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Page Curve from Holographic Moving Mirror

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Thanks to collaborations with

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Based on arXiv: 2011.12005 [Phys. Rev. Lett. **126**, 061604]
+ (a longer paper coming soon)

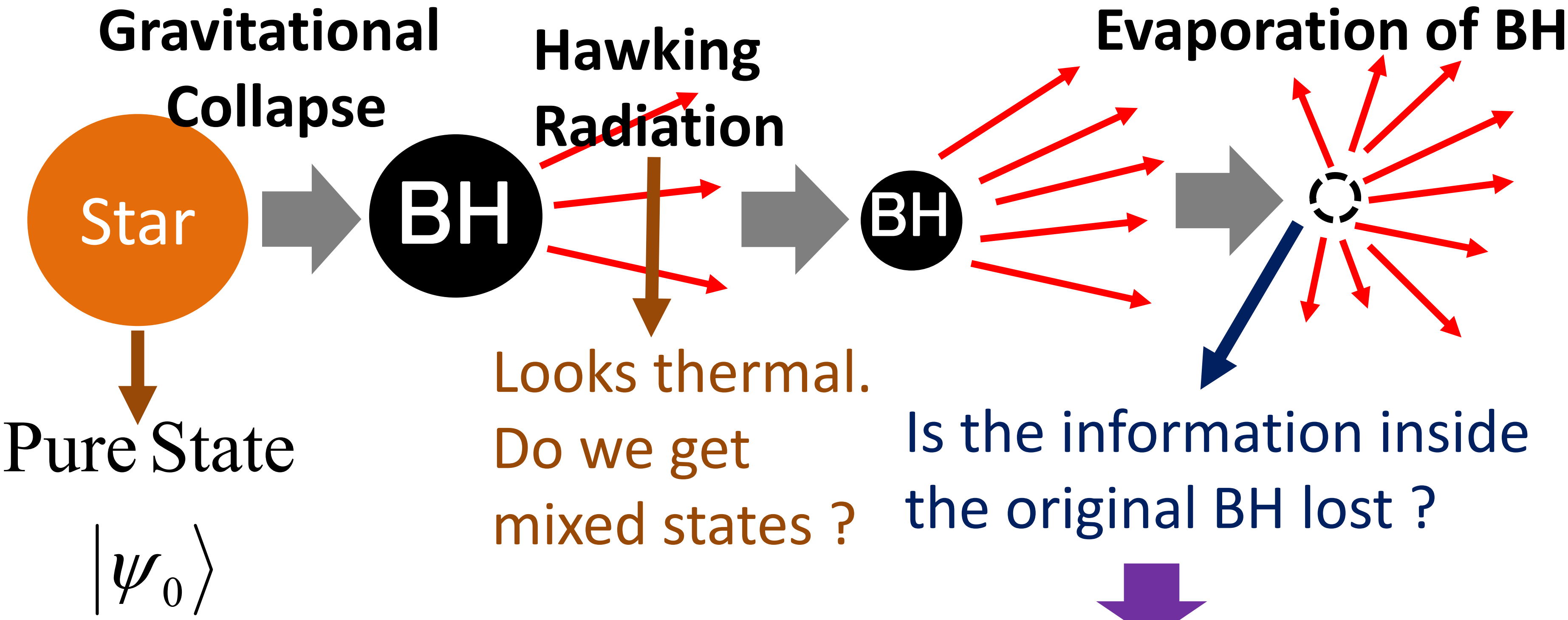
Contents

CFT (② – ③)

Holography (④ – ⑥)

① Introduction

The black hole information loss problem is a crucial key to find the meaning of "quantum" in quantum gravity.



The information should not be lost if the evolution is unitary and the final state remains pure.

A quantitative formulation of the BH information problem is obtained by considering entanglement entropy of radiations. ⇒ Page curve 1993

Total Hilbert space : $H_{tot} = H_{BH} \otimes H_{Rad}$

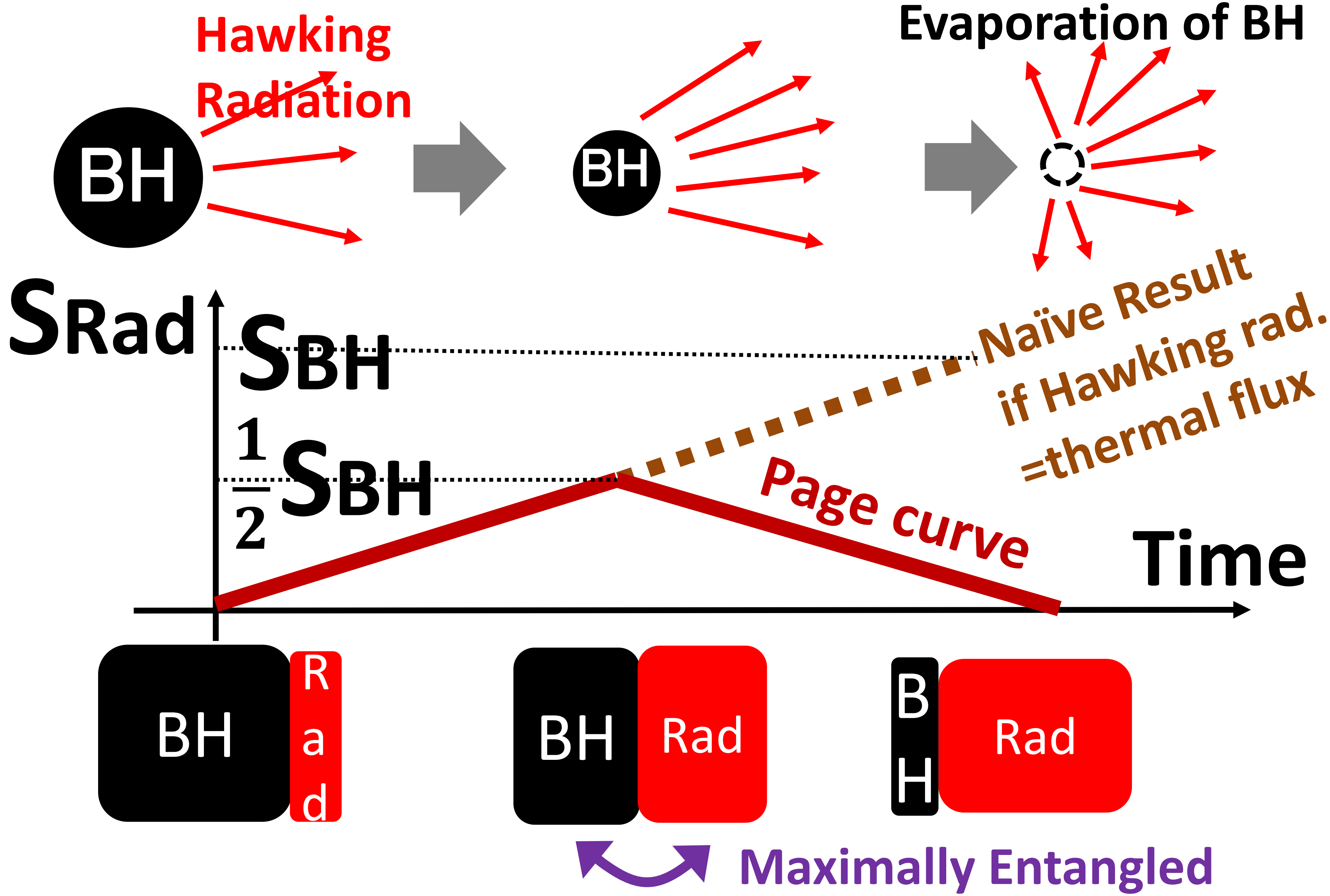
Unitary time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle \in H_{tot}$

Reduced density matrix: $\rho_{\mathrm{Rad}}(\mathrm{t}) = \mathrm{Tr}_{\mathrm{BH}}\left[|\psi(t)\rangle\langle\psi(t)|\right]$

➡ **Entanglement Entropy of Radiation**

$$S_{Rad}(t) = -\mathrm{Tr}\left[\rho_{\mathrm{Rad}}(\mathrm{t})\log\rho_{\mathrm{Rad}}(\mathrm{t})\right].$$

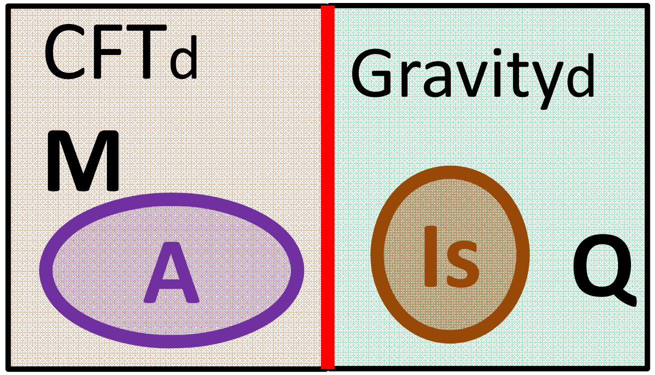
Page curve $S_{Rad}(t)$ follows the Page curve.



Recently, remarkable progresses have been made.

Page curve was explained by assuming the Island formula:

$$S_A = \underset{Is}{\text{Min Ext}} \left[\frac{\text{Area}(Is)}{4G_N} + S_{A\cup Is} \right]$$



[Penington 2019, Almheiri-Engelhardt-Marolf -Maxfield 2019, Almheiri-Mahajan-Maldacena-Zhao 2019,…]

The Island formula was originally motivated by holographic considerations:

Evaporating black hole —Holography→ **Page curve**

However, this approaches normally involve very complicated analysis of gravity + CFT system.

In this talk, we will explore a simple model in an opposite way:

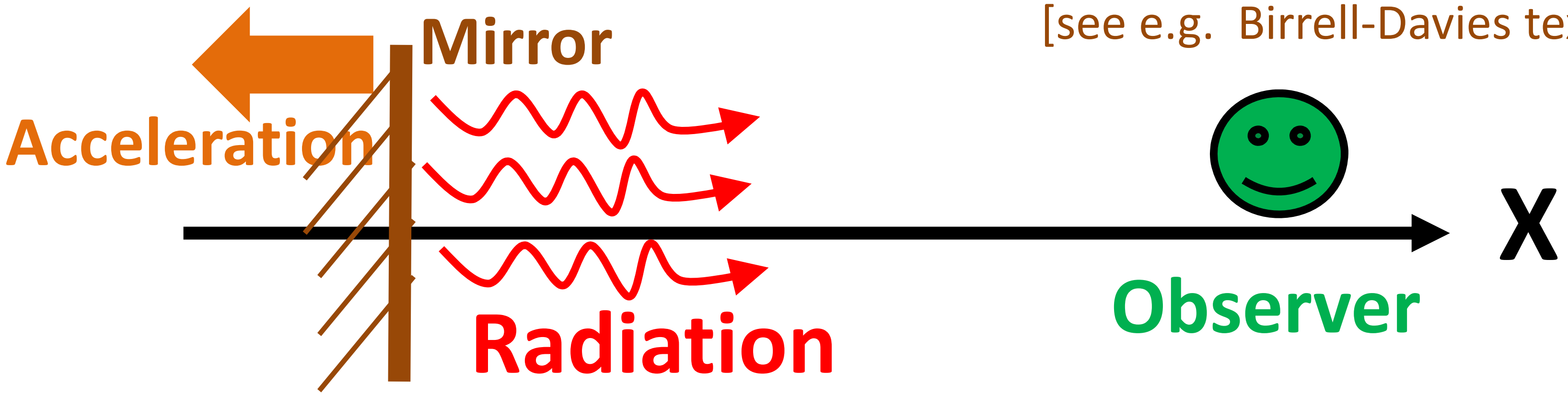
Page curve in CFT —Holography→ **Some gravity dynamics ?**

We focus on a class of models called moving mirrors.

