Hybrid Hydrodynamic Attractor and The Quark Gluon Plasma

based on arXiv:2006.09383, with Toshali Mitra, Ayan Mukhopadhyay, Anton Rebhan, and Alexander Soloviev

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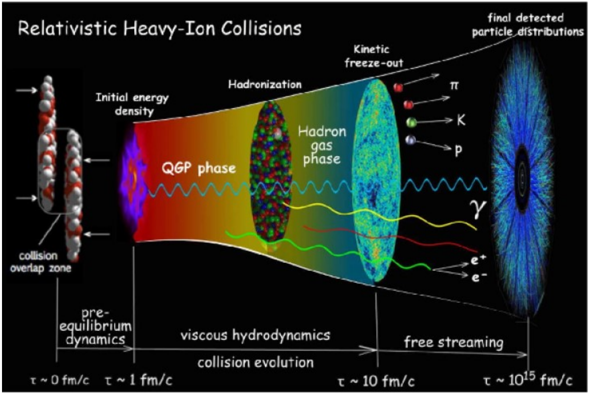
Outline

- Introduction & Motivation
  - Hydrodynamic Attractor
  - Democratic Couplings & Hybrid Fluid Model
- Hybrid Hydrodynamic Attractor
- Conclusions

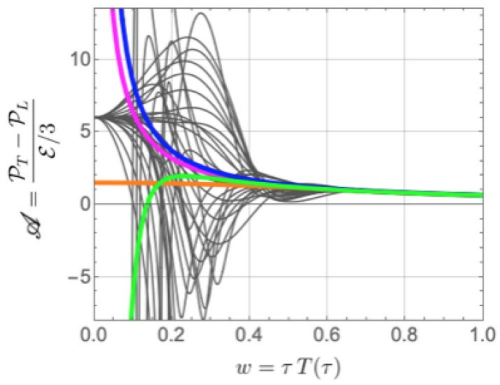
Schematic of evolution of matter produced in Heavy Ion Collision



Scharenberg, Srivastava, Hirsch, Pajares (2018)

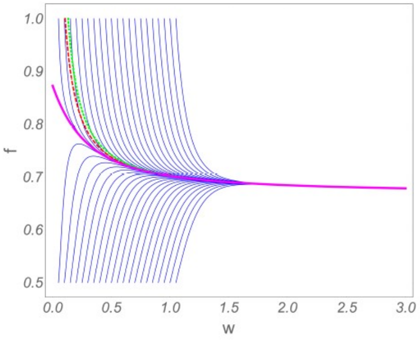
- Most of the particles detected have low values of p_T
- Low p_T particles show signs of collective behaviour from experimental measurements
- Hydrodynamic simulations of QGP successfully reproduce the experimental data for low p_T particles.

- Hydrodynamic simulations work even though they begin with highly anisotropic initial conditions
- *Hydrodynamic Attractors* provide insight into this puzzle
  Phys. Rev. Lett. 115, 072501 (2015) Heller & Spalinski
- All the out-of-equilibrium initial conditions evolve to a unique hydrodynamic attractor curve long before the system thermalizes



Solid Gray Curves: exact solutions of $\mathcal{N} = 4$ SYM using holography
Dotted Curves: solutions of truncated hydrodynamics (magenta - first order, blue - second order, green - third order)

arXiv: 2005.12299 Berges et. al.



Blue Curves - Numerical solutions of MIS equations for various initial conditions
Magenta Curve - Hydrodynamic Attractor
Red Dashed Curve - First Order Hydrodynamics
Green Dotted Curve - Second Order Hydrodynamics

Phys. Rev. Lett. 115, 072501 (2015) Heller & Spalinski

- QCD has a running coupling which decreases with increase in energy scale
- QCD weakly coupled in UV (perturbative) and strongly coupled in IR (non-perturbative)
- Matter produced in HIC involves wide range of energy scales throughout its evolution

- All stages (except perhaps very early stages) involve both weakly interacting and strongly interacting degrees of freedom (and their roles cannot be factorized)

- Phenomenology of high p_T hadrons and high p_T jets can be well understood via resummed perturbation theory

- Low p_T hadrons fit well to hydrodynamic models and can be well described by strong coupling approaches

