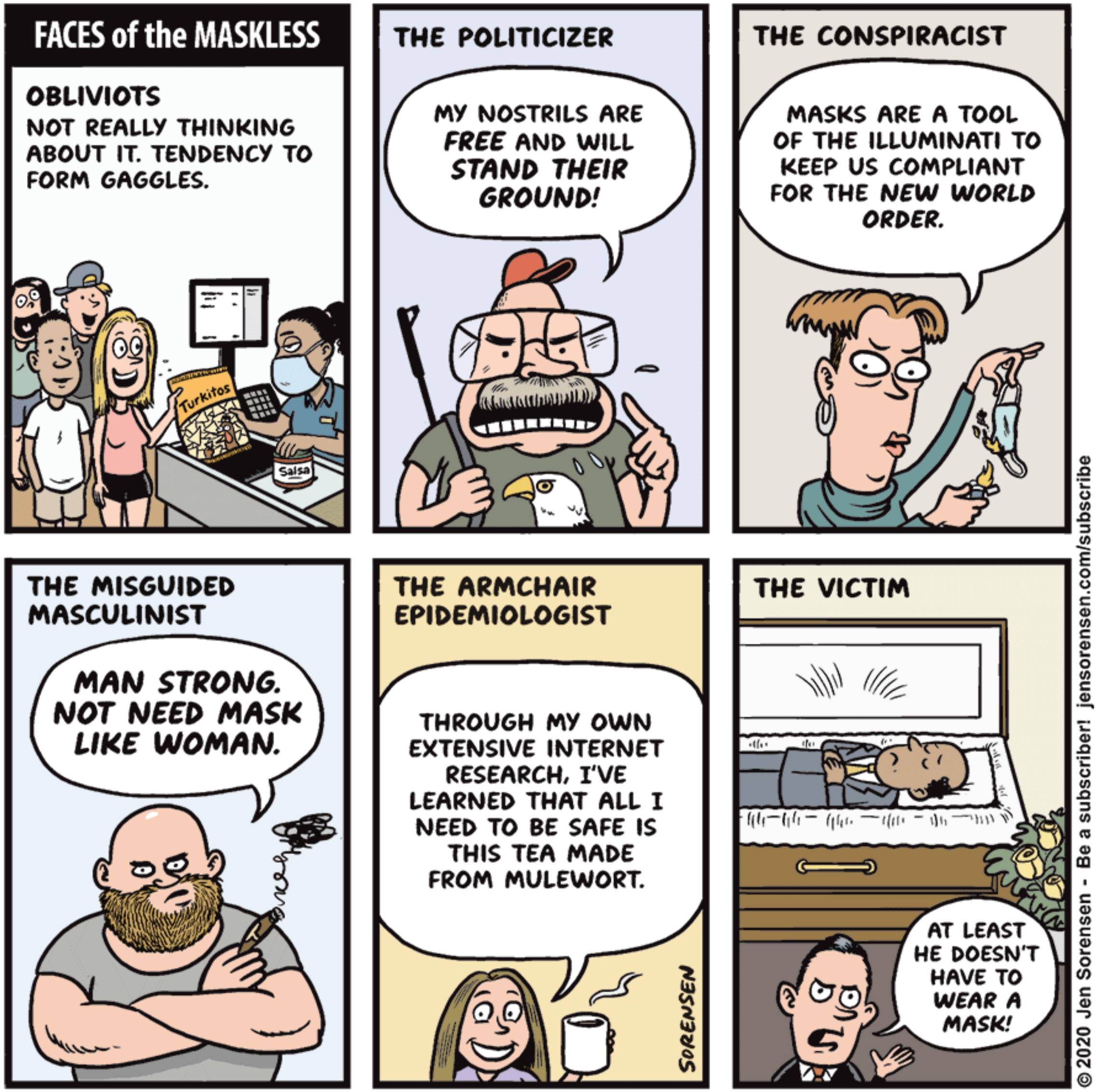
Unmasking *PT* symmetry



Carl Bender
Washington University

Holotube
13 October 2020

Thank you for your invitation to speak!!

(Although I wish I were speaking *live* – I am not happy giving a talk sitting at my desk rather than standing at a blackboard.)

These are truly depressing times…

More than any time in history mankind stands at a crossroads. One path leads to despair and utter hopelessness, the other to total extinction. Let us pray we have the wisdom to choose correctly.

-- *Woody Allen*

THEME OF THIS TALK:

Mathematicians find it enlightening to extend the ***real*** number system to the ***complex*** number system. This helps us to understand the real number system better.

In this talk we will extend conventional real physics to complex physics. By doing so we can:

(1) Understand quantization better
(2) Explain the divergence of perturbation theory
(3) Generate new *PT*-symmetric quantum theories
(4) Tame instabilities
(5) (and *much much* more…perhaps some other time!)

Outline: This talk is organized in three parts…

Outline

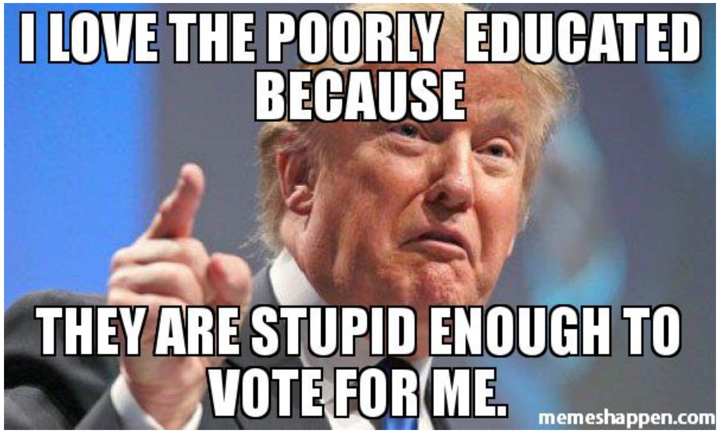
(1) Beginning

(2) Middle

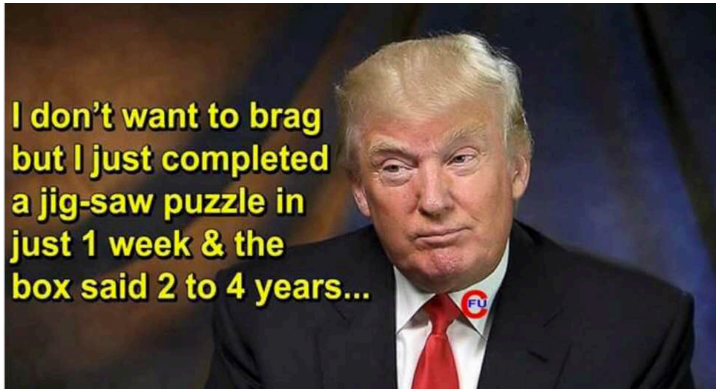
(3) End

Conventional world is described by *real* numbers:

-Election results



-IQ test results



-Money

Real measure of money …

Real quantity of money:

Complex mathematics is powerful!

(1) Explains the convergence of (real) Taylor series

(2) Determines asymptotic behavior (of real integrals)

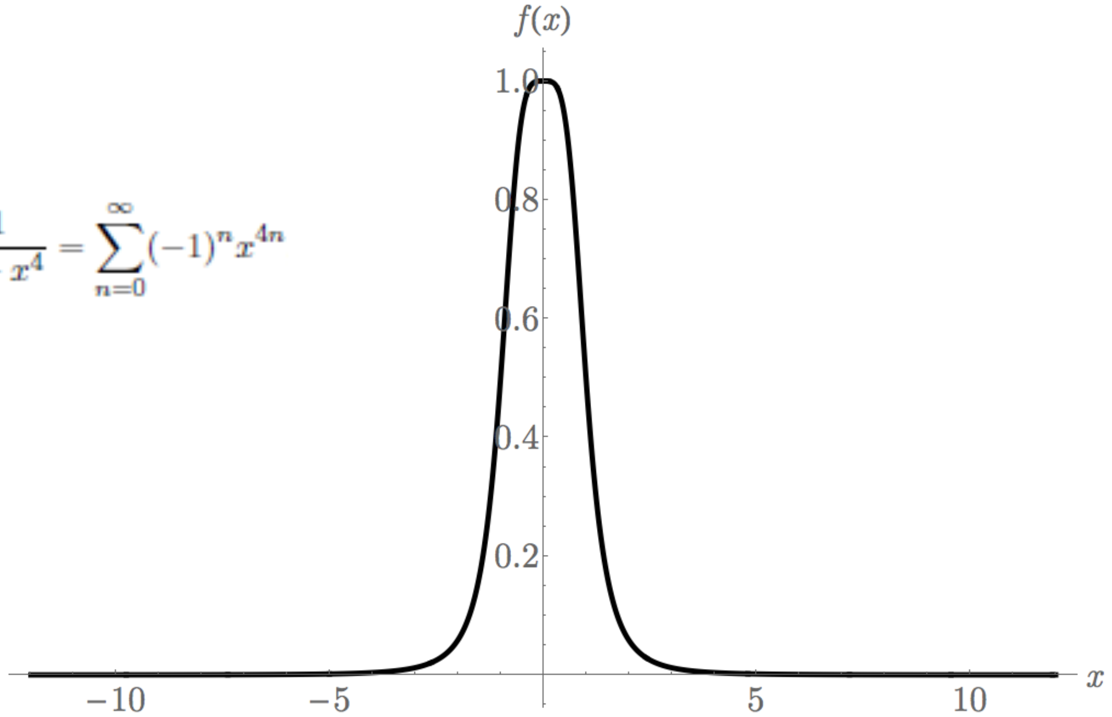
(3) Enables us to sum divergent series

(4) Explains real functions, such as square root

(5) And much much much more (some other time)

(1) Why do Taylor series stop converging?

$$f(x) = \frac{1}{1+x^4} = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$



Answer: Singularities in the *complex plane*

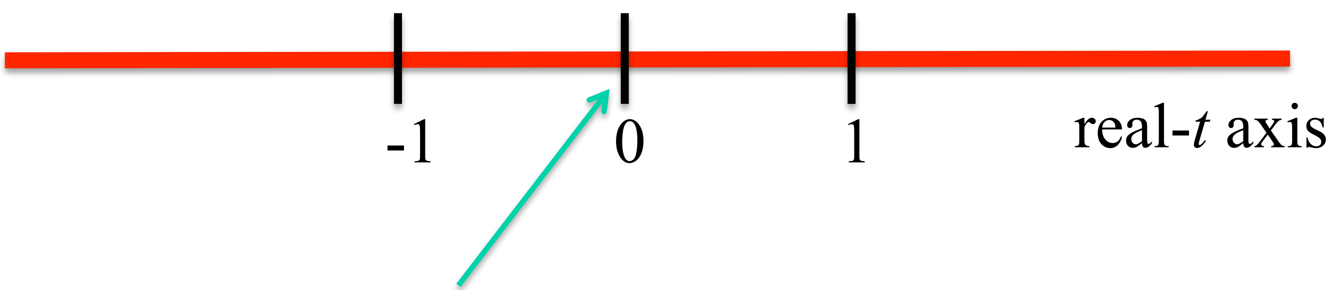
Complex plane …

Complex plane

(2) Asymptotic expansion of integrals

$$I(x) = \int_{-1}^{1} dt\, e^{-4xt^2} \cos\left(5xt - xt^3\right)$$

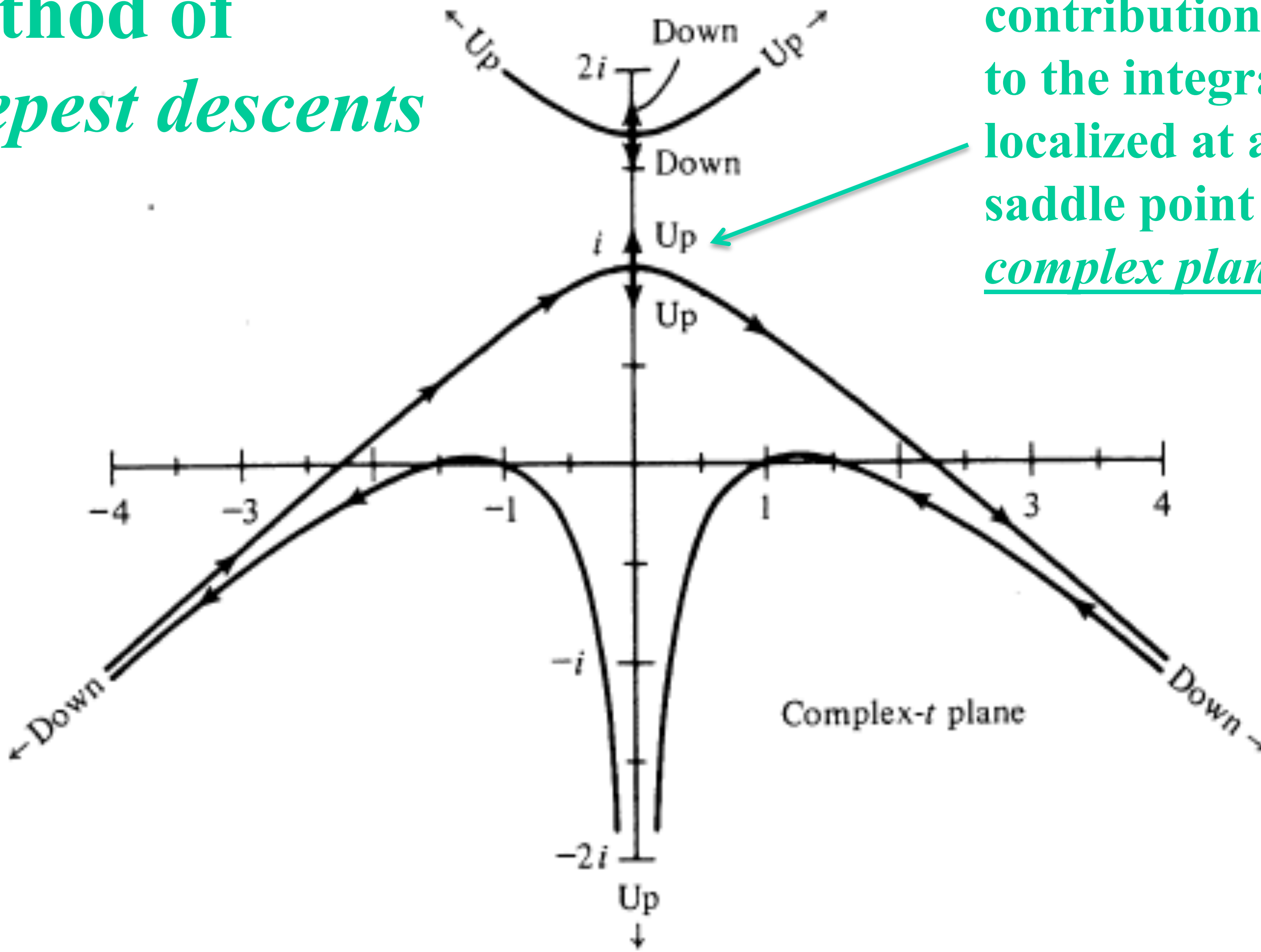
$$I(x) \sim e^{-2x}\sqrt{\pi/x} \qquad (x \to +\infty)$$



For large *x* the contribution to
I(*x*) is ***NOT*** localized at $t = 0$ (!!!)

Method of *steepest descents*

For large $|x|$ the contribution to the integral is localized at a saddle point in the *complex plane*



(3) Summing divergent series

$$1-1+1-1+1-1+1-1+\ldots = ??$$

$$1+2+4+8+16+32+\ldots = ??$$

$$1+1+1+1+1+1+1+\ldots = ??$$

$$1+2+3+4+5+6+7+\ldots = ??$$

Real-variable techniques:

$$1-1+1-1+1-1+1-1+\ldots = 1/2$$

$$1+2+4+8+16+32+\ldots = -1$$

Complex-variable techniques:

$$1+1+1+1+1+1+1+\ldots = -1/2$$

$$1+2+3+4+5+6+7+\ldots = -1/12$$

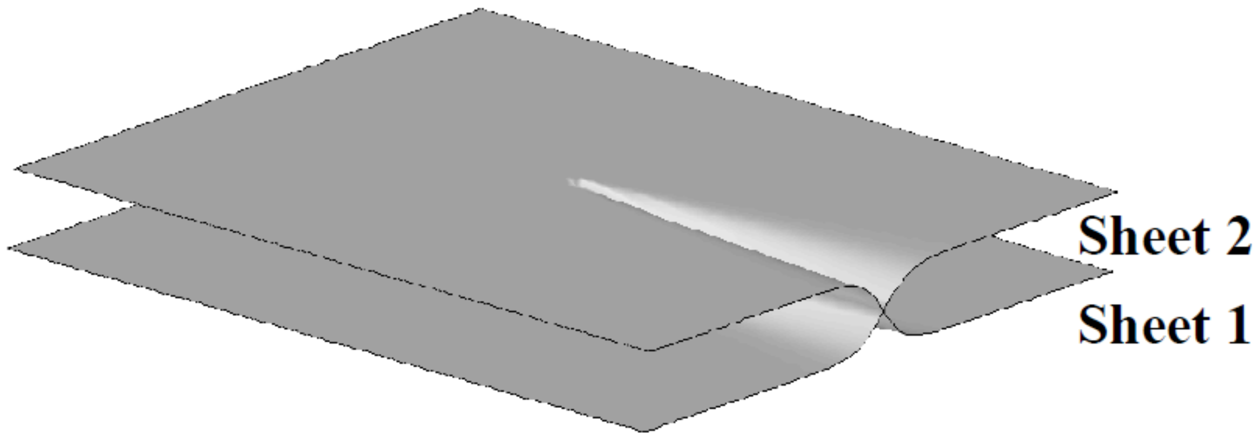
(4) Understanding real functions

Square-root function is *confusing*!

