Far-From-Equilibrium Chiral Charged Plasma Subjected to External Electromagnetic Fields

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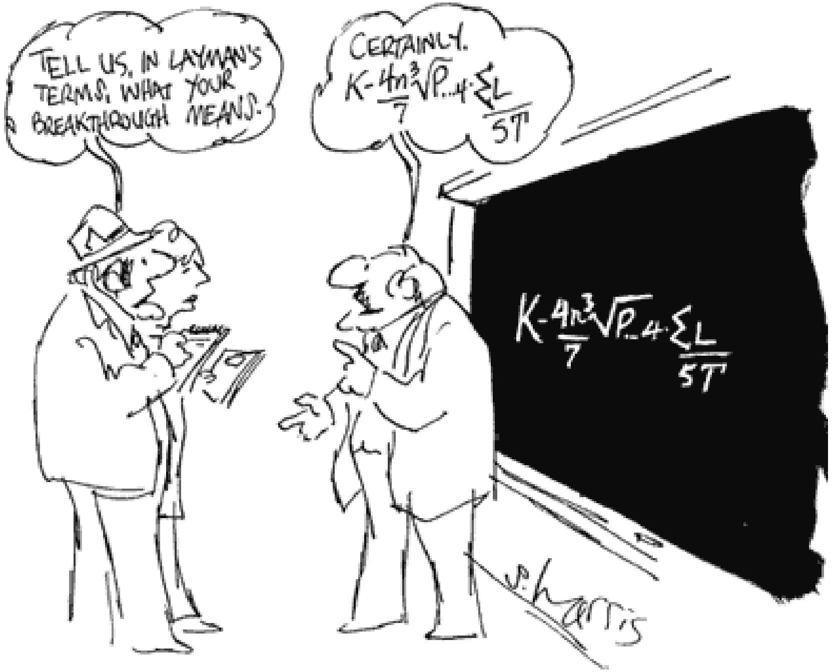
Cartwright, arXiv:2003.04325

Outline

- Motivation
- AdS Geometry
- 1 point functions
- Conservation equations
- Conclusion



All images are hyperlinked to their source

Motivation

-At high energies chiral symmetry is restored

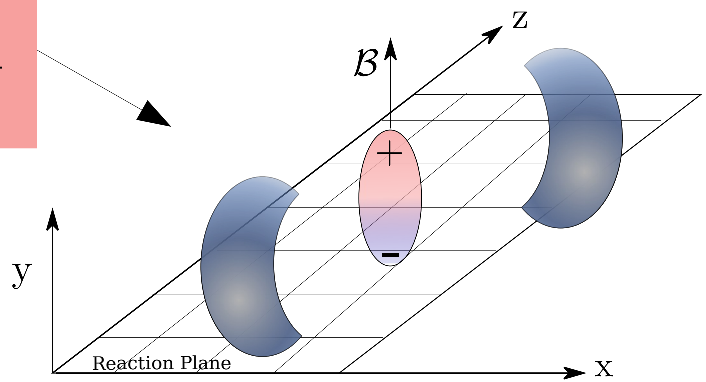
-Extremely large magnetic fields $\sim 10^{14}\mathrm{T}$

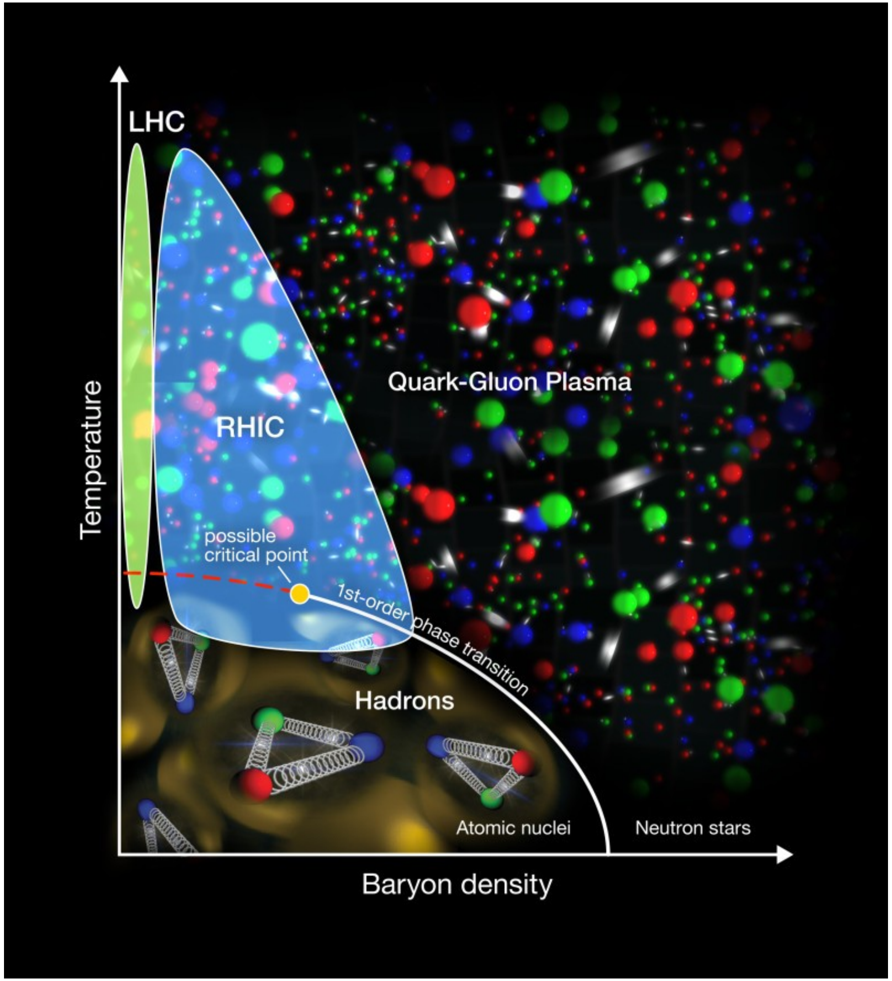
-Topological gauge configurations, $Q_w = \frac{g^2}{32\pi^2}\int \mathrm{d}^4x F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \in \mathbb{Z}$

Axial $U(1)$ is anomalous

$$N_L - N_R \sim Q_w$$

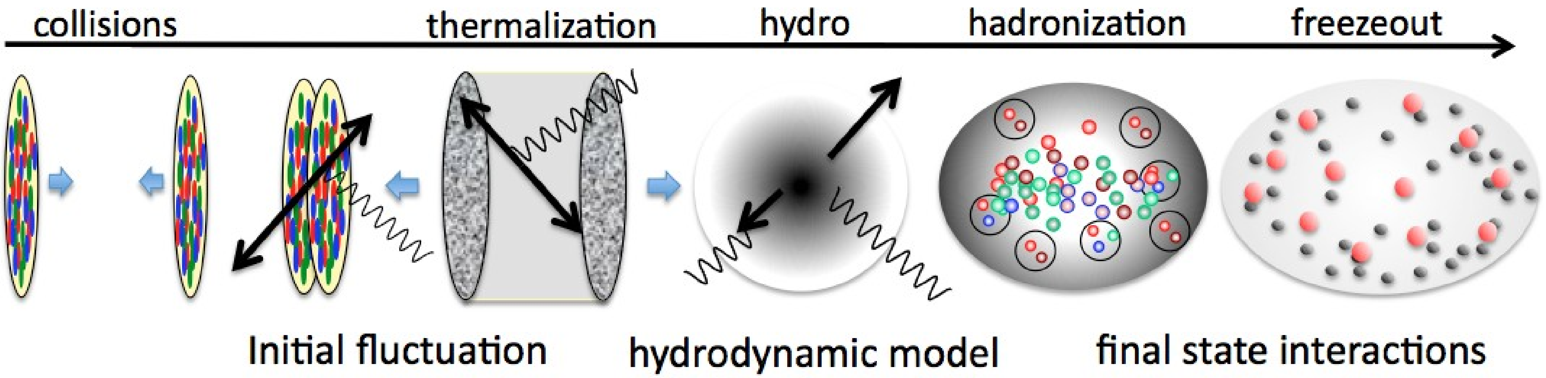
Asymmetry in the number of positive and negative charges above and below reaction plane





