Planckian Transport

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Holotube Colloquium (virtually)
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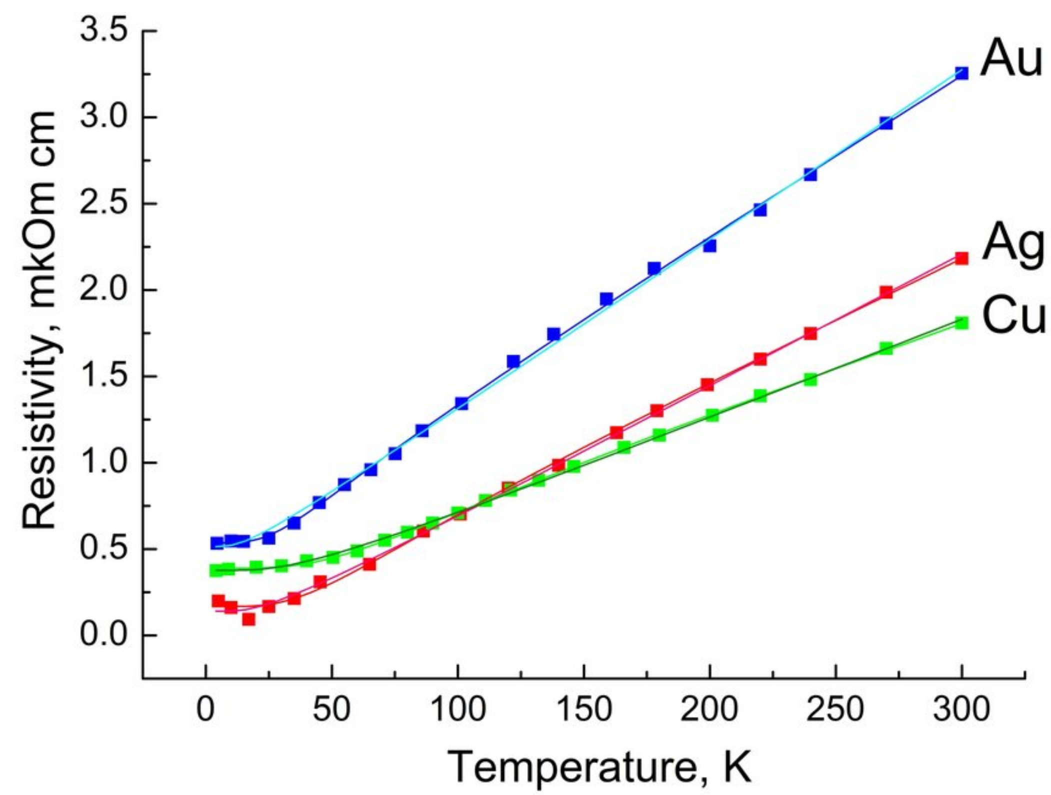
Let’s start at the beginning …

Copper, etc.



T-linear resistivity at ‘high’ temperatures due to scattering off classical atomic vibrations (phonons) with cross-section:

$$A \sim \langle (\Delta x)^2 \rangle \sim \frac{k_B T}{K}$$

[Wien 1913, Bloch 1928]

Copper, etc.

The resistivity is determined by the electronic lifetime τ according to the Drude formula:

$$\rho = \frac{m}{ne^2\tau}$$

Even for good conductors such copper, gold and silver, τ is (i) very short and (ii) quantum mechanical:

$$\tau \approx \frac{\hbar}{k_B T}$$

Planckian transport 1934

[Peierls, "Remarks on the theory of metals". 1934]

The correct condition for the usual calculational methods to be applied in the theory of conductivity, is therefore

$$\frac{h}{\tau} < 4kT\,, \tag{5}$$

If one looks at the validity of (5) for actual metals, first of all for $T > \Theta$, one finds that the inequality will in general not be satisfied in the sense of "small compared with" but that the quantities on both sides are of the same order of magnitude[1].

For the calculation of more precise numerical values of τ, one has to know something about the number of free electrons per atom, and their effective mass. For metals for which it is reasonable to set the number of conduction electrons equal to the chemical valency and their mass equal to that of free electrons, one obtains the following values for $h/(4kT\tau)$ at room temperature[2]:

Ag	Au	Cs	Cu	K	Li	Mg	Na	Rb
0.8	1.2	1.5	1.0	0.8	2.7	1.5	1.1	1.2

[1] H. Bethe has drawn my attention to the fact that this is no accident, but is generally true on dimensional grounds.

Planckian Copper etc.

- Dimensional analysis in copper etc. above T_D:

$$\frac{1}{\tau} = \frac{v_F}{\ell} \left\{ \begin{array}{ll} \dfrac{1}{\ell} = nA \sim \dfrac{1}{a^3}\dfrac{k_B T}{K} \sim \dfrac{1}{a}\dfrac{k_B T}{t} & \text{[classical]} \\ v_F \sim \dfrac{a\,t}{\hbar} & \text{[quantum]} \end{array} \right.$$

- Therefore: $$\frac{1}{\tau} \sim \frac{k_B T}{\hbar}$$

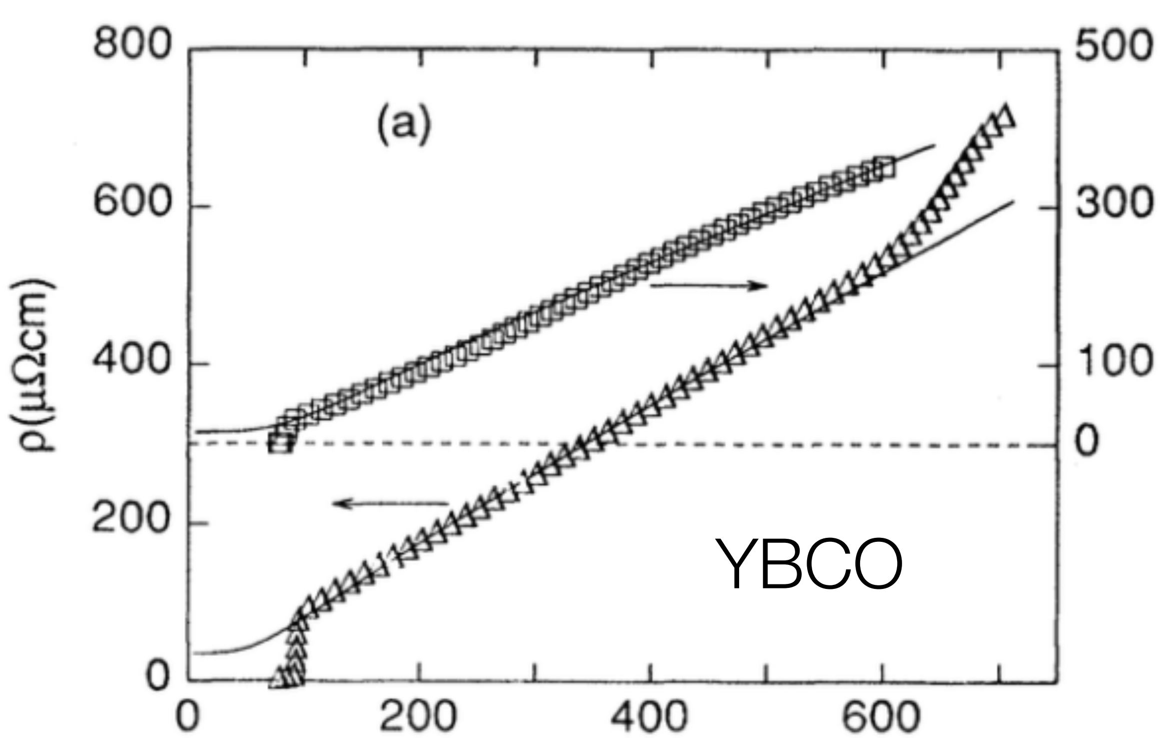
Scattering rate is large because velocity is big.

