Evaporating Black Holes Coupled to a Thermal Bath

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arXiv:1911.03402,2007.11658

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Outline

- Motivations & Background
- Generalized Entropy
- Doubly Holographic Models
- QES and Islands
- Entanglement Wedge Reconstruction

01.Background

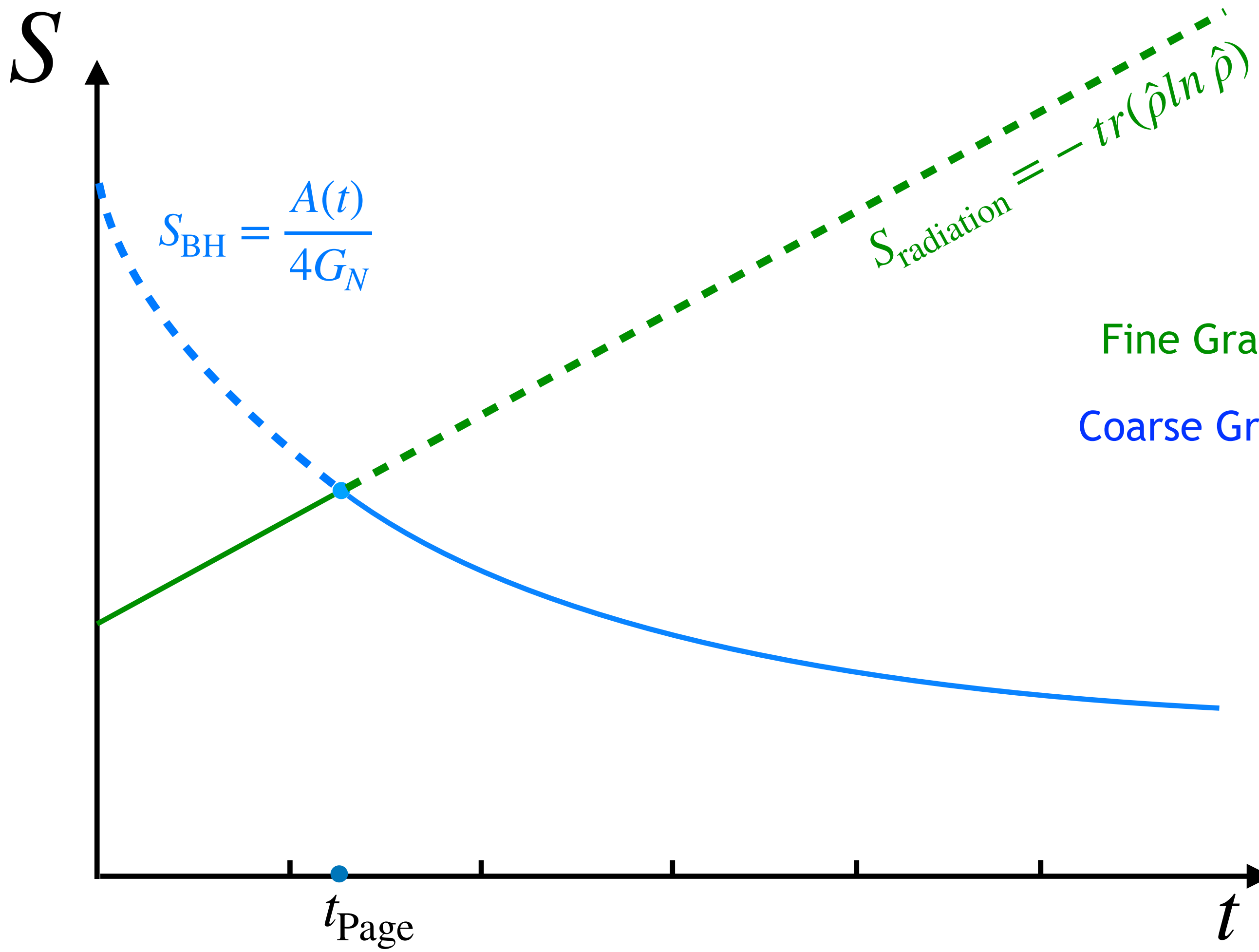
Black hole Information paradox

A quick answer from AdS/CFT

Yes! It is unitary.

An answer without answering any question

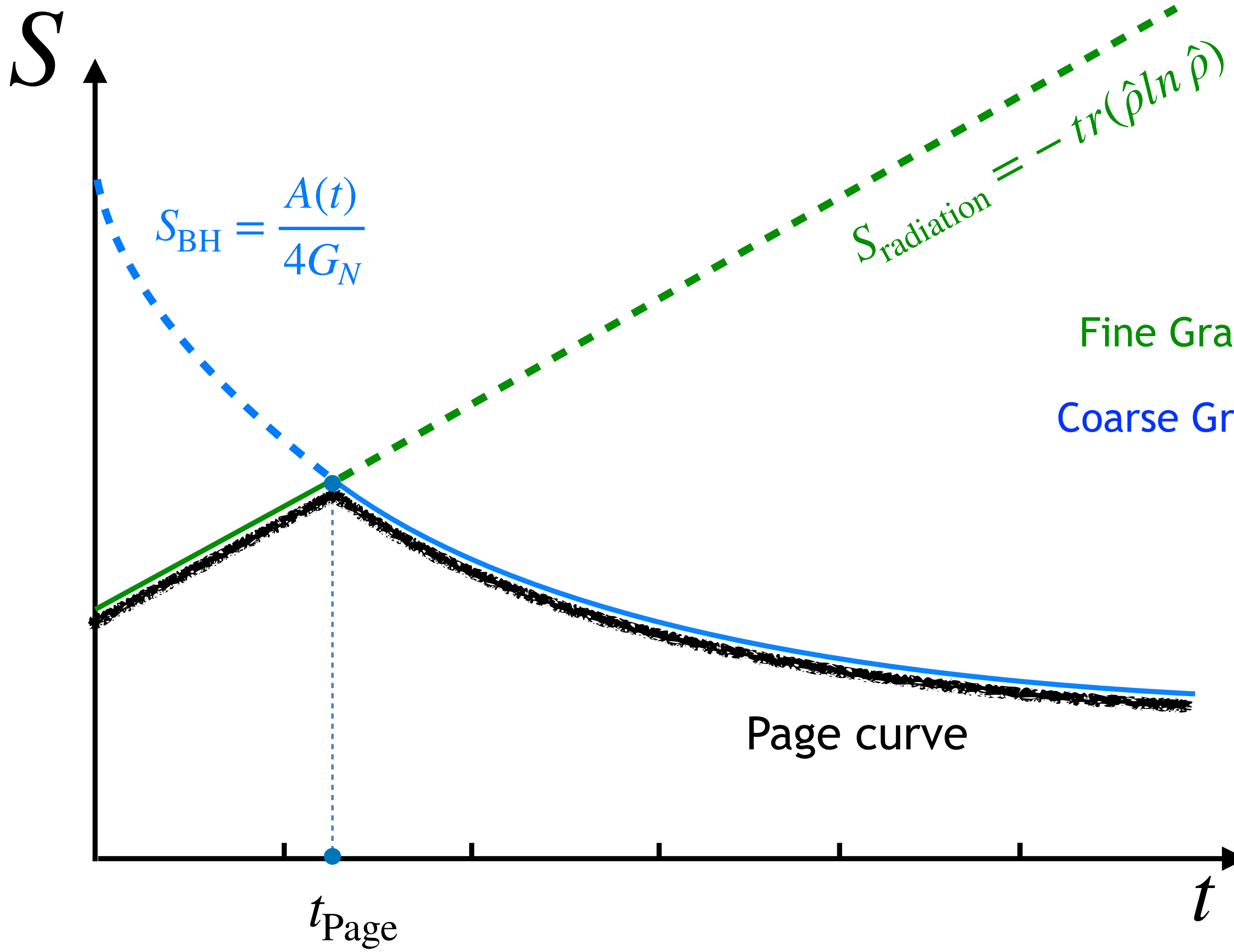
01.Background



Fine Grained entropy

Coarse Grained entropy

01.Background- Page Curve



Black hole Information paradox

First Step to the solution of information paradox

Recover Page Curve!

What is Hawking's mistake?

