Turbulent hydrodynamics in strongly correlated Kagome materials

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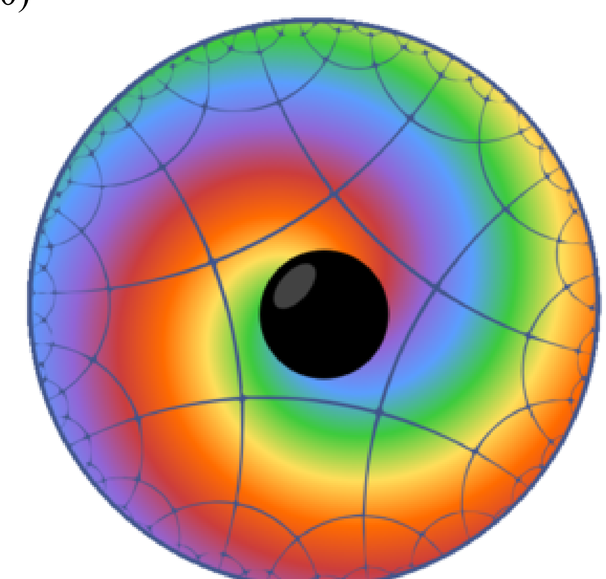
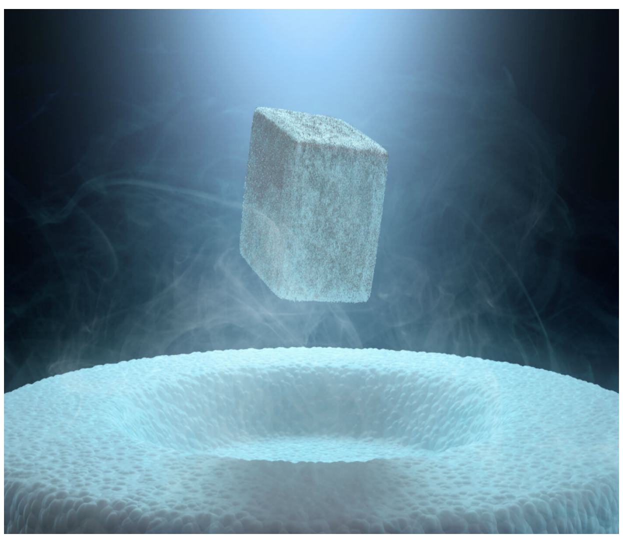
Based on
Domenico Di Sante, Johanna Erdmenger, Martin Greiter, IM, René Meyer, David Rodríguez Fernández, Ronny Thomale, Erik Van Loon, Tim Wehling,
Turbulent hydrodynamics in strongly correlated Kagome metals,
Nature Communications 11, 3997 (2020)



