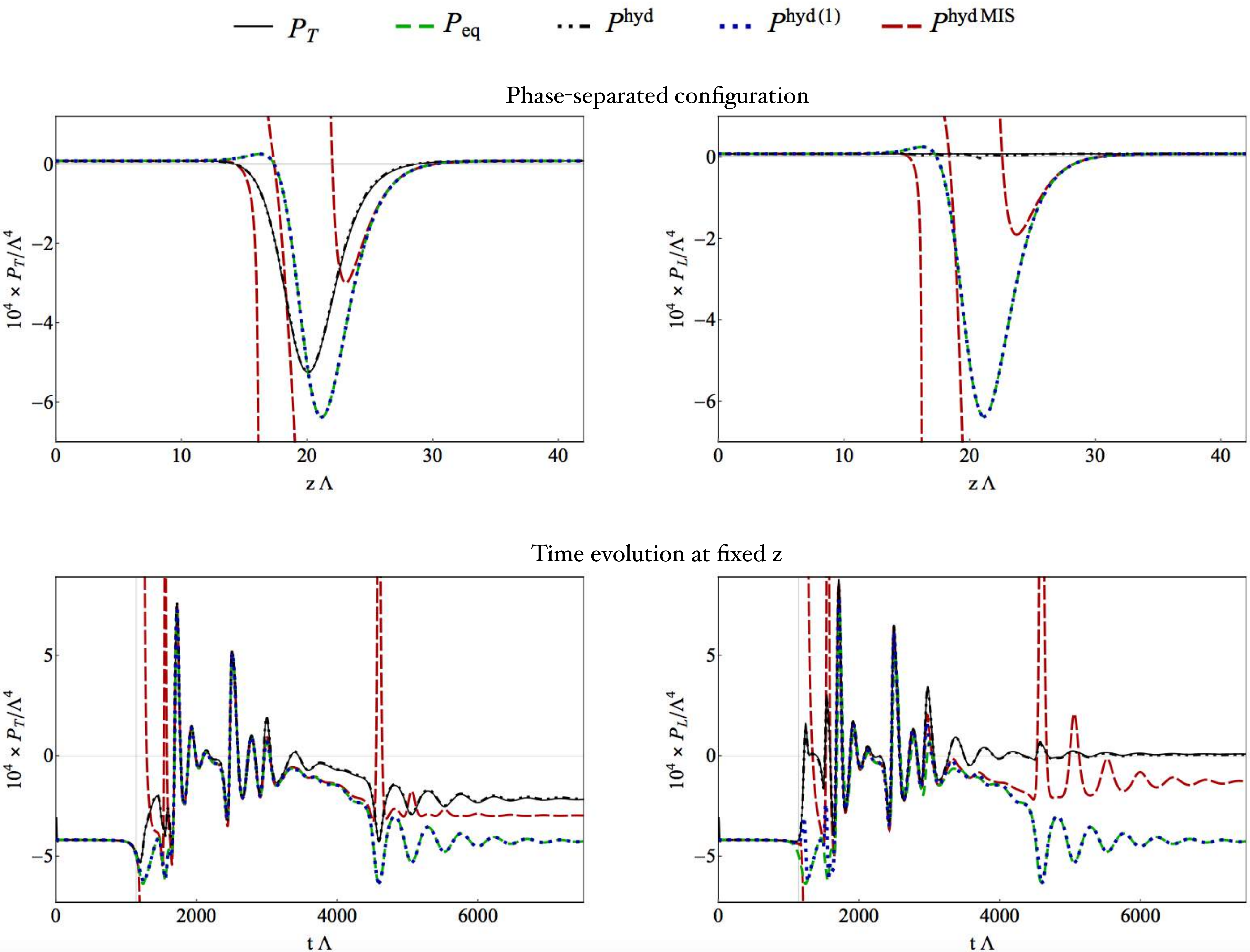
- Problem for time evolution: Hydrodynamics is acausal.

$$T_{\mu\nu} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial^2_{\text{spatial}}$$

