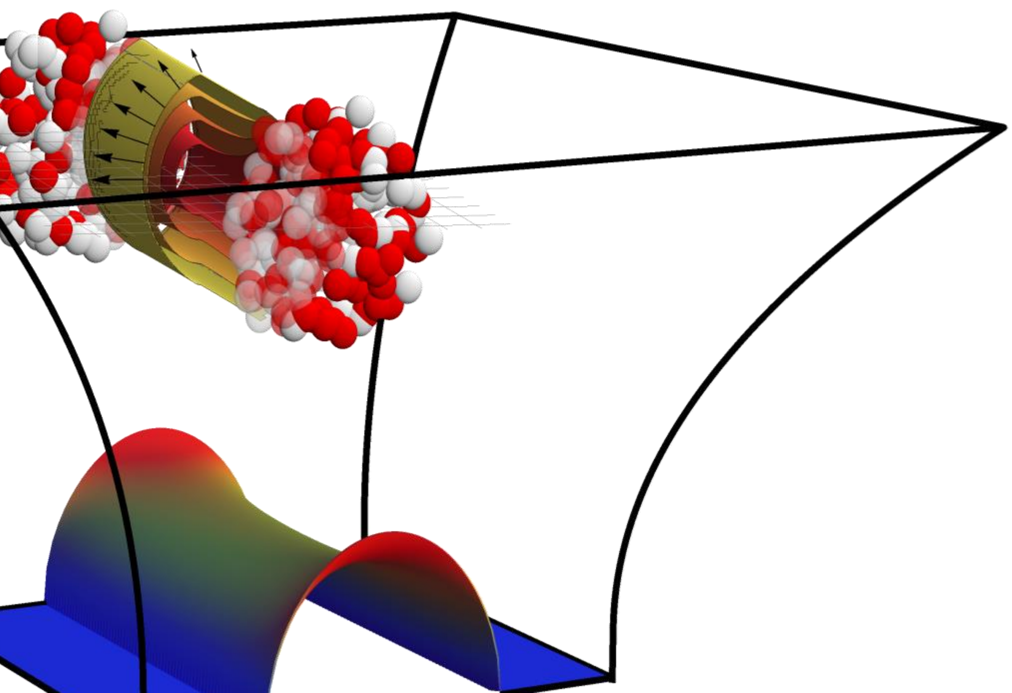
WHAT ATTRACTS TO ATTRACTORS?

FROM FAR-FROM-EQUILIBRIUM TO HYDRODYNAMICS

Based on work with Aleksi Kurkela, Urs Wiedemann and Bin Wu
Reference: 1907.08101 (PRL 2020)



Wilke van der Schee
HoloTube
Geneva, 26 January 2021

HYDRO AND QGP

Exploring the limits of hydrodynamics

BIG QUESTIONS

1. How does QGP form and hydrodynamize within 1 fm/c? What are the qualitative differences, if any, between the description of hydrodynamization in a heavy ion collision obtained by assuming a weakly coupled initial stage versus a strongly coupled holographic calculation? Note that perturbative calculations typically treat $\alpha_s = 0.3$ as small while holographic calculations treat the corresponding 't Hooft coupling $\lambda \approx 11$ as large. What can we learn about the timescales and dynamics of hydrodynamization, and hence QGP formation, by analyzing the wakes that jets leave behind as they traverse a droplet of QGP?
2. What are the limits of the applicability of hydrodynamics? Can it be applied even to systems of size a fermi or less? What is the smallest droplet of QGP that behaves hydrodynamically, and how does the answer to this question change at very high temperatures where $\eta/s > 1$ and QGP is no longer a strongly coupled liquid?

Wit Busza, Krishna Rajagopal and WS, Heavy Ion Collisions: The Big Picture, and the Big Questions (2018)

THE BIG QUESTIONS

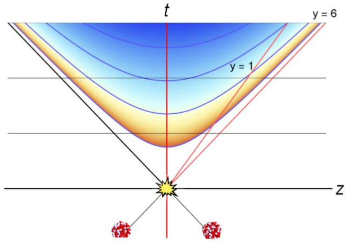
When is hydrodynamics applicable?

- And what kind of hydrodynamics? (1^{st}, 2^{nd} or infinite order?)
- Also: dependent on position

What is the initial fluid profile at this time?

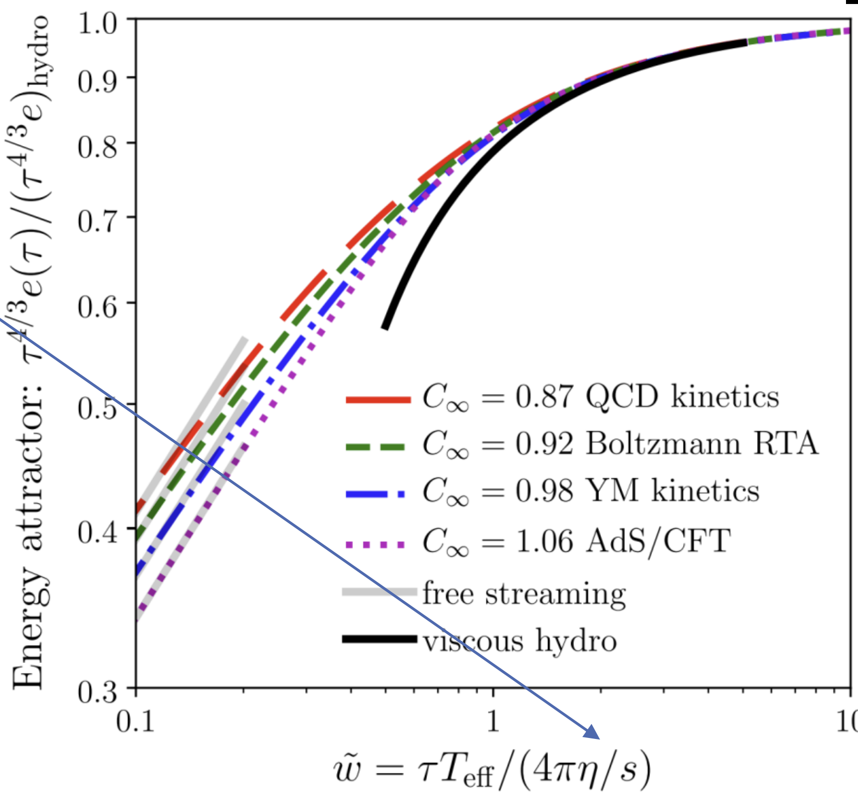
- Presumably depending (strongly) on initial stage model
  - Strong versus weak? Constituent `quarks'?
- Often argued: memory loss due to attractor
  - But is this really so?
  - → dynamics is not necessarily on the attractor
  - → even on attractor: lot of 3+1D freedom left