02. Generalized Entropy and QES

Need a correct formula for entropy

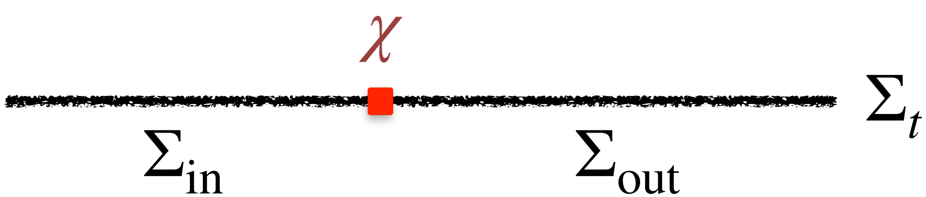
02. Generalized Entropy and QES

Generalized entropy (Cauchy surface Σ_{out})

$$S_{\text{gen}}[\Sigma_{\text{out}}] = \frac{A[\chi]}{4G_N} + S_{\text{bulk}}(\text{tr}_{\Sigma_{in}}\rho)$$

bulk entropy
(Von Neumann entropy of quantum fields)

$$S_{\text{bulk}}(\rho) = -\,\text{Tr}(\rho \ln \rho)$$



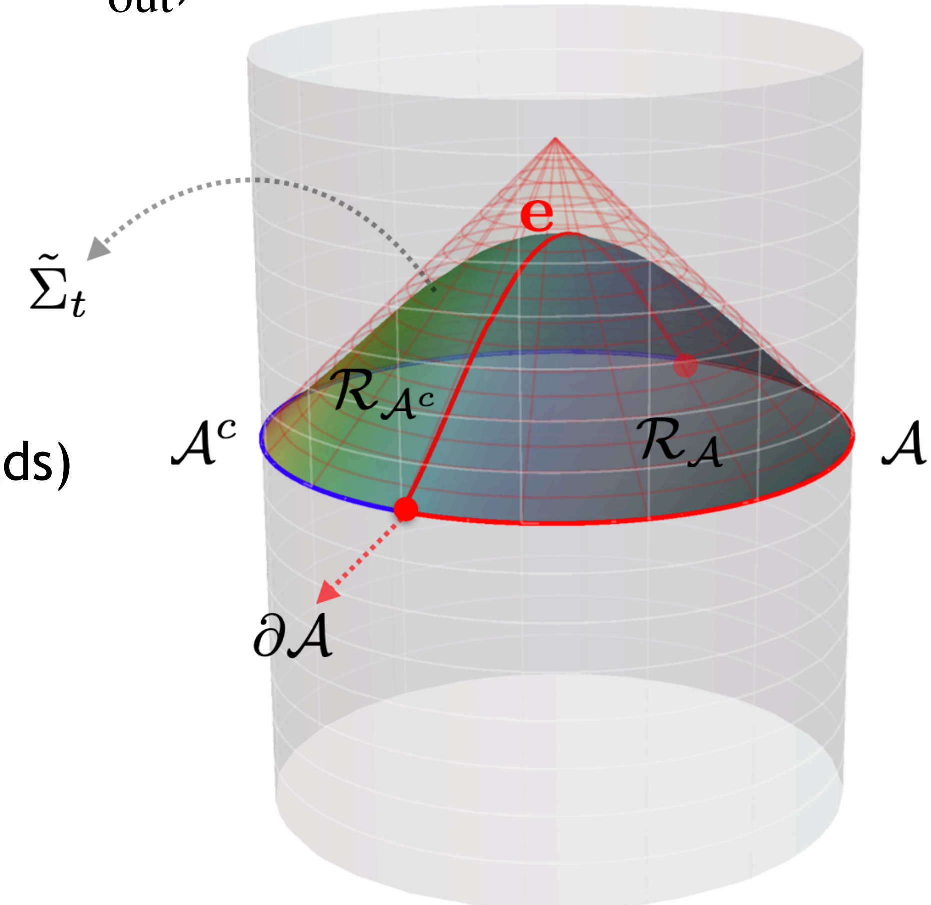


Fig 4.2 from arXiv:1609.01287

02. Generalized Entropy and QES

Generalized entropy (Σ_{out})

$$S_{\text{gen}}[\Sigma_{\text{out}}] = \frac{A[\chi]}{4G_N} + S_{\text{bulk}}(\text{tr}_{\Sigma_{in}}\rho)$$

Quantum extremal surfaces χ

Minimizing generalized entropy

$$\partial_x S_{\text{gen}} = 0 \longrightarrow \chi : x_{\text{QES}}$$

S_{bulk} is important!

$$S_{\text{gen}}[\Sigma_{\text{out}}] = \text{Min}_{\chi}\left[\text{Ext}_{\chi}\left(\frac{A[\chi]}{4G_N} + S_{\text{bulk}}(\Sigma_{\text{out}})\right)\right]$$

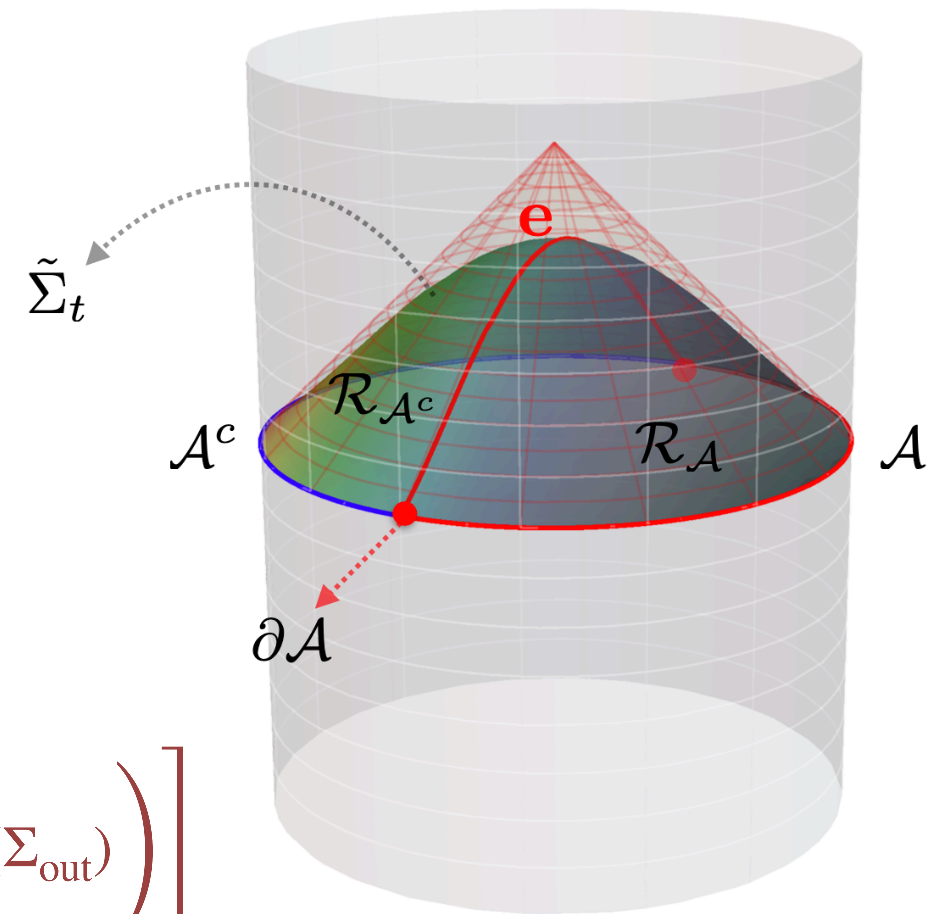


Fig 4.2 from arXiv:1609.01287

A correct formula for the fine grained entropy of Hawking radiation

Island Formula： $S_{\text{radiation}} = \text{Min}_{\chi}\left[\text{Ext}_{\chi}\left(\frac{A[\chi]}{4G_N} + S_{\text{bulk}}\left(\Sigma_{\text{radiation}} \cup \Sigma_{\text{Island}}\right)\right)\right]$

Disconnected Region

How to build a solvable model?