This ancient knowledge was largely lost in the mists of time …

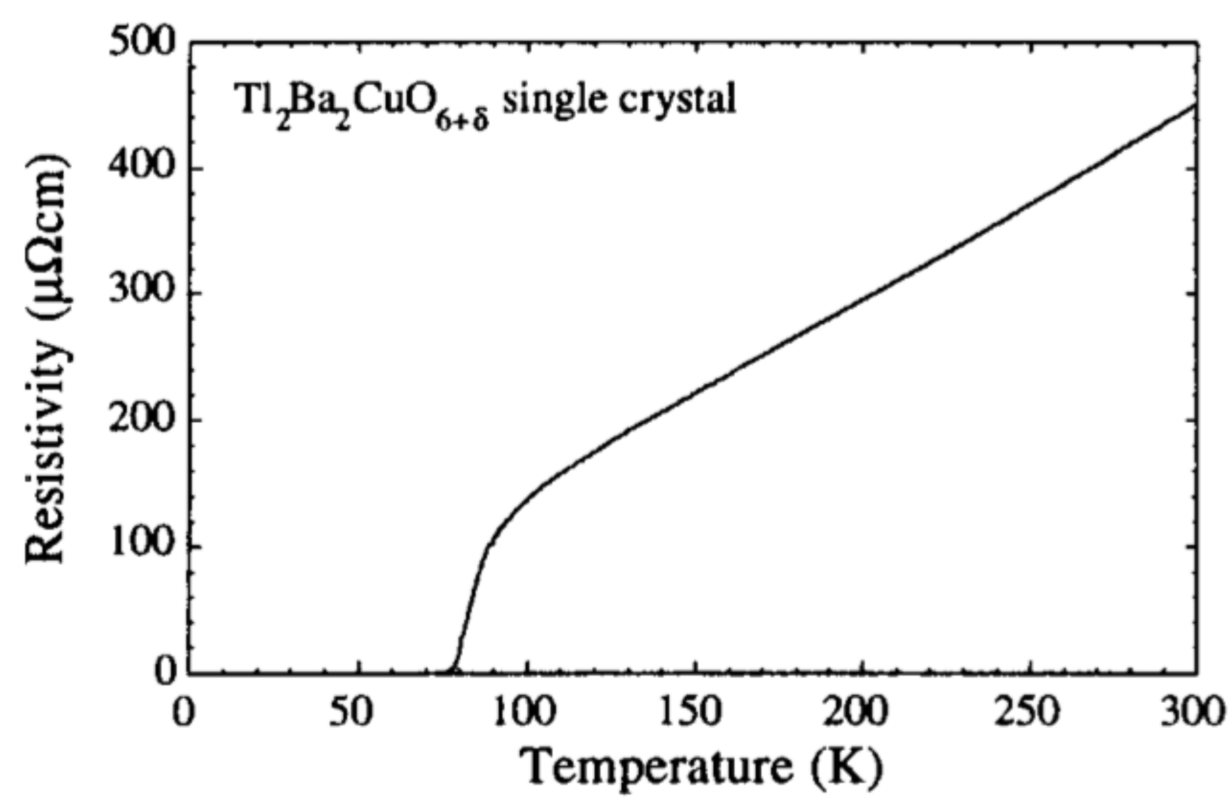
Jumping forward a few decades …

High Tc superconductors

T-linear resistivity over wide range of temperatures.
Planckian scattering rate.
Short mean free paths (small Fermi velocity)



[Martin et al. '88]

