Area and Volume in Multiboundary Wormhole Models

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SINP
Based on arXiv:2003.11870 (PRD)
&
2010.04314
(Holotube Junior)
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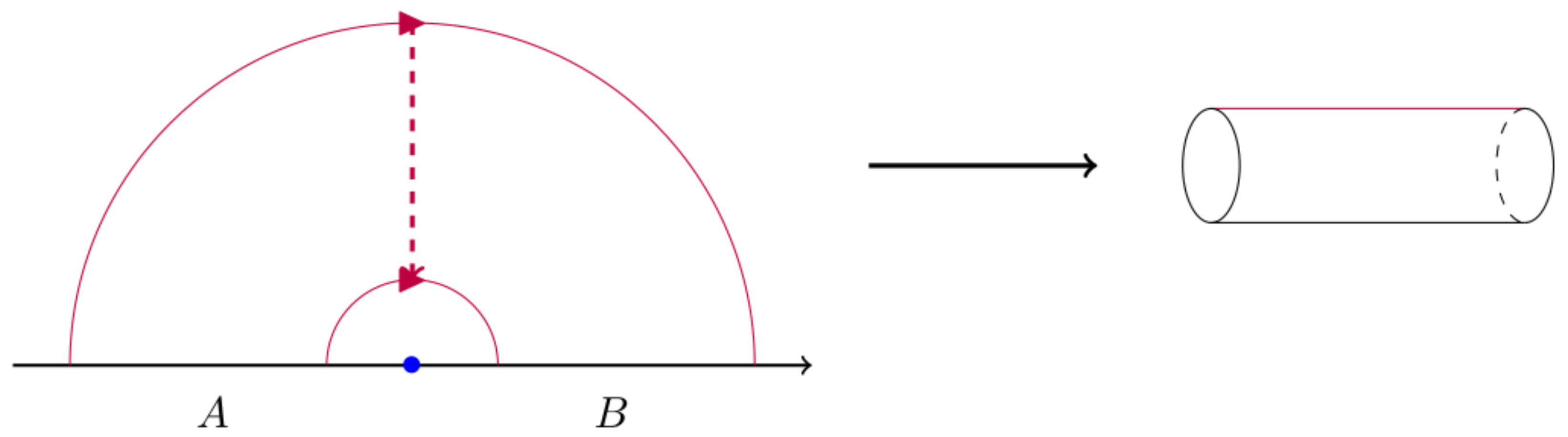
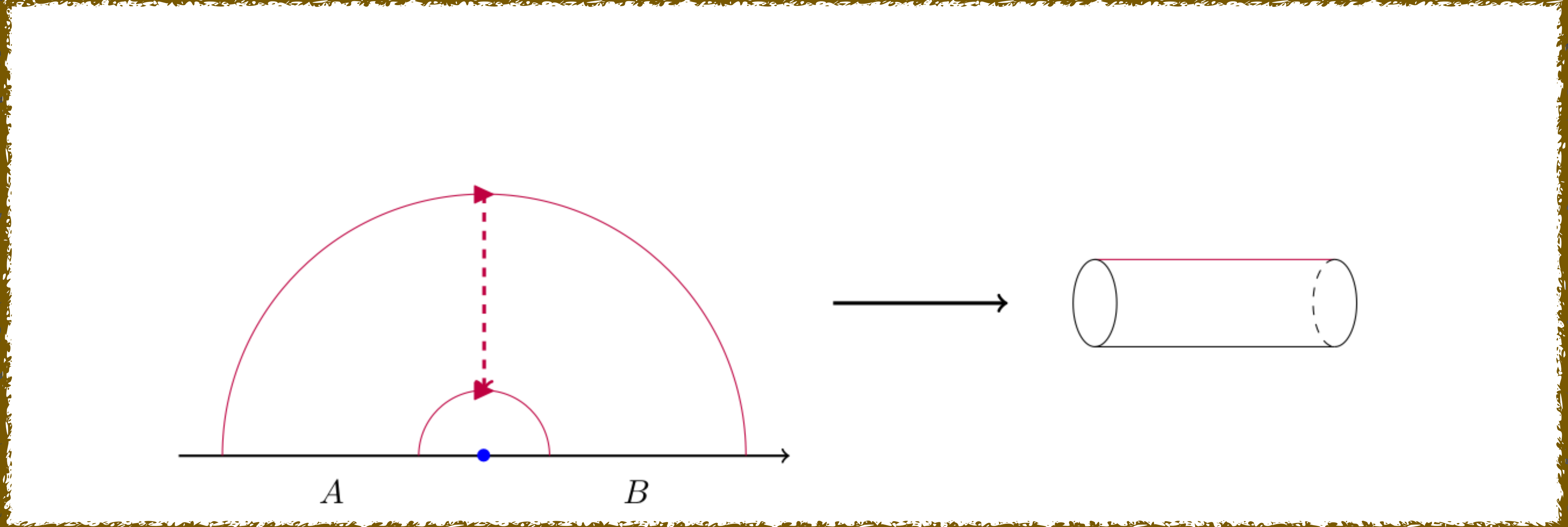
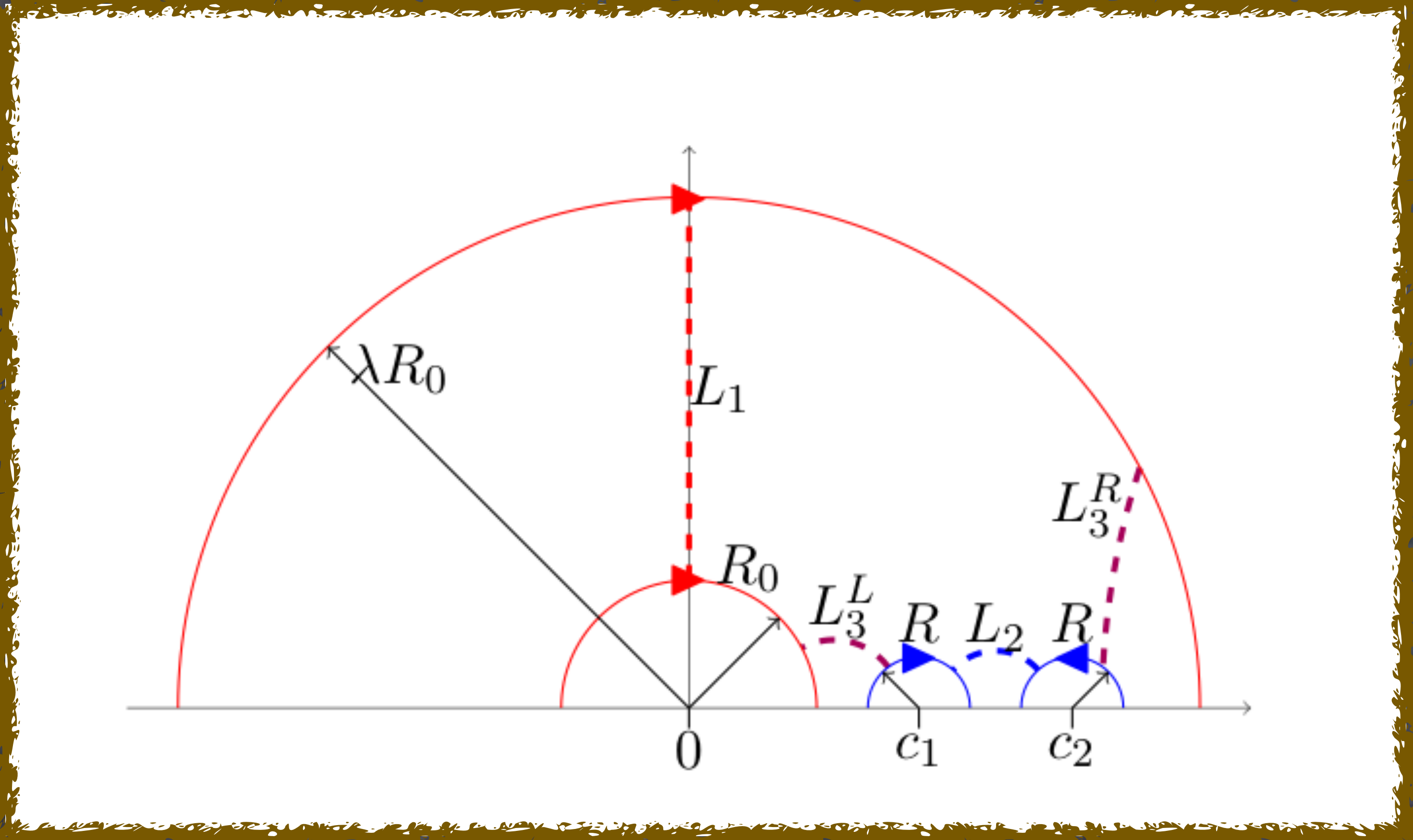
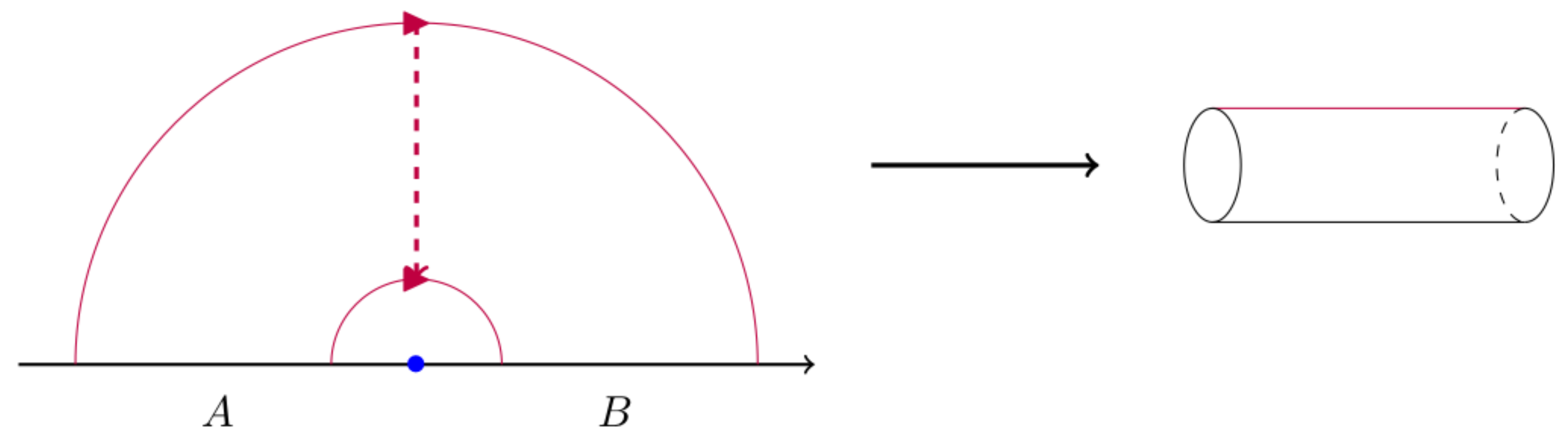
Multiboundary Wormholes (MbW) in AdS

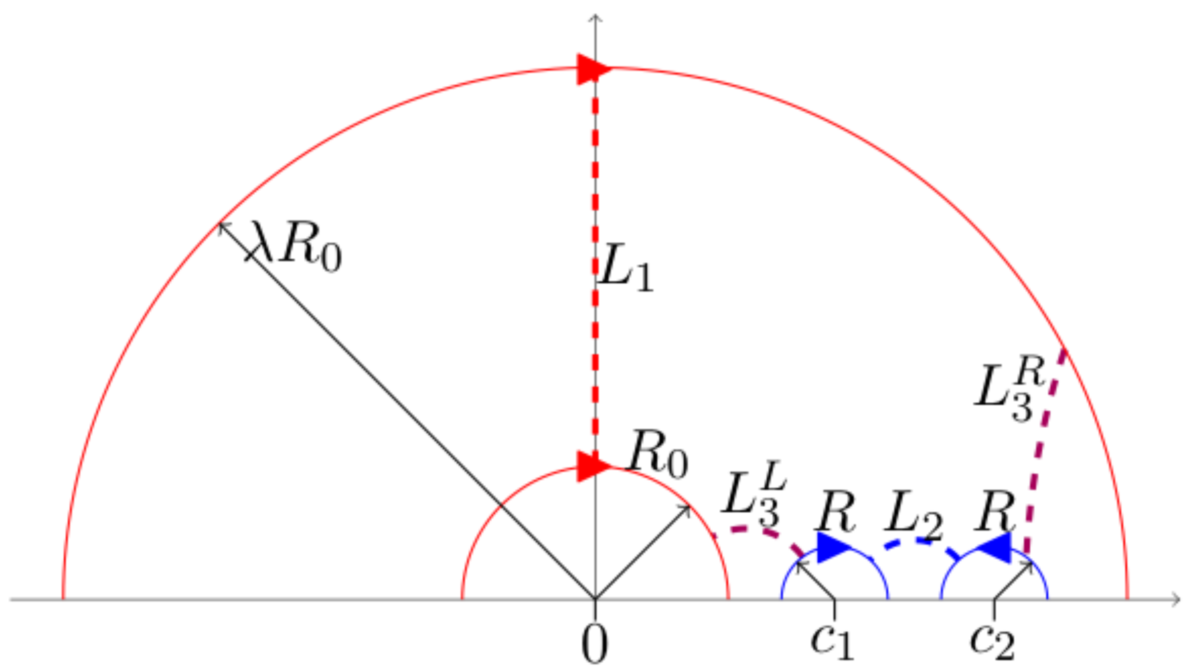
- Empty AdS_3 as a solution of Einstein's equation with constant negative curvature has maximal amount of symmetry.
- All other variants of solutions in AdS_3 can be realised by removing symmetries through quotienting or orbifolding the empty AdS_3.
- MbW geometries in AdS_3 are geometries where independent CFTs live at the boundaries of the exits and are connected by the wormhole geometry.
- Removing semicircles from a time-slice of empty AdS_3 and identifying the geodesic boundary of the removed semicircles give rise to new boundaries. (Also a way to realise BTZ from empty AdS_3)


