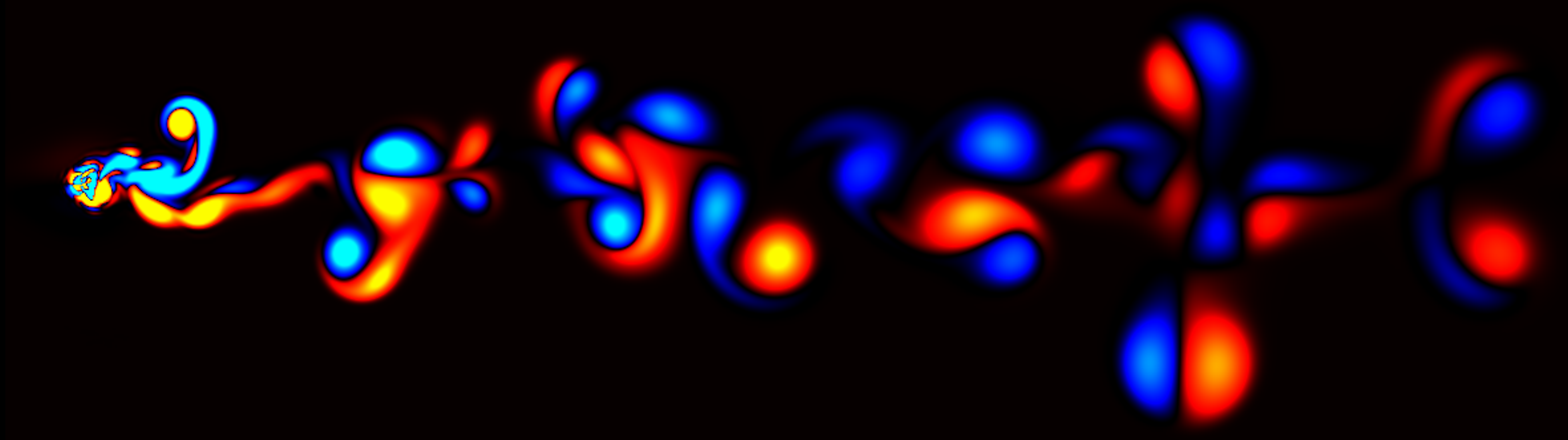

Gravitational turbulence in Large D

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Based on:
1912.00032 with T. Andrade, J. Sonner, B. Withers

Introduction

Turbulence in **fluid mechanics:**

- Widely seen in nature in systems with fluid-like behaviour.
- Has its root in the non-linear structure of NS equations:

$$\left(\frac{\partial}{\partial t} - \nu \frac{\partial^2}{\partial x^2}\right) u^i = M^i_{jk} u^j u^k \,, \qquad \nu : \text{viscosity}$$

- Non-linearities give rise to transfer of energy across scales leading to energy cascades.
- Information about initial conditions, driving washed out giving **universal behaviour** [Kolmogorov].

Natural to ask if turbulent dynamics appear in gravity too?

Introduction

Why would Einstein equations behave like fluids? Evidence that black hole event horizons behave like fluids:

- membrane paradigm: horizons behave as viscous membrane [Dramour].
- Transport properties of horizons [Policastro, Son, Starinets].
- fluid/gravity: one-to-one map between near-equilibrium black holes and solutions to relativistic viscous hydrodynamics [Rangamani, Hubeny,…].

One may hope to see turbulence in black hole dynamics and in particular, **universal features** as in turbulent cascades in fluids.

Outline

- ~~Introduction~~
- Kolmogorov’s theory
- Gravity in the Large d limit
- Results
- Conclusions

Kolmogorov's theory of turbulence (K41)

- In turbulent flows the velocity field, u, is a random variable. Consider **statistical properties of the velocity field:** reproducible, simple.
- Statistical evolution equations **unclosed:** need for assumptions.

- Kolmogorov focused on (3+1) dimensions and made the following assumptions:

  1. Turbulent motions are statistically **homogeneous and isotropic** $\Rightarrow$ pdf is translation and rotation invariant.

  2.The statistics have a **universal** form that is determined uniquely by the viscosity ν and energy transfer rate ε $\Rightarrow$ pdf=f(ν,ε).

  3. Viscous effects only influence much small scales $\Rightarrow$ there is a range where pdf=f(ε): **inertial range.**

Observables

He examined the following quantities:

- n-pt functions of u in position space, averaged over realisations:

  specialise to longitudinal 'structure functions'

$$S_n(r) \equiv \langle |(\vec{v}(\vec{x}+r\hat{y}) - \vec{v}(\vec{x})) \cdot \hat{y}|^n \rangle$$