Q: Why are there *two* answers??

A: Square-root function is defined on a *Riemann surface* …

Square-root function is defined on a two-sheeted Riemann surface:



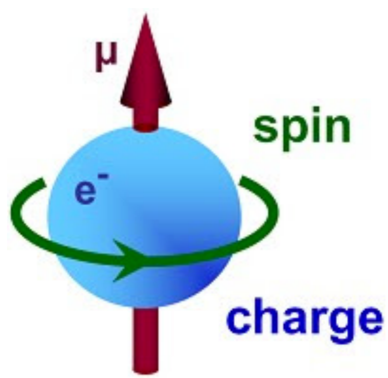
The surface is *two* complex planes cut and glued together.

**Like a Möbius strip, if you go around *twice*,
you return back to the starting point…**

Things that remain the same when you go around *twice*…

Möbius strip…

Electron…



Complex variables are already used in modern physics

Heisenberg algebra: $xp - px = i\hbar$

Schrödinger equation:

Eugene Wigner

Time reversal corresponds to complex conjugation --- changes the sign of *i*

In school you learn:

In quantum mechanics a particle in a potential well has *quantized* energy levels

Going from one level to another is a *discrete* "quantum leap"

Complex analysis provides a deeper understanding of quantization…

Imagine a two-state system having energies *a* and *b*…

$$H = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

***Couple* the states:**

$$H = \begin{pmatrix} a & g \\ g & b \end{pmatrix}$$

Energies for this two-state system

$$\det \begin{pmatrix} a-E & g \\ g & b-E \end{pmatrix} = 0$$

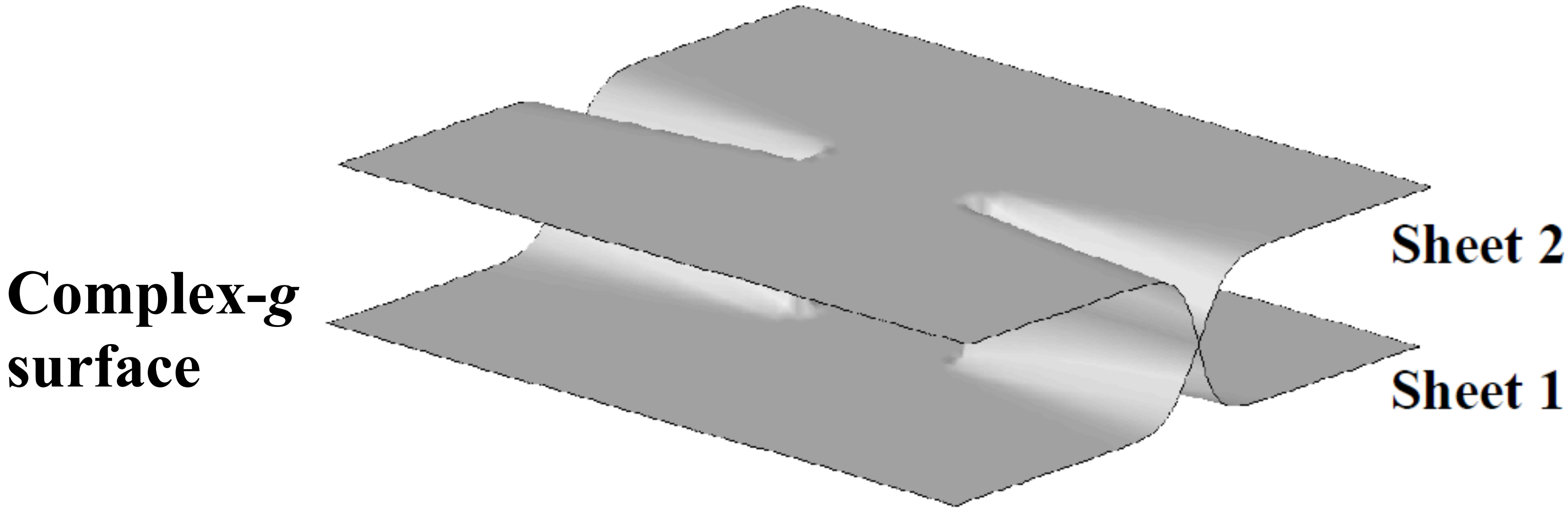
$$E^2 - (a+b)E + ab - g^2 = 0$$

$$E(g) = \frac{a+b}{2} \pm \frac{1}{2}\sqrt{(a-b)^2 + 4g^2}$$

Square-root singularities in the complex-*g* plane at $g = \pm \frac{|a-b|}{2} i$

(called *Bender-Wu singularities*)

$E(g)$ is a *smooth* function defined on a two-sheeted Riemann surface:



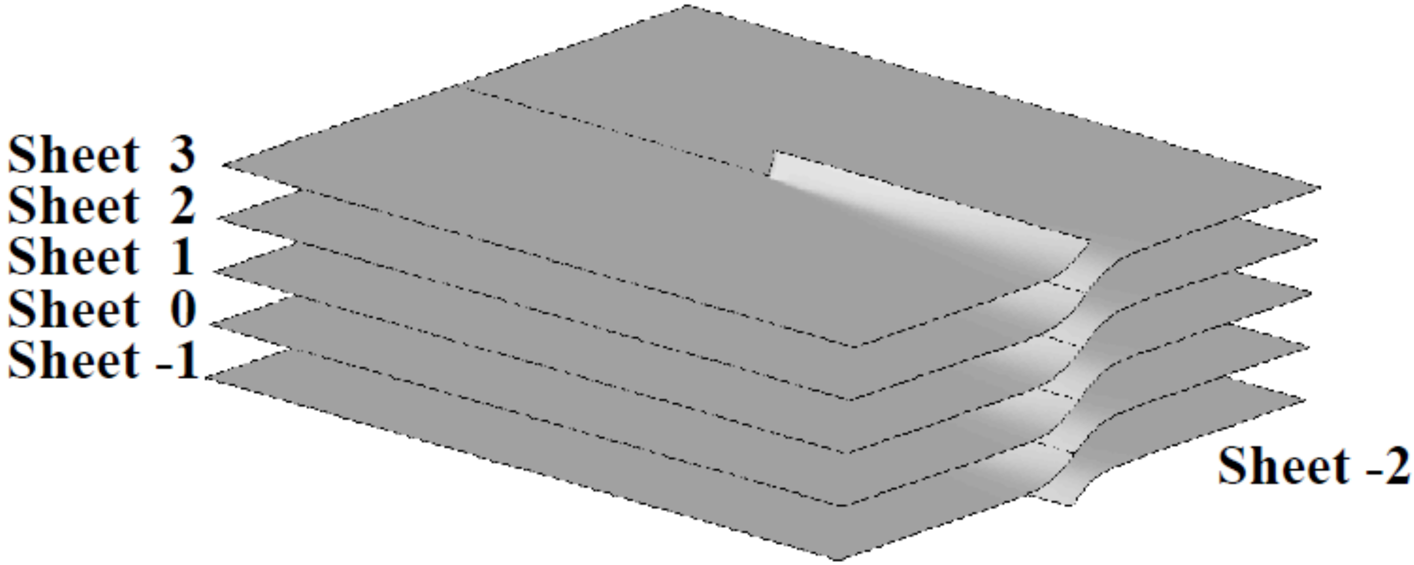
On this complex-g surface the quantum levels are ***not*** discrete!

Quantization is ***topological*** – the quantized energy levels correspond to the discrete sheets in the Riemann surface.

These singularities explain the divergence of perturbation series.

(And complex-variable techniques can be used to ***sum*** the series!)

Imagine a parking garage…



Unlike what is taught in conventional quantum theory courses, all energy levels *smoothly deform* into one another under analytic continuation!

Laboratory analytic continuation of eigenvalues

(1) PRL **108**, 024101 (2012) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2012

$\mathcal{PT}$ Symmetry and Spontaneous Symmetry Breaking in a Microwave Billiard

S. Bittner,[1] B. Dietz,[1,*] U. Günther,[2] H. L. Harney,[3] M. Miski-Oglu,[1] A. Richter,[1,4,†] and F. Schäfer[1,5]

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(Received 21 July 2011; published 10 January 2012)

We demonstrate the presence of parity-time ($\mathcal{PT}$) symmetry for the non-Hermitian two-state Hamiltonian of a dissipative microwave billiard in the vicinity of an exceptional point (EP). The shape of the billiard depends on two parameters. The Hamiltonian is determined from the measured resonance spectrum on a fine grid in the parameter plane. After applying a purely imaginary diagonal shift to the Hamiltonian, its eigenvalues are either real or complex conjugate on a curve, which passes through the EP. An appropriate basis choice reveals its $\mathcal{PT}$ symmetry. Spontaneous symmetry breaking occurs at the EP.

DOI: 10.1103/PhysRevLett.108.024101 PACS numbers: 05.45.Mt, 02.10.Yn, 11.30.Er

(2) H. Xu, D. Mason, L. Jiang, and J. G. E. Harris, *Nature* **537**, 80 (2016)

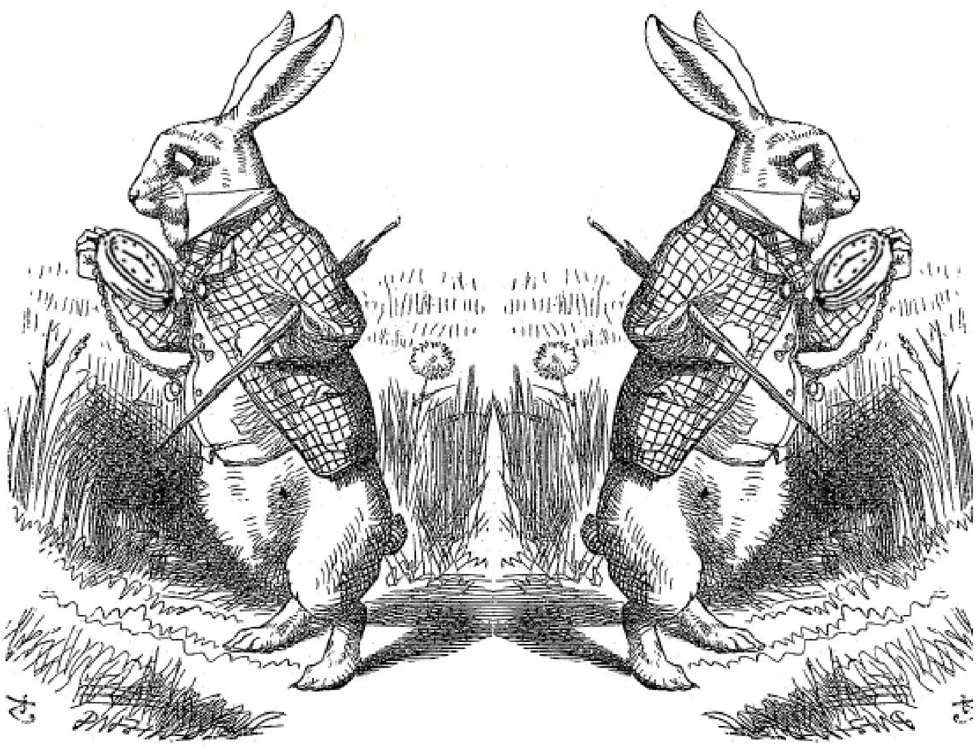
(3) J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, and S. Rotter, *Nature* **537**, 76 (2016)

Note the term ***PT symmetry*** …

PT-symmetric quantum mechanics:

Extending quantum mechanics into the complex domain.

If you respect *PT* symmetry, the eigenvalues can remain real and unitarity can be preserved even though the Hamiltonian is not Hermitian!



PT **reflection – a simultaneous reflection of space and time**

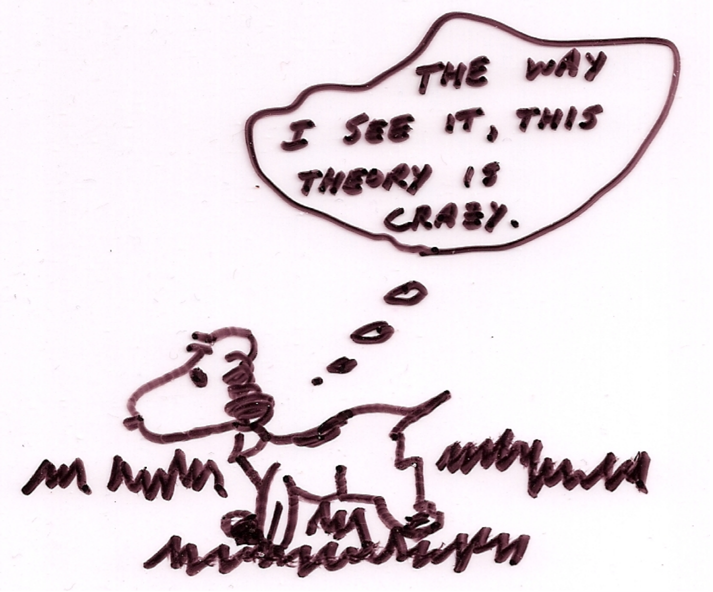
Complex origin of *PT* symmetry: Homogeneous <u>real</u> Lorentz group -- A continuous group consisting of *four* disconnected parts:

Proper orthochronous Lorentz group	Elements of POLG multiplied by ***T***
Elements of POLG multiplied by ***P***	Elements of POLG multiplied by ***PT***

The *complex* Lorentz group consists of <u>*TWO*</u> disconnected parts

Early example of a *PT*-symmetric Hamiltonian. This Hamiltonian is not Hermitian, but ---
It has REAL EIGENVALUES!
(1-D model of Lee-Yang edge singularity)

$$\boxed{H = p^2 + ix^3}$$

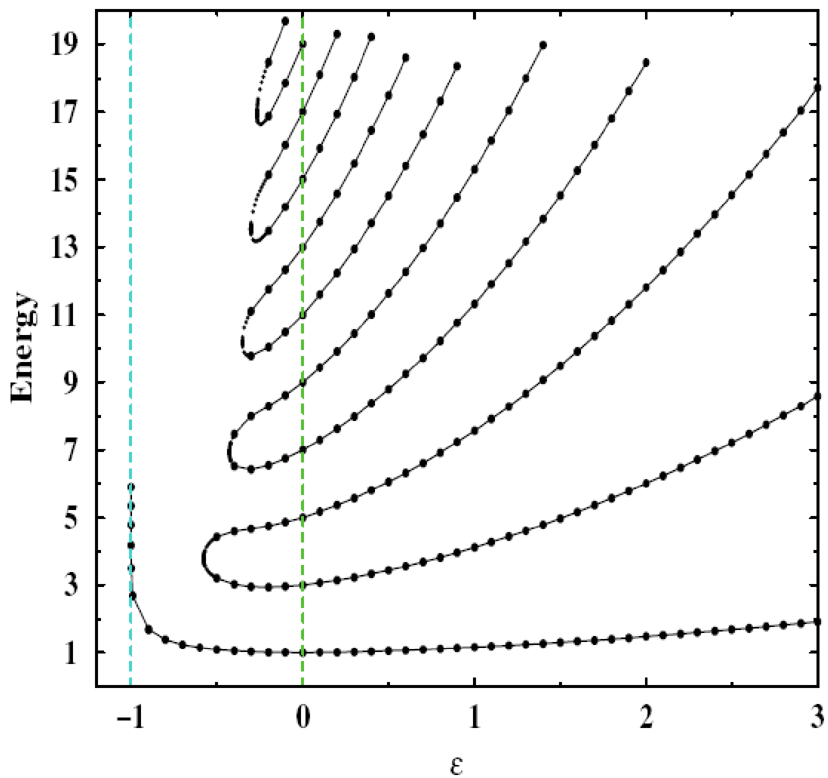


P: $x \to -x,\ p \to -p$

T: $x \to x,\ p \to -p,\ \ i \to -i$

One-parameter family of *PT*-symmetric Hamiltonians obtained by complex deformation of the harmonic oscillator

$$H = p^2 + x^2(ix)^\varepsilon \qquad (\varepsilon \text{ real})$$



Look! *H* is not Hermitian but its eigenvalues are **all real**!