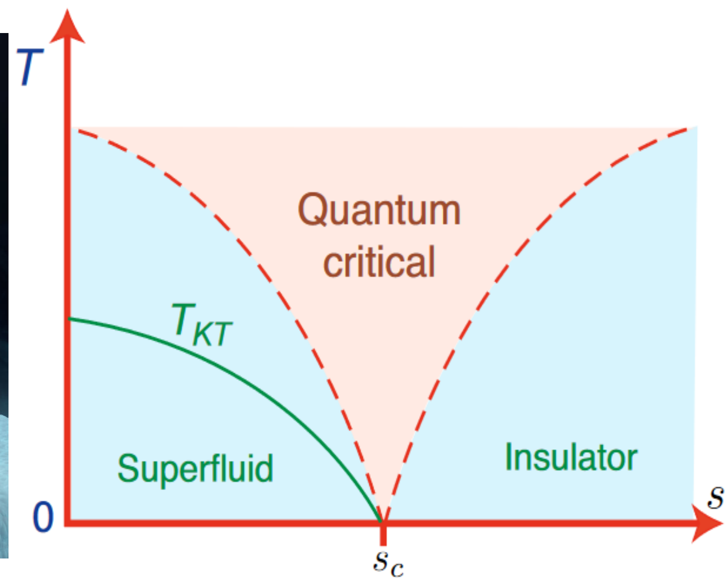
Holotube Jr. – October 2020

Motivation & Overview

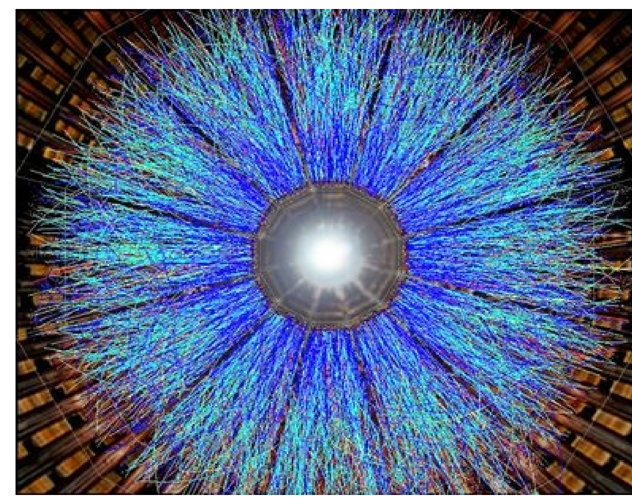
- Increase coupling strength ⟶ New and interesting transport behaviour
- E.g. High T_c superconductors, quantum critical phases, quark-gluon plasma



Source:https://singularityhub.com/2018/05/13/the-search-for-high-temperature-superconductors/



Source: Hartnoll, Lucas, Sachdev, *Holographic Quantum Matter,* The MIT Press)



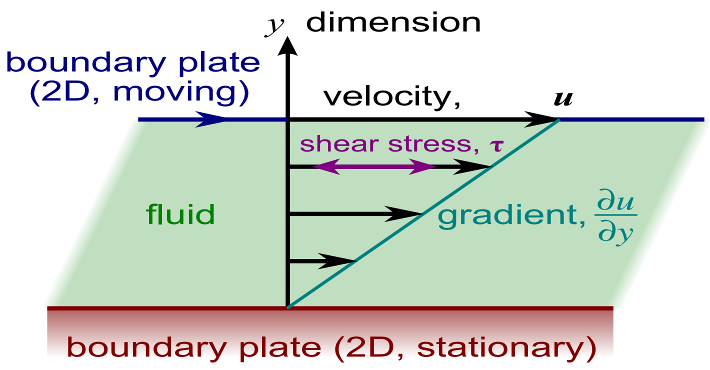
Source: https://news.mit.edu/2010/exp-quark-gluon-0609`

- Strong coupling = testing ground for AdS/CFT (and vice versa)

Transport effects due to η/s

- Here focus on shear viscosity to entropy density ratio η/s

$$\eta = \lim_{\omega\to 0} \frac{1}{\omega} \, \mathrm{Im}\langle T^{xy} \, T^{xy}\rangle_{\mathrm{R}}$$



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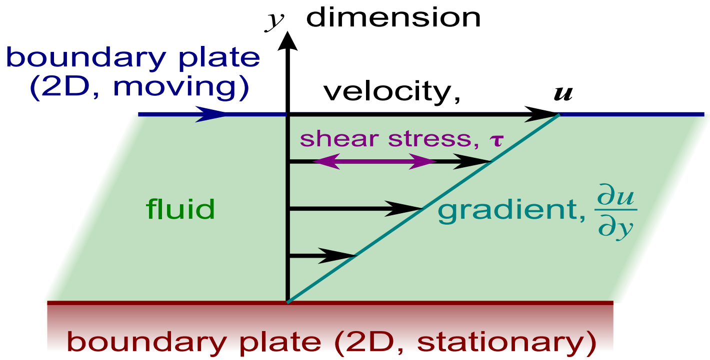

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- Known coupling dependence at “small” and “large” coupling strengths

[Müller, Schmalian, Fritz, PRL (2009); Policastro, Son, Starinets, PRL (2001); Buchel, Liu, PRL (2004); Kovtun, Son, Starinets, PRL (2005)]

$$\frac{\eta}{s} \simeq \frac{1.64}{\alpha^2} \frac{\hbar}{4\pi k_{\mathrm{B}}}$$