[Kharzeev,Tu, Zhang, Li, Phys. Rev. C (2018)]
[Kharzeev, Prog. Part. Nucl .Phys. (2014)] [STAR Collaboration, Phys. Rev. Lett. (2009)]
[Kharzeev, McLerran, Warringa Nucl. Phys. (2008)] [CMS Collaboration, Phys. Rev. Lett. (2017)]

Motivation II



[Nonaka,Asakawa; PTEP (2012)]

Fluid dynamic approximation needs to start early $\tau \approx 1 fm/c$

Time when gluons at weak coupling αs ≪ 1 reached thermal equilibrium $\tau \geq 6.9 fm/c$ [Baier,Mueller,Schiff,Son,Phys. Rev. B (2002)]

Suggestion: Perhaps full thermalization not needed...only isotropy of energy momentum tensor? [Arnold, Lenaghan,Moore, Yaffe, Phys. Rev. Lett. (2005)]

[Romatschke, EPJC (2017)]

Use holography to model isotropization

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AdS Geometry - Setup

Bulk Action $$S = -\int \mathrm{d}^5x \frac{1}{16\pi G_5}\left[\sqrt{-g}(R - 2\Lambda - L^2 F_{\mu\nu}F^{\mu\nu})\right] + \frac{\gamma}{6}\epsilon^{\alpha\beta\gamma\delta\eta}\mathcal{A}_\alpha F_{\beta\gamma}F_{\delta\eta},$$

Einstein-Hilbert U(1) Kinetic Term Chern-Simons term

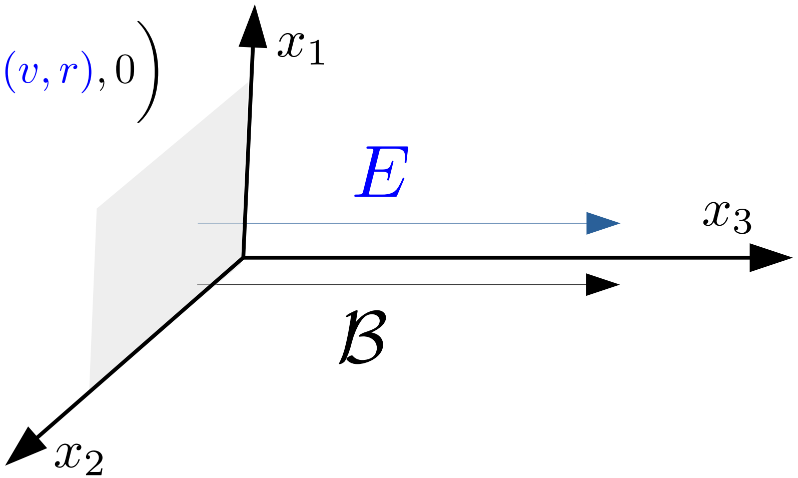
To produce chiral charges we want aligned electric and magnetic fields!

Gauge field Ansatz $$A_\mu = \left(\phi(v,r), -\frac{\mathcal{B}}{2}x_2, \frac{\mathcal{B}}{2}x_1, -P(v,r), 0\right)$$

- Magnetic field $SO(3) \to SO(2)$

- $\mathbb{Z}_2$ Parity symmetry along $\mathcal{B}$

-Broken by electric field



Metric Ansatz [Fuini, Yaffe; JHEP (2014)]
[Cartwright,Kaminski; JHEP (2019)]

$$\mathrm{d}s^2 = 2\mathrm{d}v(\mathrm{d}r - \frac{1}{2}A\mathrm{d}v) + S^2e^B(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + S^2e^{-2B}\mathrm{d}x_3^2 \; +2F\mathrm{d}v\mathrm{d}x_3$$

AdS Geometry – Setup II

Equations of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 2L^2\left(F_{\mu\lambda}F^{\lambda}_{\nu} - g_{\mu\nu}\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\right),$$

$$\nabla_\mu F^{\mu\nu} = \frac{\gamma}{8\sqrt{-g}L^2}\epsilon^{\nu\alpha\beta\lambda\sigma}F_{\alpha\beta}F_{\lambda\sigma}.$$

Using our ansatz gives this mess →

$$\begin{aligned}
6SS'' &= -3S^2\left(B'\right)^2 - 4e^{2B}\left(P'\right)^2 \\
3S^3F'' &= -3S^2S'\left(6FB' + F'\right) - 3S^3\left(2B'F' + F\left(2B'' + \left(B'\right)^2\right)\right) \\
&\quad - 4SF\left(e^{2B}\left(P'\right)^2 - 3\left(S'\right)^2\right) + 12P'(\gamma P\mathcal{B} + \rho) \\
12e^{2B}S^5\dot{S}' &= -e^{2B}S^4\left(e^{2B}F^2\left(B'\right)^2 + 4e^{2B}FB'F' + e^{2B}\left(F'\right)^2 + 24\dot{S}S'\right) \\
&\quad - 4e^{4B}S^3FS'\left(FB' + 2F'\right) - 4e^{2B}\gamma^2P^2\mathcal{B}^2 \\
&\quad - 4S^2\left(F^2\left(e^{4B}\left(S'\right)^2 - e^{6B}\left(P'\right)^2\right) + \mathcal{B}^2\right) \\
&\quad - 8e^{2B}\gamma P\rho\mathcal{B} - 4e^{2B}\rho^2 + 24e^{2B}S^6 \\
2e^{2B}S^4\dot{P}' &= -2e^{2B}S^4\left(\dot{P}B' + \dot{B}P'\right) + e^{2B}S\left(2\rho FB' + e^{2B}F^2P'S' + \rho F'\right) \\
&\quad - e^{4B}S^2F\left(F\left(4B'P' + P''\right) + 2P'F'\right) \\
&\quad + \gamma P\mathcal{B}\left(e^{2B}S\left(2FB' + F'\right) - 2e^{2B}FS' + \gamma\mathcal{B}\right) \\
&\quad - e^{2B}S^3\left(\dot{S}P' + \dot{P}S'\right) + \rho\left(\gamma\mathcal{B} - 2e^{2B}FS'\right) \\
6e^{2B}S^4\dot{B}' &= -e^{4B}SFS'\left(11FB' + 4F'\right) - 9e^{2B}S^3\left(\dot{S}B' + \dot{B}S'\right) \\
&\quad - e^{4B}S^2\left(2FB'F' + F^2\left(3B'' + 2\left(B'\right)^2\right) - 8\dot{P}P' - \left(F'\right)^2\right) \\
&\quad + 4e^{4B}F^2\left(e^{2B}\left(P'\right)^2 + \left(S'\right)^2\right) - 4\mathcal{B}^2
\end{aligned}$$

$$\begin{aligned}
6e^{2B}S^6A'' &= -6e^{2B}S^6\left(3\dot{B}B' + 4\right) + 24e^{4B}S^3FS'\left(2FB' + F'\right) \\
&\quad + e^{2B}S^4\left(12e^{2B}FB'F' + 3e^{2B}F^2\left(4B'' + 3\left(B'\right)^2\right)\right) \\
&\quad + e^{2B}S^4\left(-8e^{2B}\dot{P}P' - 3e^{2B}\left(F'\right)^2 + 72\dot{S}S'\right) + 28e^{2B}\gamma^2P^2\mathcal{B}^2 \\
&\quad + 4S^2\left(e^{4B}F^2\left(e^{2B}\left(P'\right)^2 - 3\left(S'\right)^2\right) + 5\mathcal{B}^2\right) - 48e^{4B}\rho SFP' \\
&\quad - 8e^{2B}\gamma P\mathcal{B}\left(6e^{2B}SFP' - 7\rho\right) + 28e^{2B}\rho^2
\end{aligned}$$