A
B

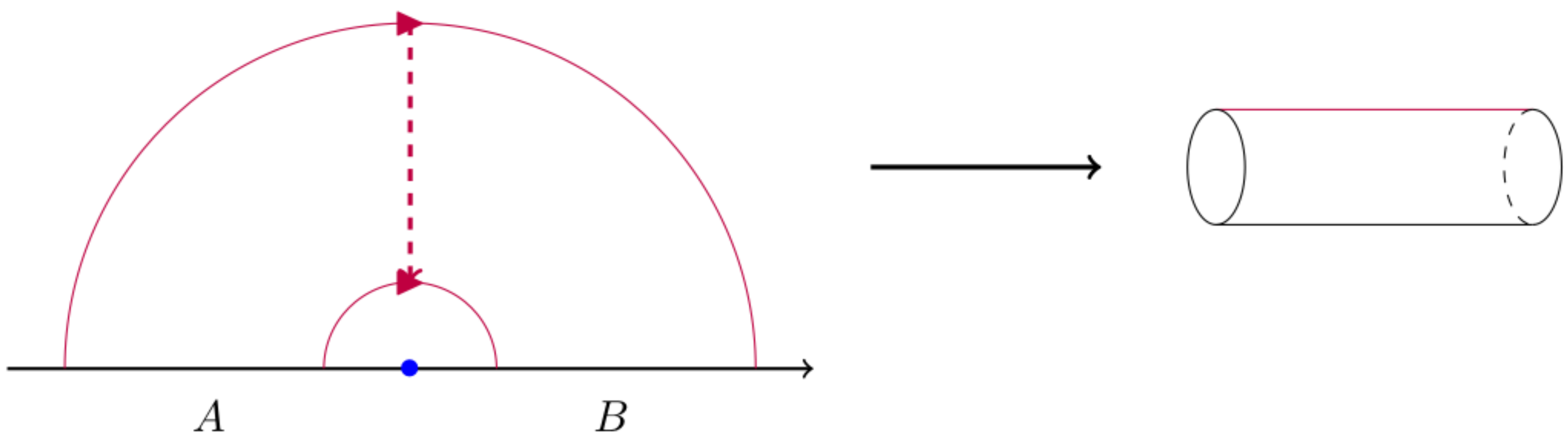

A
B

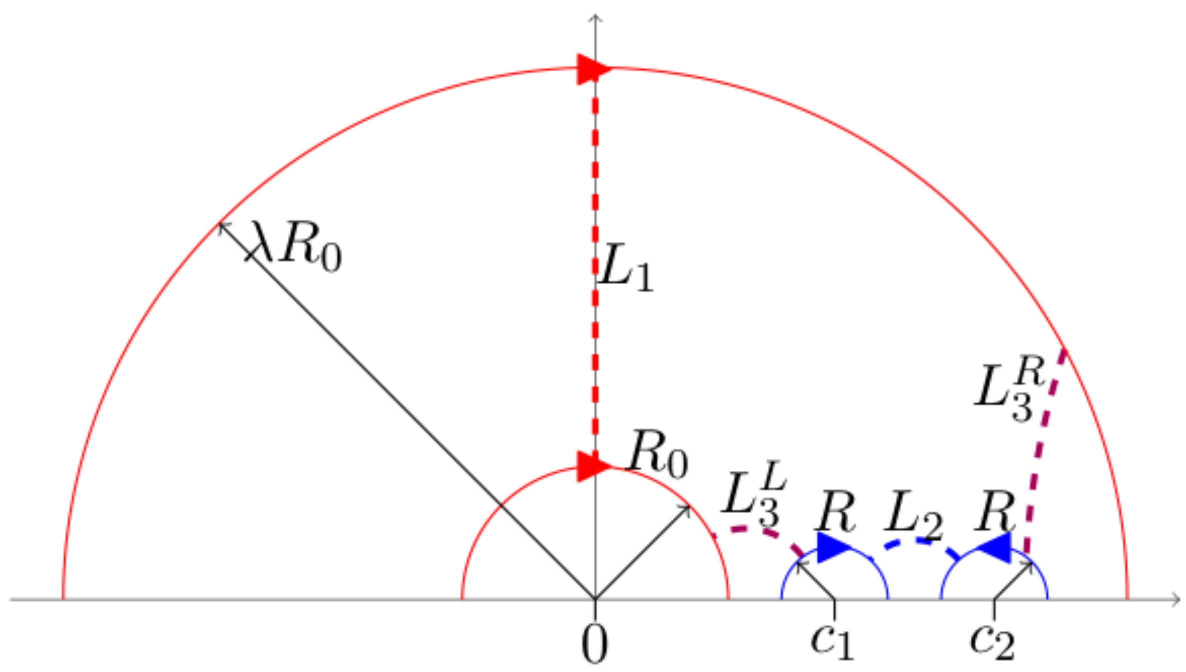

λR_0
L_1
L_3^R
R_0
L_3^L
R
L_2
R
0
c_1
c_2

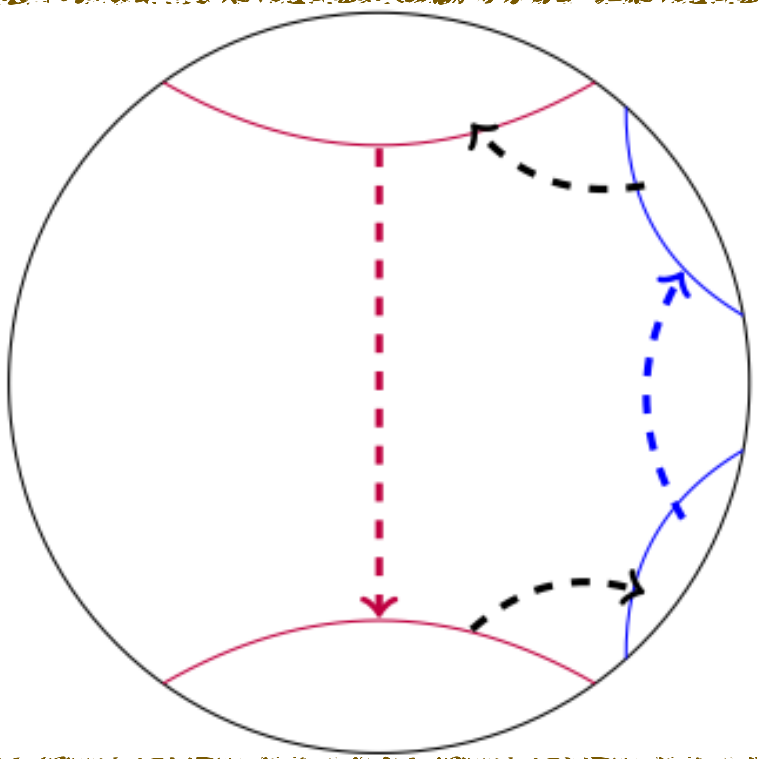




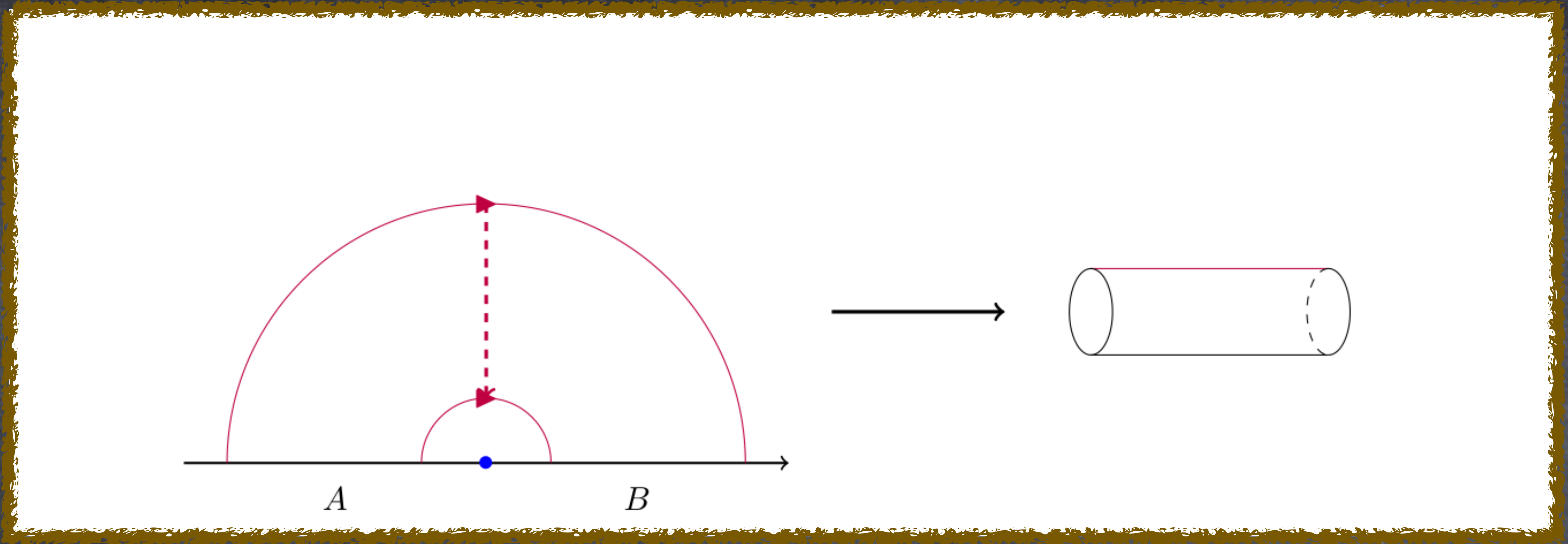
Poincaré UHP

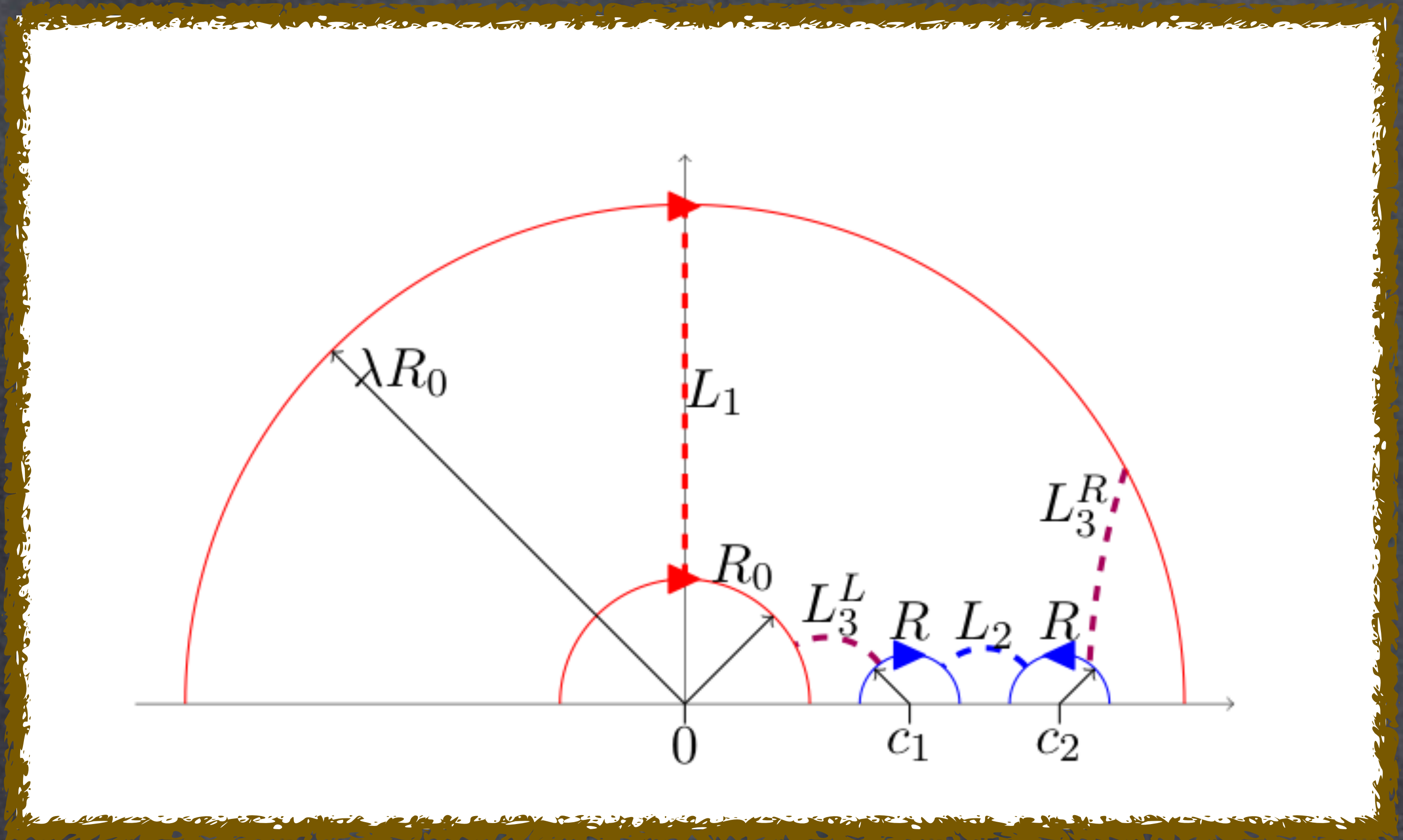


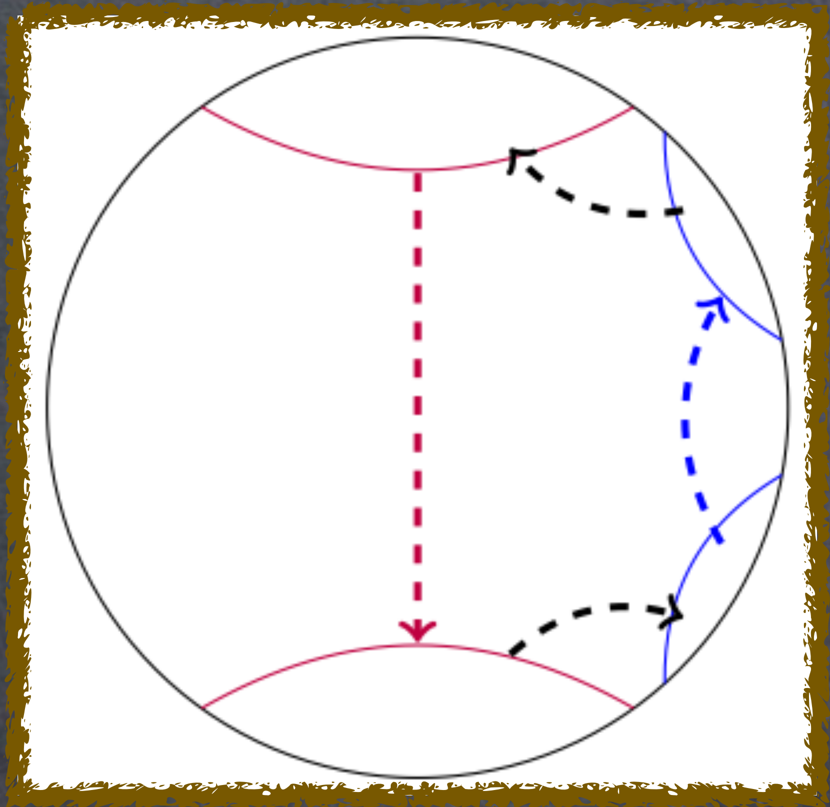




Poincaré UHP







Poincaré UHP

Poincaré disk

- Length of all the throat horizons can be tuned independently .
- Explicitly shown for Lorentzian three boundary construction.
- Genus can be understood as identifying two semicircles from the opposite sides of the smaller concentric semicircle in the Poincaré UHP.
- A general (n,g) wormhole construction possible using these machineries.(Ref. Caceres, Kundu, Patra, Sashi, JHEP (01) 2020).

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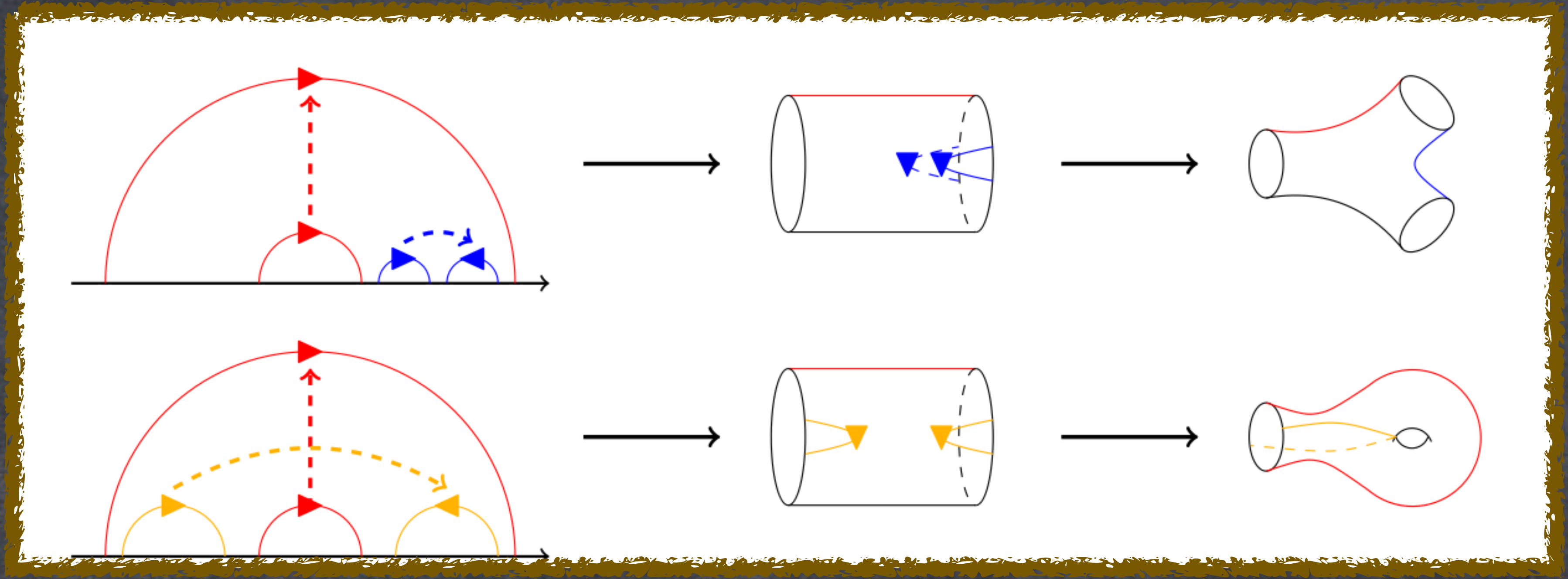

MbW model of Island

- Consider a MbW topology changing with time . # of exits (n) ~ t.
- All of these are assumed to have a larger length compared to the AdS radius and one of them to have a length much longer than the rest.
- Topology change at each unit of time and each snapshot is a valid time-reflection symmetric scenario (/ solution of Einstein's equation individually, not dynamically).
- Start from a single pure state black hole (bigger exit, keeps shrinking).