ATTRACTORS AND QGP



Energy density vs time, boost-invariant: 0+1D

- Late time: $e \propto \tau^{-4/3}$
- Attractor = approach to hydro
- Depends (crucially) on η/s
- Depends (slightly) on theory (C_∞)



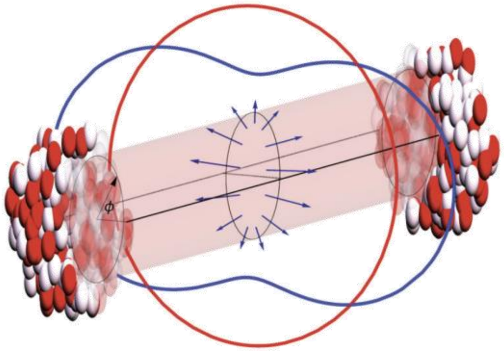
Attractor determines pressure

- Determines work done
- Can be used to estimate entropy:

$$(s\tau)_{\text{hydro}} = \frac{4}{3} C_\infty^{3/4} \left(4\pi \frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30} \nu_{\text{eff}}\right)^{1/3} (e\tau)_0^{2/3}$$

$$\frac{dN_{\text{ch}}}{d\eta} \approx \frac{1}{J} A_\perp (s\tau)_{\text{hydro}} \frac{N_{\text{ch}}}{S}$$

G. Giacalone, A. Mazeliauskas and S. Schlichting Hydrodynamic attractors, initial state energy and particle production in HIC (2019)



ATTRACTORS AND QGP

A practical application

- Estimate multiplicity `canonically':

$$(n\tau)_0(\mathbf{x}_\perp) \propto (Q_s^<)^2(\mathbf{x}_\perp),$$

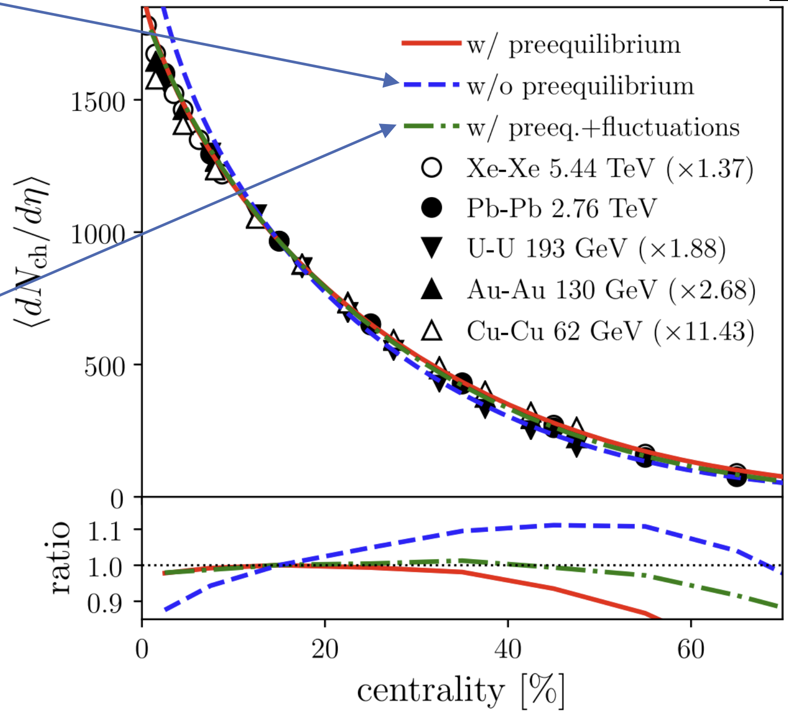
$$Q_s^2(\mathbf{x}_\perp) \propto T(\mathbf{x}_\perp),$$

$$\frac{dN_{\mathrm{ch}}}{d\eta} \propto \int d^2\mathbf{x}_\perp T^<(\mathbf{x}_\perp),$$

- Estimate using `attractor': (power 2/3!)

$$(e\tau)_0(\mathbf{x}_\perp) \propto (Q_s^<)^2(\mathbf{x}_\perp) Q_s^>(\mathbf{x}_\perp),$$

$$\frac{dN_{\mathrm{ch}}}{d\eta} \propto \int d^2\mathbf{x}_\perp \left(T^<(\mathbf{x}_\perp)\sqrt{T^>(\mathbf{x}_\perp)}\right)^{2/3}.$$



Even more practical: initial energy density

$$e_0 \approx 270\ \mathrm{GeV/fm^3} \left(\frac{\tau_0}{0.1\ \mathrm{fm}/c}\right)^{-1} \left(\frac{C_\infty}{0.87}\right)^{-9/8} \left(\frac{\eta/s}{2/4\pi}\right)^{-1/2} \left(\frac{A_\perp}{138\ \mathrm{fm}^2}\right)^{-3/2} \left(\frac{dN_{\mathrm{ch}}/d\eta}{1600}\right)^{3/2} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/2} \left(\frac{S/N_{\mathrm{ch}}}{7.5}\right)^{3/2}$$

G. Giacalone, A. Mazeliauskas and S. Schlichting Hydrodynamic attractors, initial state energy and particle production in HIC (2019)

HYDRODYNAMICS AND ATTRACTORS

Solve Muller-Israel-Stewart or Boltzmann equation

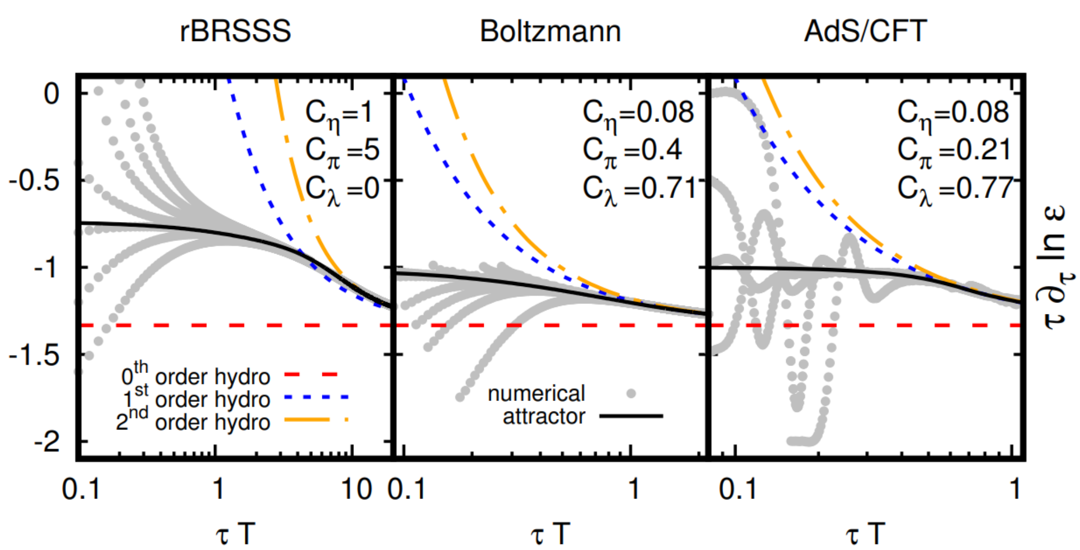
Hydro:

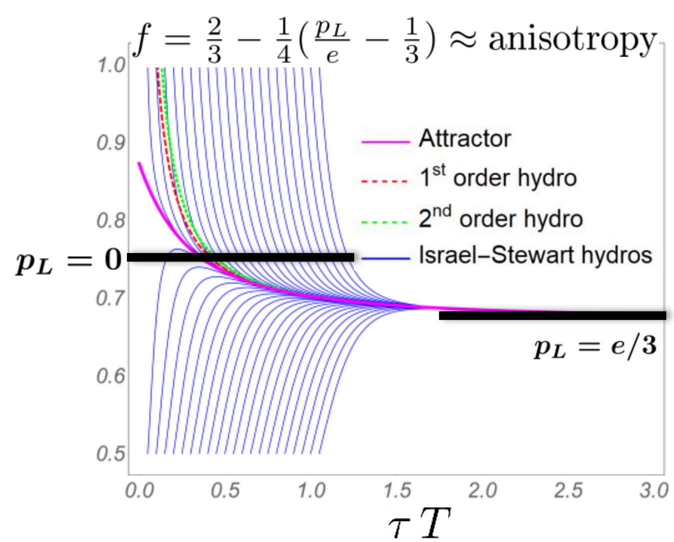
$$\partial_\tau \varepsilon = -\frac{1}{\tau}\left[\varepsilon + p_L\right]$$

$$\partial_\tau \phi + \frac{4}{3}\frac{\phi}{\tau} = -\frac{1}{\tau_R}\left[\phi - \frac{4}{3}\frac{\eta}{\tau}\right], \quad \phi \equiv \tfrac{1}{3}\varepsilon - p_L$$

RTA:

$$\partial_\tau f + \vec{v}_\perp \cdot \partial_{\vec{x}_\perp} f - \frac{p_z}{\tau}\partial_{p_z} f = -\frac{(-v_\mu u^\mu)}{\tau_R}\left[f - f_{\rm eq}\right]$$



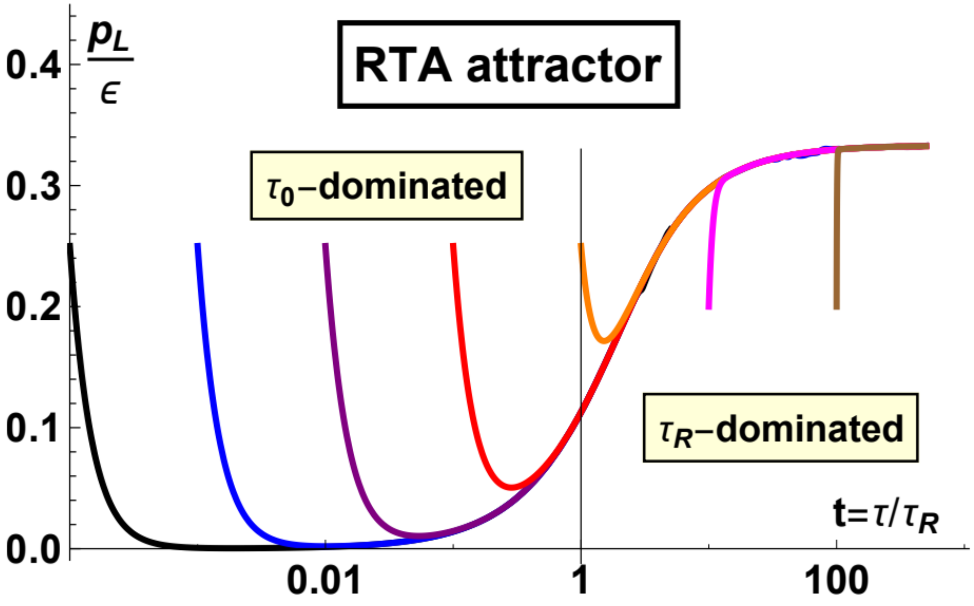


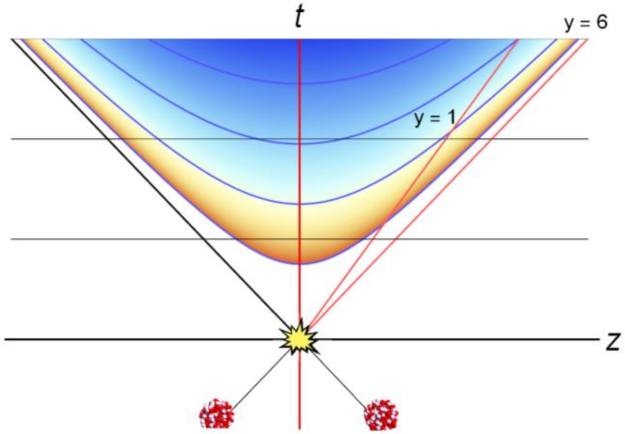
Michal P. Heller and Michal Spalinski, Hydrodynamics beyond the Gradient Expansion: Resurgence and Resummation (2015)
Paul Romatschke, Fluid Dynamics Far From Local Equilibrium (2017)

EARLY TIME DYNAMICS IN RTA

Depends crucially on initialization time:

- Decays to attractor solution in a time of order τ_0
  - Expansion `selects' particles with zero longitudinal momentum
- Curves get steeper once $\tau_0 > \tau_R$ (on log scale)