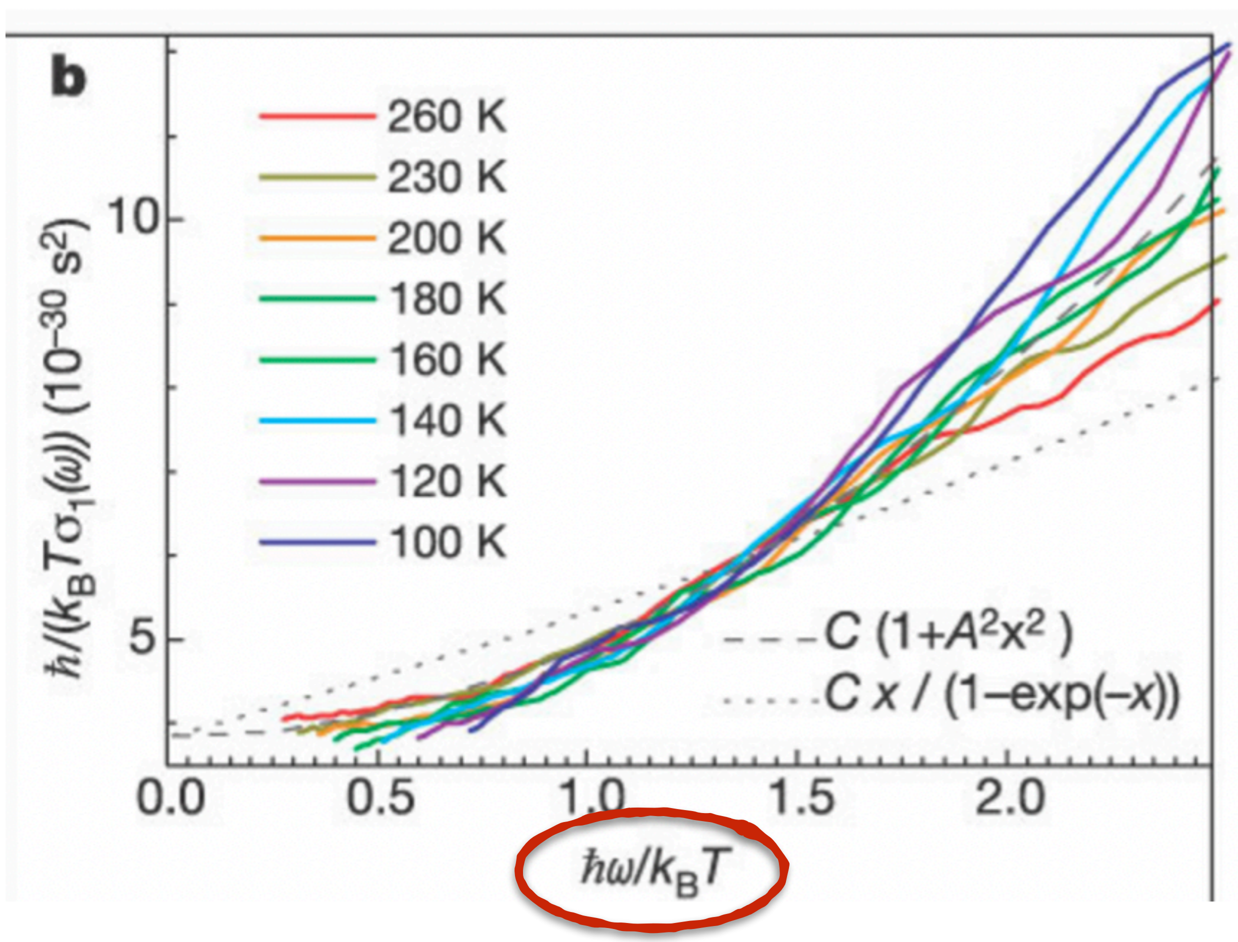


[Tyler-Mackenzie '97]

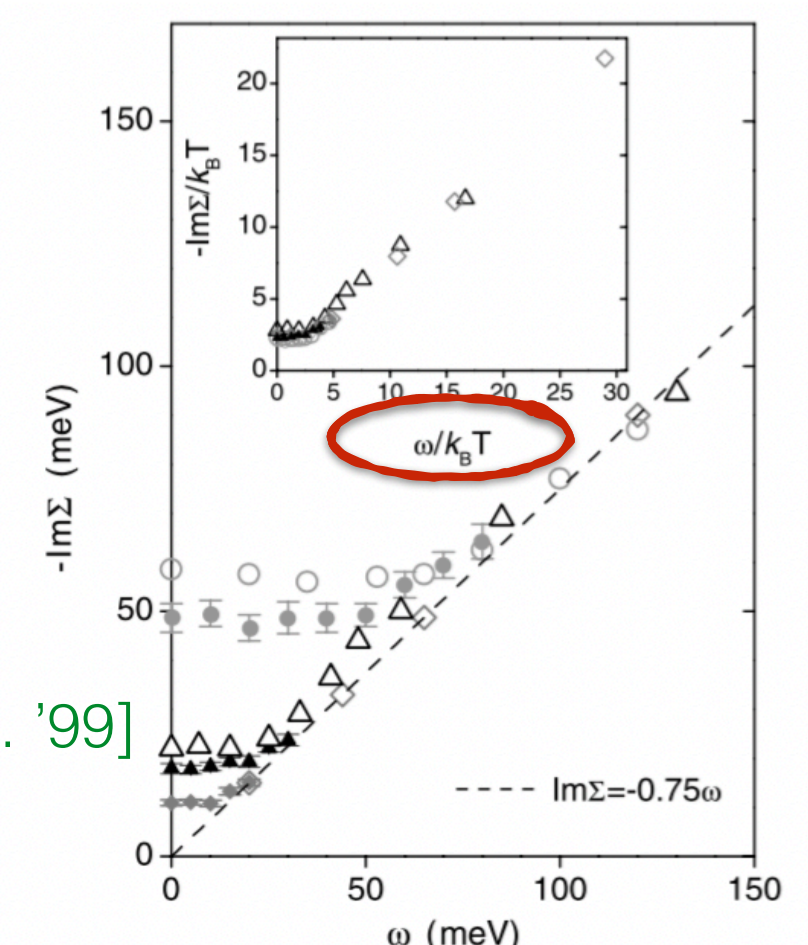
High Tc superconductors

Scaling collapse:

Planckian scattering from quantum criticality? [cf. Sachdev '99]