- Initial HRT choice $\rightarrow n\ell$. Later choice L_0' (shrinking BH throat horizon).
- L_0' Shrinks in what rate with n ? $\rightarrow$Information Conservation $L \sim \sqrt{E}$ for $AdS_3 \rightarrow L_0' = \sqrt{L_0^2 - n\ell^2}$.
- Before Page time, HRT choice length is $n\ell$, afterwards L_0'.

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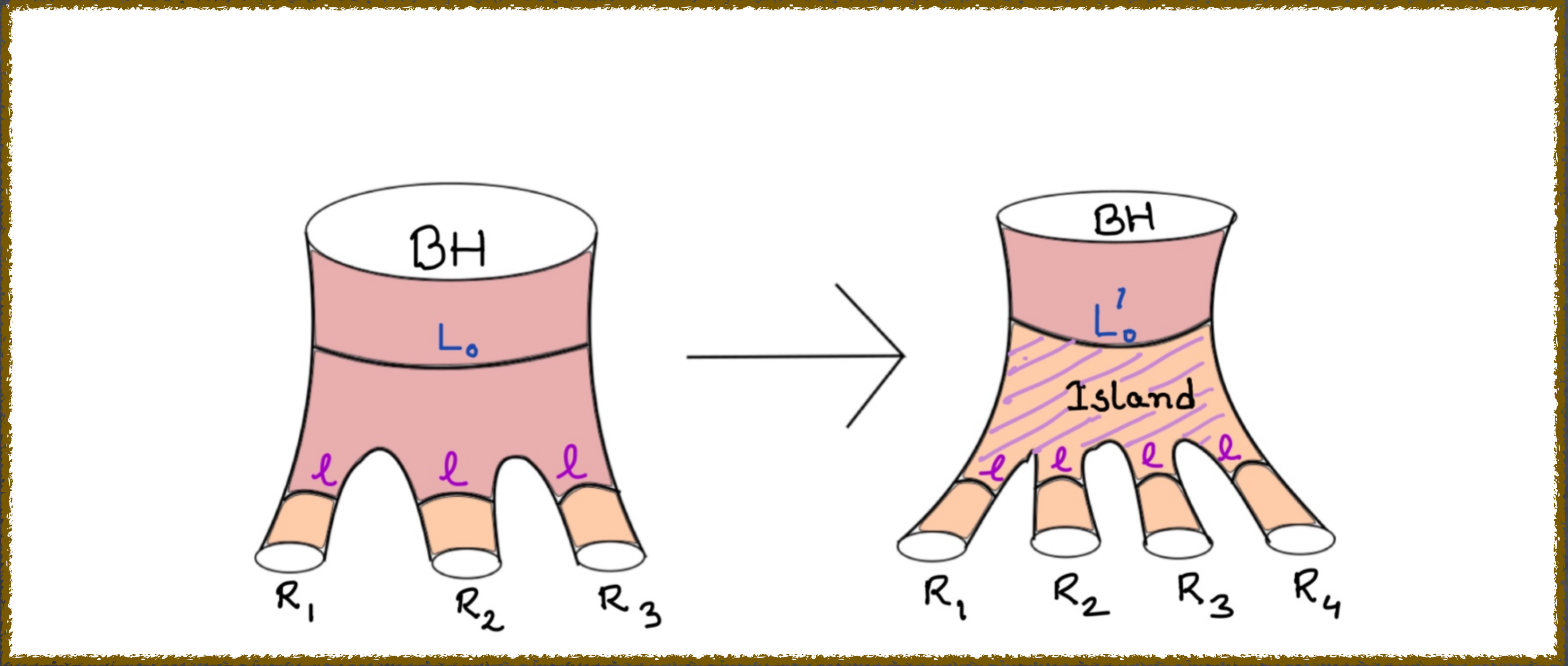

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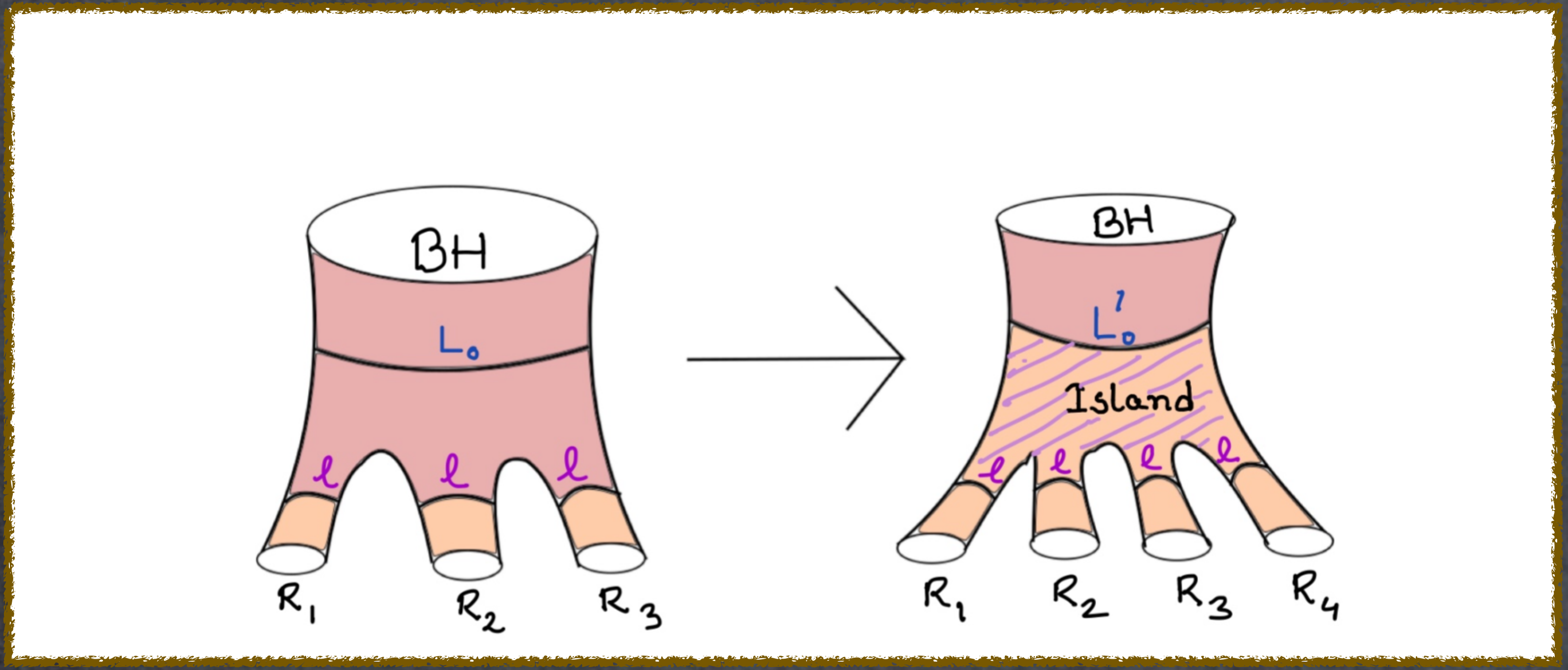
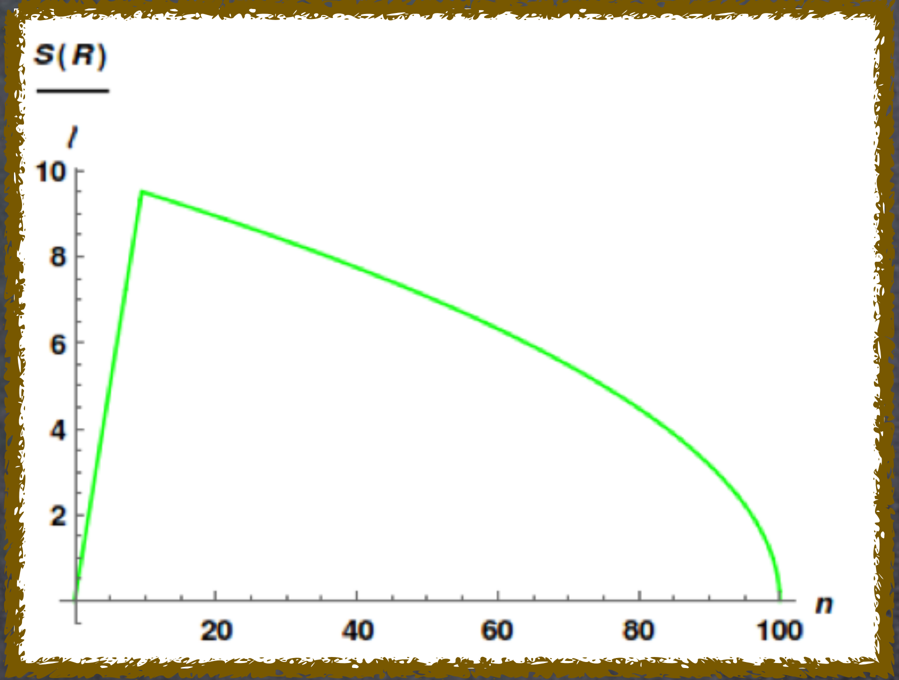
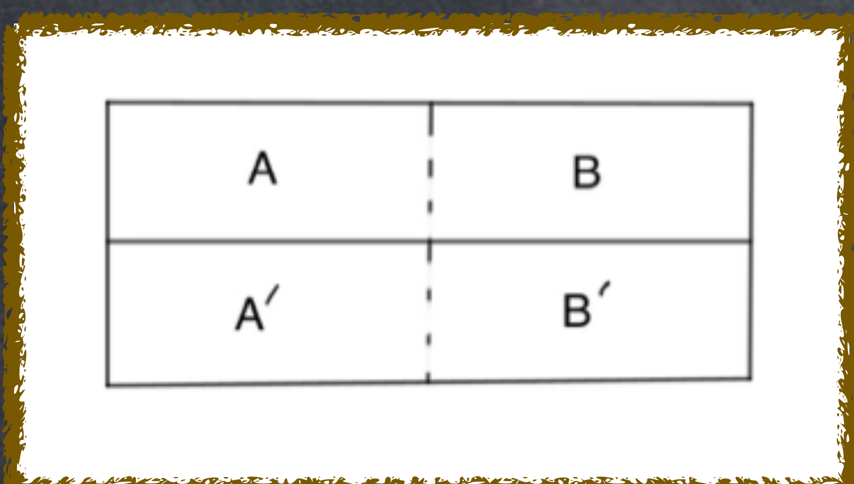




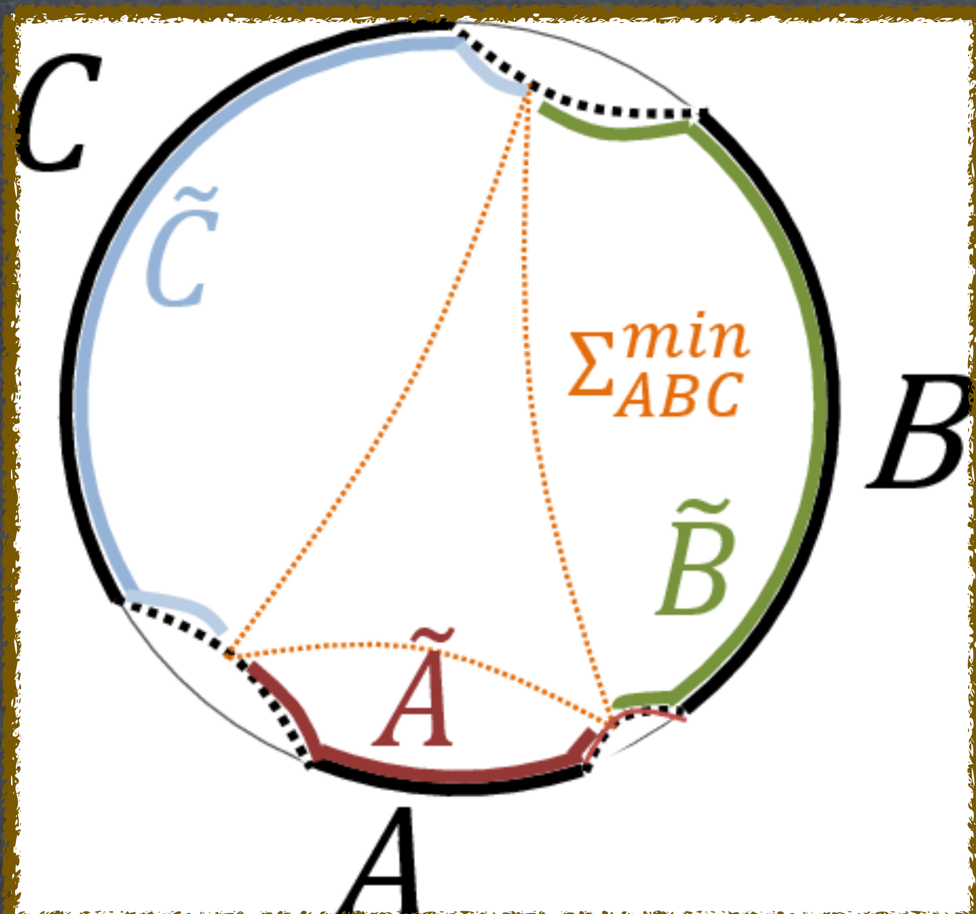
Holographic EoP

- Measure of entanglement for a given mixed state between parts of it (the compliment is unknown → all possible purification).
- Assume the mixed state is denoted by AB. Then EoP measures entanglement between A and B (Von Neumann entropy can only measure if AB is pure).
- Procedure → $AB + A'B' =$ Pure. Among all possible primed choices , choose one that minimises Von Neumann entropy between AA' and BB'.

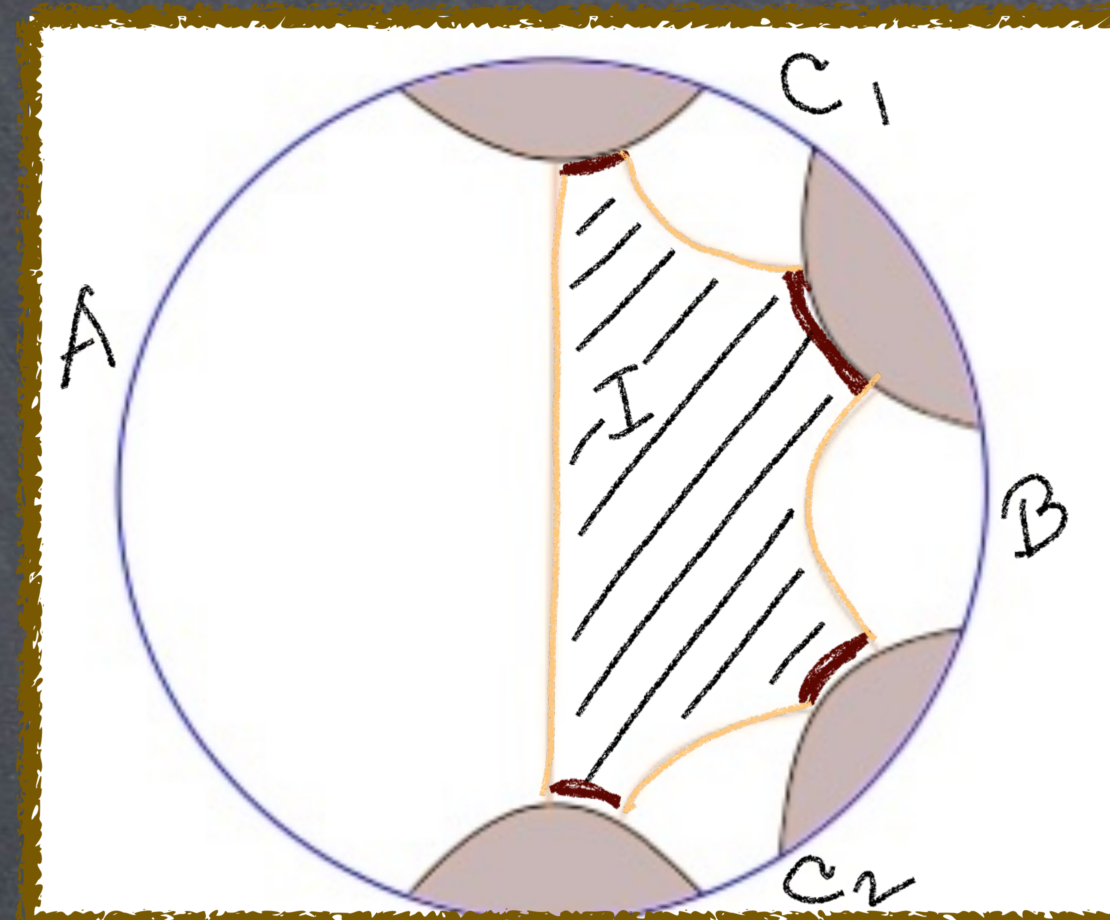


$$\Delta_{n(P)}(\rho_{A_1A_2.....A_n}) = min_{|\psi\rangle_{A_1A_1'A_2A_2'.....A_nA_n'}} \sum_{i=1}^{n} S_{A_iA_i'}.$$

- Holographic dual of EoP → Entanglement Wedge Cross Section (Takayanagi, Umemoto).
- AB is a mixed state, geometric geodesic are divided into $A'B'$ → Geometric Pure state.
- Bulk geodesics are analogue of wormhole body and EWCS is analogue of BTZ throat horizon.
- A and B in the boundary are similar to the two exits of BTZ. Also, similar to the geodesics, the wormhole body joins the mixed states at different exits to yield a pure state (n fold entangled state).
- Extending the analogy to multipartite case ??
- Starting point : Holographic dual of Multipartite EoP (Umemoto) .



