Quantum Gravity in the Lab

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HoloTube

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It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

Outline

- Holography meets quantum simulation – Quantum Gravity in the Lab

Three (quick) stories from this emerging field:

- Quantum gravity as a source of surprising phenomena
  - Traversable wormholes and size winding – arXiv:1911.06314 with A. Brown, H. Gharibyan, S. Leichenauer, H. Lin, S. Nezami, G. Salton, L. Susskind, and M. Walter
- Towards efficient algorithms
  - Sparse SYK model – arXiv:2008.02303 with S. Xu, Y. Su, and L. Susskind
- Towards quantum simulations of cosmology
  - Microstate cosmology – arXiv:1810.10601 with S. Cooper, M. Rozali, M. Van Raamsdonk, C. Waddell, and D. Wakeham and arXiv:1907.06667 with S. Antonini

What is quantum gravity in the lab?

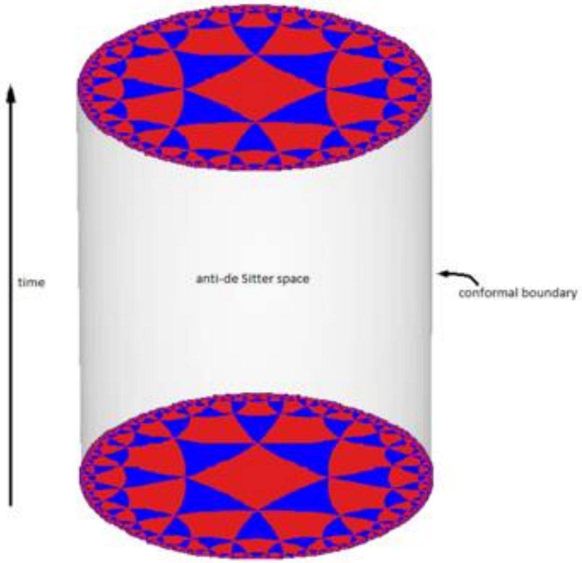
- To be honest, we’re not sure all that we want to mean by this phrase
  - We certainly hope to one day experimentally probe the quantum physics of the “naturally occurring” gravitational field
  - But we’d also like to include experiments with engineered quantum systems that exhibit emergent gravitational degrees of freedom
- Vague guiding principle: any system with a simple description in terms of a quantum spacetime geometry is a kind of quantum gravity
  - This is very likely the case for our own universe
  - But it can also be true for table-top experimental systems
  - Probably want other conditions, e.g. presence of black holes

Holographic duality

Universal quantum computer

[Maldacena, Witten, Gubser-Klebanov-Polyakov]

Experimental simulation of quantum gravity*



*in certain kinds of universes

Simulating holographic quantum gravity

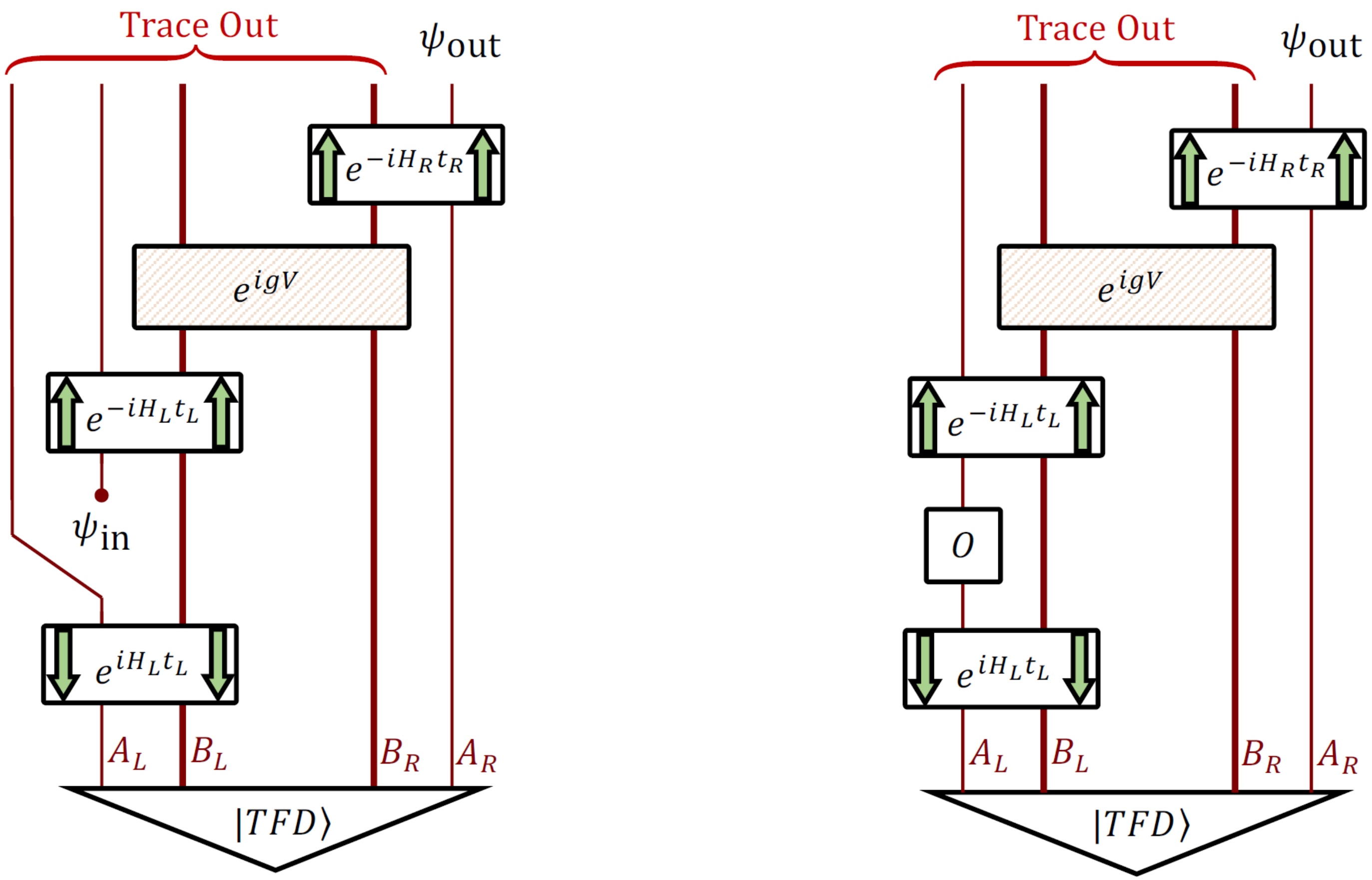
- For this talk, I’ll stick to a narrow sense of quantum gravity in the lab
  - Necessarily involves holography, the “boundary” is the thing we directly construct in the lab while the “bulk” is the quantum gravity
  - Not directly related to the gravity that holds us down and moves the planets (even if we hope to learn lessons that are more generally applicable)
- In this narrow sense, there are thus investigations that do not qualify, despite involving all three of quantum, gravity, and labs
  - Example: analog simulations of Hawking radiation
  - Such investigations are interesting, but not what I will talk about here