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_{\mathrm{B}}}$$

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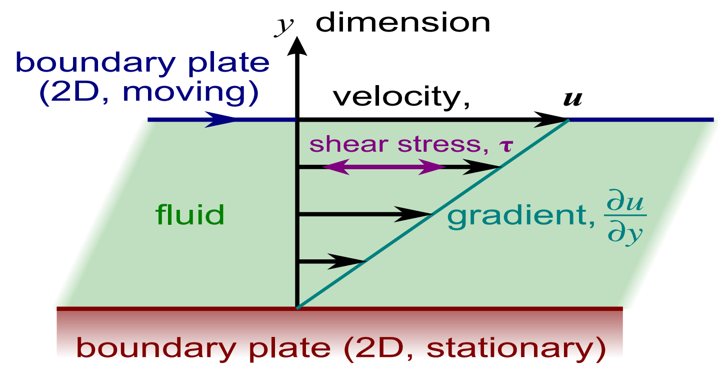

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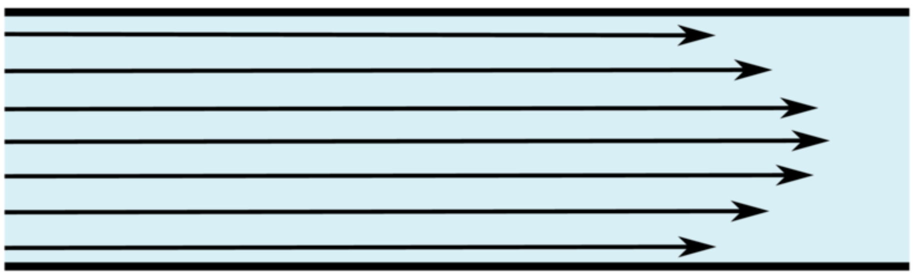
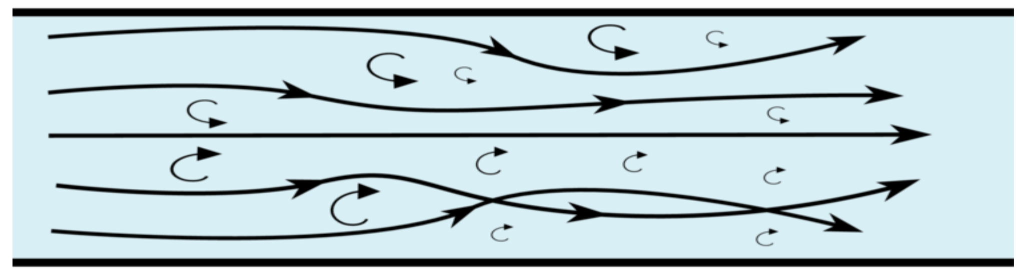
- Affects transport properties in metals:
  - Smaller differential resistivity
  - Decreased Joule heating

[Erdmenger, IM, Meyer, Rodríguez Fernández, PRB (2018)]

- Also affects the Reynolds number



$Re \ll 1000$



$Re \gtrsim \mathcal{O}(1000)$

$$Re = \left(\frac{\eta}{s}\frac{k_\mathrm{B}}{\hbar}\right)^{-1}\frac{k_\mathrm{B}T}{\hbar v_\mathrm{F}}\frac{u_\mathrm{typ}}{v_\mathrm{F}}W \propto \left(\frac{\eta}{s}\right)^{-1}$$

- Larger coupling → Larger *Re* → Turbulence?