Equations written along radial null geodesics partially nest

$$\begin{aligned}
6e^{2B}S^6\dot{F}' &= -F\left(-6e^{2B}S^6\left(A'B' + 4\right) + 6e^{2B}S^5\left(A'S' + 3\dot{S}B' - 3\dot{B}S'\right)\right. \\
&\quad +4e^{2B}\gamma^2P^2\mathcal{B}^2 + e^{4B}S^4\left(16\dot{P}P' + 3\left(F'\right)^2\right) + 8e^{2B}\gamma P\rho\mathcal{B} + 4e^{2B}\rho^2 - 4S^2\mathcal{B}^2\left.\right) \\
&\quad + 3e^{2B}S^3\left(S^3\left(A'F' - 4\dot{F}B'\right) + 8\dot{P}(\gamma P\mathcal{B} + \rho) + S^2\left(4\dot{F}S' - 6\dot{S}F'\right)\right) \\
&\quad - e^{4B}S^2F^3\left(-18SB'S' - 3S^2\left(2B'' + \left(B'\right)^2\right) + 4\left(e^{2B}\left(P'\right)^2 + 3\left(S'\right)^2\right)\right)
\end{aligned}$$

$$\begin{aligned}
12S^7\ddot{S} &= F^2\left(6e^{2B}S^5\left(A'S' + 2\dot{S}B' - 2\dot{B}S'\right) - 6e^{2B}S^6\left(\dot{B}B' + 4\right) + 4e^{2B}\gamma^2P^2\mathcal{B}^2\right. \\
&\quad +e^{2B}S^4\left(-8e^{2B}\dot{P}P' + e^{2B}\left(F'\right)^2 + 24\dot{S}S'\right) + 8e^{2B}\gamma P\rho\mathcal{B} + 4e^{2B}\rho^2 + 4S^2\mathcal{B}^2\left.\right) \\
&\quad - 2S^6\left(-3S\dot{S}A' + 4e^{2B}\dot{P}^2 + 3\dot{B}^2S^2\right) \\
&\quad - e^{4B}S^2F^4\left(-S^2\left(B'\right)^2 - 4SB'S' + 4e^{2B}\left(P'\right)^2 - 4\left(S'\right)^2\right) \\
&\quad + 4e^{4B}S^3F^3F'\left(SB' + 2S'\right) + 12e^{2B}S^5F\left(\dot{S}F' - \dot{F}S'\right)
\end{aligned}$$

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AdS Geometry - Near the boundary

Expanding near the AdS boundary ... $\longrightarrow$

$$B(v,r) = \frac{b_4(v)}{r^4} + \log(r)\cdots$$

$$A(v,r) = (r+\xi(v))^2 - 2\xi'(v) + \frac{a_4(v)}{r^2} + \log(r)\cdots$$

$$F(v,r) = \frac{f_4(v)}{r^2} + \log(r)\cdots\ ,$$

$$P(v,r) = p_0 + E\left(v + \frac{1}{r}\right) + \frac{p_2(v)}{r^2} + \log(r)\cdots$$

Also yields 2 ODE's:

$$a_4'(t) = \frac{8E}{3}(\xi(t)E + p_2(t)) \qquad f_4'(t) = E((p_0 + Et)\mathcal{B}\gamma + \rho)$$

$\Downarrow$

Cannot be solved without full evolution

$\Downarrow$

$$f_4(t) = E((p_0 t + \frac{1}{2}Et^2)\mathcal{B}\gamma + \rho t) + f_4^0$$

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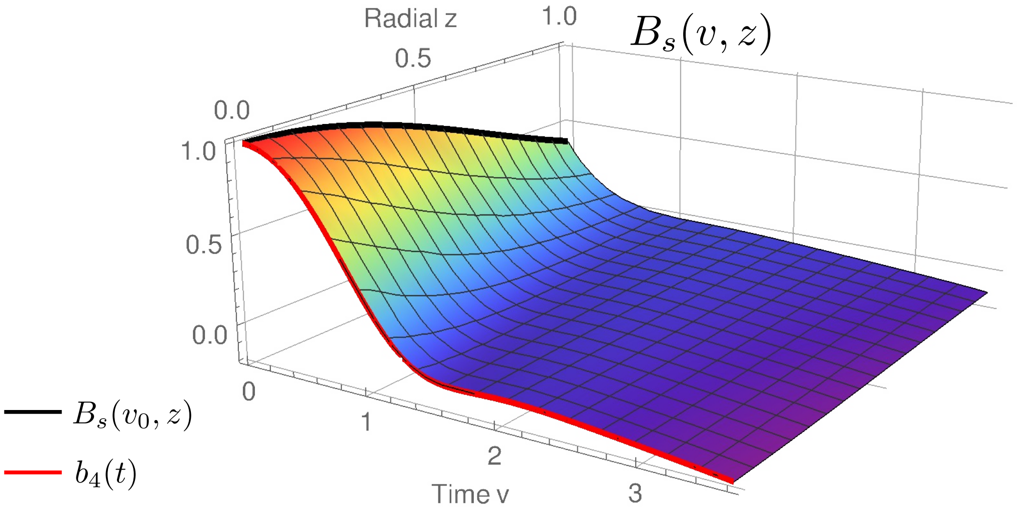
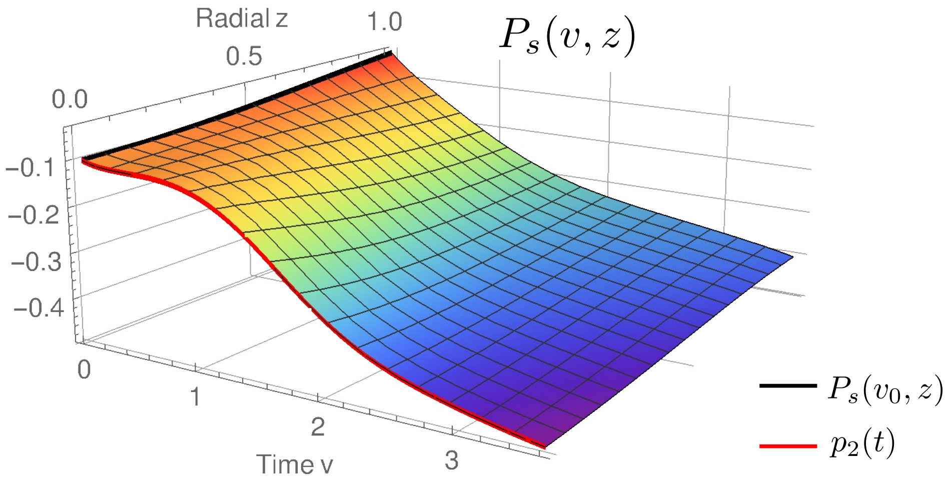
AdS Geometry - Solutions





Numerical Routine: Characteristic Formulation [Chesler,Yaffe JHEP (2014)]

- Chebyshev spectral representation
- Solve partially nested structure for $S, F, \dot{S}, \dot{B}, \dot{P}, A, \dot{F}$
- Extract time derivatives, ex. $\partial_t B = \dot{B} - A\partial_r B$
- Extract $b_4(t), a_4(t), f_4(t), p_2(t)$
- Step forward with RK4

Details:

- Solve for scaled subtracted functions, ex.