Moving mirror

Moving mirrors have been known for a while as instructive models which mimic the Hawking radiation from Black holes.

[see e.g. Birrell-Davies text book]

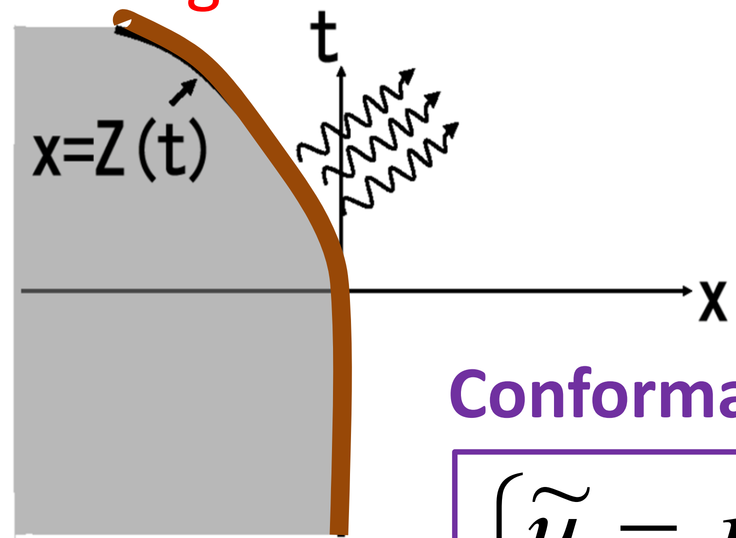


This provides an interesting class of non-equilibrium processes, where quantum entanglement gets crucial. [cf. quantum quenches]

② BCFT Description of Moving Mirror

In this talk we focus on two dim. CFTs. Then we can apply conformal mapping to solve the moving mirror problem.
We write a mirror trajectory as x=Z(t).

Moving Mirror



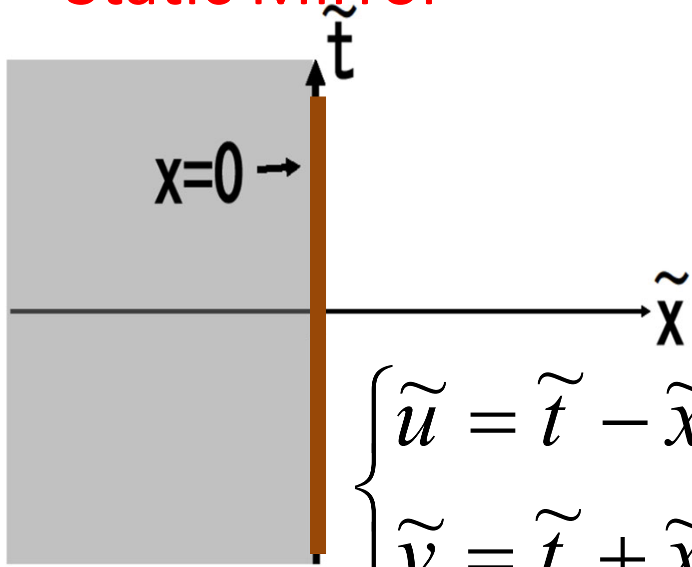
$$\begin{cases} u = t - x \\ v = t + x \end{cases}$$

Conformal Map

$$\begin{cases} \tilde{u} = p(u) \\ \tilde{v} = v \end{cases}$$

In coming vacuum

Static Mirror



$$\begin{cases} \tilde{u} = \tilde{t} - \tilde{x} \\ \tilde{v} = \tilde{t} + \tilde{x} \end{cases}$$

Standard Setup of BCFT

BCFT (Boundary Conformal Field Theory)

For special choice of boundary conditions, a part of conformal symmetries are preserved. This is called the boundary conformal field theory (BCFT).

[Cardy 1984, .., McAvity-Osborn 1995, ….]

CFTd: $SO(2,d)$

$\cup$

BCFTd : $SO(2,d-1)$

α labels
bdy conditions !

Boundary α $|B_\alpha\rangle$

When d=2, it is called boundary states (Cardy States)

$$(L_n - \widetilde{L}_{-n})|B\rangle = 0$$

Example 1 : Constant Radiation from Moving Mirror

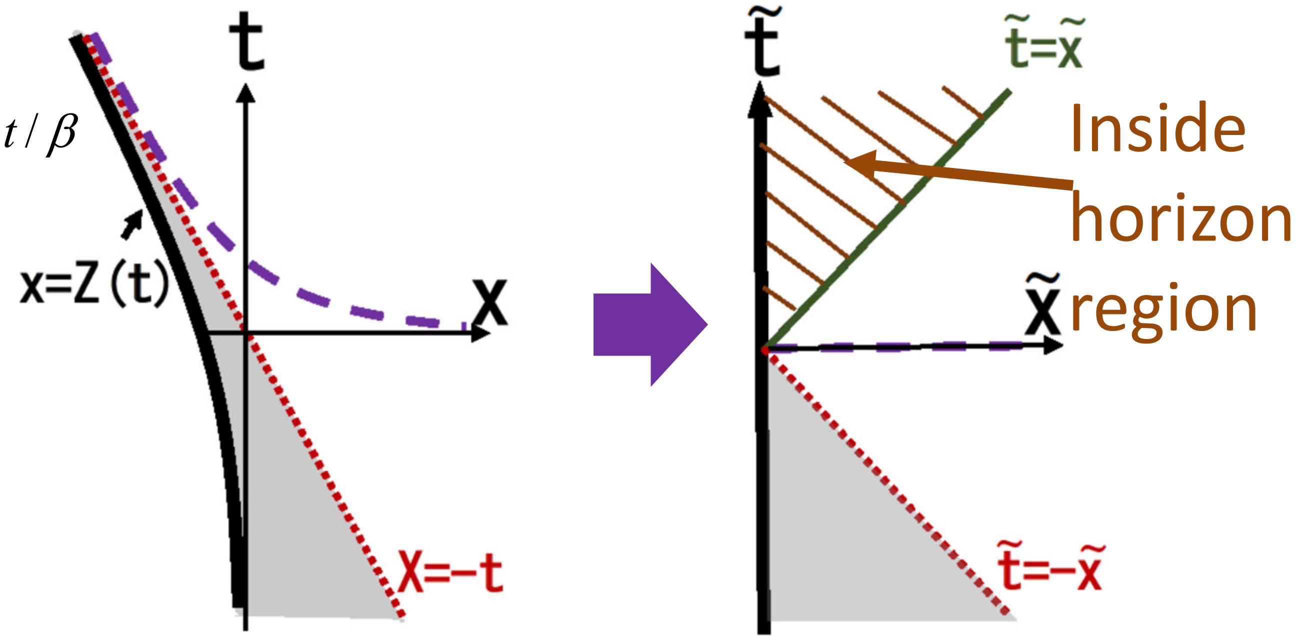
$$p(u) = -\beta \log(1 + e^{-u/\beta})$$

Thermal flux at temperature T=1/β

$$\text{Energy flux}: T_{uu} = \frac{c}{24\pi}\left(\frac{3}{2}\frac{(P'')^2}{(P')^2} - \frac{p'''}{p'}\right)$$

$$= \frac{c}{48\pi\beta^2}\left(1 - \frac{1}{(1+e^{u/\beta})^2}\right) \approx \frac{c}{48\pi\beta^2}$$

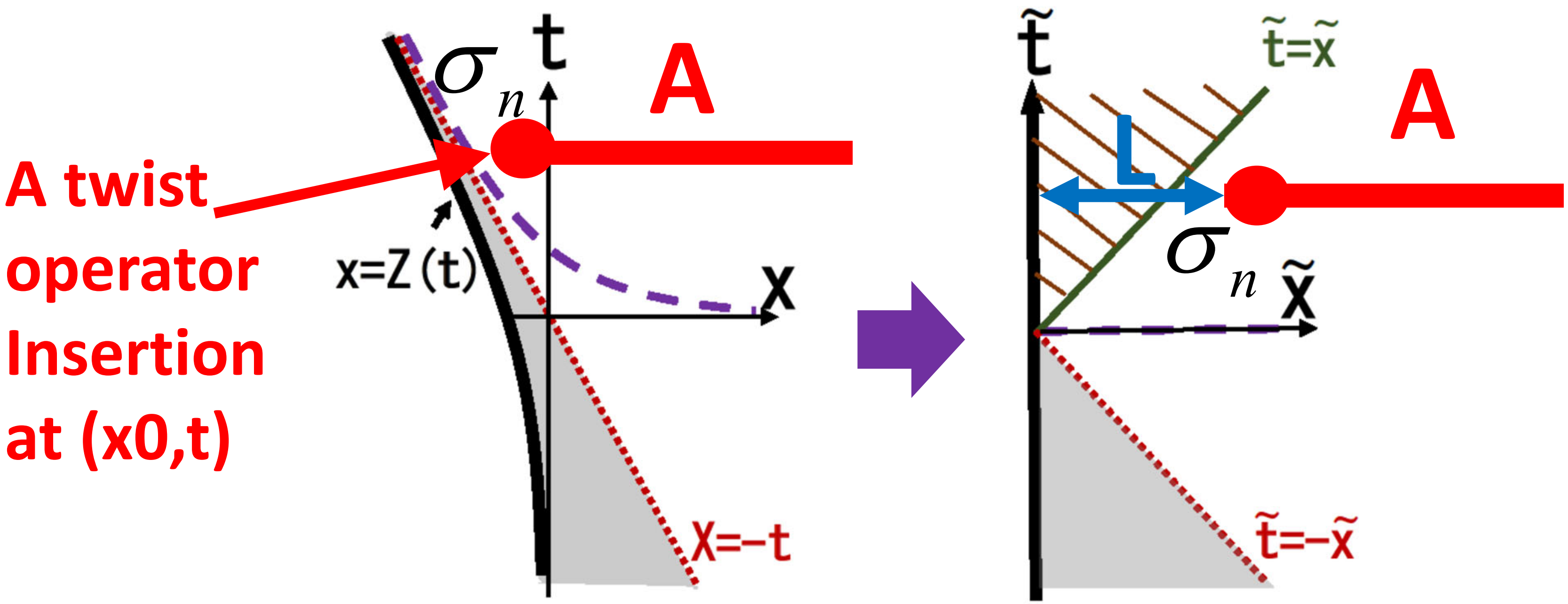
$$z(t) \underset{t\to\infty}{\approx} -t - \beta\, e^{-2t/\beta}$$



③ Page Curve from Moving Mirror

Calculation of Entanglement Entropy (EE)

To get a universal result, we choose the subsystem A to be a semi-infinite line [x0,∞] at time t. We consider the EE (SA) between A and B (=the compliment of A).