Special cases: *Cubic*: $H = p^2 + ix^3$

Quartic: $H = p^2 - x^4$

Sextic: $H = p^2 + x^6$

Proof of spectral reality:

P. Dorey, C. Dunning, and R. Tateo
J. Phys. A **34**, 5679 (2001)

P. Dorey, C. Dunning, and R. Tateo
J. Phys. A **40**, R205 (2007)

PT symmetry unmasked:

PT-symmetric Hamiltonians are *complex deformations* of Hermitian Hamiltonians

You begin with a Hermitian Hamiltonian and introduce a deformation parameter ε …

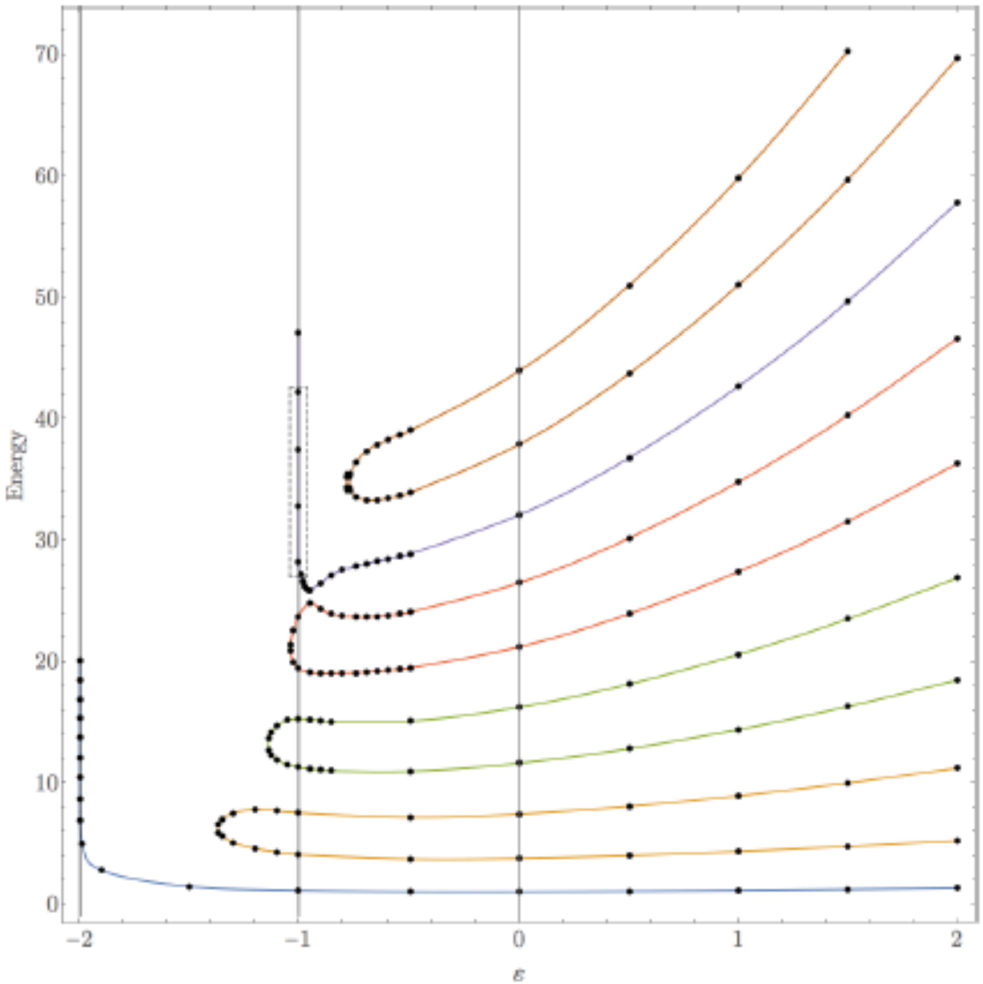
***Simple example*:** $H = p^2 + x^2 + i\varepsilon x$

$$-\phi''(x) + x^2\,\phi(x) + i\varepsilon x\,\phi(x) = E\,\phi(x)$$

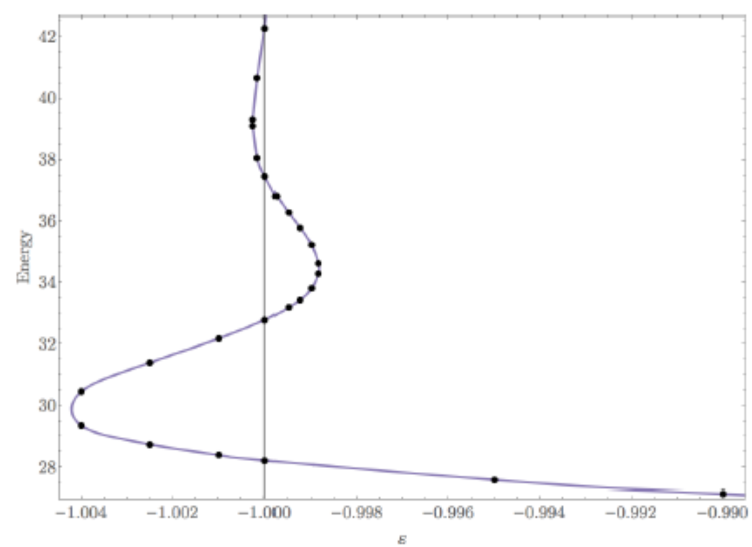
$$\phi(\pm\infty) = 0$$

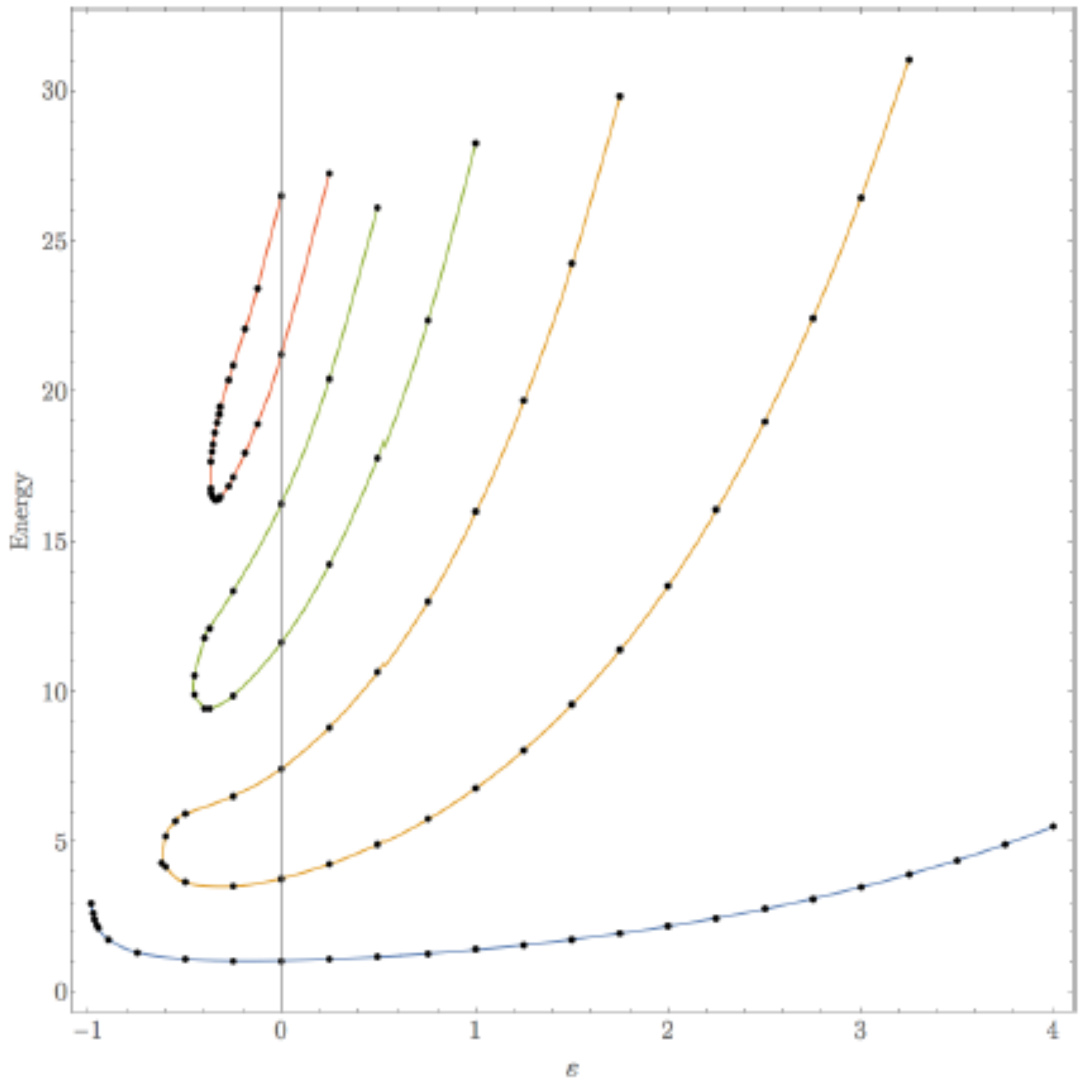
$$E_n = 2n + 1 + \varepsilon^2/4 \qquad (n = 0, 1, 2, 3, \ldots)$$

This picture of eigenvalues is generic…

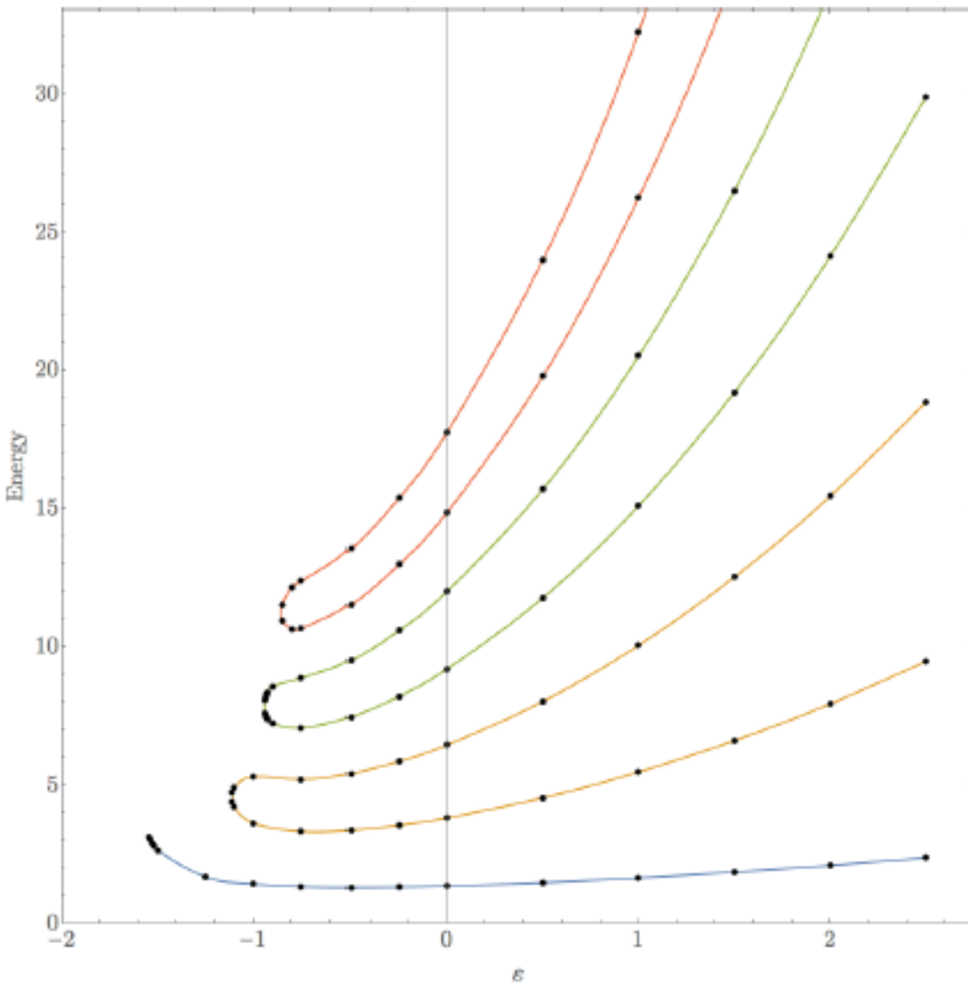


$$H = p^2 + x^4\,(ix)^{\varepsilon}$$





$$H = p^4 + x^2\,(ix)^\varepsilon$$



$$H = p^2 + x^2\,(ix)^{\varepsilon}\,\log(ix)$$

europhysicsnews

THE MAGAZINE OF THE EUROPEAN PHYSICAL SOCIETY

First direct detection of gravitational waves

PT symmetry in quantum physics

EPL for the IYL 2015

Delicious ice cream: why does salt thaw ice?

Fascinating optics in a glass of water

47/2
2016

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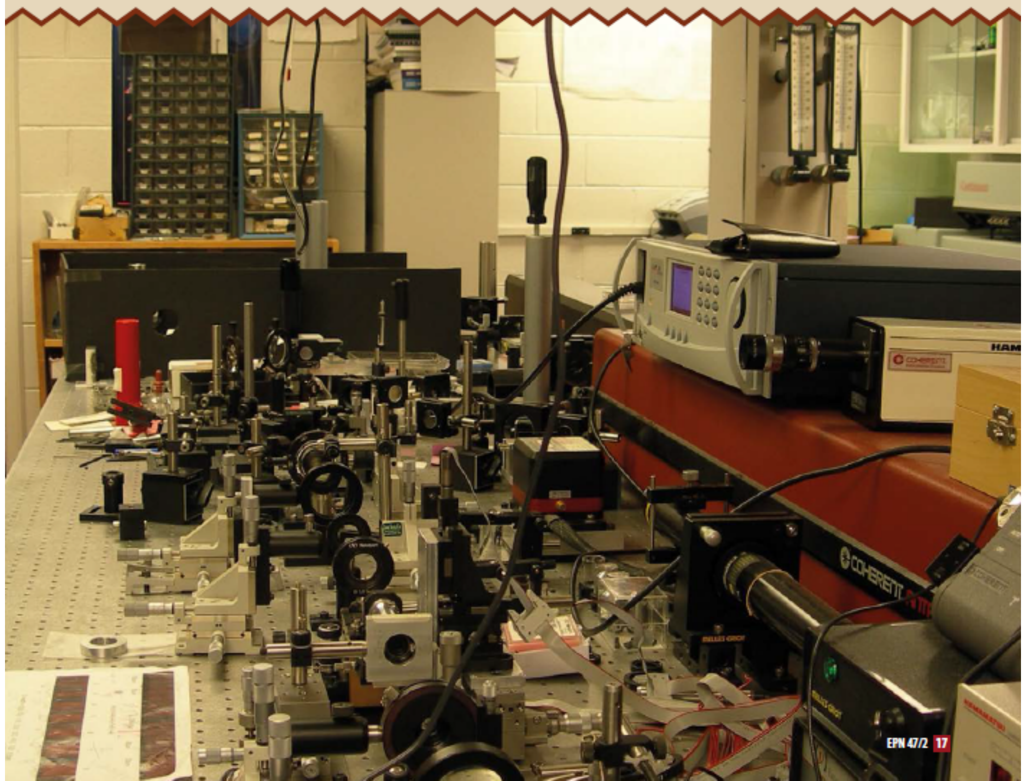
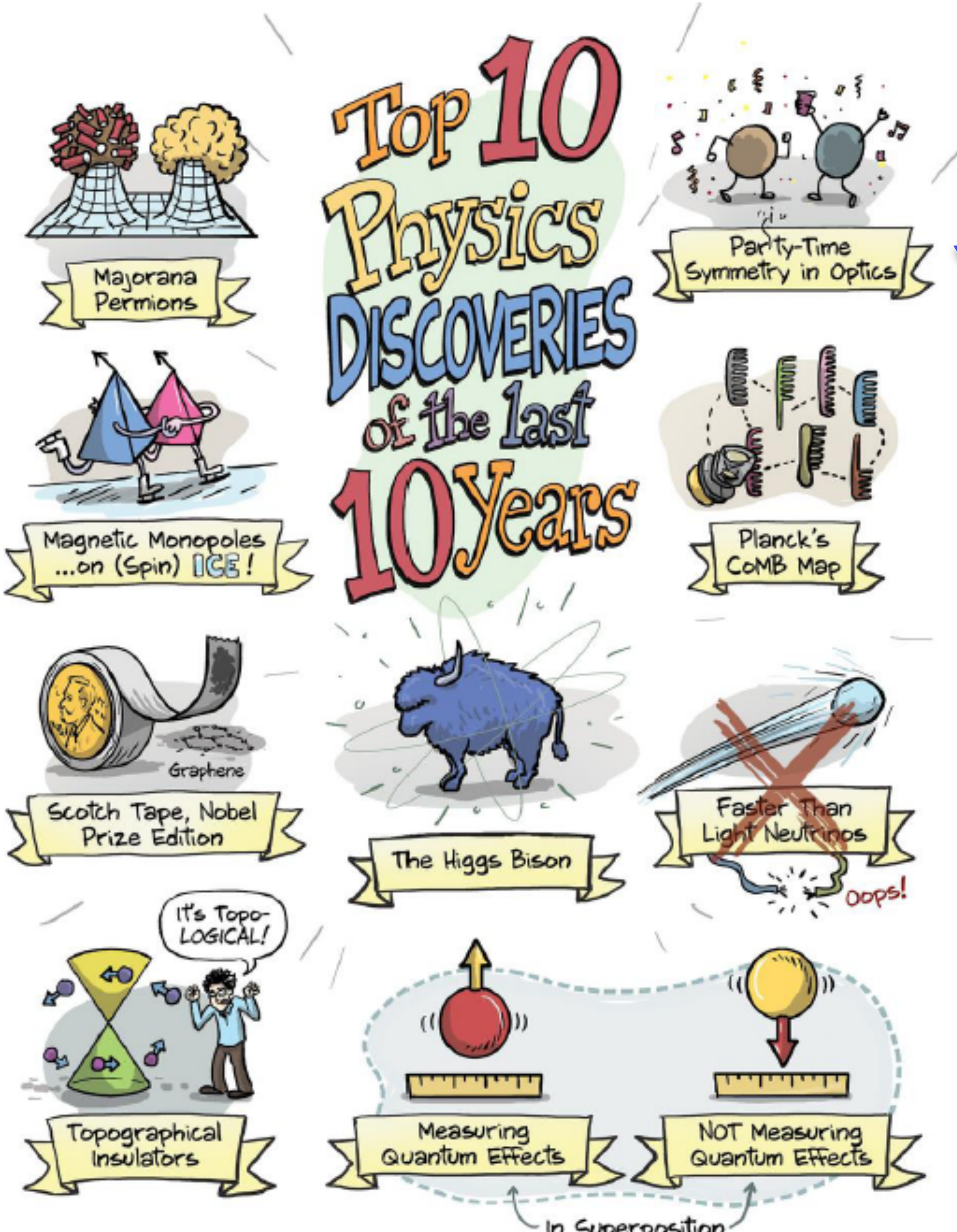
PT SYMMETRY IN QUANTUM PHYSICS: FROM A MATHEMATICAL CURIOSITY TO OPTICAL EXPERIMENTS

Carl M. Bender – Washington University in St. Louis, St. Louis, MO 63130, USA – DOI: http://dx.doi.org/10.1051/epn/2016201