- Phenomenology of intermediate p_T hadrons is not well understood

- Most existing approaches take either the exclusively strong or exclusively weak coupling paradigms
- It would be nice to have single framework which includes both perturbative and non-perturbative degrees of freedom
- Semi-holography combines perturbative and non-perturbative sectors consistently in one framework

J. High Energ. Phys. 2015, 3 (2015)
J. High Energ. Phys. 2016, 141 (2016)
Phys. Rev. D 95, 066017 (2017)
J. High Energ. Phys. 2018, 54 (2018)
J. High Energ. Phys. 2018, 74 (2018)

- To be consistent with Wilsonian RG, non-perturbative dynamics at a given energy scale should depend on perturbative dynamics only upto that energy scale

- We should be able to obtain effective macroscopic description of the combined system from coarse-grained descriptions of the sub-sectors

- This can be achieved by using democratic couplings Phys. Rev. D 95, 066017 (2017)

- Only metric couplings are relevant for fluids

- Full theory lives in actual background metric:

$$g_{\mu\nu}^{(B)}$$

- first sub-system lives in effective metric:

$$g_{\mu\nu}[\tilde{t}^{\gamma\delta}]$$

- second sub-system lives in effective metric:

$$\tilde{g}_{\mu\nu}[t^{\alpha\beta}]$$

- subsytem Ward Identities: $\nabla_\mu t^\mu{}_\nu = 0, \quad \tilde{\nabla}_\mu \tilde{t}^\mu{}_\nu = 0$

Metric Coupling Equations

$$g_{\mu\nu} = g^{(B)}_{\mu\nu} + \gamma g^{(B)}_{\mu\alpha} \tilde{t}^{\alpha\beta} g^{(B)}_{\beta\nu} \frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} + \gamma' g^{(B)}_{\mu\nu} \tilde{t}^{\alpha\beta} g^{(B)}_{\alpha\beta} \frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}}$$

$$\tilde{g}_{\mu\nu} = g^{(B)}_{\mu\nu} + \gamma g^{(B)}_{\mu\alpha} t^{\alpha\beta} g^{(B)}_{\beta\nu} \frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} + \gamma' g^{(B)}_{\mu\nu} t^{\alpha\beta} g^{(B)}_{\alpha\beta} \frac{\sqrt{-g}}{\sqrt{-g^{(B)}}}$$

The individual ward identities $\nabla_\mu t^\mu{}_\nu = 0$, $\tilde{\nabla}_\mu \tilde{t}^\mu{}_\nu = 0$ and the coupling equations together imply $\nabla^{(B)}_\mu T^\mu{}_\nu = 0$, where

$$T^\mu{}_\nu = \frac{1}{2}\left((t^\mu{}_\nu + t_\mu{}^\nu)\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} + (\tilde{t}^\mu{}_\nu + \tilde{t}_\mu{}^\nu)\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \right) + \Delta K \delta_\mu{}^\nu$$

$$=: T^\mu_1{}_\nu(\mathcal{E}_1, \mathcal{P}_1) + T^\mu_2{}_\nu(\mathcal{E}_2, \mathcal{P}_2) + T^\mu{}_{\nu,\mathrm{int}}$$

with

$$\Delta K = -\frac{\gamma}{2}\left(t^{\rho\alpha}\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\alpha\beta} \left(\tilde{t}^{\beta\sigma}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\sigma\rho} - \frac{\gamma'}{2}\left(t^{\alpha\beta}\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\alpha\beta} \left(\tilde{t}^{\sigma\rho}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\sigma\rho}$$

- Full em-tensor is a polynomial of sub-system em-tensors
- Phenomenological description of full system can be obtained from hydrodynamic descriptions of the subsystems

Spoiler!

There exists a 2 dimensional attractor surface ruled by curves. The two constants $\alpha := lim_{\tau \to \infty} \epsilon \tau^{4/3}$ and $\beta := lim_{\tau \to \infty} \tilde{\epsilon} \tau^{4/3}$ label these curves. Any initial condition evolves to one of these curves on the attractor surface.