Overview

We propose a Kagome Dirac material, Sc-Hb, with coupling strength ~ 3:

1) Proposal of AdS/CFT model

2) More robust hydrodynamic regime*

3) "Small" η/s

4) Access to turbulent flow regime

Outline

- Electron hydrodynamics
- Kagome materials and Sc-Hb
- AdS/CFT and η/s for Sc-Hb
- Conclusions and outlook

Electron Hydrodynamics

- Hydrodynamics = “long” wavelength, “small” frequency effective description of matter in thermal equilibrium

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  [Andreev, Kivelson, Spivak, PRL (2011); Polini, Geim, Physics Today (2020)]

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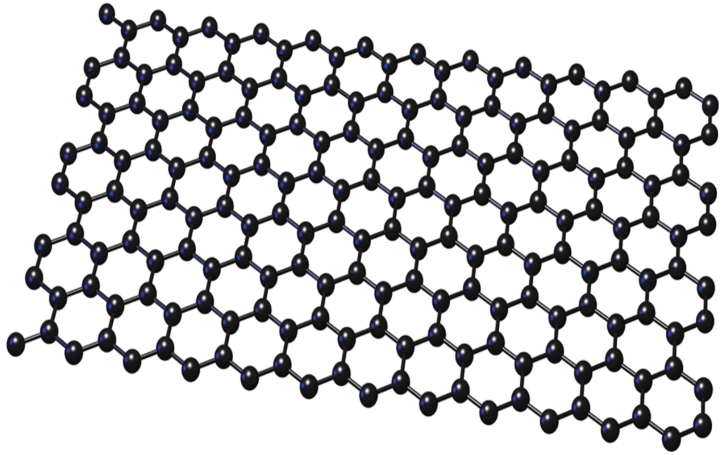
$$l_{\rm ee} \propto \frac{1}{\alpha_{\rm eff}^2} \qquad\qquad \alpha_{\rm eff} = \frac{e^2}{\epsilon_0 \epsilon_r \hbar v_{\rm F}}$$

- Electric permittivity and Fermi velocity define the coupling strength
- Stronger coupling enhances hydrodynamic behaviour*

*[Baggioli, arXiv:2010.05916; Starinets., Holotube seminar]

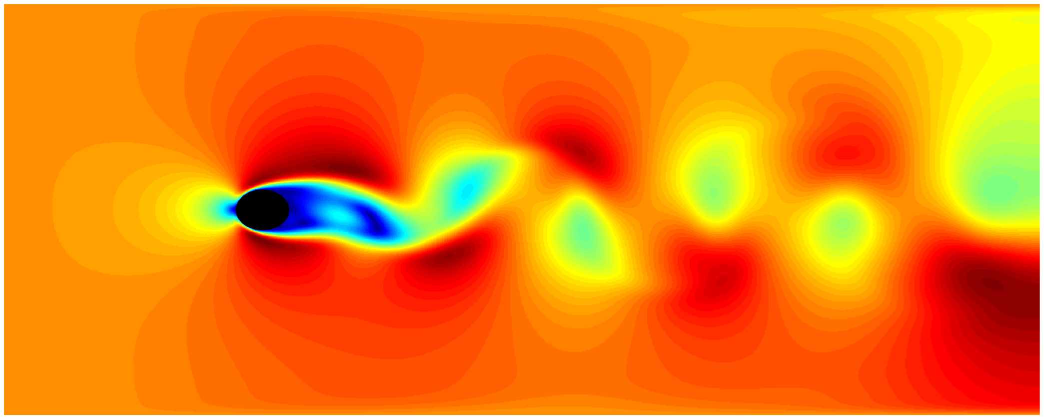
Graphene

[Polini, Geim, Physics Today (2020)]



- Prototypical relativistic electronic fluid
- Relativistic spectrum, $v_{\mathrm{F}} = c/300$
- Quantum critical around charge neutrality
- Still, only pre-turbulent *Re* ~ 100

[Mendoza, Herrmann, Succi, PRL (2011)]



Sources:
https://spectrum.ieee.org/semiconductors/materials/graphene-makes-transistors-tunable;
https://analyticalscience.wiley.com/do/10.1002/gitlab.15487