$$B(v, z) = z^{\delta_B} B_s(v, z) + \Delta_B(v, z)$$

$$P(v, z) = z^{\delta_P} P_s(v, z) + \Delta_P(v, z)$$

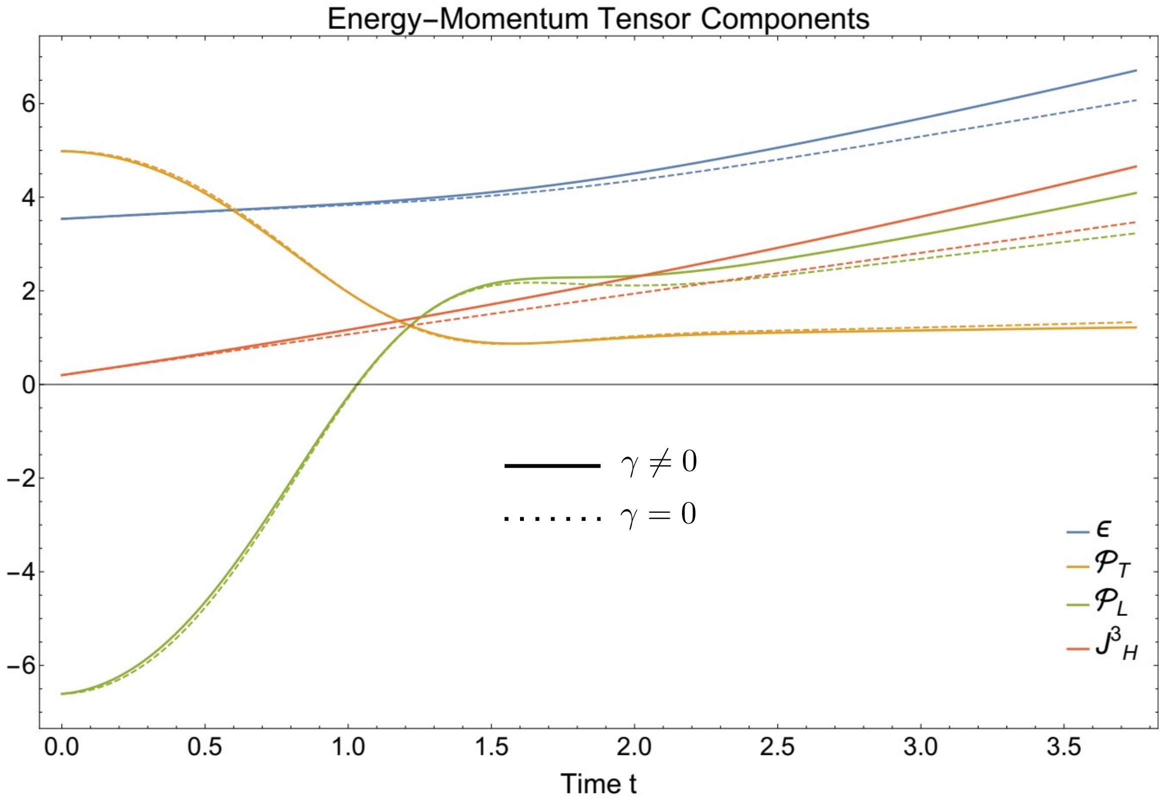
- Fix apparent horizon on every time step

1 point functions: Energy Momentum Tensor

Vary renormalized on-shell action [Fuini, Yaffe; JHEP (2014)]

$$4\pi G_5 \langle T_{\mu\nu} \rangle = g^{(4)}_{\mu\nu} - g^{(0)}_{\mu\nu} Tr(g^{(4)}) + \left(Log(\mu_r) + \mathcal{C} \right) h^{(4)}_{\mu\nu}$$

$\epsilon, \mathcal{P}_T, \mathcal{P}_L, J^3_h$
Functions of
$a_4(t), b_4(t), f_4(t)$



Very little early time change!

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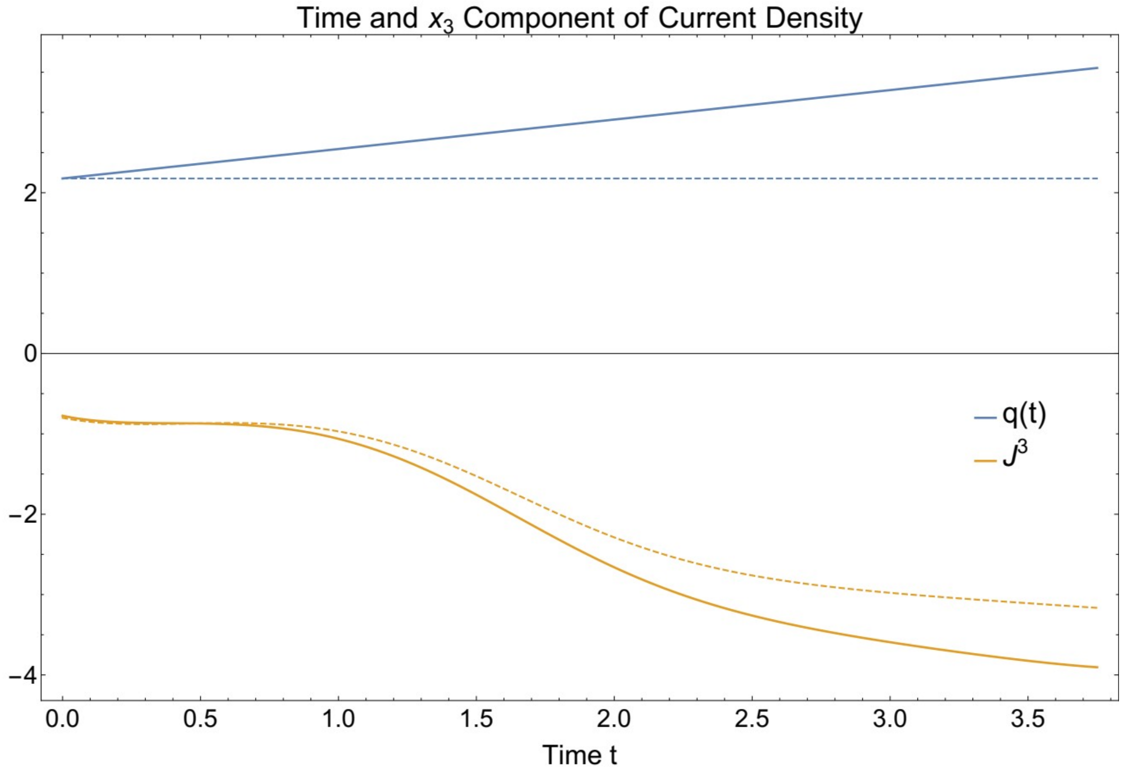
1 point functions: Axial Current

Vary renormalized on-shell action [D'Hoker,Kraus, JHEP (2010)]

$$-4\pi G_5 \langle J^\mu \rangle = \lim_{r\to\infty} -r^3 L^2 \eta^{\mu\nu}\partial_r A_\nu + \frac{k}{3}\epsilon^{\mu\nu\alpha\beta}A_\nu F_{\alpha\beta}.$$



$\gamma \neq 0$

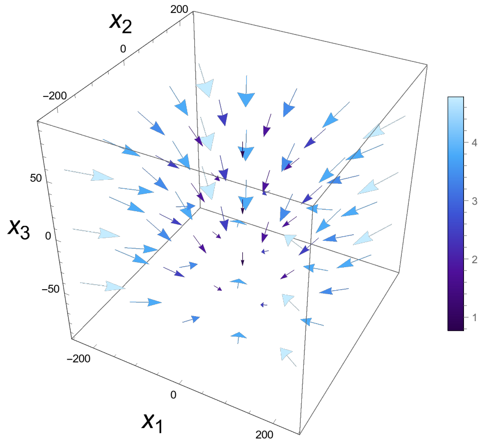
$\gamma = 0$

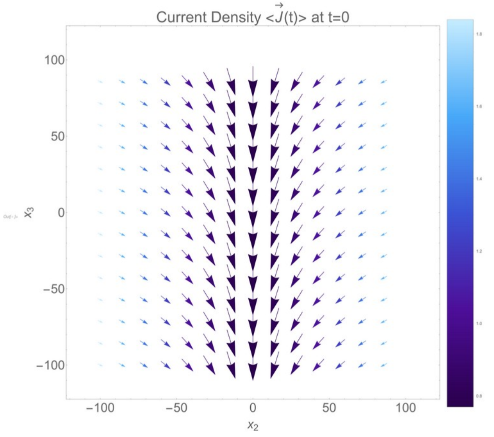
$q(t) = J^0(t)$

-Linear growth in axial charges

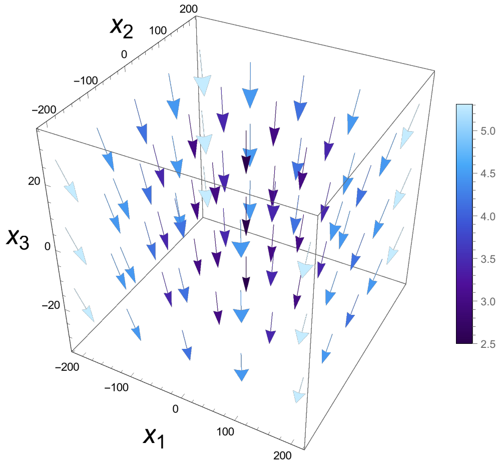
1 point functions: Spatial Current Evolution $\langle \vec{J} \rangle$