Quantum gravity as a source of surprising phenomena

arXiv:1911.06314 with A. Brown, H. Gharibyan, S. Leichenauer, H. Lin, S. Nezami, G. Salton, L. Susskind, and M. Walter; part 2 at arXiv:2102.01064 in coordination with Schuster et al. arXiv:2102.00010

Quantum gravity inspired experiments

(a) State transfer $\quad$ (b) Operator transfer

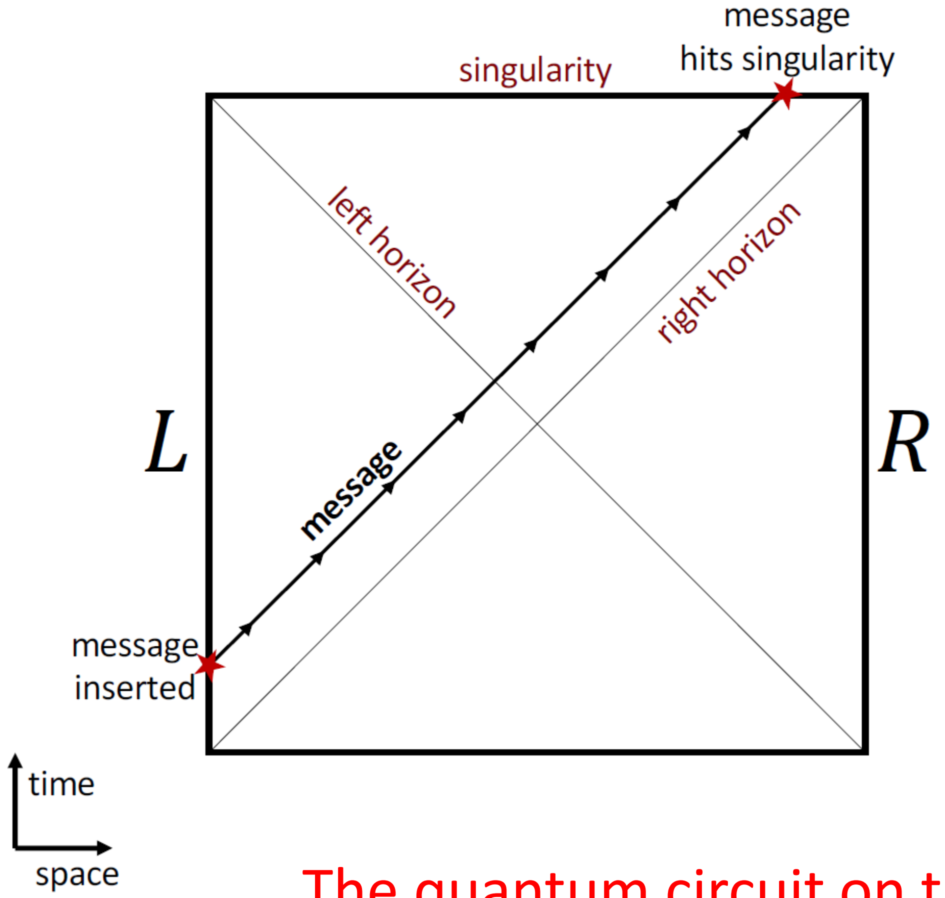


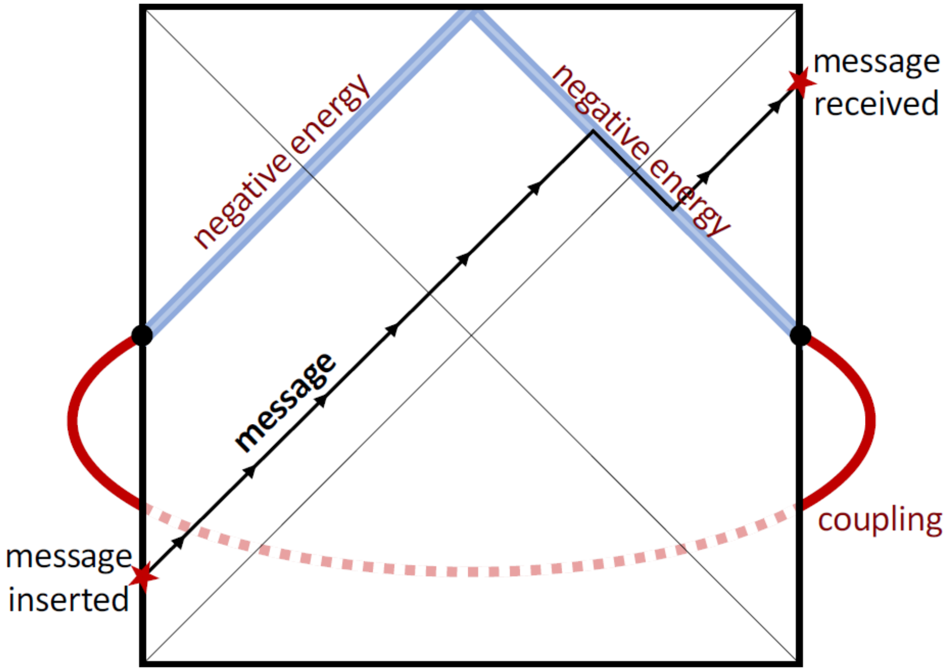
$$V = \frac{1}{n-m} \sum_{i \,\in\, \text{carrier qubits}} Z_i^L Z_i^R$$