Two-dimensional CFT and JT gravity

03. AEMM Model (arXiv:1905.0876)

The simplest gravity model **JT gravity + CFT Matter**

$$I_0 = \frac{\phi_0}{16\pi G_N}\left[\int_M d^2x\sqrt{-g}R + 2\int_{\partial M} K\right]$$

$$I_G = \frac{1}{16\pi G_N}\left[\int_M d^2x\sqrt{-g}\,\underline{\phi(R+2)} + 2\int_{\partial M}\phi_b K\right]$$

$$I_M = I_{\mathrm{CFT}}[g]$$

Poincare Coordinate

$$ds^2_{\mathrm{AdS}} = -\frac{4L^2_{\mathrm{AdS}}}{(x^+-x^-)^2}dx^+dx^-\,,$$

$$(x^\pm = t \pm s)\,.$$

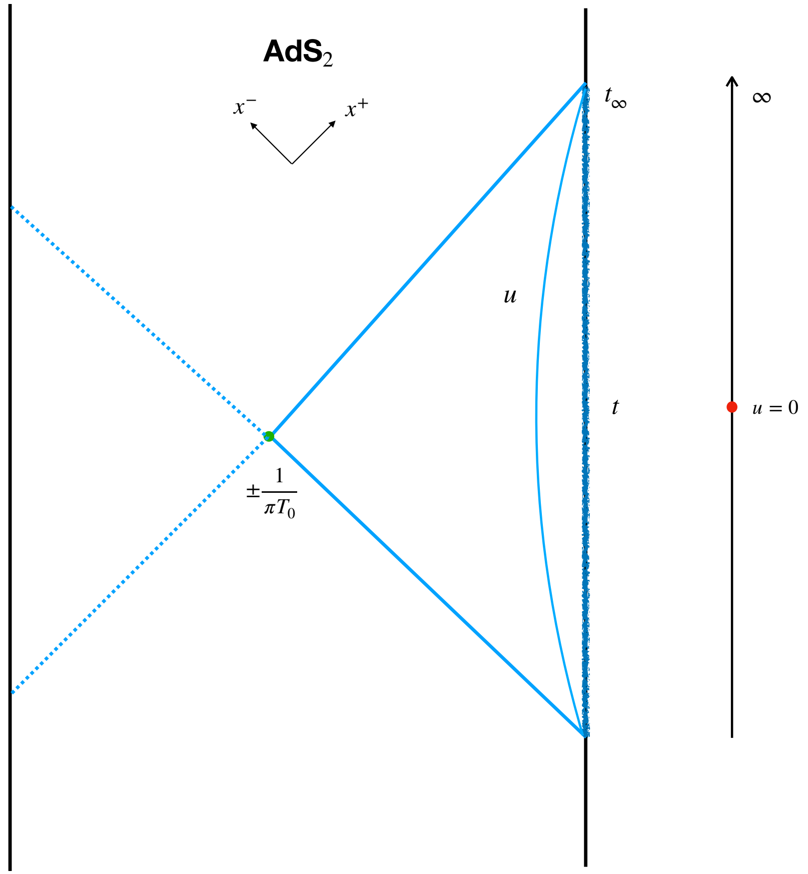
$$2\partial_{x^+}\partial_{x^-}\phi + \frac{4}{(x^+-x^-)^2}\phi = 16\pi G_N T_{x^+x^-}$$

$$-\frac{1}{(x^+-x^-)^2}\partial_{x^+}\left(\left(x^+-x^-\right)^2\partial_{x^+}\phi\right) = 8\pi G_N T_{x^+x^+}$$

$$-\frac{1}{(x^+-x^-)^2}\partial_{x^-}\left(\left(x^+-x^-\right)^2\partial_{x^-}\phi\right) = 8\pi G_N T_{x^-x^-}$$

JT gravity + CFT Matter

$$ds^2 = -\frac{4f'(y^+)f'(y^-)dy^+dy^-}{\left(f(y^+) - f(y^-)\right)^2}$$



Vacuum solution: $< T_{++} > = 0\,, < T_{--} > = 0$

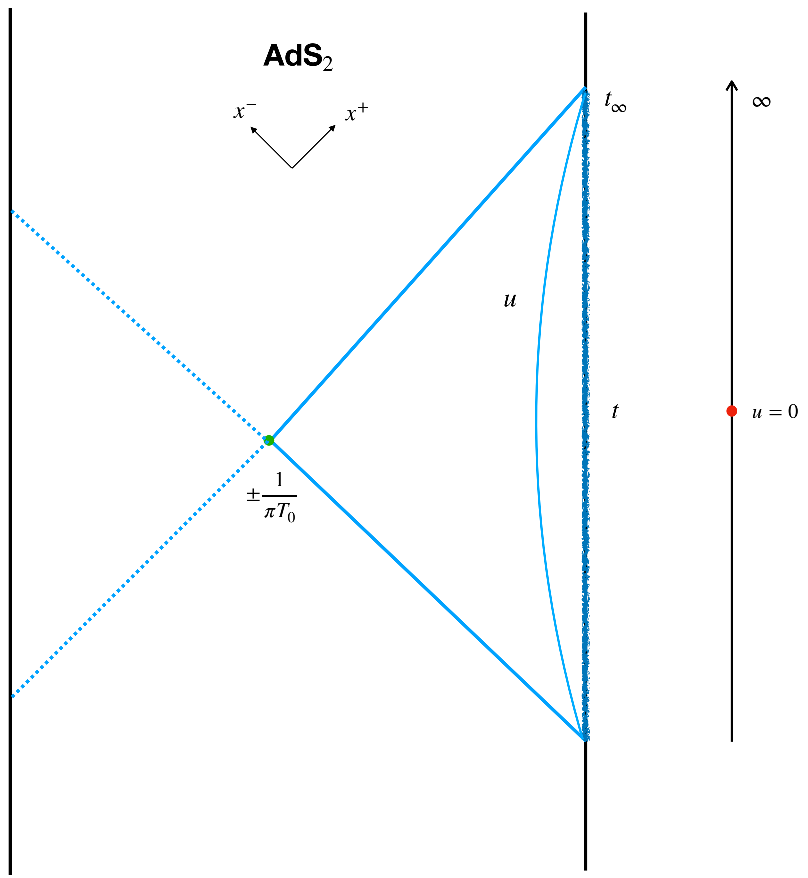
$$f(u) = \frac{1}{\pi T_0}\tanh\left(\pi T_0 u\right)$$

$$\phi = 2\bar{\phi}_r \frac{1 - \left(\pi T_0\right)^2 x^+ x^-}{x^+ - x^-}$$

$$E_0 \equiv \frac{\bar{\phi}_r}{8\pi G_N}\{f(u), u\} = \frac{\pi \bar{\phi}_r}{4G_N}T_0^2$$

eternal black hole

$$ds^2 = -\frac{4f'(y^+)f'(y^-)dy^+dy^-}{\left(f(y^+) - f(y^-)\right)^2}$$



JT gravity + CFT Matter

eternal black hole

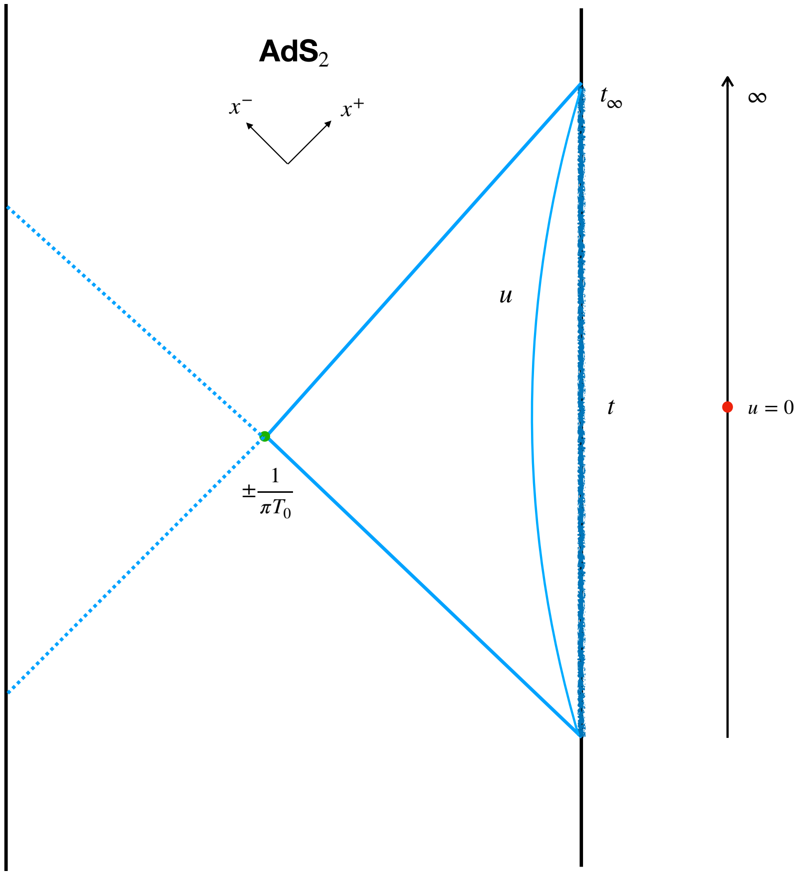
↓

Evaporating black hole

How to build a model for BH evaporation?

03. AEMM Model

$$ds^2 = -\frac{4f'(y^+)f'(y^-)dy^+dy^-}{\left(f(y^+)-f(y^-)\right)^2}$$



JT gravity + CFT Matter

eternal black hole

↓

Evaporating black hole

Put the black hole in a fridge!

