The Squashed, Stretched, Warped and Perturbed gets invaded

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Based on:
MC, G.Larios, O.Varela – arXiv: 2007.05172

Overview

- The Cubic deformation of ABJM and its dual GMPS
- The complete KK graviton spectrum over GMPS
- Full spectrum considerations: Space Invaders
- Conclusion and Outlook

Field Theory side: ABJM

ABJM theory is a $U(N) \times U(N)$ Superconformal Chern-Simons theory with levels $k,-k$. It is dual to a stack of M2 branes on a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold. For $k=1$ supersymmetry is enhanced to $\mathcal{N}=8$, with $SO(8)$ bosonic subgroup. [O.Aharony, O.Bergman, D.Jafferis, J.Maldacena '08]

$$W = 4\pi\, (\mathcal{Z}^1)^a_{\hat{a}}(\mathcal{Z}^2)^b_{\hat{b}}(\mathcal{Z}^3)^c_{\hat{c}}(\mathcal{Z}^4)^d_{\hat{d}} \left[(\mathcal{M}^{-2})^{\hat{a}\hat{c}}_{bc}(\mathcal{M}^{-2})^{\hat{b}\hat{d}}_{ad} - (\mathcal{M}^{-2})^{\hat{a}\hat{d}}_{bd}(\mathcal{M}^{-2})^{\hat{b}\hat{c}}_{ac}\right] .$$

There exist two ($\mathcal{N}=2$, $SU(3)$)-preserving relevant deformations of the theory:

$$\Delta W = (\mathcal{Z}^4)^p \,, \qquad\qquad p = 2, 3 \,,$$

leading to IR R charges

$$\mathrm{R}_1 \equiv R(\mathcal{Z}^A) = \tfrac{2(p-1)}{3p} \,, \qquad\qquad R_2 \equiv R(\mathcal{Z}^4) = \tfrac{2}{p} \,.$$

Gravity side: $\mathcal{N} = 2$ AdS geometries

$\mathcal{N} = 2$-supersymmetric geometries are obtained as

$$d\hat{s}^2_{11} = e^{2\Delta}\left(\tfrac{1}{4}ds^2(\mathrm{AdS}_4) + ds^2_7\right)\ , \qquad G = \frac{m}{16}\,(\mathrm{AdS}_4) + F\ .$$

[M. Gabella, D.Martelli, A. Passias, J.Sparks '12]

Two solutions of the supersymmetry constraint $\delta_\epsilon \Psi = 0$.

Necessary and sufficient conditions given in terms of spinor bilinears:

$$e^{-3\Delta}\mathrm{d}\left[\|\xi\|^{-1}\left(\frac{m}{6}E_1 + e^{3\Delta}|S|\sqrt{1-\|\xi\|^2}\,E_3\right)\right] = 2J_3 - 2\,\|\xi\|\,E_2\wedge E_3$$

$$\mathrm{d}\big(\,\|\xi\|^2\,e^{9\Delta}J_2\wedge E_2\big) - e^{3\Delta}|S|\mathrm{d}\big(\,\|\xi\|\,e^{6\Delta}|S|^{-1}J_1\wedge E_3\big) = 0$$

$$\mathrm{d}\big(e^{6\Delta}J_1\wedge E_2\big) + e^{3\Delta}|S|\mathrm{d}\big(\,\|\xi\|\,e^{3\Delta}|S|^{-1}J_2\wedge E_3\big) = 0$$

Gravity side: $\mathcal{N}=2$ AdS geometries

The internal geometries of interest are endowed with at least two isometries, so that the line element takes the local form [M. Gabella, D.Martelli, A. Passias, J.Sparks '12]

$$ds_7^2 = \frac{f\cdot\alpha}{4\sqrt{1+(1+r^2)\alpha^2}}\,ds^2(\mathrm{KE}_4) + \frac{\alpha^2}{16}\Big[dr^2 + \frac{r^2 f^2}{1+r^2}(d\tilde{\tau}+\sigma)^2 + \frac{1+r^2}{1+(1+r^2)\alpha^2}\Big(d\tilde{\psi} + \frac{f}{1+r^2}(d\tilde{\tau}+\sigma)\Big)^2\Big]\,,$$

with $f\,,\alpha$ metric functions satysfying the ODEs

$$\frac{f'}{f} = -\frac{1}{2}r\alpha^2\,, \qquad \frac{(r\alpha' - r^2\alpha^3)f}{\sqrt{1+(1+r^2)\alpha^2}} = -3\,.$$

For $ds^2(\mathrm{KE}_4) = ds^2(\mathbb{CP}^2)$, the isometry group gets enlarged to $SU(3)\times U(1)_{\tilde{\psi}}\times U(1)_{\tilde{\tau}}$, with $U(1)_{\tilde{\tau}}$ broken by the four-form flux.

