Tools to understand AdS_2/CFT_1

Based on 2004.07849 [Bianchi, GB, Forini, Griguolo, Seminara]
and [Bianchi, GB, Forini, Peveri], [GB,Costa, Forini,Patella]

Holotube Junior 2020, Gabriel Bliard, 30/10/2020

Plan for today

(I've left enough time, please interrupt me for questions)

- Prequel: *Current status and motivation for studying* AdS_2/CFT_1
- Part IA : *A concrete example of an* $AdS_2/dCFT_1$ *correspondence*
- Part IB: *Analytic bootstrap, Witten diagrams & Superspace*
- Part II: *From position space to Mellin Space*
- Part III: *Towards a lattice description of the String worldsheet*

Gauge

AdS_2/CFT_1?

Gravity

- Universality: $CFT_1 \in CFT_{d\geq 1}$
- Ease: Restricted sector of rich theory
- Large 'toolbox': (AdS computations, Bootstrap...)
- Natural interpretation in the language of defects:

Wilson lines & t'Hooft lines

[17 Giombi, Roiban, Tseytlin]

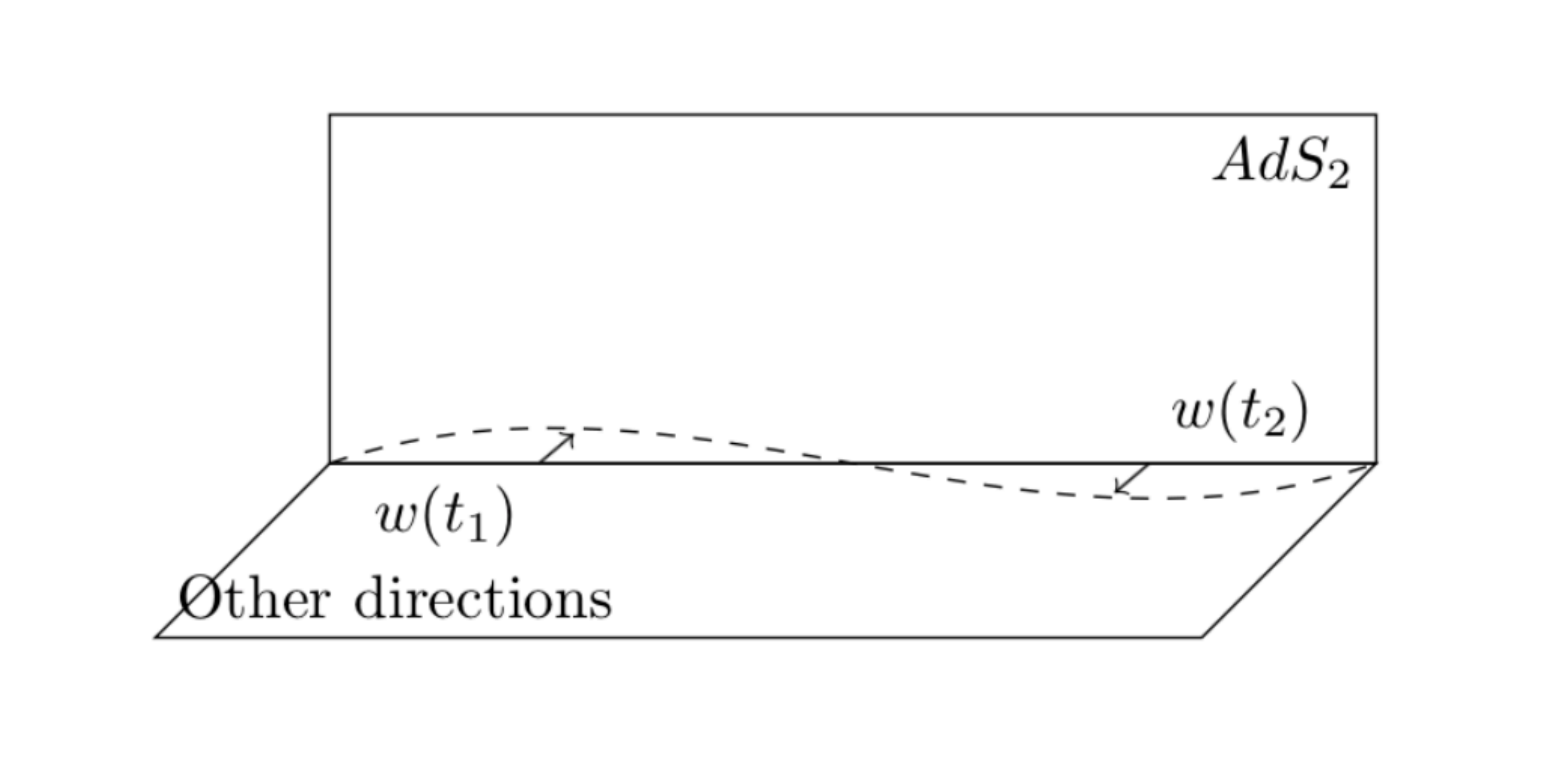
- Topological quantum mechanics connected to AdS_2 gauge theories.