We can calculate the EE using the replica method.

$$\langle \sigma_n \rangle = \frac{g}{L^{\Delta_n}}, \qquad \Delta_n = \frac{c}{12}(n - 1/n).$$

where $g = e^{S_{bdy}}$ is the g-function or boundary entropy.

By applying the conformal transformation, we obtain
(we write the UV cut off or lattice spacing as ε)

$$S_A = \frac{c}{6}\log\left[\frac{t + x_0 - p(t - x_0)}{\varepsilon\sqrt{p'(t - x_0)}}\right] + S_{bdy}$$

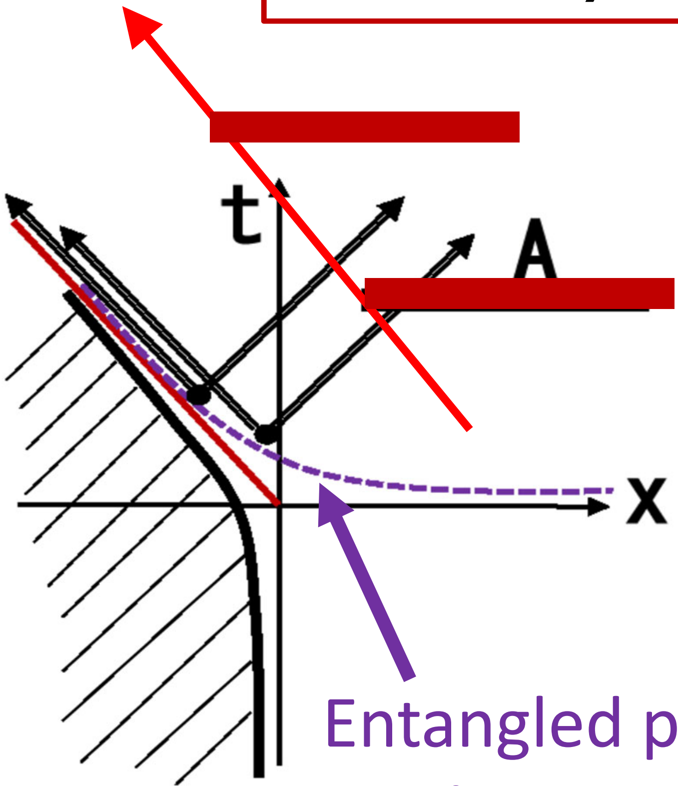
$$\underset{t\to\infty}{\approx} \frac{c}{12\beta}(t - x_0) + \frac{c}{6}\log\frac{t}{\varepsilon} + S_{bdy} \quad .$$

Note that this result is universal for any two dim. CFTs.

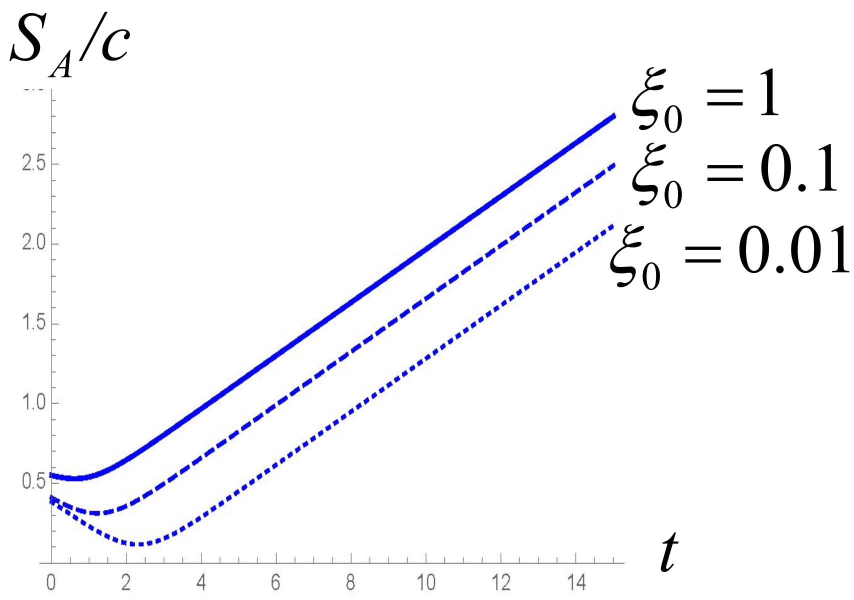
It is instructive to choose the time-dependent subsystem:

A=[x0(t),∞], where x0(t)=-t+ξ0 . In this case we find

$$S_A \underset{t\to\infty}{\approx} \frac{c}{6\beta} t + \frac{c}{6}\log\frac{\xi_0}{\varepsilon} + S_{bdy}.$$



Entangled pair productions: v+p(u)=0



$\beta = 1,\ \varepsilon = 0.1$

Entangled pair productions

For simplicity, consider a two dim. Massless free scalar.

$$|0\rangle_{in} \approx \exp\left(\# \int d\omega\, a_{\omega}^{\dagger in} a_{\omega}^{\dagger in}\right)|0\rangle_{out}.$$

Entangled pair productions

We can detect the location of entangled pair creations as

$$\langle 0_{in}|\phi(u_1,v_1)\phi(u_2,v_2)\int d\omega\, a_{\omega}^{\dagger \mathrm{in}} a_{\omega}^{\dagger \mathrm{in}}|0_{in}\rangle$$

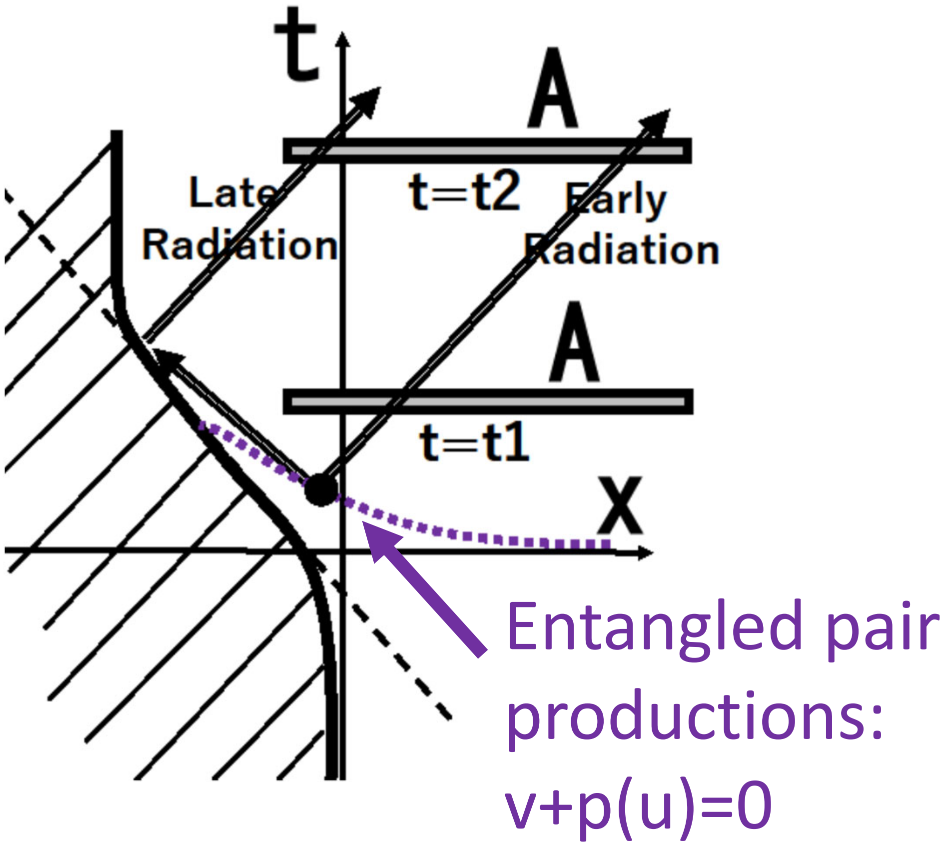
$$= \int \frac{d\omega}{\omega}\left[e^{-i\omega(v_1+p(u_2))} + e^{-i\omega(v_2+p(u_1))} - e^{-i\omega(v_1+v_2)} - e^{-i\omega(p(u_1)+p(u_2))}\right].$$

Entangled pairs are created at v+p(u)=0.

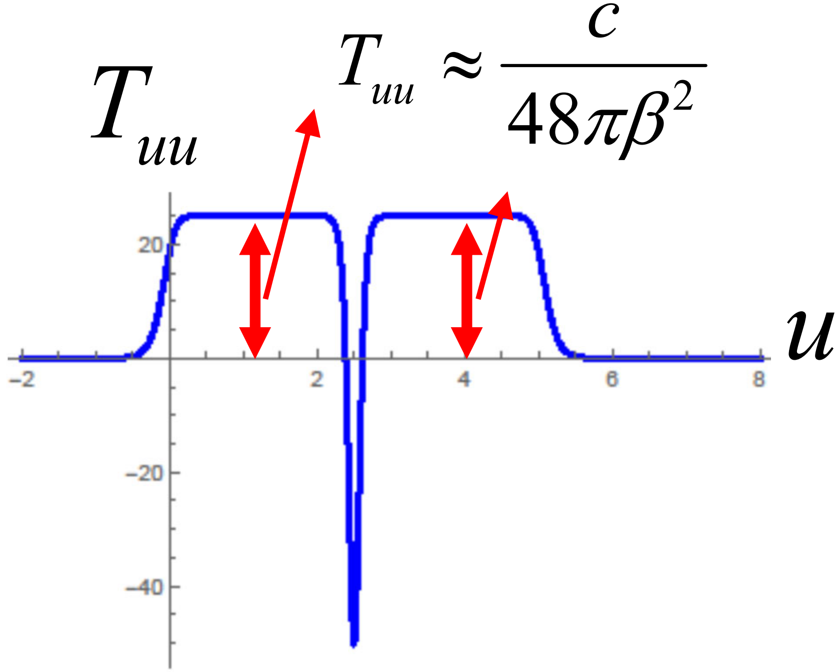
Example 2: Model mimicing a BH evaporation

$$p(u) = -\beta \log(1+e^{-u/\beta}) + \beta \log(1+e^{(u-u_0)/\beta}).$$

$$\Rightarrow Z(t) \underset{t\to-\infty}{\approx} 0, \qquad Z(t) \underset{t\to\infty}{\approx} -u_0/2.$$