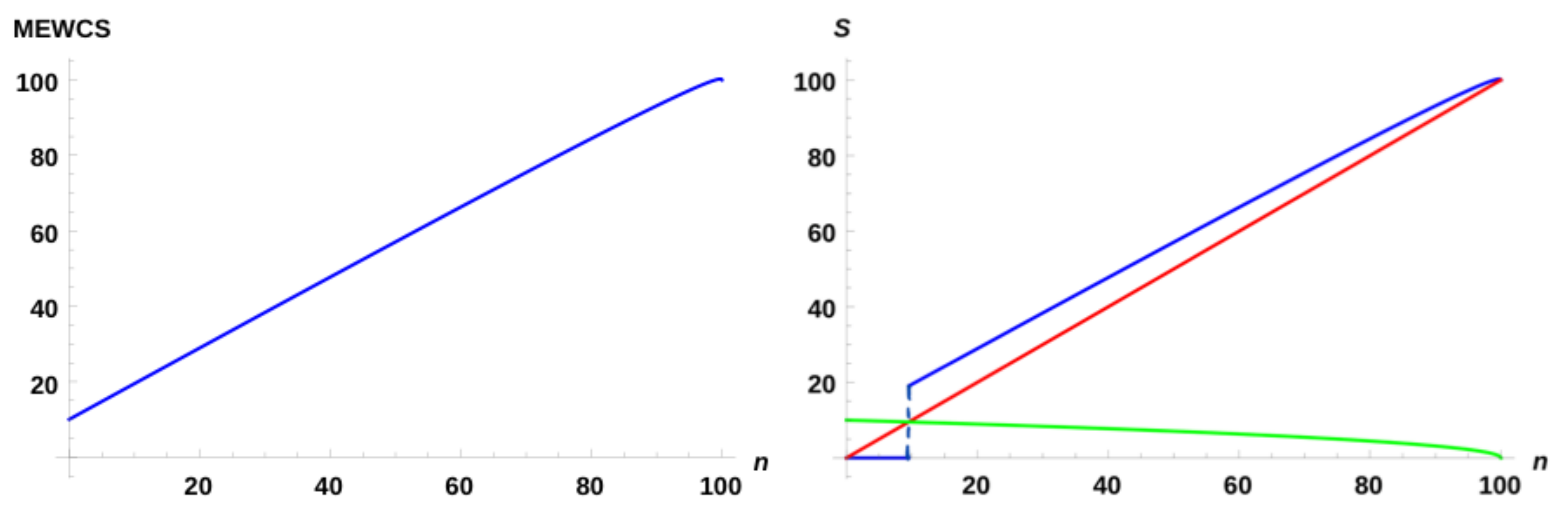
- Tripartite EoP
- The orange dotted lines by themselves close.properties checked.



- Three boundary Wormhole.
- Orange lines do not close by themselves .

Multipartite EoP and Island

- In the most general setting, the island contains regions from identifying geodesics, but the MEoP doesn't .
- In large n (# of smaller exits)limit, they do match.
- Boundary of island ~MEoP (conjecture).
- There are a few subtleties involved if considered so.
- Apply the quantum extremal surface procedure without the bulk Von Neumann entropy term ?
- $$S_{out}[R](new) = \min_{I} \left[\mathop{ext}_{I} \left\{ \frac{A(\partial I)}{4G} + S_{usual}[R \cup I] \right\} \right]$$



MEWCS
100
80
60
40
20
20
40
60
80
100
n
S
100
80
60
40
20
20
40
60
80
100
n

Questions?

Questions ?

- Boundary of the island has both $n\ell$ and L_0' terms in it, keeps growing after Page time. (Contradiction?)
- In the MbW picture, it is crucial to model the radiation as a multipartite state to get island-like situation. Other possibility $\rightarrow$ Take three boundary wormhole with one big (keeps decreasing) and two small (keep increasing) exits.
- For less number of smaller exits, the $n\ell$ term will always be the minimal HRT.
- Should we consider Multipartite EoP ?

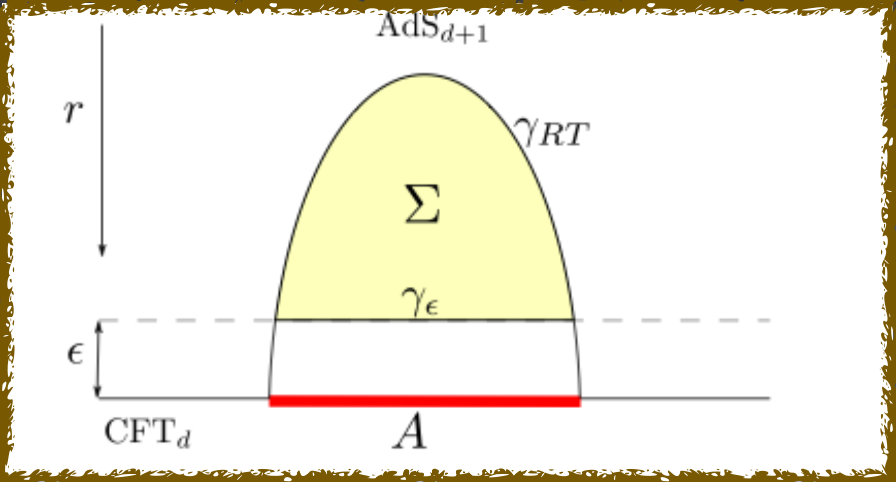
Resolutions

- Model specific problem, ultimately do not consider radiation as multipartite. BH + Radiation = Bipartite.
- Over counting, previous entanglement due to $n\ell$ is purified at later times.
- More on bipartite-ness : The smaller exits ($\ell << L_0$) means all of them are not entangled with each other in this model, only entangle with the BH state. (Balasubramanian, Hayden, Maloney, Marolf, Ross. 2014)
- The HRT surfaces at later times are the right choice, naive application of QES formula (even with just the bulk minimal surface term) leads to over counting.

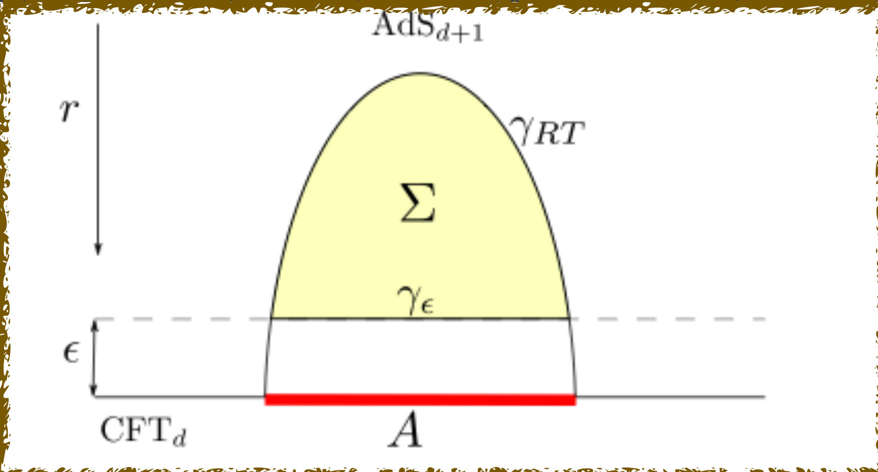
Volume dual to HRTs

- Alishahiha (2015) proposed the volume below the HRTs to be dual to mixed state Complexity.
- In AdS_3, the volume change, due to same choices of boundary region, but different HRTs, is purely topological. (Erdmenger te al. 2018)
- The volume integral can be written in a form that only knows about the geodesic curvature and the the Euler characteristics χ of the bulk region depending upon the choice of HRT.
- A smooth transition in the entanglement entropy plot depending upon the choice of HRT corresponds to a finite 2π jump in the subregion complexity plot due to the change of the Euler characteristic.

(2010.04314 AB, AC, SM, CN, SR)



$$\mathcal{C}(A) = -\frac{1}{2}\int_\Sigma R\, d\sigma = \int_{\partial\Sigma} k_g ds - 2\pi\chi(\Sigma)\ ,$$



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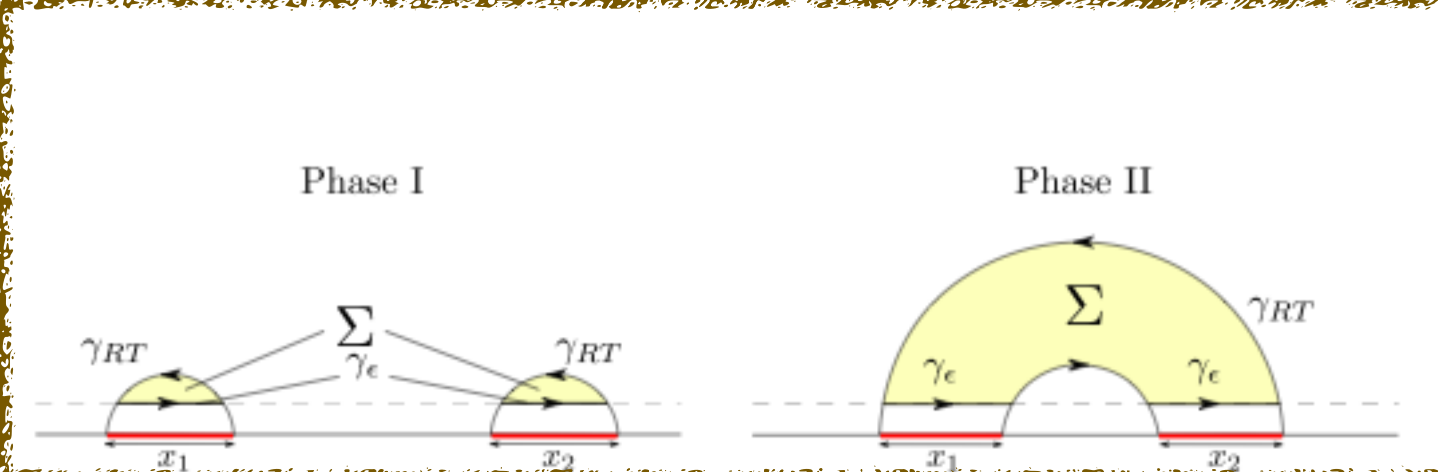

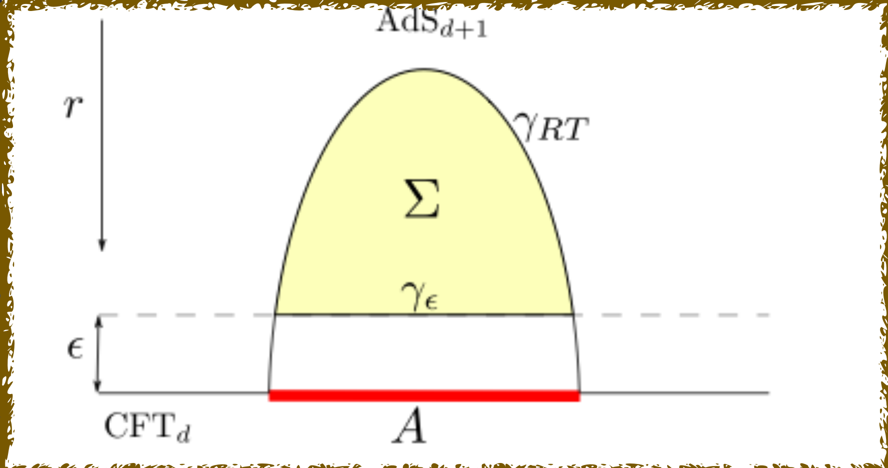

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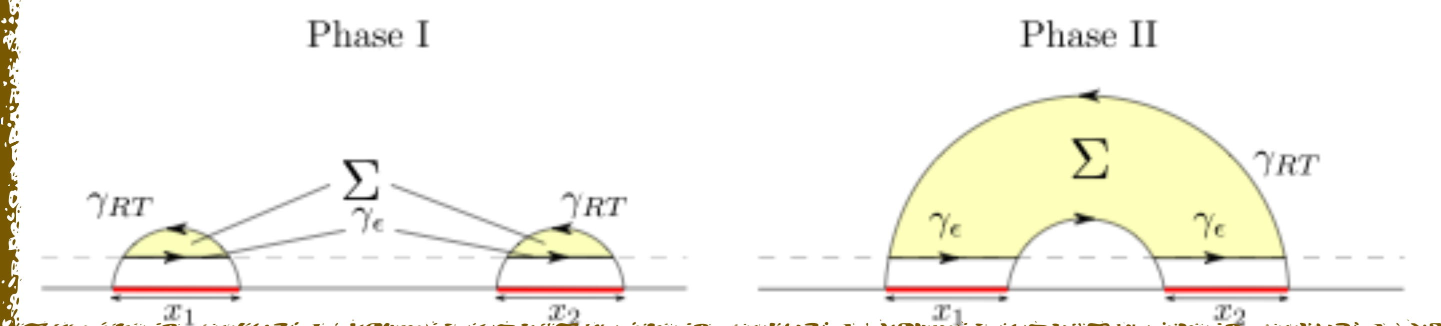