[17 Mezei, Pufu, Wang]

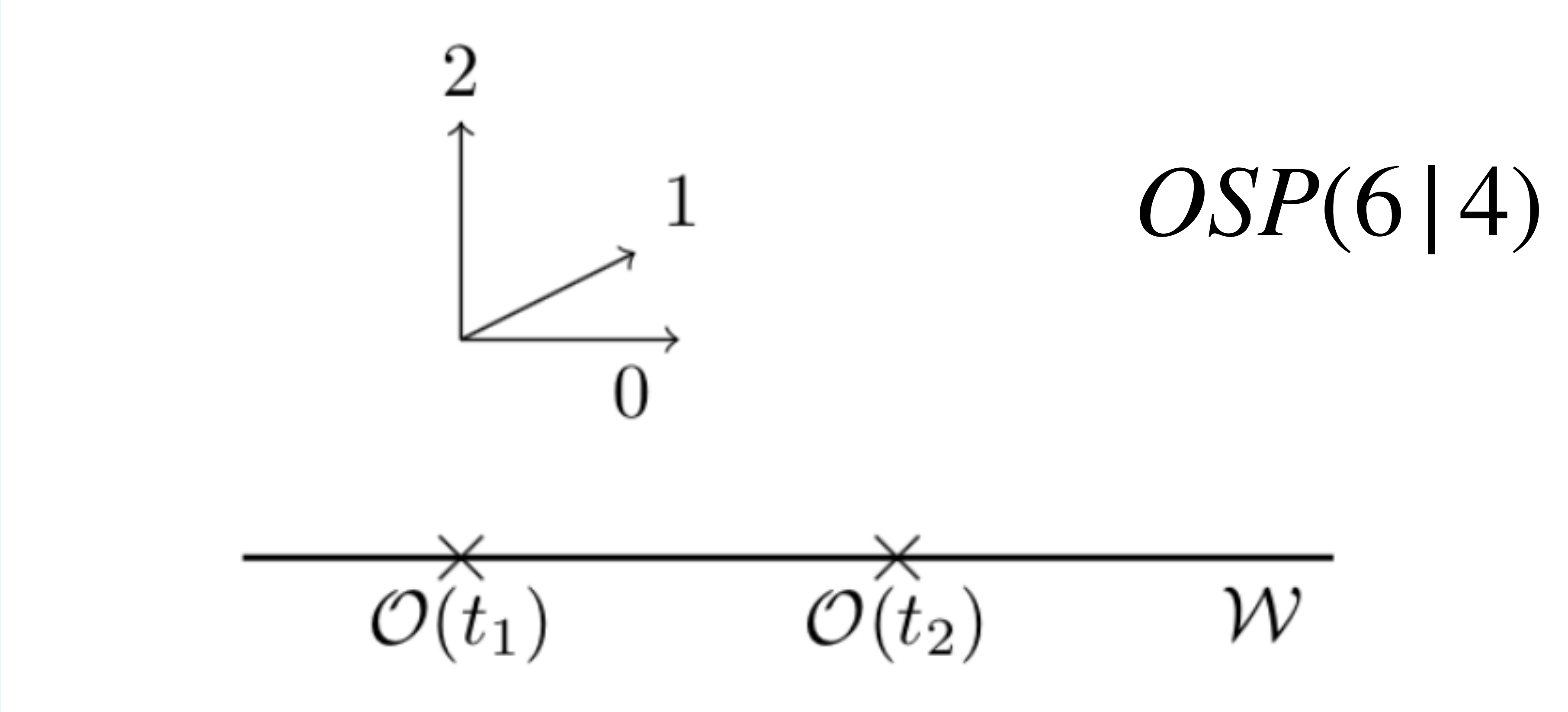
Our Setup

A concrete example of an AdS_2/CFT_1

Type IIA in $\mathrm{AdS}_4 \times \mathbb{C}P^3$ $\longleftrightarrow$ $3D, \quad N = 6$, CS with matter (ABJM)