Gravity side: $\mathcal{N} = 2$ AdS geometries

For (at least) two choices of (f, α) the geometry can be globally extended over the S^7

$$\psi = \tfrac{1}{p}\,\tilde{\psi}\,, \qquad \tau = \tilde{\tau} + \tfrac{1}{3}\big(1 - \tfrac{1}{p}\big)\tilde{\psi}\,, \qquad p = 2, 3\,.$$

Regularity, S^7-topology and AdS/CFT require the asymptotics

$$f \xrightarrow[r\to 0]{} \frac{3p}{p-1}\,, \qquad \alpha \xrightarrow[r\to 0]{} w r^{-1+1/p}\,, \qquad \text{with } w > 0\,,$$

$$f \xrightarrow[r\to r_0]{} \frac{2\sqrt{1+r_0^2}}{r_0}\,(r_0 - r)\,, \qquad \alpha \xrightarrow[r\to r_0]{} \sqrt{\frac{2}{r_0(r_0 - r)}}\,.$$

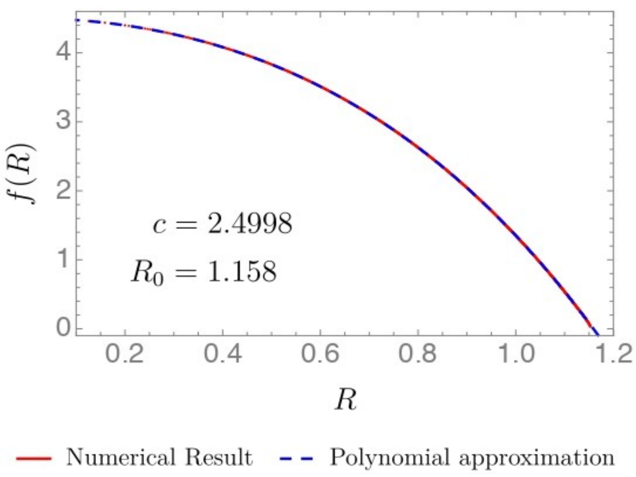
Gravity side: $p = 2$ and $p = 3$

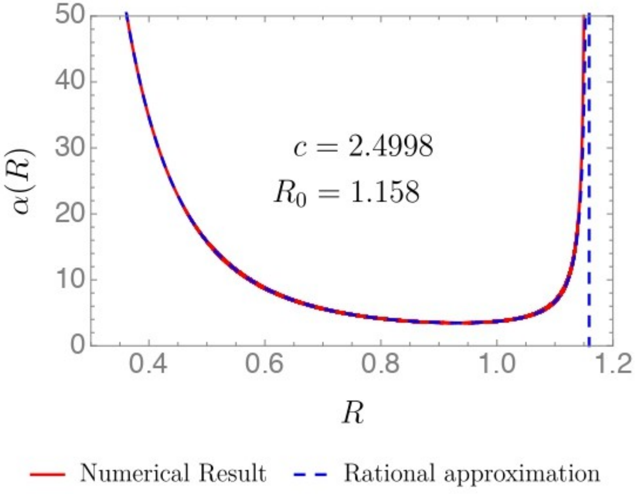
The p=2 CPW solution is known analytically [R.Corrado, K.Pilch, N.Warner '01]

$$f = 6\left(1 - \frac{r}{r_0}\right) , \qquad \alpha = \sqrt{\frac{2}{r(r_0 - r)}} , \qquad r_0 = 2\sqrt{2} .$$

The $p = 3$ GMPS solution is only known numerically [M. Gabella, D.Martelli, A. Passias, J.Sparks '12]

$$f(R) = \frac{9}{2} - cR^2 - \frac{c^2}{9}R^4 + \frac{(2187 - 128c^3)}{3888}R^6 + \frac{(19683c - 1264c^4)}{104976}R^8 + \mathcal{O}(R^{10}) , \quad R = r^{1/3} , \quad R \in [0, R_0] .$$





Kaluza-Klein Spectrum: what do we mean?

Example: One-Dimensional reduction:

$$ds^2 = ds^2_{\text{ext}}(x) + L^2 d\theta^2 \,.$$

Consider i.e. a massless scalar field and perform a Fourier expansion on the internal manifold

$$\phi(x,\theta) = \sum_{n=-\infty}^{\infty} \phi(x)_n \mathcal{Y}(\theta)_n = \sum_{n=-\infty}^{\infty} \phi(x)_n e^{\frac{i\pi n\theta}{L}} \,.$$

Then

$$\Box_D \phi(x,\theta) = 0 \longrightarrow \left(\Box_{D-1} + \partial_\theta^2\right)\phi(x,\theta) = \sum_{n=-\infty}^{\infty} \left(\Box_{D-1}\phi(x)_n - \frac{\pi^2 n^2}{L^2}\phi(x)_n\right) e^{\frac{i\pi n}{L}} = 0$$

Kaluza-Klein Gravitons on GMPS

Consider perturbations of the external metric

$$d\hat{s}^2_{11} = e^{2A}\Big[\big(\bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x,y)\big)dx^\mu dx^\nu + d\bar{s}^2_7(y)\Big]\,, \qquad e^{2A} = \tfrac{1}{4}\, e^{2\Delta}\,, \qquad d\bar{s}^2_7 = 4\, ds^2_7\,,$$

factorising as $h_{\mu\nu}(x,y) = h^{[tt]}_{\mu\nu}(x)\mathcal{Y}(y)$ and satisfying $\bar{\Box}h^{[tt]}_{\mu\nu} = (M^2L^2 - 2)h^{[tt]}_{\mu\nu}$.

