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Fluctuation and Dissipation from a Deformed String-Gauge Duality Model

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Summary

General Idea

Use a QCD-like holographic model to study fluctuation and dissipation, calculating the quantities of interest from the bulk model.

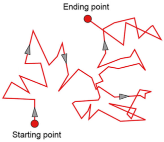


Figure: Pictorial description of Brownian Motion.

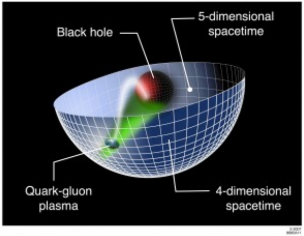


Figure: Artist's conception of holographic description of Quark-gluon plasma.

Deformed AdS-Schwarzschild metric

$$ds^2 = e^{\frac{k}{r^2}} \left[-r^2 f(r) dt^2 + r^2 \left(\eta_{ij} dx^i dx^j \right) + \frac{dr^2}{r^2 f(r)} \right]$$

$$f(r) = 1 - \frac{r_h^4}{r^4}$$

- The study of Brownian motion in AdS/CFT started with J. de Boer, V. E. Hubeny, M. Rangamani and M. Shigemori, JHEP **07**, 094 (2009). After it similar frameworks were used to study for example quantum critical points: D. Tong and K. Wong, Phys. Rev. Lett. **110**, no.6, 061602 (2013). Polynomial metric duals were analysed in D. Giataganas, D. S. Lee and C. P. Yeh, JHEP **08**, 110 (2018).
- This metric was studied as an phenomenological dual for QCD.
- In the literature there was a lack of a more complete study of the Brownian motion for this particular set-up.

Brownian Motion

Brownian motion is described by the Langevin equation

$$\frac{d\vec{p}}{dt} = -\gamma\vec{p} + \vec{R}(t) + \vec{F}(t)\,,$$

$$\langle R_i(t)\rangle = 0 \qquad\qquad \langle R_i(t)R_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')\,.$$

- $-\gamma p(t)$ ➔ Dissipation
- $R(t), \kappa$ ➔ Fluctuation

Bulk Model

The model consists of a probe fundamental string suspended from the boundary to the black hole horizon representing the Brownian particle in the bulk. The modes of the string are excited by the black hole horizon and propagate to the boundary. The string end-point executes then a Brownian Motion.

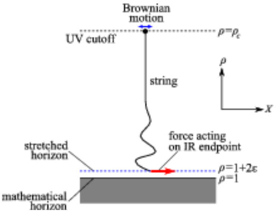


Figure: J. de Boer, V. E. Hubeny, M. Rangamani and M. Shigemori, JHEP **07**, 094 (2009).

We start studying the Nambu-Goto action for small fluctuations

$$S_{NG} \approx -\frac{1}{4\pi\alpha'}\int d\tau d\sigma \left[\dot{X}^2 \frac{e^{\frac{k}{r^2}}}{f(r)} - X'^2 r^4 f(r) e^{\frac{k}{r^2}} \right]$$

Obtaining the equation of motion

$$\frac{\partial}{\partial r}\left(r^4 f(r) e^{\frac{k}{r^2}} X'(r,t)\right) - \frac{e^{\frac{k}{r^2}}}{f(r)}\ddot{X}(t,r) = 0.$$

We solve this equation using the patching method. It consists of slicing the space-time in regions, the first one is close to the black hole horizon, the second one is an intermediate part and the last one is the near boundary zone. Close to the boundary the ingoing solution in linear regime is

$$h_\omega^C(r) = \sqrt{\frac{4\pi\alpha' r_h}{\omega \log\left(\frac{1}{\epsilon}\right)}} \left[\mathcal{C}_1 + i\omega\left(\mathcal{C}_2 + \frac{\mathcal{C}_3}{r^3}\right)\right],$$

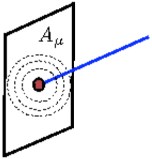
$$\mathcal{C}_1 = \frac{e^{-\frac{k}{2r_h^2}}}{r_h}, \;\; \mathcal{C}_2 = \frac{e^{-\frac{k}{2r_h^2}}}{r_h}\left(\frac{\log(2r_h)+b}{4r_h} - \frac{\pi}{8r_h}\right), \;\; \mathcal{C}_3 = \frac{r_h}{3}e^{\frac{k}{2r_h^2}}.$$

After find the classical solution we can write the quantized field as

$$X(t,r) = \sum_{\omega>0} \left(h_\omega(r) e^{-\mathrm{i}\omega t} a_\omega + h_\omega(r) e^{\mathrm{i}\omega t} a_\omega^\dagger \right) .$$

With the purpose of study the response of the system in the linear regime we turn on a electric field on the UV brane in consequence a force on the end-point of the string appears

$$F(t) = \frac{1}{2\pi\alpha'}\left[X'(t,r_b)\,(r_b^4 - r_h^4)e^{\frac{k}{r_b^2}}\right].$$



This force will act like a external force on the Brownian particle.