weak L-R coupling

Inspiration: traversable wormholes

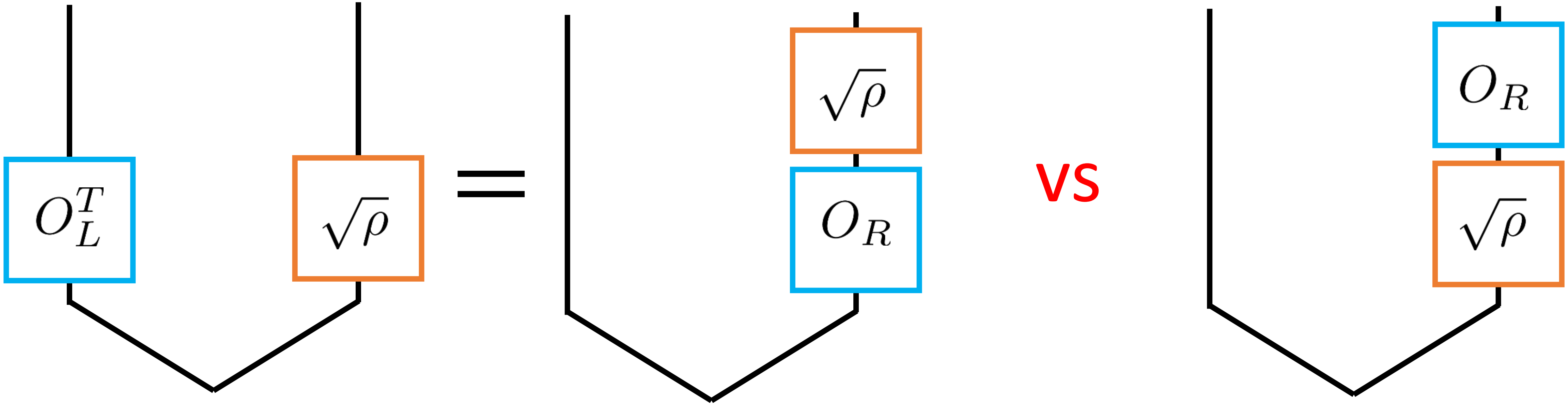
[Gao-Jafferis-Wall, Stanford-Maldacena-Yang, …]





The quantum circuit on the previous page really corresponds to a family of quantum phenomena, only some of which have a simple bulk interpretation

Operator on left vs right



$$\frac{1}{2^{n/2}} O_L^T(-t)\,|\mathrm{TFD}\rangle_{LR} = (\rho_\beta^{1/2})_R O_R(t)\,|\phi^+\rangle_{LR}$$

$$O_R(t)(\rho_\beta^{1/2})_R\,|\phi^+\rangle_{LR}$$

Size winding

- $\sqrt{\rho}O$ is generically non-Hermitian when O is Hermitian
- Consider expansion in Pauli string basis:

$$\sqrt{\rho}O = \sum_P c_P P \qquad g(\ell) = \sum_{P,|P|=\ell} c_P^2$$

- Perfect size winding ansatz:

$$\sqrt{\rho}O = \sum_P e^{i\alpha|P|/n} r_P P, \quad r_P \in \mathbb{R}$$

Size winding

- Operator action in Pauli basis assuming perfect size winding

$$O_L^T(-t)\,|\mathrm{TFD}\rangle = \sum_P e^{i\alpha|P|/n} r_P\,|P\rangle\,,$$
$$O_R(t)\,|\mathrm{TFD}\rangle = \sum_P e^{-i\alpha|P|/n} r_P\,|P\rangle\,.$$

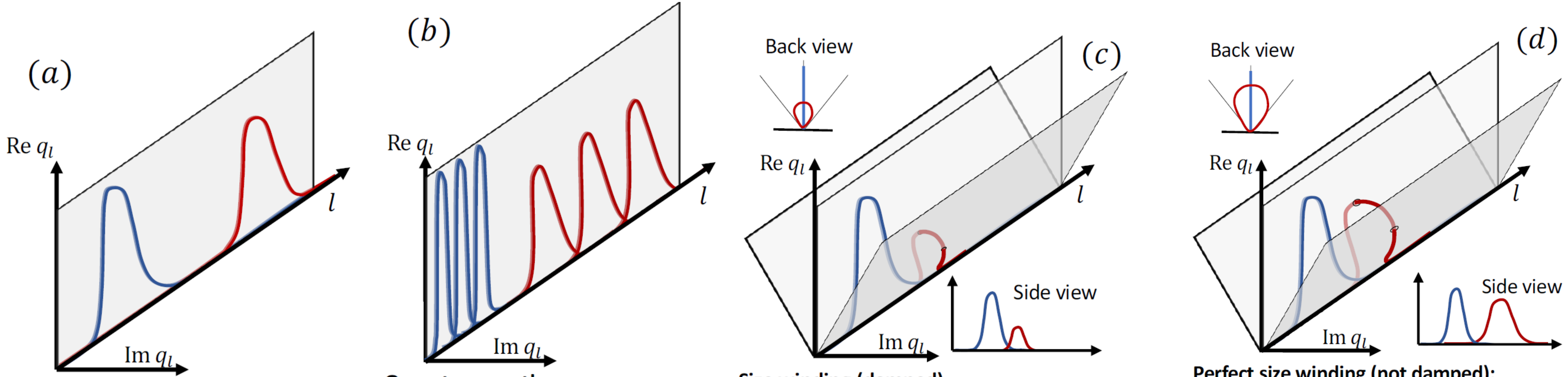
- Action of left-right coupling:

$$e^{igV}\,|P\rangle \simeq e^{-i(4/3)g|P|/n}\,|P\rangle$$

- An SYK calculation shows size winding at low temperature

[Brown-…-Nezami-Lin-…-S, see also Gao-Jafferis]

Different regimes



Operator growth:
Late time most quantum system. Teleports one qubit through state transfer mechanism. $Sign(g)$ independent. Infinite temperature.

Operator growth:
Intermediate time. Chaotic spin chains, random circuits. Teleports many qubit through state transfer mechanism. $Sign(g)$ independent. Infinite temperature.

Size winding (damped):
Low temperature. Teleports through *state transfer* mechanism. Weak $Sign(g)$ dependence. Works weakly for "operator transfer".

Perfect size winding (not damped):
Low temperature. Could teleport through *state* transfer mechanism. Strong $Sign(g)$ dependence, but limited fidelity. Works perfectly for "operator transfer" slightly before scrambling time. Strong signature of a ***geometrical wormhole***.

Towards efficient algorithms

arXiv:2008.02303 with S. Xu, Y. Su, and L. Susskind

Sachdev-Ye-Kitaev model (q-fermion)

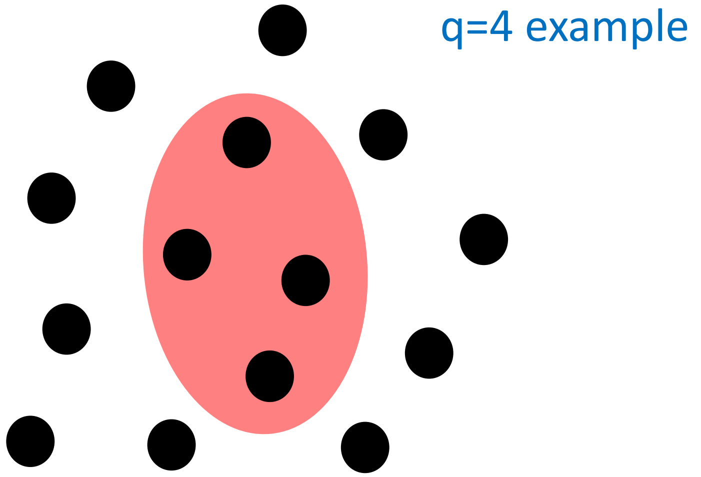
$$\chi_a^\dagger = \chi_a, \; a = 1, \cdots, N$$

$$\{\chi_a, \chi_b\} = \delta_{a,b}$$

$$\dim(\mathcal{H}) = 2^{N/2}$$

$$H = \frac{1}{q!} \sum_A J_{a_1 \cdots a_q} \chi_{a_1} \cdots \chi_{a_q}$$

$$\overline{J_A} = 0, \; \overline{J_A^2} = \frac{(q-1)! J^2}{N^{q-1}}$$



[random 2-body ensemble, Sachdev-Ye, Kitaev, Rosenhaus-Polchinski, Maldacena-Stanford, Garcia-Garcia-Verbaarschot, Bagrets-Altland-Kamenev, …]