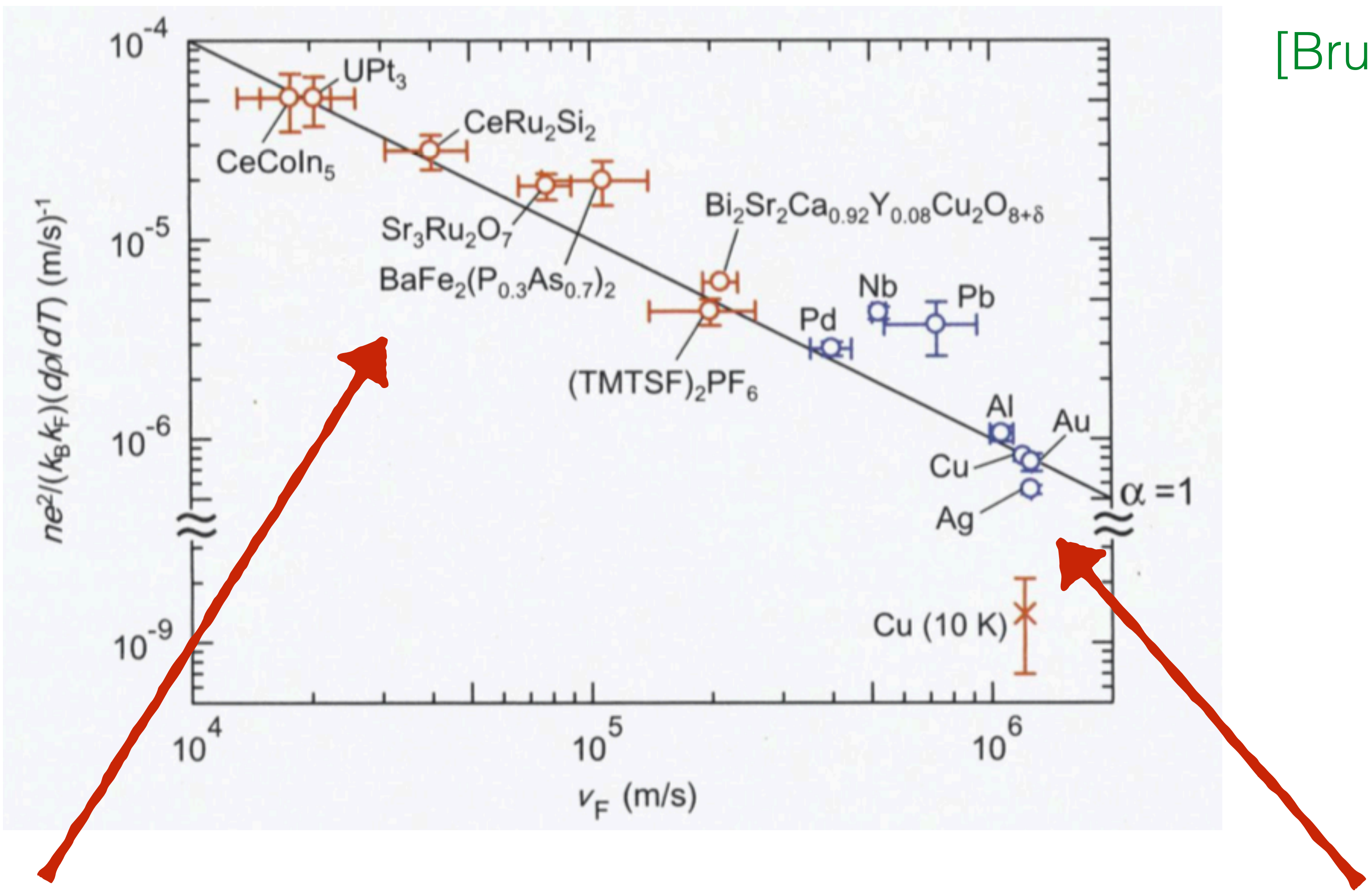
[van der Marel et al. 03]



[Valla et al. '99]

$$\rho(\omega) = T f\left(\frac{\hbar\omega}{k_B T}\right) \qquad \Sigma''(\omega) = T g\left(\frac{\hbar\omega}{k_B T}\right)$$

The unity of materials?



[Bruin et al '13]

$$\tau \approx \frac{\hbar}{k_B T}$$

Short mean free path

Long mean free path

Coincidence or a mysterious universality?

Meanwhile, in a galaxy far, far away …

Quark-gluon plasma

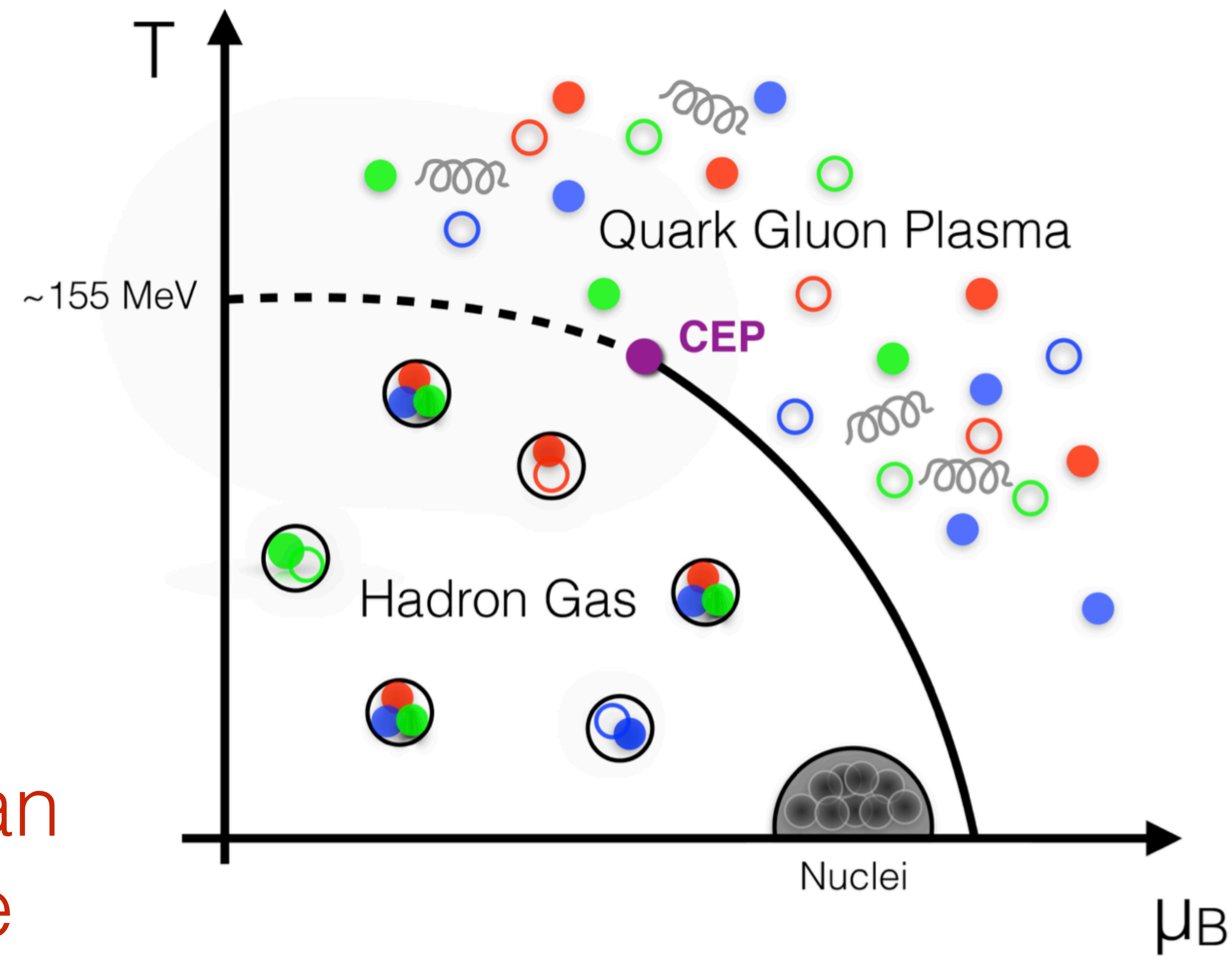
- RHIC observed the quark-gluon plasma has viscosity

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}$$

Implies the transverse momentum diffusivity:

$$D_\perp = c^2 \frac{\eta}{sT} \sim c^2 \frac{\hbar}{k_B T}$$

Planckian lifetime



Planckian bound?