- One fix (Muller-Israel-Stewart): Use lower oder equations to get:

$$T_{\mu\nu}^{\text{MIS}} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial_{\text{spatial}}\partial_{\text{time}}$$

- Produces equivalent descriptions if gradients are small, but not in our case.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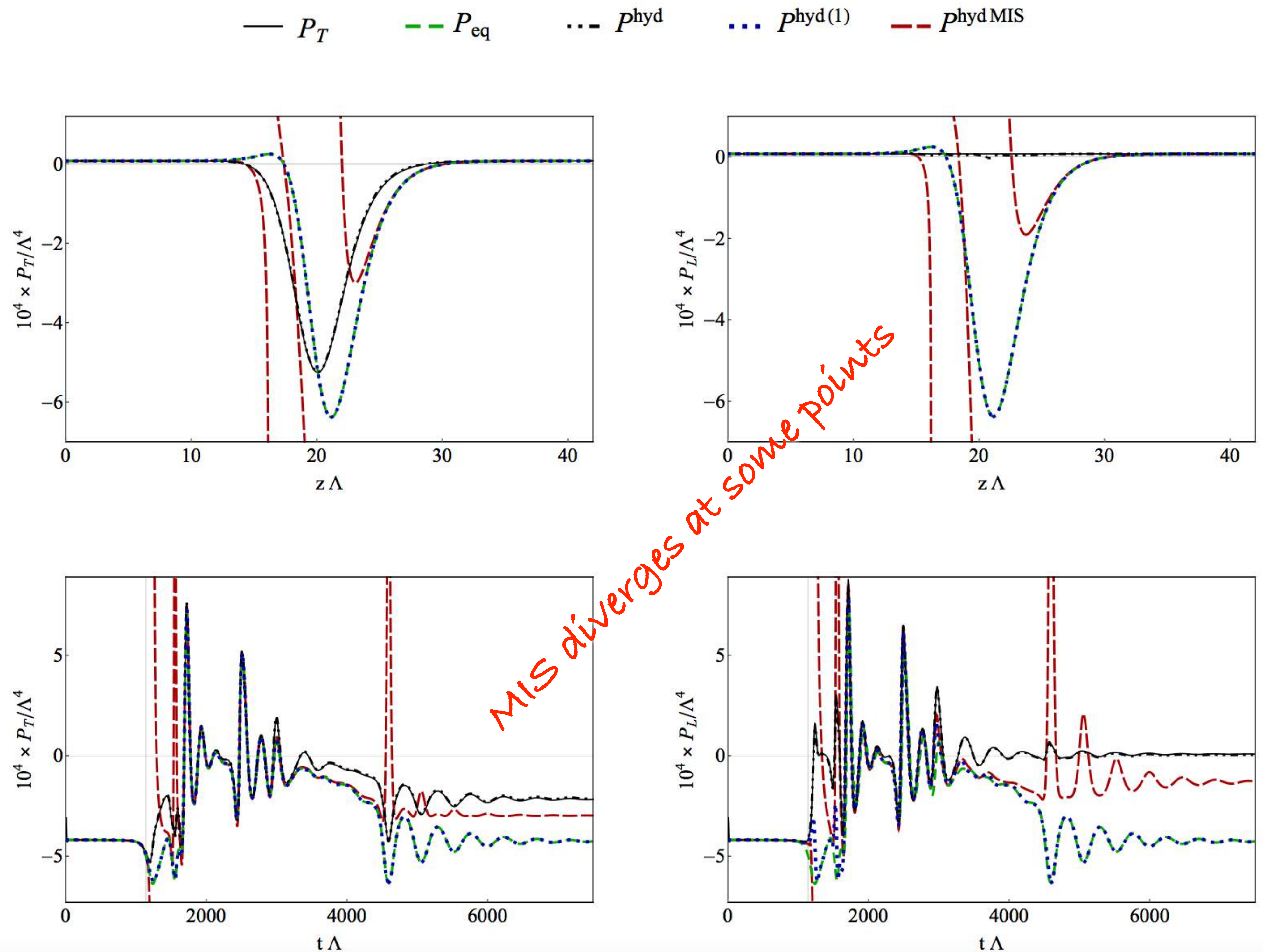
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t Λ
MIS diverges at some points