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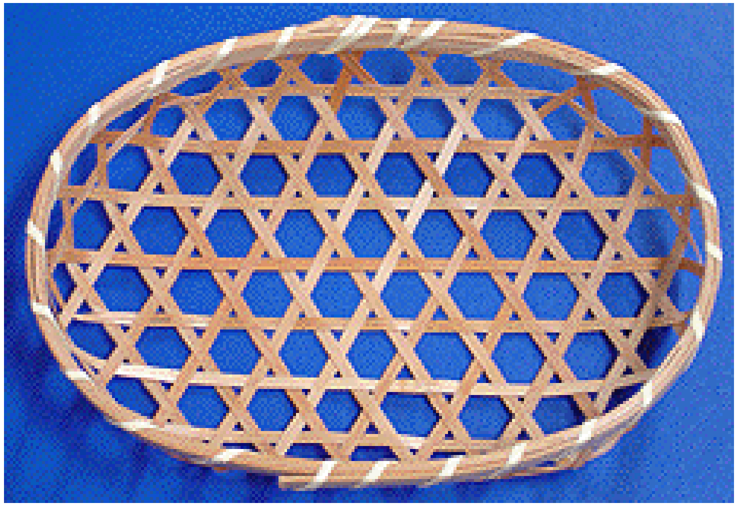
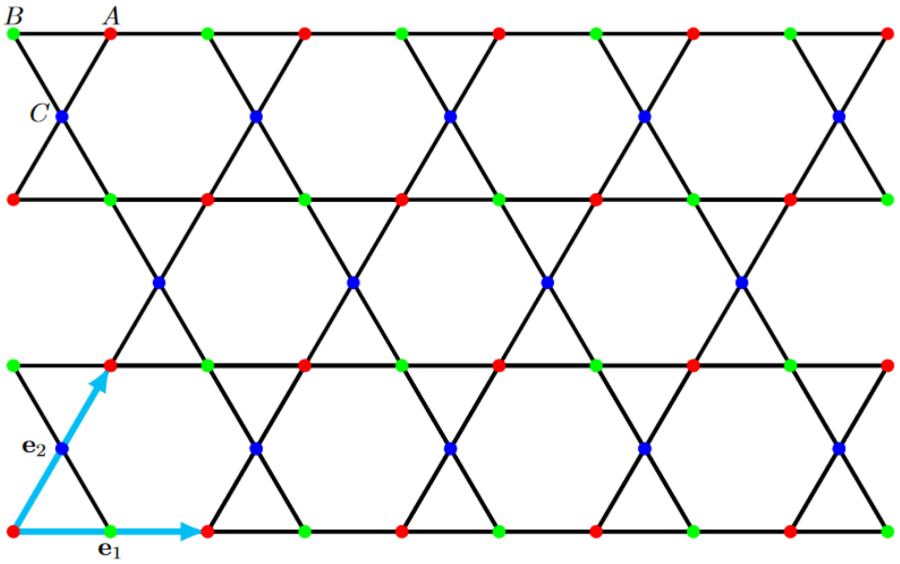
Kagome lattices