***Space-time reflection symmetry*, or *PT symmetry*, first proposed in quantum mechanics by Bender and Boettcher in 1998 [1], has become an active research area in fundamental physics. More than two thousand papers have been published on the subject and papers have appeared in two dozen categories of the *arXiv*. Over two dozen international conferences and symposia specifically devoted to PT symmetry have been held and many PhD theses have been written.**

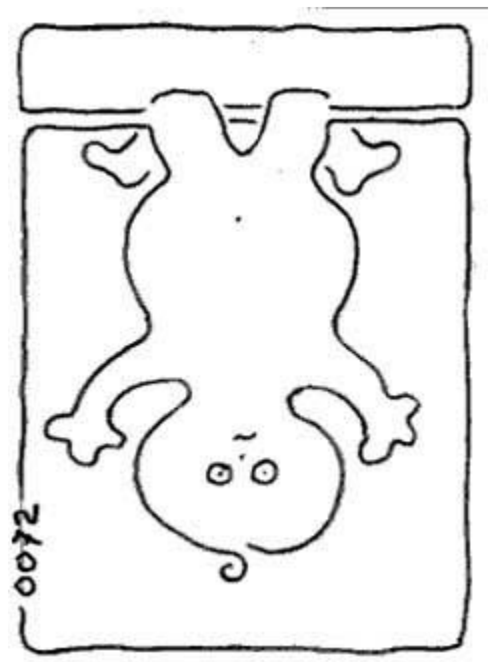




From the article:
[Top 10 physics discoveries of the last 10 years]
[Jorge Cham]
Nature Physics **11**, 799 (2015)
Published online 01 October 2015

Stability of upside-down potentials

$$V(x) = -x^4$$



This potential looks *unstable* (<u>*on the real axis*</u>)

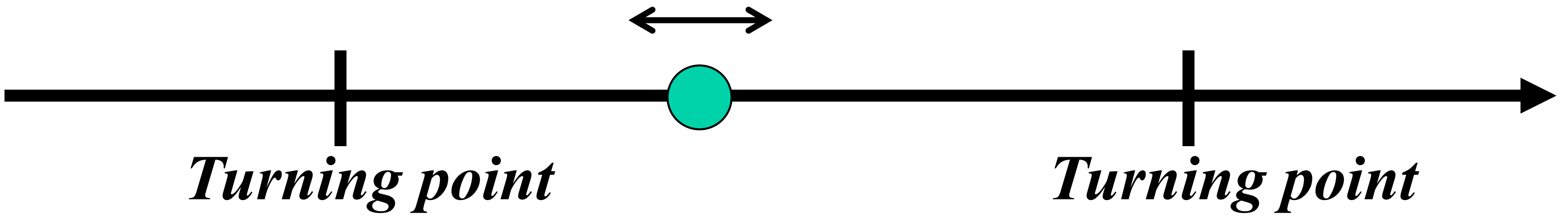
Complex variables explains why such a potential has quantum <u>bound states</u>

To explain, we first study simple classical harmonic motion in the *complex domain*.

Remember what they teach in physics 101…

Classical harmonic oscillator

Back and forth motion on the real-*x* axis:



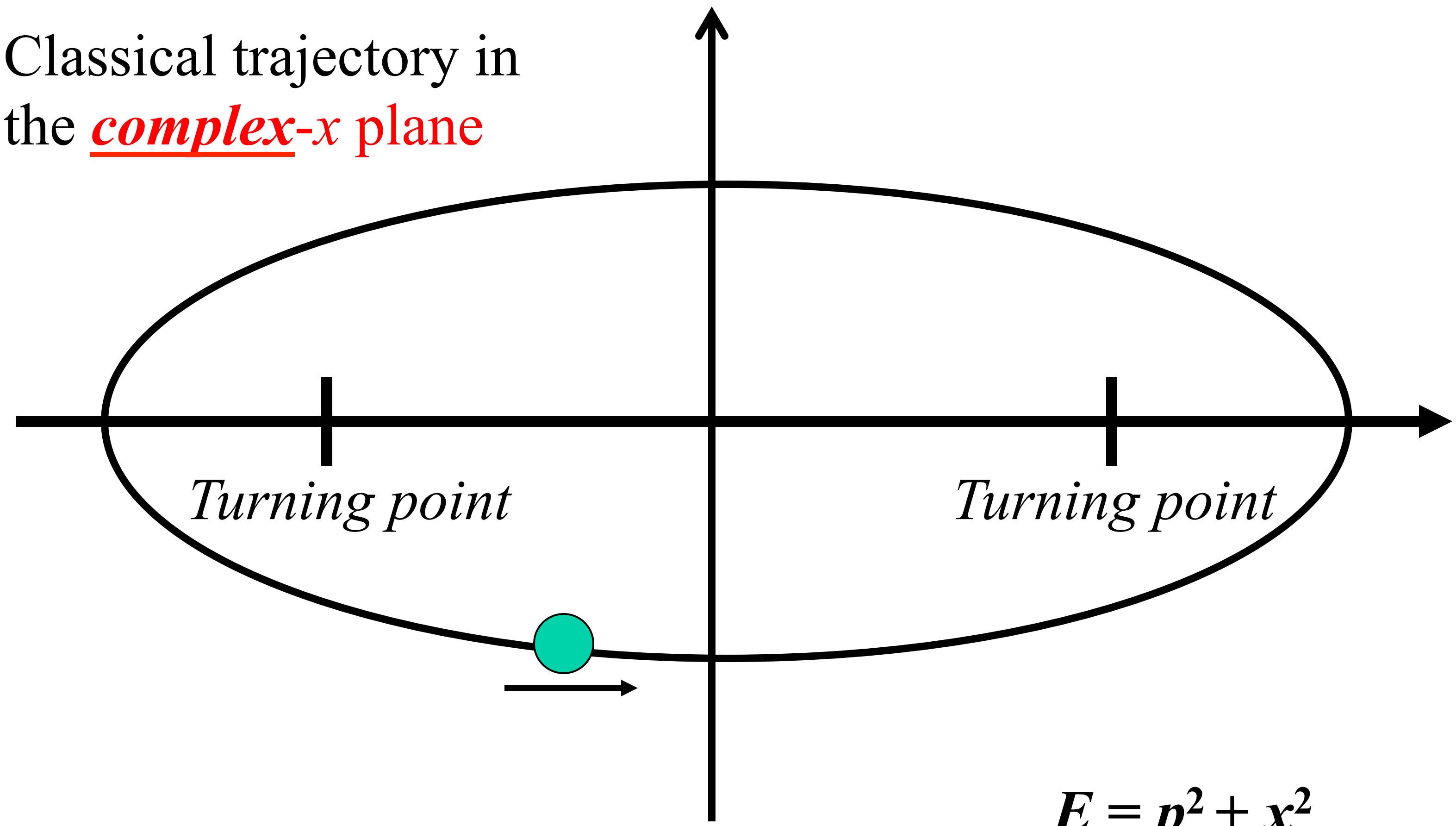
$$E = p^2 + x^2$$

Classically allowed and ***classically forbidden*** regions…

***Classically allowed* and *classically forbidden* regions**

Classical harmonic oscillator in the *complex* plane

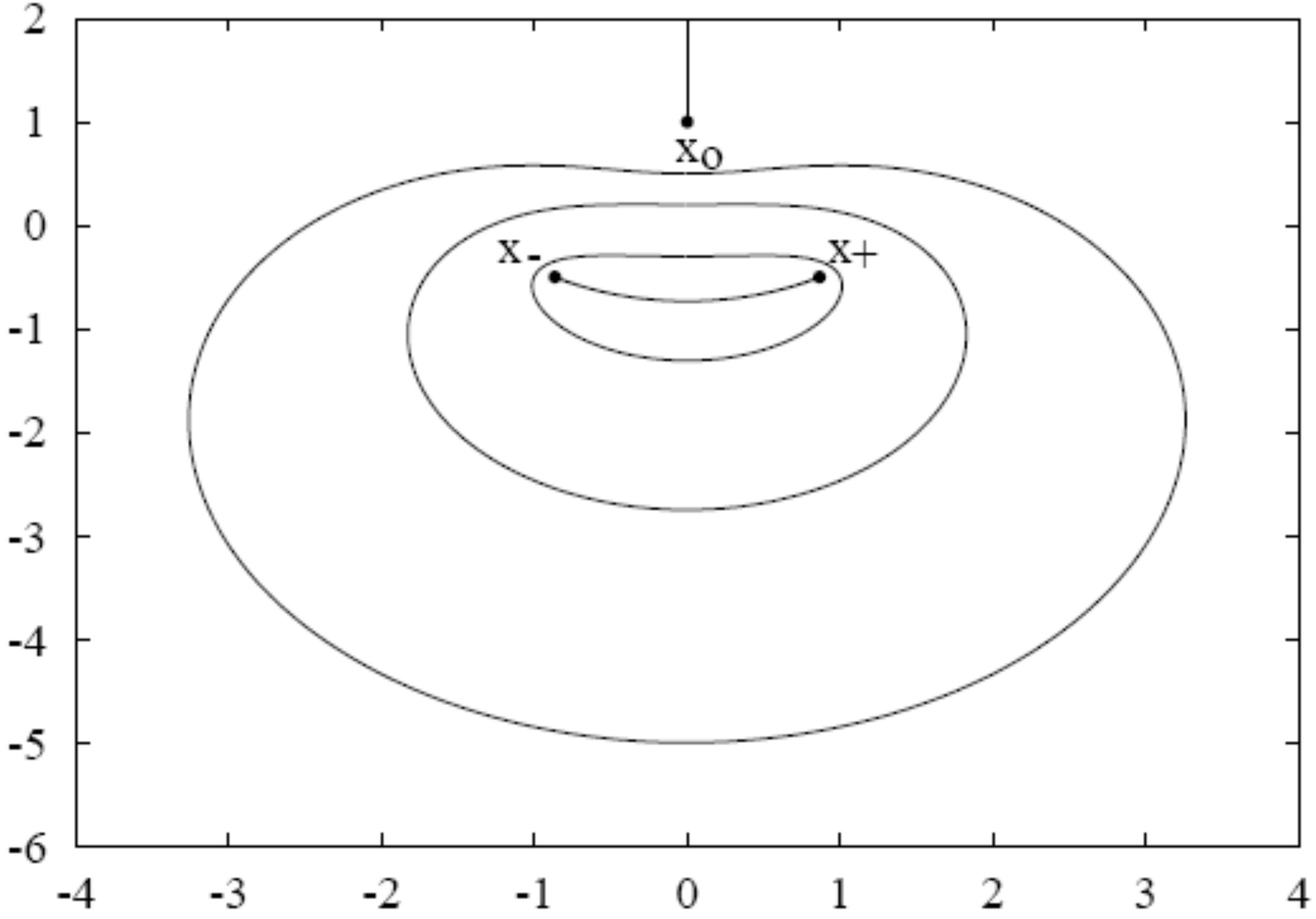
$$H = p^2 + x^2 \qquad (\varepsilon = 0)$$



(These orbits are not Keplerian!)

$H = p^2 + ix^3$ $(\varepsilon = 1)$

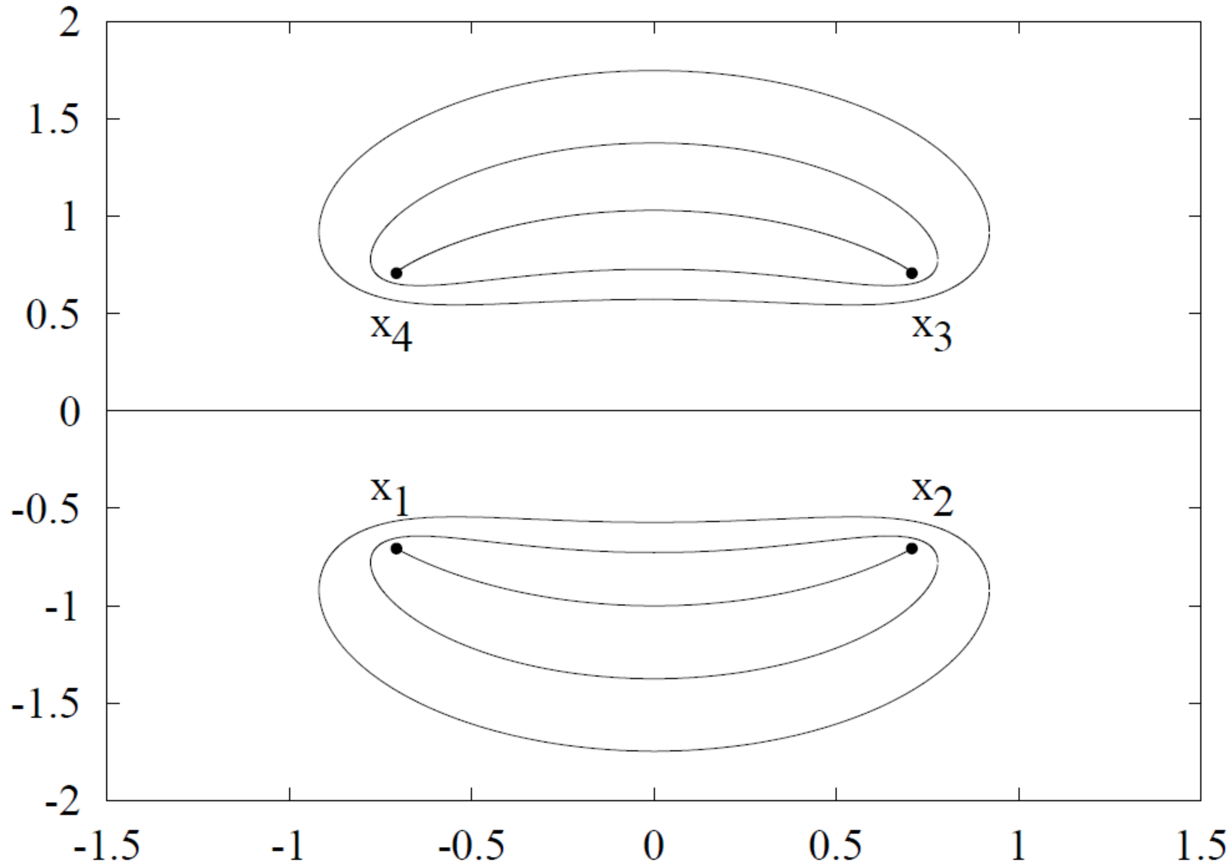
Classical trajectories in the ***complex-x* plane**



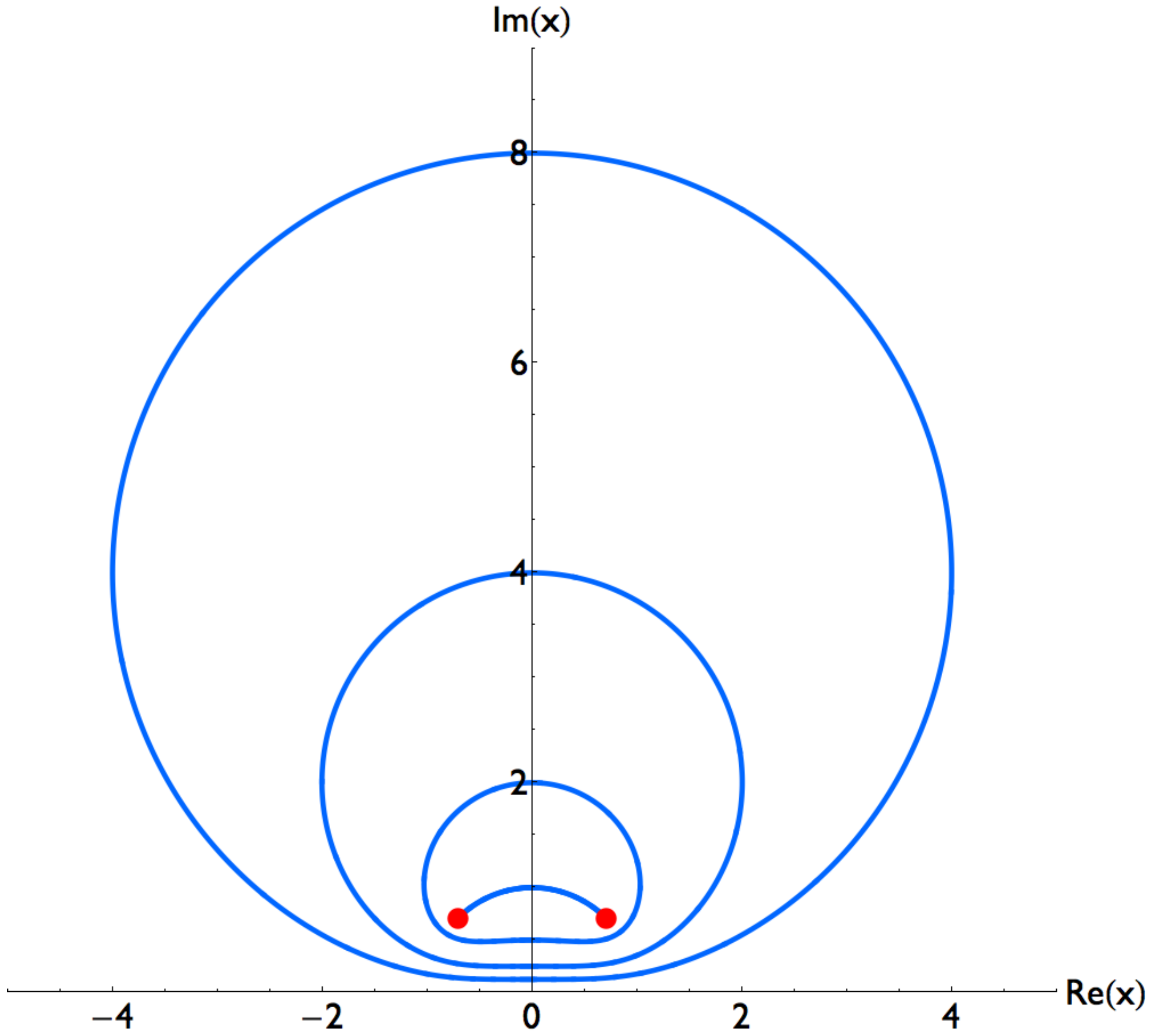
$H = p^2 - x^4 \quad (\varepsilon = 2)$

Q: On the real axis classical particles roll down to infinity in finite time *T*, so where is the particle at *T*+1??