Since isotropic

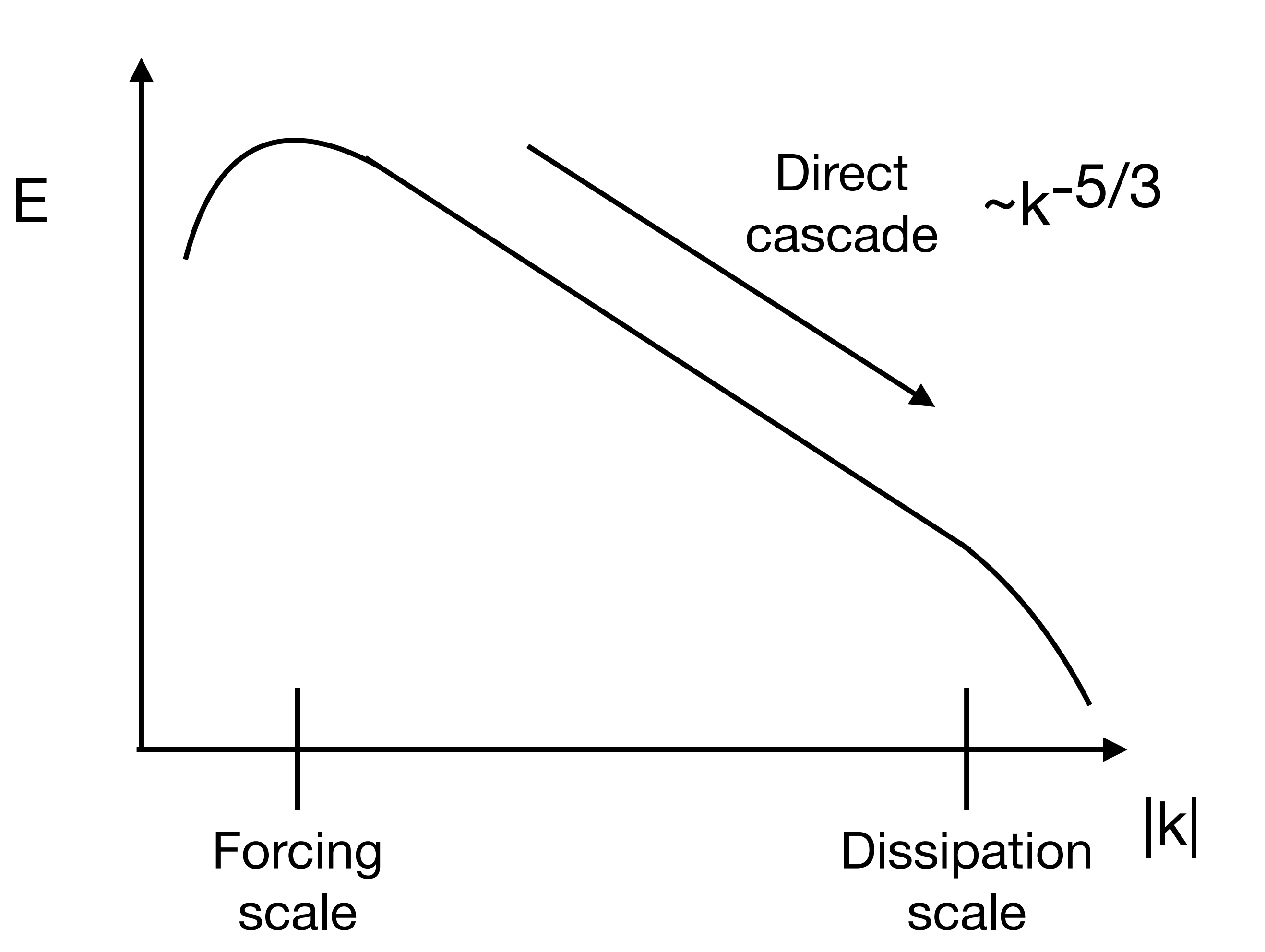
Statistical averaging

- Energy power spectrum: measures the amount of kinetic energy in a shell of radius k.

$$E(k) \equiv \partial_k \int_{|k'|\leq k} \frac{d^d k'}{(2\pi)^d} |\vec{v}_{k'}|^2$$