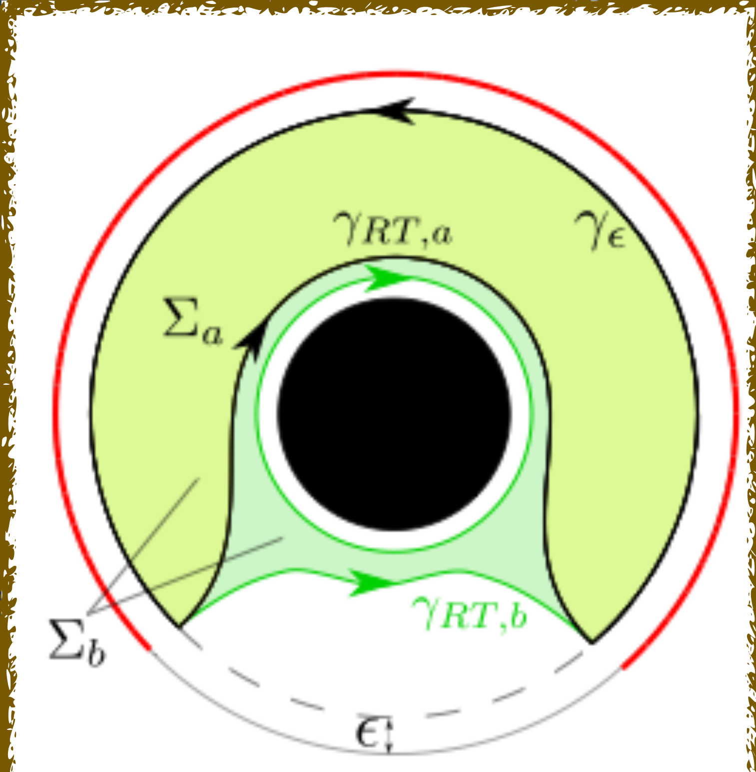

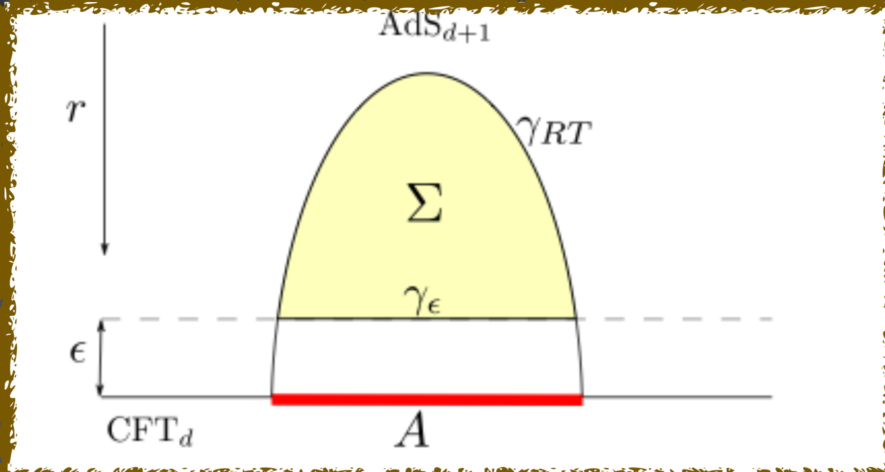

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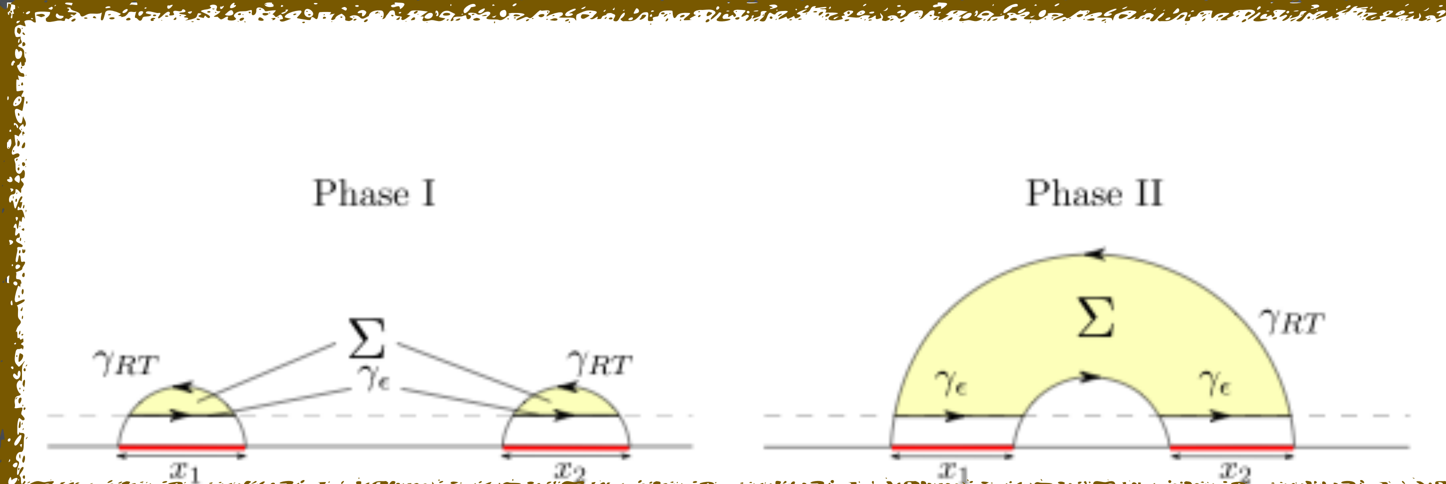
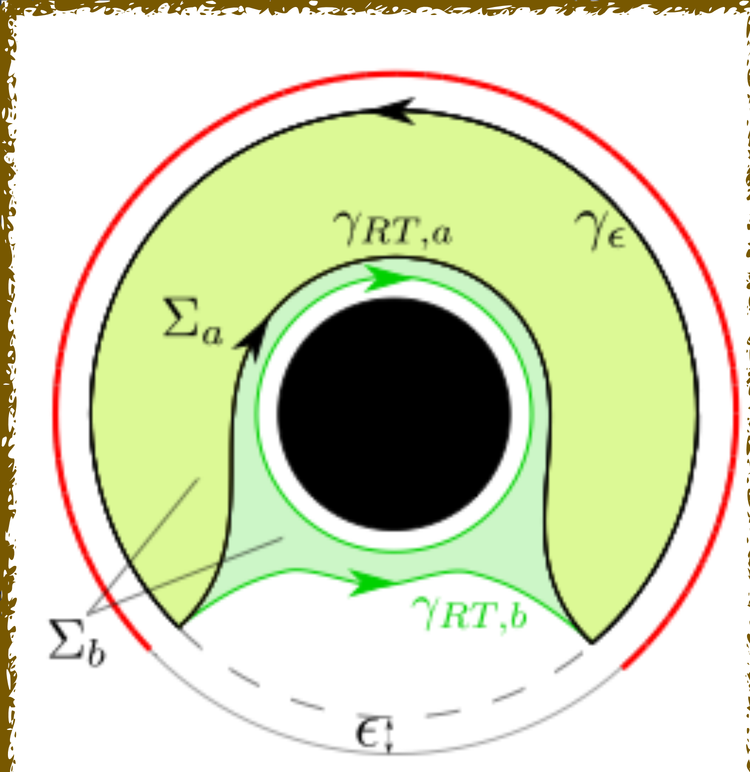
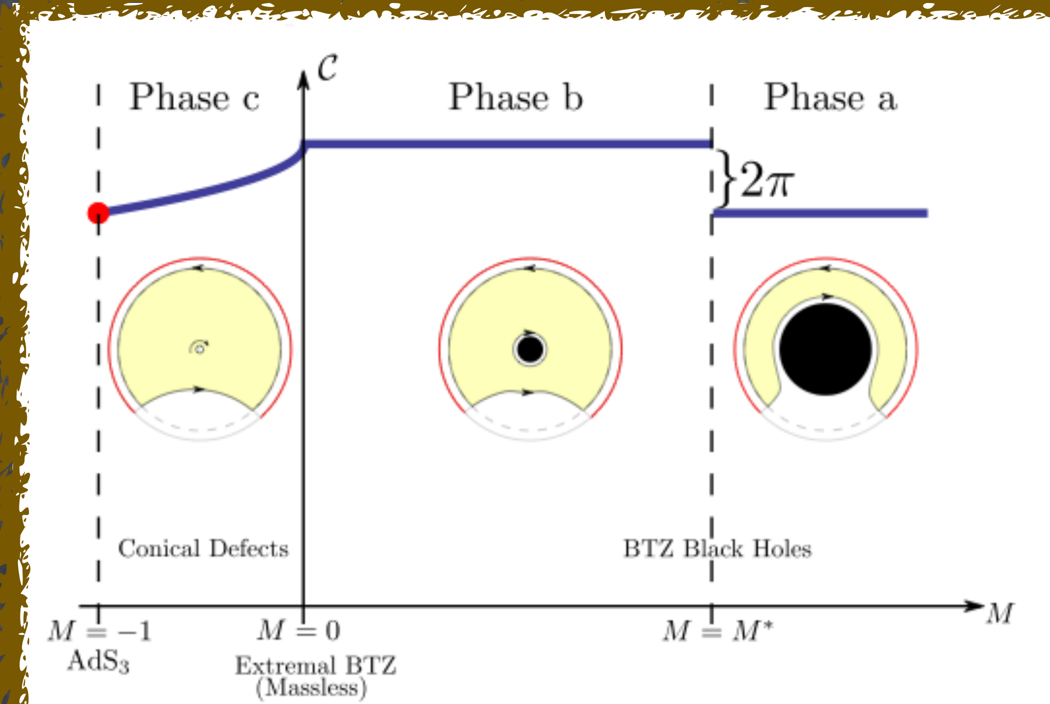
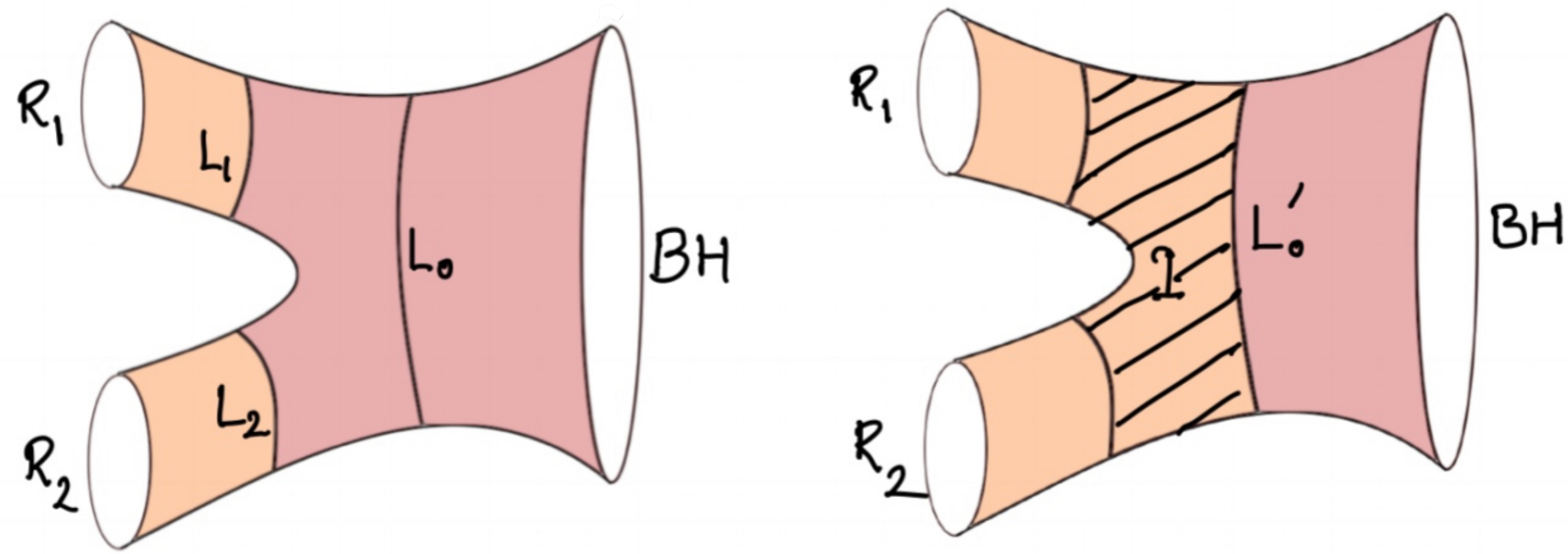