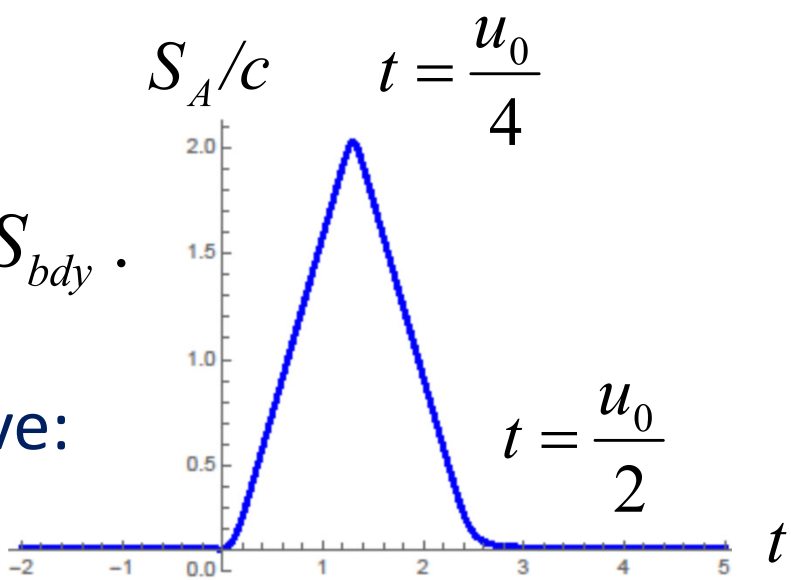
Thermal flux



The time evolution of EE

$$S_A = \frac{c}{6}\log\left[\frac{t+x_0-p(t-x_0)}{\varepsilon\sqrt{p'(t-x_0)}}\right]+S_{bdy}\,.$$

reproduces the perfect page curve:
(We chose A=[Z(t)+0.1,∞].)



$\beta=1,\ \varepsilon=0.1,\ u_0=5$

Though the above results for the semi-infinite subsystem are universal, the EE for a finite interval A depends on CFTs.

Thus, in the next part, we will focus on holographic CFTs. The above result can be reproduced from the disconnected geodesic length which is smaller than the connected one.

[cf. Earlier work : Bianchi-Smerlak 2014, Hotta-Sugita 2015
for a page curve like behavior of the connected counterpart]

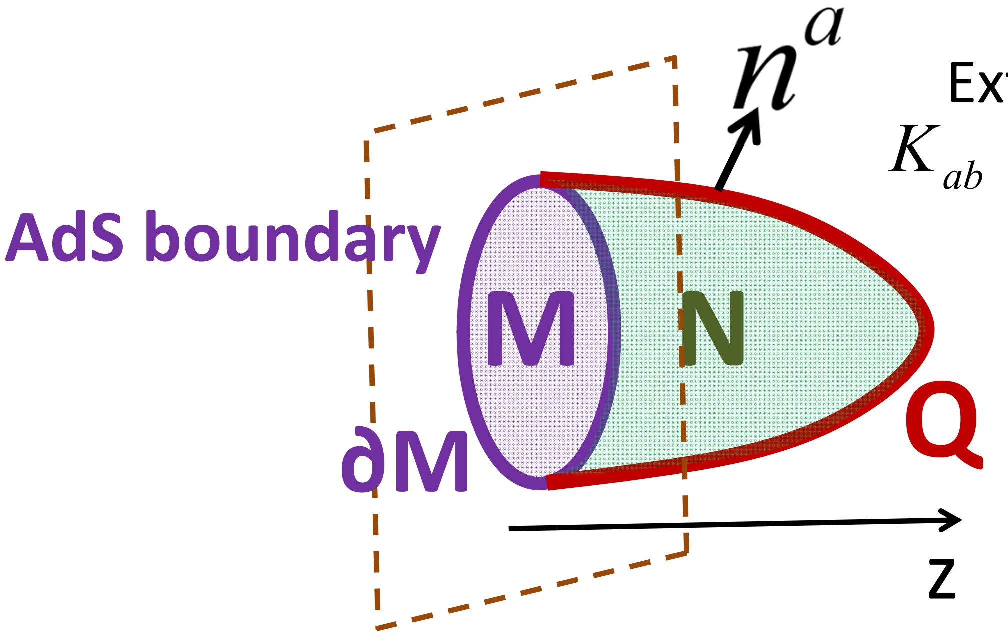
④ A Brief Review of AdS/BCFT

For a gravity dual of moving mirror we apply the AdS/BCFT.

(4-1) AdS/BCFT construction

[TT 2011, Fujita-Tonni-TT 2011, see also Karch-Randall 2001,..]

CFT on a manifold M with a boundary ∂M = Gravity on an asymptotically AdS space N, s.t. ∂N=M∪Q



Extrinsic curvature

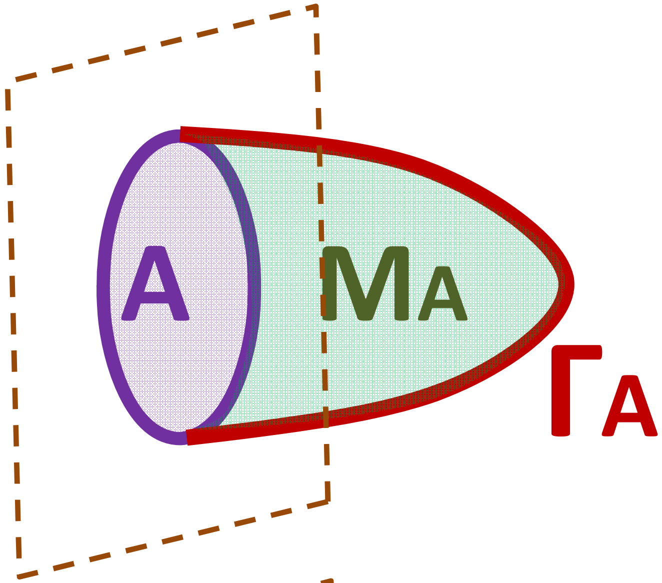
$$K_{ab} = \nabla_a n_b, \quad K = h^{ab} K_{ab}$$

←**New surface in the bulk** (End of the world brane)

We impose *Neumann b.c.*:

$$K_{ab} - K h_{ab} - T^Q_{ab} = 0$$

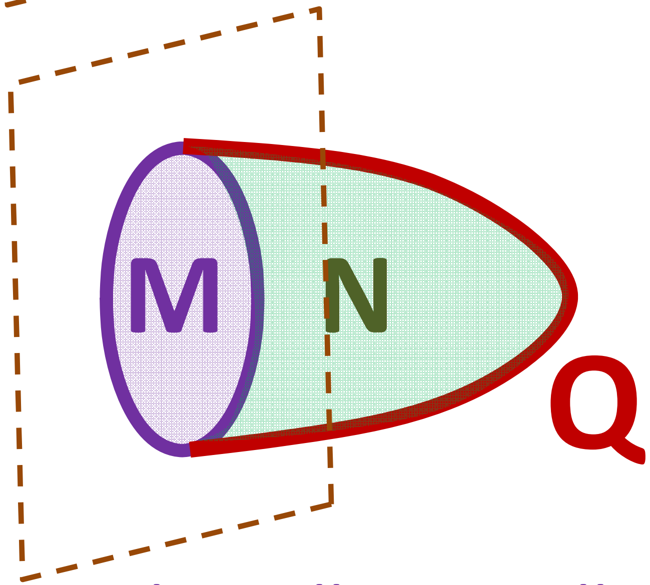
Differences between two "subregion/subregion duality"



[1] Entanglement Wedge

⇒ ΓA is extremal surface.
(no back-reactions)

$$h^{ab}K_{ab}=0$$



[2] AdS/BCFT

⇒ Q is totally geodesic surface
or it generalizations.

$$K_{ab}=\text{fixed}$$

⇒ Surface Q back-reacts !

In this talk, we will see interesting interplay between them.

Formulation of AdS/BCFT

The gravity action in Euclidean signature looks like

Gibbons
-Hawking term

$$I = \frac{1}{16\pi G_N}\int_N \sqrt{-g}(R - 2\Lambda + L_{matter}) + \frac{1}{8\pi G_N}\int_Q \sqrt{-h}(K + L^Q_{matter}).$$

Bulk matter fields

Matter fields
localized on Q

The coordinate and induced metric of Q are x^a and h^{ab}.

We define the extrinsic curvature and its trace

$K_{ab} = \nabla_a n_b, \quad K = h^{ab}K_{ab}$. (n^a is a unit vector normal to Q.)

e.g. Gaussian normal coordinate: $ds^2 = d\rho^2 + h_{ab}(\rho, x)dx^a dx^b$

$$\Rightarrow K_{ab} = \frac{1}{2}\partial_\rho h_{ab}(\rho, x).$$

Variation: $\delta I = \frac{1}{16\pi G_N}\int_Q \sqrt{-h}(K_{ab} - Kh_{ab} - T^Q_{ab})\,\delta h^{ab}.$

At the AdS boundary M, we impose Dirichlet boundary condition $\delta h^{ab} = 0$ as in the standard AdS/CFT.

On the other hand, at the new boundary Q, we argue to require the Neumann b.c. :

$$\boxed{K_{ab} - Kh_{ab} - T^Q_{ab} = 0}$$ `boundary Einstein eq.'

Why Neumann b.c. ? [closely related to brane-world models]

(1) Keep the boundary dynamical. New data at Q should not be required.

(2) Orientifolds in string theory leads to this condition.

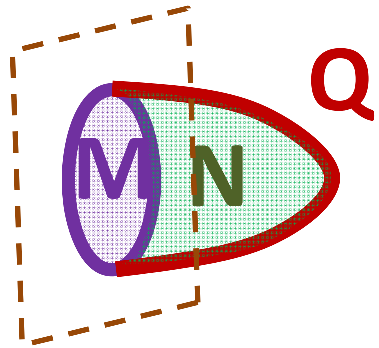
(4-2) A Basic Example of AdSd+1/BCFTd

To preserve the boundary conformal symmetry, we choose

$$T^Q_{ab} \propto h_{ab} \quad \Rightarrow \quad T^Q_{ab} = -T\, h_{ab}$$ (T is the tension of Q).

The Neumann b.c. looks like $K_{ab} = (K - T)\, h_{ab}$.