- Kagome = japanese basket weaving pattern



Source: http://www.hfmphysics.com/2006/motif.html

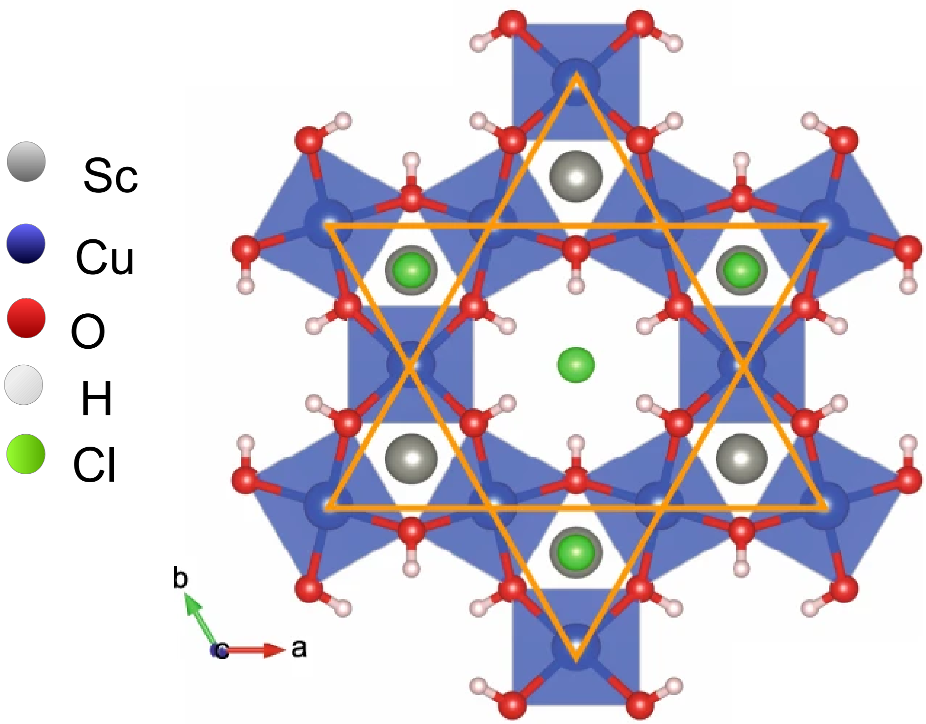


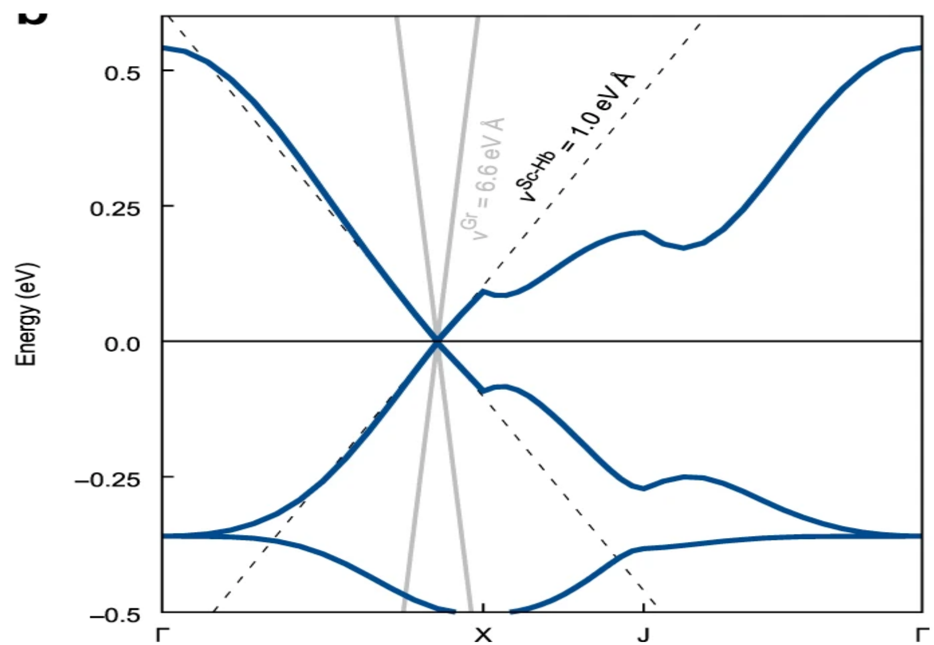
[Redder, Uhrig, PRA, 2016]

- Also, tiling of the plane → defines a 2D crystal lattice
- Relativistic spectrum due to (tri)hexagonal symmetry
- 3-sites per-unit cell → robust against ordering effects

Scandium Substituted Herbersmithite

$ScCu_3(OH)_6Cl_2$





- Sc allows d-orbital hybridization of Cu = "Flatter" spectrum
- ~7 times smaller Fermi velocity than graphene
- Fermi level at the Dirac point = physics around charge neutrality
- Away from "half-filling" = robust against long-range order

Coupling strength

$$\alpha_{\mathrm{eff}} = \frac{e^2}{\epsilon_0 \epsilon_r \hbar v_{\mathrm{F}}}$$

	v_F (eVÅ)	ϵ_r
ED in vacuum	2×10^3	1
hBN/graphene/hBN	6.6	2.2 – 4.0
graphite	–	2.5
Sc-Herbertsmithite	1.0	5.0

$$\alpha_{\mathrm{Gr}} \simeq 0.9 \qquad \alpha_{\mathrm{ScHb}} \simeq 2.9$$

Coupling enhancement by a factor of 3!