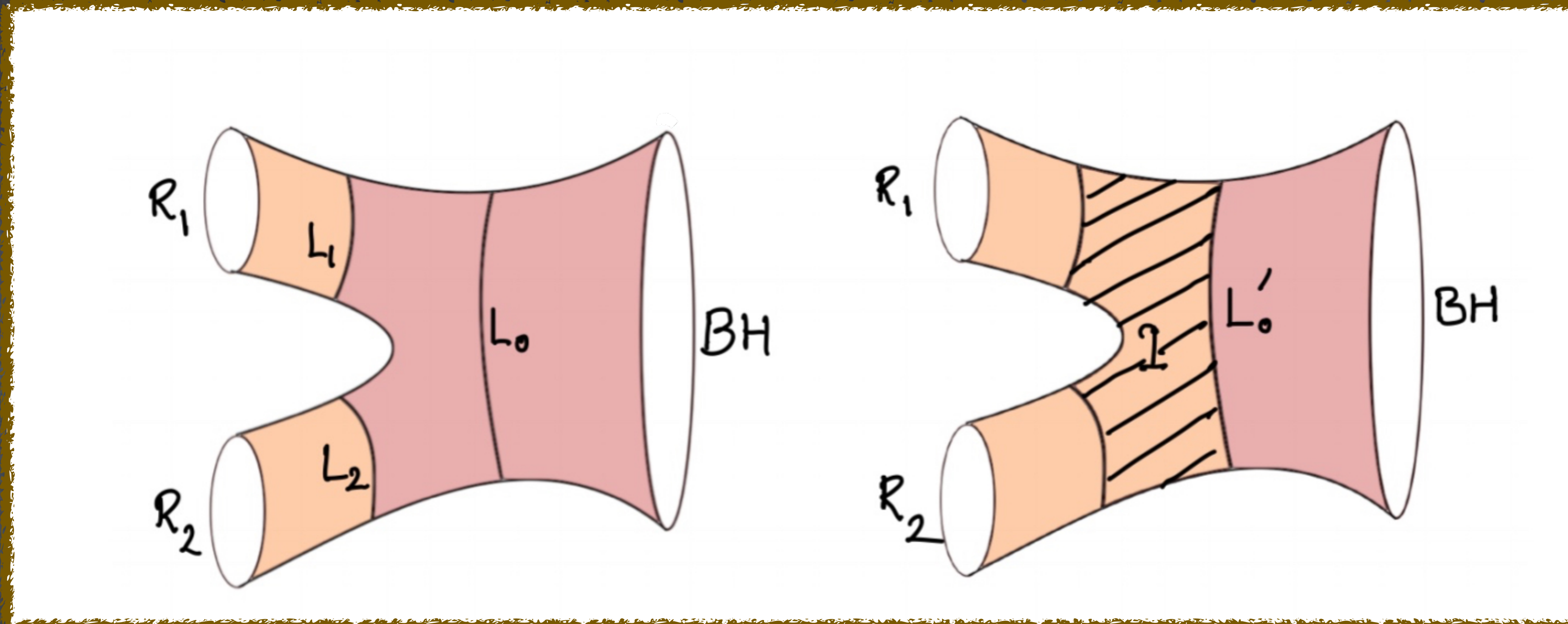


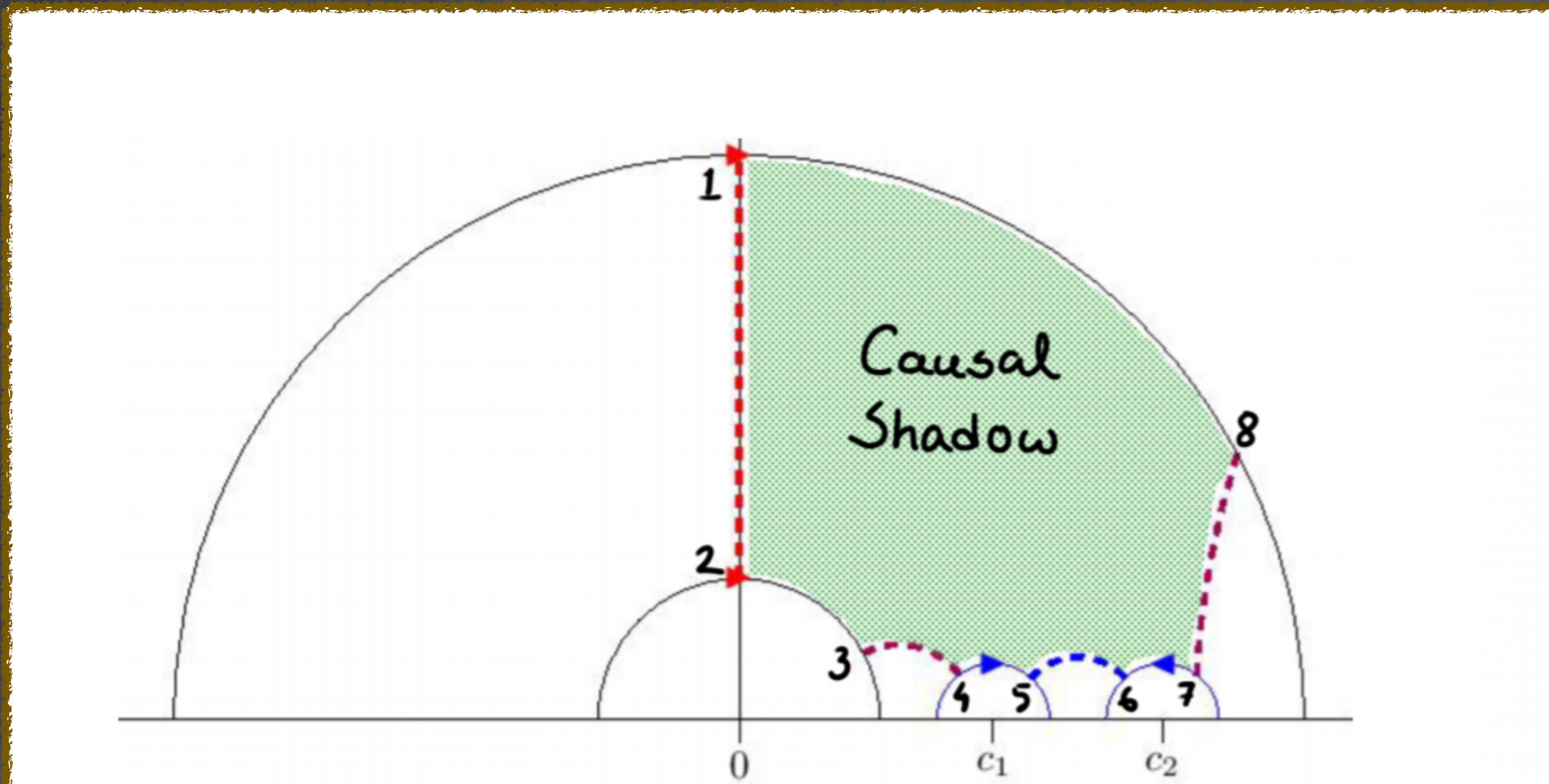
Volume dual to MBW model

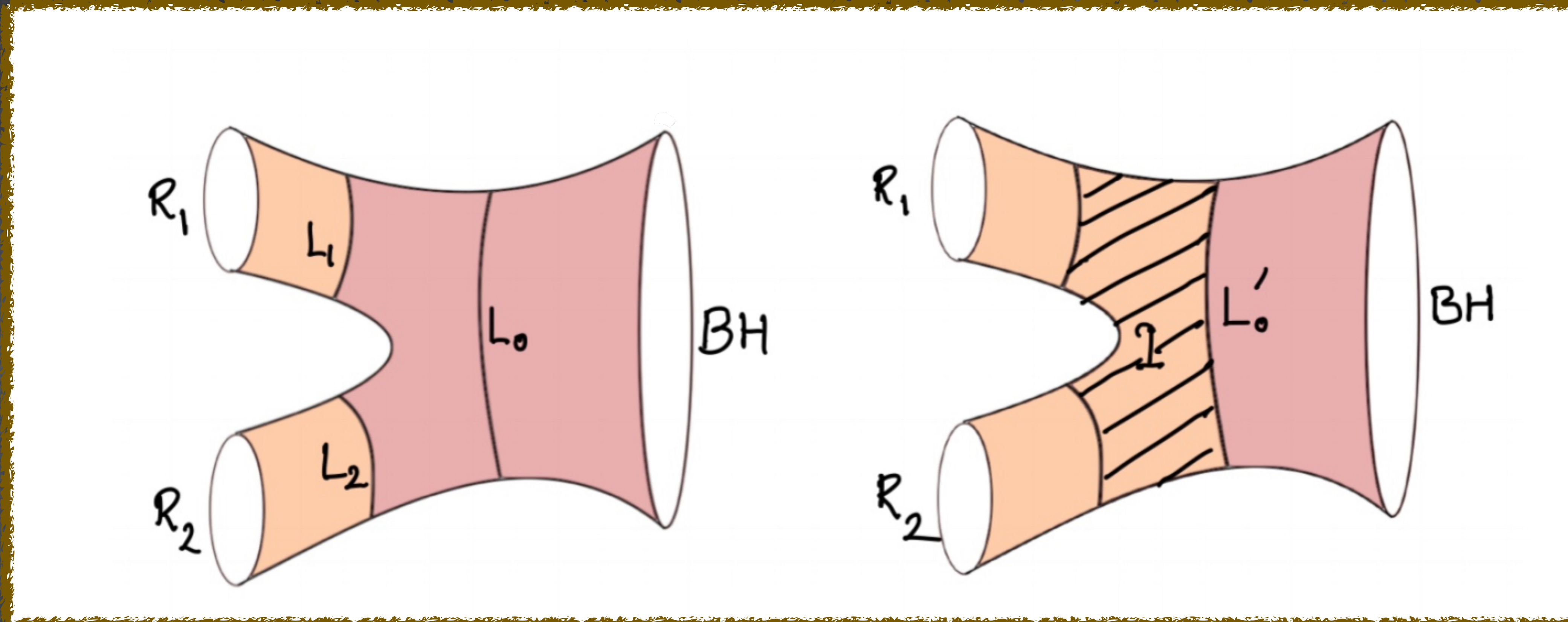
- We deal with the MbW model by using lessons from the topological complexity results.
- This is a Multiboundary wormhole, not a simple BH. But lessons from pure AdS can be used.
- The Euler characteristics and Gauss-Bonnet theorem again plays a role.
- In this case, we study two models, one three boundary and one n boundary wormhole.
- In case of the three boundary model, there is no topology change. The black hole exit shrinks and two radiation exits increase in size throughout the evaporation.