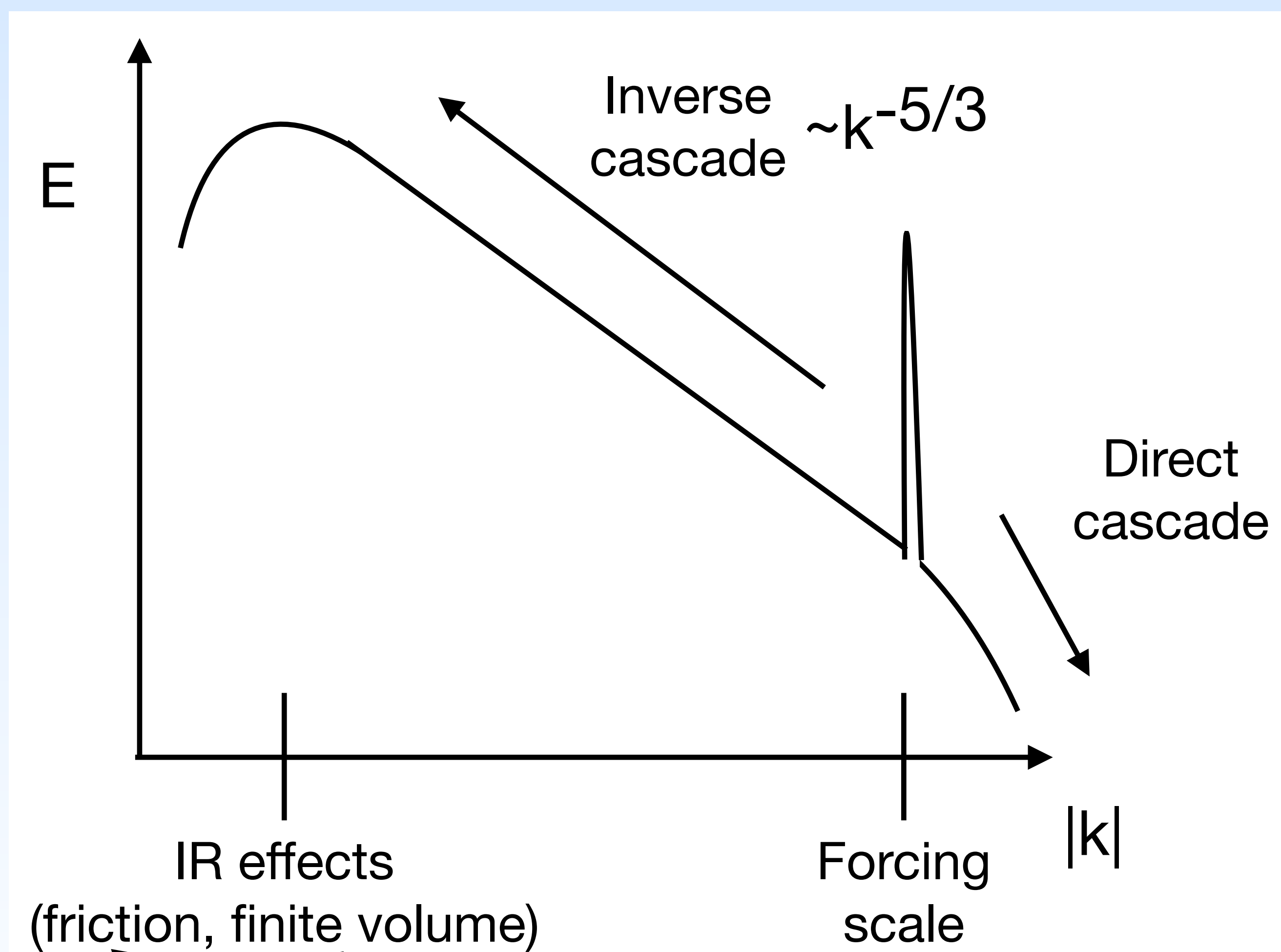
(3+1) Phenomenology: direct cascade

- Due to interaction vortices will break up to smaller and smaller ones: energy transfer to smaller scales.
- At very small scales, viscous effect set in: energy dissipates.
- Scaling behaviour in inertial range.



(2+1) Phenomenology: inverse cascade

- Inertial range grows towards larger scales, until IR effects become important.
- Direct cascade also present - ignore in this talk!



Most typical in
atmospheric simulations

Our approach.

Power laws

Scaling of power spectrum in inertial range is dimension-independent: based on simple dimensional analysis!

From similarity hypothesis, only one scale important in the inertial range

$$[\epsilon] = \frac{[u^2]}{[t]} = \frac{[l^2]}{[t^3]}$$

Then

- $$E(k) = C\epsilon^{2/3}k^{-5/3} = \frac{[l]^3}{[t]^2}$$
- $$S_n(r) = C\epsilon^{n/3}r^{n/3} = \frac{[l]^n}{[t]^n}$$

K41 predictions for homogeneous & isotropic turbulence

In this talk

Aim:

Construct turbulent black branes and check whether they exhibit a universal regime compatible with K41, focusing on inverse cascade.

Approach:

Start with Einstein's equation and

- Construct turbulent black branes in AdS_4: technically demanding.
- Consider the Large d limit, while restricting dynamics to (2+1) dimensions: simple calculation, connection with fluid.
- Need to consider forcing compatible with isotropy hypothesis to get K41.