03. AEMM Model

JT gravity + CFT Matter   Thermal Bath on half line

No gravity

Joint quench

AEMM model

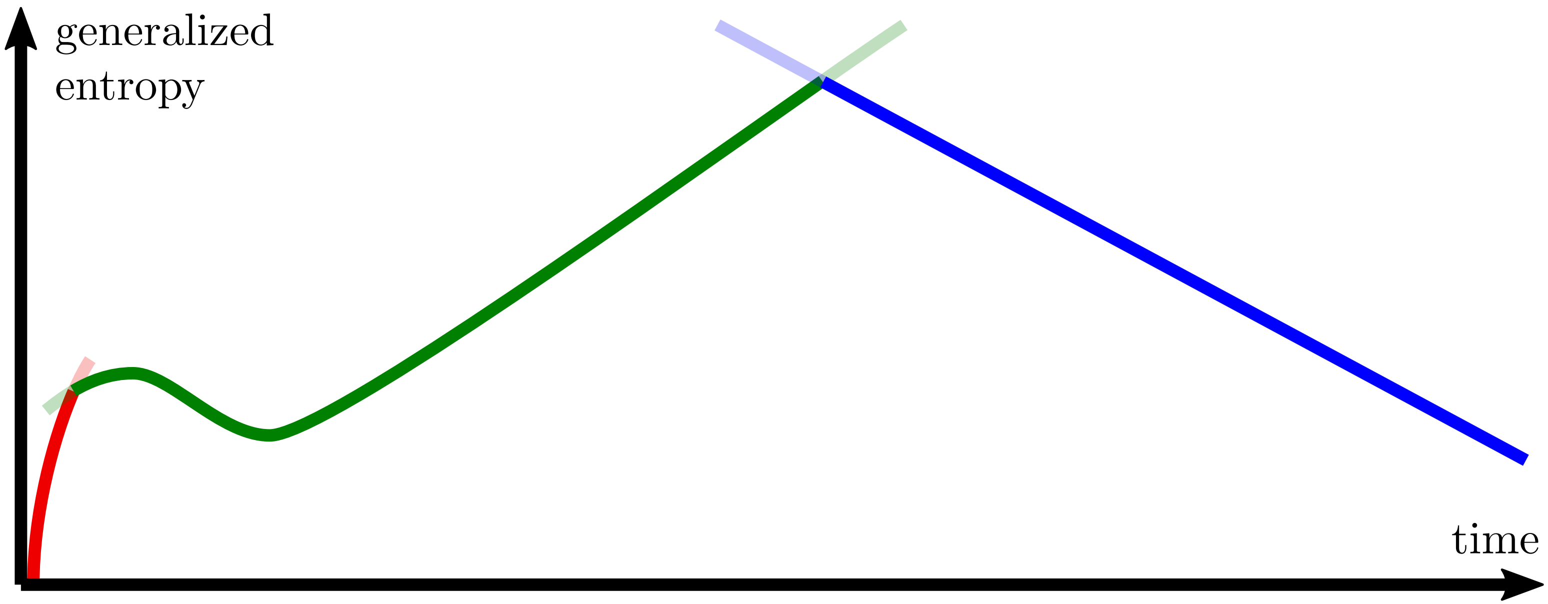
(1905.0876, Almherir, Engelhardt, Marolf, Maxfiled)

It is a holographic model

1D dual system:

03. Doubly Holographic Model

$$u_{\text{Page}} \approx \frac{2}{3}\frac{T_1 - T_0}{T_1 k} + \frac{u_{\text{HP}}}{3} + \frac{k}{6\pi T_1}\frac{5}{(T_1 - T_0)\pi} + \cdots \qquad u_{\text{HP}} = \frac{1}{2\pi T_1}\log\left(\frac{8\pi T_1}{3k}\right)$$



Page Curve

Unitary evolution of an evaporating black hole

More details: 1911.03402

put a black hole in an oven

Couple a black hole with a thermal bath

$T_b \neq 0$

arXiv:2007.11658

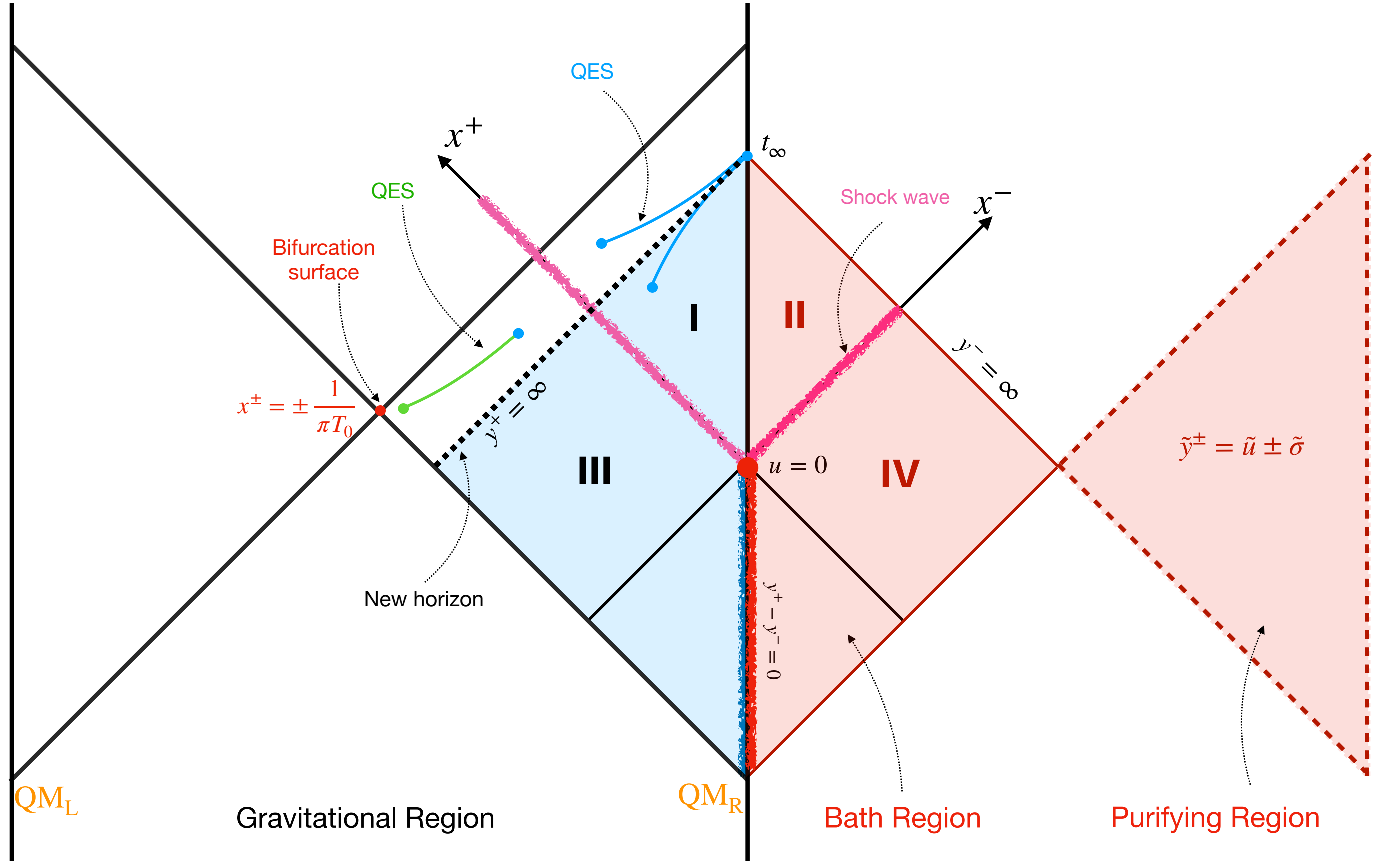
with V.Chen, Z.Fisher, J.Hernandez, and R.Myers