Analogy between the quark-gluon plasma and cuprates implicitly made by [Zaanen '04]. Introduced the word 'Planckian'.

Superconductivity

Why the temperature is high

Jan Zaanen

According to a new empirical law, the transition temperature to superconductivity is high in copper oxides because their metallic states are as viscous as is permitted by the laws of quantum physics.

Could dissipation in quantum systems be subject to a Planckian bound?:

$$\tau \gtrsim \frac{\hbar}{k_B T}$$

[cf. Sachdev '99]

For relativistic systems, implies the conjectured Kovtun-Son-Starinets bound:

$$\frac{\eta}{s} \gtrsim \frac{\hbar}{k_B}$$

[KSS '05]

Planck hits the mainstream

Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates [Nat Phys 2018]

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6*] and C. Proust[3,6*]

PHYSICAL REVIEW LETTERS **123,** 066601 (2019)
Editors' Suggestion

Theory of a Planckian Metal

Aavishkar A. Patel and Subir Sachdev
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

PHYSICAL REVIEW LETTERS **124,** 076801 (2020)
Editors' Suggestion | Featured in Physics

Strange Metal in Magic-Angle Graphene with near Planckian Dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,2,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigorda,[1] Kenji Watanabe,[3] Takashi Taniguchi,[3] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,‡]
[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Department of Physics, Cornell University, Ithaca, New York 14853, USA*
[3]*National Institute for Materials Science, Namiki 1-1, Tsukuba, Ibaraki 305-0044, Japan*

PHYSICAL REVIEW LETTERS **122,** 216601 (2019)
Editors' Suggestion

Operator Size at Finite Temperature and Planckian Bounds on Quantum Dynamics

Andrew Lucas*
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A Planckian bound was actually proven on a certain quantum Lyapunov exponent [Maldacena-Shenker-Stanford '15]

Suggests a possible connection between many-body quantum chaos and transport [Blake '16, …]

Planckian electrons and phonons in cuprates

Phonons and electrons in cuprates

- In the remainder I want to close the circle of ideas discussed so far, by discussing the role of electron-phonon scattering in cuprates.

- Phonons have every right to produce Planckian scattering of electrons in cuprates, as they do in conventional metals. Can we see it?

[Mousatov-Hartnoll, arXiv:2011.10466]

Diffusion

I suggested that diffusion was the right observable to bound. It occurs with or without quasiparticles and only requires a conserved quantity and local dynamics.

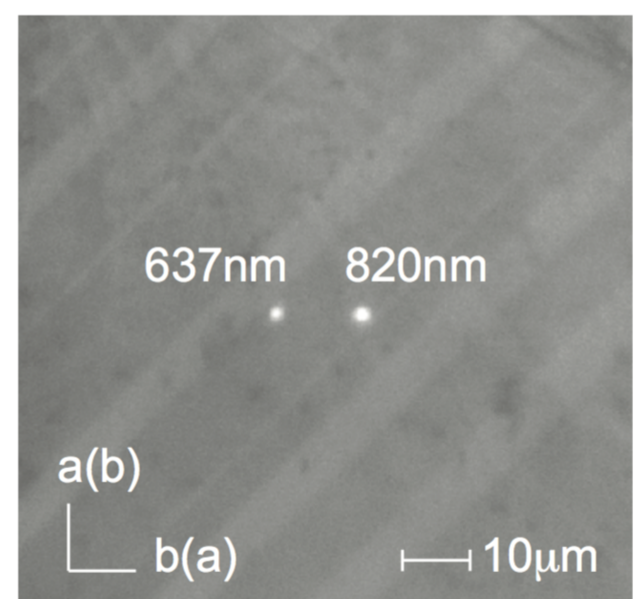
ARTICLES
PUBLISHED ONLINE: 23 DECEMBER 2014 | DOI: 10.1038/NPHYS3174
nature physics