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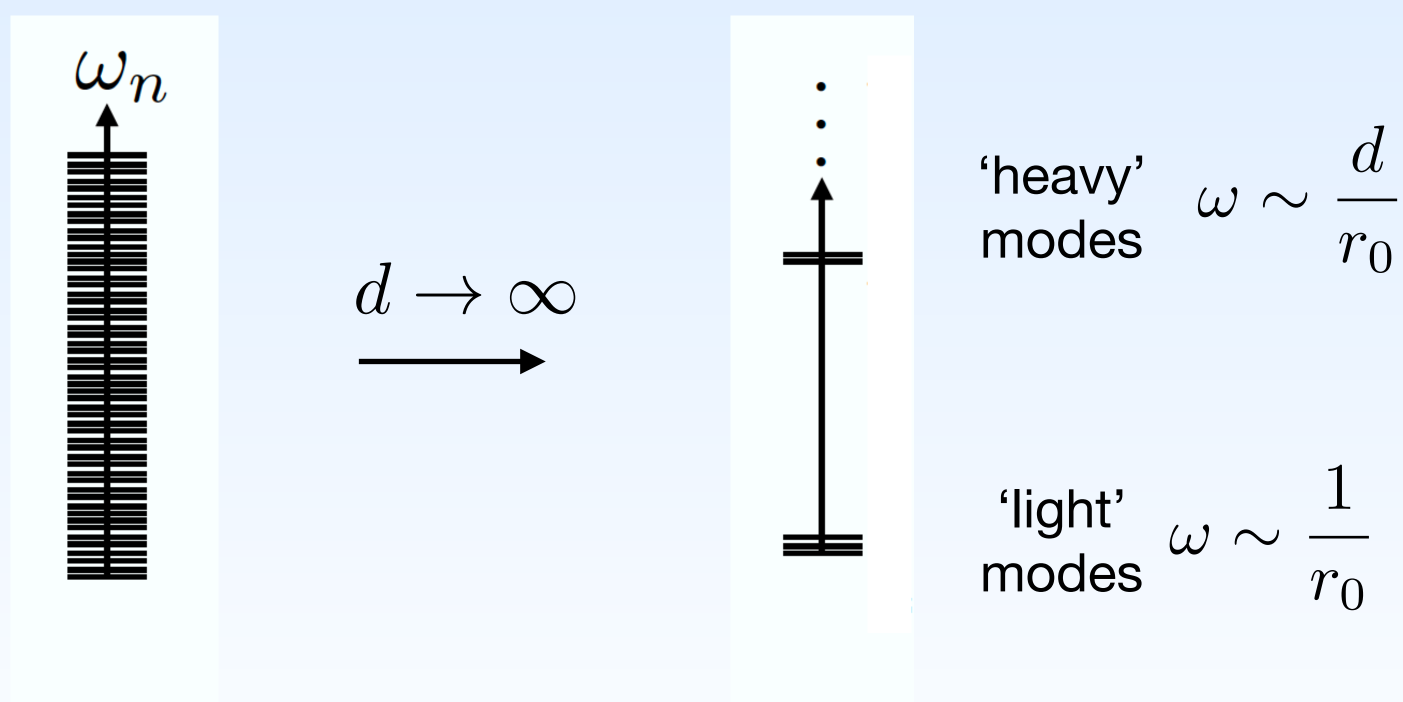
Gravity in the Large d limit

- Treating the number of dimensions (d) as a parameter, one can do a perturbative expansion in 1/d for d large. [Emparan et al, Minwala et al]
- When considering black holes, there is a separation of scales: $r_0\ ,\ \dfrac{r_0}{d}$ hinting towards the existence of an effective theory.
    - One way to see this is by considering the QNM spectrum of Sch-AdS$_{d+1}$. In the large d, these modes split into two types:



Large d Effective theory

Procedure:

1. Take boosted AdS-Sch metric: $g_0(r)$, parameters: a, p^i

2. Promote moduli to function of boundary coordinates:

$$a, p^i \to a(t, \vec{x}), p^i(t, \vec{x})$$

3. New metric, $g_0(t, r, \vec{x})$, solves Einstein's equations to $\mathcal{O}\left(\frac{1}{d}\right)$

4. Add metric correction $g_0(t, r, \vec{x}) + \frac{1}{d} g_1(t, r, \vec{x})$

5. New metric solves Einstein's equations up to $\mathcal{O}\left(\frac{1}{d^2}\right)$

Large d Effective theory

The constraints are given by

$$(\partial_t - \partial_i \partial^i) a + \partial_i p^i = 0\,,$$
$$(\partial_t - \partial_j \partial^j) p_i + \partial_i a + \partial_j \left(\frac{p_i p^j}{a} \right) = 0$$

<u>Comments:</u>

- Solutions to these eqns give rise to full black hole metrics in AdS.
- Look like NS equations, but exact in gradients- better control than hydrodynamics.
- For our purposes: we restrict dynamics to (2+1) dim.