Then 11 dimensional Einstein equations imply [C.Bachas, J.Estes '11]

$$\mathcal{L}\,\mathcal{Y} = L^2 M^2\,\mathcal{Y}\,, \qquad \text{with} \qquad \mathcal{L} = -\frac{e^{-9A}}{\sqrt{\bar{g}_7}}\partial_m\Big(e^{9A}\sqrt{\bar{g}_7}\,\bar{g}^{mn}\partial_n\Big)\,, \qquad m,n = 1,\ldots,7\,.$$

Kaluza-Klein Gravitons on GMPS

$$\mathcal{L} = -\frac{4}{r\alpha^2 f^3}\partial_r\left[r f^3 \partial_r\right] - \frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha}\Box_{S^5}$$

[MC, G.Larios, O.Varela '20]

$$- \frac{4}{9}\left(1+\frac{1}{r^2\alpha^2}\right)\partial_\psi^2 - \frac{8}{3}\left[\frac{2}{9}\left(1+\frac{1}{r^2\alpha^2}\right) - \frac{1}{r^2\alpha^2 f}\right]\partial_\psi\partial_\tau$$

$$- \left[-\frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha} + \frac{16}{81}\left(1+\frac{1}{r^2\alpha^2}\right) + \frac{4(1+r^2)}{r^2\alpha^2 f^2} - \frac{16}{9r^2\alpha^2 f}\right]\partial_\tau^2 \ .$$

Exploit the $SU(3)\times U(1)\times U(1)$ symmetry

$$\mathcal{Y} = \sum_{\ell,m,j} \xi_{\ell,m,j}(r)\, Y_{\ell,m}(z,\bar{z},\tau)\, e^{ij\psi}$$

$$\Box_{S^5} Y_{\ell,m} = -\ell(\ell+4)Y_{\ell,m}\ , \qquad \partial_\tau Y_{\ell,m} = im Y_{\ell,m}$$

Kaluza-Klein Gravitons on GMPS

$$L^2 M^2 \xi = -\frac{4}{r\alpha^2 f^3}\frac{d}{dr}\left[r f^3 \frac{d\xi}{dr}\right] + \frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha}\ell(\ell+4)\xi$$

[MC, G.Larios, O.Varela '20]

$$+\frac{4}{9}\left(1+\frac{1}{r^2\alpha^2}\right)j^2\xi + \frac{8}{3}\left[\frac{2}{9}\left(1+\frac{1}{r^2\alpha^2}\right) - \frac{1}{r^2\alpha^2 f}\right]jm\xi$$

$$+\left[-\frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha} + \frac{16}{81}\left(1+\frac{1}{r^2\alpha^2}\right) + \frac{4(1+r^2)}{r^2\alpha^2 f^2} - \frac{16}{9r^2\alpha^2 f}\right]m^2\xi\ .$$

Exploit the $SU(3)\times U(1)\times U(1)$ symmetry

$$\mathcal{Y} = \sum_{\ell,m,j} \xi_{\ell,m,j}(r)\, Y_{\ell,m}(z,\bar{z},\tau)\, e^{ij\psi}$$

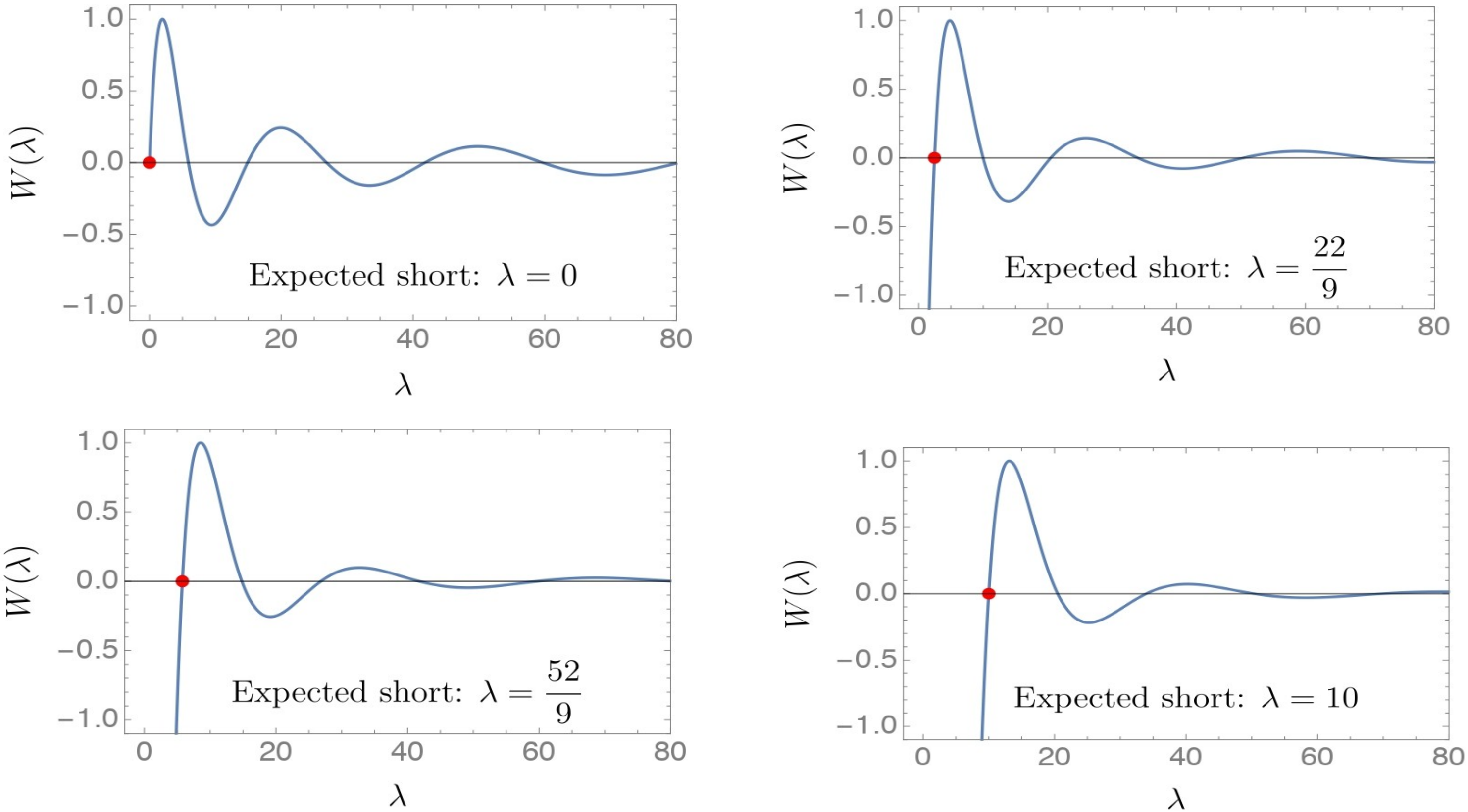
$$\Box_{S^5} Y_{\ell,m} = -\ell(\ell+4)Y_{\ell,m}\ , \qquad \partial_\tau Y_{\ell,m} = im Y_{\ell,m}$$