Quantum simulation – full SYK

- Direct Trotterization is very expensive [Garcia-Alvarez et al.]

$$\mathcal{O}\left(N^{10}(Jt)^2/\epsilon\right)$$

- "asymmetric qubitization" gives current best result for simulating fully connected SYK (q=4) [Babbush-Berry-Neven, Low-Chuang]

$$\mathcal{O}\left(N^{7/2}Jt + N^{5/2}Jt\,\mathrm{polylog}(N/\epsilon)\right)$$

far more complex than suggested by holographic complexity proposals

- Analog simulations may also be possible using cold atoms, graphene flakes, … [Danshita-Hanada-Tezuka, Pikulin-Franz, …]

Sparse SYK (pruned model, q=4 example)

$$H_p = \frac{1}{4!} \sum_{abcd} J_{abcd} x_{abcd} \chi_a \chi_b \chi_c \chi_d$$

$$\overline{J_{abcd}} = 0, \ \overline{J_{abcd}^2} = \frac{3! J^2}{p N^3}$$

$$x_{abcd} \in \{0, 1\}$$

$$\Pr(x_{abcd} = 1) = p$$

$$N_{\text{terms}} = p \binom{N}{4} \equiv kN$$

Sparse SYK (pruned model, q=4 example)

$$H_p = \frac{1}{4!} \sum_{abcd} J_{abcd} x_{abcd} \chi_a \chi_b \chi_c \chi_d$$

$$N_{\text{terms}} = p \binom{N}{4} \equiv kN$$

Fully connected limit:

$$N \to \infty,\ k \sim N^3$$

Sparse limit:

$$N \to \infty,\ k \sim \text{fixed}$$

Quantum simulation – sparse SYK

- Most efficient quantum program to holographically simulate gravity?

prior best: $\mathcal{O}\left(N^{7/2}Jt + N^{5/2}Jt\,\mathrm{polylog}(N/\epsilon)\right)$ [Babbush-Berry-Neven, Low-Chuang]

- Our new results:
  - asymmetric qubitzation

$$\mathcal{O}\left(\left(N^{q+3}/q!\right)^{1/2}\mathcal{J}t\right) \longrightarrow \mathcal{O}(k^{3/2}N^2\log N\mathcal{J}t). \qquad \mathcal{J} = \sqrt{q}\frac{J}{2^{(q-1)/2}}.$$

  - order p product formulas (order one k and q)

$$\mathcal{O}\left(\epsilon^{-\delta}N^{1+\delta}\ln N(\mathcal{J}t)^{1+\delta}\right) \qquad \delta = \frac{1}{p}$$

Path integral approach

Generalize to q-fermion interaction:

$$\overline{Z^n} = \int \prod_{\alpha} D\chi^\alpha \exp\left(-\frac{1}{2} \int d\tau \sum_{a,\alpha} \chi_a^\alpha \partial_\tau \chi_a^\alpha + \frac{(q-1)!}{2pN^{q-1}} \sum_{A,\alpha,\beta} x_A \Phi_A^{\alpha\beta} \right)$$

$$\Phi_A^{\alpha\beta} = J^2 \int d^2\tau \chi_{a_1}^\alpha(\tau)\chi_{a_1}^\beta(\tau') \cdots \chi_{a_q}^\alpha(\tau)\chi_{a_q}^\beta(\tau')$$

Lagrange multiplier fields:

$$g_a^{\alpha\beta}(\tau_1,\tau_2) = \chi_a^\alpha(\tau_1)\chi_a^\beta(\tau_2) \qquad I_E \supset \frac{1}{2} \sum_{a,\alpha,\beta} \int d^2\tau g_a^{\alpha\beta} \sigma_a^{\alpha\beta}$$

Saddle points: kq-regular hypergraph model

$$(g_a^{-1})^{\alpha\beta} = -i\omega\delta^{\alpha\beta} - \sigma_a^{\alpha\beta} \qquad \sigma_a^{\alpha\beta} = \frac{J^2}{kq}\frac{1}{(q-1)!}\sum_{a_2\cdots a_q} x_{aa_2\cdots a_q} g_{a_2}^{\alpha\beta}\cdots g_{a_q}^{\alpha\beta}$$

$$\frac{1}{(q-1)!}\sum_{a_2\cdots a_q} x_{aa_2\cdots a_q} = kq$$

hypergraph regularity condition

Uniform replica-diagonal saddle point: $g_s^{-1} = -i\omega - \sigma_s \qquad \sigma_s = J^2 g_s^{q-1}$

Quadratic fluctuations (I)

$$I_{E,2} \supset \sum_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} g_s^{\alpha_2\alpha_3} g_s^{\alpha_4\alpha_1} \delta\sigma^{\alpha_1\alpha_2} \delta\sigma^{\alpha_3\alpha_4}$$

replica structure

$$I_{\text{eff}} = -\frac{1}{2}\sum_a \ln\det(\partial_\tau - \sigma_a) + \frac{1}{2}\int d^2\tau \left[\sum_a g_a\sigma_a - \sum_{abcd} \frac{J^2}{4!4k} x_{abcd} g_a g_b g_c g_d\right]$$

focus on a single replica, convenient parameterization of fluctuations

$$|g(\tau_1,\tau_2)|^{-1}\delta g_a(\tau_1,\tau_2) = g_a(\tau_1,\tau_2) - g_s(\tau_1,\tau_2)$$

$$|g(\tau_1,\tau_2)|\delta\sigma_a(\tau_1,\tau_2) = \sigma_a(\tau_1,\tau_2) - \sigma_s(\tau_1,\tau_2)$$