Theory of universal incoherent metallic transport

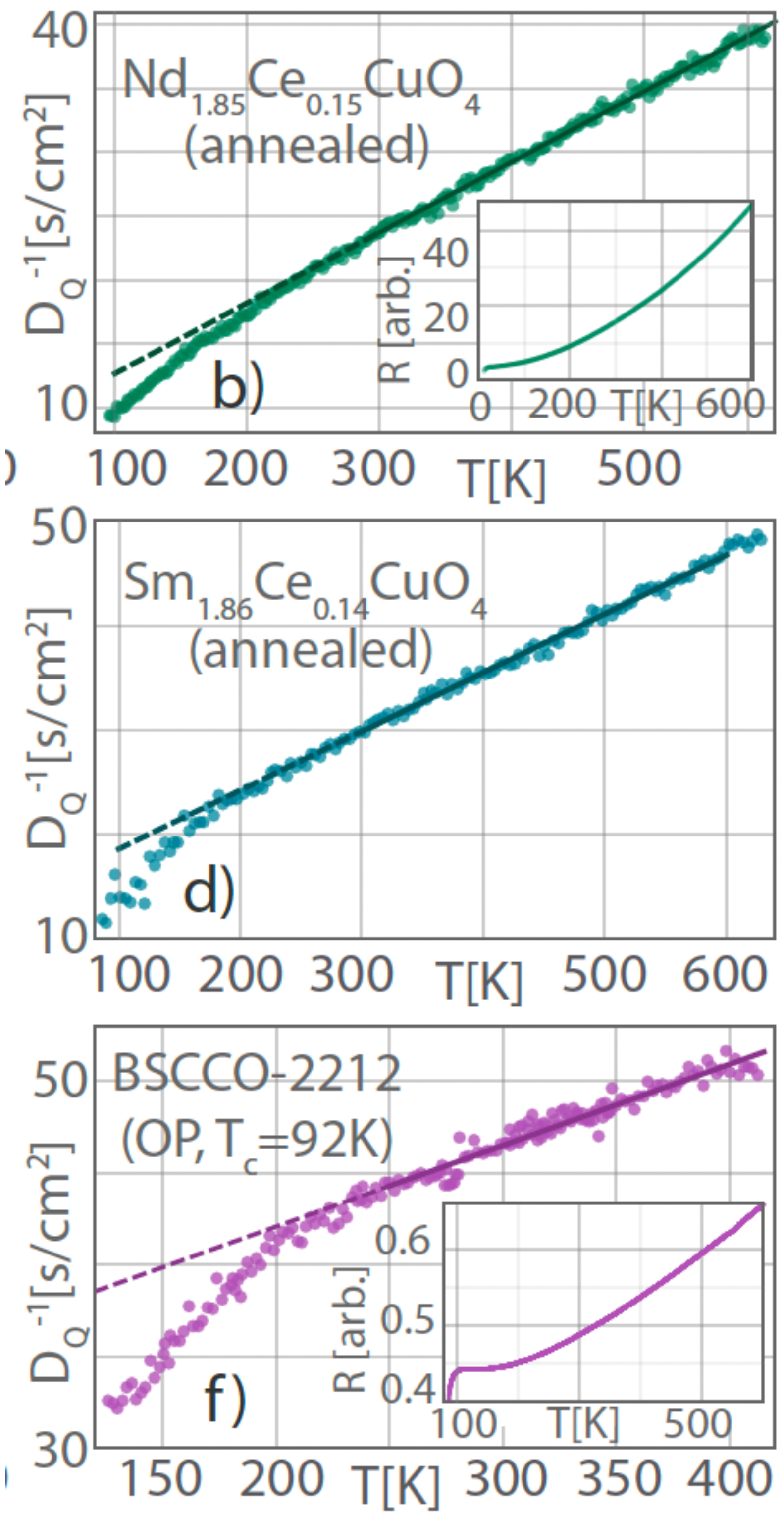
Sean A. Hartnoll

$$\begin{cases} D^{-1}_{\text{charge}} \sim \dfrac{1}{v_F^2}\dfrac{k_B T}{\hbar} \\ D^{-1}_{\text{mom.}} \sim \dfrac{1}{c^2}\dfrac{k_B T}{\hbar} \end{cases}$$

Inspired my colleague Kapitulnik to measure thermal diffusion of cuprates:



High Tc, thermally



Measurements of thermal diffusivity in cuprates showed a fascinating parallel with electronic transport.

Anomalous thermal diffusivity in underdoped $YBa_2Cu_3O_{6+x}$ [PNAS '17]

Jiecheng Zhang[a,b], Eli M. Levenson-Falk[a,b], B. J. Ramshaw[c], D. A. Bonn[d,e], Ruixing Liang[d,e], W. N. Hardy[d,e], Sean A. Hartnoll[b], and Aharon Kapitulnik[a,b,f,1]

Thermal Diffusivity Above the Mott-Ioffe-Regel Limit

Jiecheng Zhang,[1,2,*] Erik D. Kountz,[1,2] Eli M. Levenson-Falk,[3] Dongjoon Song,[4] Richard L. Greene,[5,6] and Aharon Kapitulnik[1,2,7]

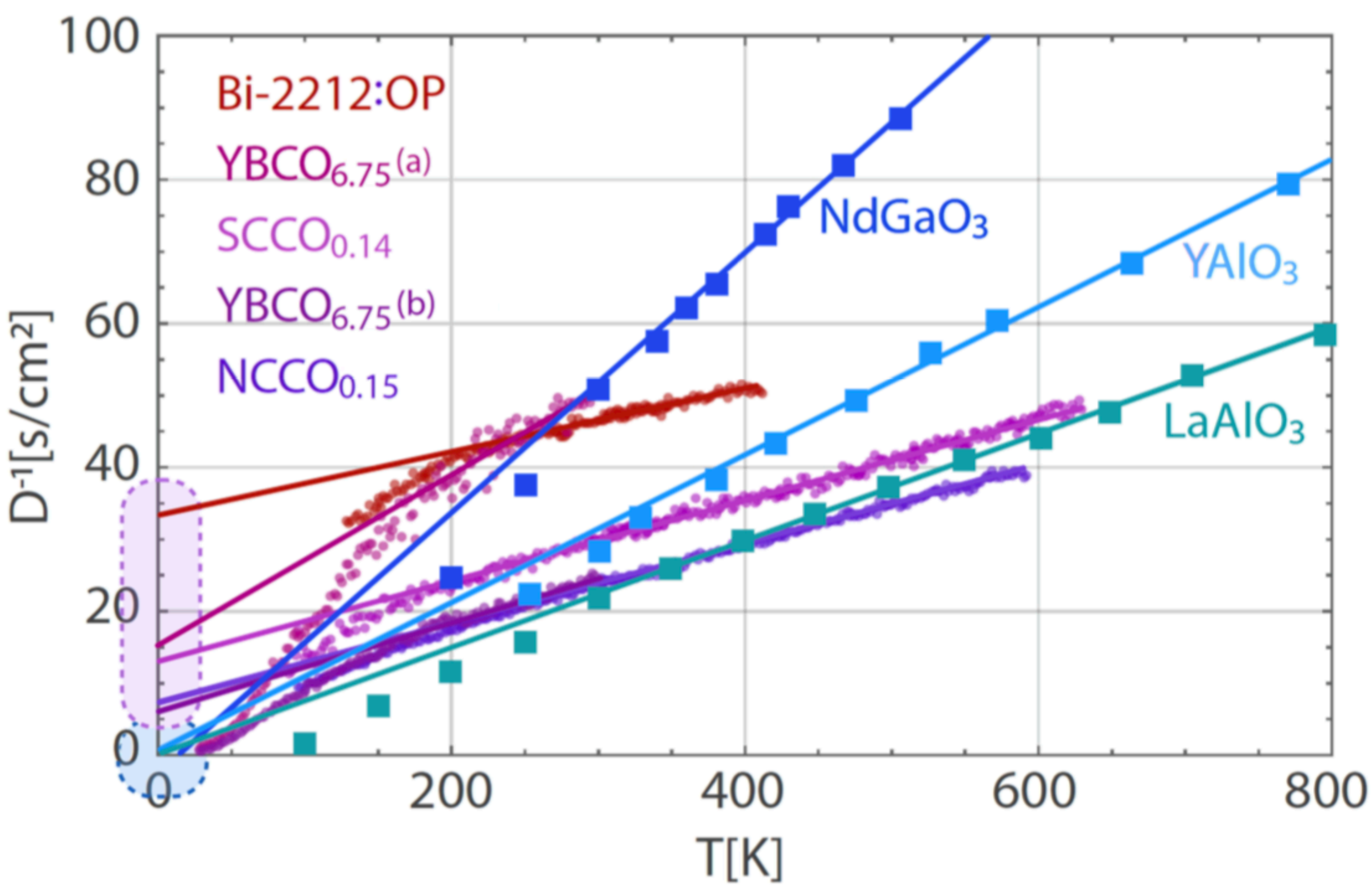
[PRB '19]

$$D^{-1} \sim \frac{1}{v_s^2}\frac{k_B T}{\hbar}$$ ⟷ Planckian!