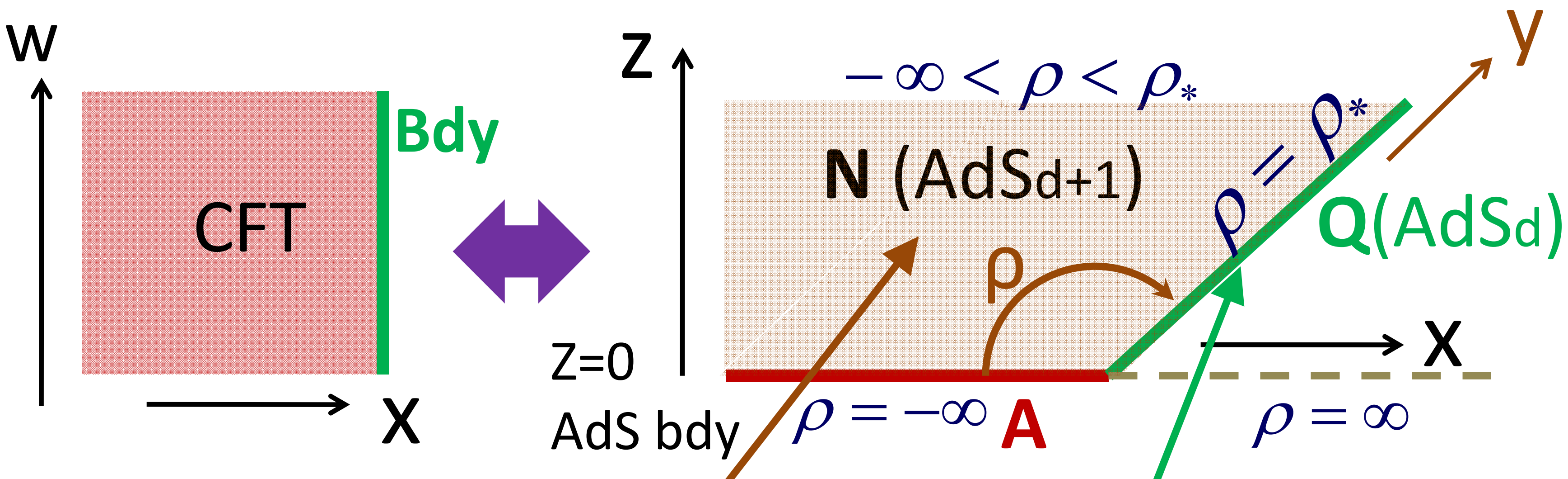
$$K_{ab} = \frac{d}{d-1} T\, h_{ab}$$



The gravity action takes the simple form

$$I_G = \frac{1}{16\pi G_N}\int_N \sqrt{-g}\,(R - 2\Lambda) + \frac{1}{8\pi G_N}\int_Q \sqrt{-h}\,(K - T).$$

Construction of a gravity dual of the BCFT



$$ds^2_{M_{d+1}} = d\rho^2 + \cosh^2\left(\frac{\rho}{R}\right) ds^2_{AdS(d)}$$

SO(2,d-1) sym.

$$= R^2\left(\frac{-dt^2 + dz^2 + dx^2 + d\vec{w}^2}{z^2}\right).$$

$$ds^2_{AdS(d)} = R^2\left(\frac{-dt^2 + dy^2 + d\vec{w}^2}{y^2}\right)$$

$$K_{ab} = \frac{1}{R}\tanh\left(\frac{\rho_*}{R}\right) h_{ab},$$

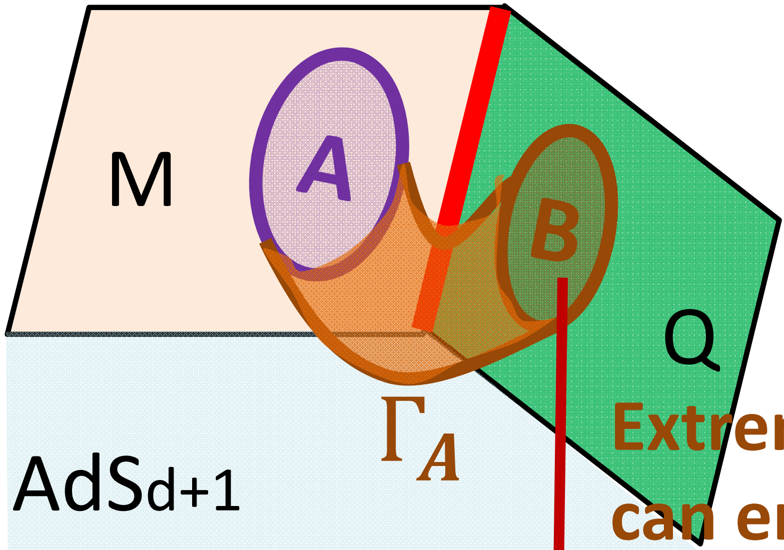
$$z = y/\cosh(\rho/R), \quad x = y\tanh(\rho/R),$$

$$\Rightarrow \quad T = \frac{d-1}{R}\tanh\frac{\rho_*}{R}.$$

(4-3) Holographic Entanglement Entropy in AdS/BCFT

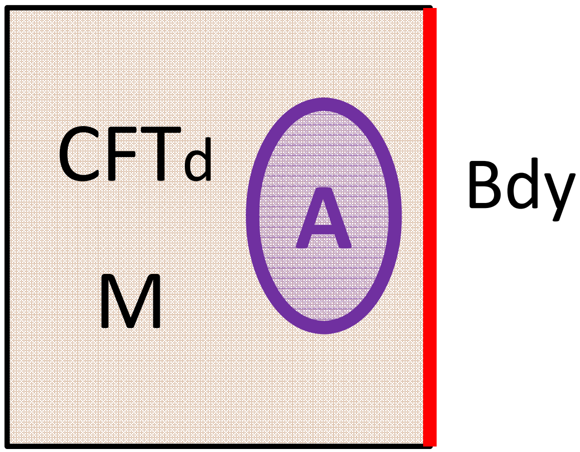
[TT 2011, Fujita-Tonni-TT 2011]

$$S_A = \underset{\Gamma_A,\, B}{\text{Min Ext}} \left[\frac{\text{Area}(\Gamma_A)}{4G_N} \right] \qquad \partial\Gamma_A = \partial A \cup \partial B$$



AdS/BCFT

=



Extremal Surfaces can end on Q !

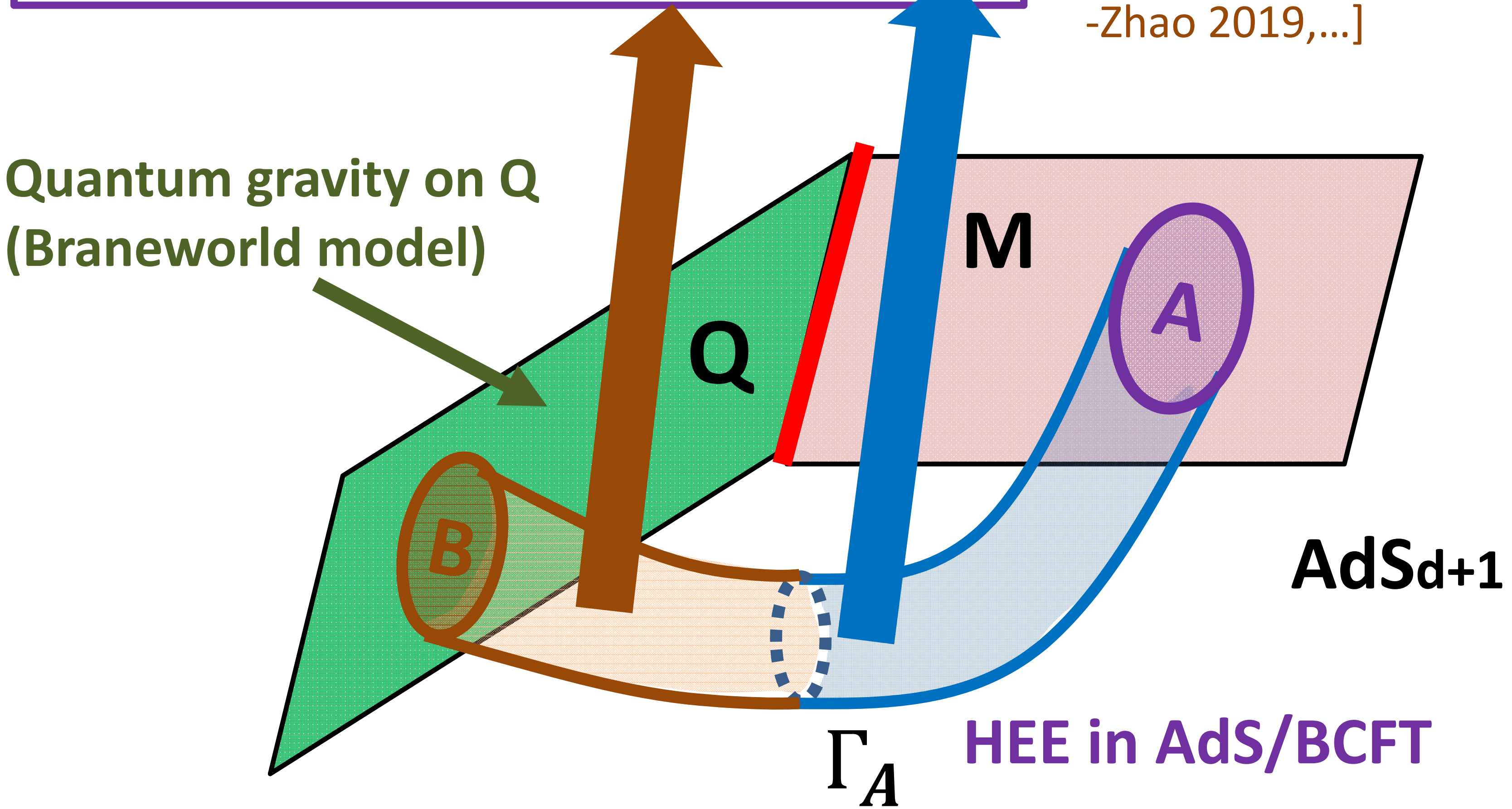
This region B is now known as an **Island** !