03. Doubly Holographic Models- T_b

Preheat the oven first! —Prepare a thermal state on cylinder

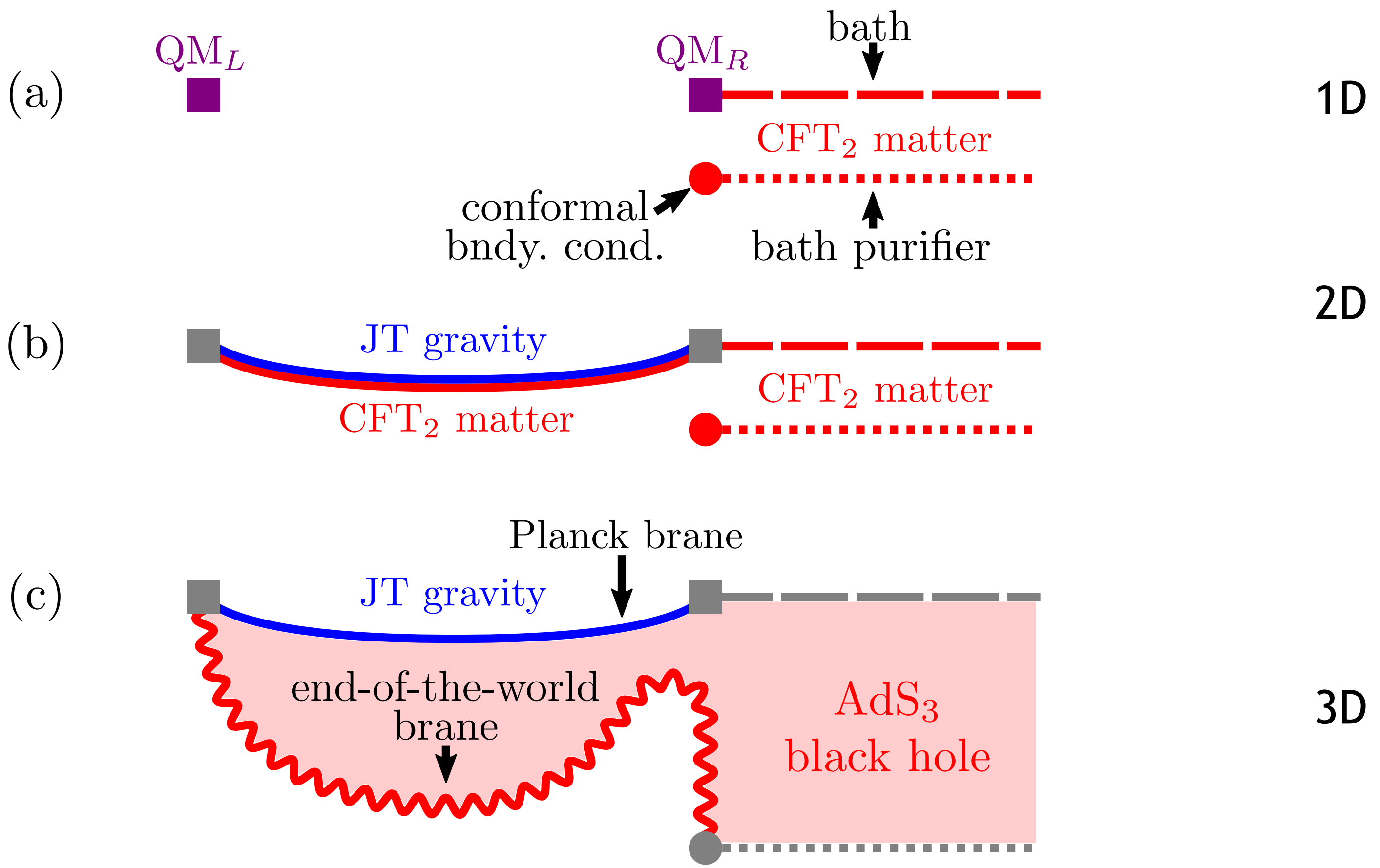
$$Y = \frac{1}{\pi T_b} \tanh(\pi T_b y)$$

Half line Half cylinder



03. Doubly Holographic Models- T_b

Three equivalent descriptions



Time Evolution of the Black Hole

ADM energy of AdS2 $E(u) \equiv \frac{\bar{\phi}_r}{8\pi G_N}\{f(u), u\}$ $\partial_u E(u) = f'(u)^2 \left(T_{x^-x^-} - T_{x^+x^+}\right)$

Stress tensor in physical coordinate

conformal anomaly

$$\left\langle T_{x^\pm x^\pm}\left(x^\pm\right)\right\rangle_{\mathrm{AdS}} = E_S \delta\left(x^\pm\right) - \frac{c}{24\pi}\left\{Y^\pm, x^\pm\right\}\Theta\left(\mp x^\pm\right)$$

$$= E_S \delta\left(x^\pm\right) - \frac{c}{24\pi}\Theta\left(\mp x^\pm\right)\left[\left\{y^\pm, x^\pm\right\} - 2\left(\frac{\pi T_{\mathrm{b}}}{f'\left(y^\pm\right)}\right)^2\right]$$

$$\{f(u), u\} = -2\pi^2\left[T_{\mathrm{b}}^2 + \left(T_1^2 - T_{\mathrm{b}}^2\right)e^{-ku}\right], \quad \text{with} \quad k \equiv \frac{cG_N}{3\bar{\phi}_r} \ll 1$$

A Solvable Model

Solutions

$$f(u, T_b) = \frac{2}{ka} \frac{I_\nu(a) K_\nu\left(a e^{-ku/2}\right) - K_\nu(a) I_\nu\left(a e^{-ku/2}\right)}{I'_\nu(a) K_\nu\left(a e^{-ku/2}\right) - K'_\nu(a) I_\nu\left(a e^{-ku/2}\right)}$$

$$a = \frac{2\pi}{k}\sqrt{T_1^2 - T_b^2}$$

$$\nu = \frac{2\pi T_b}{k}$$

$$E(u) = \frac{\bar{\phi}_r \pi}{4G_N} T_{\text{eff}}^2(u)$$

$$T_{\text{eff}}(u; T_b) = \sqrt{T_b^2 + \left(T_1^2 - T_b^2\right) e^{-ku}}$$

$T_b > T_1$ Thermalized $T_b = T_1$ Equilibrium $T_b < T_1$ Evaporating

$$\langle T_{x^- x^-} \rangle = E_S \delta(t) + \frac{c\pi}{12} \frac{1}{\left(f'(u)\right)^2} \left(T_b^2 - T_{\text{eff}}^2(u)\right)$$

Hawking radiation

Generalized Entropy

$$S_{\text{gen}}[\Sigma_{\text{out}}] = \text{Min}\left[\text{Ext}_{x^{\pm}}\left(\frac{\phi}{4G_N} + S_{\text{bulk}}(x^+, x^-)\right)\right]$$

A Solvable Model

Dilaton after coupling with bath

$$\phi = \bar{\phi}_r \frac{2 - 2\left(\pi T_1\right)^2 x^+ x^- + k I_0}{x^+ - x^-}$$

$$I_0 = -\frac{24\pi}{c} \int_0^{x^-} dt \left(x^+ - t\right)\left(x^- - t\right)\left\langle T_{x^- x^-}(t)\right\rangle$$

$$f\left(u, T_b\right) = \frac{2}{ka} \frac{I_\nu(a) K_\nu\left(a e^{-ku/2}\right) - K_\nu(a) I_\nu\left(a e^{-ku/2}\right)}{I'_\nu(a) K_\nu\left(a e^{-ku/2}\right) - K'_\nu(a) I_\nu\left(a e^{-ku/2}\right)}$$

$$\phi\left(x^\pm\right) = \phi_r \left(\frac{2 f'\left(y^-\right)}{x^+ - x^-} + \frac{f''\left(y^-\right)}{f'\left(y^-\right)}\right)$$

How to calculate S_{bulk} in the semi-classical limit?