The ODE is not analytically solvable without knowing explicitly f and α.

Strategy: [J. Richard, R. Terrisse, D. Tsimpis '14]

- For given l, m, j find the asymptotic, normalisable values of the eigenfunctions $\xi(R)$, with $R = r^{1/3}$, close to the endpoints of $[0, R_0]$.
- Use these values as seeds to trigger a numerical integration of the ODE, starting from both endpoints of $[0, R_0]$. The resulting functions are $\xi^L_\lambda(R)$, $\xi^R_\lambda(R)$, with $\lambda = L^2M^2$.
- The relevant eigenfunctions are the ones for which ξ_L and ξ_R are linearly dependent. This implies $W(\lambda, R) = \xi^L_\lambda(R)\dot{\xi}^R_\lambda(R) - \xi^R_\lambda(R)\dot{\xi}^L_\lambda(R) = 0$.

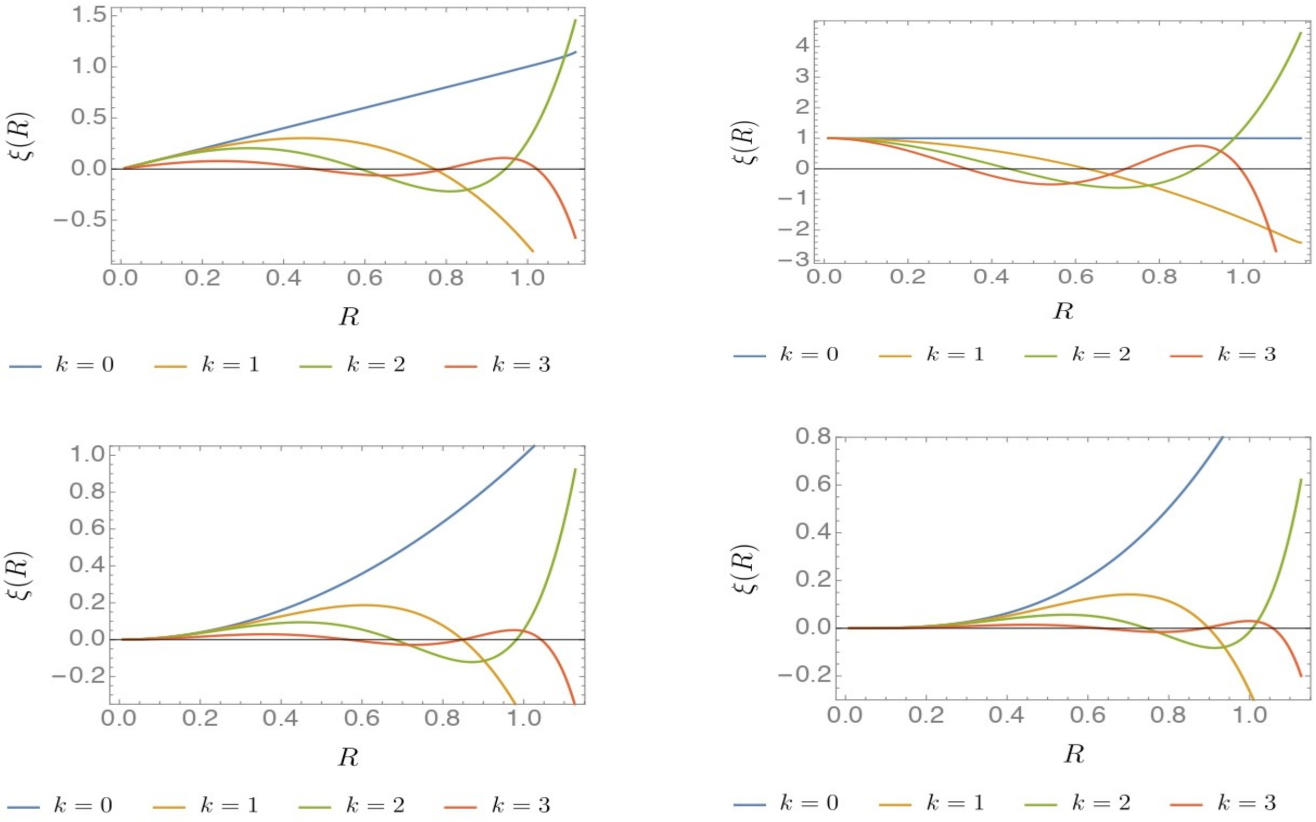
Wronskians for $l, m = 0$ and $j = 0, 1, 2, 3$.