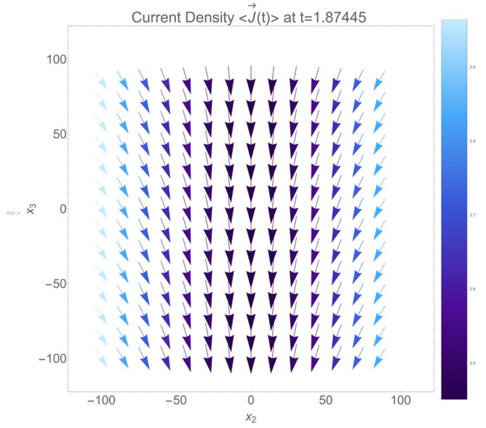
$t = 0$



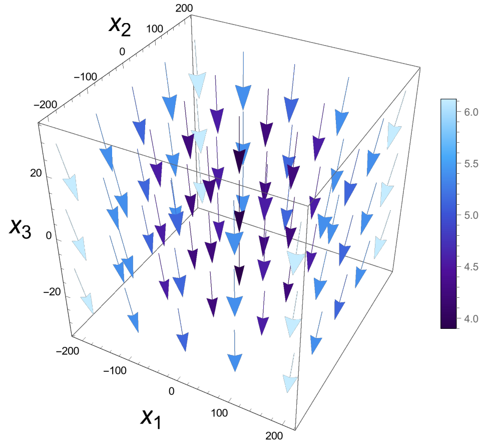


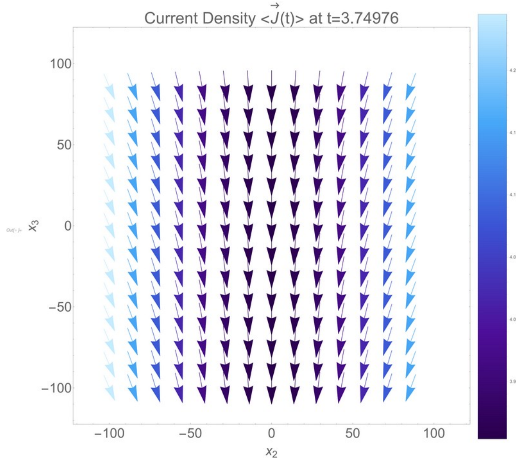
$t = 1.87455$





$t = 3.74976$





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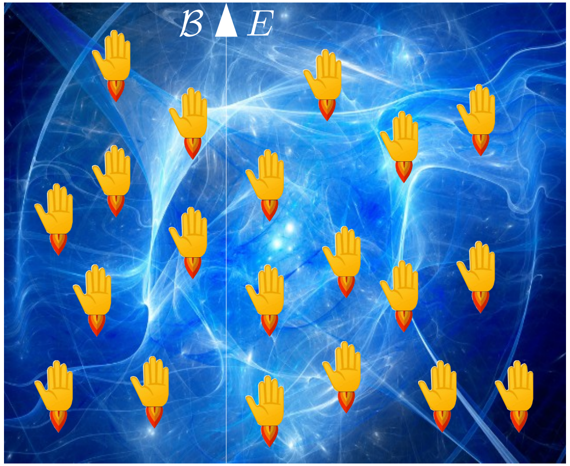
Hydrodynamics

Can we understand some of this behavior?

Field theory

Conservation equations: $\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu \qquad \partial_\mu J^\mu = \epsilon^{\mu\nu\alpha\beta} C F_{\mu\nu} F_{\alpha\beta}$



Assume: $T^{\mu\nu} = \epsilon \delta_0^\mu \delta_0^\nu + J_h \left(\delta_0^\mu \delta_3^\nu + \delta_3^\mu \delta_0^\nu\right) + p_t \delta_i^\mu \delta_j^\mu + p_l \delta_3^\mu \delta_3^\nu$

$J^\mu = (J^0, -C x_1 B E, -C x_2 B E, J^3),$

$$\partial_t J^0 = C E_3 B_3 \qquad \partial_t J_h = E_3 J^0 \qquad \partial_t \epsilon = E_3 J^3$$

Two can be solved: $J^0(t) = C E_3 B_3 t + q_0 \quad J_h(t) = E_3 \left(\frac{1}{2} C E_3 B_3 t^2 + q_0 t\right) + j_h$

Near boundary analysis was exactly field theory conservation equations

-Final equation cannot be determined without additional information

-Joule heating $\partial_t \epsilon = E_3 J^3 = \vec{E} \cdot \vec{J}$

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Observations

Heat current $J_h(t) = E_3\left(\frac{1}{2}CE_3B_3t^2 + q_0t\right) + j_h$

Axial current $J^0(t) = CE_3B_3t + q_0$

Unbounded growth!

Deep Thoughts

Whats happening?

Translational invariance $\longrightarrow$ Perfect conductor/no resistivity

[Blake,Tong,Vegh Phys.Rev.Lett (2014)]

Axial anomaly $\longrightarrow$ Anomalous production of chiral charges at rate $\vec{E}\cdot\vec{B}$

Perfect conductance+anomalous production=unbounded current growth

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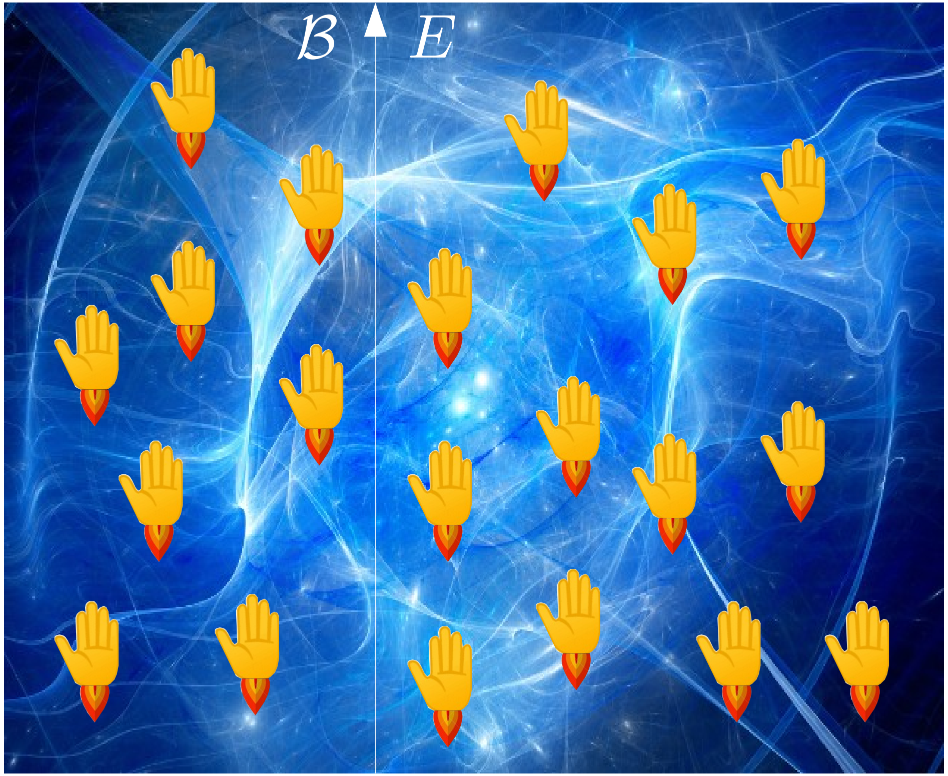
Summary of Results