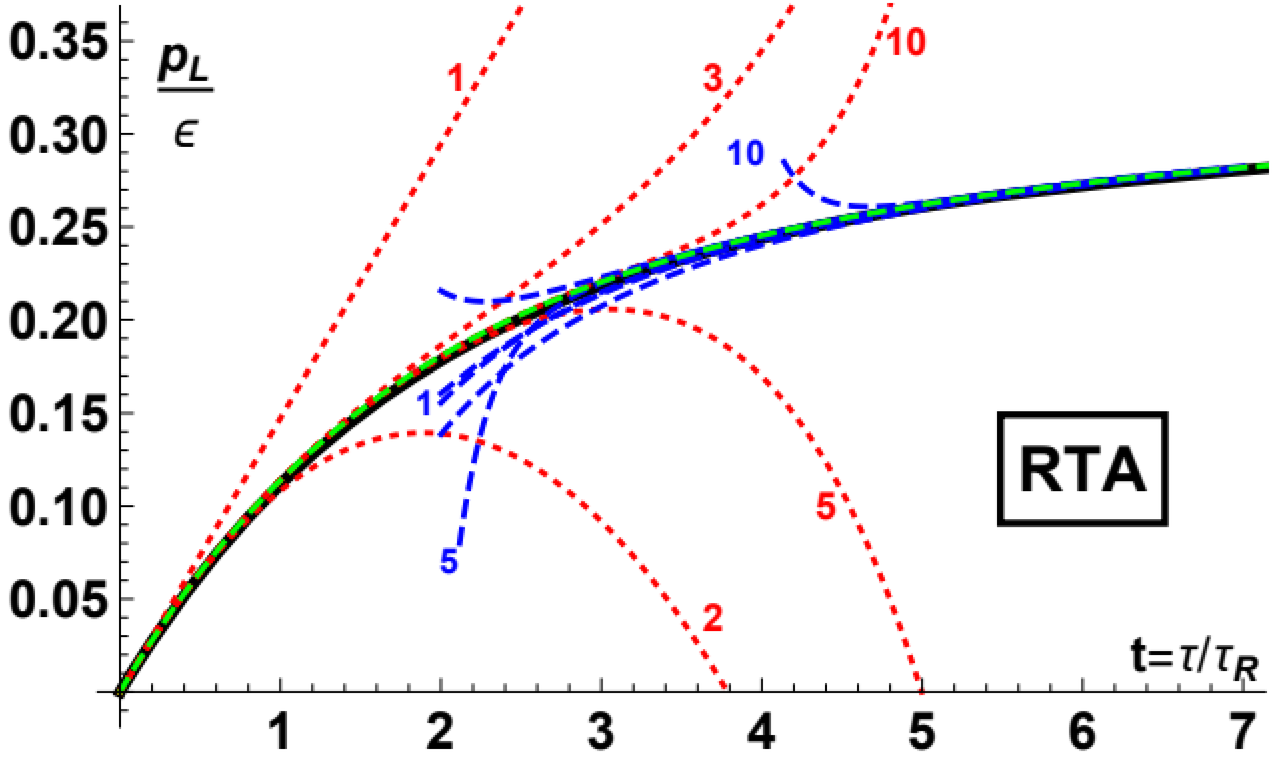




EARLY TIME EXPANSION

Expand attractor around early and late times:

- Starts at 0 (free streaming), grows (single hit), converges to isotropy
- Green-dashed: Pade approximant of early time expansion
- Converges for all times



Wilke van der Schee, CERN

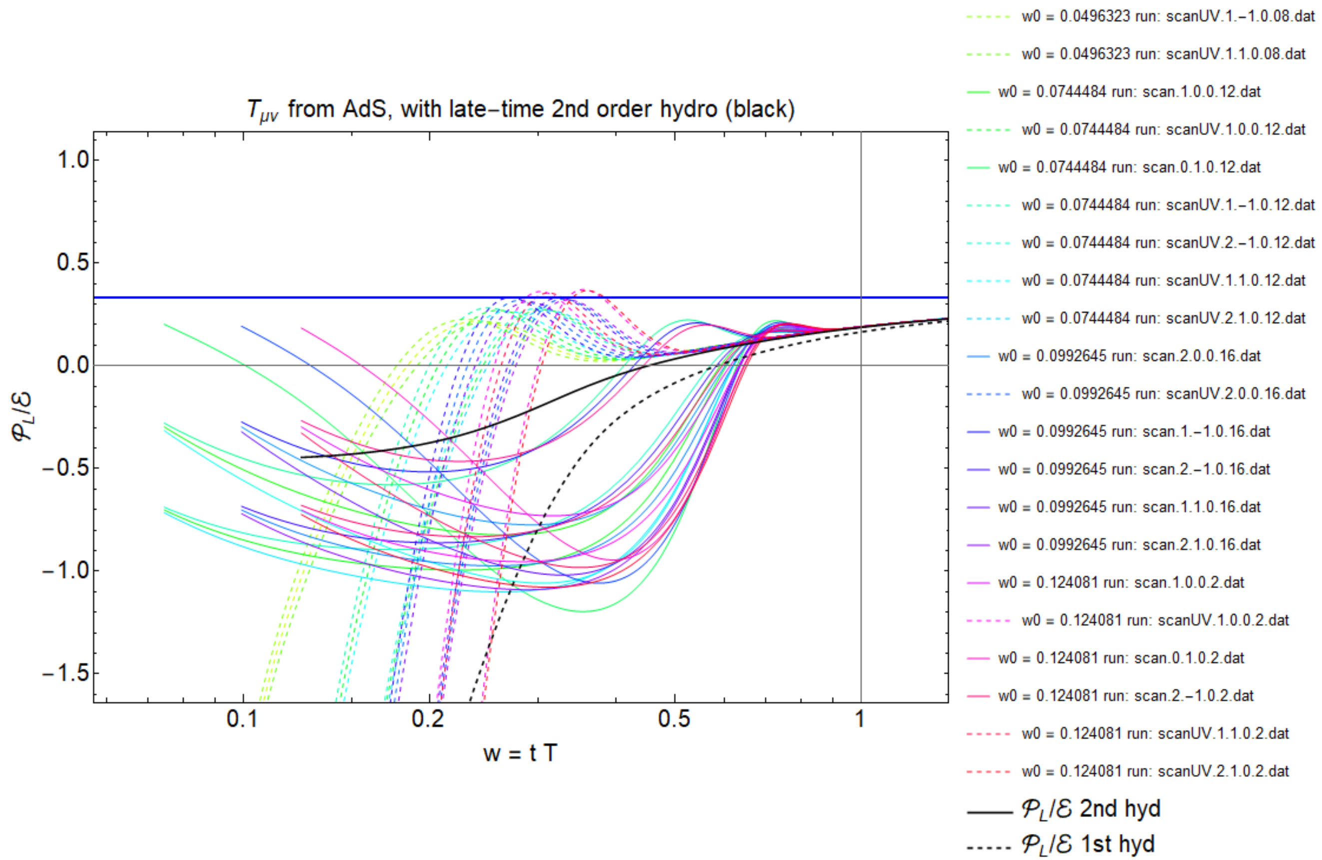
$$ds^2_{\mathrm{EF}} \;=\; -C\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + 2G\mathrm{d}z\mathrm{d}t + S^2\left(e^B\mathrm{d}\mathbf{x}_\perp^2 + e^{-2B}\mathrm{d}z^2\right)$$