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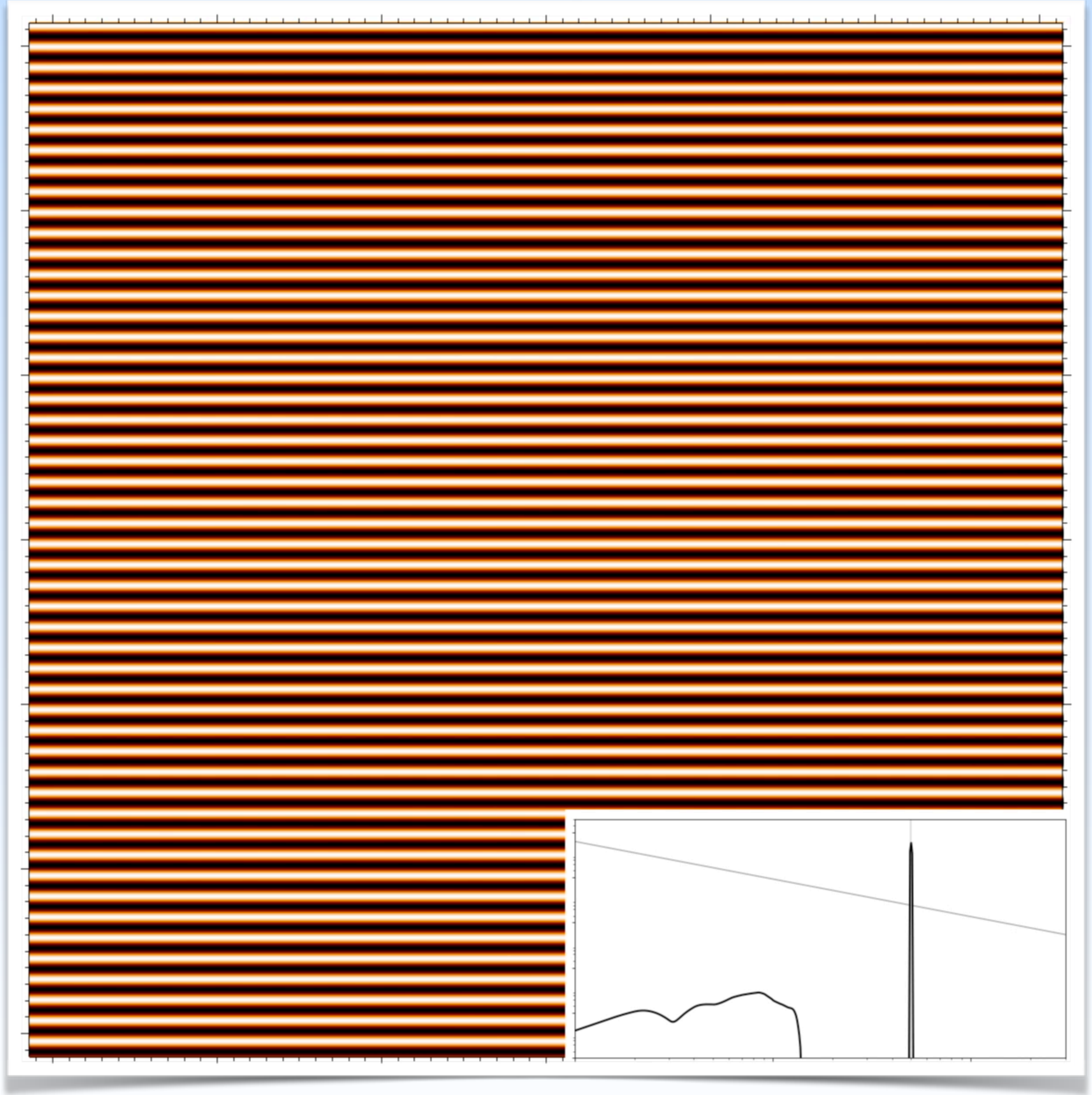
No driving: initial data induced

Study (2+1) decaying turbulence in large d [Rozali et al.].

- Start from counter flow initial data and evolve on a torus of size LxL.

$$a = c_0 \, , \; p_x = 0 \, ,$$
$$p_y = c_1 \; \cos(\frac{2\pi \, n}{L} \, x)$$

- Monitor the vorticity $\omega = \epsilon^{ij}\partial_i u_j$ and the power spectrum.
- Turbulent, but no clear power law consistent with K41.
    - Due to lack of driving.
    - Same conclusion reached by [Chesler, Yaffe] in AdS_4 in the same setting.



Homogeneous Isotropic driving

- Drive the system in a way to produce vorticity without directly sourcing the energy density

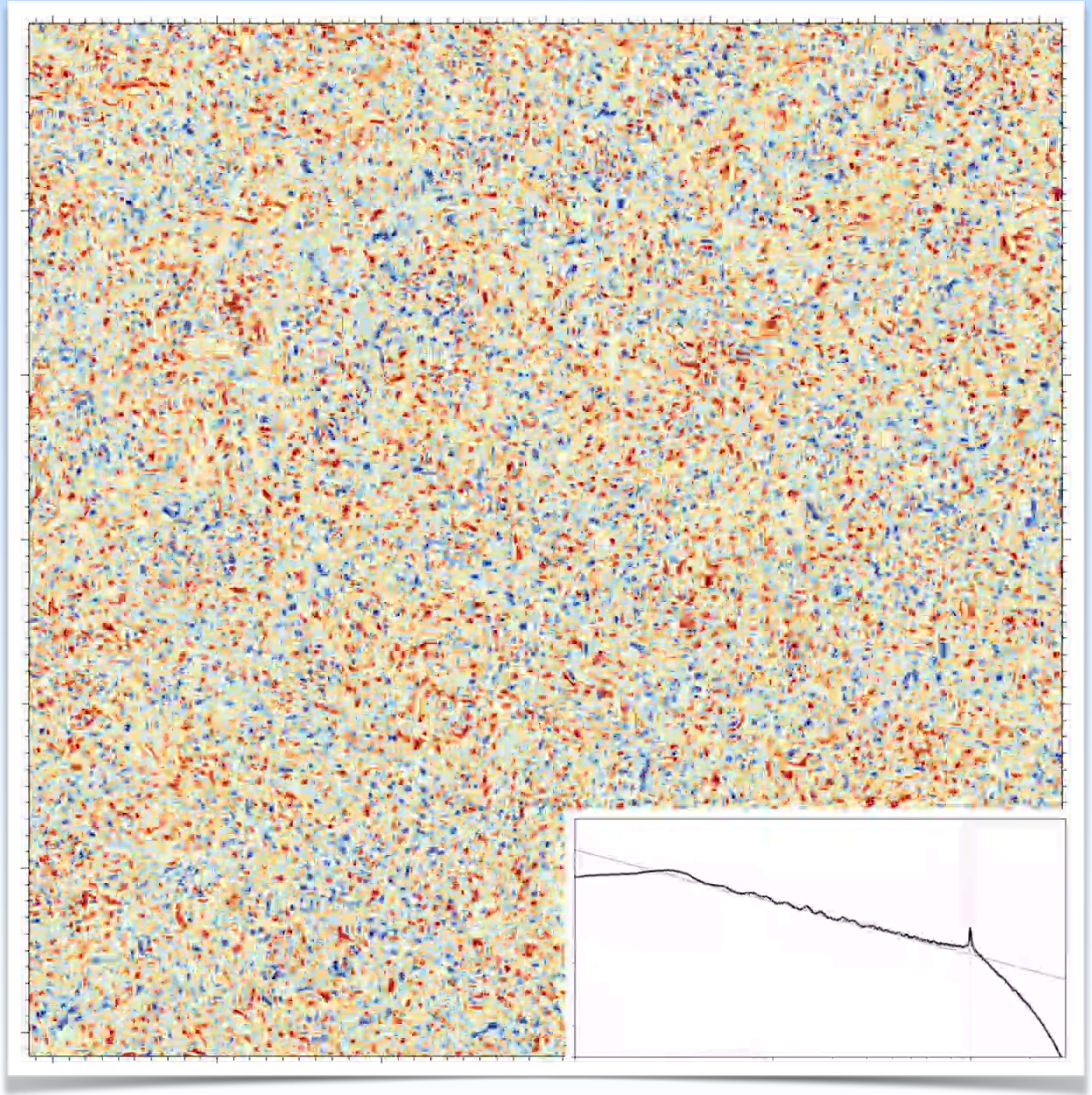
$$(\partial_t - \partial_i \partial^i) a + \partial_i p^i = 0$$