$\longleftrightarrow$



$$\langle w(t_1)\ldots w(t_n)\rangle = \frac{\mathcal{Z}[w(t_1)\,..\,w(t_n)]}{\mathcal{Z}_0} \quad \longleftrightarrow \quad \langle \mathcal{O}(t_1)\ldots\mathcal{O}(t_n)\rangle_{\mathcal{W}}$$

The 1/2 BPS Wilson Line

Explicitely, the 1/2-BPS Wilson loop operator is realized as the holonomy of a superconnection $\mathscr{L}(t)$ living in $U(N|N)$ [Drukker, Trancanelli 2009] [Cardinali,Griguolo,Martelloni, Seminara,2012][Lee,Lee,2010]:

$$\mathscr{W} = \mathrm{Str}\left[P\exp\left(-i\oint dt\mathscr{L}(t)\right)\mathscr{T}\right].$$

The Wilson Line breaks the full symmetry group:

$$OSp(6|4) \to SU(1,1|3)$$

- Remaining symmetry contains:

$$SU(1,1) \times SU(3) \times U(1)$$

$$[\Delta, j_0, j_1, j_2]$$

- Broken symmetries define operator insertions:

Broken generator	Corresponding Operator insertion
$\mathrm{P},\ \bar{P}$	$\mathbb{D},\ \bar{\mathbb{D}}$
$Q_a,\ \bar{Q}^a$	$\mathbb{\Lambda}_a,\ \bar{\mathbb{\Lambda}}^a$
$J^a,\ \bar{J}_a$	$\mathbb{O}^a,\ \bar{\mathbb{O}}_a$

Form a **chiral primary** $(\Phi, \bar{\Phi})$

Computing Quantities for the CFT1

Weak coupling:

-Perturbation theory

Non-perturbative methods

Strong coupling:

-Bootstrap

What are we computing?

$$\langle \mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\ldots\mathcal{O}_n(t_n)\rangle_{\mathcal{W}} \equiv \frac{\langle \mathrm{Tr}\mathcal{P}\left[\mathcal{W}_{t_i,t_1}\mathcal{O}_1(t_1)\mathcal{W}_{t_1,t_2}\ldots\mathcal{O}_n(t_n)\mathcal{W}_{t_n,t_f}\right]\rangle}{\langle \mathcal{W}\rangle}$$

CFT is a strong constraint!

$$< \mathcal{O}_{\Delta_1}(t_1)\mathcal{O}_{\Delta_2}(t_2) > = \frac{\delta_{\Delta_1,\Delta_2}}{(t_1-t_2)^{2\Delta}}$$

$$< \mathcal{O}_{\Delta_1}(t_1)\mathcal{O}_{\Delta_2}(t_2)\mathcal{O}_{\Delta_3}(t_3)\mathcal{O}_{\Delta_4}(t_4) > = \frac{1}{F(t_1\ldots t_4)}G(\chi)$$

Reminder:

- Broken symmetries define operator insertions
- Remaining symmetries have $SU(1,1)\times SU(3)\times U(1)$

The bootstrap

- The exchange of identical operators in the 4 point function gives a 'crossing equation' which is solved by a set of families of increasing transcendentality: [Liendo, Meneghelli, Mitev, 2018]

$$< \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4) > = \frac{1}{t_{13}t_{24}}\hat{f}(\chi) = \frac{1}{t_{13}t_{24}}\hat{f}(1-\chi)$$

$$f^0 \sim r(\chi(1-\chi)) \qquad f^1 \sim r_0(\chi) + r_1(\chi)\log(\chi) + r_1(1-\chi)\log(1-\chi) \qquad f^2 \sim Li_2(\chi) + \dots$$