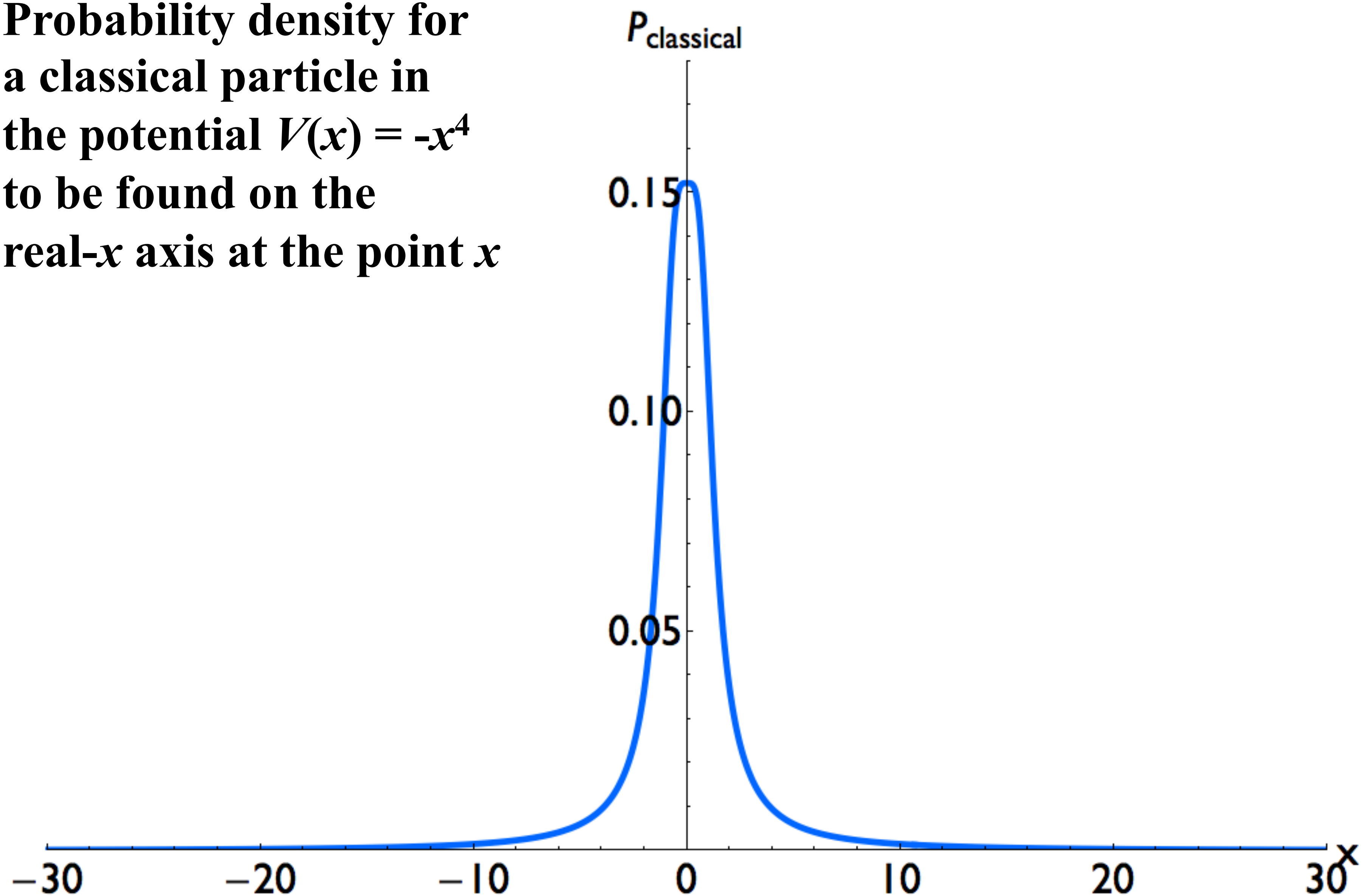
Classical trajectories in the *complex*-*x* plane

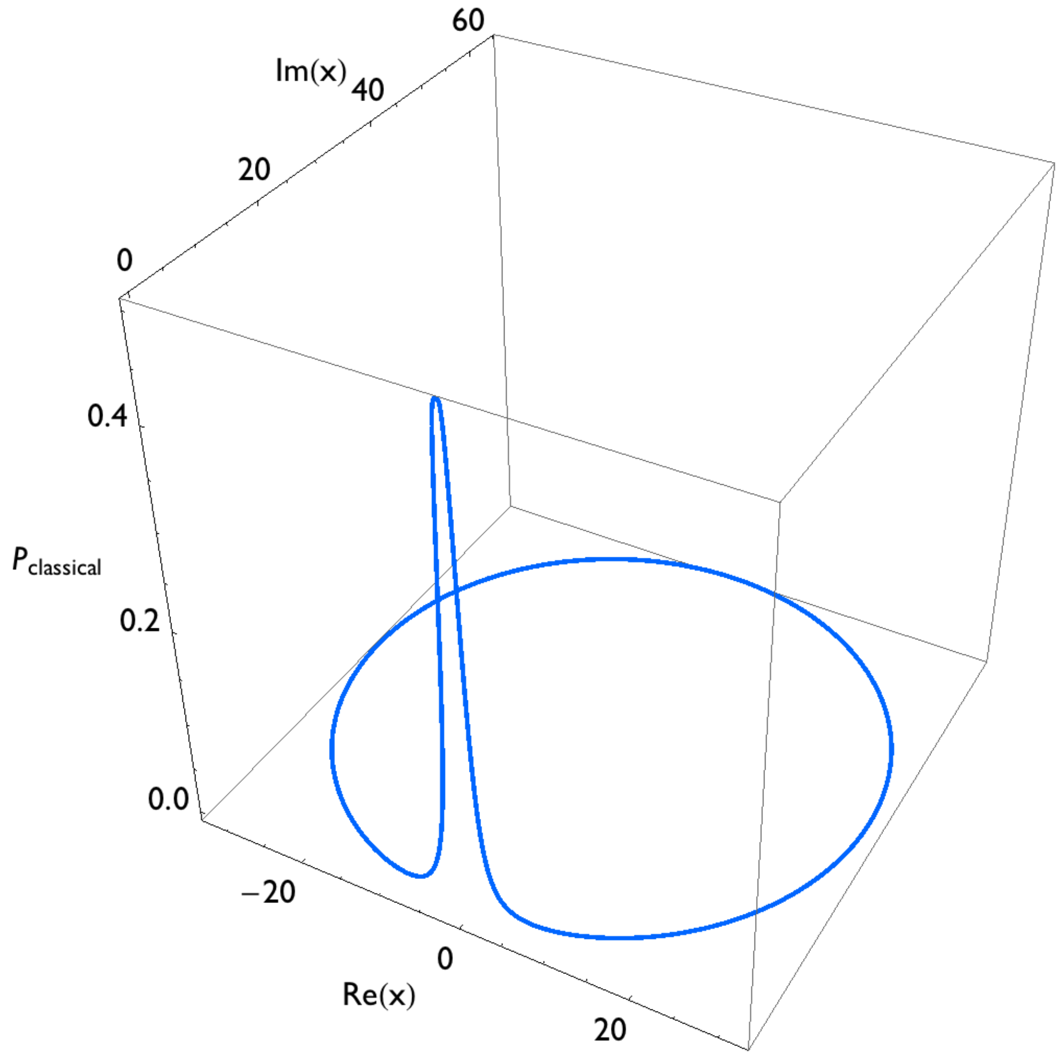


As the classical trajectories approach the real axis, the classical orbits go further out into *complex-x* plane



Probability density for a classical particle in the potential $V(x) = -x^4$ to be found on the real-x axis at the point x





The *static* instability becomes *dynamically stable* in the complex domain (like a bicycle or a top)

Bohr-Sommerfeld Quantization of a complex atom

$$\oint dx\, p = \left(n + \tfrac{1}{2}\right)\pi$$

Instability at $x = 0$ is tamed!

Complex analysis allows us to *tame instabilities*

Physical systems that seem to be unstable can become *stable* in the complex domain!

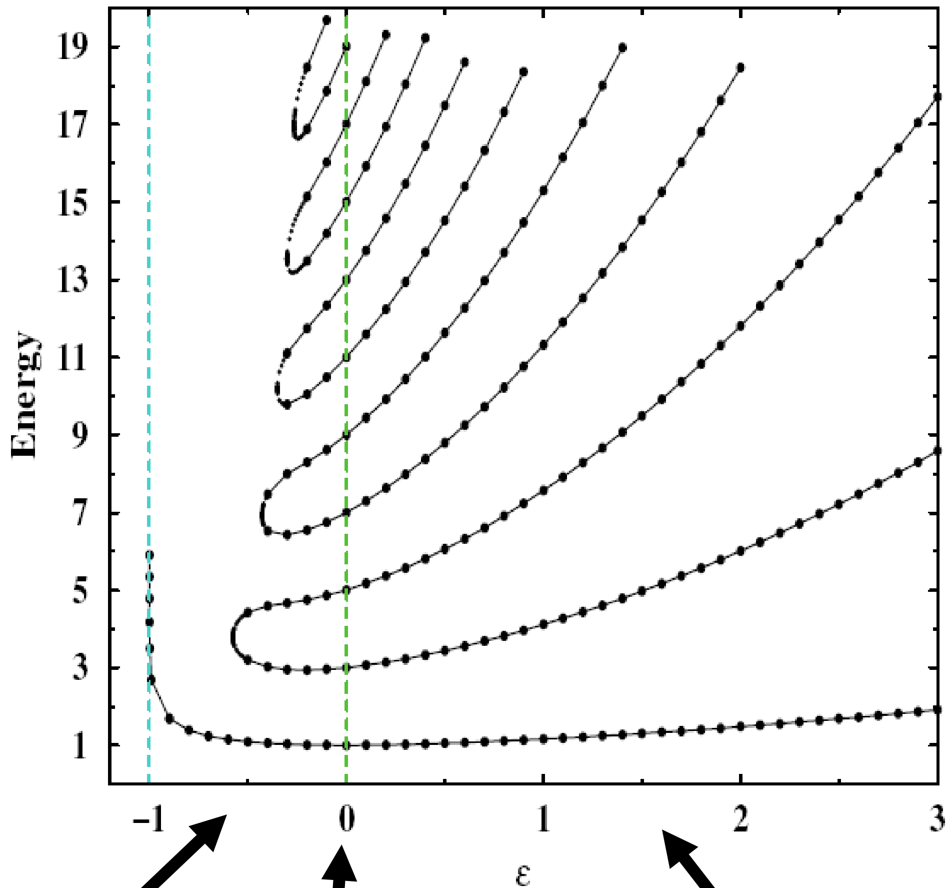
Q: WHY IS THERE NO INSTABILITY??

A: If you extend real numbers to complex numbers, you lose the *ordering* property of real numbers

You lose the concept of > and <

Physical systems that *look* unstable may be stable!

$$H = p^2 + x^2(ix)^\varepsilon \qquad (\varepsilon \text{ real})$$



Transition

at ε = 0

Region of ***broken***
PT symmetry

PT **Boundary**

Region of ***unbroken***
PT symmetry

PT **symmetry does not conflict with conventional quantum theory, but it is *weaker* than Hermiticity: All eigenvalues E of a Hermitian Hamiltonian are real. For *PT*-symmetric Hamiltonians *only the secular equation*** $\det(H - IE) = 0$ ***is real*.**

Unlike Hermitian Hamiltonians, there are

***TWO POSSIBILITIES*:**

***PT*-symmetric theories may have an *all* real or a *partly* real spectrum.**

Broken ***P***arro***T*** Unbroken ***P***arro***T***

Hermitian Hamiltonians: BORING!

Eigenvalues are always real – nothing interesting happens

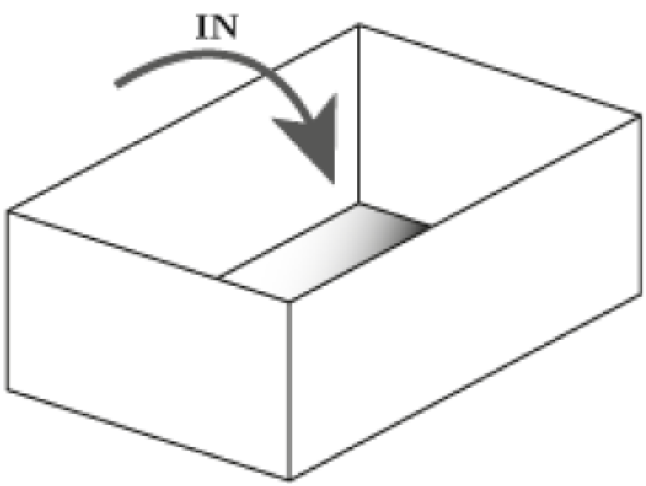
PT-symmetric Hamiltonians: ASTONISHING!

Transition between parametric regions of broken and unbroken *PT* symmetry –
Easy to observe experimentally!


Intuitive explanation of the *PT* transition …

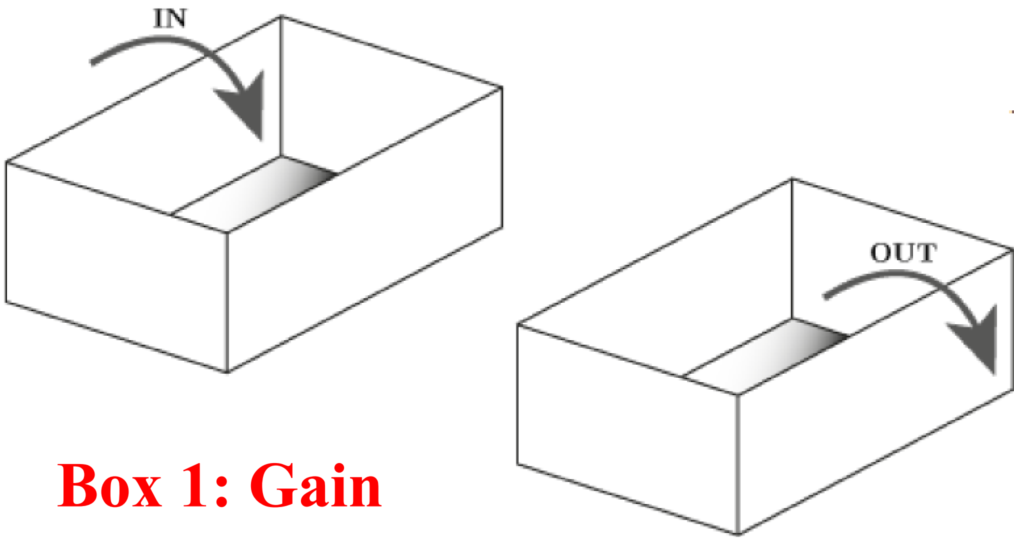
Intuitive explanation of the *PT* transition

Imagine a closed box with gain. The 1 x 1 Hamiltonian for this system is non-Hermitian: $H = [a+ib]$



Box 1: Gain

Two noninteracting closed boxes, one with gain, the other with loss:



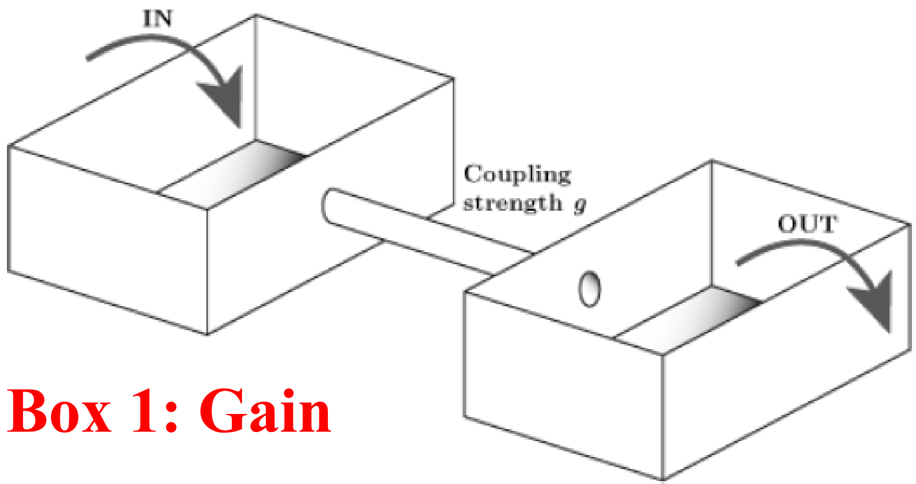
$$H_{\text{combined}} = \begin{bmatrix} a+ib & 0 \\ 0 & a-ib \end{bmatrix}$$

Box 1: Gain

Box 2: Loss

This system is *not in equilibrium*

Couple the boxes:



Box 1: Gain

Box 2: Loss

$$H_{\text{coupled}} = \begin{bmatrix} a+ib & g \\ g & a-ib \end{bmatrix}$$

This Hamiltonian is not Hermitian but it is *PT* symmetric:

Time reversal: $\mathcal{T}$ = complex conjugation

Parity: $\mathcal{P} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Eigenvalues satisfy a real secular equation:

$$\det(H_{\text{coupled}} - IE) = E^2 - 2aE + \mathrm{a}^2 + b^2 - g^2$$

$$E_{\pm} = a \pm (g^2 - b^2)^{1/2}$$

Transition at $|g| = |b|$
Energy is REAL if $|g| > |b|$

This system is in equilibrium for sufficiently large coupling!