$$(\partial_t - \partial_j \partial^j) p_i + \partial_i a + \partial_j \left(\frac{p_i p^j}{a} \right) = F_i$$

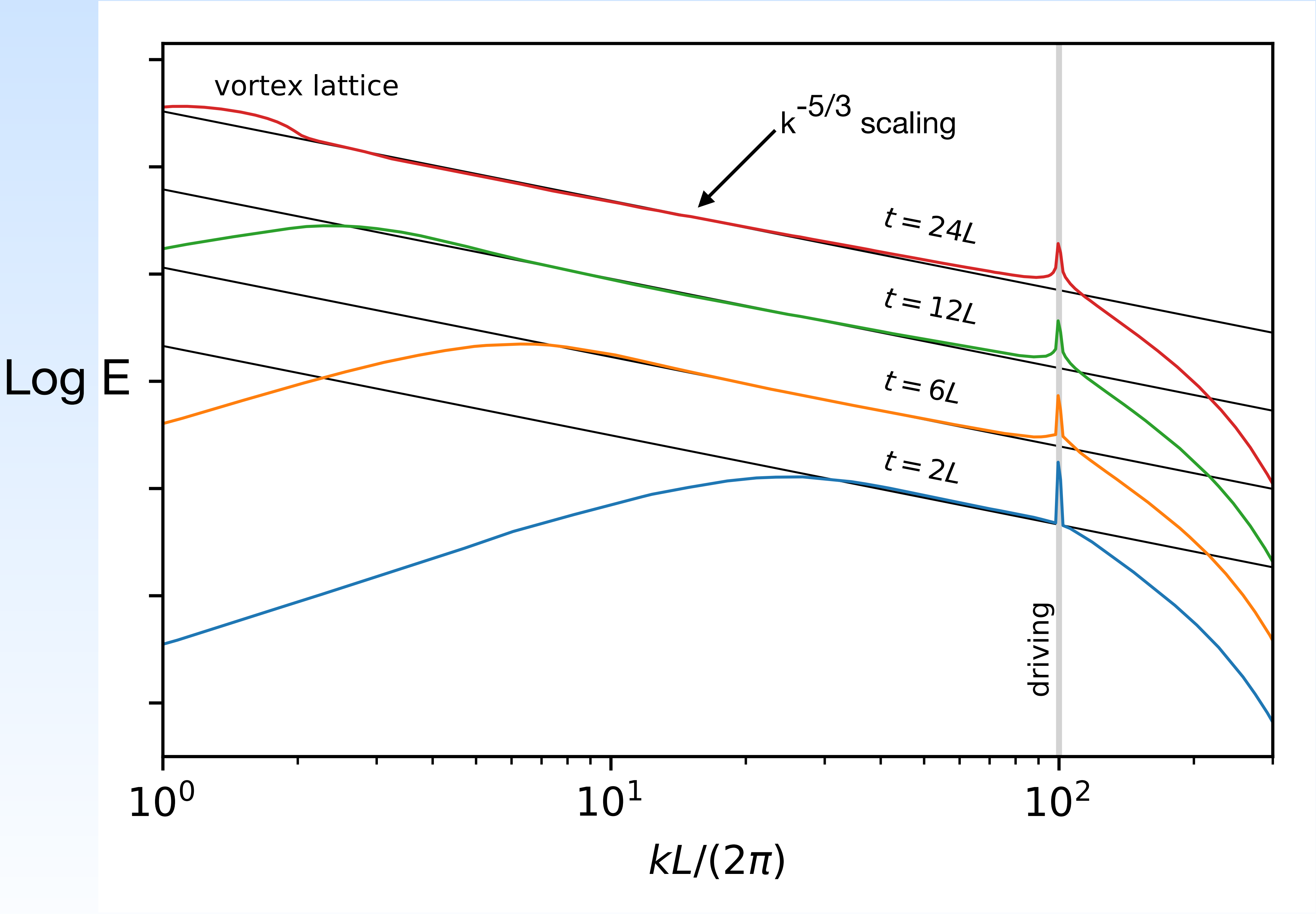
$$F^i = \epsilon^{ij} \partial_j F$$

F: isotropic sum of k-modes with random amplitudes and phases.