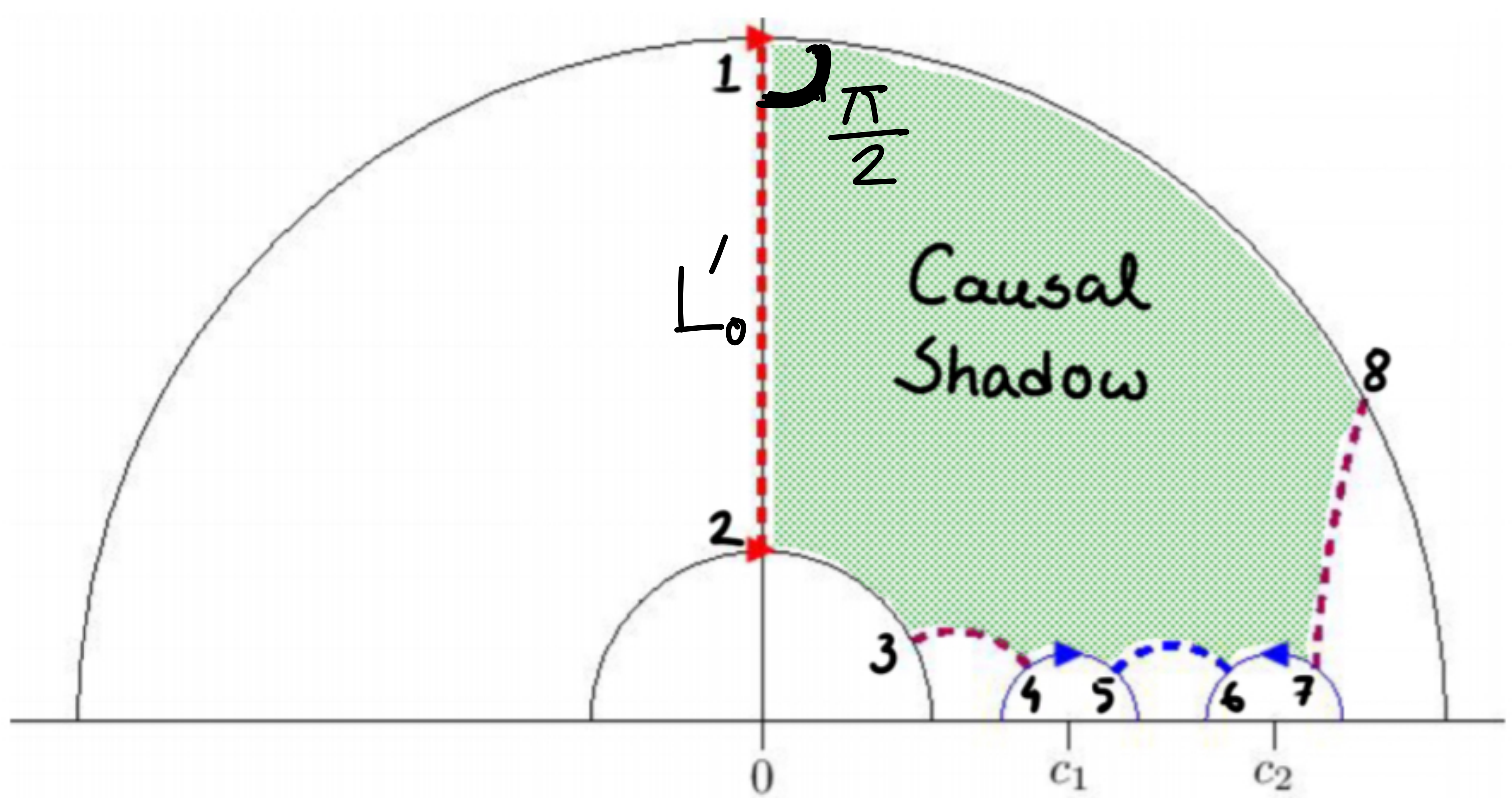


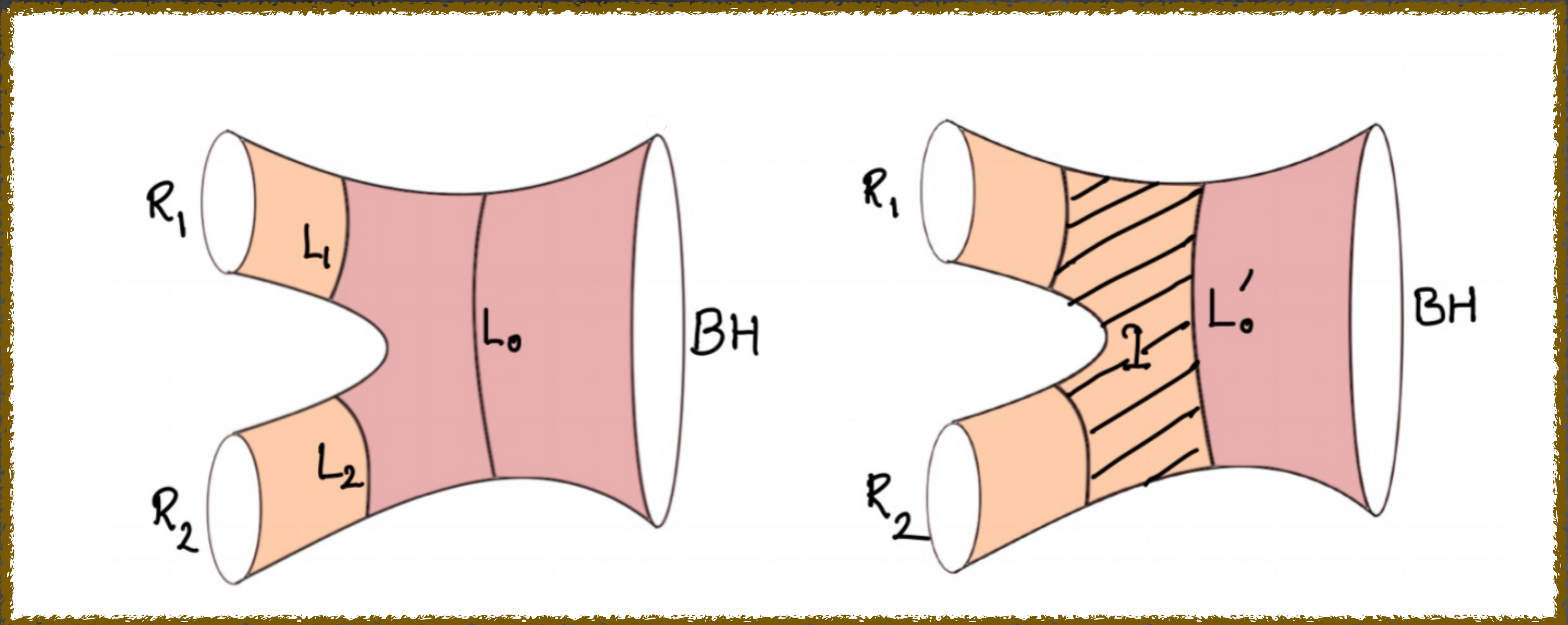
R_1
L_1
L_0
BH
R_2
L_2
R_1
L_0'
BH
R_2


R1
R2
L1
L2
L0
BH
R1
R2
L0'
BH

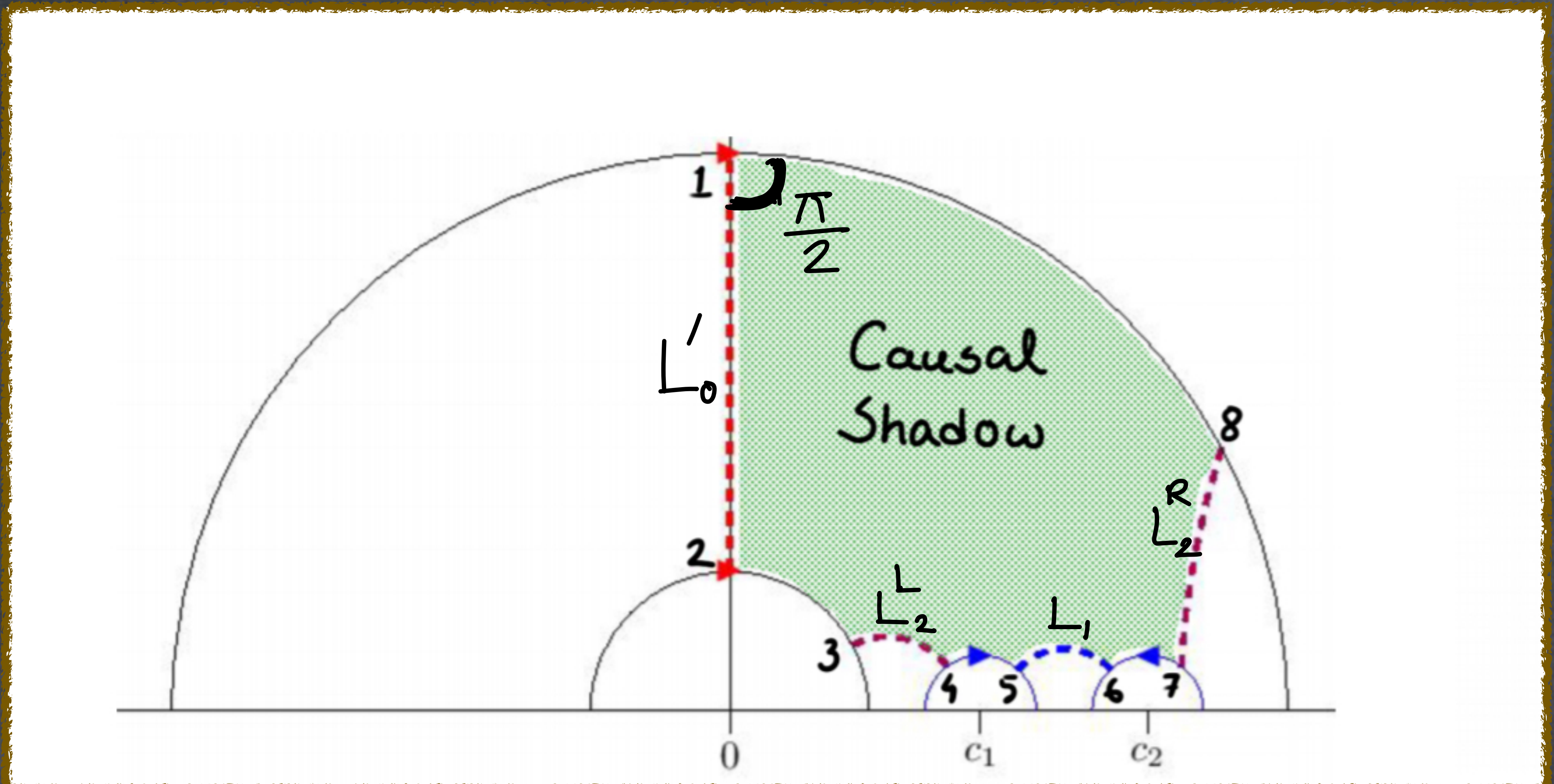

Causal Shadow
1
2
3
4
5
6
7
8
0
c_1
c_2


R1
L1
L0
BH
R2
L2
R1
I
L0'
BH
R2

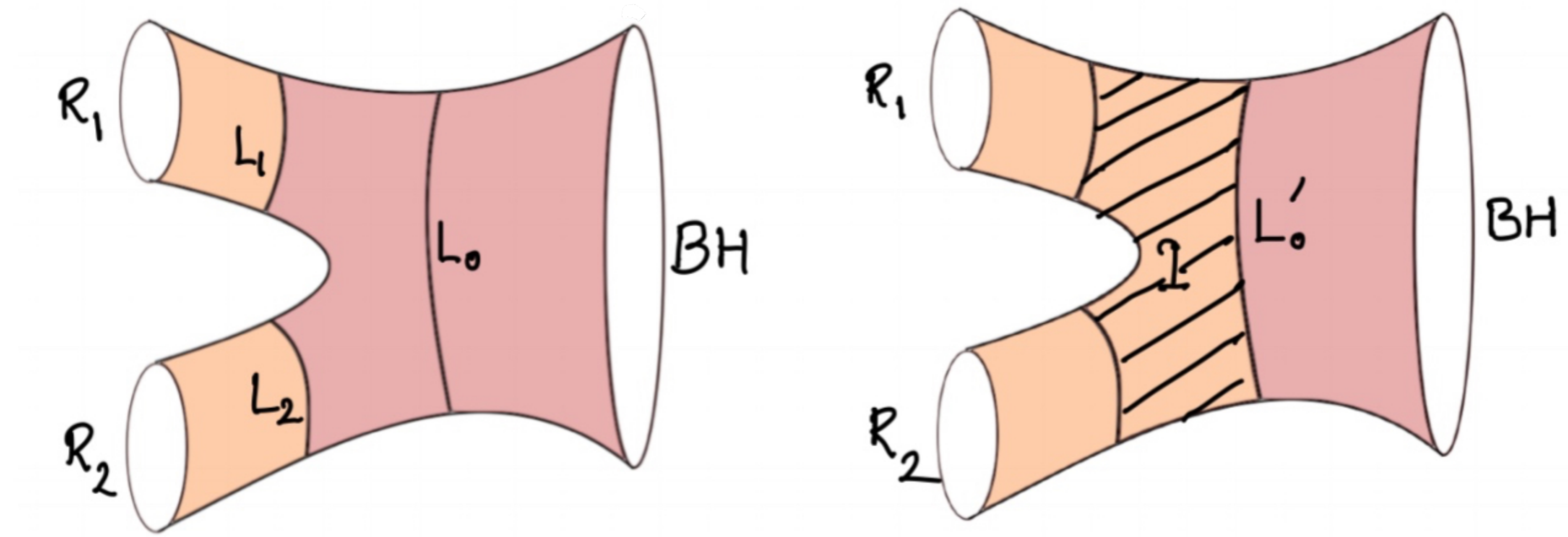

1
π/2
L0'
Causal
Shadow
8
2
3
4
5
6
7
0
c1
c2



R_1
L_1
L_0
BH
L_2
R_2
R_1
L_0'
BH
R_2

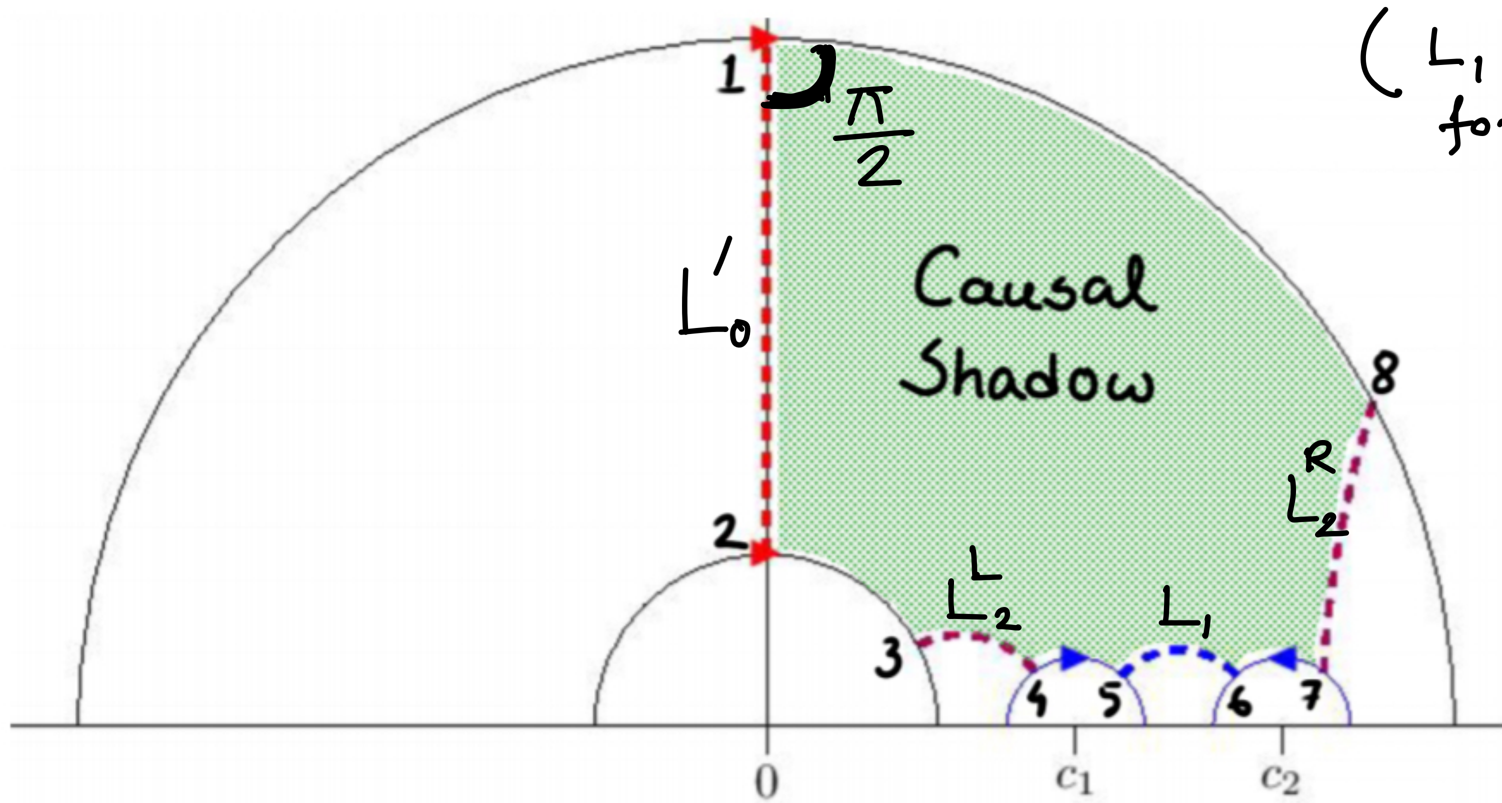


1
$\frac{\pi}{2}$
L_0'
Causal Shadow
8
L_2^R
2
L_2^L
L_1
3
4
5
6
7
0
c_1
c_2



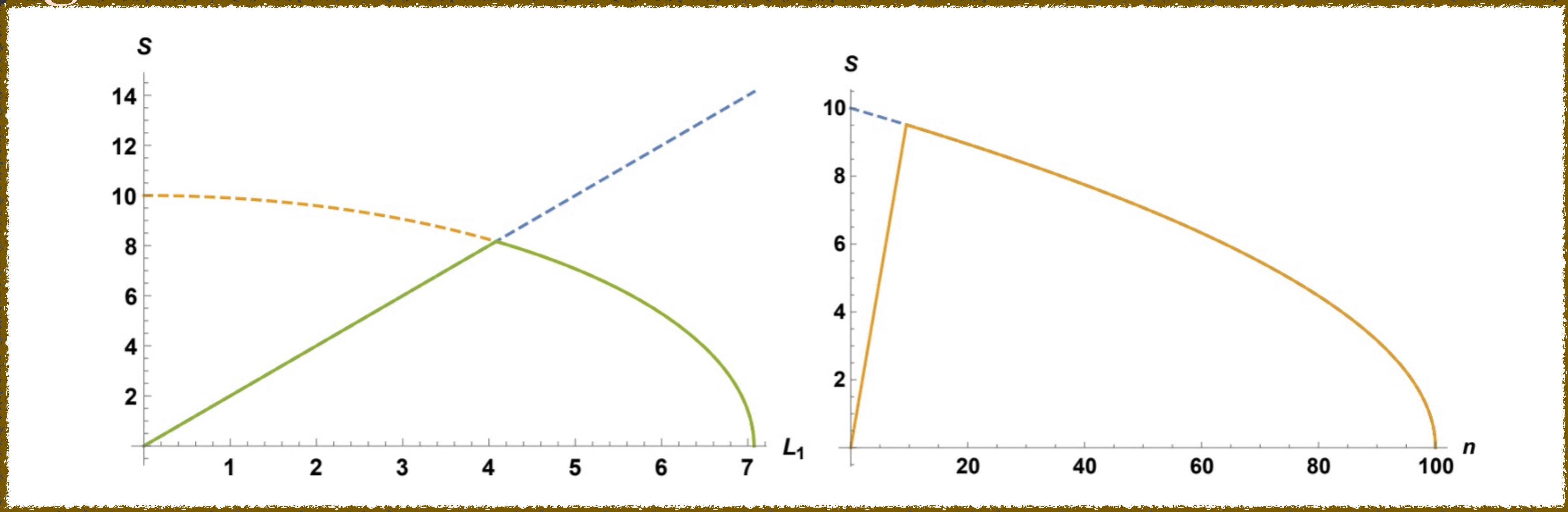
$$L_0' = \sqrt{L_0^2 - 2L_1^2}$$

$(L_1 = L_2)$ for all t.