ATTRACTOR IN ADS/CFT REVISITED

Try a wide range of initial profiles

- IR profile: $B_{IR}(\rho,\, \tau_0) = \rho^4(a + b\,\rho)$
- UV profile: $B_{UV}(\rho,\, \tau_0) = \rho^4 e^{-40\rho}(a + b\,\rho)$

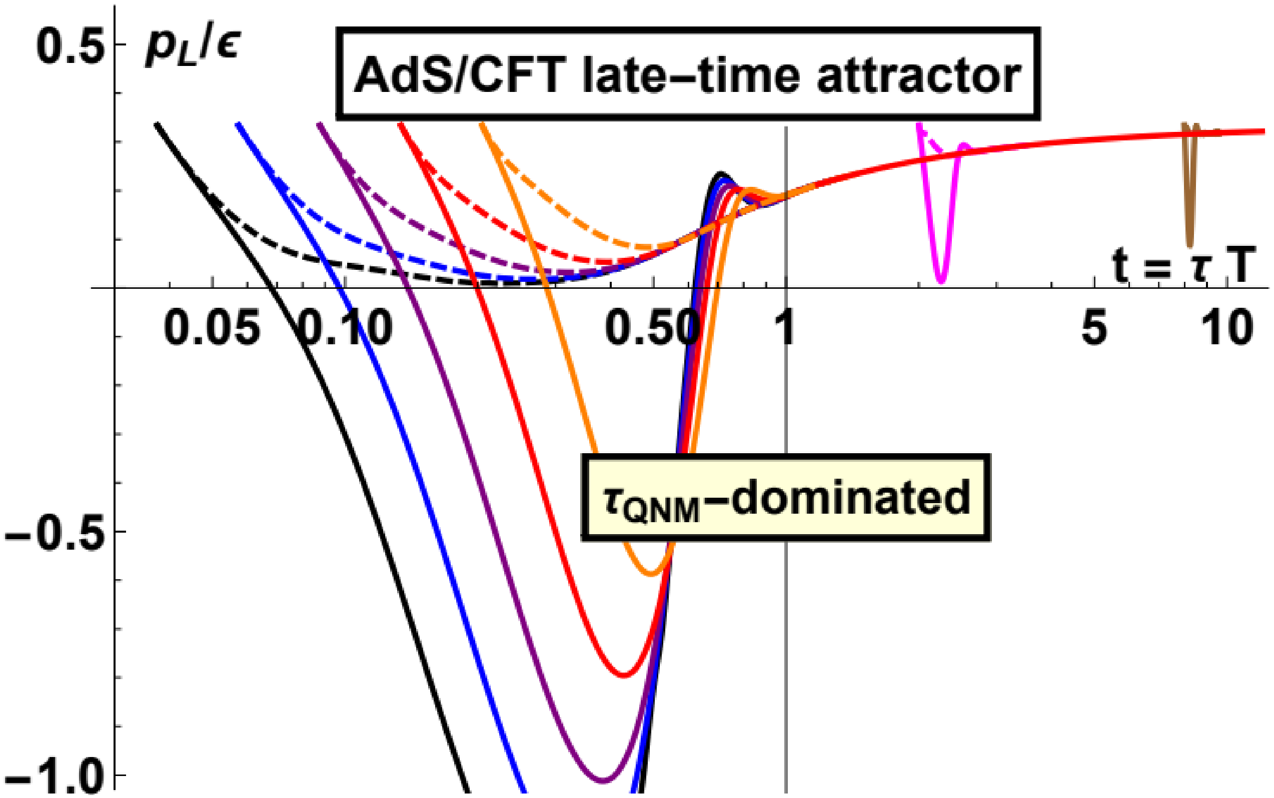
Could in principle follow from a real model, e.g. colliding sheets



ATTRACTOR IN ADS/CFT REVISITED

Start at seven different initial times

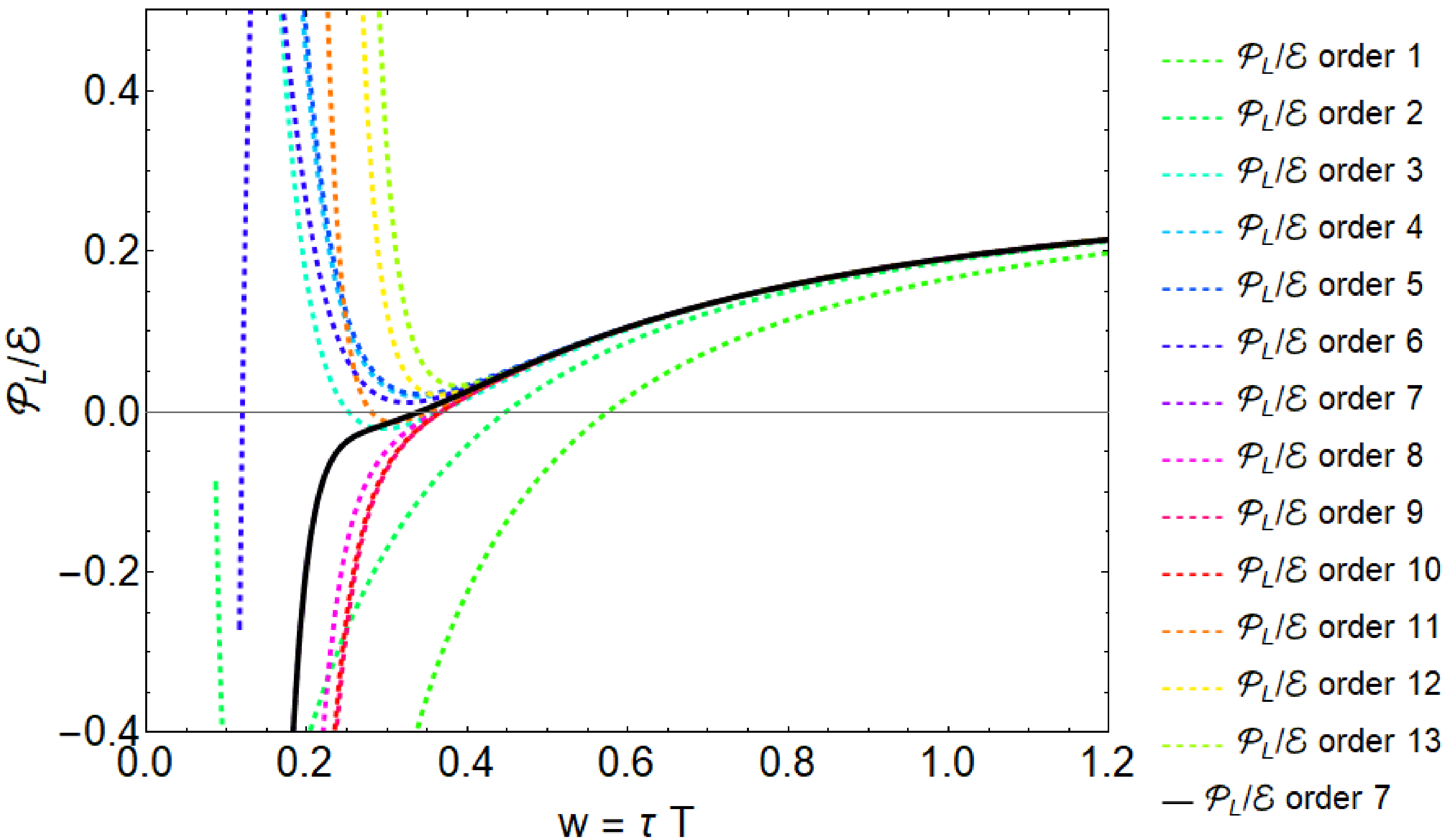
- UV + IR profiles, fixing initial isotropy + derivative
- Can distinguish a `profile-dependent' attractor (?)