Relation to Island formula

Island Formula

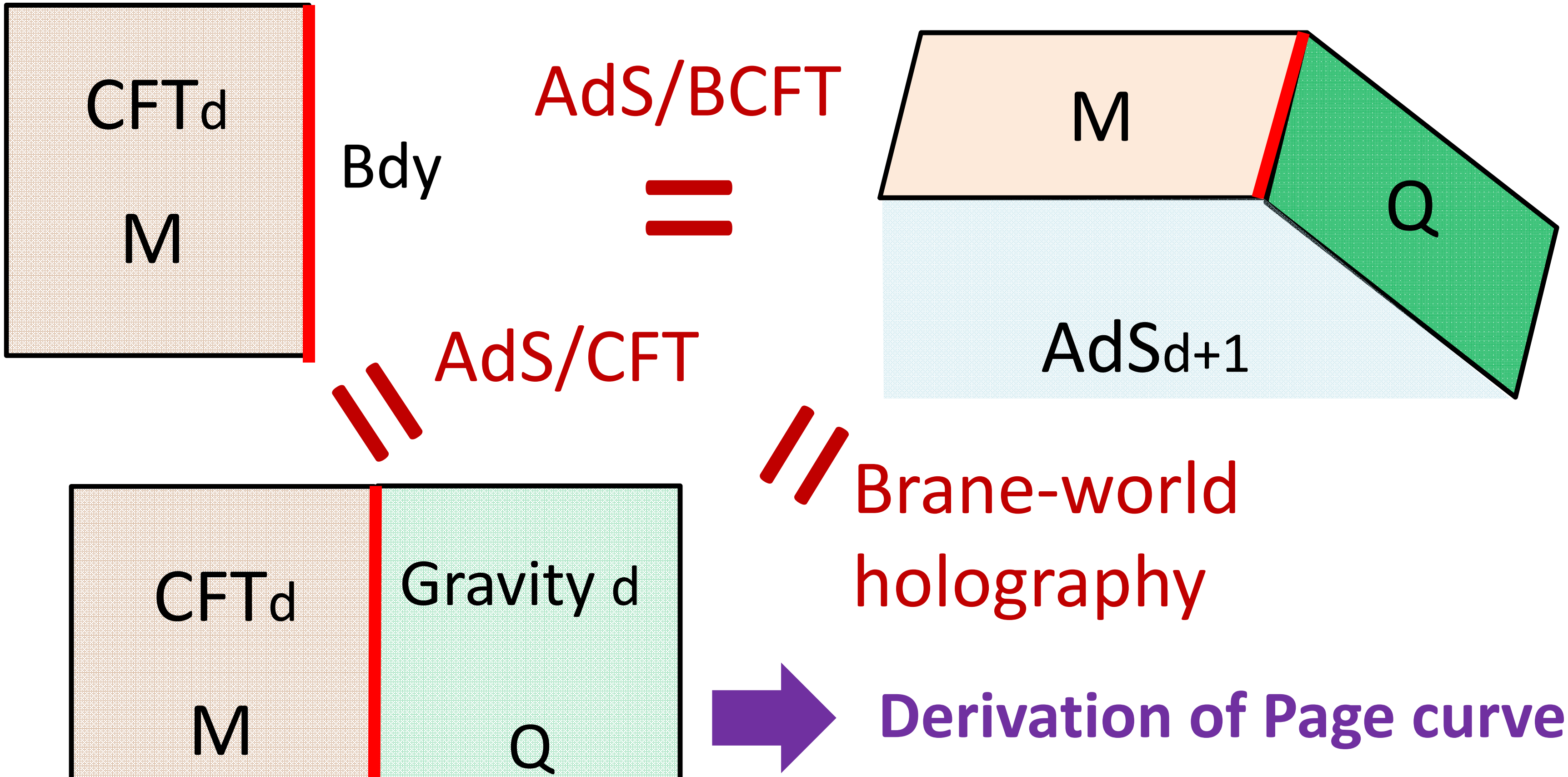
$$S_A = \underset{Is}{\text{Min Ext}}\left[\frac{Area(Is)}{4G_N} + S_{A\cup Is}\right]$$

[Penington 2019,
Almheiri-Engelhardt-Marolf
-Maxfield 2019,
Almheiri-Mahajan-Maldacena
-Zhao 2019,...]



Holographic Triality

We can apply the brane-world holography to AdS/BCFT as follows.



[Argument in Almheiri-Mahajan
-Maldacena-Zhao 2019]

[More Progresses: Rozali-Sully-Raamsdonk-Waddell-Wakeham 2019, Chen-Fisher-Hernandez -Myers-Ruan 2019, Almheiri-Mahajan-Santos 2019, Chen-Myers-Neuenfeld-Reyes-Sandor 2020, Chen-Gorbenko-Maldacena 2020, Geng-Karch 2020, Balasubramanian-Kar-Ugajin 2020, ……]

(4-4) AdS3/BCFT2 and Boundary Entropy

<u>Boundary Entropy</u>

We focus on the d=2 case (AdS3/BCFT2).

α labels bdy conditions ! → Boundary α

Boundary entropy: A measure of the degrees of freedom at the boundary [Affleck-Ludwig 1991]

$$g = e^{S_{bdy}}$$

→ **This value depends on α !**

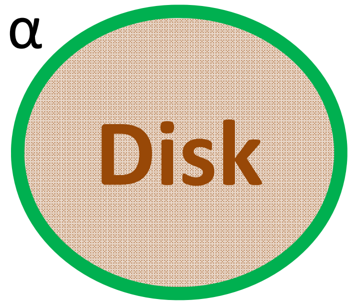
g-theorem: Sbdy or g decreases under the bdy RG flow.

[Proof: Friedan-Konechny 2004]

Three Definitions of Boundary Entropy

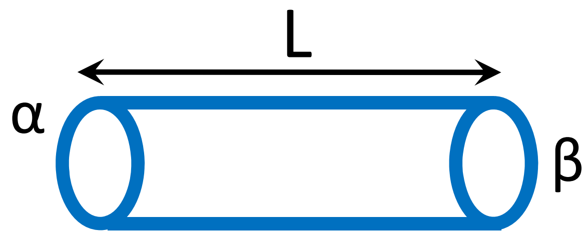
Def 1 (Disk Amplitude)

$$S_{bdy(\alpha)} = \log g_\alpha , \qquad g_\alpha \equiv \langle 0 | B_\alpha \rangle .$$



Def 2 (Cylinder Amplitude)

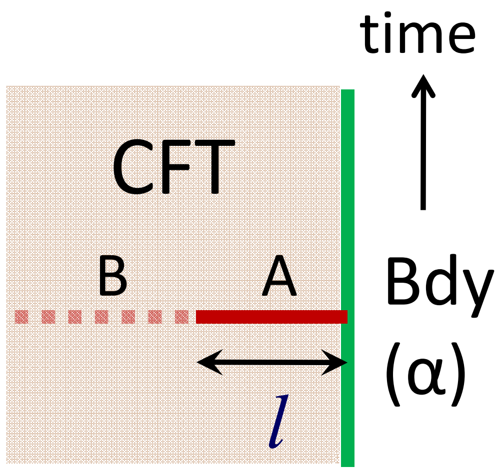
$$Z^{cylinder}_{(\alpha,\beta)} = \langle B_\alpha | e^{-HL} | B_\beta \rangle \underset{L\to\infty}{\approx} \underbrace{g_\alpha g_\beta}_{\text{Boundary Part}} \underbrace{e^{-E_0 L}}_{\text{Bulk Part}} .$$



Def 3 (Entanglement Entropy)

In 2D BCFT, the EE behaves like

$$S_A = \underbrace{\frac{c}{6}\log\frac{l}{\varepsilon}}_{\text{Bulk Part}} + \underbrace{\log g_\alpha}_{\text{Boundary Part}} .$$



[Calabrese-Cardy 2004]

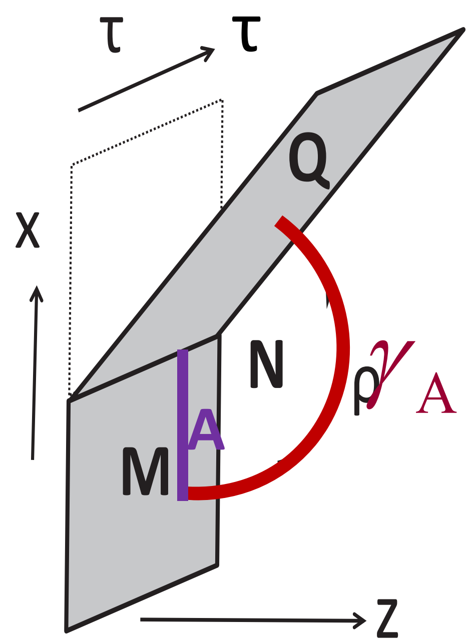
Bdy entropy from HEE in AdS/BCFT

New Aspect in AdS/BCFT: Extremal Surfaces end on Q !

The holographic EE is obtained as

$$S_A = \frac{\text{Length}}{4G_N} = \frac{1}{4G_N}\int_{-\rho_\infty}^{\rho_*} d\rho$$

$$= \frac{\rho_\infty + \rho_*}{4G_N} = \frac{c}{6}\log\frac{l}{\varepsilon} + \frac{\rho_*}{4G_N} \quad .$$

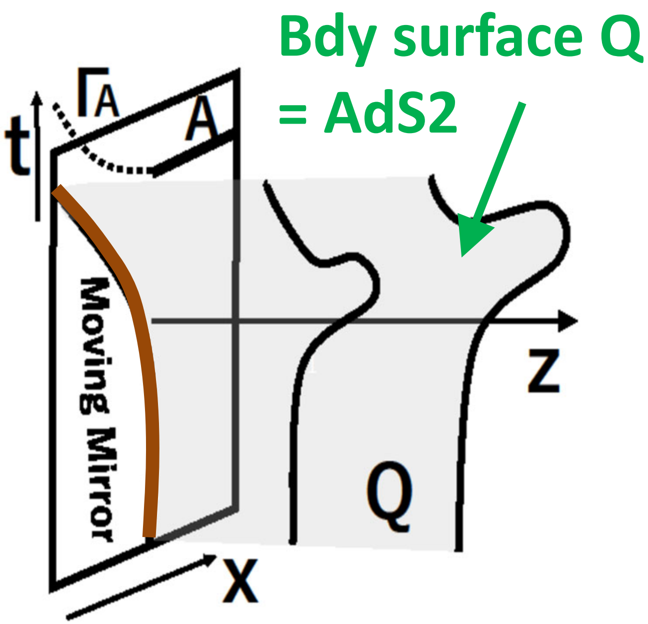


Thus we reproduced the same boundary entropy:

$$S_{bdy} = \frac{\rho_*}{4G_N}.$$

⑤ Holographic Moving Mirror

We apply AdS/BCFT to get a gravity dual of moving mirror.