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Effective action for Sc-Hb

We want:

- Estimate of η/s
- Holographic effective action of an Sc-Hb like system

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- Typical ($S_G \sim R$) AdS/CFT calculations not applicable
- Coupling corrections = higher derivative corrections
- Maxwell terms irrelevant around charge neutrality

[Myers, Paulos, Sinha, JHEP (2009)]

$$S_G[J] = S_G[g_{\mu\nu}, A_\mu] \rightarrow \; S_G[g_{\mu\nu}]$$

Effective action for Sc-Hb

$$S_G = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[R - 2\Lambda + c_1 \alpha' R^2 + c_2 (\alpha')^2 R^3 + c_3 (\alpha')^3 R^4 \right]$$

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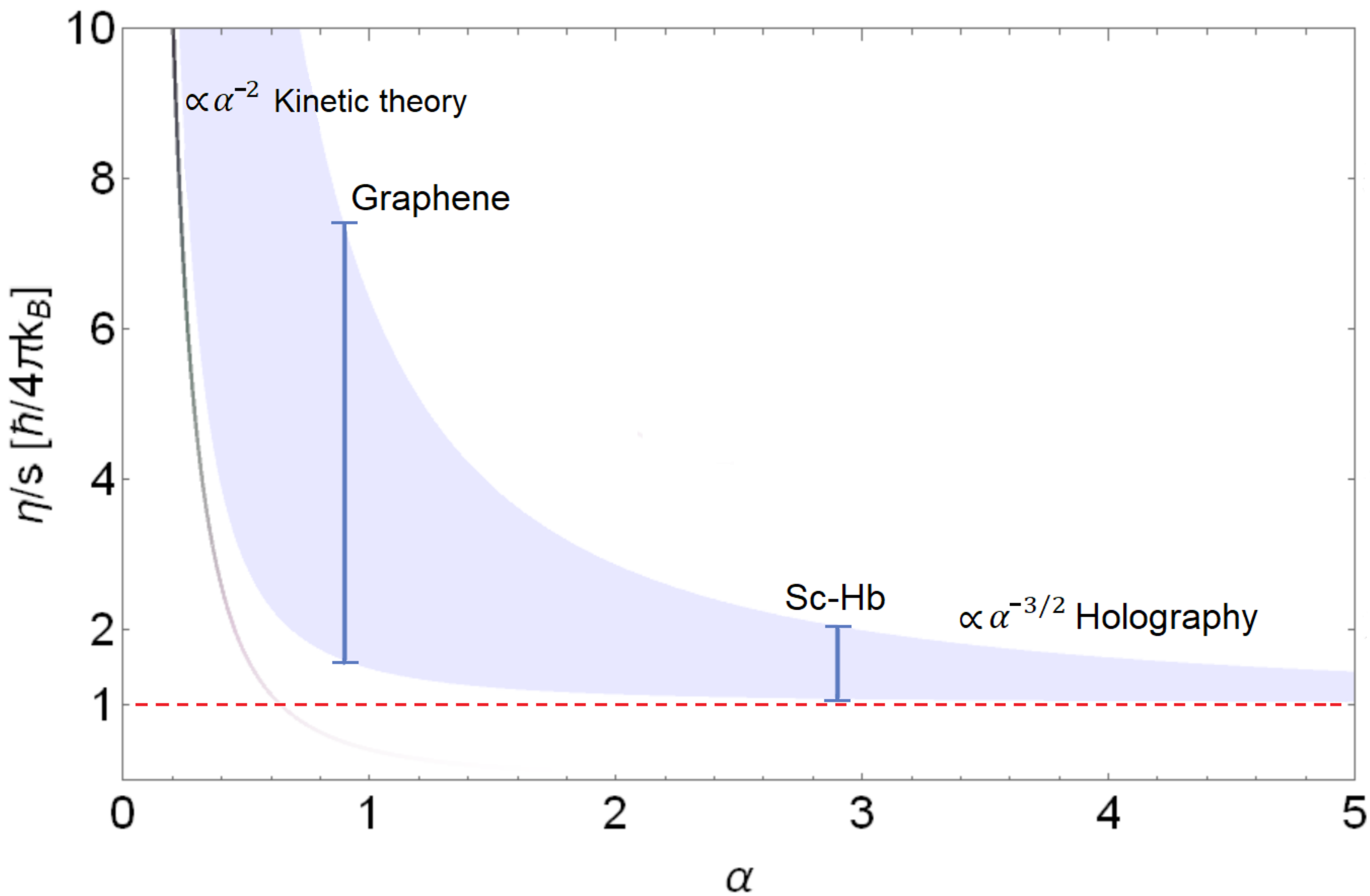
$$\downarrow$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\left(1 + \frac{\mathcal{C}}{\alpha_{\text{eff}}^{3/2}}\right)$$