Quadratic fluctuations (II)

$$I_{\text{eff},2} = \frac{3J^2}{4} \sum_{a,b} \int d^4\tau \delta g_a(\tau_1, \tau_2) \left[\delta_{ab} K^{-1}(\tau_i) - \frac{M_{ab}}{12k} \delta(\tau_{13}) \delta(\tau_{24}) \right] \delta g_b(\tau_3, \tau_4)$$

usual full SYK kernel

fermion-fermion coupling matrix

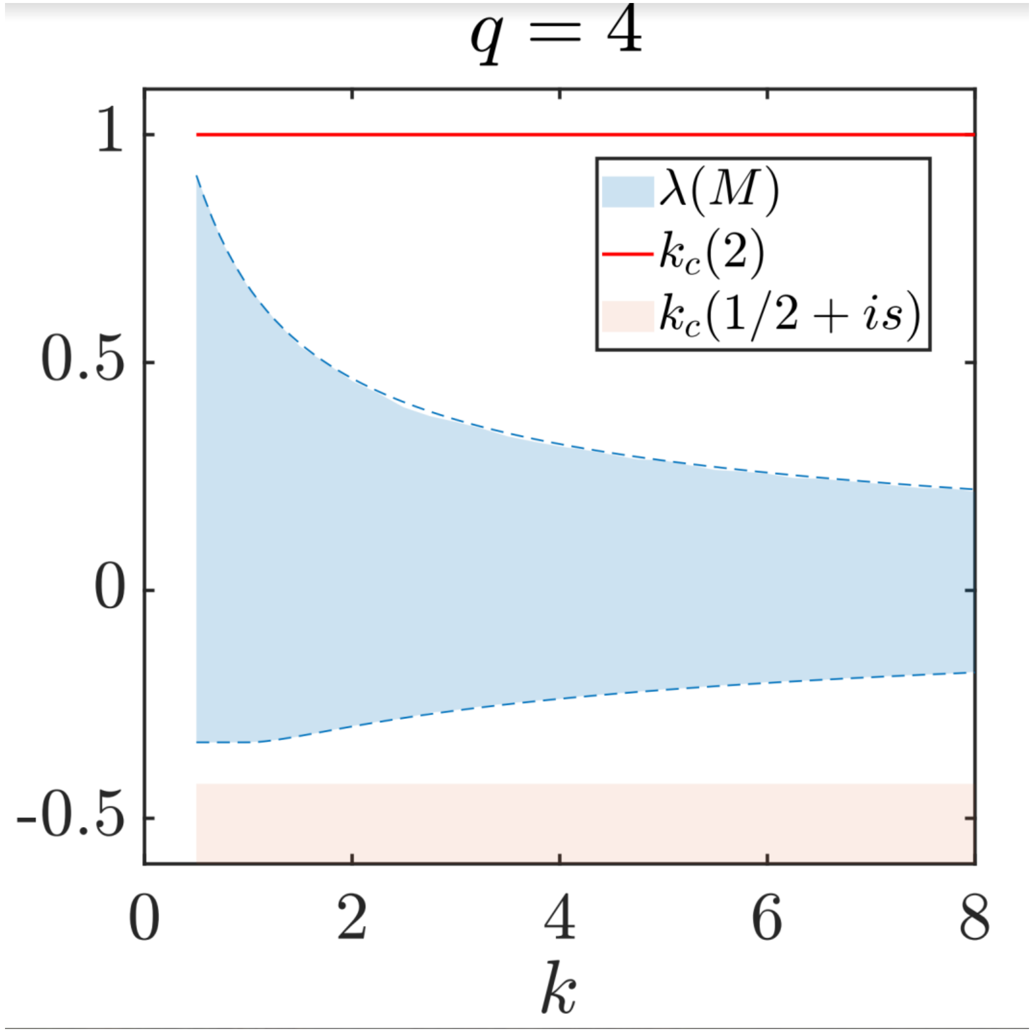
$$M_{ab} = \frac{1}{2} \sum_{cd} x_{abcd}$$

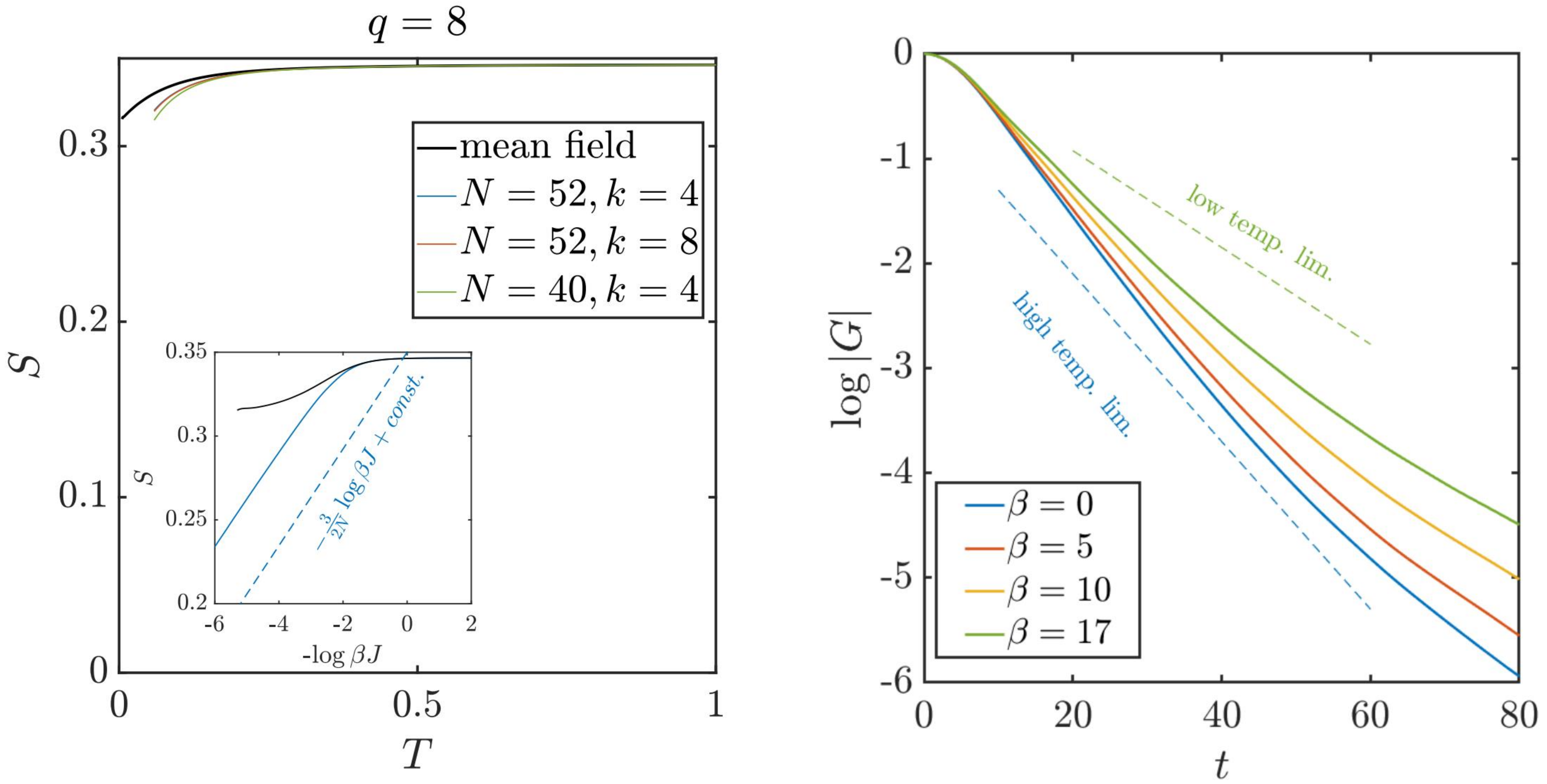
Regularity → M/(12k) has uniform eigenvector with eigenvalue 1