- f^0 matches the free field result, introduce strong coupling parameter ϵ
- Apply the symmetry of the system: Selection rules on the operator product expansion

$$f(\chi) = 1 - \chi + \epsilon(\chi - 1 + \chi(3-\chi)\log(\chi) + \frac{(1-\chi)^3}{\chi}\log(1-\chi)) + \mathcal{O}(\epsilon^2)$$

Reminder:

- Operator insertions are elements of chiral primary $(\Phi, \bar{\Phi})$ of $SU(1,1|3)$, whose first component is $\mathbb{F}$

The string worldsheet fluctuations:

$$S_B = T\int d^2\sigma \sqrt{\det\left[\frac{(1+1/2|X|^2)^2}{(1-1/2|X|^2)^2}g_{\mu\nu} + 2\frac{\partial_\mu X\partial_\nu \bar{X}}{(1-1/2|X|^2)^2} + 4\left(\frac{\partial_\mu \bar{w}_a \partial_\nu w^a}{1+|w|^2} - \frac{\partial_\mu \bar{w}_a w^a \bar{w}_b \partial_\nu w^b}{(1+|w|^2)^2}\right)\right]}$$

$X, \bar{X}, w^a, \bar{w}_a$ are the remaining AdS_4 and $\mathbb{C}P^3$ directions

$$S_B = T\int d^2\sigma\sqrt{g}\left(g^{\mu\nu}\partial_\mu X\partial_\nu\bar{X} + 2|X|^2 + g^{\mu\nu}\partial_\mu\bar{w}_a\partial_\nu w^a + L_{int}\right)$$

$w^a \leftrightarrow \mathbb{O}^a$ $\qquad \psi \leftrightarrow \mathbb{F}$

$X \leftrightarrow \mathbb{D}$ $\qquad \psi_a \leftrightarrow \Lambda_a$

$$\langle X(t_1)\bar{X}(t_2)X(t_3)\bar{X}(t_4)\rangle = L(Q_i, \bar{Q}_i) < \Phi(t_1)\bar{\Phi}(t_2)\Phi(t_3)\bar{\Phi}(t_4) > = \mathcal{L}[f(\chi)]$$

$$f(\chi) = 1 - \chi + \frac{1}{4\pi T}(\chi - 1 + \chi(3-\chi)\log(\chi) + \frac{(1-\chi)^3}{\chi}\log(1-\chi)) + \mathcal{O}(\frac{1}{T^2})$$

Reminder:

From bootstrap: $f(\chi) = 1 - \chi + \epsilon(\chi - 1 + \chi(3-\chi)\log(\chi) + \frac{(1-\chi)^3}{\chi}\log(1-\chi)) + \mathcal{O}(\epsilon^2)$

Mellin formalism

- Based on Mellin transform: $M[g](s) = \int dx g(x) x^s$
- Simpler structure linked to physically intuitive quantities such as poles
- Have an interpretation as Scattering amplitudes [09, Mack]
- In higher dimensions, we have :

$$< \phi_\Delta(\vec{x}_1)\phi_\Delta(\vec{x}_2)\phi_\Delta(\vec{x}_3)\phi_\Delta(\vec{x}_4) > (u,v) = \frac{1}{(x_{12}^2 x_{34}^2)^\Delta} \int_{c-i\infty}^{c+i\infty} \frac{ds dt}{(2\pi i)^2} u^{-s} v^{-t} M(s,t) \Gamma(s)^2 \Gamma(t)^2 \Gamma(4\Delta - s - t)^2$$

- In one dimension, there is only 1 cross ratio : redundancy of the Mellin description
- Is there a better Mellin description for 1D correlators?

Challenges:

- Non-trivial analytic behaviour
- 1D case is the limiting case of d>1
- Conformal integrals are hard

Conclusion

- AdS2/CFT1 gives a playground to explore tools and techniques (Witten diagrams, Superspace computations, bootstrap, Mellin amplitudes, lattice description)

In the first project, we found:

- Two independently computed solutions agreeing (up to the identification $\epsilon \to \frac{1}{4\pi T}$)
- A perturbative description of **Fermion** 4 point function.
- Provides a first order analysis of the Spectrum ($\gamma_n = -n^2 - n + 2$ in C-C channel)

Other tools for AdS2/CFT1 include:

- Mellin amplitudes (Still a need for a non-redundant description)
- Lattice worldsheet description: Monte-Carlo simulations

Thank you!

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