Purely spatial coefficients are smooth and finite

Attems, Bea, Casalderrey, D.M., Triana & Zilhao '17
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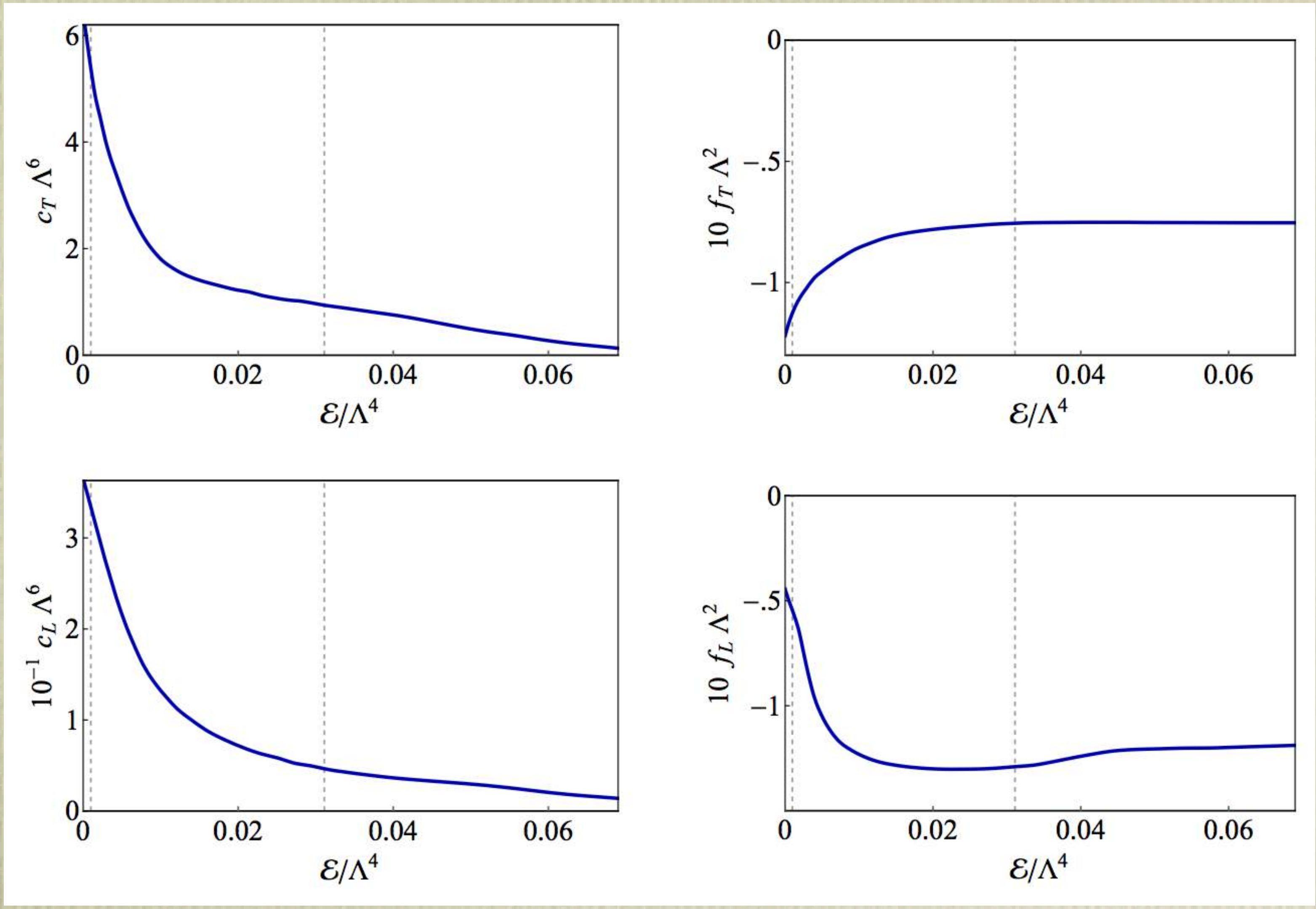