PT-symmetric systems lie between closed and open systems

Hermitian H	*PT*-symmetric H	Non-Hermitian H
		

Theoretical applications: **renormalizing makes a Hamiltonian non-Hermitian, but still *PT* symmetric**

- Lee model is unitary (there are no ghosts!)
- Pais-Uhlenbeck model (no ghosts!)
- Self-force on the electron (runaway modes)
- Double-scaling limit in QFT
- Stability of the Higgs vacuum
- Asymptotic behavior of the Painlevé transcendents
- Application to the Riemann hypothesis

…and many many many many more!

Experimental Studies of *PT* symmetry:

- ***PT***-symmetric wave guides
- ***PT***-symmetric lasers
- ***PT***-symmetric electronic and mechanical systems
- Unidirectional transmission of light
- ***PT***-symmetric atomic diffusion
- ***PT***-symmetric superconducting wires
- ***PT***-symmetric optical graphene
- ***PT***-symmetric power transfer
- ***PT***-symmetric fluid instabilities

…and many many many many more!

Experimental studies of *PT*–symmetric systems

First observation of *PT* transition using optical wave guides:

"Observation of PT-symmetry breaking in complex optical potentials," A. Guo, G. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. Siviloglou, and D. Christodoulides, *Physical Review Letters* **103**, 093902 (2009)

nature physics

LETTERS

PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515

Observation of parity–time symmetry in optics

Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]*

One of the fundamental axioms of quantum mechanics is associated with the Hermiticity of physical observables[1]. In the case of the Hamiltonian operator, this requirement not only implies real eigenenergies but also guarantees probability conservation. Interestingly, a wide class of non-Hermitian Hamiltonians can still show entirely real spectra. Among these are Hamiltonians respecting parity–time (*PT*) symmetry[2–7]. Even though the Hermiticity of quantum observables was never in doubt, such concepts have motivated discussions on several fronts in physics, including quantum field theories[8], non-Hermitian Anderson models[9] and open quantum systems[10,11], to mention a few. Although the impact of *PT* symmetry in these fields is still debated, it has been recently realized that optics can provide a fertile ground where *PT*-related notions can be implemented and experimentally investigated[12–15]. In this letter we report the first observation of the behaviour of a *PT* optical coupled system that judiciously involves a complex index potential. We observe both spontaneous *PT* symmetry breaking and power oscillations violating left–right symmetry. Our results may pave the way towards a new class of *PT*-synthetic materials with intriguing and unexpected properties that rely on non-reciprocal light propagation and tailored transverse energy flow.

($\varepsilon > \varepsilon_{th}$), the spectrum ceases to be real and starts to involve imaginary eigenvalues. This signifies the onset of a spontaneous *PT* symmetry-breaking, that is, a 'phase transition' from the exact to broken-*PT* phase[7,20].

In optics, several physical processes are known to obey equations that are formally equivalent to that of Schrödinger in quantum mechanics. Spatial diffraction and temporal dispersion are perhaps the most prominent examples. In this work we focus our attention on the spatial domain, for example optical beam propagation in *PT*-symmetric complex potentials. In fact, such *PT* 'optical potentials' can be realized through a judicious inclusion of index guiding and gain/loss regions[7,12–14]. Given that the complex refractive-index distribution $n(x) = n_R(x) + i n_I(x)$ plays the role of an optical potential, we can then design a *PT*-symmetric system by satisfying the conditions $n_R(x) = n_R(-x)$ and $n_I(x) = -n_I(-x)$.

In other words, the refractive-index profile must be an even function of position x whereas the gain/loss distribution should be odd. Under these conditions, the electric-field envelope E of the optical beam is governed by the paraxial equation of diffraction[13]:

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\frac{\partial^2 E}{\partial x^2} + k_0[n_R(x) + i n_I(x)]E = 0$$

PT-symmetric diffusion – Shanghai/Rutgers

PHYSICAL REVIEW A **81**, 042903 (2010)

Enhanced magnetic resonance signal of spin-polarized Rb atoms near surfaces of coated cells

K. F. Zhao,[1,*] M. Schaden,[2] and Z. Wu[2]
[1]*Institute of Modern Physics, Fudan University, Shanghai 200433, People's Republic of China*
[2]*Department of Physics, Rutgers University, Newark, New Jersey 07102, USA*
(Received 12 November 2009; published 21 April 2010)

We present a detailed experimental and theoretical study of edge enhancement in optically pumped Rb vapor in coated cylindrical pyrex glass cells. The Zeeman polarization of Rb atoms is produced and probed in the vicinity ($\sim 10^{-4}$ cm) of the cell surface by evanescent pump and probe beams. Spin-polarized Rb atoms diffuse throughout the cell in the presence of magnetic field gradients. In the present experiment the edge enhanced signal from the back surface of the cell is suppressed compared to that from the front surface, due to the fact that polarization is probed by the evanescent wave at the front surface only. The observed magnetic resonance line shape is reproduced quantitatively by a theoretical model and yields information about the dwell time and relaxation probability of Rb atoms on Pyrex glass surfaces coated with antirelaxation coatings.

DOI: 10.1103/PhysRevA.81.042903 PACS number(s): 34.35.+a, 75.40.Gb, 76.70.Hb, 87.57.nt

PT-symmetric optics – Caltech

SCIENCE VOL 333 5 AUGUST 2011

Nonreciprocal Light Propagation in a Silicon Photonic Circuit

Liang Feng,[1,2,4]*† Maurice Ayache,[3]* Jingqing Huang,[1,4]* Ye-Long Xu,[2] Ming-Hui Lu,[2] Yan-Feng Chen,[2]† Yeshaiahu Fainman,[3] Axel Scherer[1,4]†

Optical communications and computing require on-chip nonreciprocal light propagation to isolate and stabilize different chip-scale optical components. We have designed and fabricated a metallic-silicon waveguide system in which the optical potential is modulated along the length of the waveguide such that nonreciprocal light propagation is obtained on a silicon photonic chip. Nonreciprocal light transport and one-way photonic mode conversion are demonstrated at the wavelength of 1.55 micrometers in both simulations and experiments. Our system is compatible with conventional complementary metal-oxide-semiconductor processing, providing a way to chip-scale optical isolators for optical communications and computing.

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PT-symmetric superconducting wires – Indiana

PRL **99**, 167003 (2007) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2007

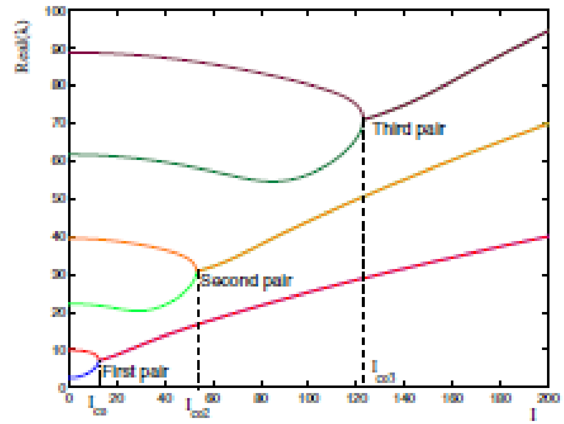
Bifurcation Diagram and Pattern Formation of Phase Slip Centers in Superconducting Wires Driven with Electric Currents

J. Rubinstein, P. Sternberg, and Q. Ma

Mathematics Department, Indiana University, Bloomington, Indiana 47405, USA

(Received 14 February 2007; published 18 October 2007)

We provide here new insights into the classical problem of a one-dimensional superconducting wire exposed to an applied electric current using the time-dependent Ginzburg-Landau model. The most striking feature of this system is the well-known appearance of oscillatory solutions exhibiting phase slip centers (PSC's) where the order parameter vanishes. Retaining temperature and applied current as parameters, we present a simple yet definitive explanation of the mechanism within this nonlinear model that leads to the PSC phenomenon and we establish where in parameter space these oscillatory solutions can be found. One of the most interesting features of the analysis is the evident collision of real eigenvalues of the associated *PT*-symmetric linearization, leading as it does to the emergence of complex elements of the spectrum.



$\mathcal{PT}$-symmetric microwave cavities – Germany

PRL 108, 024101 (2012) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2012

$\mathcal{PT}$ Symmetry and Spontaneous Symmetry Breaking in a Microwave Billiard

S. Bittner,[1] B. Dietz,[1,*] U. Günther,[2] H. L. Harney,[3] M. Miski-Oglu,[1] A. Richter,[1,4,†] and F. Schäfer[1,5]

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[4]*ECT*, Villa Tambosi, I-38123 Villazzano (Trento), Italy*
[5]*LENS, University of Florence, I-50019 Sesto-Fiorentino (Firenze), Italy*

(Received 21 July 2011; published 10 January 2012)

We demonstrate the presence of parity-time ($\mathcal{PT}$) symmetry for the non-Hermitian two-state Hamiltonian of a dissipative microwave billiard in the vicinity of an exceptional point (EP). The shape of the billiard depends on two parameters. The Hamiltonian is determined from the measured resonance spectrum on a fine grid in the parameter plane. After applying a purely imaginary diagonal shift to the Hamiltonian, its eigenvalues are either real or complex conjugate on a curve, which passes through the EP. An appropriate basis choice reveals its $\mathcal{PT}$ symmetry. Spontaneous symmetry breaking occurs at the EP.

DOI: 10.1103/PhysRevLett.108.024101 PACS numbers: 05.45.Mt, 02.10.Yn, 11.30.Er

PT-symmetric cavity lasers – Yale

PRL **106**, 093902 (2011) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2011

$\mathcal{PT}$-Symmetry Breaking and Laser-Absorber Modes in Optical Scattering Systems

Y. D. Chong,* Li Ge,† and A. Douglas Stone

Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA

(Received 30 August 2010; revised manuscript received 27 January 2011; published 2 March 2011)

Using a scattering matrix formalism, we derive the general scattering properties of optical structures that are symmetric under a combination of parity and time reversal ($\mathcal{PT}$). We demonstrate the existence of a transition between $\mathcal{PT}$-symmetric scattering eigenstates, which are norm preserving, and symmetry-broken pairs of eigenstates exhibiting net amplification and loss. The system proposed by Longhi [Phys. Rev. A **82**, 031801 (2010).], which can act simultaneously as a laser and coherent perfect absorber, occurs at discrete points in the broken-symmetry phase, when a pole and zero of the S matrix coincide.

DOI: 10.1103/PhysRevLett.106.093902 PACS numbers: 42.25.Bs, 42.25.Hz, 42.55.Ah

PT-symmetric photonic graphene – Israel

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **84**, 021806(R) (2011)

$\mathcal{PT}$-symmetry in honeycomb photonic lattices

Alexander Szameit, Mikael C. Rechtsman, Omri Bahat-Treidel, and Mordechai Segev

Physics Department and Solid State Institute, Technion, 32000 Haifa, Israel

(Received 21 April 2011; published 19 August 2011)

We apply gain and loss to honeycomb photonic lattices and show that the dispersion relation is identical to tachyons—particles with imaginary mass that travel faster than the speed of light. This is accompanied by $\mathcal{PT}$-symmetry breaking in this structure. We further show that the $\mathcal{PT}$-symmetry can be restored by deforming the lattice.

DOI: 10.1103/PhysRevA.84.021806 PACS number(s): 42.25.−p, 42.82.Et

PT lasers – Vienna/Princeton/Yale/Zurich

PRL **108,** 173901 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Pump-Induced Exceptional Points in Lasers

M. Liertzer,[1,*] Li Ge,[2] A. Cerjan,[3] A. D. Stone,[3] H. E. Türeci,[2,4] and S. Rotter[1,†]

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[2]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
[3]*Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA*
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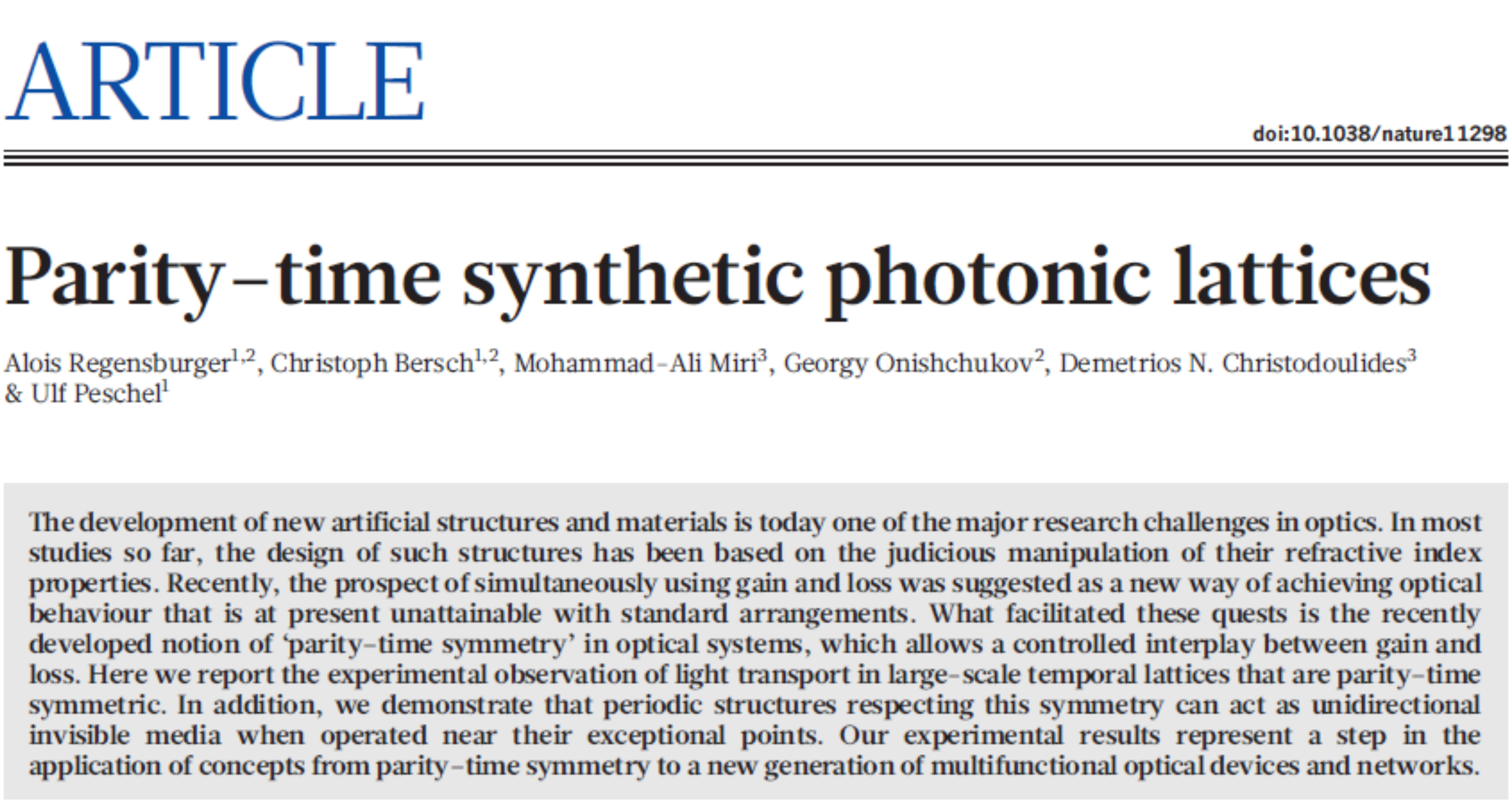
(Received 2 September 2011; revised manuscript received 20 January 2012; published 24 April 2012)

We demonstrate that the above-threshold behavior of a laser can be strongly affected by exceptional points which are induced by pumping the laser nonuniformly. At these singularities, the eigenstates of the non-Hermitian operator which describes the lasing modes coalesce. In their vicinity, the laser may turn off even when the overall pump power deposited in the system is increased. Such signatures of a pump-induced exceptional point can be experimentally probed with coupled ridge or microdisk lasers.

Multiple *PT*-symmetric waveguides – Germany/Florida


ARTICLE

doi:10.1038/nature11298

Parity–time synthetic photonic lattices

Alois Regensburger[1,2], Christoph Bersch[1,2], Mohammad-Ali Miri[3], Georgy Onishchukov[2], Demetrios N. Christodoulides[3] & Ulf Peschel[1]

The development of new artificial structures and materials is today one of the major research challenges in optics. In most studies so far, the design of such structures has been based on the judicious manipulation of their refractive index properties. Recently, the prospect of simultaneously using gain and loss was suggested as a new way of achieving optical behaviour that is at present unattainable with standard arrangements. What facilitated these quests is the recently developed notion of 'parity–time symmetry' in optical systems, which allows a controlled interplay between gain and loss. Here we report the experimental observation of light transport in large-scale temporal lattices that are parity–time symmetric. In addition, we demonstrate that periodic structures respecting this symmetry can act as unidirectional invisible media when operated near their exceptional points. Our experimental results represent a step in the application of concepts from parity–time symmetry to a new generation of multifunctional optical devices and networks.

$\mathcal{PT}$-symmetric superconducting wires – Argonne

PRL 109, 150405 (2012) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2012

Stimulation of the Fluctuation Superconductivity by $\mathcal{PT}$ Symmetry

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[2]*Department of Theoretical Physics, Moscow Institute of Physics and Technology, 141700 Moscow, Russia*
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[4]*Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
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(Received 6 May 2012; published 9 October 2012)

We discuss fluctuations near the second-order phase transition where the free energy has an additional non-Hermitian term. The spectrum of the fluctuations changes when the odd-parity potential amplitude exceeds the critical value corresponding to the $\mathcal{PT}$-symmetry breakdown in the topological structure of the Hilbert space of the effective non-Hermitian Hamiltonian. We calculate the fluctuation contribution to the differential resistance of a superconducting weak link and find the manifestation of the $\mathcal{PT}$-symmetry breaking in its temperature evolution. We successfully validate our theory by carrying out measurements of far from equilibrium transport in mesoscale-patterned superconducting wires.