The two systems never equilibrate and yet the full system em-tensor can be described as a single fluid at late time. The EoS and shear viscosity of the full system is determined by the curve on the attractor surface to which the system evolves at late time.

Spoiler!

Bottom-up thermalization is universal as long as one of the systems is weakly coupled and another is strongly coupled.

The weak system hydrodynamizes later than the strong system unless the strong system has extremely tiny fraction of energy at a reference time $\tau_0 = \gamma^{1/4} \approx Q_s^{-1}$. However, the ratio of hydrodynamization times becomes extremes extreme as the total energy of the system at reference tie is decreased. Gives insight into small vs large system collision.

The equilibrium of the hybrid system is described by coupling two perfect fluids. (J. High Energ. Phys. 2018, 54 (2018))
Extend this model to capture non-equilibrium dynamics
Describe each sub-sector by MIS theory (arXiv:2006.09383)

- In MIS theory, $\pi^{\mu\nu}$ is promoted to an independent dynamical variable (on the same footing as $\mathcal{E}$ and u^μ).
- $\pi^{\mu\nu}$ satisfies the following relaxation type equation:

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi u^\alpha \nabla_\alpha \pi^{\mu\nu}$$

τ_π is new transport coefficient called as relaxation time.

- Two sets of Equations of motion for the fluid:

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= 0 && \text{Ward Identity} \\ (1 + \tau_\pi u^\alpha \nabla_\alpha)\pi^{\mu\nu} &= -\,\eta\sigma^{\mu\nu} && \text{MIS equation} \end{aligned}$$

The background metric is flat Minkowski metric in Bjorken flow

$$g_{\mu\nu}^{(B)} = diag(-1, 1, 1, \tau^2)$$

boost invariant ansatz for the effective metrics of subsectors:

$$g_{\mu\nu} = diag(-a^2, b^2, b^2, c^2)$$
$$\tilde{g}_{\mu\nu} = diag(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{c}^2)$$

$a,\ b,\ c,\ \tilde{a}, \tilde{b}, \tilde{c}$ are functions of τ.

Assume conformal equations of state for both the subsystems:

$$\epsilon = 3P, \quad \tilde{\epsilon} = 3\tilde{P}$$

energy-momentum tensors of subsystems:

$$t^{\mu}_{\nu} = \mathrm{diag}\left(-\epsilon, P, P, P\right) + \pi^{\mu}_{\nu}$$
$$\tilde{t}^{\mu}_{\nu} = \mathrm{diag}\left(-\tilde{\epsilon}, \tilde{P}, \tilde{P}, \tilde{P}\right) + \tilde{\pi}^{\mu}_{\nu}$$

$$\pi^{\mu}_{\nu} = \mathrm{diag}\left(0, \frac{\phi}{2}, \frac{\phi}{2}, -\phi\right),$$
$$\tilde{\pi}^{\mu}_{\nu} = \mathrm{diag}\left(0, \frac{\tilde{\phi}}{2}, \frac{\tilde{\phi}}{2}, -\tilde{\phi}\right)$$

EOMs for two subsystems

$$\nabla_\mu t^{\mu\nu} = 0, \quad \Big(\tau_\pi u^\alpha \nabla_\alpha + 1\Big)\pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$
$$\tilde{\nabla}_\mu \tilde{t}^{\mu\nu} = 0, \quad \Big(\tilde{\tau}_\pi \tilde{u}^\alpha \tilde{\nabla}_\alpha + 1\Big)\tilde{\pi}^{\mu\nu} = -\tilde{\eta}\tilde{\sigma}^{\mu\nu}$$

We parametrize transport coefficients as follows

$$C_\eta = \frac{\eta}{s}, \quad C_\tau = \tau_\pi \epsilon^{1/4}$$
$$\tilde{C}_\eta = \frac{\tilde{\eta}}{\tilde{s}}, \quad \tilde{C}_\tau = \tilde{\tau}_\pi \tilde{\epsilon}^{1/4}$$

C_η, C_τ, $\tilde{C}_\eta$, $\tilde{C}_\tau$ are all dimensionless parameters which are given by the underlying microscopic theory