Kaluza-Klein Gravitons on GMPS

Numerics

Numerical wavefunctions for $l, m = 0$ and $j = 0, 1, 2, 3$.



The gravitons' $SO(8)$ representations at KK level n branch under $SU(3)\times U(1)_3$, where $U(1)_3$ such as to recover the correct $R_1 \equiv R\left(\mathcal{Z}^A\right), R_2 \equiv R\left(\mathcal{Z}^4\right)$.

$$[n,0,0,0] \xrightarrow{\mathrm{SU(3)\times U(1)_3}} \bigoplus_{\ell=0}^{n} \bigoplus_{t=0}^{n-\ell} \bigoplus_{p=0}^{\ell} [p,\ \ell-p]_{-R_1(\ell-2p)+R_2(n-\ell-2t)}$$

so that, by redefining quantum numbers as

$$n = 2k + |j| + \ell\ , \qquad m = 2p - \ell\ , \qquad j = n - \ell - 2t$$

we are guaranteed to sweep over the complete gravitons' spectrum.

Kaluza-Klein Gravitons on GMPS

n	$[p, \ell - p]_{\frac{4}{9}(2p-\ell)+\frac{2}{3}(n-\ell-2t)}$	$d_{p,\, \ell-p}$	$L^2 M^2_{n,\ell,t,p}$	$\Delta_{n,\ell,t,p}$	Dual operator	Short?
0	$[0,0]_0$	1	0	3	$\mathcal{T}^{(0)}_{\alpha\beta}\|_{s=2}$	✓
1	$[0,0]_{\pm\frac{2}{3}}$	1	$\frac{22}{9}$	$\frac{11}{3}$	$\mathcal{T}^{(0)}_{\alpha\beta}\,\mathcal{Z}^4\|_{s=2}$, c.c.	✓
	$[1,0]_{\frac{4}{9}}, [0,1]_{-\frac{4}{9}}$	3	1.76	3.50	$\mathcal{T}^{(0)}_{\alpha\beta}\,\mathcal{Z}^a\|_{s=2}$, c.c.	
2	$[0,0]_{\pm\frac{4}{3}}$	1	$\frac{52}{9}$	$\frac{13}{3}$	$\mathcal{T}^{(0)}_{\alpha\beta}\,(\mathcal{Z}^4)^2\|_{s=2}$, c.c.	✓
	$[1,0]_{-\frac{2}{9}}, [0,1]_{\frac{2}{9}}$	3	4.68	4.13	$\mathcal{T}^{(0)}_{\alpha\beta}\,\mathcal{Z}^a\bar{\mathcal{Z}}_4\|_{s=2}$, c.c.	
	$[2,0]_{\frac{8}{9}}, [0,2]_{-\frac{8}{9}}$	6	3.88	3.97	$\mathcal{T}^{(0)}_{\alpha\beta}\,\mathcal{Z}^{(a}\mathcal{Z}^{b)}\|_{s=2}$, c.c.	
	$[1,0]_{\frac{10}{9}}, [0,1]_{-\frac{10}{9}}$	3	5.07	4.21	$\mathcal{T}^{(0)}_{\alpha\beta}\,\mathcal{Z}^a\mathcal{Z}^4\|_{s=2}$, c.c.	
	$[0,0]_0$	1	5.92	4.36	$\mathcal{T}^{(0)}_{\alpha\beta}\,(1 - 4a^2\mathcal{Z}^4\bar{\mathcal{Z}}_4 + b\mathcal{Z}^a\bar{\mathcal{Z}}_a)\|_{s=2}$	
	$[1,1]_0$	8	4	4	$\mathcal{T}^{(0)}_{\alpha\beta}\,(\mathcal{Z}^a\bar{\mathcal{Z}}_b - \frac{1}{3}\delta^a_b\mathcal{Z}^c\bar{\mathcal{Z}}_c)\|_{s=2}$	

For $[0,0]_{\pm R_2 n}$, the masses are well approximated by $L^2 M_n^2 = R_2 n \left(R_2 n + 3\right)$. As $\Delta(\Delta - 3) = M^2 L^2$, their conformal dimension is locked to the R charge as

$$\Delta_n = |R_n| + 3$$

Also present in CPW [I.Klebanov, T.Klose, A.Murugan '09]

Group Theory determines the dual CFT operator:

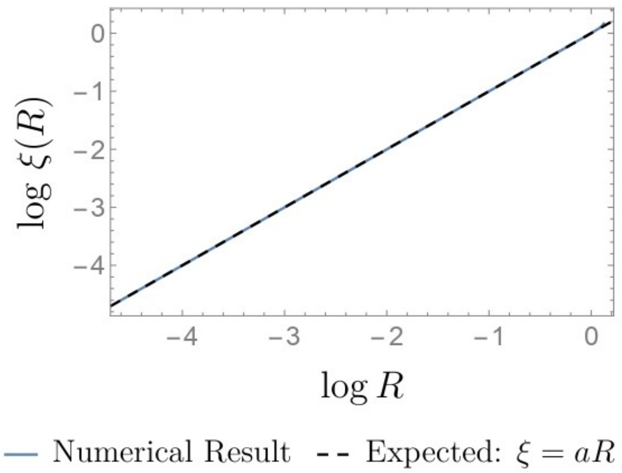
$$\mathcal{T}^{(0)}_{\alpha\beta} \left(\mathcal{Z}^4\right)^n \,, \quad n = 0, 1, 2, \dots \,,$$

while the correspondent eigenfunction is

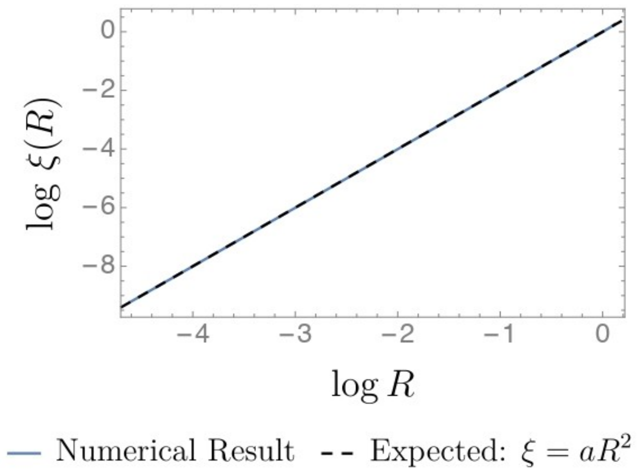
$$\mathcal{Y}_j = (\xi_1)^j e^{ij\psi} \,.$$

$$\xi_1 = r^{1/3} \equiv R$$

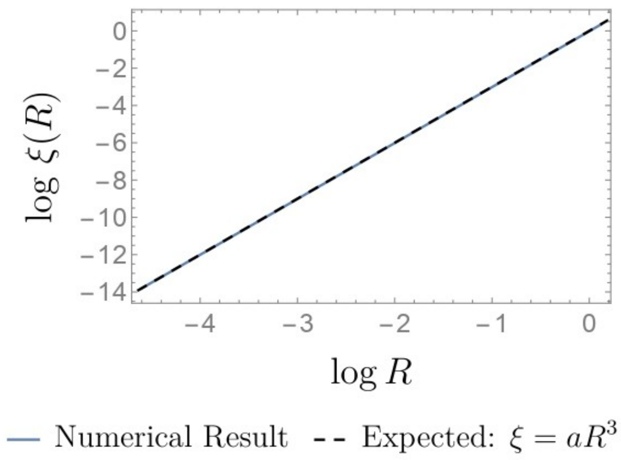
Kaluza-Klein Gravitons on GMPS
Analytics: short gravitons



$j = 1$



$j = 2$



$j = 3$

For $[1,1]_{\pm R_2(n-2)}$, masses are well approximated by

$$L^2 M_n^2 = \big((n-2)R_2+4\big)\big((n-2)R_2+1\big)\,, \qquad n=2,3,\dots \qquad \Delta_n = (n-2)R_2+4$$

[M. Billo et al. '00]

Group Theory tells us the dual CFT operator

Also present in CPW [I.Klebanov, T.Klose, A.Murugan '09]

$$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^A_B - \tfrac{1}{3}\delta^A_B \mathcal{Z}^C \bar{\mathcal{Z}}_C\right)(\mathcal{Z}^4)^{n-2}\,,$$

with eigenfunction

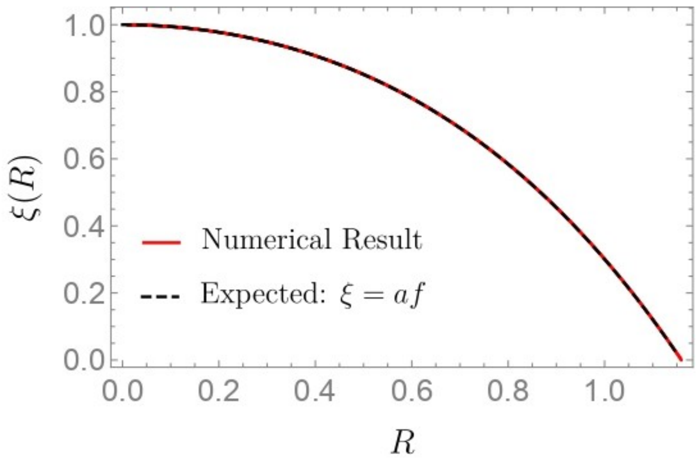
$$\mathcal{Y}_j = \xi_8\, r^{j/3}\, Y_{2,0}\, e^{ij\psi}\,.$$

The j-dependent infinite set of ODEs can be shown to amount to the set of two ODEs

$$\xi_8 + \frac{2}{r\alpha^2}\xi_8' = 0\,, \qquad \xi_8 - \frac{3\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha}\xi_8 + \frac{1}{r\alpha^2 f^3}\left(r f^3 \xi_8'\right)' = 0\,.$$