MIS coefficients diverge at points where $c_s=0$

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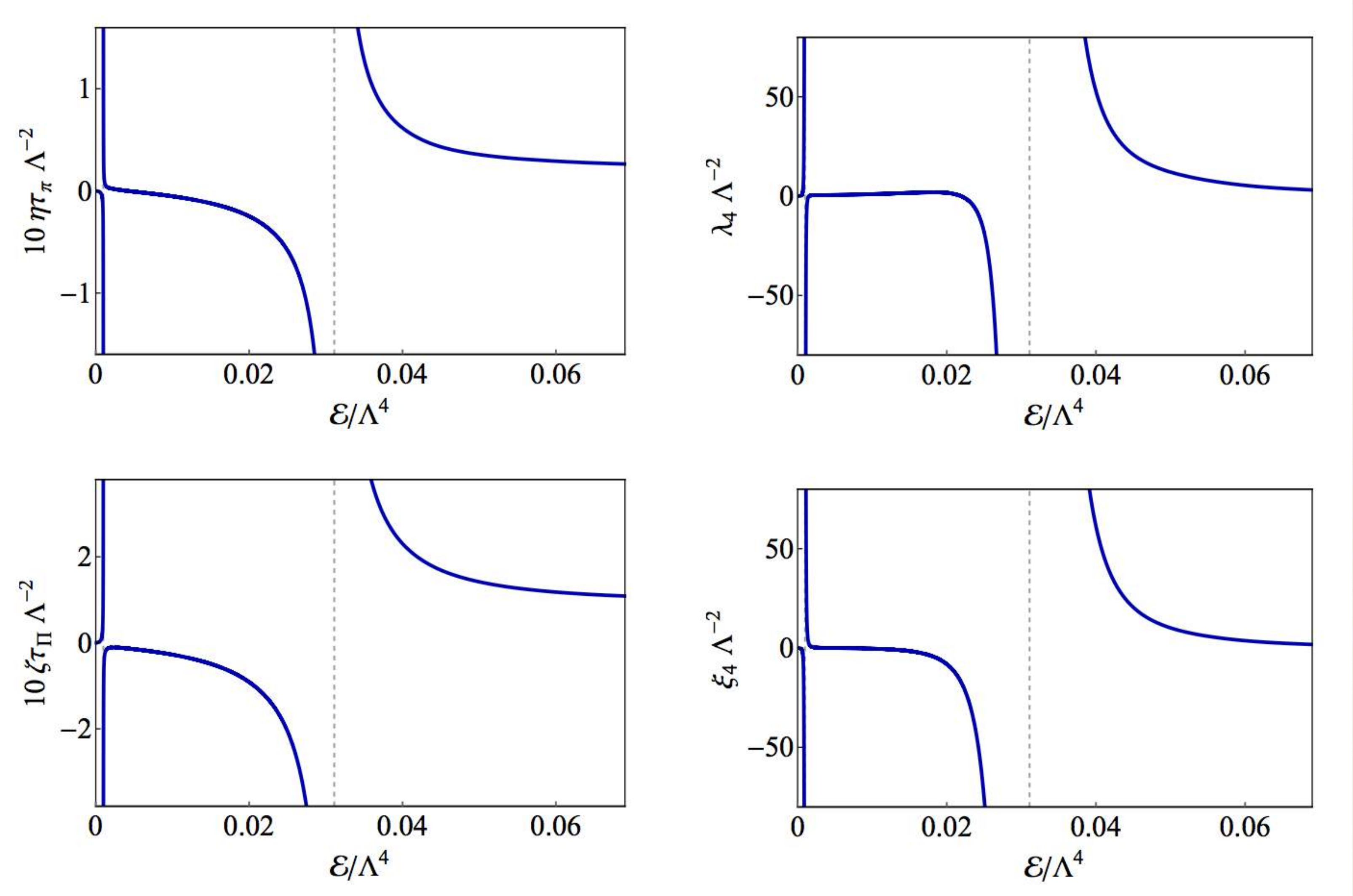

Change of basis involves powers of $1/c_s$