[Buchel, Liu, Starinets, Nucl. Phys. B (2005); Benincasa, Buchel, JHEP (2006)]

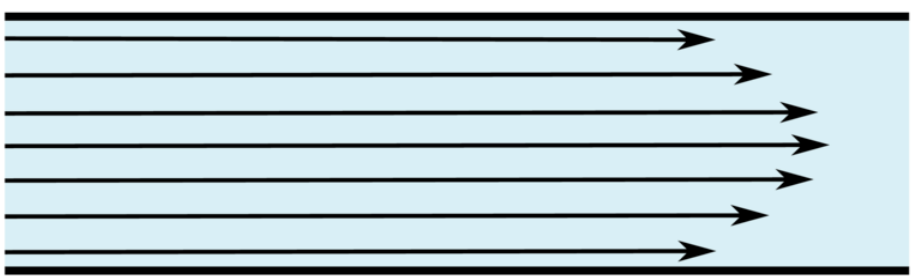


- Parametrize corrections by $\mathcal{C} \in [5 \times 10^{-4}, 5]$
- For N = *4* SYM, equivalent to $N \in [2, 10^3]$

Turbulence

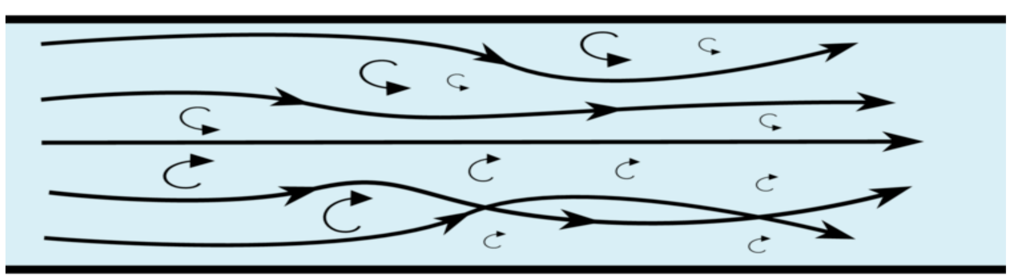
$$Re = \left(\frac{\eta}{s}\frac{k_{\mathrm{B}}}{\hbar}\right)^{-1}\frac{k_{\mathrm{B}}T}{\hbar v_{\mathrm{F}}}\frac{u_{\mathrm{typ}}}{v_{\mathrm{F}}}W \propto \left(\frac{\eta}{s}\right)^{-1}$$

laminar flow



$$Re \ll 1000$$

turbulent flow



$$Re \gtrsim \mathcal{O}(1000)$$

- Enhanced *Re* due to smaller η/s, v_{F}

$$Re_{\mathrm{ScHb}} \sim 100 Re_{\mathrm{Gr}} \sim 10^3 - 10^4$$

Conclusions and Outlook

1. Sc-Hb has an $\alpha \sim 3$ well into the non-perturbative regime
2. Well-suited for a holographic description
3. Enables transition to hydro regime*
4. Enables transition to turbulent regime

- Turbulence from gravity?

  [Adams, Chesler, Liu P R L (2014)]
  [See also Paul Wittmer's talk and Christiana Pantelidou's Holotube seminar 27/10]

- Similar materials?

  [Fuchs, Liu, Schwemmer, Sangiovanni, Thomale, Franchini, Di Sante, J. Phys. Mater. (2020)]

Thank you!