DOI: 10.1103/PhysRevLett.109.150405 PACS numbers: 11.30.Er, 03.65.Ge, 73.63.−b

PT-symmetric NMR – Beijing

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

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Research

Cite this article: Zheng C, Hao L, Long GL. 2013 Observation of a fast evolution in a parity-time-symmetric system. Phil Trans R Soc A 371: 20120053. http://dx.doi.org/10.1098/rsta.2012.0053

One contribution of 17 to a Theme Issue '$\mathcal{PT}$ quantum mechanics'.

Observation of a fast evolution in a parity-time-symmetric system

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[2]Tsinghua National Laboratory for Information Science and Technology, Beijing 100084, People's Republic of China

In parity-time-symmetric ($\mathcal{PT}$-symmetric) Hamiltonian theory, the optimal evolution time can be reduced drastically and can even be zero. In this article, we report our experimental simulation of the fast evolution of a $\mathcal{PT}$-symmetric Hamiltonian in a nuclear magnetic resonance quantum system. The experimental results demonstrate that the $\mathcal{PT}$-symmetric Hamiltonian system can indeed evolve much faster than the quantum system, and the evolution time can be arbitrarily close to zero.

PT-symmetric metasurfaces – Texas

PRL **113**, 023903 (2014) PHYSICAL REVIEW LETTERS week ending 11 JULY 2014

Negative Refraction and Planar Focusing Based on Parity-Time Symmetric Metasurfaces

Romain Fleury, Dimitrios L. Sounas, and Andrea Alù[*]
Department of Electrical & Computer Engineering, The University of Texas at Austin, Austin, Texas 78712, USA
(Received 19 April 2014; published 10 July 2014)

We introduce a new mechanism to realize negative refraction and planar focusing using a pair of parity-time symmetric metasurfaces. In contrast to existing solutions that achieve these effects with negative-index metamaterials or phase conjugating surfaces, the proposed parity-time symmetric lens enables loss-free, all-angle negative refraction and planar focusing in free space, without relying on bulk metamaterials or nonlinear effects. This concept may represent a pivotal step towards loss-free negative refraction and highly efficient planar focusing by exploiting the largely uncharted scattering properties of parity-time symmetric systems.

DOI: 10.1103/PhysRevLett.113.023903 PACS numbers: 42.30.Wb, 03.65.Nk, 11.30.Er, 78.67.Pt

PT-symmetric photonic crystals – Stanford

PRL 116, 203902 (2016) PHYSICAL REVIEW LETTERS week ending 20 MAY 2016

Exceptional Contours and Band Structure Design in Parity-Time Symmetric Photonic Crystals

Alexander Cerjan, Aaswath Raman, and Shanhui Fan*
Department of Electrical Engineering, and Ginzton Laboratory, Stanford University, Stanford, California 94305, USA
(Received 22 January 2016; published 20 May 2016)

We investigate the properties of two-dimensional parity-time symmetric periodic systems whose non-Hermitian periodicity is an integer multiple of the underlying Hermitian system's periodicity. This creates a natural set of degeneracies that can undergo thresholdless $\mathcal{PT}$ transitions. We derive a $\mathbf{k} \cdot \mathbf{p}$ perturbation theory suited to the continuous eigenvalues of such systems in terms of the modes of the underlying Hermitian system. In photonic crystals, such thresholdless $\mathcal{PT}$ transitions are shown to yield significant control over the band structure of the system, and can result in all-angle supercollimation, a $\mathcal{PT}$-superprism effect, and unidirectional behavior.

DOI: 10.1103/PhysRevLett.116.203902

PT-symmetric wireless power transfer – Stanford

LETTER

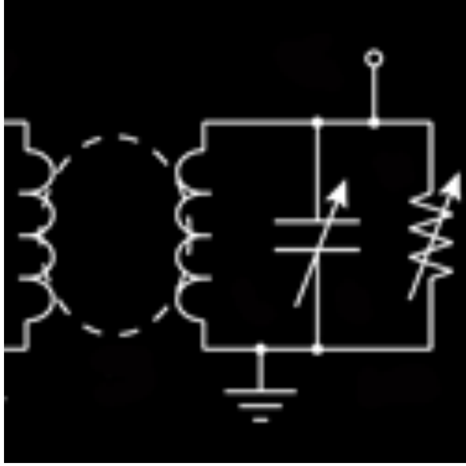
doi:10.1038/nature22404

Robust wireless power transfer using a nonlinear parity–time-symmetric circuit

Sid Assawaworrarit[1], Xiaofang Yu[1] & Shanhui Fan[1]

Considerable progress in wireless power transfer has been made in the realm of non-radiative transfer, which employs magnetic-field coupling in the near field[1–4]. A combination of circuit resonance and impedance transformation is often used to help to achieve efficient transfer of power over a predetermined distance of about the size of the resonators[3,4]. The development of non-radiative wireless power transfer has paved the way towards real-world applications such as wireless powering of implantable medical devices and wireless charging of stationary electric vehicles[1,2,5–8]. However, it remains a fundamental challenge to create a wireless power transfer system in which the transfer efficiency is robust against the variation of operating conditions. Here we propose theoretically and demonstrate experimentally that a parity–time-symmetric circuit incorporating a nonlinear gain saturation element provides robust wireless power transfer. Our results show that the transfer efficiency remains near unity over a distance variation of approximately one metre, without the need for any tuning. This is in contrast with conventional methods where high transfer efficiency can only be maintained by constantly tuning the frequency or the internal coupling parameters as the transfer distance or the relative orientation of the source and receiver units is varied. The use of a nonlinear parity–time-symmetric circuit should enable robust wireless power transfer to moving devices or vehicles[9,10].

APS: Spotlighting exceptional research



J. Schindler *et al.*, Phys. Rev. A (2011)

Experimental study of active *LRC* circuits with *PT* symmetries

Joseph Schindler, Ang Li, Mei C. Zheng, F. M. Ellis, and Tsampikos Kottos

Phys. Rev. A **84**, 040101 (2011)

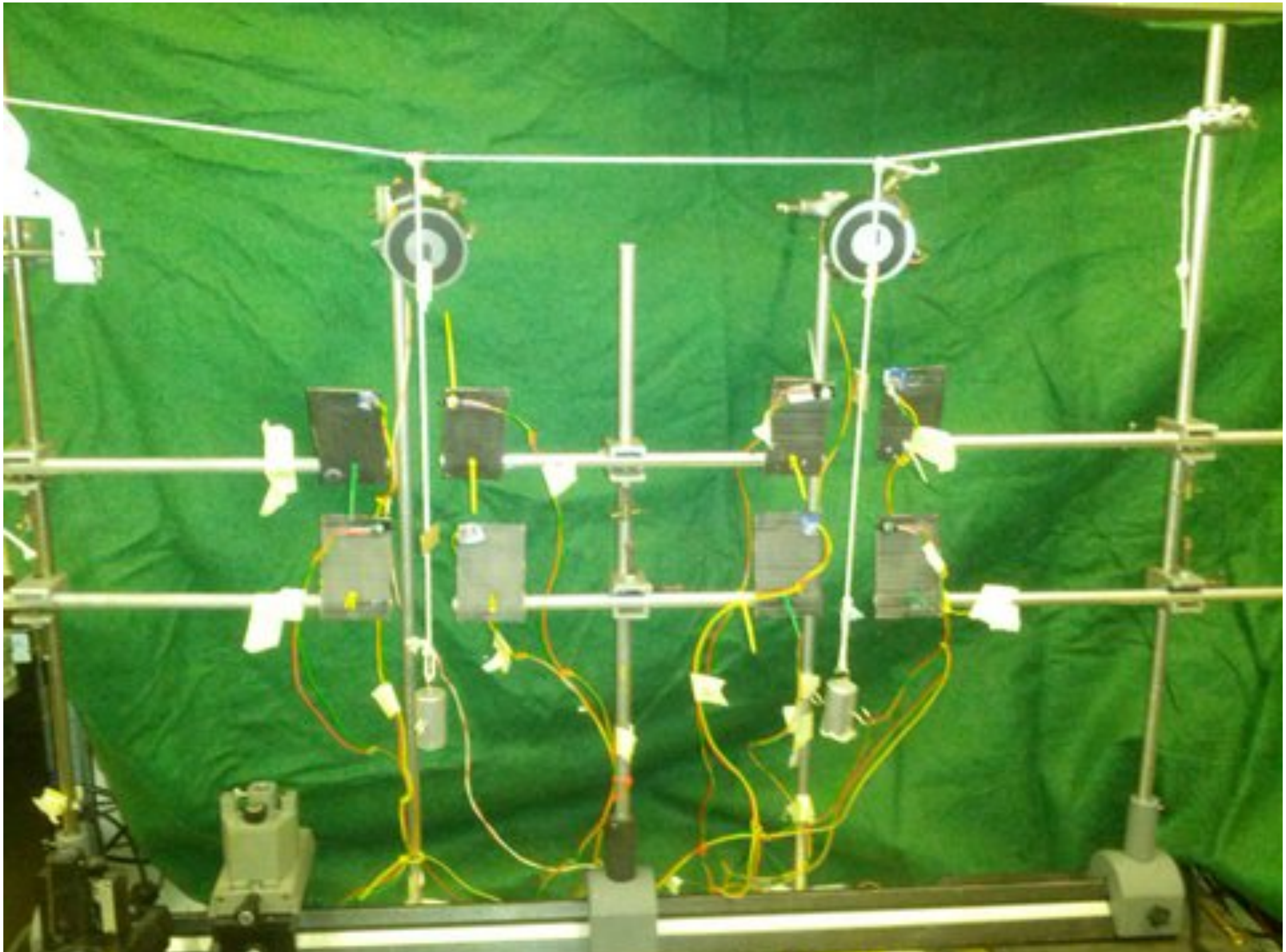
Published October 13, 2011

Everyone learns in a first course on quantum mechanics that the result of a measurement cannot be a complex number, so the quantum mechanical operator that corresponds to a measurement must be Hermitian. However, certain classes of complex Hamiltonians that are not Hermitian can still have real eigenvalues. The key property of these Hamiltonians is that they are parity-time (*PT*) symmetric, that is, they are invariant under a mirror reflection and complex conjugation (which is equivalent to time reversal).

Hamiltonians that have *PT* symmetry have been used to describe the depinning of vortex flux lines in type-II superconductors and optical effects that involve a complex index of refraction, but there has never been a simple physical system where the effects of *PT* symmetry can be clearly understood and explored. Now, Joseph Schindler and colleagues at Wesleyan University in Connecticut have devised a simple *LRC* electrical circuit that displays directly the effects of *PT* symmetry. The key components are a pair of coupled resonant circuits, one with active gain and the other with an equivalent amount of loss. Schindler *et al.* explore the eigenfrequencies of this system as a function of the "gain/loss" parameter that controls the degree of amplification and attenuation of the system. For a critical value of this parameter, the eigenfrequencies undergo a spontaneous phase transition from real to complex values, while the eigenstates coalesce and acquire a definite chirality (handedness). This simple electronic analog to a quantum Hamiltonian could be a useful reference point for studying more complex applications.

– *Gordon W. F. Drake*

"Observation of **PT** phase transition in a simple mechanical system,"
CMB, B. Berntson, D. Parker, E. Samuel, *American Journal of Physics* **81**, 173 (2013)



PT-symmetric system of coupled pendula

$$
\begin{aligned}
x''(t) + ax'(t) + x(t) + \varepsilon y(t) &= 0 \\
y''(t) - ay'(t) + y(t) + \varepsilon x(t) &= 0
\end{aligned}
$$

Loss and gain:

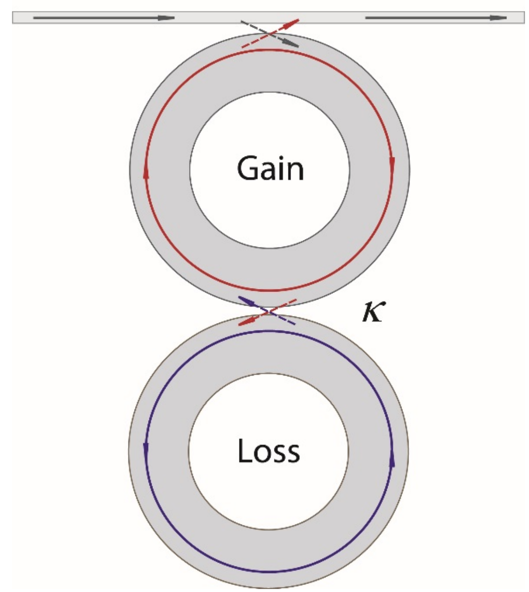
Remove energy from the *x* pendulum and transfer it to the *y* pendulum.

Fancy experiments involving whispering-galery microcavities

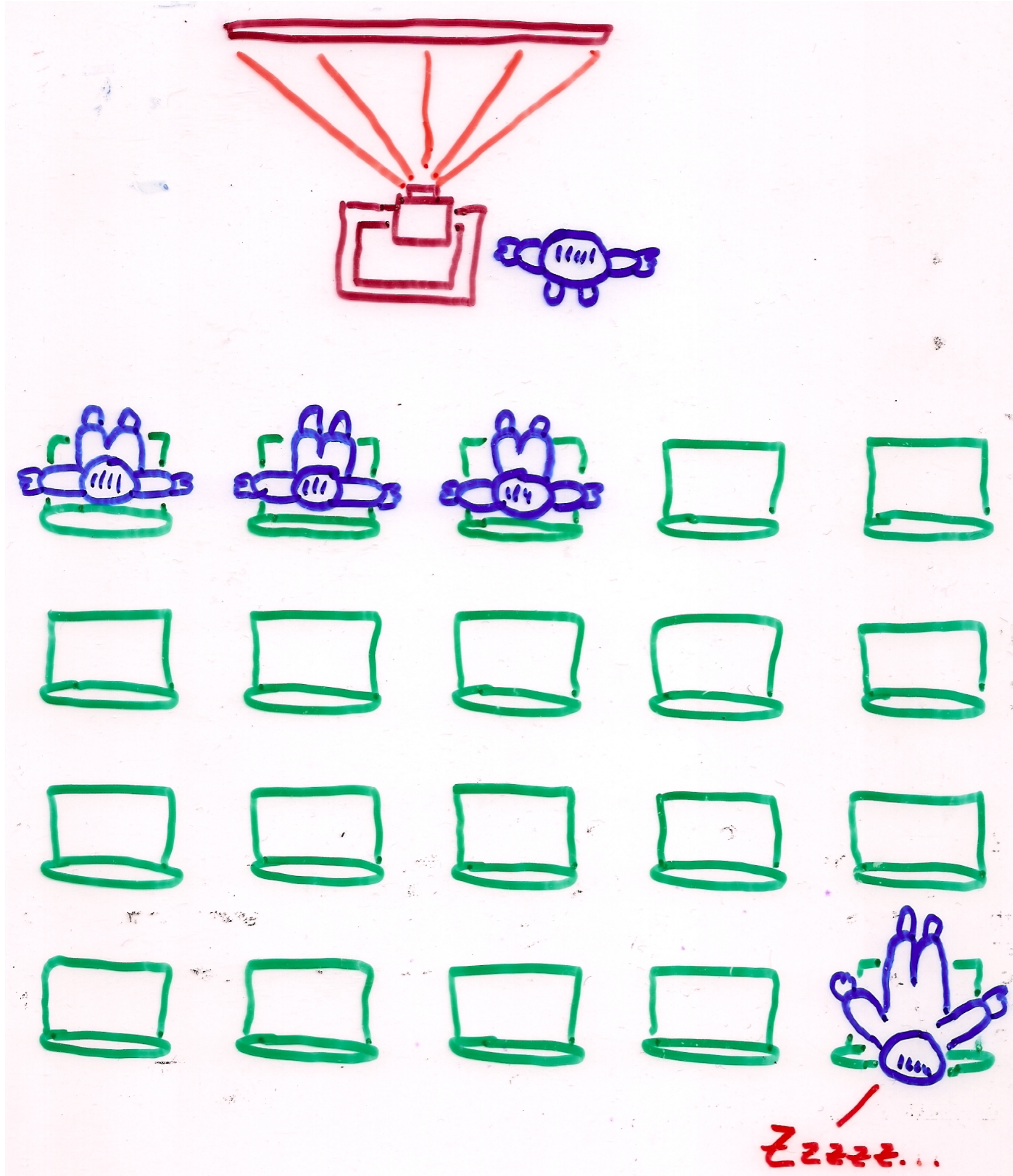
"Nonreciprocal light transmission in parity-time-symmetric whispering-gallery microcavities," B. Peng, S. K. Ozdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, CMB, L. Yang, *Nature Physics* **10**, 394 (2014)

"Twofold transition in **PT**-symmetric coupled oscillators," CMB, M. Gianfreda, B. Peng, S. K. Ozdemir, and L. Yang, *Physical Review A* **88**, 062111 (2013)

"Loss-induced suppression and revival of lasing," B. Peng, S.K. Ozdemir, S. Rotter, H. Yilmaz, M. Liertzer, CMB, F. Nori, L. Yang, *Science* **346**, 328 (2014)



Overview of my talk:



A few theoretical examples

Example 1: Lee model

$$V \to N + \theta, \qquad N + \theta \to V.$$

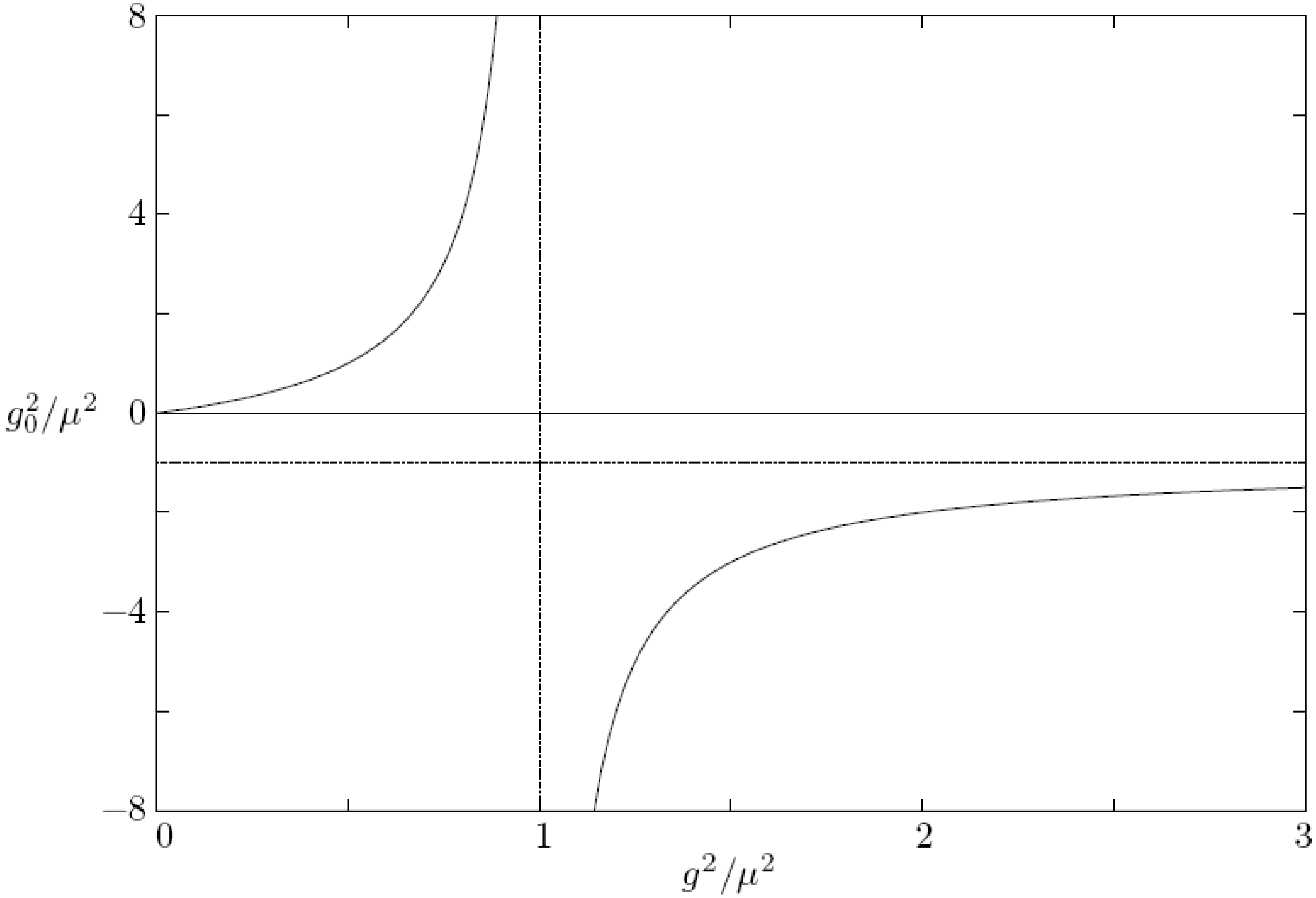
$$H = H_0 + g_0 H_1,$$

$$\begin{aligned} H_0 &= m_{V_0} V^\dagger V + m_N N^\dagger N + m_\theta a^\dagger a, \\ H_1 &= V^\dagger N a + a^\dagger N^\dagger V. \end{aligned}$$

T. D. Lee, Phys. Rev. **95**, 1329 (1954)

G. Källén and W. Pauli, Dan. Mat. Fys. Medd. **30**, No. 7 (1955)

Problem with the Lee model



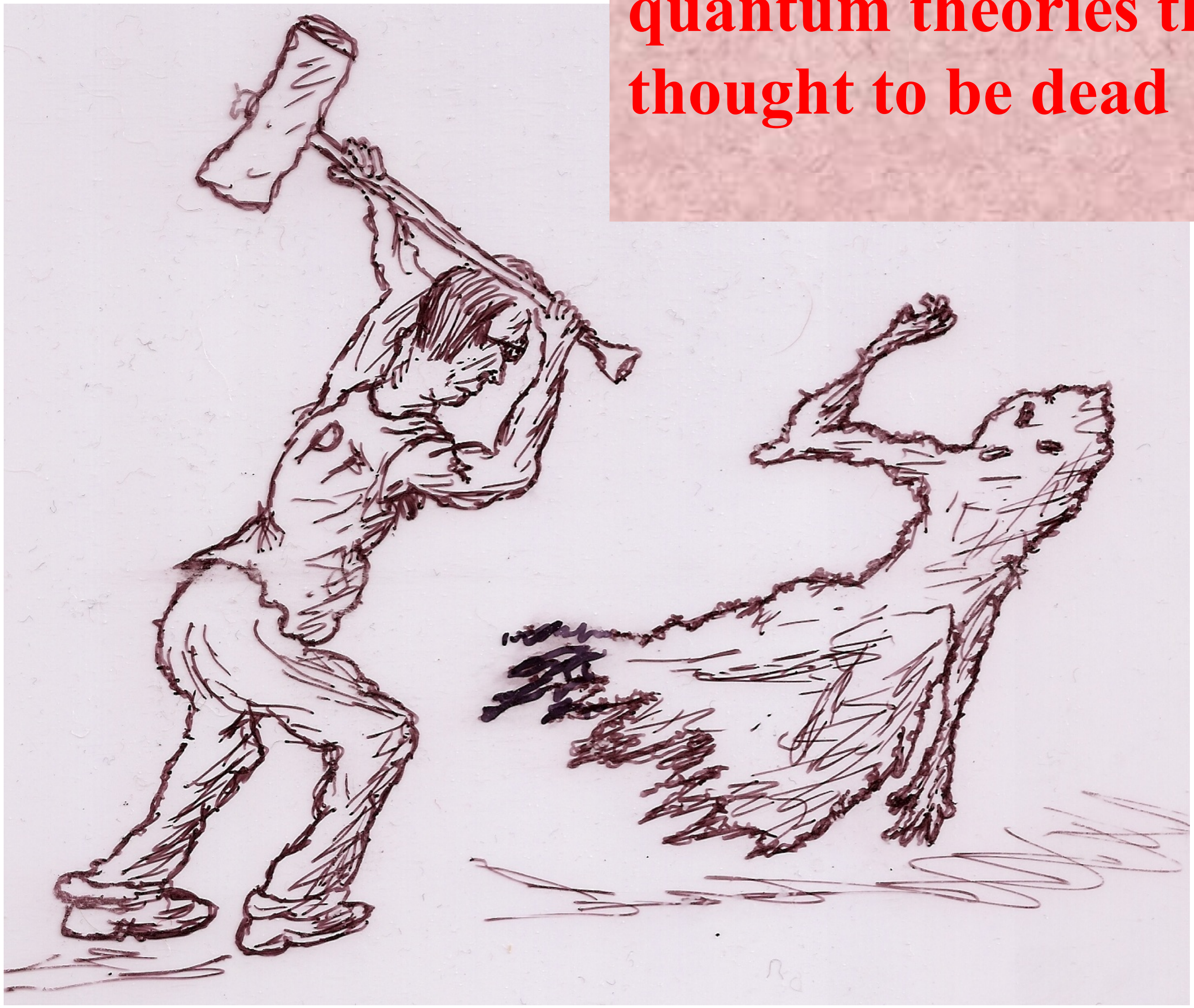
$$g_0^2 = g^2 / \left(1 - g^2/\mu^2\right)$$

"A non-Hermitian Hamiltonian is unacceptable partly because it may lead to complex energy eigenvalues, but chiefly because it implies a non-unitary S matrix, which fails to conserve probability and makes a hash of the physical interpretation."

G. Barton, *Introduction to Advanced Field Theory* (John Wiley & Sons, New York, 1963)

Renormalization creates instability.
This is a *really* hard problem. Pauli, Heisenberg, Wick, Sudarshan, … worked on it, but no cigar.

GHOSTBUSTING: Reviving quantum theories that were thought to be dead



"Ghost busting: ***PT***-symmetric interpretation of the Lee model,"
CMB, S. Brandt, J.-H. Chen, and Q. Wang, *Phys. Rev. D* **71**, 025014 (2005)

Example 2:
Double-scaling limit in QFT

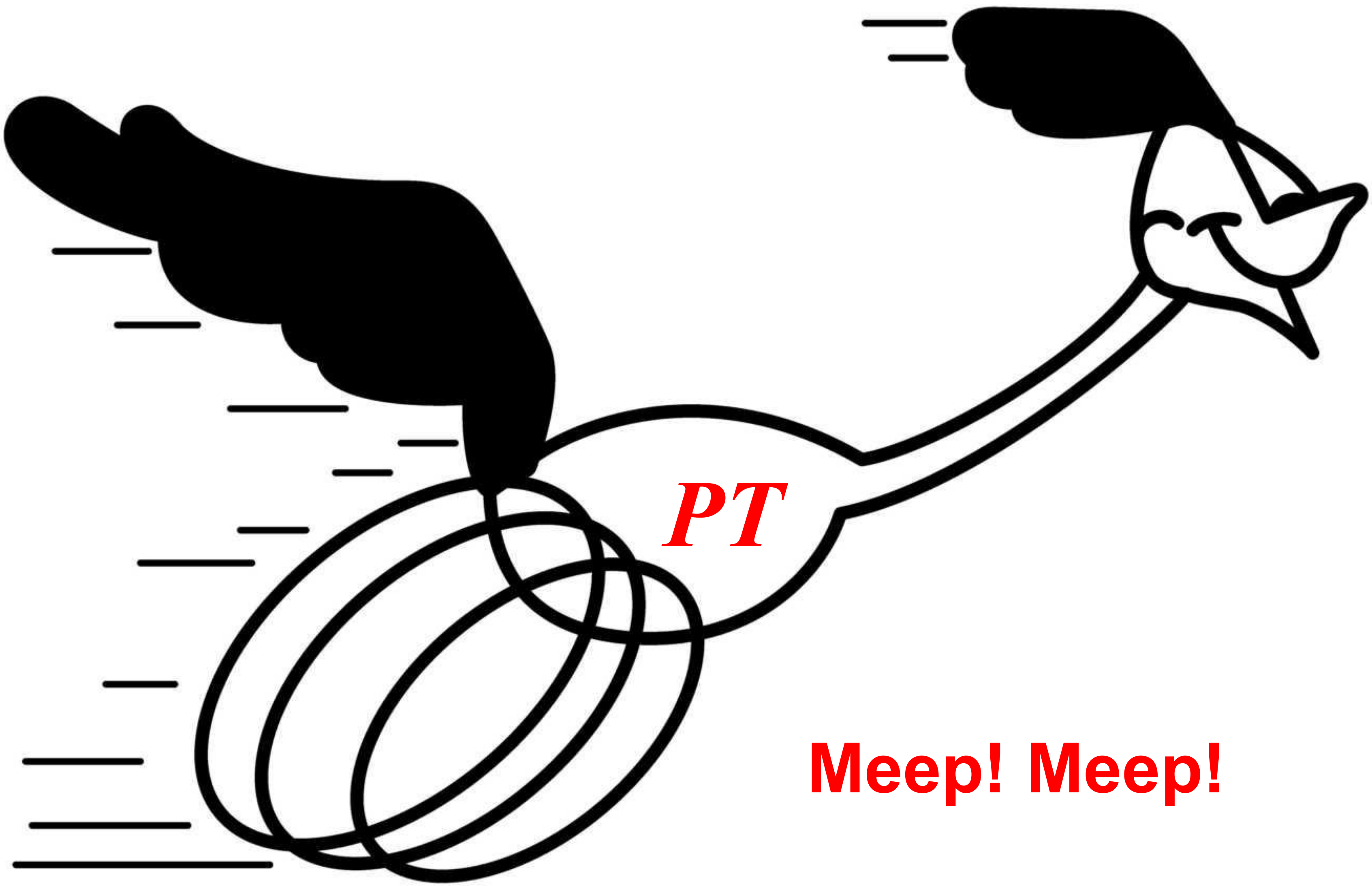
The double-scaling limit is a *correlated limit*; it is universal and produces an entire function of scaled variable

BUT!
The double-scaling limit of a conventional quartic theory gives a "wrong-sign" (*upside-down* potential) universal theory: $-g\phi^4$

CMB, M. Moshe, and S. Sarkar, *J. Phys. A: Math. Theor.* **46**, 102002 (2013)

CMB and S. Sarkar, *J. Phys. A: Math. Theor*. **46**, 442001 (2013)

Again: PT-symmetric quantum mechanics to the rescue!



Example 3: Instabilities of nonlinear differential equations

Painlevé transcendents have fundamental instabilities that can be tamed and understood quantitatively by using *PT*-symmetric quantum theory

"Nonlinear eigenvalue problems"
CMB, A. Fring, and J. Komijani
Journal of Physics A: Mathematical and Theoretical **47**, 235204 (2014) [arXiv: 1402.1158]

"**PT**-symmetric Hamiltonians and the Painlevé transcendents"
CMB and J. Komijani
Journal of Physics A: Mathematical and Theoretical **48**, 475202 (2015) [arXiv: 1502.04089]

"Nonlinear eigenvalue problems for generalized Painlevé equations"
CMB, J. Komijani, and Q.-h. Wang
Journal of Physics A: Mathematical and Theoretical **52**, 315202 (2019) [arXiv: 1903.10640]

Instability of Painlevé I explained from large eigenvalues of

cubic PT-symmetric Hamiltonian

$$H = p^2 + ix^3$$

Painlevé I corresponds to ε = 1

(Do you remember
the cubic *PT*-symmetric
Hamiltonian?)

Instability of Painlevé II explained from large eigenvalues of

quartic PT-symmetric Hamiltonian

$$H = p^2 - x^4$$

Painlevé II corresponds to ε = 2

(Do you remember the quartic upside-down *PT*-symmetric Hamiltonian?)

Instability of Painlevé IV explained in terms of the

sextic PT-symmetric Hamiltonian

$$H = p^2 + x^6$$

Painlevé IV corresponds to ε = 4

(Do you remember the sextic ***PT***-symmetric Hamiltonian?)

Example 4:
PT-symmetric quantum field theory

D-dimensional Euclidean-space quantum field theory with a pseudoscalar field

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}\phi^2(i\phi)^\varepsilon \quad (\varepsilon \geq 0)$$

Objective: Calculate the vacuum energy density, renormalized mass, Green's functions G_1, $G_2(x\text{-}y)$, $G_3(x\text{-}y,x\text{-}z)$, ... as series in powers of ε

"*PT*-symmetric quantum field theory in *D* dimensions"
CMB, N. Hassanpour, S. P. Klevansky, and S. Sarkar
Physical Review D **98**, 125003 (2018) [arXiv: 1810.12479]

If we expand in ε we get logarithmic terms in the Lagrangian:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}\phi^2 + \tfrac{1}{2}\varepsilon\phi^2\log(i\phi) + \mathrm{O}(\varepsilon^2)$$

Unperturbed Lagrangian is the usual free theory:

$$\mathcal{L}_0 = \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}\phi^2$$

How do we interpret the logarithm term $\log(i\phi)$ **??**

$$\log(i\phi) = \tfrac{1}{2}i\pi + \log(\phi) \quad (\phi > 0)$$

$$\log(i\phi) = -\tfrac{1}{2}i\pi + \log(-\phi) \quad (\phi < 0)$$

$$\boxed{\log(i\phi) = \tfrac{1}{2}i\pi\frac{|\phi|}{\phi} + \log(|\phi|) = \tfrac{1}{2}i\pi\frac{|\phi|}{\phi} + \tfrac{1}{2}\log(\phi^2)}$$

The imaginary term is odd in ϕ and the real term is even in ϕ so <u>this is how to ensure ***PT*** symmetry!</u>

n-point Green's function for *n* = 1, 3, 4, 5, …

$$G_n(x_1, x_2, \ldots x_n) = -\tfrac{1}{2}\varepsilon(-i)^n\Gamma\left(\tfrac{1}{2}n-1\right)$$
$$\times\left[\tfrac{1}{2}\Delta(0)\right]^{1-n/2}\int d^D u\prod_{k=1}^{n}\Delta(x_k-u)$$

Exact to order ε

Free propagator in coordinate space:

$$\Delta(x_1-x_2) = (2\pi)^{-\frac{D}{2}}|x_1-x_2|^{1-\frac{D}{2}}\mathrm{K}_{1-\frac{D}{2}}(|x_1-x_2|)$$

$$\Delta(0) = (4\pi)^{-D/2}\Gamma(1-D/2)$$

Renormalized mass

$$\tilde{G}_2(p) = \frac{1}{p^2 + 1 + \varepsilon K + \mathrm{O}\big(\varepsilon^2\big)}$$

$$M_{\mathrm{R}}^2 = 1 + K\varepsilon + \mathrm{O}\big(\varepsilon^2\big)$$

$$\begin{aligned} K &= \tfrac{1}{2} + \tfrac{1}{2}\Gamma'\big(\tfrac{3}{2}\big)/\Gamma\big(\tfrac{3}{2}\big) + \tfrac{1}{2}\log[2\Delta(0)] \\ &= \tfrac{3}{2} - \tfrac{1}{2}\gamma + \tfrac{1}{2}\log\left[\tfrac{1}{2}\Delta(0)\right] \end{aligned}$$

Comments on renormalization

for listening to my talk!

I am happy to answer questions…