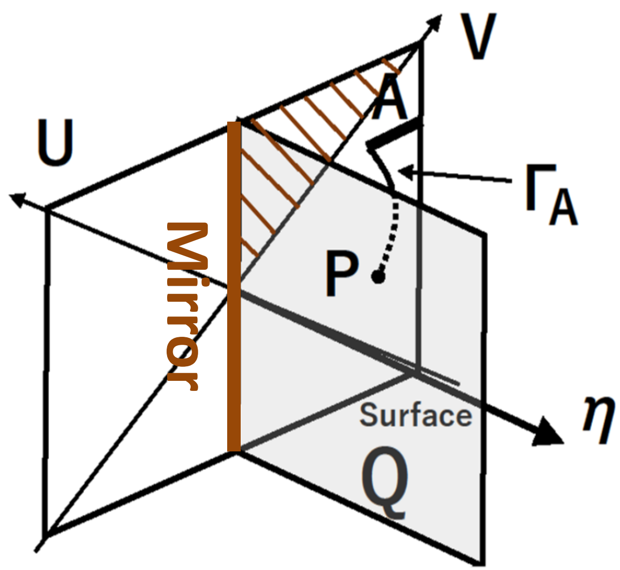


$$\begin{cases} U = p(u) \\ V = v + \frac{p''(u)}{2p'(u)} z^2 \\ \eta = z\sqrt{p'(u)} \end{cases}$$

$$ds^2 = \frac{dz^2}{z^2} - \frac{dudv}{z^2} + \frac{12\pi}{c} T_{uu}(u) du^2$$

Coordinate transformation

[Banados 1999, Roberts 2012]



$$ds^2 = \frac{d\eta^2 - dUdV}{\eta^2}$$

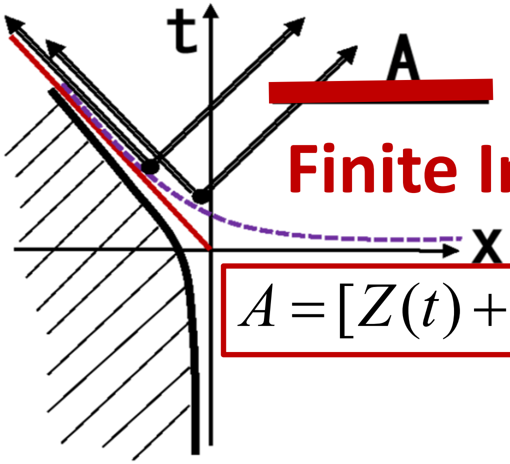
Standard AdS/BCFT setup for BCFT on a half plane

Example 1: Constant Radiation from Moving Mirror

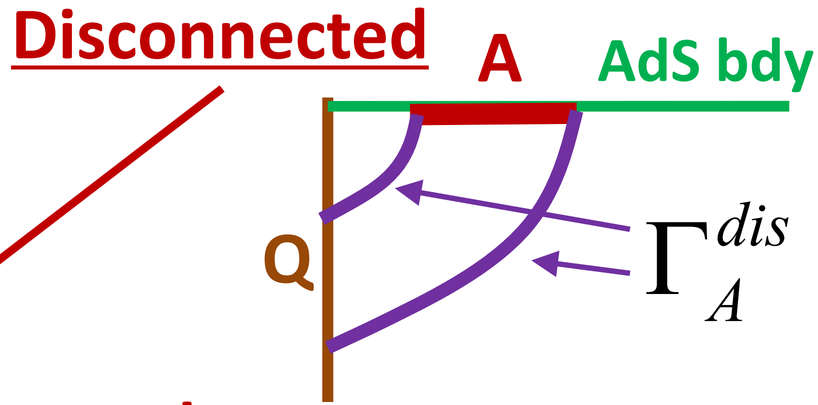
$$p(u) = -\beta \log(1 + e^{-u/\beta})$$

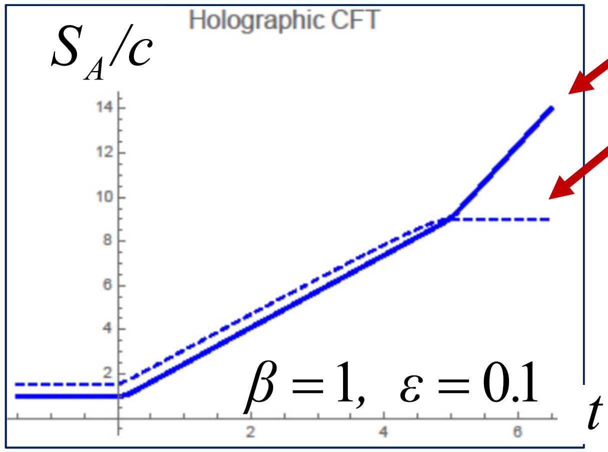
The HEE can be computed as

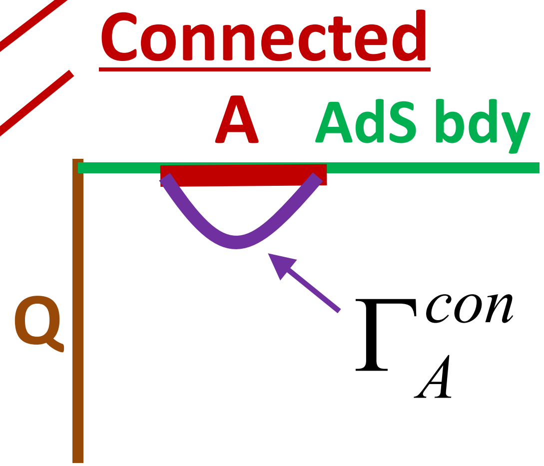
$$S_A = Min\left\{ \frac{L(\Gamma_A^{con})}{4G_N}, \frac{L(\Gamma_A^{dis})}{4G_N} \right\}.$$

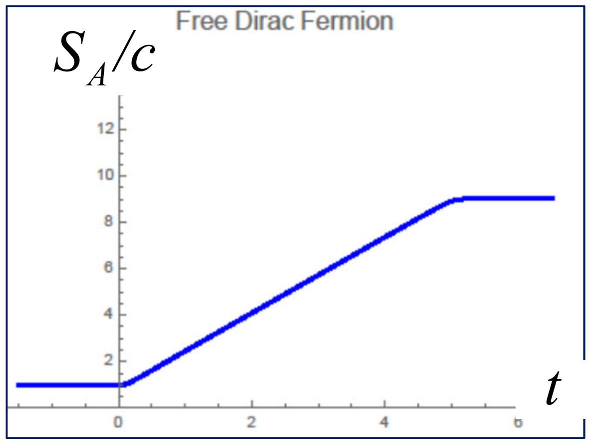


$$A = [Z(t) + 0.1, Z(t) + 10]$$





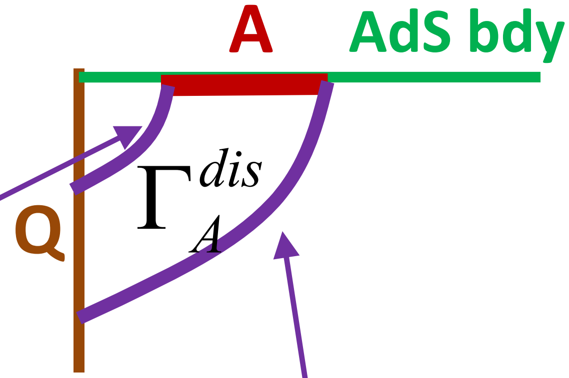




Explicit Formula for the HEE

A=[x0,x1] at time t

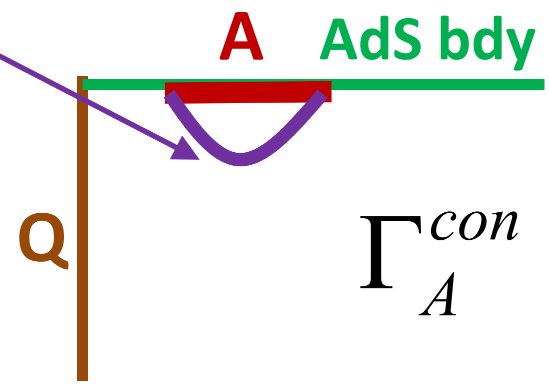
Disconnected



$$S_A^{dis} = \frac{c}{6}\log\left[\frac{t+x_0-p(t-x_0)}{\varepsilon\sqrt{p'(t-x_0)}}\right] + \frac{c}{6}\log\left[\frac{t+x_1-p(t-x_1)}{\varepsilon\sqrt{p'(t-x_1)}}\right] + 2S_{bdy}$$

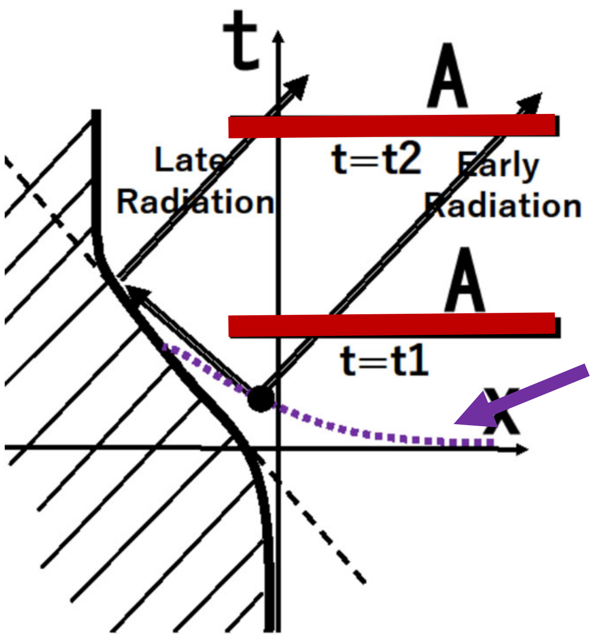
$$S_A^{con} = \frac{c}{6}\log\frac{(x_1-x_0)\left(p(t-x_0)-p(t-x_1)\right)}{\varepsilon^2\sqrt{p'(t-x_0)p'(t-x_1)}}.$$