- Azimuthal influx of axial charges
- Conservation equations determine evolution of (non)-conserved quantities
- Can we include local gauge fields in the boundary? [Grozdanov,Poovuttikul, JHEP (2019)]
- Can we model the 1 point functions in a more realistic boundary theory? Radial flow? [van der Schee, Phys. Rev. D. (2013)]

Appendix

1 point functions: Energy Momentum Tensor

Vary renormalized on-shell action [Fuini, Yaffe; JHEP (2014)]

$$4\pi G_5 \langle T_{\mu\nu}\rangle = g^{(4)}_{\mu\nu} - g^{(0)}_{\mu\nu} Tr(g^{(4)}) + (Log(\mu_r) + \mathcal{C})\, h^{(4)}_{\mu\nu}$$

-True theory is $S_{SYM+EM} = S_{SYM,Min.Coupled} + S_{EM}$

-Total stress energy tensor is renormalization point independent

$$T_{tot} = T_{EM}(\mu_r) + \Delta T_{SYM}(\mu_r)$$

Energy momentum tensor contains energy density, transverse and longitudinal pressures and heat current

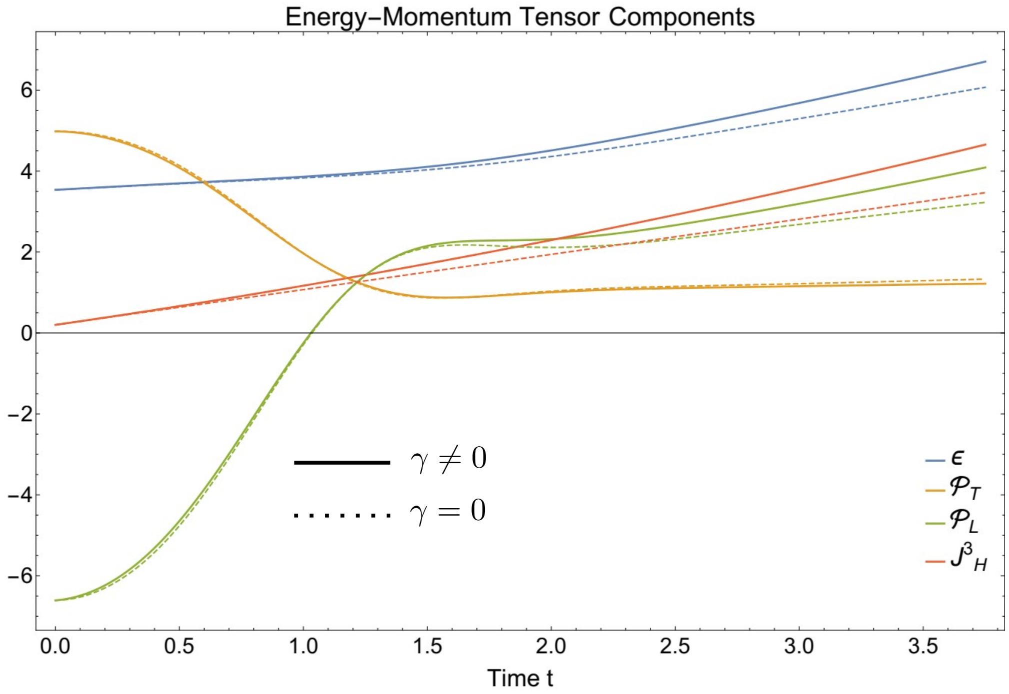
$$\epsilon = -3a_4(t) - \frac{4E^2}{3} + 2(E^2 + \mathcal{B}^2)\log(\mu_r)$$

$$J^3_H = 4f_4(t)$$

$$\mathcal{P}_T = -a_4(t) + 4b_4(t) - \frac{E^2}{9} - \mathcal{B}^2 + 2(E^2 + \mathcal{B}^2)\log(\mu_r)$$

$$\mathcal{P}_L = -a_4(t) - 8b_4(t) + \frac{8E^2}{9} - 2(E^2 + \mathcal{B}^2)\log(\mu_r)$$

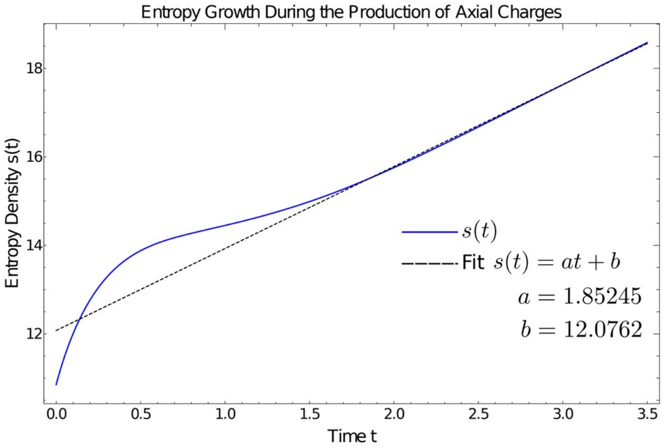
Trace anomaly $Tr(T) = -\frac{1}{4}F^2 = E^2 - \mathcal{B}^2$



Very little early time change!

Appendix

Entropy Production



Entropy density

$$s = 4\pi S(v, r_h)^3$$

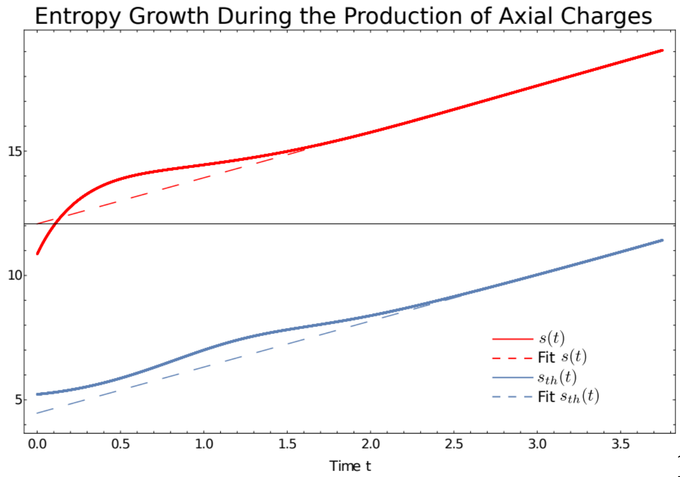
- r_h is apparent horizon location
- Late time linear growth

How does this compare to thermodynamic entropy?

$$s_{th} = \frac{a}{\langle T^{00}\rangle^{1/4}}\left(\langle T^{00}\rangle + \frac{1}{2}\left(\frac{1}{2}\left(\langle T^{11}\rangle + \langle T^{22}\rangle\right) + \langle T^{33}\rangle\right)\right)$$

-Late time linear growth is due to Joule heating of continuously produced axial charges

-Choose a to match late time growth $a = 0.437209$



Entanglement Entropy

Ryu-Takayanagi conjecture $S_A = \frac{\mathcal{A}(\gamma_A)}{4G_N^5}$

$$\mathcal{A} = \int \mathrm{d}^3\sigma \sqrt{\det\left(g_{\mu\nu}\frac{\partial\chi^\mu}{\partial\sigma^a}\frac{\partial\chi^\nu}{\partial\sigma^b}\right)}$$