04. QES and Islands

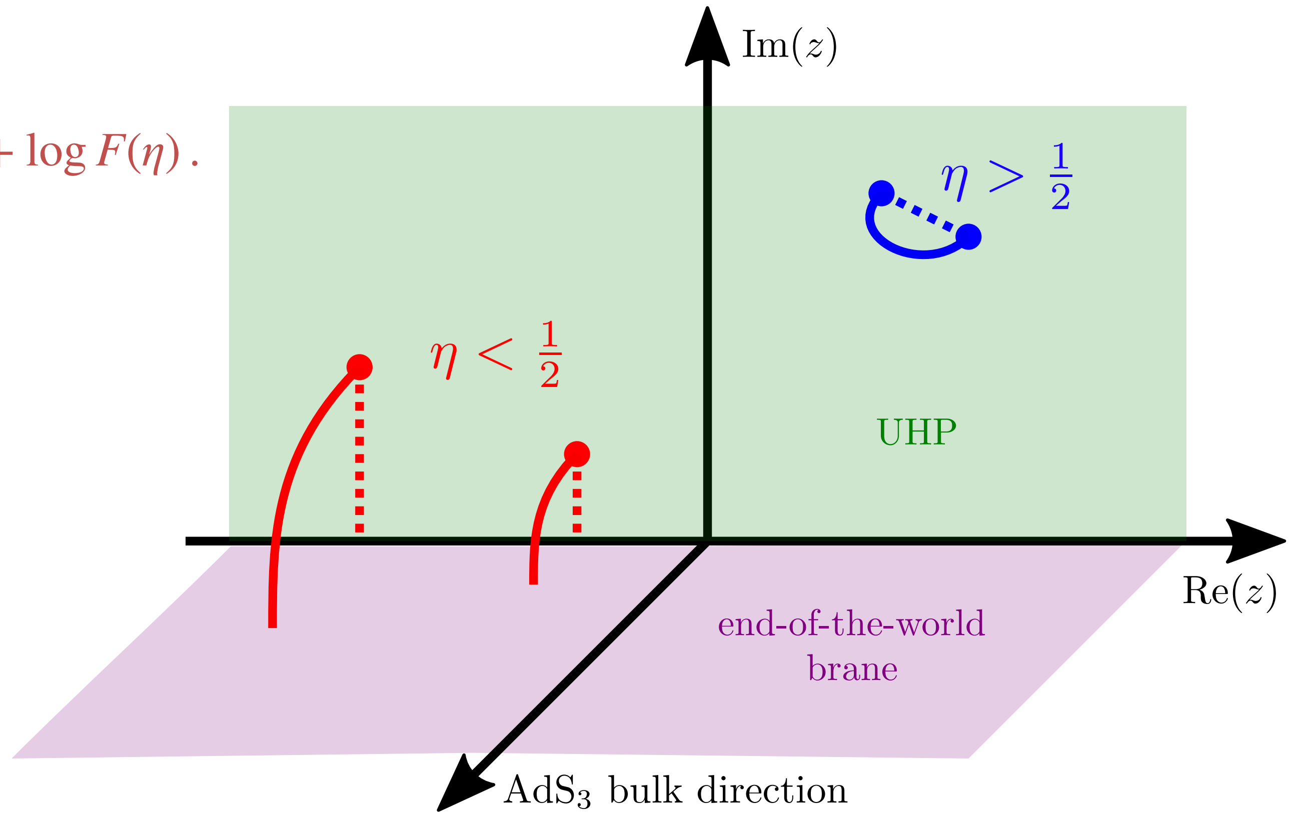
Takayanagi, AdS/BCFT, 1105.5165

BCFT on upper half plane (UHP)

$$S_{\text{UHP}} = \frac{c}{6}\log\left(\frac{|z_0 - z_1|^2}{\delta^2}\eta\right) + \log F(\eta)\,.$$

$$\eta = \frac{(z_1 - \bar{z}_1)(z_0 - \bar{z}_0)}{(z_1 - \bar{z}_0)(z_0 - \bar{z}_1)},$$

Holographic BCFT | Large c



$$S_{\text{UHP}} = \begin{cases} \frac{c}{6}\log(\frac{|z_1-z_0|^2}{\delta^2}) & \text{for } \eta > \eta_* \\ \frac{c}{6}\log\left(\frac{|z_1-\bar{z}_1|}{\delta}\cdot\frac{|z_0-\bar{z}_0|}{\delta}\right) + 2\log g & \text{for } \eta < \eta_* \end{cases}$$

$$\eta_* = (1 + g^{12/c})^{-1}$$

QES with Islands

$$S_{\text{gen}}(x^+, x^-) = \frac{\phi}{4G_N} + S_{\text{bulk}}\,,$$

$$\partial_{x^\pm} S_{\text{gen}} = 0$$

$\longrightarrow$

$$x^+_{\text{QES}} \approx t_\infty + \frac{\Gamma_{\text{eff}}}{4-\Gamma_{\text{eff}}}\left(t_\infty - t\right)$$

$$x^-_{\text{QES}} \approx t_\infty - \frac{8\pi T_{\text{eff}}}{k\left(4-\Gamma_{\text{eff}}\right)}\left(t_\infty - t\right)$$

$$\Gamma_{\text{eff}}\left(y^-_{\text{QES}}\right) \equiv \left(1 - \frac{T_{\text{b}}}{T_{\text{eff}}\left(y^-_{\text{QES}}\right)}\right)^2$$

Island Phase: $$\frac{dS_{\text{gen,late}}}{du} \approx -\frac{\bar{\phi}_r}{4G_{\text{N}}}\left(1-\frac{T_{\text{b}}^2}{T_{\text{eff}}^2(u)}\right)k\pi T_{\text{eff}}(u)$$

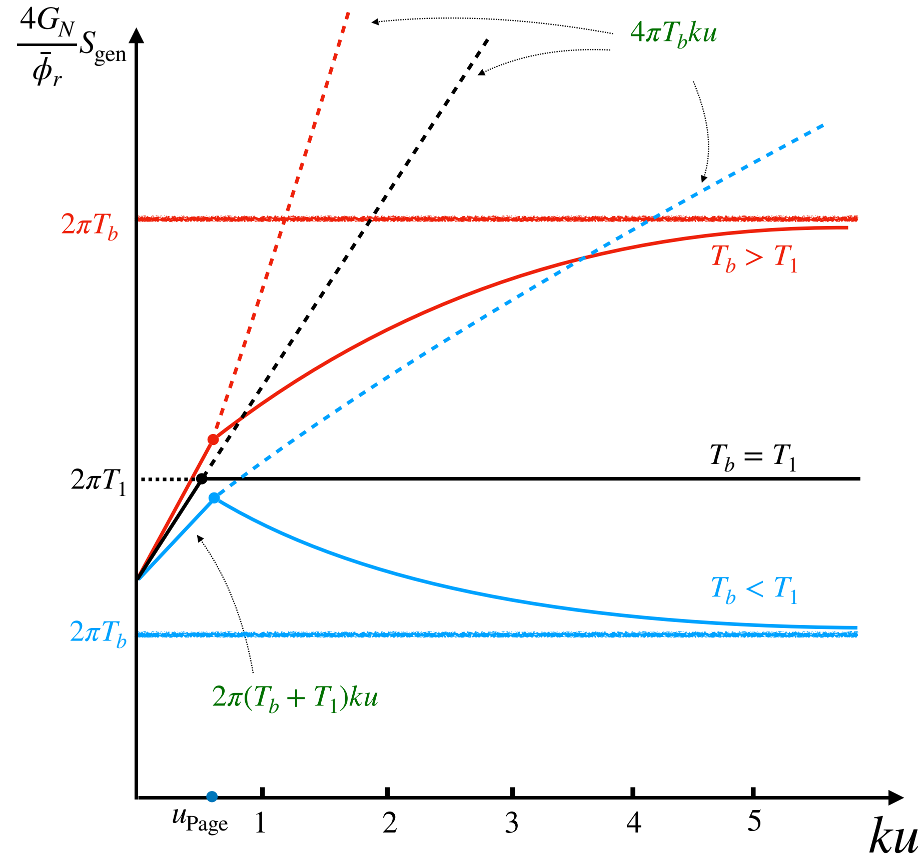
04. QES and Islands

Island Phase: $$\frac{dS_{\text{gen,late}}}{du} \approx -\frac{\bar{\phi}_r}{4G_\text{N}}\left(1-\frac{T_\text{b}^2}{T_\text{eff}^2(u)}\right)k\pi T_\text{eff}(u)$$

$T_b > T_1$ Thermalized

$T_b = T_1$ Equilibrium

$T_b < T_1$ Evaporating



arXiv: 2007.11658

04. QES and Islands

Equilibrium Status $T_1 = T_b$

$$x^+_{\text{QES}}(t) = \frac{\sqrt{k^2+\pi^2T_1^2}\left((\pi T_1 t)^2-1\right)+k\left((\pi T_1 t)^2+1\right)}{\pi^2T_1^2\left(\pi^2T_1^2t^2+2kt-1\right)}$$

$$x^-_{\text{QES}}(t) = \frac{\sqrt{k^2+\pi^2T_1^2}\left((\pi T_1 t)^2-1\right)+k\left((\pi T_1 t)^2+1\right)}{\pi^2T_1^2\left(-\pi^2T_1^2t^2+2kt+1\right)}$$

$$S_{\text{gen,late}}\left(T_1\right) = \frac{\bar{\phi}}{2G_{\text{N}}}\left(\sqrt{k^2+\pi^2T_1^2} - k\log\left[\epsilon\left(k+\sqrt{k^2+\pi^2T_1^2}\right)\right]\right)$$

Constant

Island outside horizon

$$x^+_{\text{QES}}(t) < t_\infty = x^+_{\text{QES}}(t_\infty); \quad \frac{dx^+_{\text{QES}}(t)}{dt} > 0$$

Island outside horizon

$$T_{c_1}(u) \approx \left(1 - \sqrt{\frac{2k}{\pi T_1}}\right) T_{\text{eff}}\left(\bar{y}_{\text{QES}}\right) \qquad T_{c_2}(u) \approx \left(1 + \sqrt{\frac{2k}{\pi T_1}}\right) T_{\text{eff}}\left(\bar{y}_{\text{QES}}\right)$$