Connected



Example 2: Model mimicing a BH evaporation

$$p(u) = -\beta \log(1+e^{-u/\beta}) + \beta \log(1+e^{(u-u_0)/\beta})$$



Entangled pair productions: v+p(u)=0

A=Semi infinite interval

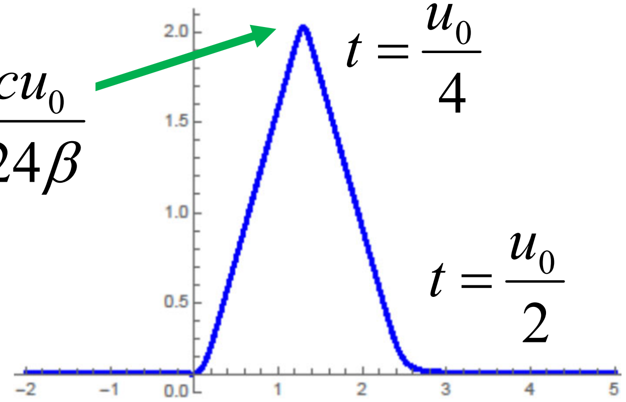
$$A = [Z(t)+0.1, \infty]$$

S_A/c

$$S_A = \frac{cu_0}{24\beta}$$

$t = \frac{u_0}{4}$

$t = \frac{u_0}{2}$



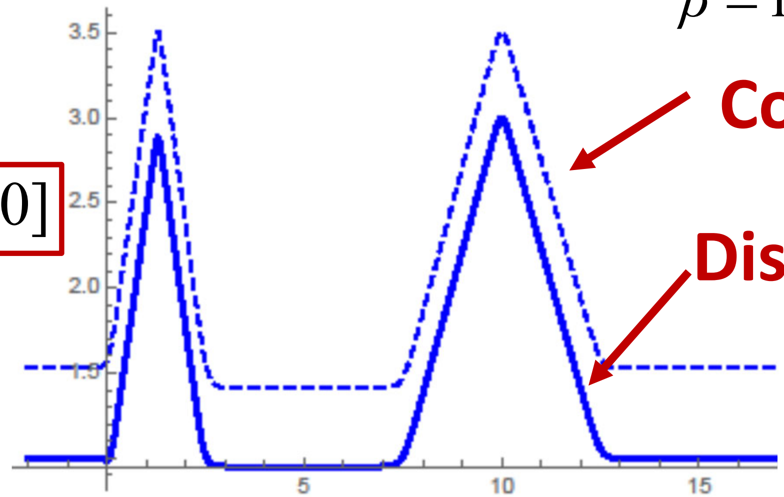
t

$\beta = 1, \ \varepsilon = 0.1, \ u_0 = 5$

A=Finite Interval

$$A = [Z(t)+0.1, Z(t)+10]$$

S_A/c



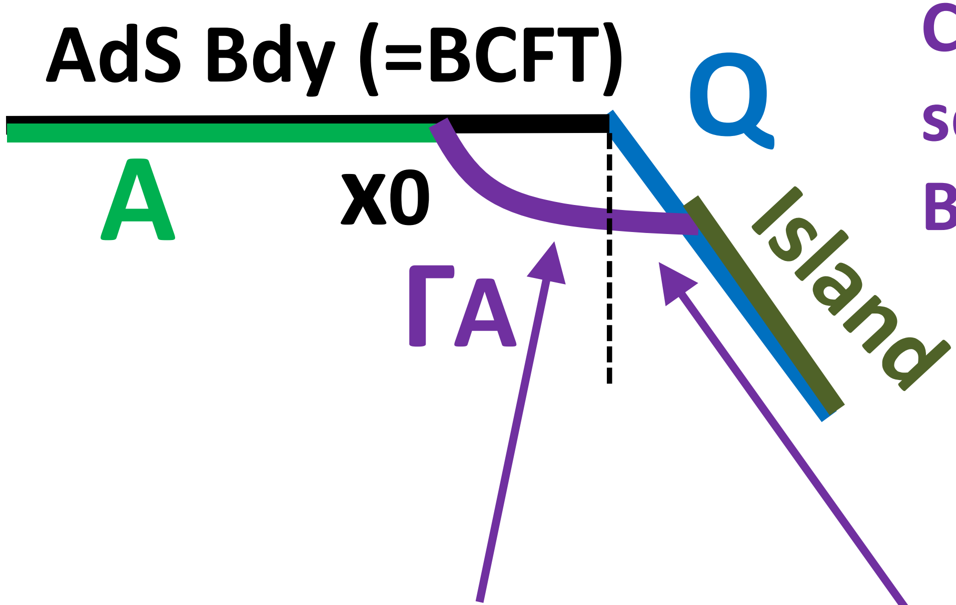
t

Connected Γ_A^{con}

Disconnected Γ_A^{dis}

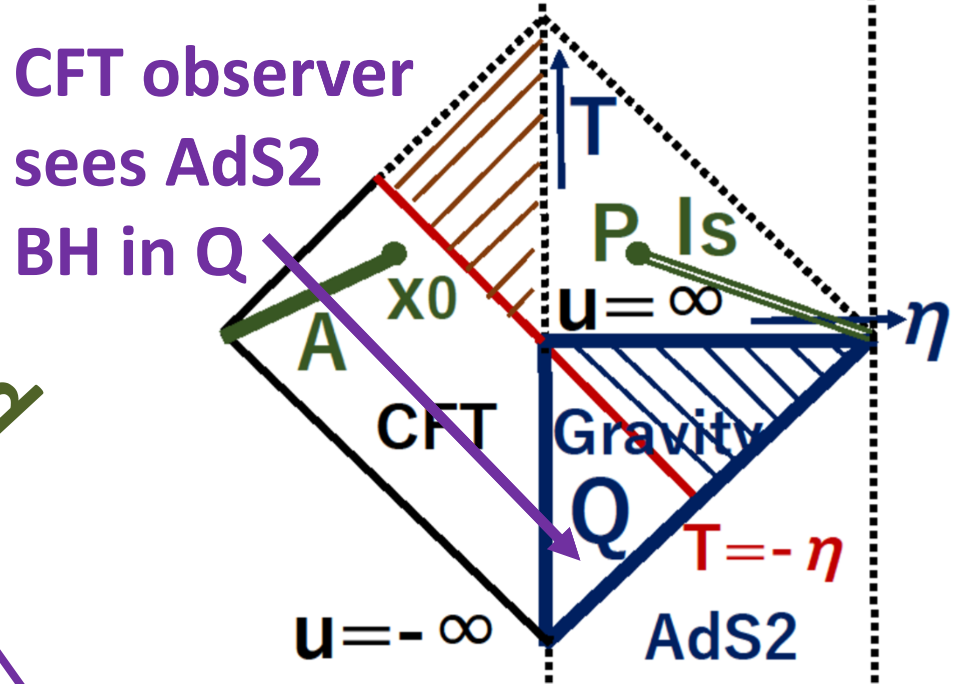
⑥ Brane-world and Island for Moving Mirror

Let us examine the spacetime structure of our gravity dual of moving mirror.

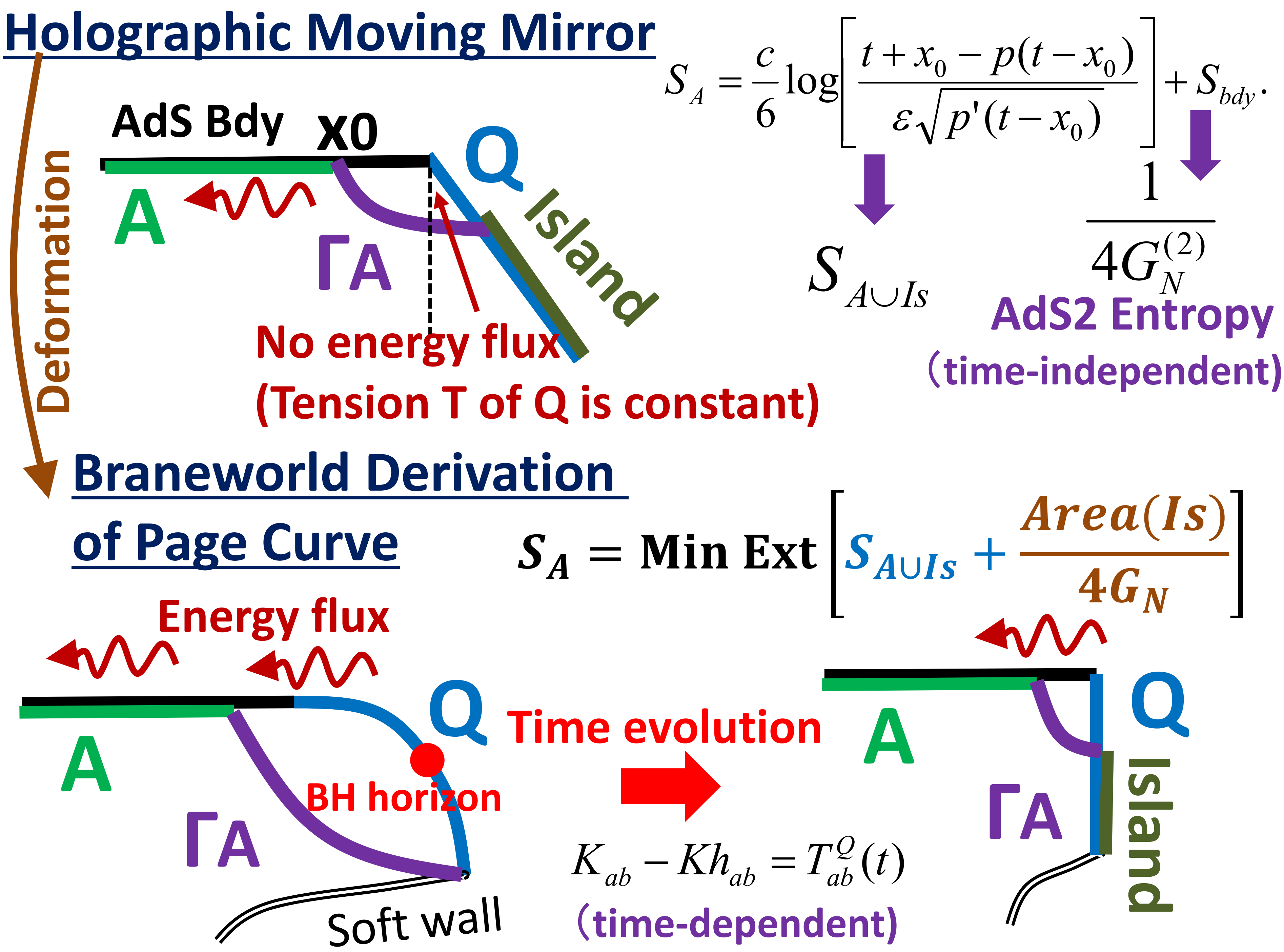


$$S_A = \frac{c}{6}\log\left[\frac{t+x_0-p(t-x_0)}{\varepsilon\sqrt{p'(t-x_0)}}\right]+S_{bdy}.$$

Causal Structure of Ex.1



When A=semi-infinite, an Island always exists ! The island is in horizon.


Holographic Moving Mirror
$S_A = \frac{c}{6}\log\left[\frac{t+x_0-p(t-x_0)}{\varepsilon\sqrt{p'(t-x_0)}}\right] + S_{bdy}.$
AdS Bdy
X0
Q
Island
A
ΓA
No energy flux
(Tension T of Q is constant)
$S_{A\cup Is}$
$\frac{1}{4G_N^{(2)}}$
AdS2 Entropy
(time-independent)
Deformation
Braneworld Derivation
of Page Curve
$S_A = \text{Min Ext}\left[S_{A\cup Is} + \frac{Area(Is)}{4G_N}\right]$
Energy flux
A
ΓA
Q
BH horizon
Soft wall
Time evolution
$K_{ab} - Kh_{ab} = T^Q_{ab}(t)$
(time-dependent)
A
ΓA
Q
Island

⑦ Conclusions

- Moving mirrors provide a class of non-equilibrium setups, analogous to Hawking radiations and BH evaporations.
- We computed the time evolution of entanglement entropy (EE) and gave its clear explanation in terms of entangled pair productions.
- In a moving mirror model which mimics a BH evaporation, we showed that the EE follows an ideal Page curve.
- We presented a gravity dual of moving mirror via the AdS/BCFT. Our moving mirror setup may be interpreted as a deformation of brane-world derivation of Page curve.

Further directions

- Double Moving Mirrors ?
  [→Our forthcoming longer paper]
- Higher Dimensional Generalizations
- Precise connection between an evaporating BH and
  a gravity dual of a moving mirror ?
- BH singularities ?
  [→ Space-like Boundary in CFT ?]
- Condensed Matter Applications ?
- Tensor Network Interpretation ?