Strongly Coupled System ($\mathcal{N} = 4$ SYM values)

$$\tilde{C}_\tau = \frac{2 - \log(2)}{2\pi}, \quad \tilde{C}_\eta = \frac{1}{4\pi}$$

Weakly Coupled System

$$C_\tau = 5 C_\eta, \quad C_\eta = 10 \tilde{C}_\eta$$

6 algebraic equations (coupling equations)
4 first order ODEs (EOMs)
10 variables: $a, b, c, \tilde{a}, \tilde{b}, \tilde{c}, \epsilon, \phi, \tilde{\epsilon}, \tilde{\phi}$

Six variables in effective metrics can be solved in terms of ϵ, ϕ, $\tilde{\epsilon}$, $\tilde{\phi}$ using coupling equations

Four dimensional phase space spanned by ϵ, ϕ, $\tilde{\epsilon}$, $\tilde{\phi}$

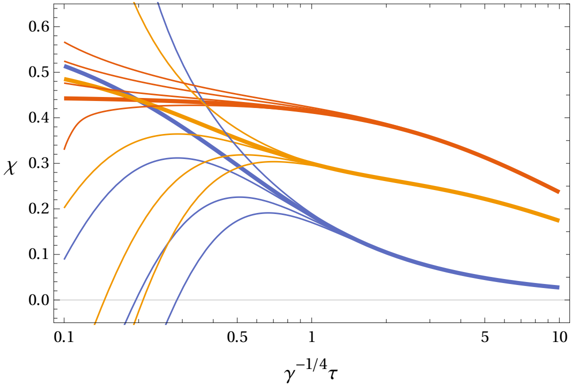
Two dimensional attractor surface exists in this four dimensional phase space

dimensionless anisotropy variable: $\chi = \frac{\phi}{\epsilon + P}$, $\tilde{\chi} = \frac{\tilde{\phi}}{\tilde{\epsilon} + \tilde{P}}$

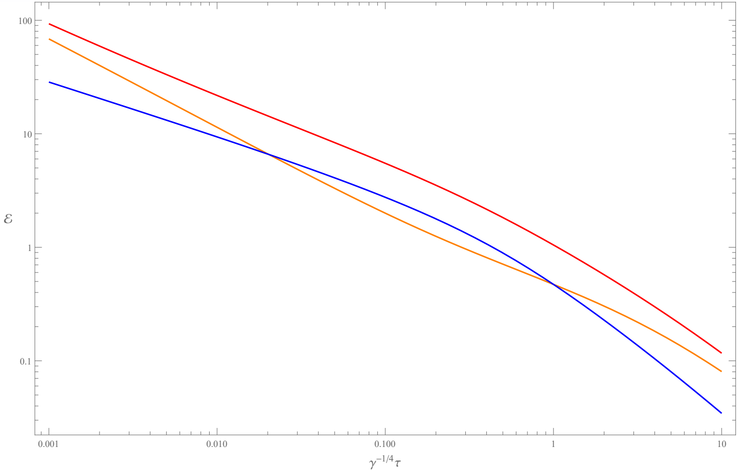
At $\tau \to 0$

$$\chi \to \sigma := \sqrt{\frac{C_\eta}{C_\tau}} \approx 0.45 \text{ (weakly coupled)}$$

$$\tilde{\chi} \to \tilde{\sigma} := \sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}} \approx 0.62 \text{ (strongly coupled)}$$



strongly coupled, weakly coupled, total system



Energy Densities

strongly coupled sector, weakly coupled sector, full system

Bottom-up Thermalization

At early times, energy in the weakly coupled sector is always greater than the energy in the strongly coupled sector.

At early times (near $\tau = 0$), the subsystem energy densities have the following behaviour:

$$\mathcal{E}_1 := (ab^2c/\tau)\epsilon \sim \tau^{4(\sigma-1)/3}, \;\; \mathcal{E}_2 := (\tilde{a}\tilde{b}^2\tilde{c}/\tau)\tilde{\epsilon} \sim \tau^{4(2\tilde{\sigma}-\sigma-1)/3}$$

$$\mathcal{E}_2/\mathcal{E}_1 \sim \tau^{8(\tilde{\sigma}-\sigma)/3} \qquad \tilde{\sigma} > \sigma$$