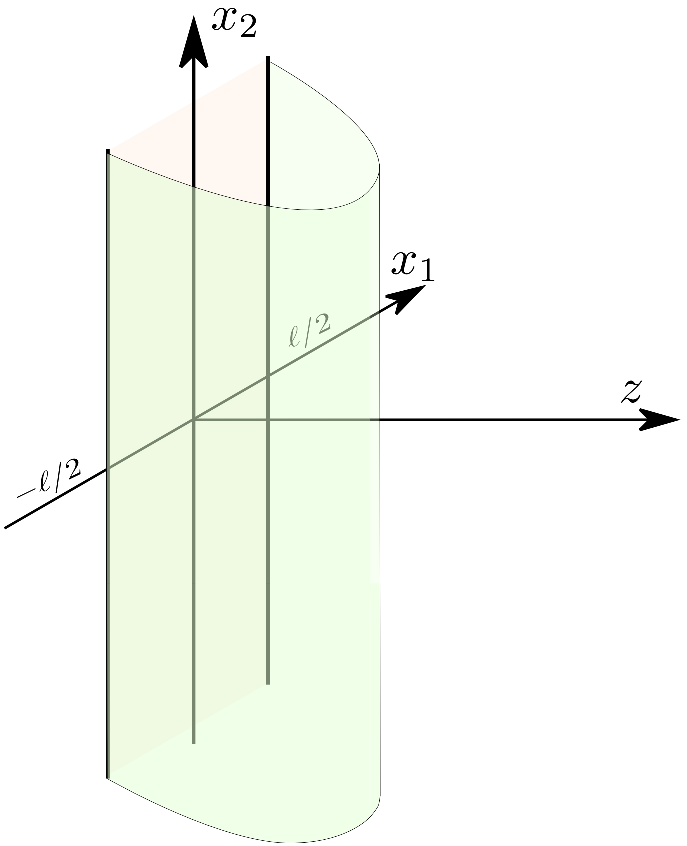
We compute for a strip in the field theory,

Perpendicular/Transverse: $\chi^\mu = (v(\sigma), x_1(\sigma), x_2, x_3, z(\sigma))$

$$\mathrm{S}_\perp = \frac{A_\perp}{V_\perp} = \int \mathrm{d}\sigma \sqrt{-\dot{v}^2 A e^{-B} S^4 - \frac{2\dot{v}\dot{z}e^{-B}S^4}{z^2} + \dot{x_1}^2 S^6}$$

Parallel/Longitudinal: $\chi^\mu = (v(\sigma), x_1, x_2, x_3(\sigma), z(\sigma))$

$$S_\parallel = \frac{A_\parallel}{V_\parallel} = \int \mathrm{d}\sigma \sqrt{-Ae^{2B}S^4\dot{v}^2 + 2e^{2B}FS^4\dot{v}\dot{x_3} - \frac{2e^{2B}S^4\dot{v}\dot{z}}{z^2} + S^6\dot{x_3}^2}.$$



[Ryu, Takayanagi, Phys. Rev. Lett. (2006)]

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Appendix

Minimal Surfaces-Time dependence

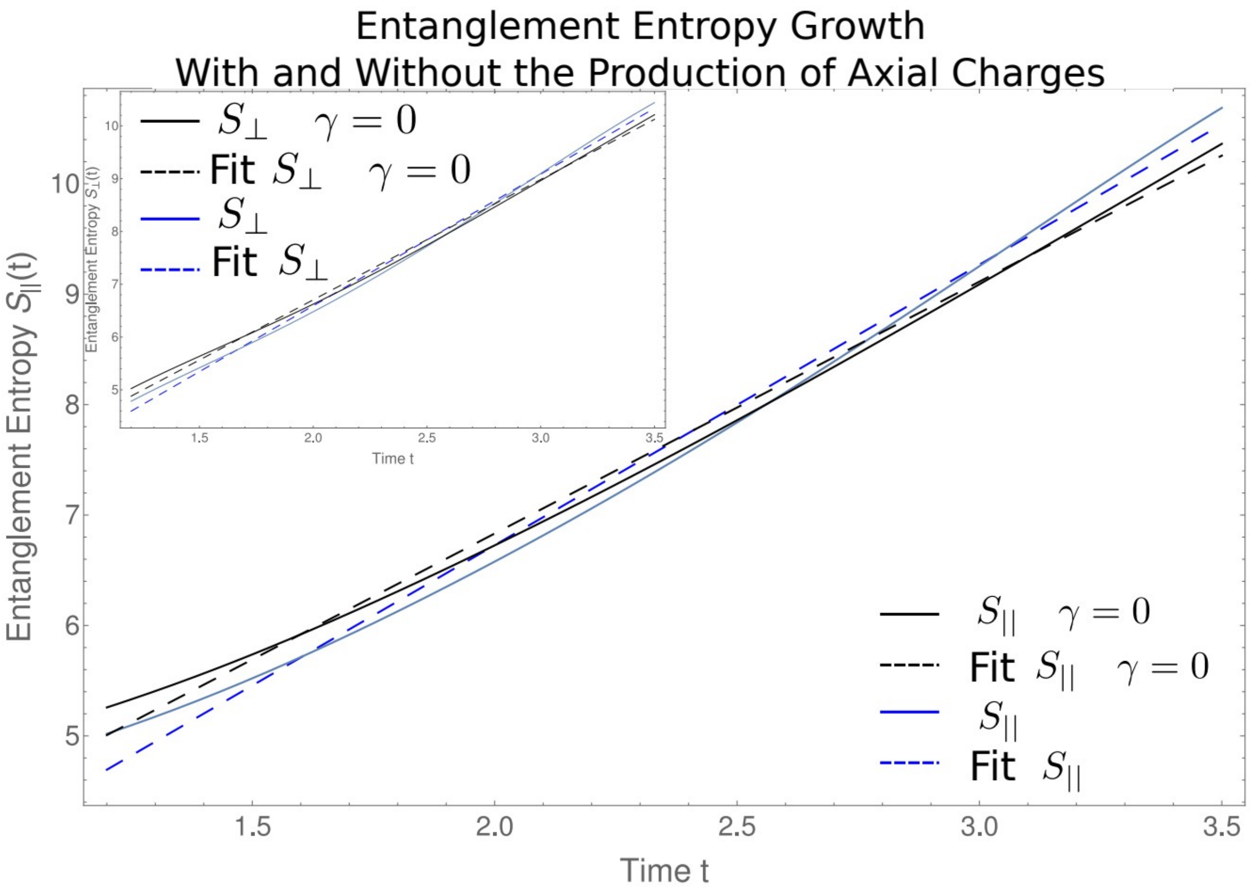
-Oscillations around Linear growth

-Fit form $S = at + b$

	$a_\perp$	$b_\perp$	$a_{\|\|}$	$b_{\|\|}$
$\gamma = 0$	2.28	2.15	2.28	2.27
$\gamma = 1/2$	2.49	1.60	2.54	1.64

$$\frac{\mathrm{d}S_\perp}{\mathrm{d}t} \approx \frac{\mathrm{d}S_{||}}{\mathrm{d}t}$$

-Entanglement velocity $v_E \propto a$



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Appendix

Minimal Surfaces-Time dependence

Transverse direction:

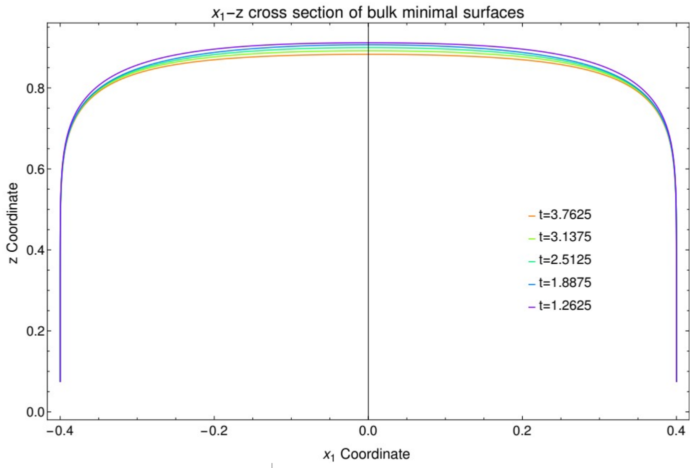
Numerical method: Relaxation

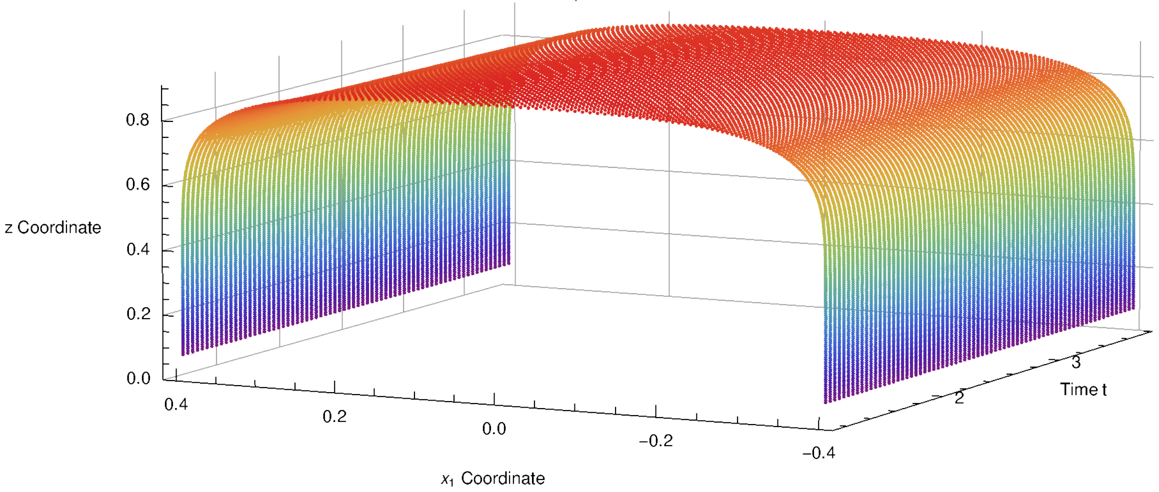
Embedding Coordinates: $\chi^{\mu} = (v(\sigma), x_1(\sigma), x_2, x_3, z(\sigma))$

Strip width: $\ell = 0.8$

Ultra-violet cutoff: $z_{UV} = 0.075$

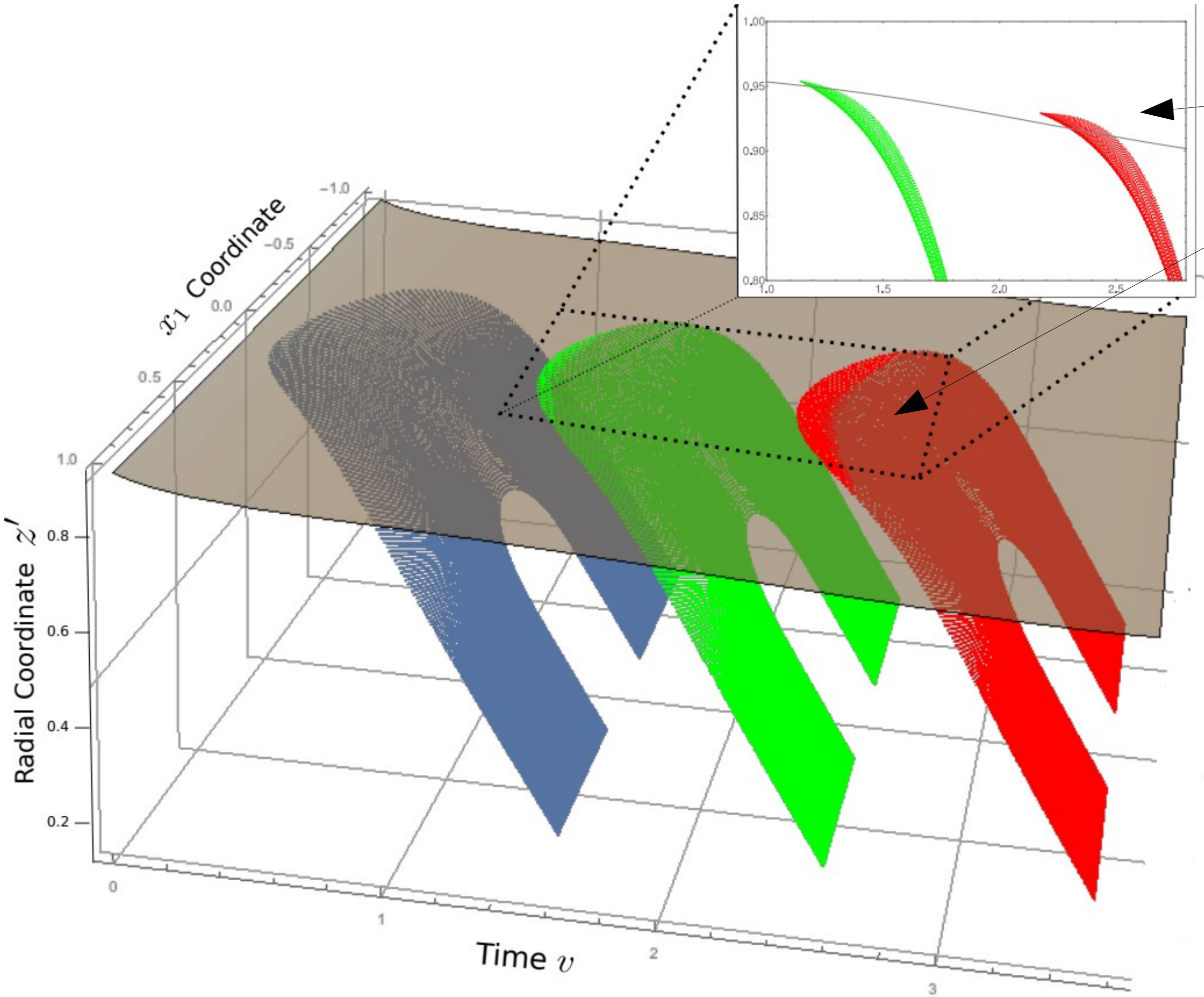
Grid size: 350 points



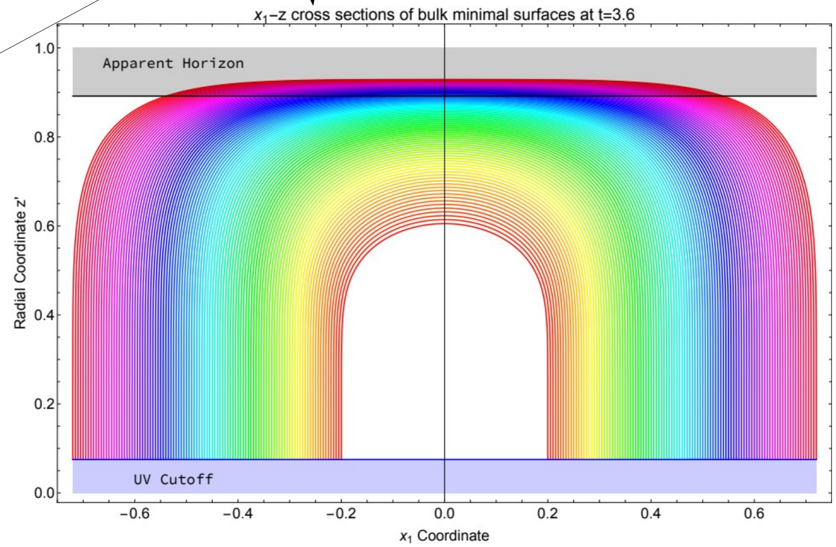


Appendix

Minimal Surfaces – Length dependence



Horizon crossings



Apparent horizon does not act like a wall for minimal surfaces, [Abajo-Arrastia,Aparicio,Lopez, JHEP (2010)]