APPROACH TO HYDRODYNAMICS

Many order hydrodynamics in Bjorken strongly coupled $\mathcal{N} = 4$

- Order 7 seems optimal (?)

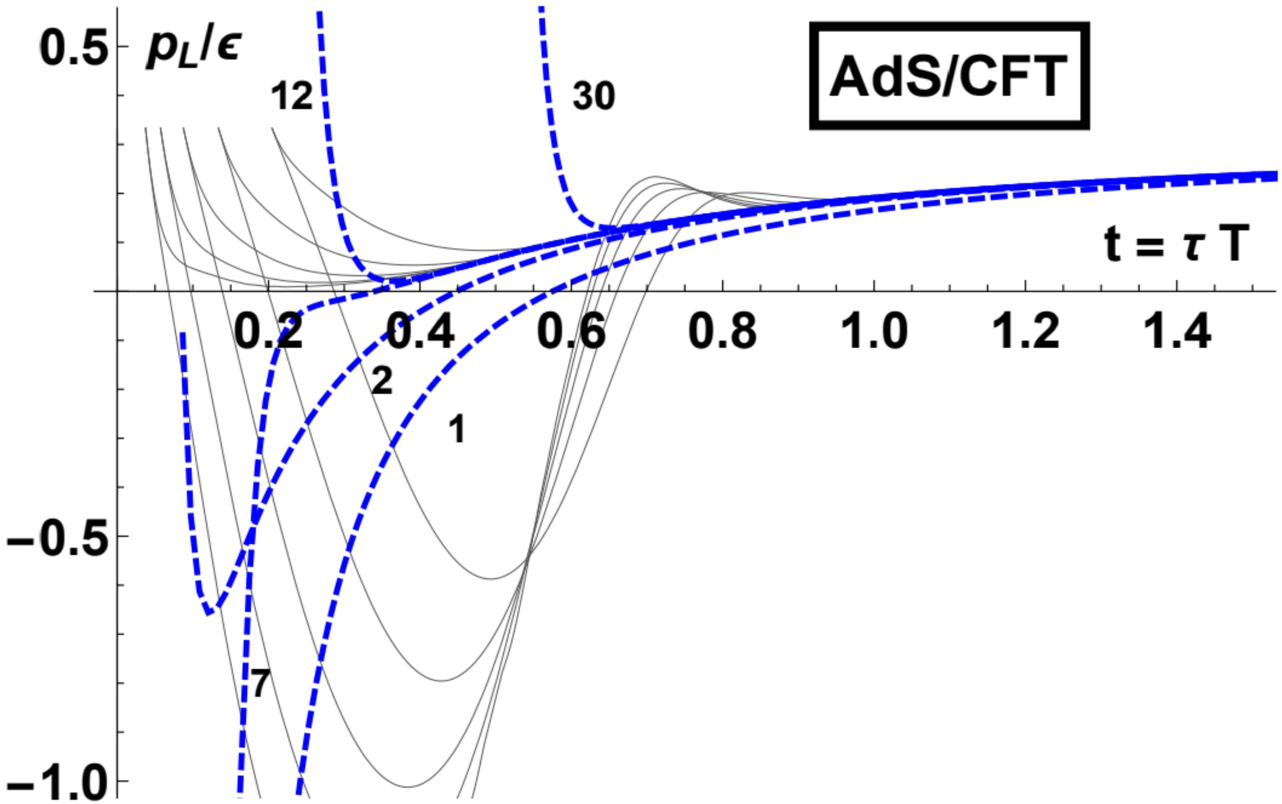


Michal P. Heller, Romuald A. Janik and Przemyslaw Witaszczyk, Hydrodynamic gradient expansion in gauge theory plasmas (2013)