Randomness (over hypergraph) → spec(M) has a gap, uniform mode is isolated

Emergent gravity is still present

- At the quadratic level, only the Schwarzian mode is soft; all others are gapped
- Combined with numerical simulations, we have strong evidence that the sparse model continues to exhibit gravitational dynamics down to $k \sim 1/q$!





q=8: full model has roughly 750 million terms; sparse model has 208 or 416 terms

Towards cosmological simulations

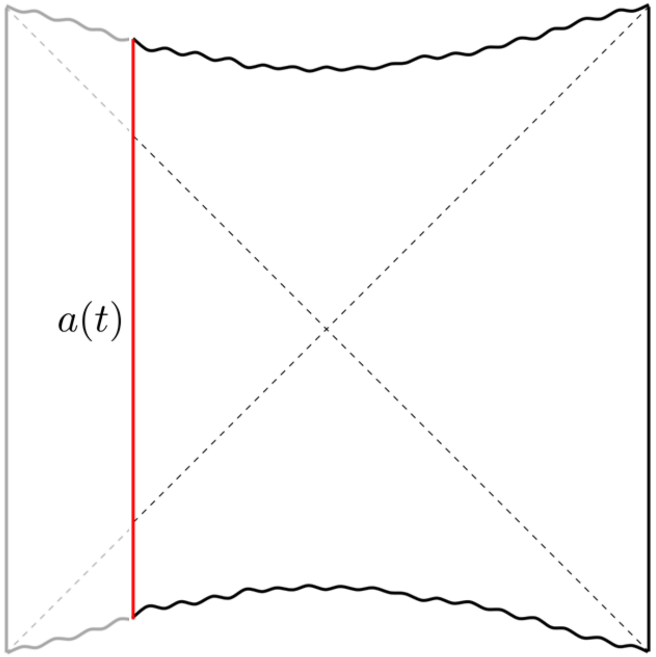
arXiv:1810.10601 with S. Cooper, M. Rozali, M. Van Raamsdonk, C. Waddell, and D. Wakeham and arXiv:1907.06667 with S. Antonini

Other universes?

- Many ideas related to flat space, de Sitter, but still early days
- One idea -- microscopic braneworld cosmology in AdS/CFT:

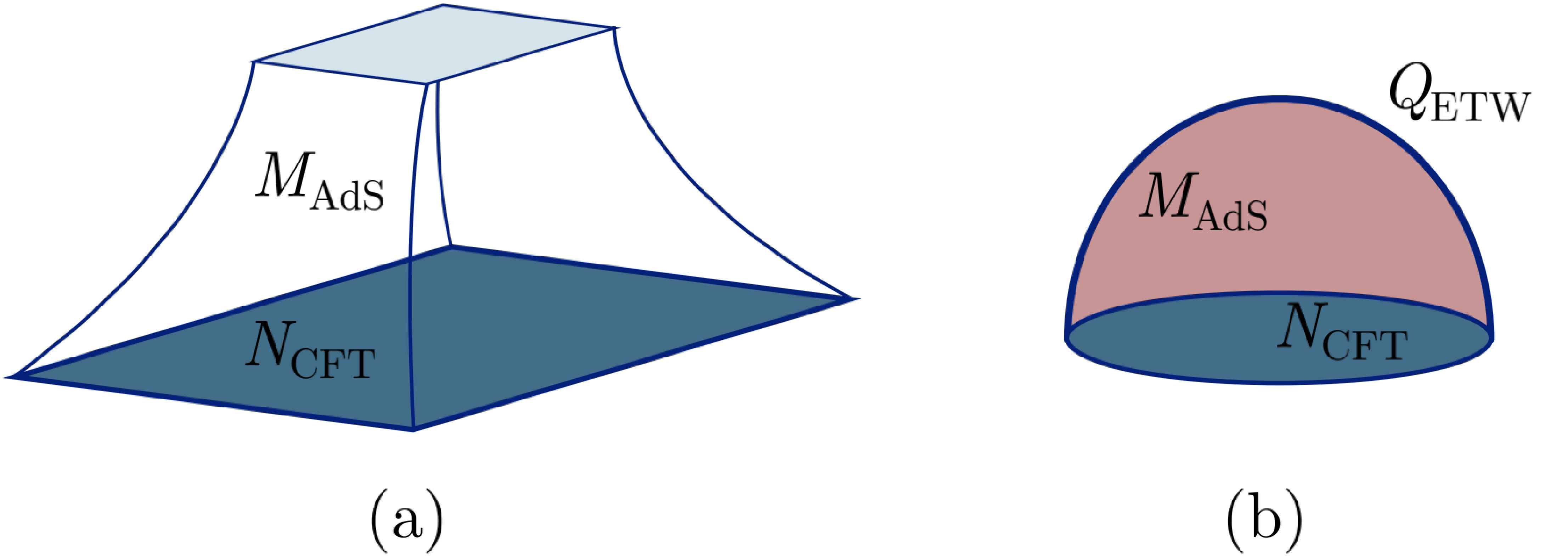
Randall-Sundrum braneworld (roughly):

- Our universe is a brane coupled to gravity in a higher dimensional bulk
- Bulk gravity spectrum has modes localized to the brane that approximate ordinary GR