Caveat

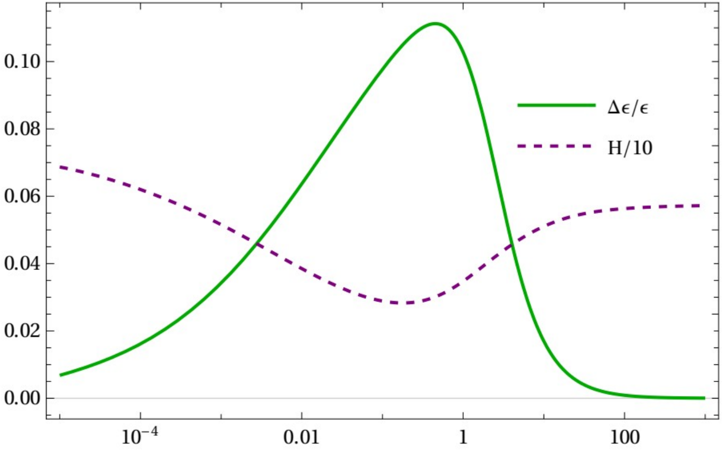
- In semi-holography, the transfer of energy is irreversible from hard sector to soft sector.
  (J. High Energ. Phys. 2018, 74 (2018))
- However, when the semi-holographic coupling is small, this transfer is very slow. (J. High Energ. Phys. 2018, 74 (2018))
- Two fluid model can capture some features of semi-holography at intermediate times.
- In two fluid model, hard sector becomes dominant again (as there is no mechanism for irreversible transfer of energy to soft sector in two fluid model)
- This behaviour is reminiscent of transition from deconfined QGP to a weakly coupled hadron gas in heavy ion collisions

- At late times, variables $\epsilon, \phi, \tilde{\epsilon}, \tilde{\phi}$ admit hydrodynamic expansions involving two parameters $\alpha := lim_{\tau\to\infty}\epsilon\tau^{4/3}$ and $\beta := lim_{\tau\to\infty}\tilde{\epsilon}\tau^{4/3}$.
- The two sub-systems do not equilibrate but the full system can be described as a single fluid

$$\left(\frac{\eta}{s}\right)^{\text{full}} = C_\eta^{eff} := \lim_{\tau\to\infty} H(\tau)$$

$$C_\eta^{eff} = \frac{C_\eta \alpha^{4/3} + \tilde{C}_\eta \beta^{4/3}}{(\alpha+\beta)^{4/3}}$$

$$H(\tau) = \frac{C_\eta \epsilon^{4/3} + \tilde{C}_\eta \tilde{\epsilon}^{4/3}}{(\epsilon+\tilde{\epsilon})^{4/3}}$$



Interaction Measure and Effective Shear Viscosity

Linear fluctuations about the hydro solutions have the following structure:

$$\frac{\delta\phi}{\epsilon_{pf}} = c_1 e^{-\frac{3}{2}\frac{w}{C_\tau}} w^{\sigma^2}(1+\mathcal{O}(w^{-1})) + \tilde{c}_2 e^{-\frac{3}{2}\sqrt{\frac{\alpha}{\beta}}\frac{w}{\tilde{C}_\tau}} w^{-3+\tilde{\sigma}^2}(1+\mathcal{O}(w^{-1})),$$

$$\frac{\delta\tilde{\phi}}{\epsilon_{pf}} = c_2 e^{-\frac{3}{2}\sqrt{\frac{\alpha}{\beta}}\frac{w}{\tilde{C}_\tau}} w^{\tilde{\sigma}^2}(1+\mathcal{O}(w^{-1})) + \tilde{c}_1 e^{-\frac{3}{2}\frac{w}{C_\tau}} w^{-3+\sigma^2}(1+\mathcal{O}(w^{-1}))$$

condition $\quad \tilde{C}_\tau\sqrt{\beta} \neq C_\tau\sqrt{\alpha}.$

If $\frac{1}{C_\tau} < \frac{1}{\tilde{C}_\tau}\sqrt{\frac{\alpha}{\beta}}$, then $\delta\tilde{\phi}$ decays faster

If $\frac{1}{C_\tau} > \frac{1}{\tilde{C}_\tau}\sqrt{\frac{\alpha}{\beta}}$, then $\delta\phi$ decays faster

- The weak system hydrodynamizes later than the strong system unless the strong system has extremely tiny fraction of energy at a reference time $\tau_0 = \gamma^{1/4} \approx Q_s^{-1}$
- However the ratio of hydrodynamization times becomes extreme as the total energy of the system at reference time is decreased. Gives insight into small vs large system collision.