Inside horizon	$T_{c_1}(u) < T_{\text{b}} < T_{c_2}(u)$
On the horizon	$T_{\text{b}} = T_{c_1}(u)$ or $T_{\text{b}} = T_{c_2}(u)$
Outside horizon	$T_{\text{b}} < T_{c_1}(u)$ or $T_{\text{b}} > T_{c_2}(u)$

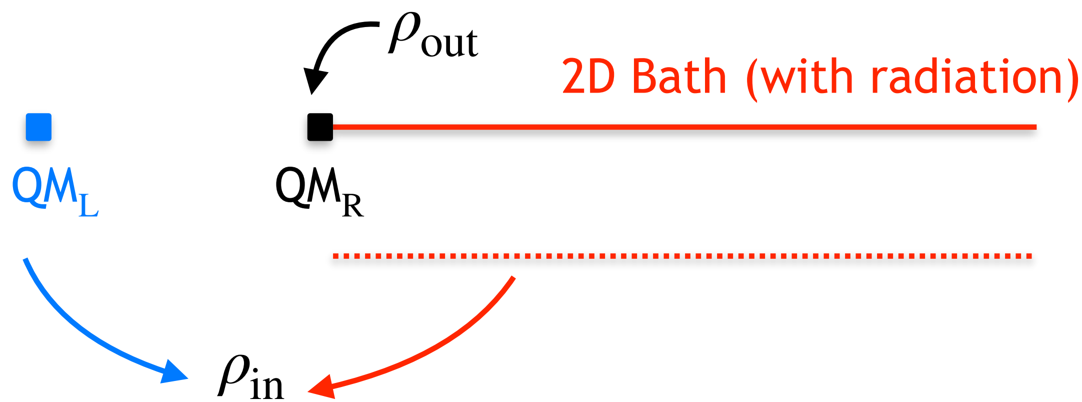
At very late time, QES always moves outside horizon

$$ku \gtrsim \log\left(\left|1 - \frac{T_1^2}{T_{\text{b}}^2}\right| \sqrt{\frac{\pi T_1}{8k}}\right)$$

A correct formula for Radiation

Island formula

Pure State:



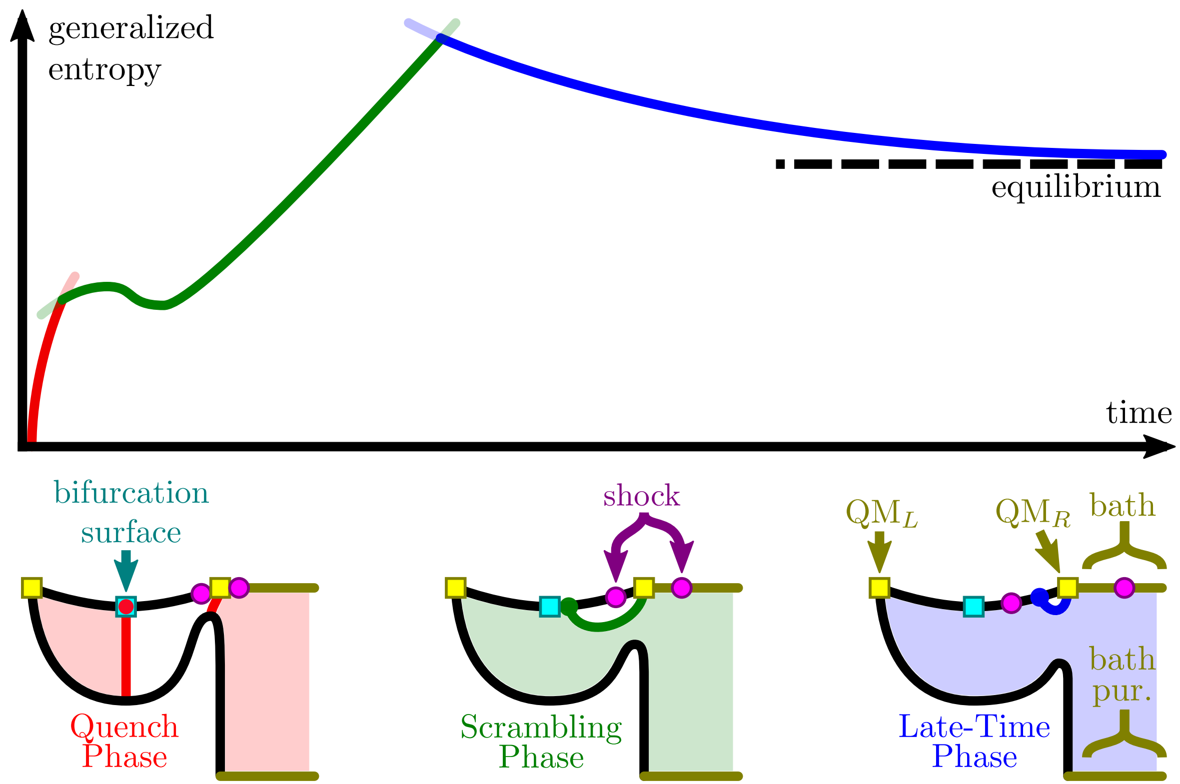
$$S_{\text{gen}}[\rho_{\text{out}}] = \text{Min}_{\chi}\left[\text{Ext}_{\chi}\left(\frac{A[\chi]}{4G_N} + S_{\text{bulk}}(\Sigma_{\text{out}})\right)\right]$$

Equivalent

$$S_{\text{gen}}[\rho_{\text{in}}] = \text{Min}_{\chi}\left[\text{Ext}_{\chi}\left(\frac{A[\chi]}{4G_N} + S_{\text{bulk}}\left(\Sigma_{\text{in}} \cup \Sigma_{\text{Island}}\right)\right)\right]$$

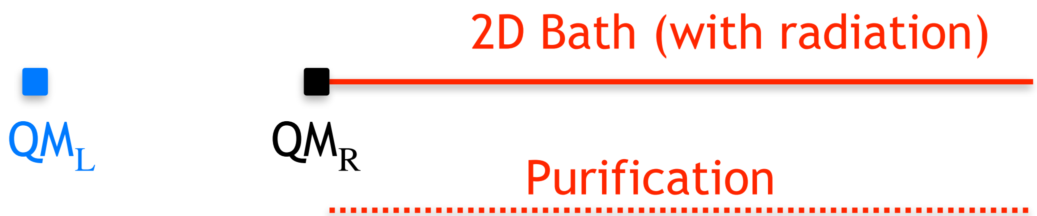
Island formula

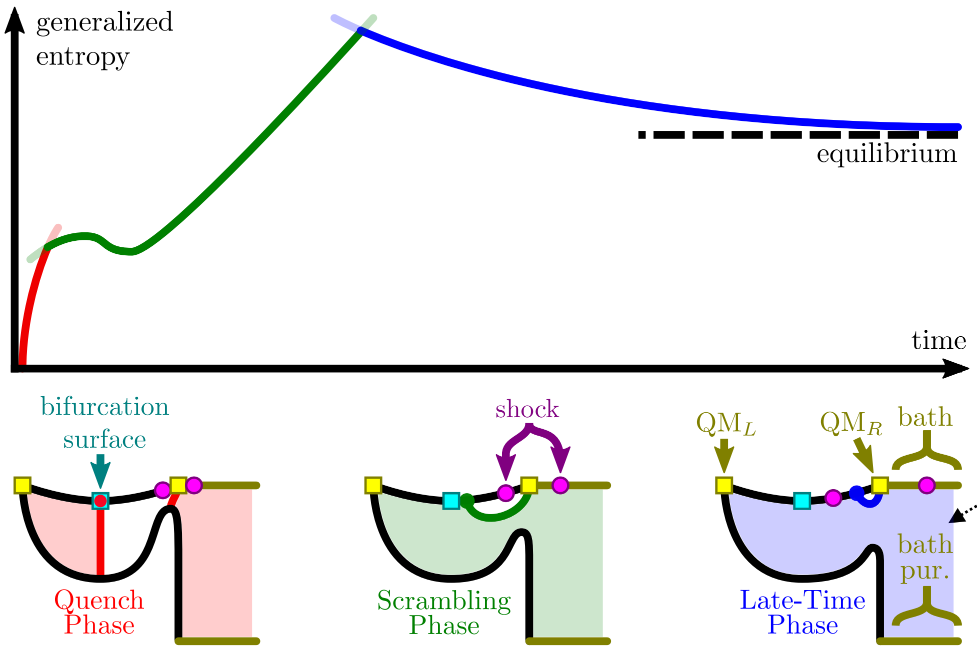
Generalized Entropy of QM_R



Entanglement Wedge of QM_L + bath (Hawking radiation)+Purification

05. Entanglement Wedge Reconstruction





Entropy of
QM$_L$ + bath (Hawking radiation)
+Purification

Entanglement Wedge of
QM$_L$ + bath (Hawking radiation)
+Purification

Reconstruct the interior of BH after Page transition (Island Phase)