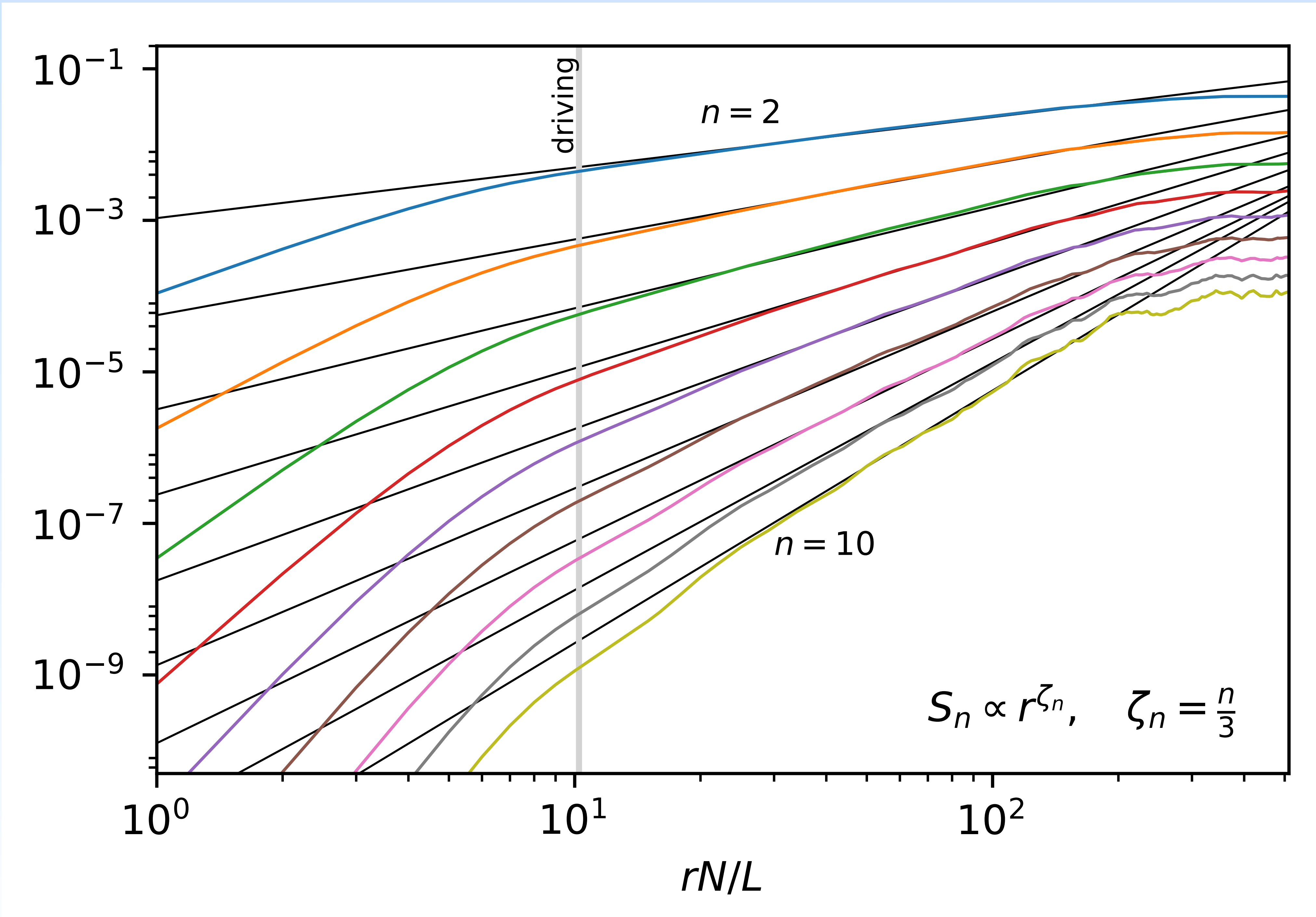
- Start from thermal equilibrium - initial conditions: $a = 2\,, p_i = 0$
- Also work on the 2-torus.
- Evolve in time using 4th-order Runge-Kutta time-stepping. For spatial derivatives we use 4th order finite difference.



• Energy power spectrum - 256 realisations



- Structure functions - 256 realisations



Gravitational driving

Can we see turbulence in a fully gravitational description?