APPROACH TO HYDRODYNAMICS

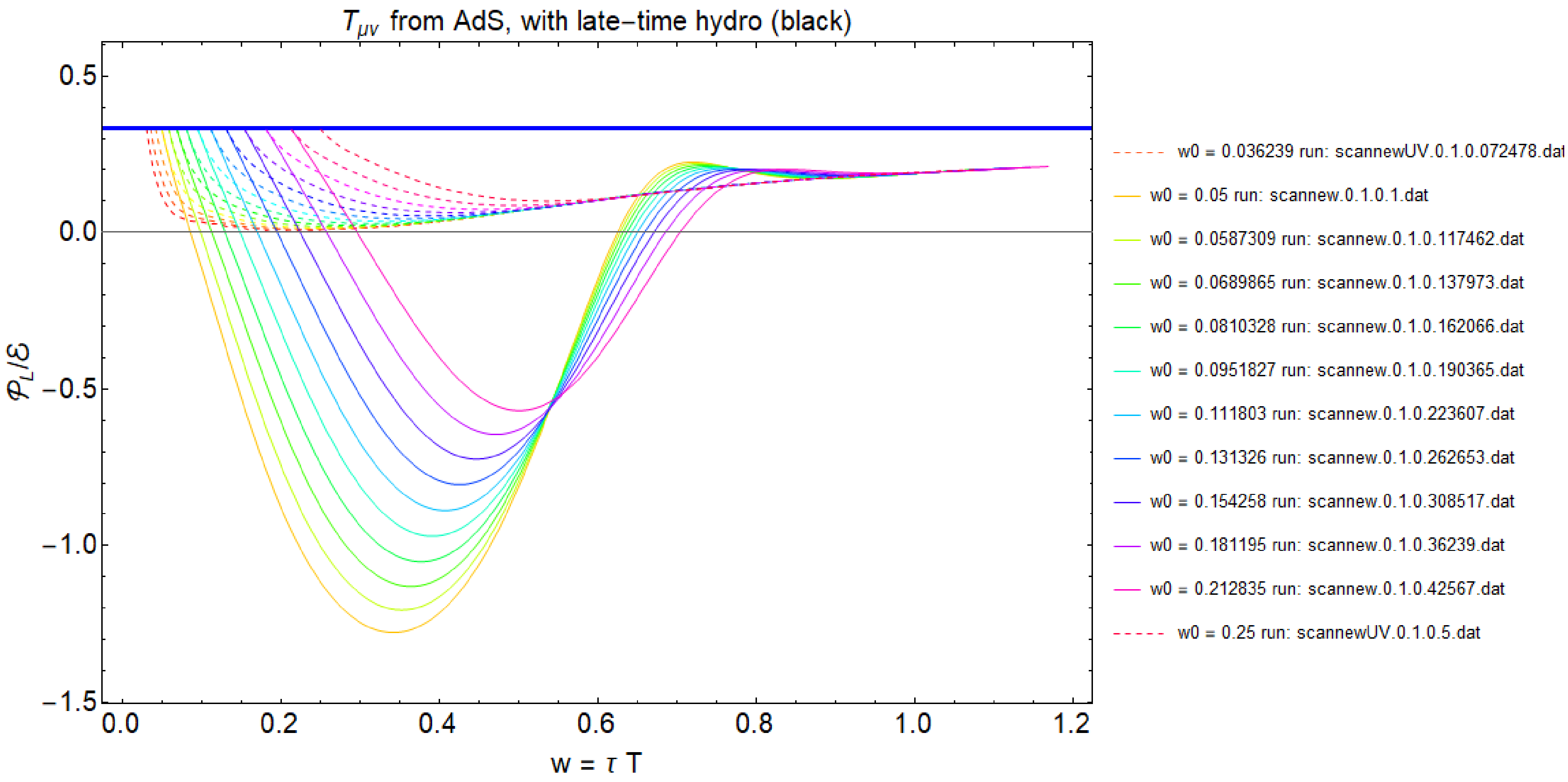
Same AdS/CFT evolutions as before

- Quite curious: UV agree with 7^{th} order at ~ t = 0.4
- For general profiles going beyond 2^{nd} order does not improve
- Even more curious: is there an early time attractor with free streaming? But why? (see also 1704.08699)



ADS/CFT ON A LINEAR SCALE

Some dynamics is clearer on a linear scale:



ATTRACTOR SUMMARY

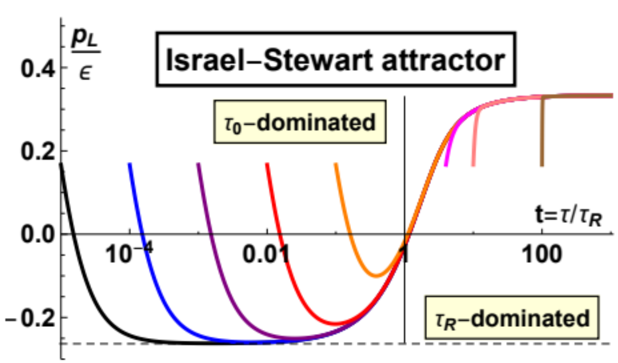
Israel-Stewart and RTA:

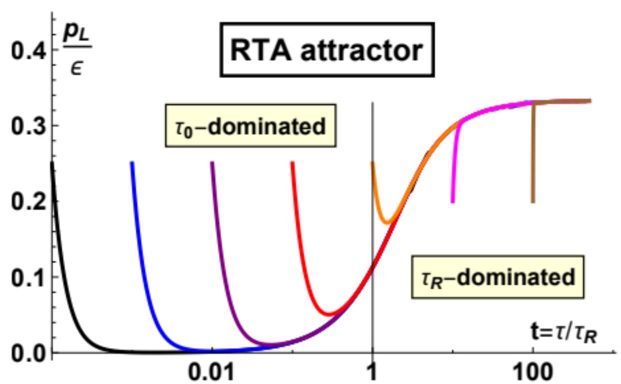
- Decay to attractor on time scale τ_0
- RTA: expansion dominated: free streaming (~$p_L = 0$)

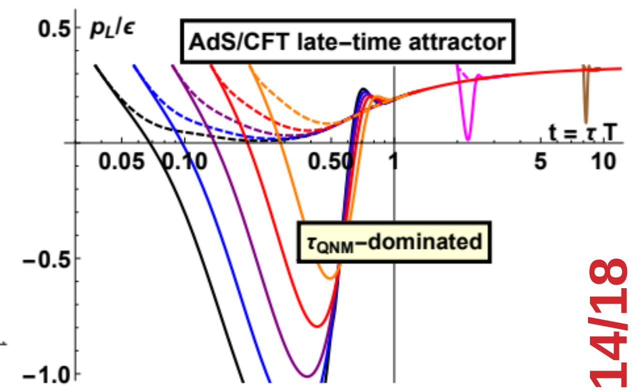
Strong coupling

- Decay to attractor on time scale 1/T
- Initial dynamics determined by initial condition (IC)
    - `UV' profile converges faster

Attractor itself dominated by interaction/transport (τ_R)







WHY DOES ADS/CFT DECAY `SLOWLY'?

UV and IR profiles contain higher-point correlation functions

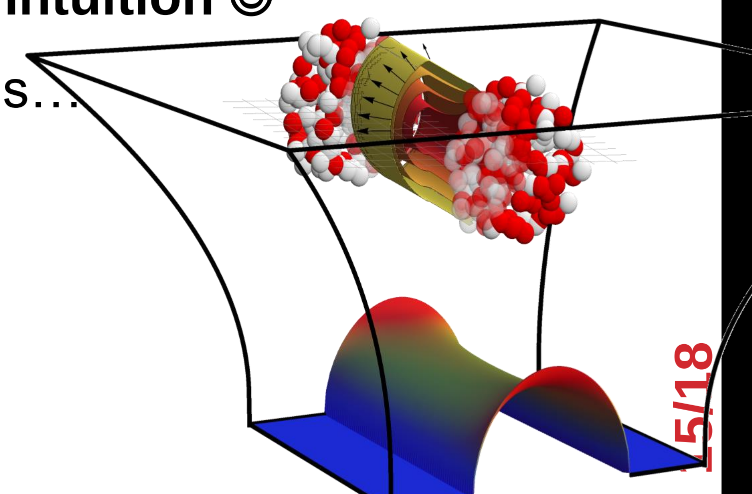
- Impossible to put correlations on lengths >> 1/T (because of horizon)
- IR: strong correlation on scale 1/T
- UV: constructed to have only small-scale correlations