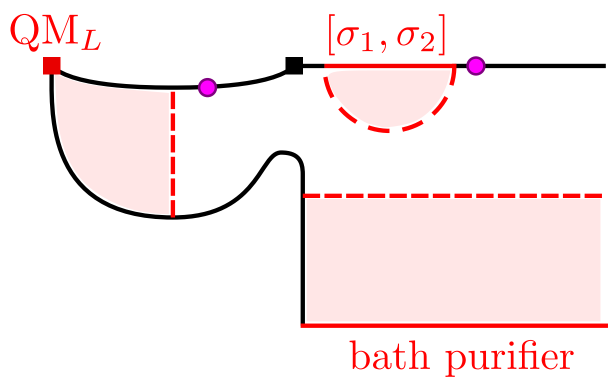
05. EWR- a finite bath interval

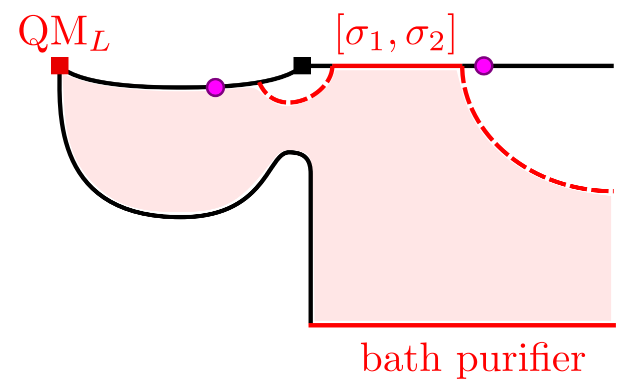
Who knows information of the BH interior?

Stored in the Hawking radiation living in the Bath?

How much Hawking radiation we need to reconstruct BH interior?

subsystem: QM$_L$ + part of the bath $[\sigma_1, \sigma_2]$ + Purification





$S_{\mathrm{N}} = S^{\mathrm{gen}}_{\mathrm{QES}''} + S_{1-2} + S_{\frac{1}{2}-\mathrm{line}}$

Competing Channels

$S_{\mathrm{R}} = S^{\mathrm{gen}}_{\mathrm{QES}-1} + S_{2-\mathrm{IR}}$

Island Phase

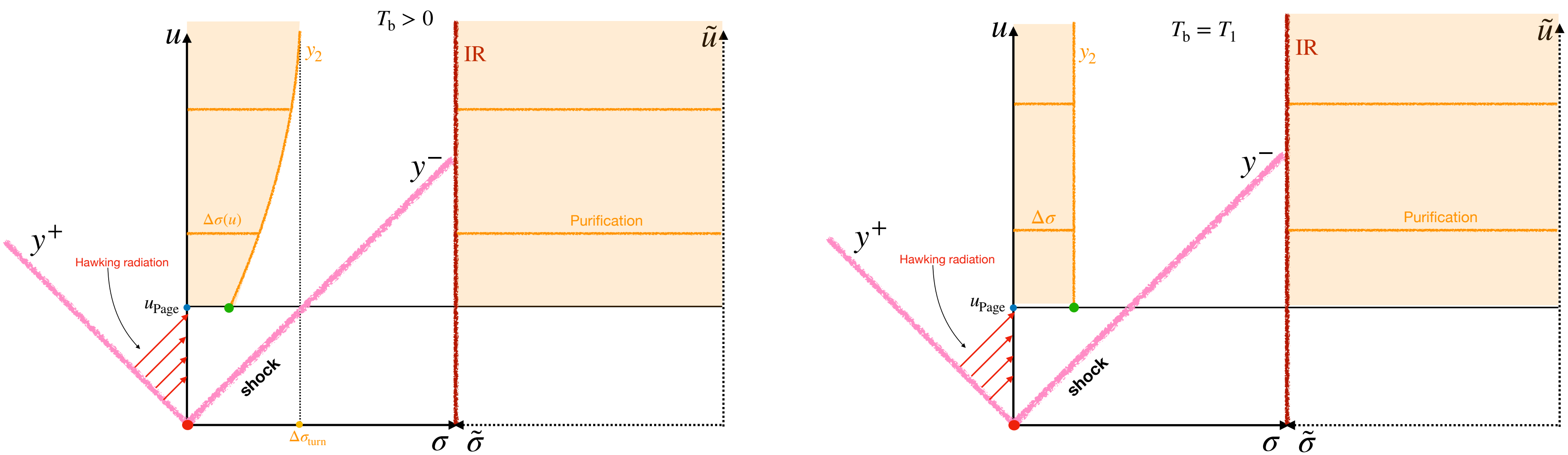
Reconstruct the interior of BH !

$S_{\mathrm{R}} \leq S_{\mathrm{N}}$

05. EWR- a finite bath interval

subsystem: QM_L + part of the bath $[\sigma_1, \sigma_2]$ + Purification

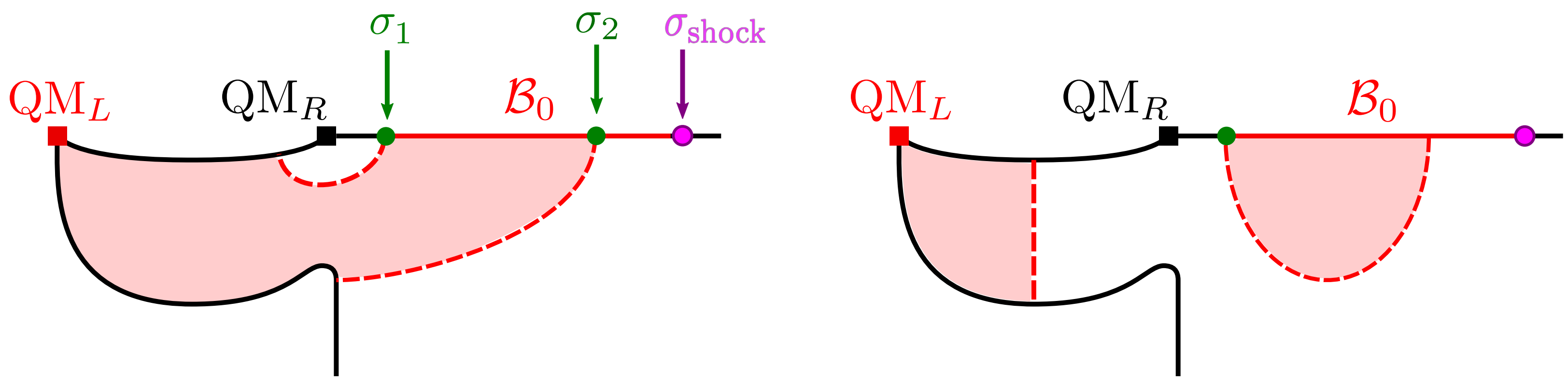
$$\sigma_2(u) \gtrsim \frac{T_1 - T_0}{2k\left(T_1 + T_b\right)} + \frac{T_1}{4\left(T_1 + T_b\right)}\left(u\left(1 - \frac{T_b}{T_1}\right)^2 + u_{HP}\left(1 - \frac{T_b^2}{T_1^2}\right)\right) + \frac{\log\left(\frac{6E_s}{cT_1}\right)}{2\pi\left(T_1 + T_b\right)} + \cdots$$



The information encoded in Hawking radiation is degenerate!

05. EWR- the role of purification

subsystem: $\mathrm{QM_L}$ + part of the bath $[\sigma_1, \sigma_2]$



$S^{\rm gen}_{\rm QES-1} + S_2$

$S^{\rm gen}_{\rm QES''} + S_{1-2}$

Reconstruct the interior of BH ! $S^{\rm gen}_{\rm QES-1} + S_2 < S^{\rm gen}_{\rm QES''} + S_{1-2}$

Only If :

$$T_{\rm b} \lesssim T_p \approx \frac{T_1 + T_0}{2} + \frac{k}{2\pi}\log\left(\frac{6E_s}{cT_1}\right)$$

We also need the purification part when $T_b \geq T_p$

Remarks and Conclusions

- The information of BH is not lost!
- Surprising Result: Unitarity from semi-classical limit
- Unitarity in the evolution of BH is universal !/?
  (details of bath, evaporation, equilibrium, growing-up, boundary entropy log g, higher dimensions….)
- Important role of Quantum Extremal Surface (QES)
- Appearance of Island region at late time
- Degeneracy of information in Hawking radiation
- Secret Sharing Scheme
- ………….

Lots to Explore!

Thanks for your attention!