- Drive the system through a deformation of the boundary metric.
- The Large d effective field theory is modified [Andrade, CP, Withers].
- Focus on turning on source γ_{tt}

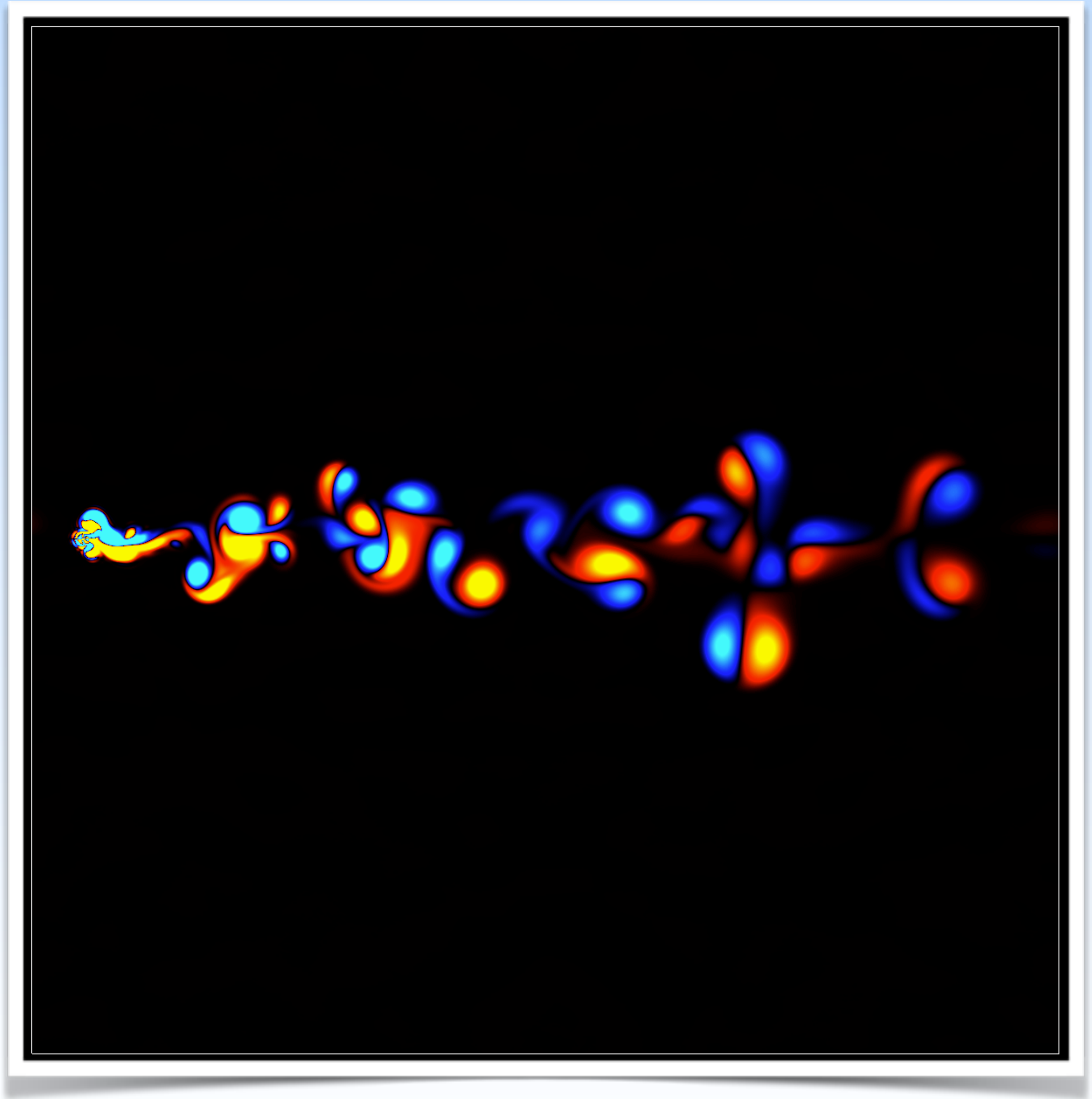
$$(\partial_t - \partial_i\partial^i)a + \partial_i p^i = 0$$

$$(\partial_t - \partial_j\partial^j)p_i + \partial_i a + \partial_j\left(\frac{p_i p^j}{a}\right) = F_i$$

$F_i = a\nabla_i\gamma_{tt}/2$
Gradient of a scalar,
Does not inject vorticity

Computation:

- Start with 'fluid' is in uniform motion, quench γ_{tt} from zero to a symmetric Gaussian profile at a fixed location: obstacle in a flow.
- Neither homogeneous nor isotropic: do not expect K41. It gives turbulent wake, at the back of the obstacle.



Outline

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Summary

- We studied turbulent black hole dynamics in the presence of explicit, homogeneous and isotropic forcing.
    - Large d expansion: gained analytic control.
    - The solutions correspond to full AdS black holes.
    - Universality in turbulent black holes consistent with K41.

- We briefly studied a set-up with an explicit gravitational source, giving rise to tidally-induced turbulent wakes.

Outlook and future directions

Physical relevance:

- Via AdS/CFT, relevant for turbulence at strong coupling, e.g. quark-gluon plasmas, condensed matter.

- From purely gravitational perspective:
  - Which *geometric* quantities capture turbulence? what is the gravitational origin of turbulent cascades?
  - Relevant for the plunge dynamics for BH-BH or BH-NS mergers as measured by [LIGO]: Finite d, spherical horizons, asymptotically flat?

Thanks for your attention!