Causality: system cannot relax before a time 1/T has passed (IR)

- RTA can relax much faster

Somewhat against weak/strong coupling intuition ☺

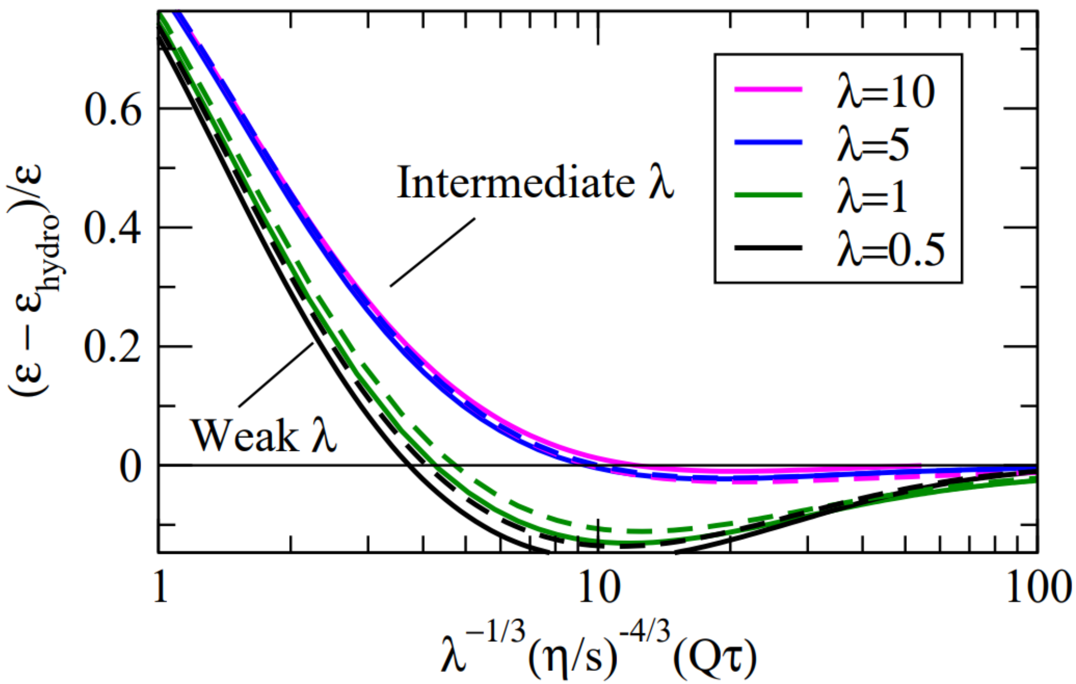
- But perhaps it's all about initial conditions…



WEAK COUPLING BEYOND RTA

Relaxation time approximation gives limited dynamics

- All moments decay at same time: quick decay to attractor
- Different in full-fledged kinetic theory (YM):
  (also showing bottom-up thermalization)

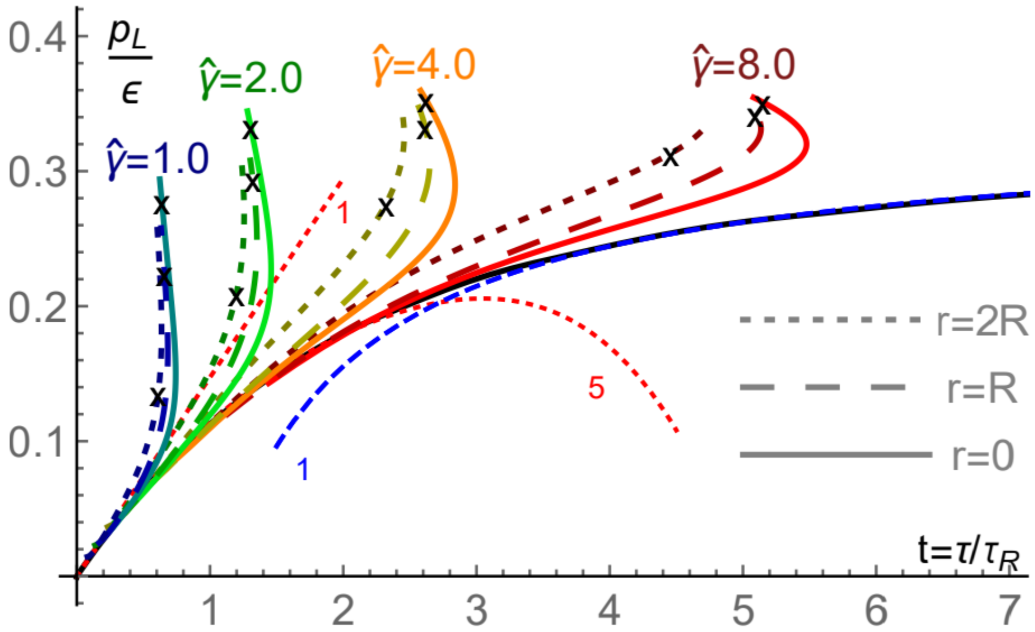


Aleksi Kurkela and Yan Zhu, Isotropization and hydrodynamization in weakly coupled heavy-ion collisions (2015)

3+1 ADVERTISEMENT

Follow anisotropy across system sizes position in plane

- Relevant parameter: opacity: $\hat{\gamma} = \gamma R^{3/4} (\varepsilon_0 \, \tau_0)^{1/4}$
- Depending on coupling strength γ
- Crosses indicate t = 2R, after which system decouples



THE BIG QUESTIONS / ANSWERS (?)

When is hydrodynamics applicable?

- RTA: within $2/\tau_0$, strong coupling: within $1/T$
- And what kind of hydrodynamics? (1^{st}, 2^{nd} or infinite order?)
- RTA: attractor (τ_R), strong coupling: 2^{nd} order, or 7^{th} order in rare cases
- Also: dependent on position

What is the initial fluid profile at this time?

- Presumably depending (strongly) on initial stage model
  - Strong coupling: evolution depending on higher-point correlations: may need refined knowledge initial stage (this work: arbitrary IC)
- Often argued: memory loss due to attractor
  - In the end everything goes to hydro (at least in 1+1D CFT)
  - RTA: IC at early enough τ_0 is truly lost (but note assumption of RTA)
  - Strong coupling: IC will influence final distribution of energy
    - *Conservation of misery remains true…*