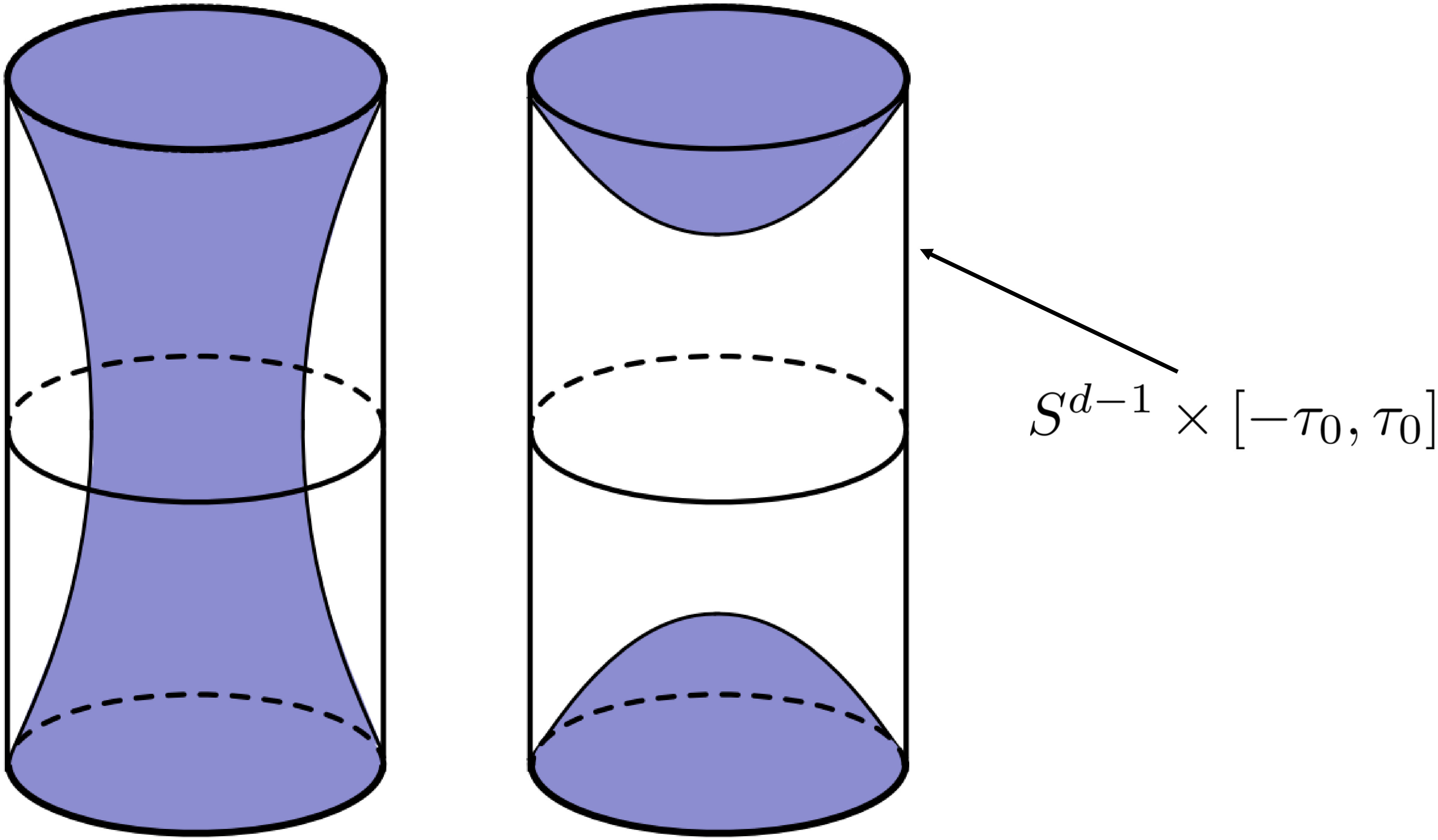
$$|\Psi\rangle = e^{-\tau_0 H}|B\rangle$$ boundary state

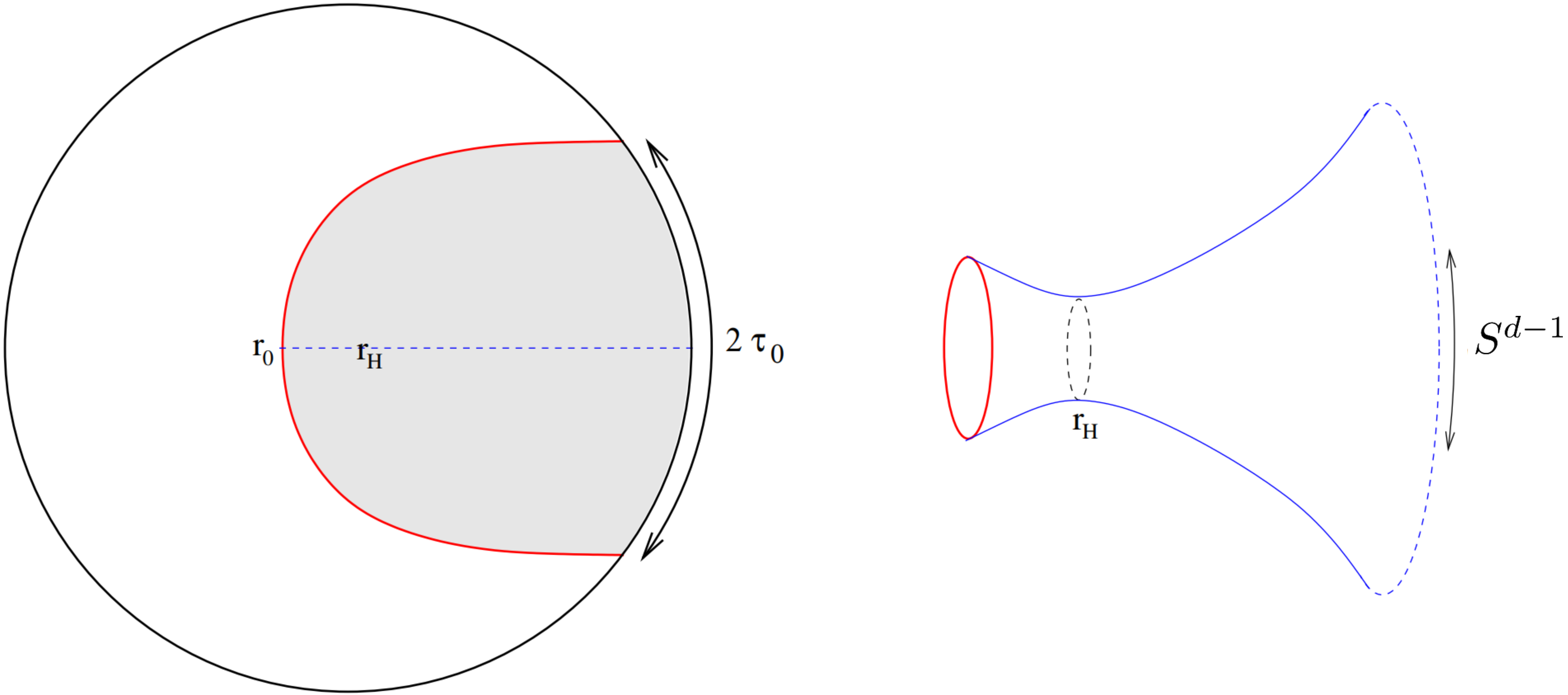
[Idea: Cooper-Rozali-S-Van Raamsdonk-Waddell-Wakeham, Antonini-S, TFD prep: Martyn-S, Wu-Hsieh, Cottrell-Freivogel-Hofman-Lokhande, Zhu-...-Monroe]