From the retarded solution and force we find that the response function can be written as

Admittance

$$\chi(\omega) = \frac{h_\omega^{(C)}(r_b)}{F(\omega)} = \frac{1}{-i\omega}\frac{2\alpha'}{\pi T^2}e^{-\frac{k}{\pi^2 T^2}}$$

This corresponds to a exponential correction to the pure AdS-Schwarzschild value.

From the response function we readily get the diffusion coefficient

Diffusion Coefficient

$$D = T \lim_{\omega \to 0} (-i\omega\chi(\omega)) = \frac{2\alpha'}{\pi T} e^{\frac{-k}{\pi^2 T^2}}.$$

Dependence of sign of k

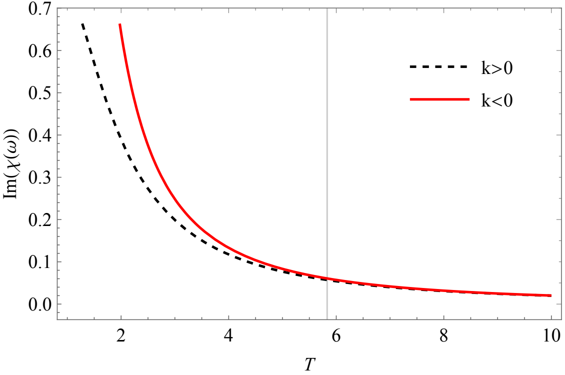


Figure: The imaginary part of admittance $\chi(\omega)$, for a fixed ω, as a function of the temperature for both $k = \pm 1$ in arbitrary energy units. The vertical line represents the approximate value for the temperature $(T \approx 5.8)$. From this temperature forward (high temperatures) the sign of k is no longer relevant.

Friction Coefficient

$$\gamma = \pi^2 T^2 e^{\frac{k}{\pi^2 T^2}}$$

Thermal Mass

$$\Delta m = \left(\frac{\log(2\pi T) + b}{4} - \frac{\pi}{8} \right) \pi T e^{\frac{k}{\pi^2 T^2}}.$$

Two Point Functions

$$\langle x(t)x(0)\rangle = \alpha' \int_0^\infty \frac{d\omega}{\omega} \left(\mathcal{C}_1^2 + \omega^2 \mathcal{C}_2^2\right) \left(\frac{2\cos(\omega t)}{e^{\beta\omega} - 1} + e^{-i\omega t}\right).$$

$$\langle x(t)x(t)\rangle = \alpha' \int_0^\infty \frac{d\omega}{\omega} \left(\mathcal{C}_1^2 + \omega^2 \mathcal{C}_2^2\right) \left(\frac{2}{e^{\beta\omega} - 1} + 1\right).$$

$$\mathcal{C}_1 = \frac{e^{-\frac{k}{2r_h^2}}}{r_h}, \;\; \mathcal{C}_2 = \frac{e^{-\frac{k}{2r_h^2}}}{r_h} \left(\frac{\log(2r_h) + b}{4r_h} - \frac{\pi}{8r_h}\right).$$

After we obtain the two point functions is possible to calculate the mean square displacement

$$s^2_{\text{reg}}(t) = \langle : [x(t) - x(0)]^2 : \rangle = \langle : [X(t, r_b) - X(0, r_b)]^2 : \rangle.$$

Mean Square Displacement: Ballistic Regime

$$s^2_{\text{reg}}(t) = \frac{\alpha' e^{\frac{-k}{T^2\pi^2}}}{6}\left[\frac{1}{2} + \frac{1}{80}\left(\log(2\pi T) - \frac{\pi}{2}\right)^2\right] t^2$$

Mean Square Displacement: Diffusive Regime

$$s^2_{\text{reg}}(t) = \frac{\alpha' e^{\frac{-k}{\pi^2 T^2}}}{2\pi T} t \sim Dt.$$

Fluctuation-Dissipation Theorem

Figure: Ryogo Kubo

Fluctuation-Dissipation Theorem Check

There are many "avatars" for the fluctuation-dissipation theorem, here we use

$$\langle \{x(\omega), x(0)\} \rangle = (2n_B(\omega) + 1)\mathrm{Im}\,\chi(\omega).$$

From the two-point functions and admittance with some manipulation we see that this expression is satisfied. This shows that the Fluctuation-Dissipation theorem is valid for this set-up.

Conclusion

In this work we calculated several quantities of interest for the description of the Brownian motion from the set-up presented.
We made a check of the fluctuation-dissipation theorem.
It is compared the behavior of the admittance for the different signs of k.

Thank You!