! $$\frac{f'}{f} = -\frac{1}{2} r\alpha^2$$

$$\xi_8 \propto f$$



GMPS is not isometrically embedded in $\mathbb{R}^8$

If GMPS S^7 were isometrically embedded in $\mathbb{R}^8$ then

$$\bar{Z}_C Z^C + \bar{Z}_4 Z^4 = 1 \qquad (\xi_3)^2 + (\xi_1)^2 = 1$$

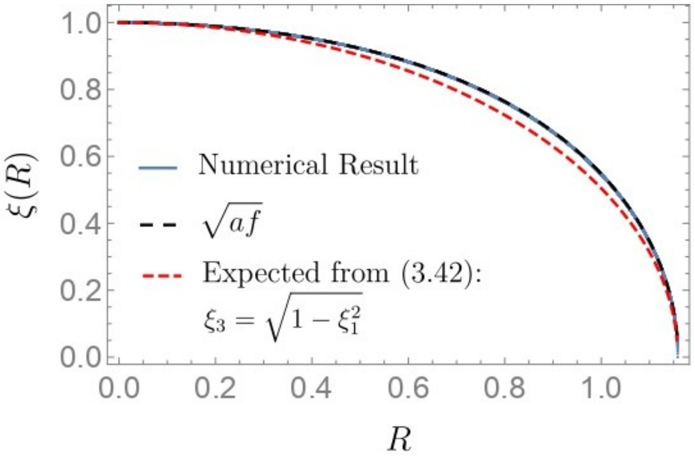
$$\xi_8 \propto f = \frac{9}{2}\left(1 - \left(\frac{r}{r_0}\right)^{2/3}\right) \longrightarrow \alpha^2 = \frac{4}{3r^2\left[\left(\frac{r}{r_0}\right)^{-2/3} - 1\right]}$$

Although satisfying the required asymptotics, there is no real value of r_0 for which GMPS ODEs are solved.

Remark:

This is not the only example of compactification on such a kind of manifold.

[M.Awada, M.Duff, C.Pope '83]



Full Spectrum considerations: Space Invaders

One could branch all the $SO(8)$ representations under $SU(3) \times U(1)_3$ and try to allocate in $OSp(4|2)_3$ supermultiplets states with different spins and same $SU(3)$ representation. For CPW [I.Klebanov, T.Klose, A.Murugan '09]

$$\begin{aligned}
\text{gravitini} &: \quad \mathbf{8}_s \xrightarrow{\mathrm{SU(3)\times U(1)_R}} \mathbf{3}_{\frac{1}{9}} + \mathbf{1}_1 + \text{"c.c."} \,, \\
\text{vectors} &: \quad \mathbf{28} \xrightarrow{\mathrm{SU(3)\times U(1)_R}} \mathbf{8}_0 + \mathbf{3}_{-\frac{2}{9}} + \mathbf{3}_{-\frac{8}{9}} + \mathbf{3}_{\frac{10}{9}} + 2\cdot\mathbf{1}_0 + \text{"c.c."} \,, \\
\text{fermions} &: \quad \mathbf{56}_s \xrightarrow{\mathrm{SU(3)\times U(1)_R}} \mathbf{8}_1 + \mathbf{6}_{-\frac{1}{9}} + 2\cdot\mathbf{3}_{\frac{1}{9}} + \mathbf{3}_{-\frac{11}{9}} + \mathbf{3}_{\frac{7}{9}} + \mathbf{1}_1 + \mathbf{1}_{\frac{1}{3}} + \text{"c.c."} \,, \\
\text{scalars} &: \quad \mathbf{35}_v \xrightarrow{\mathrm{SU(3)\times U(1)_R}} \mathbf{8}_0 + \mathbf{6}_{\frac{8}{9}} + \mathbf{3}_{\frac{10}{9}} + \mathbf{3}_{-\frac{2}{9}} + \mathbf{1}_0 + \mathbf{1}_{\frac{4}{3}} + \text{"c.c."} \,, \\
\text{pseudoscalars} &: \quad \mathbf{35}_c \xrightarrow{\mathrm{SU(3)\times U(1)_R}} \mathbf{8}_0 + \mathbf{6}_{-\frac{10}{9}} + \mathbf{3}_{-\frac{8}{9}} + \mathbf{3}_{-\frac{2}{9}} + \mathbf{1}_0 + \mathbf{1}_{\frac{2}{3}} + \text{"c.c."} \,.
\end{aligned}$$