$$I_{\rm bulk}+I_{\rm ETW}=\frac{1}{16\pi G}\int_{N_{AdS}}\mathrm{d}^{d+1}x\,\sqrt{-g}(R-2\Lambda)+\frac{1}{8\pi G}\int_{Q_{\rm ETW}}\mathrm{d}^{d-1}y\,\sqrt{-h}(K-(d-1)T)$$

$$8\pi G T_{ab}=(1-d)T g_{ab}/L_{\rm AdS}\ ,$$


$S^{d-1} \times [-\tau_0, \tau_0]$



r_0
r_H
$2\,\tau_0$
r_H
S^{d-1}

Criteria for gravity localization

- Randall-Sundrum II: gravity localized on brane due to zero mode in a warped (AdS5) geometry

$$ds_5^2 = dz^2 + e^{-2A(z)} g_{\mu\nu}(x) dx^\mu dx^\nu \ , \qquad V(r) \approx \frac{GM}{r}\left(1 + \frac{2\ell^2}{3r^2}\right)$$

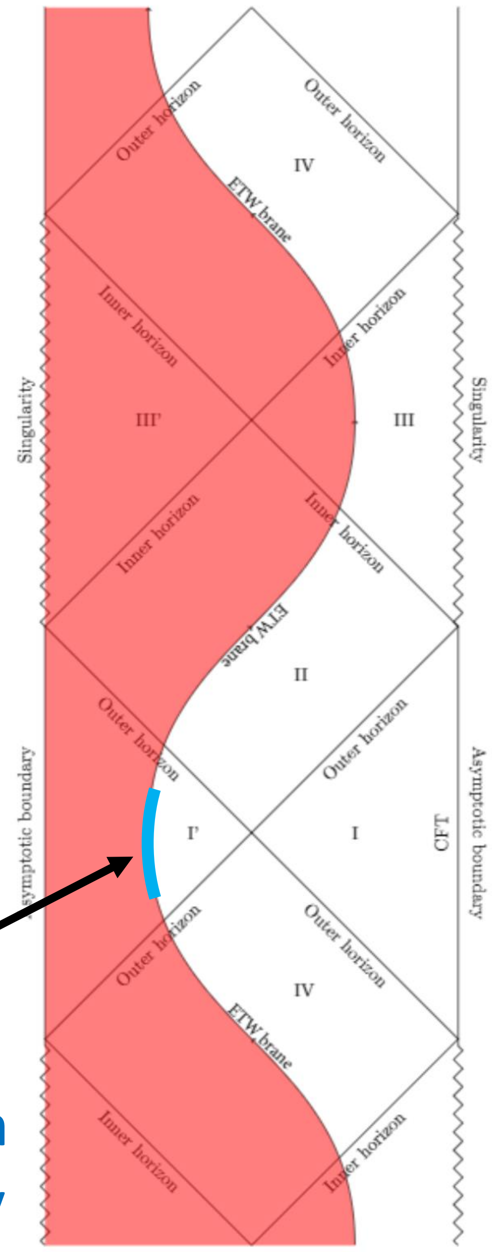
- However, localization is not perfect in general (e.g. Karch-Randall) and we have a black hole hole (e.g. Seahra-Clarkson-Maartens)
- Minimal requirements:
  - Brane position much bigger than black hole radius
  - FRW brane Hubble constant much smaller than inverse AdS radius

Need another ingredient

- Simplest constant tension brane solutions don't meet all criteria
  - Essentially, we seem to need a tension approaching one
  - Lorentzian solution looks fine
  - Euclidean solution only makes sense up to some tension < 1 (in d>2)
- It seems likely that one could do better in a more complex brane model, or perhaps by better understanding Euclidean situation
- w/ Stefano Antonini: constant tension brane in charged black hole background can approach T=1 by tuning the black hole near extremality

$$I_{ETW} = -\frac{1}{8\pi G}\int_{ETW} d^d x\sqrt{h}\,[K-(d-1)T] + I_{ETW}^{em}$$

$$\left(\frac{\dot{r}}{r}\right)^2 = -\frac{1}{r^2} + \frac{2\mu}{r^d} - \frac{Q^2}{r^{2d-2}} + \left(T^2 - \frac{1}{L_{AdS}^2}\right)$$



Can achieve approximate localization for a significant part of the trajectory

y_b	T_H	t_o	t_d	$(y')^2 / [f(y)]^2$	$\lambda(y_b)/\lambda_{tot}$
30	$1.762 \cdot 10^{-3}$	$9.172 \cdot 10^{-4}$	0.08370	$3.581 \cdot 10^{-6}$	0.9764
45	$2.698 \cdot 10^{-3}$	$9.190 \cdot 10^{-4}$	0.1880	$6.786 \cdot 10^{-7}$	0.9465
60	$3.716 \cdot 10^{-3}$	$9.229 \cdot 10^{-4}$	0.3342	$2.012 \cdot 10^{-7}$	0.9035
75	$4.875 \cdot 10^{-3}$	$9.282 \cdot 10^{-4}$	0.5226	$7.484 \cdot 10^{-8}$	0.8459
90	$6.282 \cdot 10^{-3}$	$9.350 \cdot 10^{-4}$	0.7516	$3.129 \cdot 10^{-8}$	0.7713
105	$8.178 \cdot 10^{-3}$	$9.432 \cdot 10^{-4}$	1.026	$1.356 \cdot 10^{-8}$	0.6748
120	0.01124	$9.530 \cdot 10^{-4}$	1.339	$5.495 \cdot 10^{-9}$	0.5462
135	0.01895	$9.645 \cdot 10^{-4}$	1.713	$1.527 \cdot 10^{-9}$	0.3540
145.1 (max)	∞	$9.733 \cdot 10^{-4}$	1.956	0	0

Table 1: **Adiabatic approximation**. Time scales of oscillation, decay and motion of the brane, ratio $(y')^2 / [f(y)]^2$ and fraction of proper time left between y_b and y_b^{max}. The chosen parameters are: $\gamma = 0.01$, $L_{AdS} = 1$, $r_+ = 100$, $r_- = 99.9$, $l = 10$, $T = 0.999999999$. The position accompanied by "max" is the maximum position reached by the brane for the chosen parameters. The Euclidean solution corresponding to this choice of parameters is sensible and dominant in the thermodynamic ensemble.

Conclusion

- Quantum gravity in the lab is a rich subject and isn't just about trying to detect tiny signals from gravitons etc.
- Quantum simulations of holographic models of gravity are likely much closer to realization and synergize with the broader effort to understand complex, chaotic, highly entangled quantum systems
- There are many open directions, ranging from questions about how to simulate general spacetimes to the search for good algorithms to the study of stringy effects [e.g. string/M-theory Gharibyan-Hanada-Honda-Liu]