Suggests phonons

Comparison to insulators

- High temperature behavior of thermal diffusivity is in fact familiar from insulators (anharmonic phonons). But there is an additional offset.



[Zhang et al PNAS 2020]

Looks electronic

$$D_{\rm th}^{-1} \approx \frac{k_B T}{\hbar}\frac{1}{v_s^2} + \frac{m_\star}{\hbar}$$

Explain this in the remainder

Heat in cuprates

- In conventional metals heat transport at room temperature is dominated by electrons. This is manifested in the Wiedemann-Franz law:

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\frac{k_B^2}{e^2} = L_0\,.$$

- In cuprates find at high temperatures $L > L_0$ (often by a factor of 3 or more), so that heat transport is dominated by phonons.

- Explains why leading behavior is as in insulators. But what about the offset?

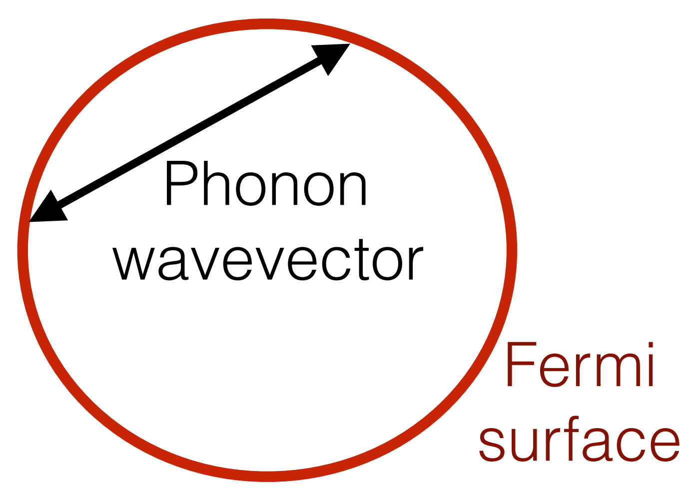
Heat in cuprates

- In a metal phonons can also scatter off electrons (particle-hole pairs). Matthiessen's rule suggests

$$D_{\rm th}^{-1} \sim D_{\rm th,ph}^{-1} \sim \frac{d}{v_s^2}\frac{1}{\tau_{\rm ph}} = \frac{d}{v_s^2}\left(\frac{1}{\tau_{\rm ph\to ph}} + \frac{1}{\tau_{\rm ph\to el}}\right)$$

- A simple textbook scattering computation gives:

$$\frac{1}{\tau_{\rm ph\to el}} \sim {\rm const.} \times \frac{m_\star v_s^2}{\hbar}$$



Action and reaction of electron and phonons

- The observations imply that the const. ~ O(1).
- This constant is determined by the strength of electron-phonon interactions.
- The same constant determines the converse scattering of electrons by phonons:

$$\frac{1}{\tau_{\mathrm{el}\to\mathrm{ph}}} \sim \mathrm{const.} \times \frac{k_B T}{\hbar}$$

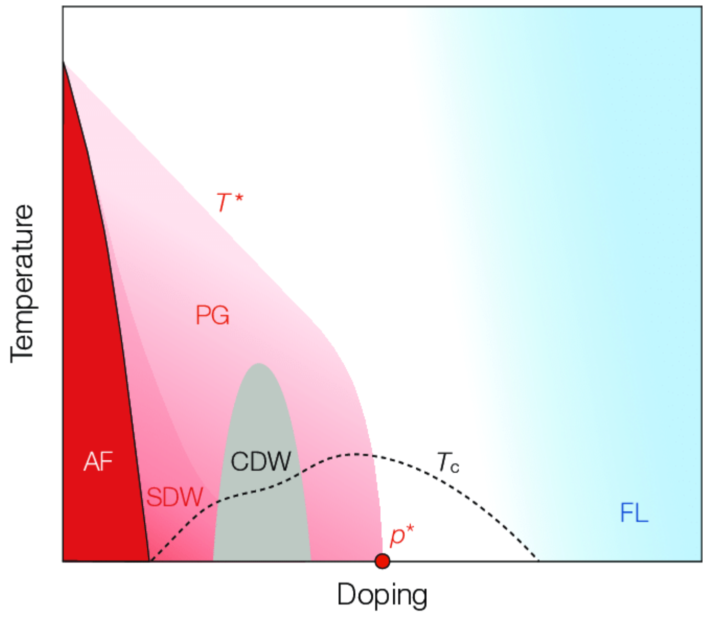
$$\wr\wr$$

$$\frac{1}{\tau_{\mathrm{ph}\to\mathrm{el}}} \sim \mathrm{const.} \times \frac{m_\star v_s^2}{\hbar}$$