Anisotropy, $A = \frac{P_\perp - P_L}{P} \sim 6\chi$
w scales as $\frac{4\pi\eta}{s}$
$\frac{|\Delta P_L|}{P} := \frac{|\phi - \phi_{1st}|}{P} < 0.1$, $\tau > \tau_{hd}$

Subsystem hydrodynamization times and the concurrent values of the dimensionless quantities $w = \mathcal{E}_1^{1/4}\tau$, $\tilde{w} = \mathcal{E}_2^{1/4}\tau$, χ, and $\tilde{\chi}$ for three scenarios with different values of $\mathcal{E}_1(1) = \mathcal{E}_2(1) =: \mathcal{E}(1)$, where all dimensionful quantities are given in units of γ. The last column gives the ratio $R_{\rm hd} := \tau_{\rm hd}/\tilde{\tau}_{\rm hd}$.

$\mathcal{E}(1)$	$\tau_{\rm hd}$	$w_{\rm hd}/10$	$\chi_{\rm hd}$	$\tilde{\tau}_{\rm hd}$	$\tilde{w}_{\rm hd}$	$\tilde{\chi}_{\rm hd}$	$R_{\rm hd}$
0.26	12.0	0.609	0.215	2.08	1.42	0.101	5.76
0.32	10.2	0.705	0.203	3.90	2.82	0.0525	2.62
0.052	25.5	0.608	0.210	1.39	0.613	0.211	18.4

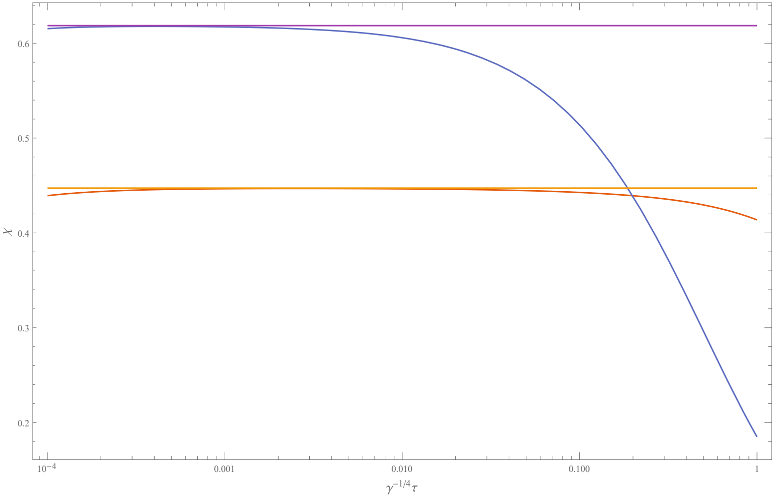
Conclusions

- Hybrid two fluid model in combination with MIS equations provides a model for non-equilibrium dynamics of two component system with different amounts of self interactions
- Hybrid system exhibits a two dimensional attractor surface ruled by curves. Any initial condition evolves to one of these curves on the attractor surface.
- Bottom-up thermalization is universal as long as one of the systems is weakly coupled and another is strongly coupled.
- At later times weakly coupled system dominates again as in QGP to hadron gas crossover

Conclusions

- Full system behaves as a single fluid at late times even though the two subsystems never equilibrate. EoS and shear viscosity of the full system are determined by the curve on the attractor surface to which the system evolves at late time.
- The ratio of hydrodynamization times depends strongly on the total energy in the system at reference time. Gives insight into small vs large system collision
- Our model is successful in capturing only certain features of heavy ion collision and open to further generalizations

Thank You For Your Attention



Attractor, $\tilde{\sigma} \approx 0.62$ (strongly coupled), $\sigma \approx 0.45$ (weakly coupled)