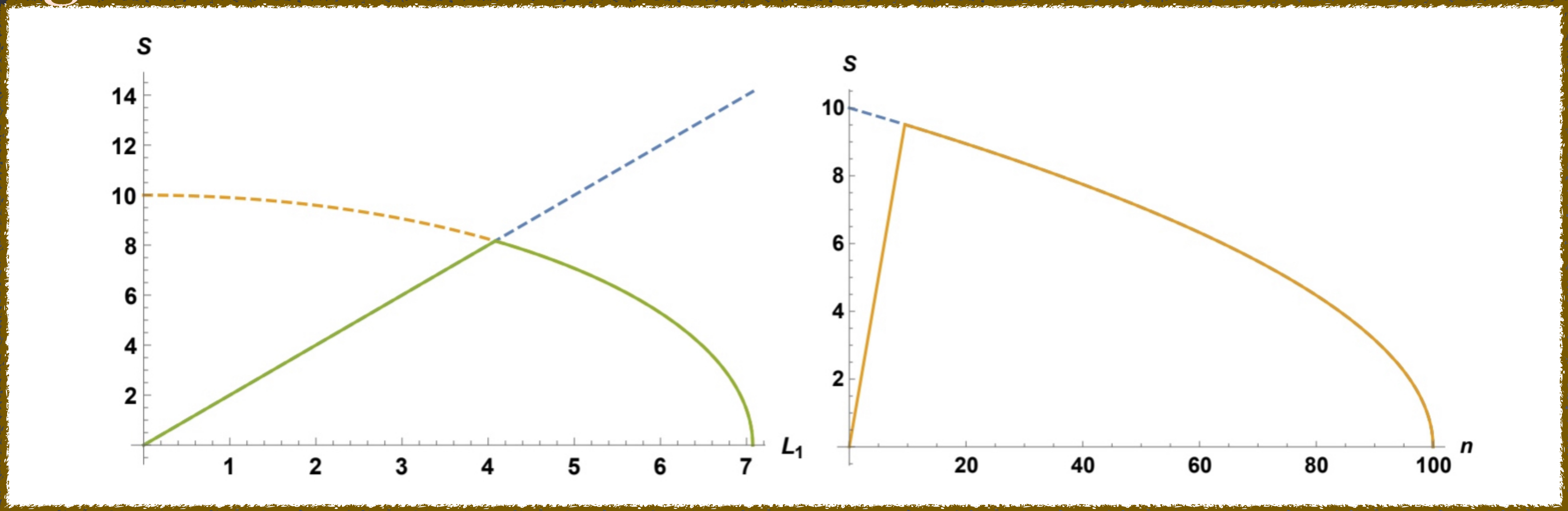


Page Curves

Page Curves

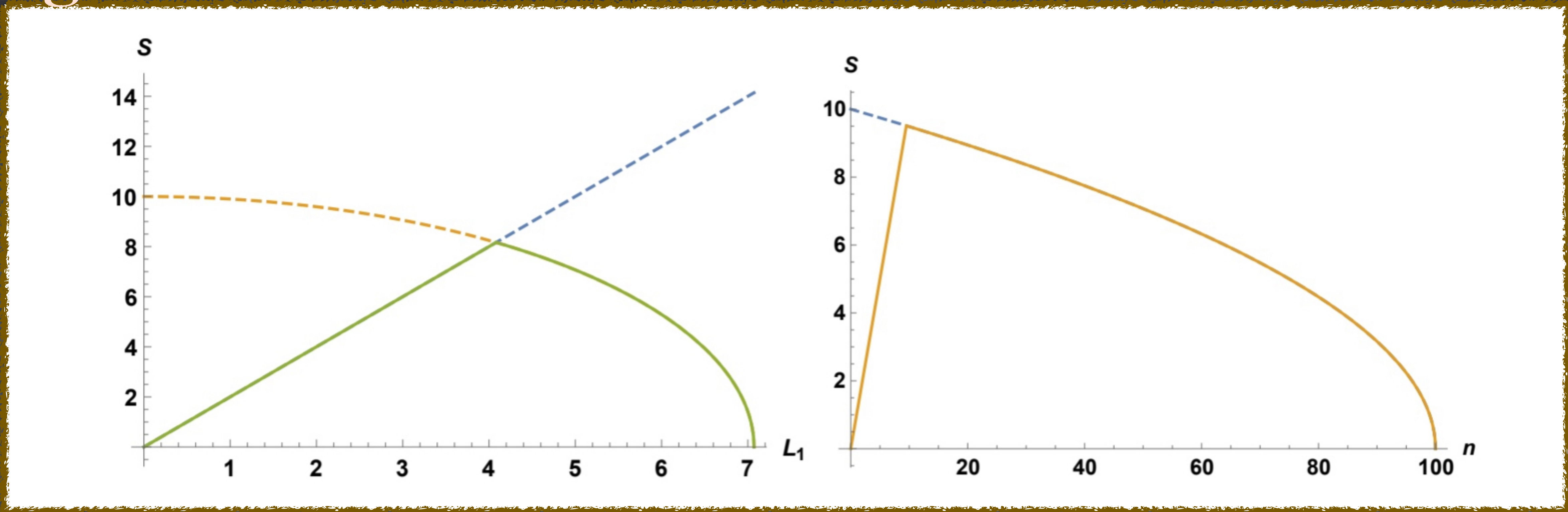


Page Curves



3-bdy

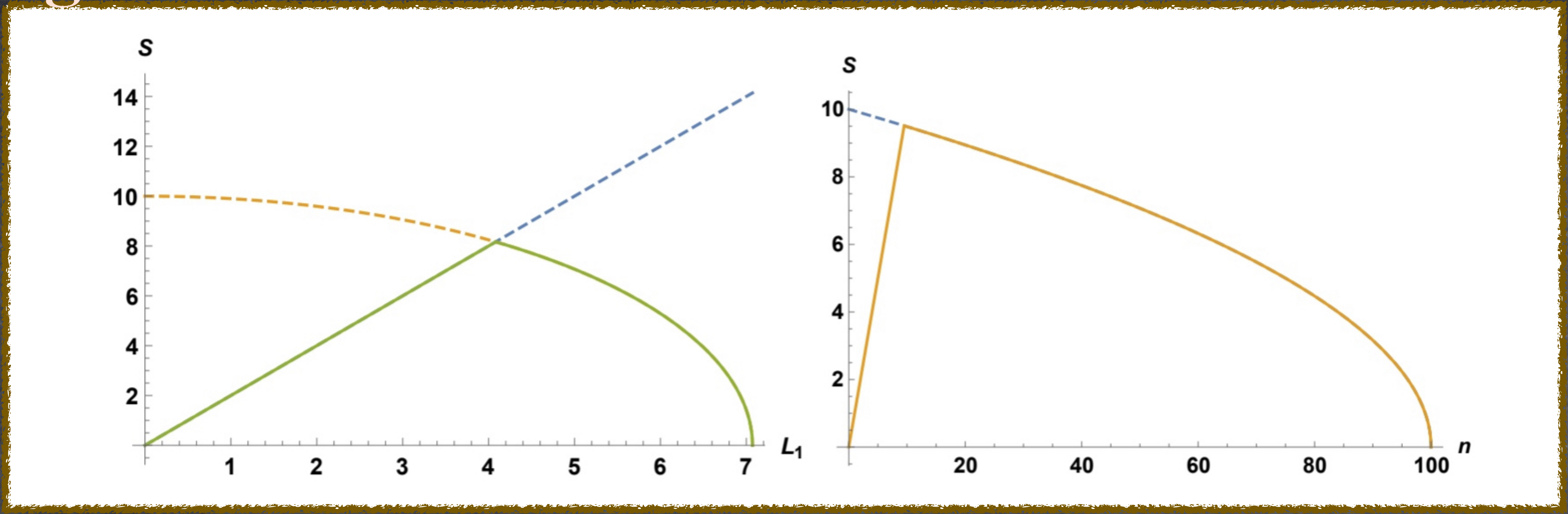
Page Curves



3-bdy

n-bdy

Page Curves

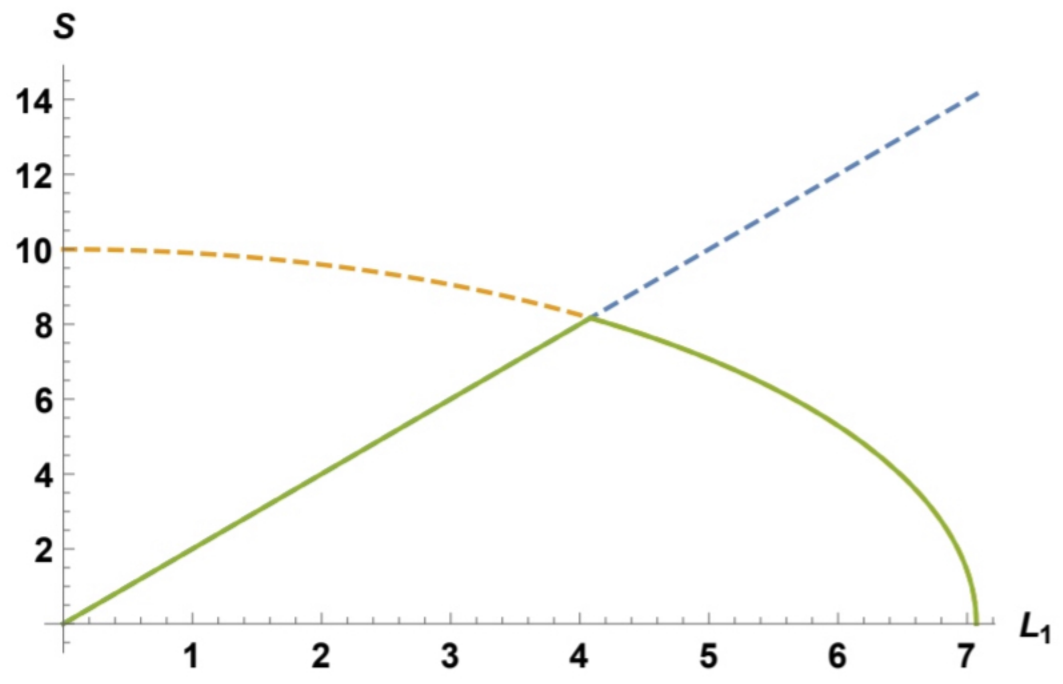


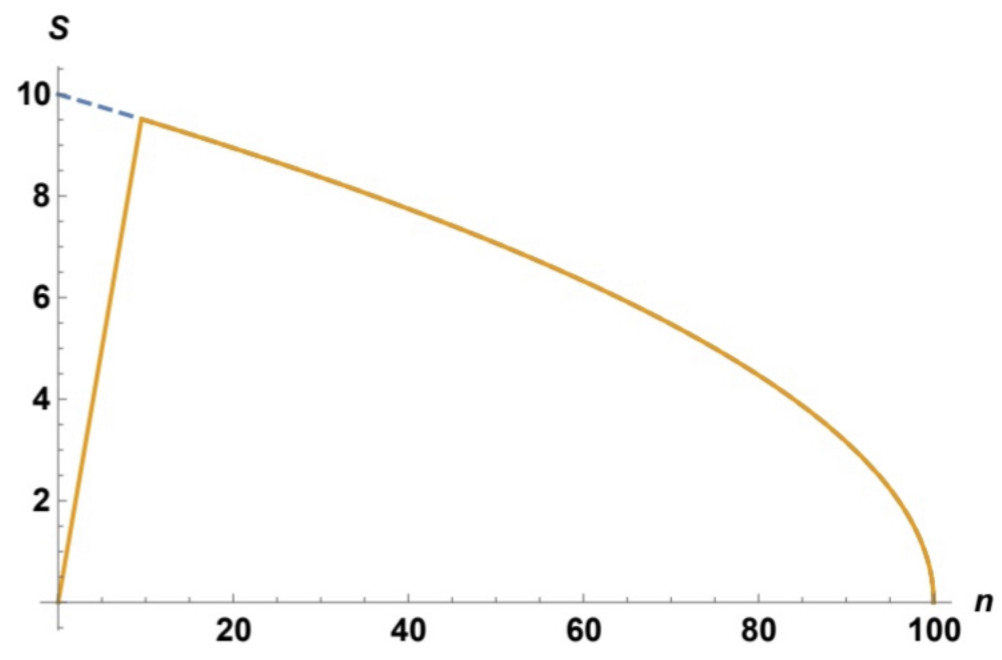
3 - bdy

n - bdy

Volume plots : 3 bdy

Page Curves

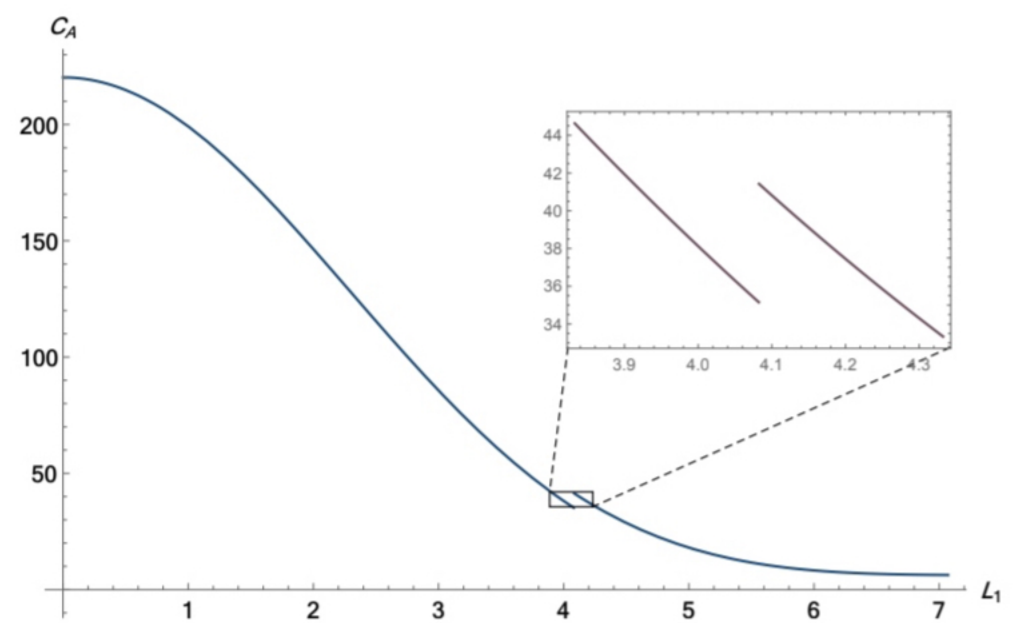


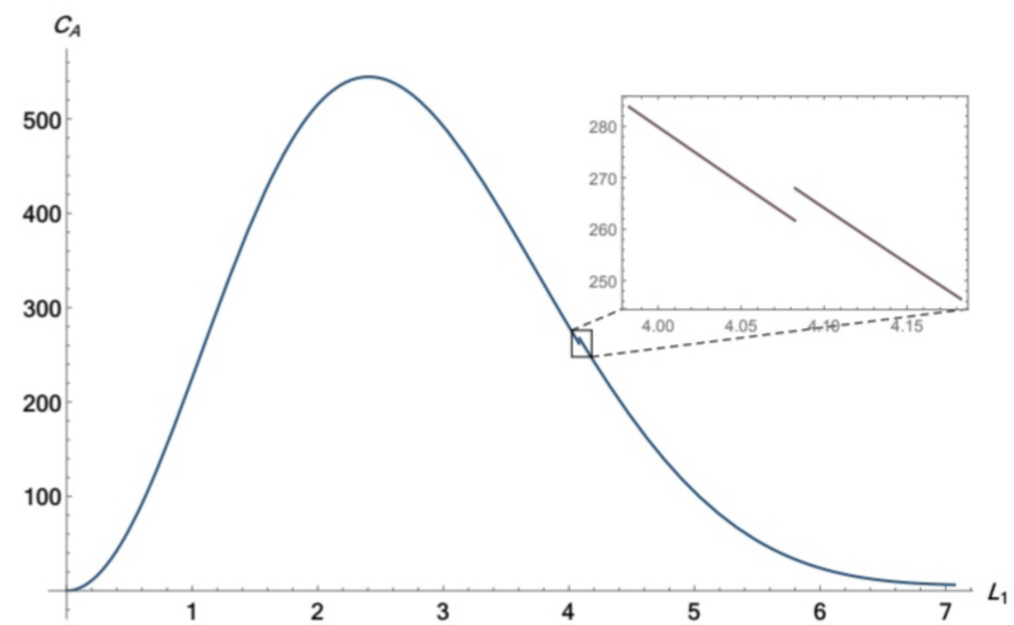


3 - bdy

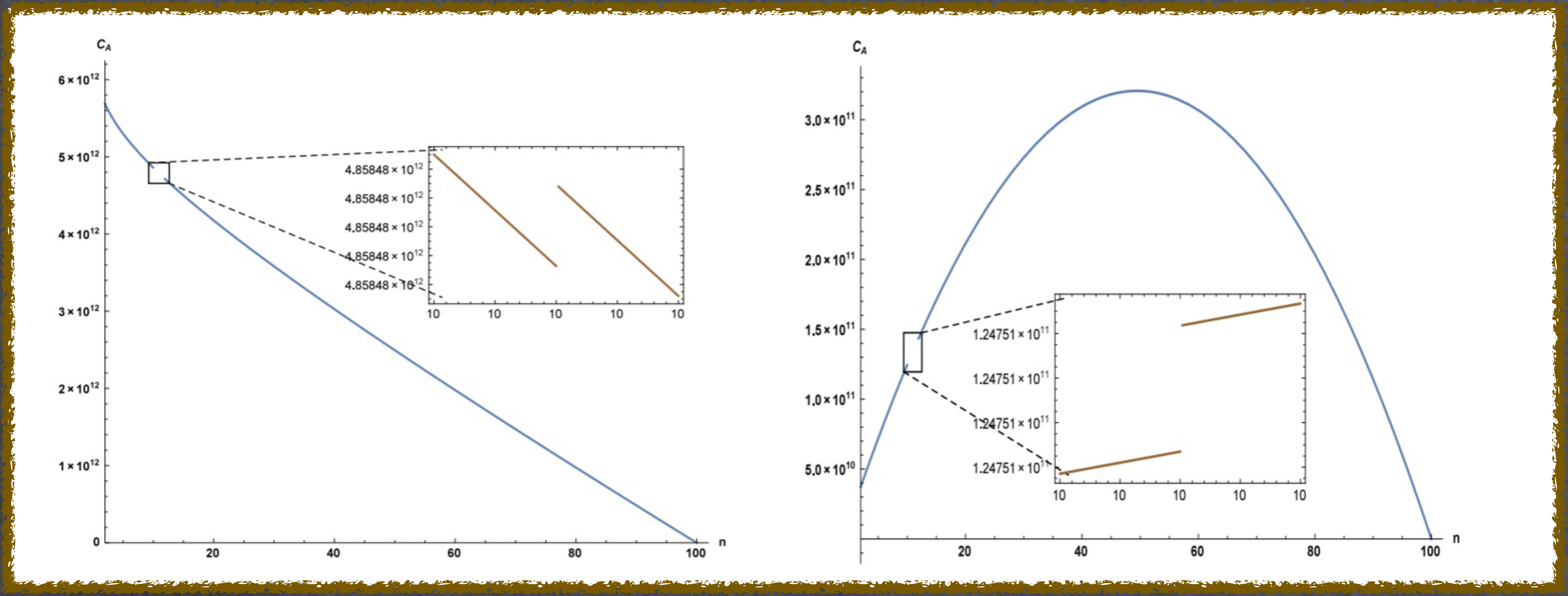
n - bdy

Volume plots : 3 bdy

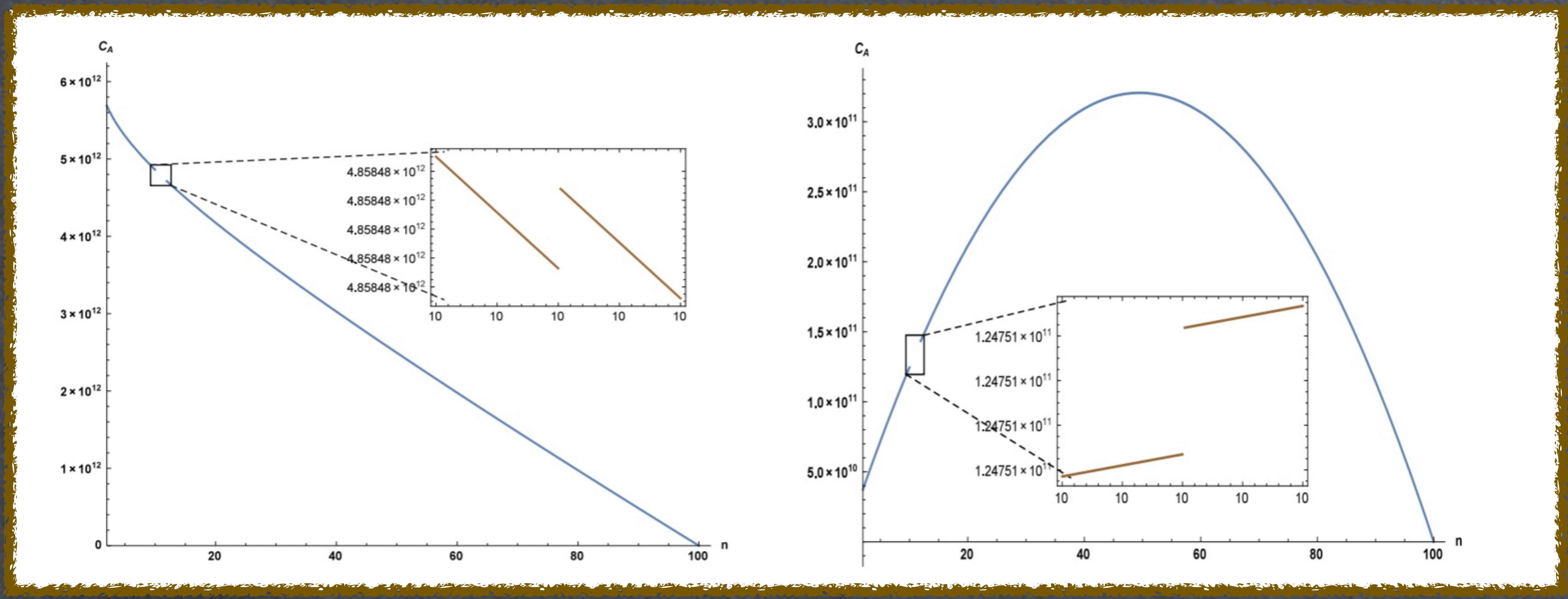




Volume plots : n - bdy



Volume plots : n - bdy



- For 3-bdy model, causal shadow is hyperbolic octagon. 2π volume gets added at the time of island inclusion.
- For n-bdy, causal shadow is hyperbolic $4(n_{page}-1)$ gon, $(n_{page}-2)\times 2\pi$ volume gets added.
- For n boundary model, the volumes are numerically approximated and the functional approximations are always within one of the two above shown universality classes. The nature of the plots are universal.

Lessons

- We derive a Page curve analogue plot for subregion complexity. (Radiation POV)
- The smooth transition point in Page curve is realised by a finite jump (dip) in volume for radiation (BH).
- Although the Page curve is same for radiation and BH, the volume plot is not, reflecting the difference of difficulty in producing corresponding time dependent states.
- The volume plots for both the models show similar plots. Both have two variants. (“London bridge falling” and “What goes up must come down”)
- The volume added at the point of island inclusion matches multipartite purification complexity. (Previously studied by Caceres et al and Maxfield et al)
- The islands lie in the entanglement shadow (behind all the throat horizons) and dual to Multipartite EoP in MbW models.
- Since EoP-Quantum error correction connections are well studied, this strengthens the island-QEC connections.

Other lessons & Way forward

- Tensor Network: Topological subregion complexity change reproduced by tensor network studies (using 2d Ising). Can the same be done for MBW jump in volume?
- For area, compute number of tensor/bonds cutting the minimal surface. For volume, total number of tensor bonds (along with associated costs) below the minimal surface.
- Kinematic Space: In AdS3/CFT2, bulk areas and volumes can be reproduced in terms of boundary entanglement entropies using all the bulk geodesics connecting two points in the boundary.
- Area: Number of geodesics crossing the minimal one. Volume: number of geodesics $\times$ cord lengths.
- Which sets of geodesics contribute to area and volumes in MBW model at different times?

Thank You.