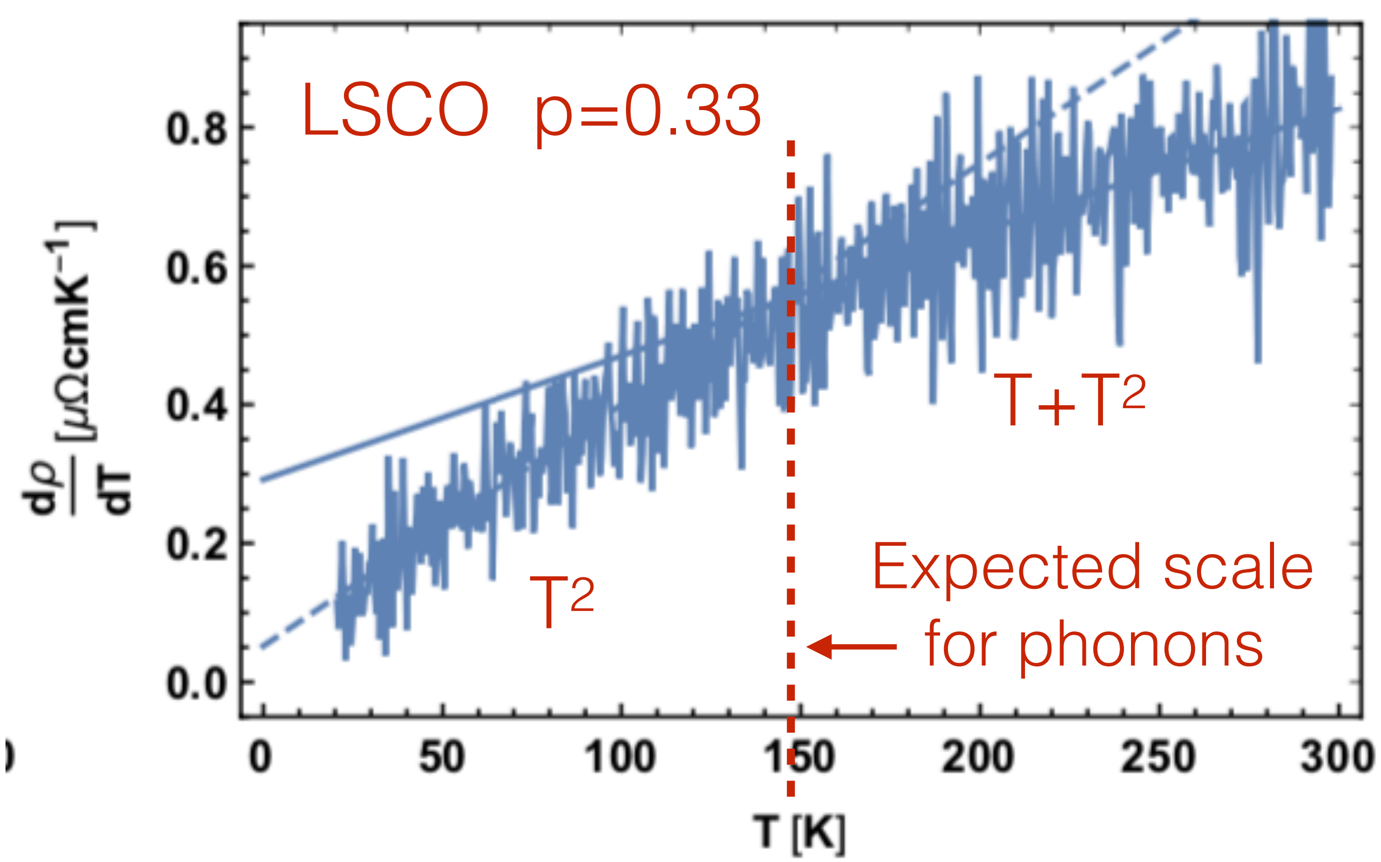
Planckian!

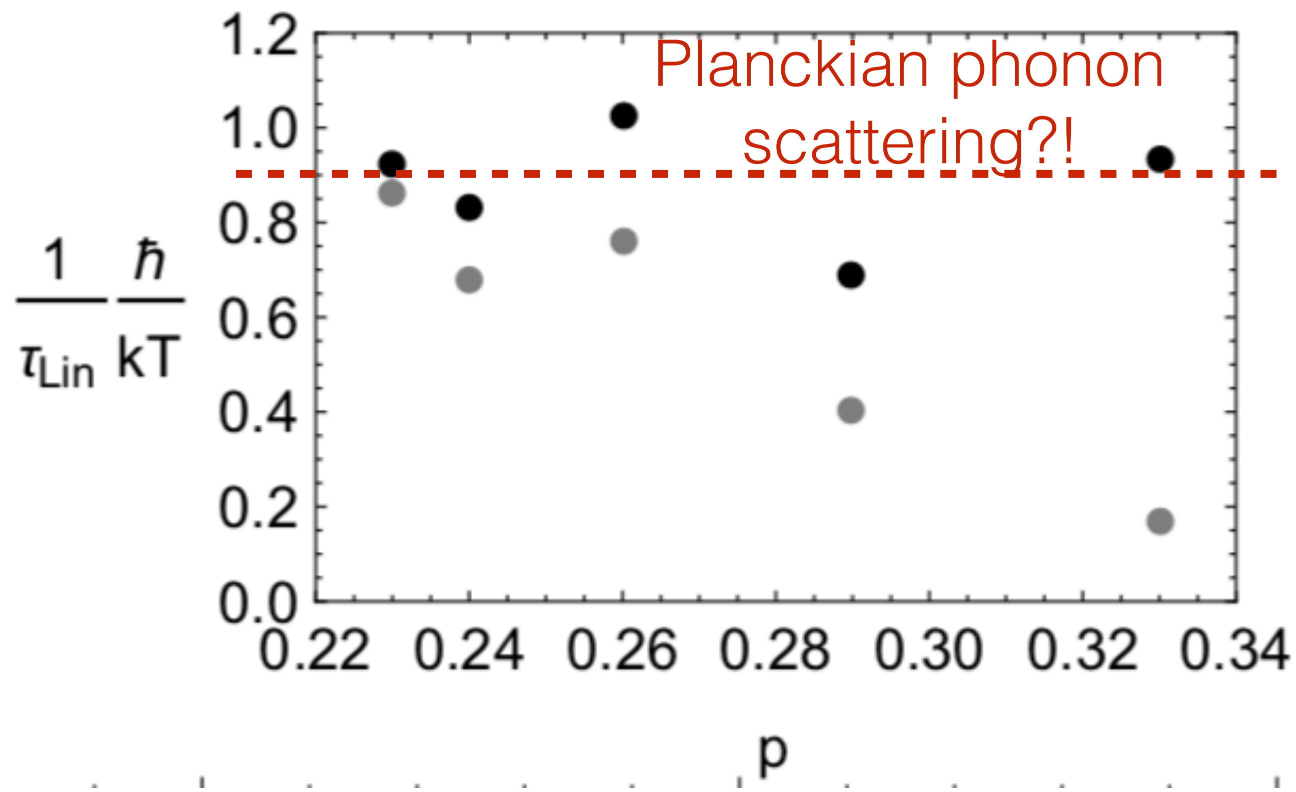
Electrons and phonons in cuprates

- Just seen that phonons should produce Planckian scattering of electrons in cuprates.
- Sharp paradox: At optimal doping Planckian scattering continues to T=0 with no feature at the scale where phonon scattering should appear.
- We decided to re-examine old data from overdoped cuprates, which should be more conventional…



Electrons and phonons in cuprates





Overdoped: See onset of Planckian phonon scattering exactly where it should be! Seems to merge with quantum critical scattering at p ~ 0.2!

[Data from Ando et al '04 and Cooper et al '09]

[Analysis similar but different to Hussey et al '11]

Summary

- Planckian scattering arises in both the most conventional ('phonon') and the least conventional ('quantum critical') metals. [and elsewhere!]
- Seen that in high Tc cuprates, furthermore, these two mechanisms merge into each other. Possibly supports the notion of a fundamental bound.
- The scale $k_B T/\hbar$ is insensitive to details, depending only on statistical and quantum mechanics. If a Planckian bound does exist, should be provable!