Spin	SO(8)	$SU(3)\times U(1)_3$											
2	$\mathbf{1}$	$\mathbf{1}_0$											
$\frac{3}{2}$	$\mathbf{8}_s$	$\mathbf{1}_{+1}$ $\mathbf{1}_{-1}$	$\mathbf{3}_{\frac{1}{9}}$	$\bar{\mathbf{3}}_{-\frac{1}{9}}$									
1	$\mathbf{28}$	$\mathbf{1}_0$	$\mathbf{3}_{-\frac{8}{9}}$ $\mathbf{3}_{\frac{10}{9}}$	$\bar{\mathbf{3}}_{\frac{8}{9}}$ $\bar{\mathbf{3}}_{-\frac{10}{9}}$	$\mathbf{8}_0$	$\mathbf{3}_{-\frac{2}{9}}$	$\bar{\mathbf{3}}_{\frac{2}{9}}$	$\mathbf{1}_0$					
			$\mathbf{3}_{-\frac{8}{9}}$	$\bar{\mathbf{3}}_{\frac{8}{9}}$									
$\frac{1}{2}$	$\mathbf{56}_s$		$\mathbf{3}_{\frac{1}{9}}$ $\mathbf{3}_{\frac{1}{9}}$	$\bar{\mathbf{3}}_{-\frac{1}{9}}$ $\bar{\mathbf{3}}_{-\frac{1}{9}}$	$\mathbf{8}_{+1}$ $\mathbf{8}_{-1}$	$\mathbf{3}_{\frac{7}{9}}$ $\mathbf{3}_{-\frac{11}{9}}$	$\bar{\mathbf{3}}_{-\frac{7}{9}}$ $\bar{\mathbf{3}}_{\frac{11}{9}}$	$\mathbf{1}_{+1}$ $\mathbf{1}_{-1}$	$\mathbf{6}_{-\frac{1}{9}}$	$\bar{\mathbf{6}}_{\frac{1}{9}}$	$\mathbf{1}_{\frac{1}{3}}$	$\mathbf{1}_{-\frac{1}{3}}$	
			$\mathbf{3}_{-\frac{17}{9}}$	$\bar{\mathbf{3}}_{\frac{17}{9}}$		$\mathbf{3}_{\frac{7}{9}}$	$\bar{\mathbf{3}}_{-\frac{7}{9}}$	$\mathbf{1}_{+1}$ $\mathbf{1}_{-1}$					$\mathbf{3}_{\frac{1}{9}}$ $\bar{\mathbf{3}}_{-\frac{1}{9}}$
0	$\mathbf{35}_v$				$\mathbf{8}_0$	$\mathbf{3}_{-\frac{2}{9}}$	$\bar{\mathbf{3}}_{\frac{2}{9}}$	$\mathbf{1}_0$	$\mathbf{6}_{\frac{8}{9}}$	$\bar{\mathbf{6}}_{-\frac{8}{9}}$	$\mathbf{1}_{\frac{4}{3}}$	$\mathbf{1}_{-\frac{4}{3}}$	$\mathbf{3}_{\frac{10}{9}}$ $\bar{\mathbf{3}}_{-\frac{10}{9}}$
	$\mathbf{35}_c$		$\mathbf{3}_{-\frac{8}{9}}$	$\bar{\mathbf{3}}_{\frac{8}{9}}$	$\mathbf{8}_0$	$\mathbf{3}_{-\frac{2}{9}}$	$\bar{\mathbf{3}}_{\frac{2}{9}}$	$\mathbf{1}_0$	$\mathbf{6}_{-\frac{10}{9}}$	$\bar{\mathbf{6}}_{\frac{10}{9}}$	$\mathbf{1}_{-\frac{2}{3}}$	$\mathbf{1}_{\frac{2}{3}}$	
						$\mathbf{3}_{\frac{16}{9}}$	$\bar{\mathbf{3}}_{-\frac{16}{9}}$	$\mathbf{1}_0$ $\mathbf{1}_{+2}$ $\mathbf{1}_{-2}$					$\mathbf{3}_{-\frac{8}{9}}, \bar{\mathbf{3}}_{\frac{8}{9}}$ $\mathbf{3}_{-\frac{8}{9}}, \bar{\mathbf{3}}_{\frac{8}{9}}$ $\mathbf{3}_{-\frac{2}{9}}, \bar{\mathbf{3}}_{\frac{2}{9}}$ $\mathbf{1}_0$
		Massless graviton	Short gravitino	Short gravitino	Massless vector	Short vector	Short vector	Long vector	Massive hyper	Massive hyper	Massive hyper	Massive hyper	Eaten modes

Summary and Outlook

Summary:

- We computed numerically the full KK graviton spectrum over GMPS.
- We provided analytic results about short states and shadow octects.
- We showed that GMPS S^7 is not isometrically embedded in $\mathbf{R}^8$.
- Furthermore, the full KK spectrum over GMPS displays *space invasion*.

Yet to be addressed:

- Look for an isometric embedding for the GMPS S^7.
- Correct invasion pattern.
- No consistent uplift from D=4 $\longleftrightarrow$ $\nexists f : S^7 \hookrightarrow R^8$ $\longleftrightarrow$ Space Invaders.

$SO(8)$ representations of fields at Kaluza-Klein level n

$$
\begin{aligned}
\text{graviton} &: \quad G_n \equiv [n,0,0,0] \ , \\
\text{gravitini} &: \quad \mathcal{G}_n \equiv [n,0,0,1] \oplus [n-1,0,1,0] \ , \\
\text{vectors} &: \quad V_n \equiv [n,1,0,0] \oplus [n-1,0,1,1] \oplus [n-2,1,0,0] \ , \\
\text{fermions} &: \quad \mathcal{F}_n \equiv [n+1,0,1,0] \oplus [n-1,1,1,0] \oplus [n-2,1,0,1] \oplus [n-2,0,0,1] \ , \\
\text{scalars} &: \quad S_n^+ \equiv [n+2,0,0,0] \oplus [n-2,2,0,0] \oplus [n-2,0,0,0] \ , \\
\text{pseudoscalars} &: \quad S_n^- \equiv [n,0,2,0] \oplus [n-2,0,0,2] \ ,
\end{aligned}
$$

Lagrangian of the cubic deformation

The effect of the cubic deformation of the superpotential at the Lagrangian level is to introduce interaction terms:

$$\mathcal{L} = \frac{1}{2}|\alpha|^2 \left(Z^4\right)^2 \left(\bar{Z}_4\right)^2 + \frac{1}{2}\alpha\chi^4\chi^4 Z^4 + h.c.\,,$$

with relevant dimensions $\Delta = 2$ and $\Delta = \frac{5}{2}$ and whose $SO(8)$ representations are the $\mathbf{294}_v$ and the $\mathbf{224}_{cv}$.